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Pre-play Negotiations, Learning and Nash Equilibrium

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I, Topi Olli Oskari Miettinen, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Abstract

A solution concept maps strategic games into strategy predictions. Nash equilibrium is the most widely used solution concept in game theory. Three main explanations have been used to argue why players should end up playing Nash equilibrium: 1) introspective reasoning, 2) communication 3) learning. Careful study of these has shown that the case for the Nash equilibrium is not entirely unambiguous. In this thesis, we conclude with new insights into why Nash equilibrium may be too restrictive a prediction in the context of pre-play communication and learning.

Experiments suggest that communication increases the contribution to public goods. There is also evidence that, when contemplating a lie, people trade off their private benefit from the lie with the harm it inflicts on others. In the first chapter, we develop a theory of bilateral pre-play negotiation that assumes the latter and implies the former. We show that a preference for not lying enables non-Nash outcomes. In symmetric games, pre-play negotiations crucially depend on whether actions are strategic complements or substitutes. With strategic substitutes commitment power tends to decrease in efficiency whereas the opposite may be true with strategic complements.

In the second chapter we consider negotiation with an alternating offer protocol. As opposed to previous contributions we show that impatience may be beneficial for a player.

In the third chapter we illustrate how the complexity of conjectures about opponents' strategies in the analogy-based expectation equilibrium (ABEE) corresponds to various other equilibrium concepts in the learning literature. We also introduce a payoff-confirming refinement of the ABEE where the sample of own payoffs induced by the true equilibrium strategies must confirm the conjectures about opponents' strategies. We show that there may be non-Bayesian-Nash payoff-confirming ABEE. We provide a sufficient condition for this and show that the condition is also necessary in an interesting class of games.

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Chapter 1

Introduction

A solution concept maps strategic games into strategy and outcome predictions. Nash equilibrium is the most widely used solution concept in non-cooperative game theory. Three main explanations have been used to argue why players should end up playing a Nash equilibrium: 1) introspective reasoning, 2) pre-play communication and 3) learning. Yet, careful formal studies of these ideas have shown that the case for the Nash equilibrium may not be that strong. Firstly, Bernheim (1984) and Pearce (1984) show that introspective reasoning (rationalising strategies) leads to a larger set of predictions than Nash equilibria. Secondly, Aumann (1974), Farrell (1987) and Aumann and Hart (2003) point out in pre-play communication frameworks (even if language is common knowledge) how players can condition their choices on pre-play messages chosen either by players or by exogenous randomisation devices. This results in a larger set of predictions and potentially improved expected payoffs than mere (symmetric) Nash equilibria of the underlying game. Thirdly, learning processes where players update their conjectures about others' behaviour based on their experience, may have various other strategy profiles than Nash equilibria as steady states, depending on specific assumptions on the process. For instance, expectations about off-equilibrium-path behaviour may be incorrect in extensive forms games (Fudenberg and Levine, 1993). Alternatively, players may not observe the equilibrium path but rather a less informative signal (Battigalli, 1987; Dekel Fudenberg and Levine, 2004). Finally, even if players observe decisions at each node along the path, cognitive limitations may necessitate simplifications of opponents' behaviour by bundling several decision nodes of the opponents together and keeping track of opponents' average behaviour in each such class (Jehiel, 2005).

In this thesis one of our main concerns is to study the effects of pre-play communication (pre-play negotiations in particular) and learning on outcome predictions. We conclude with new insights into why Nash equilibrium may be too restrictive a notion.

The second chapter presents a theory of pre-play agreements, reached by pre-play negotiations, conventions or social norms. We assume that people dislike lying about their intentions, breaching informal agreements and transgressing social norms. Moreover, we assume that people dislike a transgression of an agreement more if they inflict more harm on others by doing so. These assumptions are in line with stylised findings of reasearch in social psychology and supported by findings in experimental economics. If players dislike breaching promises, pre-play negotiations transform the payoffs of any given game prior to which players negotiate: deviations from promises become less profitable. As a consequence, players may be able to commit to strategy profiles in the new game that are not Nash equilibria of the original game. In addition, any Nash equilibrium of the original game remains a Nash equilibrium of the transformed game where players promise to play that Nash equilibrium.

Let us discuss how our contribution relates to previous literature on pre-play communication. Aumann (1974) and Aumann and Hart (2003) illustrate how players can use lotteries to correlate their choices and thereby reach better than underlying game Nash equilibrium outcomes for both. These contributions assume correct conjectures about each others' strategies and thus equilibrium play. The assumption of equilibrium play rules out non-equilibrium behaviour where players rationalise, out of equilibrium, that opponent might misunderstand the agreement (miscoordinate on strategies of two different equilibria as a special case). Farrel (1987) shows that pre-play communication in a common known language can improve symmetric equilibrium expected payoffs and Farrell (1988) illustrates how pre-play negotiations can induce Nash equilibrium play when players are assumed to rationalise their strategy choices rather than correctly anticipate their opponents' strategies. Frank (1988) argues that people have a tendency to feel bad about breaching agreements. Bad feelings about breaching agreements transform payoffs of the game and non-Nash strategies of the original game may be played. Frank illustrates in a stylised model that the tendency to feel bad may survive evolutionary pressures. Ellingsen and Johanneson (2004) elaborate this approach and show both theoretically and experimentally how dislike to breach agreements induces play which is predicted neither by standard cheap talk models nor by cheap talk models where players are inequity averse (Fehr and Scmidt, 1999; Bolton and Ockenfels, 2000). In their model the emotional cost of breaching the agreement is assumed to be constant. In particular, the cost is independent of the harm that breaching inflicts on the opponent. Our contribution in chapter two is to generalise the approach of Ellingsen and Johanneson by allowing for, first, any negotiation protocol, second, a large class of underlying games, and third, the cost of breaching to increase in the harm inflicted on the other.

Our approach allows us to explain interesting previously unexplained phenomena: theoretically, we show that there is a conflict between the efficiency of the agreement and the incentives to respect it in symmetric games with strategic substitutes.[1] This class of games contains boh the public good provision game where the public good is produced under decreasing returns to scale and the Cournot duopoly, for instance. On the other hand, in an important class of symmetric games where actions are (weak or strong) strategic complements, such a conflict is circumvented: a symmetric efficient agreement can be made, if any. This class includes the public good provision game with constant returns to scale and some other team work and partnership designs as well as Bertrand duopolies with imperfect substitutes.

Public good experiments with communication lend strong support for our theoretical finding: Isaac and Walker (1988) adopt a constant-returns-to-scale technology and find a strong positive effect of communication on efficiency. Average contribution levels are practically first-best efficient. Isaac, McCue and Plott (1985) consider decreasing returns to scale. Despite the positive effect of communication on efficiency, they find that the average contribution levels are well below first-best efficient[2]. A model where breaching cost is independent of the harm inflicted on the other, as in Ellingsen and Johanneson (2004), cannot explain the difference in the effect of communication between technologies of constant and decreasing to scale.

In chapter three we take a more careful look at the effect of the negotiation protocol on the outcome prediction. Chapter two provides a set-wise prediction potentially including non-Nash equilibria of the original game. In chapter three in order to make a more precise prediction, we introduce time preferences and a particular widely studied negotiation protocol: alternating offer negotiations which parallel the alternating offer bargaining (Stahl, 1972; Rubinstein, 1982). This sharper prediction may also lead to non-Nash outcomes of the underlying game.

In the alternating offers bargaining players negotiate about how to divide a surplus. One of the players first proposes a division to the other player who then either accepts or rejects the offer. If he accepts, then players divide the surplus accordingly. If he rejects,

[1](Bulow, Roberts and Klemperer, 1985) introduce the concepts of strategic substitutes and complements.

[2]These two studies allow subjects to play repeatedly and learn about the game. Isaac and Walker (1988) have one design with constant returns to scale and another with decreasing returns to scale. With the former design, the first-best efficiency is reached whereas the latter falls short of the first best.

then he gets to propose a division to the player who proposed first. Players alternate in their offers until they reach an agreement.

Rubinstein (1982) shows that players' time preferences allow for a unique subgame perfect equilibrium (Selten, 1975) prediction on how the surplus will be split. Moreover, he shows that a player's equilibrium share increases in her patience. A more impatient player has a stronger cardinal preference for the present to the future. Being more impatient is detrimental for a player's payoff, since the threat of a postponed agreement renders a player more willing to accept the current proposal even with worse terms.

Intuition suggests that in real life bargaining, a player's equilibrium payoff may not monotonically increase in her patience. An impatient bargaining party tends to get frustrated more easily. This added emotional motivation triggered by a greater impatience might provide the impatient player with an advantage: she might be able to threaten credibly whereas a more patient and thus a less frustrated player cannot.

Impatience operates together with reciprocity. A player is reciprocally motivated if she has an intrinsic incentive to sacrifice her own payoff in favour of the opponent's payoff if the opponent is willing to do so. We show that if players are reciprocally motivated, the player may gain from being more impatient in the alternating offer pre-play negotiations: a postponed agreement is equivalent to the opponent not behaving reciprocally, since the opponent is reducing the player's payoff by postponing the agreement. The opponent behaves even less reciprocally if the player is impatient, since an impatient player suffers more than a patient player if the agreement is postponed. Thus, a reciprocally motivated impatient player can credibly threaten an opponent who contemplates postponing an agreement with actions that are not credible for a patient player.

Chapter four studies the third justification for Nash equilibrium - learning. Nash and Bayesian-Nash equilibrium suppose that players have correct conjectures about each others' strategy choices; and if exogenous randomness is involved, they have correct probability estimates about it, too. In an equilibrium, given what others do, no player has an incentive to deviate. A researcher does not need to worry that conjectures about others' and nature's choices might be incorrect and yet plausible.

But we lose a lot in terms of realism in assuming correct conjectures. One reason why conjectures may end up being correct is learning: if each player plays the game repeatedly, she may acquire experience about the uncertainty she faces and about how others play. Formal models of learning relax the correct conjecture hypothesis. The evolution of conjectures and play becomes a stochastic process. In a steady state of the process, best replies to conjectures generate behaviour which does not contradict the conjectures. This is the core of the *conjectural equilibrium* (Battigalli, 1987). It is easy to see that Nash equilibria with correct conjectures satisfy this steady state condition. The question then is whether there are non-Nash steady states.

Previous contributions have shown that the answer to this question is affirmative, in general (Battigalli (1987), Fudenberg and Levine (1993), Dekel et al (2004), Jehiel (2005)). Yet, exactly which outcomes correspond to a steady state, depends on the assumptions about the learning environment: what players know in the beginning of the process, what players observe, how many other people are involved in the process, how they handle information etc. Depending on specific assumptions on the environment, various steady state concepts, i.e. equilibrium predictions, emerge.

In chapter four, we review such equilibrium concepts and some of the literature on learning. In section 4.2, we introduce the underlying game, the context of learning. In section 4.3, we study learning when the set of players remains fixed over the entire learning process. We start from two benchmark approaches: the first, complete information and correct conjectures; the second, incomplete information and Bayesian learning with a common prior. In these approaches strategies constitute a Nash and a Bayesian-Nash equilibrium of the game, respectively. Section 4.3.2 relaxes the assumption of mutually consistent initial conjectures and studies learning with a fixed set of players who start with possibly mutually inconsistent initial conjectures about the behaviour of others and update their conjectures as game continues (Kalai and Lehrer (1993), Jordan (1995), Nahcbar (1997) Foster and Young (2001)).

Section 4.4 is the core of our contribution. It studies learning in large populations with random matching of players before each stage game is played - subsection 4.4.1 focuses on static games of incomplete information and section 4.4.2 on extensive form games. We suppose throughout that opponents' actions are observed and kept track of in the learning process. In addition, opponents' types profiles are also observed but possibly not as precisely as the opponents themselves observe these when choosing their strategies. Our first contribution is to illustrate how we can reach various equilibrium concepts in the anonymous learning literature (Bayesian-Nash equilibrium, Harsanyi (1967-68); self-confirming equilibrium, Fudenberg and Levine (1993); self-confirming equilibrium, Dekel et al (2004); cursed equilibrium, Eyster and Rabin (2005)) by changing the complexity of beliefs about the strategies of others in the analogy-based expectation equilibrium. This complexity is driven by the precision with which players observe the opponents' type profile after each round of play. (ABEE; Jehiel (2005), Jehiel and Koessler (2006)). Thus, the ABEE provides a way to analyse the complexity of various other equilibrium concepts.

If we wish to model learning where each player plays repeatedly, it is plausible to assume that each player observes and keeps track of her own payoffs. This is because the payoff is what players ultimately care for: if they do not, why should they strive to best-reply in the first place? Successes and failures, which are measured in payoffs, are what players vividly experience and tend to remember. This is what motivates our refinement of the ABEE, the payoff-confirming ABEE (PCABEE), where each player observes and keeps track of her private payoffs. Player's own payoffs provide further information about opponents' equilibrium strategies. Yet, incorrect conjectures may survive and non-Bayesian-Nash PCABEE may exist. We provide a sufficient condition for an ABEE to be PCABEE. This condition is also necessary in an interesting class of games.

Chapter 2

A Theory of Pre-play Negotiations

There is no commonly honest man ...who does not inwardly feel the truth of the great stoical maxim, that for one man to deprive another unjustly to promote his own advantage by the loss or the disadvantage of the another, is more contrary to nature, than death, than poverty, than pain, than all the misfortunes which can affect him, either his body, or his external circumstances.

-Adam Smith (The Theory of Moral Sentiments, p. 159, 2002 (1759))

2.1 Introduction

Ray and Cal have a magic pot and ten dollars each. Each dollar put into the pot gives $\frac{3}{4}$ dollars to both of them. Ray and Cal have to decide how many dollars to put into the pot and how many to keep to themselves. Ray figures that, whatever Cal puts into the pot, for each dollar he puts into the pot, he gets only $\frac{3}{4}$ dollars back and, hence, should put nothing into the pot.

Before they decide, they can talk to each other. They may agree on how many dollars each of them will put into the pot. The agreement is not binding. Yet, having talked to Cal for a while, he seems like a nice guy to Ray. Ray starts to think that he would feel bad if he lied about how many dollars he will put into the pot. He also figures that Cal may well think similarly about him. Eventually Ray and Cal agree on putting ten dollars each into the pot and neither violates the agreement.

Most people would think that the story above is vaguely plausible but doubt that such magic pots exist. An economist is certain about the existence of the magic pot, but has doubts whether people care about inflicting harm on the other by not doing as agreed.

Two findings in experimental economics give a reason to believe that the magic pots and the dislike to breach oral agreements are worth taking seriously: First, communication increases contributions in public good games.[1] Second, if people lie, they tend to dislike it; and they seem to dislike it more if they inflict more harm on others by doing so. This is shown by Gneezy (2005) and studies in social psychology. In the public good games agreeing to contribute more than one actually intends to contribute amounts to a lie which harms others. Therefore deviating from the promise is less profitable and promises contribute may be credible. Thus a theory that assumes the latter finding (dislike breaching if harming others) provides an explanation for the former finding (increased contributions in public good games).

This chapter presents a theory of pre-play agreements, by pre-play negotiations, conventions or social norms. We assume that people dislike lying about their intentions, breaching informal agreements and transgressing social norms. Moreover, we assume that people dislike transgression of an agreement more if they inflict more harm on others by doing so. We show that given the game prior to which players negotiate (underlying game), players may agree on and play non-Nash strategy profiles (even non-correlated equilibria). We also show that there is a conflict between the efficiency of the agreement and the incentives to respect it in symmetric games with strategic substitutes such as the public good production with decreasing returns to scale and the Cournot duopoly. On the other hand, in an important class of symmetric games where actions are (weak) strategic complements [2] such a conflict is circumvented: a symmetric efficient agreement can be made, if any. This class includes the public good production with constant returns to scale and other team work and partnership designs as well as Bertrand duopolies with imperfect substitutes.

Public good experiments with communication lend strong support for our theoretical finding: Isaac and Walker (1988) adopt a constant-returns-to-scale technology and find a strong positive effect of communication on efficiency. Average contribution levels are practically first-best efficient. Isaac, McCue and Plott (1985) consider decreasing returns to scale. Despite the positive effect of communication on efficiency, they find that the

[1]See Ledyard (1995) for a review of experimental research on public goods. This result holds for the public good games without a threshold. The evidence that communication would increase contributions in the public good games with thresholds is more mixed - the increase in contributions is not always significant.

[2](Bulow, Roberts and Klemperer, 1985)

average contribution levels are well below first-best efficient.[3]

Our theory considers bilateral agreements in a wide array of strategic two-player interactions. The *underlying game* whose strategies are being agreed upon can be any normal form game. We assign the guilt cost properties that experimental and narrative research in economics and psychology has discovered. We assume that the general principles that govern guilt are the same for all players. Players may differ only in their *proneness to guilt*, i.e. how much weight they put on the guilt cost. We abstract from how an agreement is established (in pre-play negotiations, the negotiation protocol) but assume that the *agreement* is either an action profile of the underlying game or disagreement. Having an agreement on an action profile, a player who breaches may feel guilty, which lowers her utility.

Given a game and players' proneness to guilt, each agreement maps the game into another game with the same strategy sets but different payoffs. We are interested in which action profiles are *agreeable,* which action profiles can be enforced by guilt. Also, we are interested in how agreeability is affected by changes in (1) the underlying game, (2) the agreement, and (3) players' proneness to guilt.

Agreeability is defined in terms of incentive compatibility. An action profile is *incentive compatible* if neither player prefers breaching. That is, for any unilateral deviation from the profile, the guilt cost is larger than or equal to the underlying game benefit for the deviator. We call the difference between the underlying game benefit and the guilt cost the *incentive to breach.*

Which agreements are agreeable will depend crucially on the properties of the guilt cost. We adopt the following properties which are based on stylized facts in research in social psychology and experimental economics[4]:

{A} Guilt costs are weakly increasing in the harm a player inflicts on his opponent by breaching an agreement.

{B} If the opponent breaches, then there is no guilt cost.

{C} Guilt costs are weakly increasing in the player's agreed payoff.

[3] These two studies are the only one's that allow subjects to play repeatedly and learn about the game. Isaac and Walker (1988) have one design with constant returns to scale and another with decreasing returns to scale. With the former design first-best efficiency is reached whereas the latter falls short of the first best.

[4] In addition to their intuitive appeal we present experimental evidence and psychological theory that supports these assumptions in section 2.2.

{D} If no agreement is reached, there is no guilt cost.

Property {A} captures the idea that if my breaching the agreement causes my opponent to lose a toe, I do not suffer more than if my breaching the agreement causes my opponent to lose a leg. Gneezy (2005) finds strong support for property {A}: his experiments suggest that people trade off the benefits of lying against the harm that lying inflicts on the opponent. Property {B} is a no-sucker property: I will not feel guilty about breaching an agreement if my opponent breaches the agreement, too. According to property {C} the agreement's generosity induces stronger guilt. To understand this property, notice that by property {B}, there is guilt only if the opponent does not breach the agreement. If the opponent respects and moreover the payoff is high if I respect, too, then the opponent is not only kind and but also generous. Property {C} says that breaching the agreement and not reciprocating will induce stronger guilt than if the agreement had been less generous.[5] Properties {B} and {C} render guilt reciprocal. Property {D} says that if there is no agreement about how the game should be played then there is no guilt.

If the agreement is established by pre-play negotiations, it is natural to think that each player can veto any agreement. We say that an action profile is *individually rational* if it ensures that each player gets more than in her least preferred Nash equilibrium of the underlying game. In pre-play negotiations, upon deciding whether to signal disagreement, each player acts as if she knew that doing so will imply that her worst Nash equilibrium will be played.

Crucial for our finding in games with strategic complements and substitutes and an interesting result in its own right is that, in games where actions are ordered and the payoff is concave in each of the two actions, *checking that a marginal deviation from the agreement does not pay off is necessary and sufficient for incentive compatibility.*

Further towards our main conclusion we find unambiguous effects on the incentive to breach when the terms of the agreement are altered (if the agreement is agreeable in the first place): *in symmetric games with strategic complements, changing either agreed action so as to improve a player's agreed payoff decreases her marginal incentive to breach.* These effects are quite natural and intuitive: if the terms of the agreement are better for me, I have a lower incentive to breach. Yet, the result does not hold generally.

In *symmetric games with strategic substitutes*, as far as changes in player's own agreed action are concerned, the player's payoff and her incentives to respect agreements are

[5] In the context of conventions or social norms this effect may be weaker but even there a player may be more willing to breach a convention she considers unjustified.

still naturally aligned. Yet, changing the opponent's agreed action implies quite the opposite effect: *the marginal benefit increases and the marginal harm on the opponent decreases when the opponent's action is changed so as to improve player's payoff.* This is the source of our result, identifying a conflict between efficiency and incentives in symmetric games with strategic substitutes, such as the standard Cournot duopoly or public good provision when the public good is produced under decreasing returns to scale.

We also describe the agreeable set in a more general class of games and characterize the smallest and largest such set: Nash equilibria are always agreeable and nothing but Nash equilibria are agreeable for players with no proneness to guilt. Yet, a player who is sufficiently prone to guilt can agree on any individually rational profile that she cannot alone Pareto-improve and strictly benefit herself.

The chapter is organized as follows. Section 2.2 presents related literature in economics and psychology. Section 2.3 presents the model. Section 2.4 studies public good games. Section 2.5 presents general results and section 2.6 studies games with ordered strategy spaces. Section 2.7 considers a Cournot duopoly example. Section 2.8 concludes and discusses some further research problems.

2.2 Related literature

Economics. Evidence from experiments in the public good games shows that even without communication subjects contribute positive amounts when purely monetary incentives make zero contribution a strictly dominant strategy. Existing social preference models nicely capture this effect (Rabin, 1993; Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000). Yet, a largely unexplained finding is that communication raises the contributions well above the amounts observed without communication (Ledyard, 1995). Earliest experiments show this in prisoner's dilemma games (Loomis,1959; Radlow and Weidner, 1966). Recent studies for the two-person prisoner's dilemma case are provided by Duffy and Feltowich (2002) and (2005).[6]

A way forward in explaining the effect of communication would be to combine one of the inequity aversion theories (Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000) with Farrell's (1987) idea that agreements will be stuck by if there is no incentive not to do so.

[6]Extensions to public good provision games have been considered and the robustness of this result is verified by various experiments, for instance, Dawes, McTavish, and Shaklee (1977), Isaac, McCue, and Plott (1985), and Isaac and Walker (1988).

Yet, this fusion of theories is not completely convincing. First, it can only account for the experimental findings as long as the payoffs are not too asymmetric, since if they are, symmetric contribution profiles lead to unequal payoffs and players with payoffs below the average cannot commit to these profiles. Second, even in symmetric environments, if the more efficient symmetric equilibria exist in the underlying game, the learning process never reaches these equilibria in laboratory experiments when communication is not present and, yet, these outcomes are reached when communication is allowed for (Isaac, McCue and Plott (1985); Isaac and Walker (1988)). Third, Gneezy (2005) and Ellingsen and Johanneson (2004), carry out further communication experiments and find behavioural patterns that cannot be explained by inequity aversion theories alone but which point to a preference for not lying.

The extensive form extension of Rabin's (1993) theory of reciprocity as introduced by Dufwenberg and Kirchsteiger (2004) is another candidate for capturing the phenomenon. Nevertheless, Charness and Dufwenberg (2003) show that sequential reciprocity cannot fully account for the detected behavioural patterns related to communication. They conclude that there must be a separate preference related to lying and introduce, independently of the contribution in this chapter, the guilt-aversion equilibrium, where a player suffers a cost when she acts counter to the opponent's expectation about her behaviour. Thus, like the theories of reciprocity, the theory falls into the category of psychological game theory (Geanokoplos, Pierce, and Stachetti 1989) where players' payoffs depend on beliefs explicitly (see also Dufwenberg (2002)).

The guilt-aversion or let-down aversion theory (Charness and Dufwenberg, 2003) is closely related to our approach. In their model, promising to carry out an action is assumed to strengthen the belief that the opponent expects corresponding behaviour. Thereby the promise creates further incentives to behave accordingly. Nevertheless, the role of communication is only implicit in their model. Furthermore, however unjustified the opponent's expectation is, guilt is constant whenever the harm on the opponent is the same.

Our model can be considered as a tractable model of let-down aversion where a player may be averse to let down a *justified* expectation but less averse to let down an unjustified expectation. To see this, notice first that in a guilt-aversion equilibrium, first and second order expectations coincide with the actual actions. Equilibrium expectations in the let-down aversion model are equivalent to the agreement in our model. We argue that the expectations must satisfy three criteria for let-down aversion equilibrium to emerge: they must be 1) justified 2) mutually consistent and 3) commonly known.

First, if a player thinks that the opponent's expectation is not fully justified, the player might have less of an intrinsic motivation to act according to the expectation. Important applications of this feature are captured by properties {B} and {C} of our model. If a player expects the opponent to breach the agreement but she expects the opponent to expect that she respects the agreement, she is likely to feel that the opponent's expectation is unjustified and she does not have an aversion to let-down the opponent. This idea is captured by property {B}. So as to property {C}, even if the opponent intends to respect the agreement and the player expects the opponent to do so, she may feel that what the opponent expects her to do gives such a low payoff that the opponent's expectation is not justified. Therefore she may be less averse to let down the opponent.

Second, guilt-aversion equilibrium only considers cases where equilibrium expectations coincide with equilibrium actions. Thus guilt-aversion equilibria correspond to all agreements that neither player prefers to breach in our model. Notice moreover that guilt-aversion is an equilibrium theory and, yet due to added incentives to comply with opponents' expectations, it is very likely that there are multiple equilibria. The set of rationalizable action profiles might be even larger. If players lack a coordination device, then it may be difficult for the players to coordinate on a focal equilibrium. Pre-play agreements and conventions circumvent the selection problem by pointing out to a focal equilibrium. A pre-play agreement generates commonly known focal expectations, which will be correct if there are sufficient incentives to comply with them. In conclusion, guilt-aversion equilibrium may be a more accurate prediction when pre-play agreements are present. Moreover, agreements by pre-play negotiations are natural devices to generate justified expectations: expectations derive their justification from not being vetoed by either party.

In addition, our approach is explicit about the effect of communication and the agreement. This view is supported by experimental evidence: Lev-on (2005) reviews communication experiments in public good games and concludes that mere identification or discussion which lacks explicit promising loses some of its effectiveness in supporting cooperation.[7] The model is general. It captures many features of reciprocity, yet avoiding problems of tractability in models where payoffs depend on beliefs explicitly.[8] The guilt

[7] Furthermore, mere face-to-face identification increases cooperation especially in simple prisoner's dilemma games where coordination on group optimum is easy (Bohnet and Frey (1998)). Yet, cooperation rates are significantly weaker than when interactive communication is allowed for.

[8] Some feasible guilt cost functions imply that the preferences in the cases where an agreement is in place are tractable social preferences of Cox and Friedman (2002).

in our model bases its properties on research in social psychology and allows for most of the features relevant to pre-play negotiations and conventions.

Guilt has been discussed in several papers since Frank (1988) who argues that it may well be materially profitable for an agent to have a conscience - a dislike for disobeying social norms. A recent model on emotional cost of breaching social norms is provided by Huck, Kübler, and Weibull (2003). These models involve no communication. Ellingsen and Johanneson (2004) do allow for communication and study the interplay of inequity aversion and guilt in a specific hold-up problem between a seller and a buyer. Their model is similar to ours in that guilt does not depend on the beliefs explicitly. In their model also, guilt is suffered if one breaches a promise. However, their model of guilt is simpler, since it does not take into account the reciprocal elements of opponent's behaviour and it assumes that breaching a promise inflicts a constant guilt cost.

Psychology. In addition to their intuitive appeal, properties {A} to {D} are supported by experimental evidence and by psychological theory. As to property {A}, Hoffman (1982) suggests that guilt has its roots in a distress response to the suffering of others. The main empirical finding of Gneezy (2005) is that 1) lying is directly costly and 2) people do not care only about their own gain from lying: they are also sensitive to the harm that lying may inflict on others.

As far as property {B} is concerned, Baumeister, Stillwell, and Heatherton (1995) find that people feel more guilty about transgressions involving an "esteemed" person than about transgressions involving someone they hold in low regard. It is rather appealing to suppose that, if the opponent breaches the agreement, the esteem of a player towards the opponent is smaller than if the opponent respects. We go to an extreme and assume that the player does not suffer from guilt if the opponent breaches the agreement.

Property {C} operates together with property {B}: agreements that are respected and give a high payoff to a player, signal opponent's concern for player's welfare and such opponents are likely to be esteemed. According to Clark and Mills (1984) and Clark (1979), concern for the other's welfare is the defining feature of communal relationships as opposed to exchange relationships. According to Baumeister, Stillwell, and Heatherton (1995), guilt is more likely to arise in the former type than in the latter type of relationships.

So as to property {D}, an agreement or an action-norm explicitly states an expectation and a standard of behaviour for the play phase. Not reaching an agreement indicates players' inability to establish such a standard and a shared expectation. Millar and Tesser (1988) note that guilt depends on a concurrence of one's own expectations of behaviour and those of the other person. Guilt appears mainly when there is a match in expectations of behaviour. Such a match of expectations is established either by an exogenous action-norm or a pre-play agreement to an action profile. On the other hand, some experimental studies of the public good game show that a single message for not contributing is sufficient to make an agreement invalid.[9] This body of research suggests each player should have an ability to veto an agreement and that if there is no agreement in place, guilt should be lower. We take this to an extreme and assume that there is guilt only if there is an agreement or a commonly known action-norm.

More generally, research in psychology identifies three types of emotional distress associated with lying: guilt, shame and fear of punishment. From a game theoretical perspective, the latter two have a reputation and repetition flavour respectively whereas guilt may be suffered even if the act of lying is unobservable and unverifiable to others, or the victim or a third party is in no position to retaliate.

According to Baumeister, Stillwell, and Heatherton (1994), "guilt can be distinguished from fear of punishment on the basis that the distress pertains to the action itself rather than to the expectation of hedonically aversive consequences of the action. ...One can clearly feel guilt..., even if the victim is in no position to retaliate."

Baumeister, Stillwell, and Heatherton (1994) are concerned with what makes people feel guilt and what that feeling, or the motivation to avoid that feeling, causes them to do (p.245). They argue that:

- From an interpersonal perspective, the prototypical cause of guilt would be the infliction of harm, loss, or distress on a relationship partner. Although guilt may begin with close relationships, it is not confined to them; guilt proneness may become generalized to other relationships. ... In particular, a well-socialized individual would presumably have learned to feel guilty over inflicting harm to even a stranger.

Based on this view, we elaborate on the idea of guilt as an internalized punishment payoff in a repeated game prior to which players agree on a stationary pattern of play in the appendix.

In the present model, as in theories of fairness, players internalize the opponent's payoff *but only conditional on reaching an agreement*, *conditional on the opponent respecting the agreement* and *conditional on the opponent suffering from breaching.* Thus, the model shares some of the features of the models of fairness but differs from those in important dimensions.

[9] See Ledyard (1995) and Pavitt and Shankar (2002).

2.3 The model

Let Γ be a two-player simultaneous move normal form game, below referred to as the *underlying game.*[10] Before the game is played, an agreement - a mutual expectation - is established either by pre-play negotiations or by convention. Generally, the pre-play negotiations may have an arbitrary strategic structure or the agreement may be exogenous - the only requirement is that there is an agreement or disagreement on how to play. [11]

We rule out the use of mixed strategies in the underlying game. If we allowed for mixed strategies, we should determine whether guilt is a function of consequences only or whether guilt is felt even if a mixed strategy different from the agreed one is chosen but the random draw picks up a pure strategy that is in the support of the agreed mixed strategy.[12]

2.3.1 The underlying game

The two-player *underlying game* is given by $\Gamma = \{S_i,\ u_i(s) : S \to R\}$. The action set of player i is S_i. A combination of actions is an *action profile* $s = (s_i, s_j) \in S = S_i \times S_j$. The *underlying game payoff* of player i is $u_i(s)$. Notice that this payoff may well include social preference terms.

The lowest Nash payoff of player i is defined by $u_i^* \doteq \min_{s \in NE(\Gamma)} u_i(s)$ where $NE(\Gamma)$ is the set of pure Nash equilibria in the underlying game. The vector of such payoffs is $u^* = (u_i^*, u_j^*)$. If rational players play without pre-play negotiations and they have correct expectations about the behaviour of the other, then a Nash equilibrium should result. Thus, the lowest Nash payoff is the worst case scenario if negotiations fail (and players believe in equilibrium play).

The negotiations or the convention establishes an agreement, m, on how to play, or disagreement. Thus, we restrict $m \in S \cup \{d\}$ where d denotes disagreement. If $m \in S$ is the agreement, then m_1 and m_2 are the *agreed actions* of players one and two respectively. The *agreed payoff*[13] indicates how much more than u_i^* the player gets if both respect

[10] The theory allows for a straightforward extension to sequential two stage games.

[11] *Preplay negotiation* is a finite extensive form game tree. The terminal histories are associated with an oral (non-binding) agreement, or with disagreement.

[12] On the other hand, we could easily allow for correlated strategies where players agree on a given random draw on how to play: guilt would be a function of the expected agreed payoff.

[13] Most of our results would be unaltered if we alternatively suppose that the reference point in the agreed payoff is the player's worst Pareto-efficient Nash payoff which is the lower bound for a long pre-play negotiation payoff derived in Rabin (1994).

the agreement, $v_i(m) \doteq u_i(m) - u_i^*$. If player i deviates from the agreement, we get the *harm* on j by subtracting j's payoff at the deviation profile from the payoff at the agreed action profile, $h_j(m, s_i) \doteq u_j(m) - u_j(m_j, s_i)$. Similarly, i's *benefit from breaching* is $b_i(m, s_i) \doteq u_i(m_j, s_i) - u_i(m)$.

In this chapter, we restrict focus to simultaneous move games. Notice, that we could easily extend our theory to corresponding Stackelberg games, say, with player one the leader and player two the follower. That player one moves first gives her perfect commitment power. If the leader breaches, the follower does not suffer from guilt and her payoff coincides with the UG payoff. Thus, the follower will choose an UG best reply to the leader's action. In the Stackelberg version of the theory, we should replace the worst Nash payoff with the worst Stackelberg payoff.

2.3.2 The entire game

Players are prone to guilt. If there is an agreement in place, they feel bad about not doing their part of the deal. Player i's *guilt cost*, $g_i(v_i(m), h_j(m, s_i))$, depends on the inflicted harm and on the agreed payoff. The utility function in the entire game is assumed to be additively separable in guilt and the underlying game payoff.

$$U_i(m,s) = \begin{array}{l} u_i(s) - \theta_i g(v_i(m), h_j(m, s_i)) \text{ if } s_i \neq m_i \text{ , } s_j = m_j \\ u_i(s) \text{ otherwise} \end{array} \qquad \text{(BD)}$$

The entire game payoff now depends on m and, due to guilt, talk is not cheap. The guilt cost is represented by $\theta_i g(v_i(m), h_j(m, s_i))$ which is assumed to be non-negative. This rules out revengeful feelings or spite, on the one hand, and positive emotions related to respecting agreements, on the other hand. This is somewhat restrictive, but here we want to focus on guilt.

The parameters $\theta = (\theta_1, \theta_2)$ capture players' *proneness to guilt.* For a given deviation, a player with a higher proneness to guilt suffers more. We only allow for non-negative proneness to guilt, $\theta_i \in [0, \infty)$. If it is common knowledge that the proneness to guilt of both players equals zero, the model coupled with a communication protocol is one of cheap talk.[14]

Notice first, that the guilt cost depends on the agreement and on the deviation only indirectly through the agreed payoff and the harm. Second, choosing the agreed action m_i minimizes the guilt cost at the second stage. Furthermore, (BD) implies that if

[14]As in Farrell (1987) but with any finite extensive form communication protocol ending up in an agreement - an action profile of the game.

disagreement is reached, then there is no guilt cost. We assume that each player can unilaterally enforce disagreement, d. Also, there are no bad feelings about own cheating if the opponent cheats too. Thus (BD) introduces properties {B} and {D} into the guilt cost.

Moreover, we assume, that the guilt cost $g(v_i, h_j)$ is weakly increasing in the agreed payoff and in the harm. This introduces properties {A} and {C} into the guilt cost.

$$g(v_i, h_j) \text{ is weakly increasing in } v_i \text{ and in } h_j \tag{AC}$$

Obviously, if the guilt function is differentiable then these monotonicity properties simply amount to positive derivatives, $\frac{\partial g}{\partial v_i} \geq 0$ and $\frac{\partial g}{\partial h_j} \geq 0$.

Also, we assume that if the player inflicts no harm on the opponent[15] or if the agreed payoff equals the worst Nash payoff, then there is no guilt cost. Yet, we assume that if strictly positive harm is inflicted and the agreed payoff is strictly positive, then the guilt cost is strictly positive:

$$\begin{aligned} g(v_i, h_j) > 0 \text{ if } h_j > 0,\ v_i > 0 \\ g(v_i, h_j) = 0 \text{ if } h_j = 0 \text{ or } v_i = 0 \end{aligned} \tag{EF}$$

Notice that these assumptions allow for a number of possible cost functions. For instance, a fixed guilt cost

$$g(v_i, h_j) = \begin{cases} \gamma \text{ if } h_j > 0,\ v_i > 0 \\ 0 \text{ otherwise} \end{cases}, \tag{2.1}$$

or a guilt cost that only depends on one of the arguments is allowed for. Another example of a guilt cost function with all the properties assumed in this section is [16]

$$g(u_i(m), h_j(m, s_i)) = \max\{v_i(m), 0\}^{\gamma} \max\{h_j(m, s_i), 0\}^{\varphi} \tag{2.2}$$

This function is zero if the harm is non-positive or if the agreed payoff is below 0. Otherwise, it is strictly positive. It is increasing in the harm and in the agreed payoff.

[15] Andreoni (2005) provides some indirect evidence for this. In his extension of the buyer-seller trust game where sellers can make non-binding promises of refunds, the sellers who promise a refund, increase the return rates (quality) above no-buy utility so that no harm is inflicted, if a promised refund request is rejected. Thus, for any realised rejection of refund, guilt is avoided, and the present theory (or its straightforward extension to sequential two-stage games) predicts rejection conditional on refund request and return rate above one which the data in Andreoni seems to confirm.

[16] The entire game preferences of this form with $\gamma = \varphi = 1$ belong to the class of Cox-Friedman (2002) preferences with $\alpha = 1$ with the emotional state depending on the agreed payoff $v_i(m)$.

We suppose that the proneness to guilt types and the language are common knowledge. Thus, players have correct point predictions about their opponent's proneness to guilt and beliefs of all degrees coincide. Also, players do not have to worry that the opponent might interpret an agreement to 'meet at noon' as an agreement to 'meet at quarter past noon.' Both these considerations are relevant but at this first step we abstract from this.[17]

Let us now introduce some further notation. Denote by $BR_i(s_j)$ the underlying game best reply correspondence of player i. Denote by $\Gamma(m;\theta)$ a subgame where m is agreed and players' proneness to guilt is given by θ. Denote by $s^*(m;\theta) = (s_i^*(m;\theta), s_j^*(m;\theta))$ the equilibrium correspondence in that subgame.

Let us write the payoffs of player i and player j respectively when player i deviates to s_i and player j respects the agreement, $s_j = m_j$, as

$$U_i(m_i, m_j, s_i, m_j) = u_i(m) + b_i(m, s_i) - \theta_i g(u_i(m), h_j(m, s_i)) \tag{2.3}$$

and

$$U_j(m_j, m_i, m_j, s_i) = u_j(m) - h_j(m, s_i). \tag{2.4}$$

where the first two entries of $U_i(.,.,.,.)$ are the agreed actions and the last two entries are the played actions of i and j respectively. These expressions give players' entire game payoffs in terms of the agreed payoff, the benefit from breaching, and the harm inflicted on the other when i breaches but not j. Player's *incentive to breach* an agreement m is the difference between the benefit from breaching and the guilt cost, $B_i(m, s_i; \theta_i) \equiv b_i(m, s_i) - \theta_i g(u_i(m), h_j(m, s_i))$.

An agreement m is called *incentive compatible* if neither benefits from a unilateral deviation from the agreement,

$$\text{for all } s_i \in S_i \quad B_i(m, s_i; \theta_i) \leq 0 \tag{IC_i}$$

When this incentive compatibility condition holds for both players, the agreement m is a Nash equilibrium of the subgame where m is agreed upon, $\Gamma(m;\theta)$. On the other hand, an agreement m is called *individually rational* if no player prefers enforcing disagreement (pre-play negotiations) over playing m, i.e. if for $i = 1, 2$

$$u_i(m) \geq u_i^*. \tag{IR_i}$$

Here, the threat for the player who enforces d is the lowest payoff Nash equilibrium, u_i^*.

[17] Notice also that since guilt depends on the ageement only indirectly, any permutation of the meanings of the agreements leaves the guilt unaltered.

We now define *player i's potential to agree* as $A_i(\Gamma, \theta_i) \equiv \{m|\ m$ satisfies (IC_i) and $(IR_i)\}$ and *the agreeable set* is defined as the intersection of the two potentials to agree, $A(\Gamma, \theta) \equiv \cap_{i=1,2} A_i(\Gamma, \theta_i)$. We call an action profile in i's potential to agree *agreeable for i* and we call an action profile in the agreeable set simply *agreeable*.

2.4 A public good game

The prisoner's dilemma is a stylized version of a public good game. In the prisoner's dilemma there are two players who decide whether to contribute to the production of the public good or not. It is efficient that both contribute but it is a strictly dominant strategy not to contribute. We consider a prisoner's dilemma with the following payoffs:

	C	N
C	u_1, u_2	$u_1 - h_1, u_2 + b_2$
N	$u_1 + b_1, u_2 - h_2$	$0, 0$

(Prisoner's dilemma)

where $h_i > u_i > 0$ and $b_i > 0$ for $i = 1, 2$. Supposing that the guilt cost takes the simple form of the example given in (2.2) with $\gamma = \varphi = 1$, player i respects an agreement to contribute, $m = (C, C)$, (given that the opponent does) if and only if

$$\theta_i \geq \frac{b_i}{u_i h_j} \tag{2.5}$$

An agreement on cooperation satisfying (2.5) is incentive compatible. Moreover, both contributing is individually rational by the structure of the prisoner's dilemma. So, an agreement on (C, C) should be particularly easy to reach if b_i is small and h_j is large - just as Gneezy (2005) suggests. Also, a large u_i facilitates cooperative agreements. This gives us comparative statics results that are testable.

In the prisoner's dilemma, the payoff of one of the player's is below the zero payoff in equilibrium at outcomes (C, N) and (N, C). Thus, these agreements are not individually rational. Both not contributing, (N, N) , is incentive compatible and individually rational for all types since it is the unique Nash equilibrium. Hence, (N, N) is always agreeable and (C, C) is agreeable if (2.5) holds for both players.

Moreover, notice that the individual rationality condition is actually redundant. it is implied by incentive compatibility: if individual rationality is violated, the agreed payoff falls below zero and guilt is zero thus any deviation which is beneficial in terms of underlying game payoff will be made. This property is more general as we shall see in section 2.5.

In our model, proneness to guilt may transform a prisoner's dilemma into a coordination game. This is a familiar property of fairness models (Rabin, 1993; Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000). Yet here, first, the transformation is explicit; and second, the ability to commit to contribute does not depend on how much more or less the opponent gets when players cooperate, $u_i - u_j$. It depends on how much more the player gets when players contribute than when they do not, $u_i - 0$. On the other hand, player's payoffs are other regarding only to the of how much a player's defection affects the opponent's payoff.

Guilt is an emotional cost of defection. In the prisoner's dilemma, it is trivial that if this cost is sufficiently large to balance off the benefit from breaching, the player can credibly commit not to defect. The prisoner's dilemma is a rather degenerate game for our interests: there is only one action profile that Pareto dominates the underlying game Nash equilibrium. Thus, the set of agreements under negotiation is very limited. Our pre-play negotiations model may have bite in any game with an inefficient equilibrium. In more general games the model has richer implications as we shall see next.

We can easily generalize the prisoner's dilemma type of argumentation to public good games. Each player has an endowment of ten dollars. Each player decides how many dollars to contribute, $s_i \in \{0, ..., 10\}$. The payoff of player i reads

$$u_i(s) = G(\sum_{k=1,2} s_k) + 10 - s_i$$

where the production technology $G(.)$ maps the sum of contributions into the amount of public good produced. We suppose that for all (s_1, s_2), $G'(\sum s) < 1$ where G' is the *marginal per capita return* (MPCR). Hence, it is a strictly dominant strategy, and thus a Nash equilibrium strategy, to contribute nothing. Whenever *marginal group return* equals $2G' > 1$, it is socially optimal to increase one's contribution.

Let us suppose for the time being that the guilt cost is given by (2.2) with $\gamma = \varphi = 1$ and let the production technology have constant or decreasing returns to scale, $G'' \leq 0$. By our definition of agreeability, players can agree to any agreement where both get a positive payoff (indivudual rationality) and the there is sufficient guilt (incentive compatibility) to prevent breaching. As in the prisoner's dilemma, the individual rationality condition

$$u_i(m) = G(\sum_{k=1,2} m_k) + 10 - m_i \geq 0. \tag{2.6}$$

is implied by the incentive compatibility condition.

Notice in addition, that due to the concavity of payoff in each action, the agreement is incentive compatible if a one-dollar underprovision does not pay off: the benefit from breaching is concave and the harm on the other is convex as a rescaled negative of opponent's payoff. Thus, we only need to verify that the public good will not be marginally underprovided, and we need not to worry about other deviations.

Let us call the differential between the marginal benefit from breaching and the marginal guilt cost player i's *marginal incentive to breach*,

$$\begin{aligned}&1 - G(\textstyle\sum m_i) + G(\sum m_i - 1) \\ &-\theta_i[G(\textstyle\sum m_i) - G(\sum m_i - 1)]^{\varphi} \max\{u_i(m), 0\}^{\gamma}.\end{aligned} \tag{2.7}$$

Supposing that an indifferent player respects the agreement, a player will breach if and only if (2.7) is positive. These marginal incentive compatibility conditions imply the incentive compatibility conditions which in turn imply the individual rationality conditions. Thus, the marginal incentive compatibility conditions are necessary and sufficient for agreeability (2.7).

A property explicit in (2.7) is worth emphasizing: there is a conflict between the efficiency of the agreement and the incentives to respect. To see this, notice that the harm on j due to a unit underprovision by i reads $h_j(m, m_i - 1) = G(\sum m_i) - G(\sum m_i - 1)$ which is decreasing in the sum of contributions and thus in efficiency when too little is contributed. The marginal benefit for i from her unit underprovision vis-à-vis the agreement is $1 - h_j(m, m_i - 1)$. This is increasing in the sum of contributions. Since efficiency increases in the sum of contributions but the marginal harm on others decreases and the marginal benefit from breaching increases in the sum of contributions, the conflict is evident.

More importantly, the conflict is strict if $G'' < 0$. Only in this case the fact that a player trades off the marginal harm with the marginal benefit strictly increases the marginal incentive to breach when the sum of contributions is increased.

To better understand how increasing the sum of contributions affects the marginal incentive to breach, let us now isolate the effect of each agreed action. Let us first consider the effect of the own agreed action, m_i, on the incentive to breach. If we increase m_i, the benefit from breaching increases and the harm on the other decreases. The increasing effect of m_i on the marginal incentive to breach is amplified by the negative effect of m_i on i's agreed payoff, which dampens guilt. Yet, considering now the effect of increasing the opponent's agreed action, m_j , it is clear that this has a positive effect on i's agreed payoff and, since by assumption efficiency is increased, the overall agreed payoff effect on guilt is positive. This tends to decrease i's incentive to breach.

Thus, whether or not incentive to breach increases in the sum of contributions depends on G'', on the one hand (effect on marginal benefit and harm), and on G' and $\frac{\partial g}{u_i}$, on the other hand (effect on agreed payoff). If G'' is close to zero the trading off of marginal benefit and harm is unaffected but the agreed payoff effect decreases incentives to breach. Yet if G'' is substantially below zero and the agreed payoff does not much affect guilt, the effect of trading off benefit and harm increases the incentives to breach. Furthermore, if G'' is negative the agreed payoff effect tends to fade away with efficiency. Eventually, if we have an interior group optimum, there will be a conflict between efficiency and incentives as we are sufficiently close to the group optimum.

Yet, as a special case, if there are constant returns to scale, $G' = \alpha$, the marginal payoffs are constant and the changes in breaching incentives are driven only by the agreed payoff effects: incentives to breach decrease in efficiency. With constant returns to scale incentives and efficiency are aligned: if some disequilibrium strategy profile is agreeable, than an efficient profile is.

Some other remarks can be made on (2.7). First, a player with a higher proneness to guilt can agree on a larger set of agreements. Second, the relative contributions matter (but not the relative payoffs). Moreover, if there are constant returns to scale, $G' = \alpha$, the marginal benefit from breaching decreases in α and the marginal harm increases in α and the agreed payoff of any agreeable action profile increases in α. Thus, it is easier for the players to agree when the marginal per capita return is higher.

Let us collect the findings of this section in a proposition.

Proposition 1. *Let g satisfy (2.2) with $\varphi \geq 1$. In the public good game,*

- *an agreement is agreeable iff the marginal incentive to breach is non-positive for $i = 1, 2$.*
- *player i's marginal incentive to breach is increasing in m_i.*
- *if $G' = \alpha$, player i's marginal incentive to breach is decreasing in α and in m_j and in $\sum_{k=1,2} m_k$.*
- *if $G'' < 0$ and $\gamma = 0$, player i's marginal incentive to breach is increasing in m_j and in $\sum_{k=1,2} m_k$.*

Proof. To prove the first claim, it is straightforward that

$$m \text{ satisfies } IC_i \text{ for } i = 1, 2 \;\Leftrightarrow\; m \text{ is agreeable},$$

since IC_i implies IR_i. It is easy to see that an upward deviation never pays off. Thus, it suffices to show that a non-positive marginal incentive to breach is equivalent to a non-positive incentive for deviating to any $s_i \in S_i$. We have for all $s_i < m_i$

$$
\begin{aligned}
& m_i - G(\sum_{k=1,2} m_k) - s_i + G(m_j + m_i - s_i) - \theta_i g(v_i(m), [G(\sum_{k=1,2} m_k) - G(m_j + m_i - s_i)]) \\
\leq \; & [1 - G(\sum_{k=1,2} m_k) + G(m_j + m_i - 1)][m_i - s_i] \quad (2.8) \\
& -\theta_i g(v_i(m), [G(\sum_{k=1,2} m_k) - G(m_j + m_i - 1_i)](m_i - s_i)) \quad (2.9) \\
\leq \; & [1 - G(\sum_{k=1,2} m_k) + G(m_j + m_i - 1)] \quad (2.10) \\
& -\theta_i g(v_i(m), [G(\sum_{k=1,2} m_k) - G(m_j + m_i - 1_i)]) \quad (2.11) \\
\leq \; & 0 \quad (2.12)
\end{aligned}
$$

where the first inequality follows from the fact that the opponent's payoff is increasing in s_i and that g is convex in h_j, and the second inequality follows from the fact that $[m_i - s_i] \geq 1$.

To prove the second claim, notice that increasing m_i will, (1) decrease $u_i(m)$ and thus $v_i(m)$, (2) increase $b_i(m, m_i - 1) = 1 - G(\sum_{k=1,2} m_k) + G(m_j + m_i - 1)$ and (3) decrease $h_j(m, m_i - 1) = G(\sum_{k=1,2} m_k) - G(m_j + m_i - 1)$.

To prove the third claim, notice that increasing α will (1) increase $u_i(m)$ and thus $v_i(m)$, (2) decrease $b_i(m, m_i - 1)$ and (3) increase $h_j(m, m_i - 1)$. Increasing m_j or $\sum_{k=1,2} m_k$ will (1) increase $u_i(m)$ and thus $v_i(m)$ and leave (2) $b_i(m, m_i - 1)$ and (3) $h_j(m, m_i - 1)$ unaffected.

To prove the fourth claim, notice that increasing m_j or $\sum_{k=1,2} m_k$ will increase $u_i(m)$ and thus $v_i(m)$ but since $\gamma = 0$ this will not affect g. Increasing m_j or $\sum_{k=1,2} m_k$ will increase $b_i(m, m_i - 1)$ and decrease $h_j(m, m_i - 1)$. □

Proposition 1 establishes that instead of checking for all possible deviations it is necessary and sufficient simply to check for a local deviation. Moreover, if $G' = \alpha$, the marginal incentive to breach is monotone in each of the two agreed actions. Thus, to determine a player's potential to agree, we can look for agreements where the player is indifferent between respecting and deviating marginally. Any agreement where a player's action is smaller or an opponent's action is larger than at the boundary is agreeable for that player. Figure 2.1 shows the agreeable set for $G' = \alpha = \frac{3}{4}$, $\theta_i = 4$ and $g(v_i, h_j)$ as in (2.2) with $\gamma = \varphi = 1$.

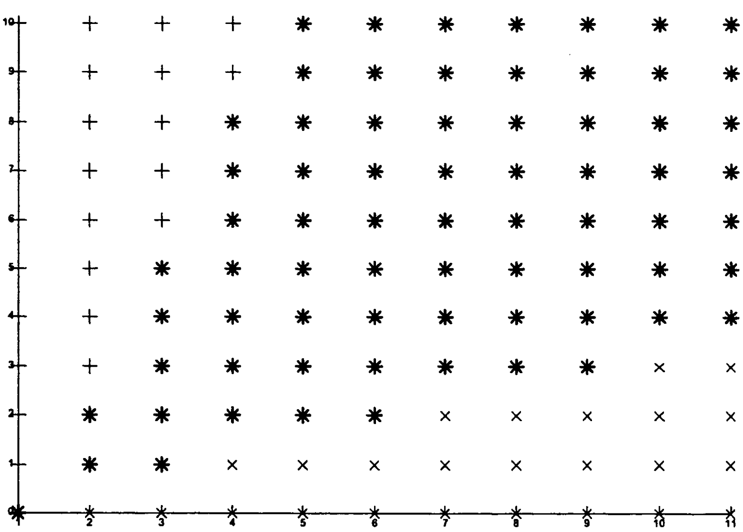


Figure 2.1: The agreeable set

The action profiles that belong to player one's potential to agree are marked with plus signs and the action profiles that belong to player two's potential to agree are marked with crosses. Thus the action profiles marked with asterisks are agreeable action profiles, $A(\Gamma_{PG}(\frac{3}{4}), (4,4))$. Notice, that the best reply curves lie on the axes and that each player's best reply curve is agreeable for each player. Thus, the Nash equilibrium, $(0,0)$, is agreeable. Notice also that some efficient action profiles are agreeable, for instance, the symmetric efficient action profile where both give a full contribution, $m = (10,10)$.

Figure 2.2 illustrates how easy it is to agree on this symmetric efficient action profile, $m = (10,10)$. Specifically, it plots the critical θ that makes a player indifferent between breaching and respecting as a function of α. As stated above, increasing α makes the incentive compatibility constraint less stringent and, thus, the function is decreasing.

Indeed, we have shown in this section that when communication is allowed for in public good games and players are prone to guilt, players may agree to contribute positive amounts and guilt may provide the necessary incentives to commit to the agreement. Further in regards to the experiments by Isaac, Mccue and Plott (1985) and Isaac and Walker (1988), we have suggested that a likely explanation for the differences in their results may not be that it is more difficult for the players to identify an interior group optimum than a boundary one, as has been suggested. Rather that incentives to respect more efficient agreements, especially those close to the group optimum, may be smaller. This conflict is typically absent when it is optimal to contribute everything to the public good as in Isaac and Walker (1988).

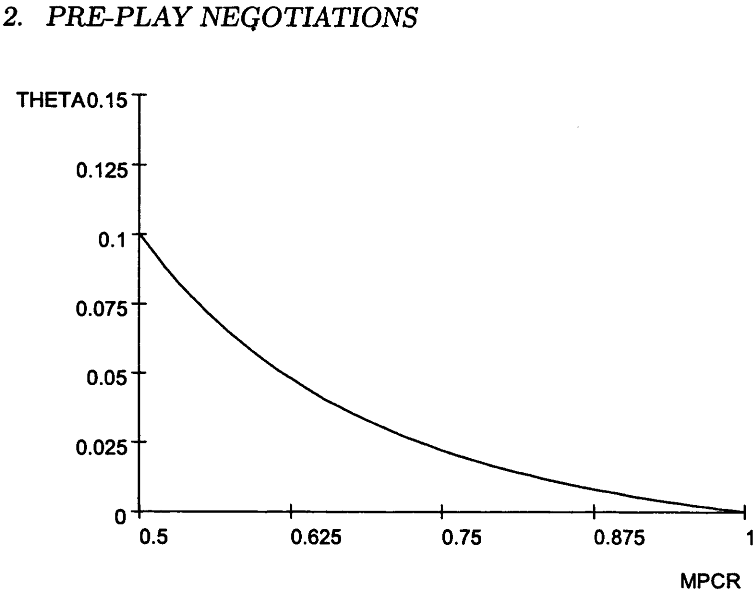


Figure 2.2: Indifferent player at symmetric efficient profile

Notice that it is crucial here that guilt cost is convex in the harm on the other. For instance fixed guilt cost due to a deviation, (2.1), cannot account for the difference since with that specification guilt is concave in the harm on other overall.[18]

Other social preference models (Rabin, 1993; Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000) can explain positive contributions to public goods but none have explained why communication further increases contributions. In this section we have illustrated how communication may increase contributions of players who without communication have a strictly dominant strategy to contribute nothing. This does not seem to comply with the empirical finding that even without communication positive amounts are contributed. Yet, the next section develops the theory in the more general case where the underlying game preferences may take an arbitrary form (and may thus involve social preferences) and equilibria of the game are inefficient. The section shows in particular how the present theory can account for the fact that communication increases contributions from the levels that prevail without communication.

Moreover, section 2.6 generalizes the sharp contrast in feasibility of first-best efficiency between constant returns to scale technology and decreasing returns to scale technology in public good production: we shall show that there is a conflict between incentives and efficiency in symmetric games with strategic subsitutes where payoff is monotone in opponent's action. Such a conflict tends to be absent in symmetric games with strategic complements where payoff is monotone in opponent's action.

[18] This will imply, the model of Ellingsen and Johanneson (2004) cannot account for the differences in efficiciency results of Isaac, McCue and Plott (1985) and Isaac and Walker (1988).

2.5 Properties of the agreeable set

This section derives some simple properties that apply to any normal form underlying game. First, any UG Nash equilibrium is agreeable. Thus, the agreeable set is never smaller than the set of Nash equilibria of the UG. Second, a Nash equilibrium remains a Nash equilibrium of most subgames that follow an agreement. Yet, if an agreement is such that a player can unilaterally deviate to an UG Nash equilibrium, then this UG Nash equilibrium may no longer be a Nash equilibrium when the agreement is made. Third, if a player can deviate from an agreement and thereby benefit both players, the action profile is not agreeable. Yet, any individually rational profile that does not satisfy this property can be agreed upon if proneness to guilt is sufficiently high. This characterizes the largest possible agreeable set as opposed to the smallest such set - the set of UG Nash equilibria.

In the prisoner's dilemma, both (D, C) and (D, D) are agreeable for the row player. Both profiles are individually rational for the row player and the row player's agreed action is an UG best reply to the agreed action of the column player. Underlying game preferences drive the player to choose the best reply. If a player's agreed action is a best reply to the agreed action of the other player, the guilt cost of deviating would only add to the forgone UG payoff . The first part of the following lemma establishes this general finding.

On the other hand, if a player's agreed action is not a best reply to the opponent's agreed action, then the agreement belongs to the player's potential to agree if and only if it is incentive compatible. The UG benefit from breaching is positive at least for the deviation to the best reply; if individual rationality is violated, the guilt cost is zero and the agreement is not incentive compatible.

Lemma 1. *Let $m_i \in BR_i(m_j)$. Then $m \in A_i(\Gamma, \theta_i)$ iff (IR_i) holds.*
Let $m_i \notin BR_i(m_j)$. Then $m \in A_i(\Gamma, \theta_i)$ iff (IC_i) holds.

Proof. See appendix. □

This lemma is useful for characterizing each player's potential to agree: on the best reply curve, all individually rational agreements are agreeable. Off the best reply curve, all incentive compatible agreements are agreeable and no other agreement is. Thus, for non-equilibrium conventions only incentive compatibility matters. On the other hand, lemma 1 enables us to generalize the finding that, in the prisoner's dilemma, the defection equilibrium is agreeable for any proneness to guilt types. By definition, any

Nash equilibrium payoff is individually rational. Thus by the first part of lemma 1, any Nash equilibrium belongs to each player's potential to agree. Thus, a Nash equilibrium is agreeable.

Proposition 2. *If $m \in NE(\Gamma)$, then $m \in A(\Gamma, \theta)$.*

Proof. See appendix. □

First, for zero proneness to guilt types, Nash equilibria are the only agreeable action profiles.[19] Second, guilt never reduces the menu of agreements available to the players. To the contrary, the public good example shows that positive proneness to guilt can dramatically increase the set of profiles that are agreeable.

Recall that we ruled out mixed strategies and thus an agreeable profile may not exist. Notice, that allowing for mixed strategies would ensure that an agreeable profile always exists (whichever way we think about guilt): an underlying game Nash equilibrium is always agreeable and with mixed strategies a Nash equilibrium always exists in finite games.

Yet, pre-play negotiations may create an equilibrium selection problem when there is an agreement in place and players are prone to guilt. For instance, when players agree on cooperation in the prisoner's dilemma, defection remains an equilibrium of the transformed game. If both players defect, neither feels guilt and payoffs involve only underlying game payoffs. This insight is easily generalized: it is straightforward that an underlying game equilibrium where neither respects the agreement, m, is an equilibrium of the subgame $\Gamma(m;\theta)$. This shows that even if m is a Nash equilibrium of $\Gamma(m;\theta)$, there may be other equilibria as well.

Lemma 2. *If for $i = 1, 2$, $m_i \neq s_i^*$ and $s^* \in NE(\Gamma)$ then $s^* \in NE(\Gamma(m;\theta))$*

Proof. See appendix. □

The equilibrium selection problem apparent in lemma (2) is avoided however if we suppose that players will conform to the agreement, if there is no incentive not to do so, as assumed in Farrell (1987).[20] Lemma (2) shows that an UG Nash equilibrium may be

[19] Aumann (1990) argues that cheap talk is credible only for a subset of Nash equilibria.

[20] Applying Farrell (1987), we may refine the Nash equilibrium concept in the subgame $\Gamma(m;\theta)$ by assuming that if m is a Nash equilibrium of $\Gamma(m;\theta)$, then m will be played, $s^*(m;\theta) = m$.

Farrell and Rabin (1996) discuss messages that are self-enforcing. There are three reasons to be suspicious about a message (or an agreement). First, players may have different understanding what the message means. Second, even if messages are understood correctly, players may have incentives to a Nash equilibrium of a subgame where players do not agree on that Nash equilibrium. Notice yet, that this is not true for any agreement. Nash equilibria may be removed from the game.

Consider the following game of chicken:

	L	R
T	$0,0$	$3,1$
B	$1,3$	$2,2$

(2.13)

The Nash equilibria of this game are (B, L) and (T, R). Let us suppose that player one's proneness to guilt is two, $\theta_1 = 2$ and the guilt cost function is as in (2.1) with $\gamma = 1$. Let us suppose that players agree on playing (B, R) which gives an agreed payoff of 2 for player one. Now, if player one breaches the agreement and chooses T instead, she gets $3 - 2 = 1$ which is smaller than 2 and, thus, (T, R) is not an equilibrium when players have agreed on (B, R) even if it is a Nash equilibrium of the underlying game.

Next, we show that an agreement where one of the players can make both players better off by deviating unilaterally from the agreement (even if the opponent respects the agreement) does not belong to the agreeable set.

Lemma 3. *For any m, if there is a player i such that there exists s_i such that $u_i(s_i, m_j) > u_i(m)$ and $u_j(s_i, m_j) \geq u_j(m)$ then $m \notin A(\Gamma, \theta)$ for any θ.*

Proof. See appendix. □

Lemma 3 follows immediately from the monotonicity (AC) and the strict cost (EF) conditions: when the harm inflicted on the other is non-positive, there is no guilt cost. Since a player can make herself better off, she will do so and the agreement is not incentive compatible.

Thus, for instance pattern (B, L) is never agreeable in the following game:

	L	R
T	$2,2$	$0,100$
B	$1,1$	$1,1$

(2.14)

since if player one breaches and chooses T, both players are better off. One could argue that player one does not breach (B, L) because she understands that then player two has an incentive to choose R which would make her worse off than in (B, L). But of course,

mislead their opponents. Self-signalling messages are sent, if and only if they are true. Self-commiting messages are such that if believed, the sender will have an incentive to do accordingly.

player one would then be inclined to choose B. Agreeing on (B, L) would thus leave a lot of room for rationalizing various kinds of play and truth is no more focal in the sense of Farrell (1987). Indeed, this type of plurality may question whether (B, L) is agreeable in the first place. But for our analysis, it is sufficient to notice that since player 1 can make both better off, the agreement is not incentive compatible.

In (2.14), players cannot agree on (T, R) either, since player 1 gets a smaller payoff than in the underlying game equilibrium, (B, R). On the other hand, if player 2's proneness to guilt is small, players cannot agree on (T, L) either due to player two's high gain from choosing R instead. But if we let player two's proneness to guilt become sufficiently high, (T, L) becomes agreeable. As the proneness to guilt becomes infinite, the guilt cost becomes infinite for deviations that cause a positive harm. Hence, whenever deviation causes harm, it will not be made. In general, if UG payoffs are finite, with sufficiently high proneness to guilt all individually rational profiles are agreeable for which a Pareto-improving deviation does not exist (the deviator must strictly benefit), and no other profile is.

Proposition 3. *Let the underlying game payoffs be finite. Let* $v_i(m) > 0$ *for* $i = 1, 2$. *Then* $m \in lim_{\theta_1 \to \infty, \theta_2 \to \infty} A(\Gamma, \theta)$ *iff for* $i = 1, 2$ *and for all* s_i, $u_i(m) \geq u_i(s_i, m_j)$ *or* $u_j(m_j, s_i) < u_i(m)$

Proof. See appendix. □

If the set of Nash equilibria is the smallest set that is agreeable (cheap-talk), proposition 3 describes the largest possible agreeable set, the agreeable set for types that are infinitely prone to guilt.

Lemma 3 has another implication, which is mentioned here without a proof. Namely, within the agreeable set, the interests of the players are opposed for any change in one of the agreed actions.

Corollary 1. *Let* $(m_i, m_j), (m_i', m_j) \in A(\Gamma, \theta)$ *then*

$$\begin{aligned} u_i(m_i, m_j) > u_i(m_i', m_j) &\Rightarrow u_j(m_i', m_j) > u_j(m_i, m_j) \\ u_j(m_i', m_j) > u_j(m_i, m_j) &\Rightarrow u_i(m_i, m_j) \geq u_i(m_i', m_j) \end{aligned} \tag{2.15}$$

2.6 Finite games with ordered strategy spaces

Let us now focus on finite games with *ordered* strategy spaces, $S_i = \{s_i^1, ..., s_i^n\}$. Inspired by the results in the public good game where actions are ordered in terms of contributed amounts, we seek to generalize two results gained there: First, that the non-positive marginal incentives to breach are necessary and sufficient for a strategy profile to be agreeable. Second, trading off the marginal harm of a deviation with its marginal benefit implies a conflict between efficiency and incentives to respect when there are decreasing returns to scale in the public good production whereas such a conflict is absent with constant returns to scale. We show that, when the guilt cost is convex in the harm, the first result generalizes to underlying games with concave payoff functions in each action. We generalise the second result as follows, there is a conflict between incentives and efficiency in symmetric games with strategic substitutes where payoffs are monotone in opponent's action. Such a conflict tends to be absent in symmetric games with strategic complements and monotone payoffs in opponent's action.

We now adopt some new concepts and notational simplifications. We denote the action s_i^n by its order label n so that for $k \in \mathbb{Z}$, $s_i^n + k \doteq s_i^{n+k}$. Also for $s \in S$ we let $s + k \doteq (s_i + k, s_j + k)$. We let the marginal benefit from breaching be defined as $\beta_i(m_i, m_j) \doteq b_i(m_i, m_j, m_i - 1)$, and the marginal harm as $\eta(m_i, m_j) \doteq h_i(m_i, m_j, m_i - 1)$. Thus $\beta_i(m + k) = \beta_i(m_i + k,\, m_j + k)$, $\eta_i(m + k) = \eta_i(m_i + k, m_j + k)$, and $u_i(m + k) = u_i(m_i + k, m_j + k)$ for $k \in \mathbb{Z}$.

We first set the scene by making *further assumptions on the underlying game.* In addition to supposing that the game is finite, we suppose that

{1} The payoff of player i is increasing in the action of player j

{2} The player's payoff is concave in her own action and in that of the opponent. That is, for all s

$$\delta_i(s) \doteq u_i(s_i + 1, s_j) - u_i(s_i, s_j) - [u_i(s_i, s_j) - u_i(s_i - 1, s_j)] \leq 0$$

and for all s

$$\sigma_i(s) \doteq u_j(s_j + 1, s_i) - u_j(s_j, s_i) - [u_j(s_j, s_i) - u_j(s_j - 1, s_i)] \leq 0$$

{3} The payoff functions are supermodular (so that actions are strategic complements). That is for all s

$$\phi_i(s) \doteq u_j(s_j, s_i) - u_j(s_j - 1, s_i) - [u_j(s_j, s_i - 1) - u_j(s_j - 1, s_i - 1)] \geq 0.$$

These properties are satisfied in the public good game, but in a degenerate manner: for all s, $\delta_i(s) = \sigma_i(s) = \phi_i(s) = 0$. The first assumption is without loss of generality. Indeed, if we reverse the ordering of strategies of both players, the payoff will be decreasing in opponent's action and, yet, concavity and supermodularity of the payoffs are unaltered. Thus, symmetric games with decreasing payoffs in opponent's action can be analyzed using the same artillery.

Also, we make *further assumptions on the guilt cost.* We assume that it is convex in the harm, h_j, and in the agreed payoff, v_i, and that it is supermodular in its two arguments

{4} g is convex in h_j

{5} g is convex in v_i and supermodular in its arguments.

Notice that the fact that the payoff is concave in the opponent's action implies that the harm h_j is a convex function of s_i, since the harm is just a rescaled negative of the underlying game payoff, $h_j(m, s_i) \doteq u_j(m) - u_j(m_j, s_i)$. Thus, by assumption {4}, the guilt cost is convex in s_i as a composite of two convex functions. Notice that assumption {4} rules out constant guilt cost, (2.1), for instance, since with that specification guilt is concave in harm.[21] On the other hand, the underlying game payoff u_i is concave in s_i. Consequently, the problem of choosing the optimal deviation given that the opponent respects is a simple convex optimisation problem. Hence, checking that neither prefers to breach the agreement marginally is necessary and sufficient for an agreement to be incentive compatible.

To simply formulate such a condition, we extend the concept of the marginal incentive to breach from the public good game example.

Definition 1. *(Marginal incentive to breach)*

If $u_i(m_i - 1, m_j) - u_i(m) \geq 0$

$$\mathbb{B}_i(m, \theta_i) \doteq \beta_i(m) - \theta_i g(v_i(m), \eta_j(m))$$

If $u_i(m_i - 1, m_j) - u_i(m) < 0$

$$\mathbb{B}_i(m, \theta_i) \doteq \beta_i(m)$$

When $u_i(m_i - 1, m_j) - u_i(m) < 0$, there is certainly no incentive to deviate downwards but there may be an incentive to deviate upwards. The fact that $\mathbb{B}_i(m, \theta_i)$ does not involve any guilt cost when $u_i(m_i - 1, m_j) - u_i(m) < 0$ is due to the fact that, by assumption {1}, an upward deviation does not make the opponent worse off and thus the player does not suffer from guilt. Consequently, assumption {1} on the underlying

[21] This will imply that the model of Ellingsen and Johanneson (2004) cannot account for the differences in efficiciency results of Isaac, McCue and Plott (1985) and Isaac and Walker (1988).

game payoffs together with lemma 3 gives us a necessary condition for an action profile to be agreeable. The agreement must belong to the following set[22]

$$M_F = \{m | u_i(m_i, m_j) \text{ is non-increasing in } m_i \text{ for } i = 1, 2\} \tag{2.16}$$

If it does not, then the player can gain in UG payoff by breaching and, by doing so, she does not harm the opponent.

Next, we establish a necessary and a sufficient condition for agreeability that generalises our finding in the public good game. Above we made the remark that, due to the convexity of the problem, there is no incentive to breach the agreement at the margin if and only if there is no incentive to breach at all. Second, incentive compatibility implies individual rationality when off the underlying game best reply curves by lemma 1. Thus, we have the following.

Proposition 4. *Let $m_i \neq BR_i(m_j)$ and $m_i \notin \{s_i^1, s_i^n\}$. Let {1}, {2} and {4} hold. Then an action profile is agreeable for i if and only if the marginal incentive to breach is non-positive.*

Proof. See appendix. □

As the terms of the agreement are altered, the marginal incentive to breach is affected through three channels: i) the direct effect through the marginal benefit from breaching; ii) an indirect effect through the marginal harm on the opponent; iii) an indirect effect through the agreed payoff. The latter two are indirect in that they affect the marginal incentive to breach through the marginal guilt cost.

In the public good game, we found that the marginal incentive to breach is monotone in each agreed action. We can generalize this property. Let us first consider how a change in one agreed action affects the trading off of benefit and harm from breaching.

Let us start with the effect of the agreed action of player i, m_i. It is necessary that an agreeable action profile lies in M_F. But within M_F, the player's payoff must be decreasing in her action. Thus, the effect of a player's agreed action on her marginal benefit from breaching is nothing but the negative of the second derivative, $-\delta$. Thereby, increasing a player's agreed action increases her marginal benefit from breaching. Similarly, the effect of m_i on η_j is simply the second derivative, σ, since the harm is itself a rescaled negative of u_j and breaching takes place downwards. Thus increasing m_i increases β_i and decreases η_j and both these effects have a positive impact on the marginal incentive to breach.

[22]Except for $m_i = s_i^n$ of course.

The effect of m_j on β_i and η_j rests on the strategic complementarity[23] of actions. Due to strategic complementarity, if the opponent increases her action, then the player has a stronger incentive to increase her own action. Since breaching takes place downwards, increasing the opponent's agreed action dampens the underlying game benefit from breaching. On the other hand, the higher the opponent's action, the more harm is inflicted on her by marginally decreasing the own action. Strictly supermodular games, where $\phi > 0$, constitute a set of games where such complementarities are present. The following lemma summarizes the effects of changing the terms of the agreement on the trade-off between the benefit and the harm of breaching.

Lemma 4. $\beta_i(m_i + 1, m_j) - \beta_i(m_i, m_j) = -\delta_i(s)$
$\eta_j(m_i + 1, m_j) - \eta_j(m_i, m_j) = \sigma_j(s)$
$\beta_i(m_i, m_j + 1) - \beta_i(m_i, m_j) = -\phi_i(s_i, s_j + 1)$
$\eta_j(m_i, m_j + 1) - \eta_j(m_i, m_j) = \phi_j(s_j + 1, s_i)$

Proof. See appendix. □

Now consider the third effect - the agreed payoff effect - of m_i and m_j on the marginal incentive to breach. This effect goes through the agreed payoff. Corollary 1 together with {1} imply that the agreed payoffs change monotonically in the agreeable set: increasing own agreed action decreases the agreed payoff and increasing the opponent's action increases payoff. Thus, when m_i is increased, also the agreed payoff effect has a positive impact on the marginal incentive to breach. On the other hand, there is a negative impact when m_j is increased. Thus the agreed payoff effects are aligned with the marginal harm and benefit effects. Thus, in supermodular games, increasing an opponent's action decreases the marginal incentive to breach.[24] Similarly, increasing the own agreed action increases the marginal incentive to breach.

Proposition 5. *Let {1}, {2}, {3} ,{4} and {5} hold. Then i's marginal incentive to breach is increasing in m_i and decreasing in m_j in the agreeable set.*

Proof. See appendix □

Notice that the agreed payoff reflects a player's preference ordering of agreements conditional on both respecting. Thus, keeping one of the actions fixed and changing the

[23]See Bulow et al. (1985).

[24]Also, supermodularity of g is needed so that the interplay between the agreed payoff and the harm effect in the guilt cost does not contradict other effects.

other in symmetric games with strategic complements, the preference over agreements and the incentive to respect them are aligned in the agreeable set. Yet, in symmetric games with strategic substitutes where $\phi < 0$, there is some conflict in the preference over agreements and the incentive to respect them.

Furthermore, this implies that, apart from the agreed payoff effect, in symmetric submodular games where {2} holds, efficiency and incentives to respect are in conflict. To see this, notice that no agreement where a player is required to choose an action smaller than her underlying game best reply is efficient. Symmetric profiles that Pareto-dominate the equilibrium are such that both agreed actions are higher than in equilibrium. But, increasing both actions by one step, increases the marginal benefit from breaching and decreases the marginal harm on the opponent. Thus, abstracting from the agreed payoff effect and only focusing on the trading-off of harm and benefit, the incentive to breach is increased.

Theorem 1. *Let Γ satisfy {1}, {2} and $\phi < 0$. Let s^* be its unique symmetric equilibrium with $\beta_i(s_i^*, s_j^*) = 0$. If $u_i(s^* + k) - u(s^*) > 0$ for $k \in \mathbb{Z}$ then $\beta_i(s^* + k) > 0$ and $\eta_i(s^* + k) < \eta_i(s^*)$.*

Proof. See appendix. □

While theorem 1 establishes a conflict between efficiency and incentives to respect, we know on the other hand that in the public good game efficiency and the incentives to breach may well be aligned: with constant returns to scale, an efficient action profile can be agreed upon if and only if an interior non-equilibrium action profile can be agreed upon. In theorem 2, we establish that this holds more generally in symmetric supermodular games.

The result is not as robust as the conflict result, however. We need some further, not very restrictive assumptions which are satisfied in many examples. Either we need to suppose that guilt is unaffected by the agreed payoff ($\gamma = 0$ in the public good example above) or we suppose that the UG payoff is convex in identical changes of both actions. For the latter case, when the UG payoff is convex in this way and the payoff is increasing in such changes, it is increasing in symmetric changes from the symmetric interior equilibrium up to the symmetric efficient profile where actions cannot be increased any further. Thus, a symmetric non-equilibrium action profile is agreeable if and only if an efficient action profile is.

This argument suffices for the case that best reply curves are not particularly steep. When they are steep, there may be multiple equilibria and we can use Milgrom and

Roberts (1990) result that in supermodular games when payoffs are increasing in opponent's action, the equilibria are ordered in terms of efficiency. Thus, the profile of maximal contributions is efficient and also agreeable as an underlying game equilibrium.

Theorem 2. *Let* {*1*},{*2*} *and* {*3*} *hold. Let* $\delta_i(s), \sigma_i(s)$ *and* $\phi_i(s)$ *be constant for* $i = 1, 2$. *Let* Γ *be symmetric and let* s^* *be its inefficient Nash equilibrium such that* $\beta_i(s^*) = 0$ *for* $i = 1, 2$. *Let* g *satisfy* {*4*}. *Suppose either (a) that* $\phi + \sigma \geq 0$ *and* $g(v', \eta) = g(v, \eta)$ *for all* η *and* $v', v > 0$ *or (b) that* $2\phi + \delta + \sigma \geq 0$ *and* g *satisfies* {*5*}.

Then, a symmetric efficient s *is agreeable iff a symmetric* $s \neq s^*$ *is agreeable*

Proof. See appendix. □

Proposition 5 shows that in games with strategic complements the marginal incentive to breach has intuitive monotonicity properties: as the action of the opponent is increased, a player's incentive to breach decreases whereas the opposite is true when the player's own action is increased. On the other hand, theorem 2 shows that in supermodular games, players are able to reach symmetric efficient agreements if anything else can be agreed upon that is symmetric and that is not an interior UG equilibrium.

Notice again, that assumption {1} was made without loss of generality. All we need is symmetry. If the payoff is decreasing in the opponent's action, we can restore assumption {1} by reversing the ordering of each strategy set. This will affect neither the concavity of the UG payoff in each action nor the super- or submodularity of the underlying game payoff.

In addition to the linear public good game studied above, examples of symmetric supermodular games include, for instance, team work designs and partnerships, or the Bertrand duopoly with imperfect substitutes. Yet, as we have seen the monotonicity properties and efficiency results do not generally hold in symmetric submodular games where the payoff is increasing in opponent's action.

Examples of symmetric games with strategic substitutes are the game of chicken (see section 2.5) and public good provision with a concave production technology. The chicken is a stylized version of a public good game with a provision threshold. Experimental evidence on the effect of communication in the public good games with a threshold is mixed (Ledyard, 1995). On the other hand, Isaac, McCue and Plott (1985) find rather weak effects of communication on efficiency in a public good game with decreasing returns technology whereas Isaac and Walker (1988) find a very strong positive effect of communication on efficiency with a constant returns to scale technology. Thus, our theory organizes rather well the differences in the effects of communication in public good games.

The next section studies a Cournot duopoly as an example of a symmetric game with strategic substitutes. Thus, the incentives to respect more collusive agreements tend to be weaker.

2.7 Cournot duopoly

Let us now study an example to see what happens when supermodularity of the underlying game is violated. We transform a linear Cournot duopoly with imperfect substitutes where profits read as $\pi_i(q) = (\frac{19}{2} - \frac{1}{2}q_i - q_j)q_i$ and the strategy set is $q_i \in \{0, ..., 10\}$ into an equivalent game[25] where the strategy sets are $s_i \in \{-10, ..., 0\}$ and the underlying game payoff of player i reads

$$u_i(s_i, s_j) = \max\{-(\frac{19}{2} + \frac{1}{2}s_i + s_j)s_i, 0\} \tag{2.17}$$

This transformation makes i's payoff increasing in opponent's action but preserves symmetry, concavity of payoffs {2}, and submodularity {3}. First, increasing player $i's$ action by one unit from s_i increases the payoff of the opponent:

$$u_i(s_i, s_j + 1) - u_i(s_i, s_j) = -s_i > 0 \tag{2.18}$$

Second, $\delta = -1$, $\sigma = 0$. And third, $\phi = -1$. Thus, all other assumptions hold but is {3} violated.

Condition (2.16) requires that $i's$ marginal payoff, $-10 - s_j - s_i$, is non-positive if s is agreeable for i. Thus, an agreeable action profile satisfies $m \in \{s | 10 + s_j + s_i \geq 0,\ i = 1, 2\}$. Notice, that player i's underlying game best reply to s_j is

$$BR_i(s_j) = -10 - s_j \tag{2.19}$$

[25]Notice that despite the negative strategies, this is indeed a game equivalent to a Cournot duopoly with imperfect substitutes. In an equivalent game, $\widetilde{s}_i \in [0, 10]$ and $\widetilde{u}_i(\widetilde{s}_i, \widetilde{s}_j) = \max\{(\frac{19}{2} - \frac{1}{2}\widetilde{s}_i - \widetilde{s}_j)\widetilde{s}_i, 0\}$ where $\widetilde{s}_i = -s_i$. The transformation is done in order to satisfy assumption {1}. Both the transformation and the orginal game are submodular. The payoffs are chosen to make the best reply mapping simple. Vives (1989) shows that it can be transformed into an equivalent game which is supermodular by setting $\widetilde{s}_2 = -s_2$. Such a transformation would yield $\phi = 1 > 0$ and $\delta + 2\phi + \sigma = 1$. However, then both payoffs are not increasing in the action of the opponent and we would lose the symmetry of the game.

Thus the unique underlying game equilibrium is $s_1 = -5 = s_2$ which gives payoff $u_i^* = u_i(5,5) = 10$ to both players. At this equilibrium, the benefit from breaching is exactly zero, $\beta(5,5) = 0$ as required in theorem 2.

Let's suppose that the guilt cost is as in (2.2). This guilt cost is supermodular in its arguments and convex in u_i as required in proposition 5. The proof of proposition 4 states that, off the best reply correspondences, a non-positive marginal incentive to breach is necessary and sufficient for incentive compatibility. Each player wants to deviate downwards. The marginal incentive to breach writes

$$10 + s_j + s_i + \theta_i[u_i(s) - 10]s_j \tag{2.20}$$

This is increasing in a player's own action but the effect of the opponent's action is ambiguous (as opposed to proposition 5 which assumes that the game is supermodular).

So as to the effect of the own action, since $\delta = -1$, $\sigma = 0$ increasing a player's agreed action increases the player's marginal benefit from breaching and leaves the marginal harm unaffected. Within the agreeable set, the agreed payoff effects are as before: thus, the agreed payoff decreases in the player's own action. To summarize, the marginal incentive to breach is indeed increasing in a player's own action.

Yet, if we consider the effect of the opponent's action, now since the game is submodular, $\phi = -1$, rather than supermodular, increasing the opponent's action decreases the marginal harm on the opponent and decreases a player's marginal benefit from breaching. Agreed payoff increases in a player's own action, as before. The agreed payoff effect and the other two effects now run counter to each other. Thus, the effect on the opponent's incentive to breach is ambiguous: the monotonicity of the marginal incentive to breach in agreed actions (proposition 5) is lost.

Now, let us move on and consider theorem 2 which studies whether efficient agreements can be made, if any. Figure 2.3 studies the positive quantity equivalent of the game.[26] There, we suppose that the proneness to guilt is $\theta_i = \frac{1}{7}$ for both players. The action profiles marked with a plus sign are agreeable for player 1 and the action profiles marked with a cross are agreeable for player 2. Thus, the action profiles marked with an asterisk belong to the agreeable set. There are two symmetric action profiles in this set: the equilibrium $(5,5)$ and $(4,4)$. Yet, the efficient symmetric action profile $(3,3)$ (marked

[26]The relevant figure for the negative quantity game studied analytically is the projection of figure 2.3 through the orgin to the negative quadrant.

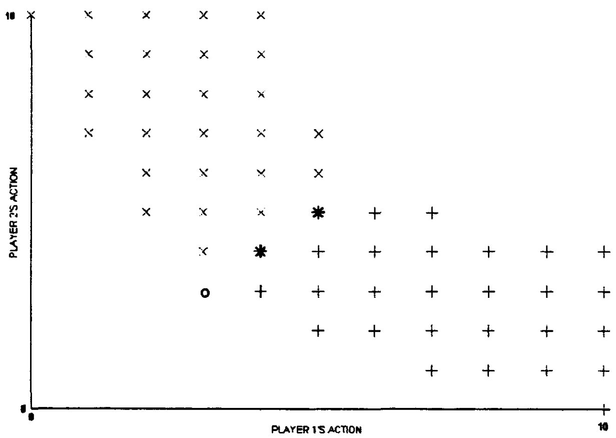


Figure 2.3: The agreeable set in the Cournot duopoly

with a circle) does not belong to the agreeable set.[27]

The underlying game equilibrium $(-5, -5)$ is agreeable by proposition 2. To see that $(-4, -4)$ is agreeable, we check that the marginal incentive to breach is negative, $10 - 4 - 4 - \frac{4}{7}[14 - 10] < 0$. For $s = (-3 - 3)$, the marginal incentive to breach reads $10 - 3 - 3 - \frac{3}{7}[15 - 10] = \frac{13}{7} > 0$ and thus, for $\theta_i = \frac{1}{7}\ i = 1, 2$, players can agree on $s = (-4, -4)$ but not on $s = (-3, -3)$.

This is because marginal symmetric changes of both actions (i) increase the marginal benefit by $-\delta - \phi = 2$ where both terms are strictly positive, (iib) decrease the marginal harm by $\sigma + \phi = -1$, and (iia) change the marginal effect of an increasing agreed payoff by $\delta + 2\phi + \delta = -4 < 0$. The negative marginal effect on the marginal incentive to breach (iia) is vanishing but the positive marginal effects are constant and thus getting relatively stronger as the agreed payoff is increased by symmetric changes of both actions. Thus, even if there is a non-equilibrium action where guilt offsets the underlying game incentive to breach, the incentives to respect a more efficient action profile are smaller. Consequently, we also lose any efficiency property akin to that in theorem 2.

[27]To see that $(3, 3)$ is efficient, maximize

$$\max_{\sigma}\{-(\frac{19}{2} + \frac{3}{2}\sigma)\sigma\} \tag{2.21}$$

This is indeed concave in σ. Looking at first order effects, a unit increase in both actions increases the expression in the brackets if and only if $\sigma \leq -\frac{11}{3}$. The agreed payoffs for the symmetric action profiles corresponding to the nearest two integers of $\sigma = -\frac{11}{3}$are $u(-3, -3) = 15$ and $u(-4, -4) = 14$. Thus $s = (-3, -3)$ is efficient.

2.8 Discussion

The main contribution of this chapter is to provide a game theoretic approach to pre-play agreements by negotiations, conventions or social norms when people may feel guilty about breaching an agreement. The model incorporates the most important stylized facts that research in social psychology and experimental economics has established about guilt.

We show that guilt, conventions and pre-play negotiation may have dramatic effects on strategic interaction. Trivially the set of agreeable outcomes may be larger than the set of underlying game Nash equilibria, since the guilt cost provides an extra incentive to comply to an agreed action profile. Agreements that are not equilibria in the underlying game are credible and respected in the enlarged game.

Moreover, the dramatic effects may prevail even if monetary stakes are high: in the prisoner's dilemma, increasing the benefit of defection sufficiently while keeping the harm on the opponent constant will restore the cheap talk prediction that an agreement on cooperation will be breached; yet, no matter how large the benefit of lying, an agreement on cooperation will be credible when the harm that the defection inflicts on the opponent is sufficiently high. Notice also that a player does not become more reluctant to agree on cooperation when she suffers more from defection. Quite the opposite: greater potential harm on herself increases the opponent's relative preference for cooperation since the opponent's promise to cooperate may become more credible.

The theory presented is in line with results from public good experiments without contribution thresholds where communication significantly increases contribution levels (Ledyard, 1995). Our theory tells us that for sufficiently high marginal per capita return, the benefit of breaching a cooperative agreement is offset by the harm on opponents. Thus, cooperative agreements become credible.[28]

This result extends to a large class of games with a public good structure: teams production, collusion in Bertrand and Cournot duopolies etc. Yet, there is an important

[28] Game theory generally abstracts from where utilities come from. That all agents contribute nothing to the public good implicitly assumes that players are money maximizers. Empirical evidence shows that people contribute positive amounts even when the game is played without communication. Distributional preference models map monetary payoff profiles to individual utilities. Thus, the underlying game (in utils payoffs) may differ from a public good game and it may have equilibria where positive amounts are contributed. Even if distributional preferences may be present in the game that is played without communication, guilt combined with pre-play negotiation complements distributional preference motivations in providing the extra incentive to contribute that is in line with what public good experiments have found.

distinction as to whether the theory predicts an efficient or an inefficient agreement: in symmetric games with strategic complements, a symmetric non-underlying game equilibrium is agreeable if and only if a symmetric efficient action profile is agreeable. On the other hand, in symmetric games with strategic substitutes, there tends to be a conflict between the incentives to respect an agreement and the efficiency of the agreement: trading off private marginal benefit and the marginal harm on the other makes it harder to agree on more efficient actions.

Experiments provide a strong support for this finding. Isaac, McCue and Plott (1985) adopt a decreasing-returns-to-scale production technology for the public good. This implies that actions are strategic substitutes. Despite the positive effect of communication on efficiency, they find that the average contribution levels fall far below the first-best (15% efficient). Isaac and Walker (1988) adopt a constant-returns-to-scale technology implying that actions are weak strategic complements. They find a strong positive effect of communication on efficiency. Average contribution levels are up to 99% efficient.

Furthermore, notice that some public good games with a contribution threshold have subsets of the strategy space where actions are strategic substitutes rather than strategic complements. For instance, the stylized version of a public good game with a threshold, the chicken game, has strategic substitutes. In the threshold public good experiments the effect of communication on contributions has not always turned out to be significant. Thus even experiments in threshold environments seem to lend support for our theory.

Second, as indicated above, our results can be extended to analyze the enforcement power of commonly known conventions and social norms.[29] This is because we abstract from the negotiation protocol and only analyze the interaction when an agreement is in place. Norms here require choosing a particular action in a given situation.[30] In this case, of course, no lies are told per se. Yet, research in social psychology suggests that guilt about transgressing such exogenous norms is stronger if more harm is inflicted on others[31] and, thus, property {A} among others remains valid. Thus the theory can be interpreted as a tractable model of let-down aversion (Charness and Dufwenberg, 2003) where pre-play negotiation or conventions establish commonly known, coinciding and

[29] I thank Joel Sobel for poiting this out.

[30] Social norms can be considered to be established by a community's moral discourse - - grand scale pre-play negotiations: When John Doe violates a social norm, the violation launches a vivid discourse by others in the community. This discussion may involve arguments for and against John Doe's action. If the social norm is well established arguments are mostly against and parties quickly converge into an agreement on how John should have behaved.

[31] For evidence see related literature in psychology in section 2.2.

justified mutual expectations about behaviour.

The theory presented in this chapter has a further interpretation in addition to face-to-face communication and conventions in one-shot games. Analogous results to those presented in this chapter would be obtained if we suppose that players have zero proneness to guilt and they informally agree on a stationary outcome in an infinitely (and infinitely often) repeated analog of the underlying game. The punishment paths are not negotiated, however, but they are exogenously determined (in a commonly known social contract, for instance). If the agreement is breached, it takes some time to detect breaching, and, when detected, players revert to mutual minmax strategies for a length of time that depends on the deviator's agreed payoff and the harm she inflicts on the other. As stated in the introduction, the origin of guilt, according to psychologists, resides in such close communal relationships (repeated games) where the prevailing social contract gets internalized.[32]

This chapter has not analyzed the effect of the negotiation protocol on the agreement. A cooperative solution concept or a bargaining protocol can be applied in predicting which agreement will be chosen from the set of agreeable profiles. When proneness to guilt is zero, the smallest agreeable set is the set of (non-cooperative) Nash equilibria of the UG. When the proneness to guilt is infinite any agreement that no player can unilaterally Pareto-improve upon is agreeable. Thus as we increase the players' proneness to guilt from zero to infinite, we move from an entirely non-cooperative prediction to a largely cooperative one. We study the effect of the negotiation protocol in the next chapter.

Another dimension for future research is the relaxation of the assumption of complete information of proneness to guilt types. The choice of an optimal agreement when information is private requires trading off the own agreed payoff with the probability that the opponent breaches the agreement.[33] On the other hand, a dynamic setup of incomplete information on proneness to guilt would allow for the players to build up reputations. First, it may be optimal for types with high proneness to guilt to build up a reputation for a lower proneness to guilt so that they are proposed higher shares of the surplus in the future. Second, types with a low proneness to guilt may be willing to build up a reputation for a higher proneness to guilt in order to be able to reach

[32]See appendix A for further details.

[33]Notice, yet, that if the information on proneness to guilt is private, signalling is not an issue: the maximisation problem conditional on respecting is the same independently of the type, and thus all types that intend to respect behave identically. Any type who intends to breach is thus detected. Thus her opponent knows that she will not suffer guilt.

agreements with a larger fraction of types. From a similar perspective one can study the evolution of proneness to guilt for a given stochastic process of games and matches.

Chapter 3

Beneficial Impatience

3.1 Introduction

Time is money. Other things equal people prefer getting what they want sooner than later. Also pre-play negotiations are generally a dynamic process where people prefer an early agreement to a late one.

In alternating offers bargaining (Stahl, 1972; Rubinstein, 1982) players negotiate about how to divide a surplus. One of the players first proposes a division to the other player who then either accepts or rejects the offer. If he accepts then players divide the surplus accordingly. If he rejects then he gets to propose a division to the player who proposed first. Players alternate in their offers until they reach an agreement.

Rubinstein (1982) shows that players' time preferences allow for a unique subgame perfect equilibrium (Selten, 1975) prediction on how the surplus will be split in this game. Time preference is defined as patience: an impatient player has a stronger preference for current to future than a patient player. Rubinstein shows that a player's equilibrium share increases in her patience. This is because the threat of a postponed agreement renders an impatient player more willing to accept the current proposal even with worse terms.

Yet, intuition suggests that in a more plausible model of negotiation a player's equilibrium payoff might not monotonically increase in her patience. An impatient negotiator tends to get frustrated more easily. This added emotional motivation triggered by greater impatience might provide the impatient player with an advantage: she might be able to threaten credibly when a more patient and thus a less frustrated player cannot.

More specifically impatience may operate together with reciprocity. A player is reciprocally motivated if she has an intrinsic incentive to sacrifice her own payoff in favour of the opponent's payoff, if the opponent is willing to do so. We show that if players are reciprocally motivated, the player may gain from being more impatient in the alternating offer pre-play negotiations: A postponed agreement is equivalent to the opponent not behaving reciprocally since the opponent is reducing the player's payoff by postponing the agreement. The opponent behaves even less reciprocally if the player is more impatient since an impatient player suffers more than a patient player if the agreement is postponed. Thus a reciprocally motivated impatient player can credibly threaten an opponent who contemplates postponing an agreement with actions that are not credible for a patient player.

This chapter elaborates this idea in the two-player pre-play negotiation framework put forward in chapter 1. Before an underlying game is played, players negotiate how to play the game. If an agreement on how to play the game is reached, players may feel guilty about breaching the agreement. Player's guilt has two reciprocal components: The first, guilt is increasing in the player's payoff conditional that both respect the agreement: "the nicer you are towards me the more guilty I feel about breaching the agreement". The second, guilt is increasing in the harm that the player inflicts on the other by breaching.

In this chapter we introduce time preferences and the specific dynamic pre-play negotiation protocol described above to the framework of chapter 1. We ask whether a player benefits from being more impatient in such pre-play negotiations. We show that, indeed, impatience may be beneficial.

Yet, we also show that a player may gain from being more patient. Moreover, she may gain even more than suggested by Rubinstein (1982). To see this, notice that a guilt-prone opponent suffers from guilt more if she harms the player more by breaching. But a given deviation by the opponent from a future agreement reduces more a patient player's payoff than an impatient player's payoff. Thus, since the opponent suffers more if she harms the player more, a guilt-prone opponent is less willing to betray a patient player if the agreement is postponed.

This chapter is organized as follows. Section 3.2 presents the model. Section 3.2.1 introduces the underlying game. Section 3.2.2 introduces the negotiation protocol. Section 3.2.3 introduces the entire game and the preferences therein. Section 3.2.4 parametrises time preferences in terms of discount factors. Section 3.3 provides an example showing that impatience may benefit a player. Section 3.4 has the general results. Section 3.5 discusses.

3.2 Model

3.2.1 The underlying game

In chapter 1 we allow for any normal form underlying game. Here we choose to consider a very specific *underlying game* in order to build a close parallel to the bargaining problem of dividing a pie of size one (Rubinstein, 1982). The two-player game is given by $\Gamma = \{S_i,\ u_i(s) : S \to R\}$ where each player's action set is $[0,1]$. A combination of actions is an *outcome* $s = (s_i, s_j) \in S = S_i \times S_j$. The *underlying game payoff* of player i is $u_i(s)$

$$u_1(s) = \begin{array}{ccc} 1 - s_1 & \text{if} & s_1 = s_2 \in (0,1) \\ 1 - s_1 - \eta & \text{if} & s_1 > s_2 \text{ or } s_2 = 1 \text{ and } s_1 > 0 \\ 1 - s_2 + \kappa & \text{if} & s_1 < s_2 < 1 \\ 0 & \text{if} & s_1 = 0 \text{ and } s_2 = 1 \end{array}$$

and that of player 2 is

$$u_2(s) = \begin{array}{ccc} s_2 & \text{if} & s_1 = s_2 \\ s_2 - \eta & \text{if} & s_1 < s_2 < 1 \\ s_1 + \kappa & \text{if} & s_1 > s_2 \text{ or } s_2 = 1 \text{ and } s_1 > 0 \\ 0 & \text{if} & s_1 = 0 \text{ and } s_2 = 1 \end{array}$$

where $\kappa > 0$ and $\eta > 1$.

There is a unique equilibrium, (0,1), where players' payoffs are zero. This is an equilibrium in weakly dominant strategies. Payoffs are strictly positive for both if and only if actions are identical. If actions are not identical, a negative payoff results for one of the players.

To fix ideas, suppose that players have full commitment power and they can negotiate how to play. Suppose further that without an agreement, players play the unique equilibrium in strictly dominant strategies. One of the players would thus always veto an agreement where actions are not identical since one of the players gets a negative payoff if she commits to such an agreement and this player can guarantee zero payoff by vetoing.

The set of strictly positive payoffs is the two dimensional unit simplex in figure 4. Thus the pre-play negotiation problem is analogous to the problem of sharing a pie of size one.

The underlying game does not provide incentives to stick to an agreement on identical actions, however. An agreement where each player gets a positive payoff can be breached by choosing a different action than the opponent. If the actions chosen are not identical

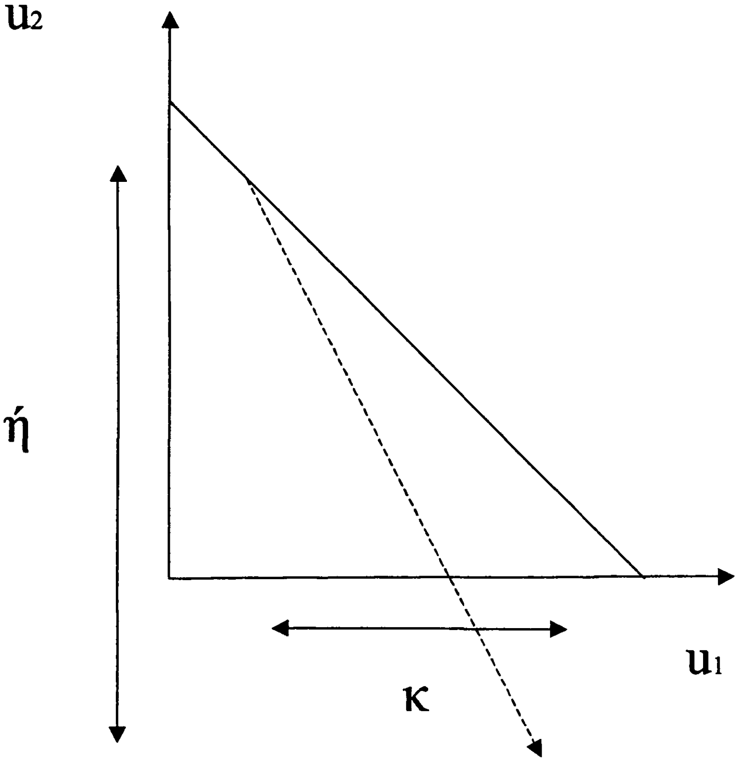


Figure 3.1: Negotiable payoffs and breaching by player one

(not on the diagonal of the strategy space), one of the player gains κ and the other loses $\eta > 1$.

Yet, here players may be able to commit to the agreement. The commitment power depends on the emotional bonds provided by guilt feelings about breaching the agreement. We will return to this issue in section 3.2.3.

In figure 2.1 $\kappa = \frac{3}{4}$ and $\eta = \frac{9}{8}$. We consider an action profile $(\frac{7}{8}, \frac{7}{8})$ which gives payoffs $(\frac{1}{8}, \frac{7}{8})$ if respected by both. This is the origin of the vector from the top right hand quadrant to the bottom right hand quadrant. The vector points to payoffs $(\frac{1}{8} + \kappa, \frac{7}{8} - \eta)$ that result if player 1 chooses her weakly dominant equilibrium strategy $s_1 = 0$ and player 2 sticks to playing $\frac{7}{8}$.

3.2.2 The Negotiation Protocol

The players can negotiate before playing the underlying game. The negotiation protocol that players use parallels the bargaining protocol of Rubinstein (1982). In particular, the protocol is an infinite horizon extensive form game where:

- Each player decides first whether to unilaterally enforce disagreement d

At period $t = 0$:

1. Player 2 proposes an agreement $m \in M \doteq S \cup \{d\}$ (M is the set of agreements, see chapter 1).

2. Player 1 accepts or rejects.

3. If the proposal is accepted, then m is the agreement.

4. If the agreement is rejected, then the roles are reversed, and the game continues from (1.) with period $t + 1$

- If no agreement is ever accepted, then the payoff for each player is 0.

Players talk about how to play the game: the terminal nodes of negotiation are associated with agreements $m \in S \cup \{d\}$ which are either action profiles of the underlying game or disagreement, d. Yet, since the negotiation is dynamic, the agreement is also characterized by the timing of the agreement. Thus the terminal nodes are mapped into $M \times \{0, 1, ...\}$. If $m \in S$, then m_1 and m_2 are the *agreed actions* of players one and two respectively.

The *agreed payoff* of an agreement reached at time t is the payoff that the player gets if both respect the agreement, $u_i^t(m)$ where $m \in S$. If player i deviates from the agreement, we get the *harm* on j by subtracting j's payoff at the deviation profile from the payoff at the agreed action profile, $h_j^t(m, s_i) \doteq u_j^t(m) - u_j^t(m_j, s_i)$. Similarly, i's *benefit from breaching* is $b_i^t(m, s_i) \doteq u_i^t(s_i, m_j) - u_i^t(m)$. Notice that, since the underlying game payoff depends on the timing of the agreement, so do these two expressions. Also notice that each player can guarantee zero payoff either by enforcing disagreement or by always proposing the equilibrium in UG and rejecting any offer.

3.2.3 The entire game

As in chapter 1 players are prone to guilt. If there is an agreement in place, they feel bad about not doing their part of the deal. Chapter 1 considers general static negotiation protocols with patient players: the underlying game payoff is unaffected by the timing of the agreement and a player's guilt, $g_i(u_i(m), h_j(m, s_i))$, depends on the harm and the agreed payoff.

Yet, if the negotiations are truly dynamic, it is plausible to suppose that the parties prefer an early agreement to a late agreement. As in Rubinstein's alternating offer bargaining players have time preferences. Thus the agreed payoff of a given agreement, the harm and the benefit decreases over time. Indirectly, guilt is a function of time, $g_i(u_i^t(m), h_j^t(m, s_i))$.

The entire game payoff is additively separable in guilt and the underlying game payoff and takes the simple form

$$U_i(m_i, m_j, s_i, s_j; t) = \begin{cases} u_i^t(s) - \theta_i(\max\{u_i^t(m), 0\})^{\varphi} \times (\max\{h_j^t(m, s_i), 0\})^{\gamma} & \text{if } s_j = m_j \\ u_i^t(s) & \text{otherwise} \end{cases} \tag{BD}$$

where $\gamma, \varphi \geq 0$ and where the first two entries of $U_i(.,.,.,.)$ are the agreed actions and the last two entries are the played actions of i and j respectively.

Let us write the payoffs of player i and player j respectively when player i deviates to s_i and player j respects the agreement, $s_j = m_j$, as

$$U_i(m_i, m_j, s_i, m_j; t) = u_i^t(m) + b_i^t(m, s_i) \tag{3.1}$$

$$-\theta_i(\max\{u_i^t(m), 0\})^{\varphi}(\max\{h_j^t(m, s_i), 0\})^{\gamma} \tag{3.2}$$

and

$$U_j(m_j, m_i, m_j, s_i; t) = u_j^t(m) - h_j^t(m, s_i). \tag{3.3}$$

These expressions give players' entire game payoffs in terms of the agreed payoff, the benefit from breaching and harm inflicted on the other when i breaches but not j. The *incentive to breach* an agreement m to s_i is the difference between the benefit from breaching and the guilt cost, $B_i^t(m, s_i; t, \theta_i) \equiv b_i^t(m, s_i) - \theta_i(\max\{u_i^t(m), 0\})^{\varphi}(\max\{h_j^t(m, s_i), 0\})^{\gamma}$.

An agreement m is called *incentive compatible* if neither benefits from a unilateral deviation from the agreement

$$\text{for all } s_i \in S_i \quad B_i^t(m, s_i; \theta_i) \leq 0 \tag{IC_i}$$

When this incentive compatibility condition holds for both players, the agreement m is a Nash equilibrium of the subgame where m is agreed upon, $\Gamma(m; \theta)$. On the other hand, an agreement m is called *individually rational* if neither prefers enforcing disagreement to agreeing on m when both respect the agreement

$$u_i^t(m) \geq 0 \tag{IR_i}$$

Here it is assumed that if negotiation ends up with disagreement, players will play the Nash equilibrium of the underlying game and payoff 0 results. Notice that only profiles on the diagonal, $s_1 = s_2$, are agreeable since, at any other profile, the payoff of one of the players is negative - lower than in the equilibrium.

Lemma 5. *An agreement that differs from UG equilibrium is individually rational for both iff* $m_1 = m_2$.

Since only individually rational agreements are ever agreed upon, without loss of generality, we can characterize the proposals and the agreement in terms of m_2.

In figure 2.1 $\kappa = \frac{3}{4}$ and $\eta = \frac{9}{8}$. At period zero players have agreed upon playing $(\frac{7}{8}, \frac{7}{8})$ which gives payoffs $(\frac{1}{8}, \frac{7}{8})$ if respected by both. If player 1 deviates to $s_1 = 0$, the underlying game payoffs are $(\frac{1}{8} + \kappa, \frac{7}{8} - \eta)$. Yet, if player 1 is prone to guilt, the her payoff equals $\frac{1}{8} + \kappa - \theta_1(\frac{1}{8})^{\varphi}(\frac{7}{8})^{\gamma}$ and thus $(\frac{7}{8}, \frac{7}{8})$ is incentive compatible for player 1 if and only if $\kappa \leq \theta_1(\frac{1}{8})^{\varphi}(\frac{7}{8})^{\gamma}$.

Constant benefit and harm allow us to abstract from the consideration of an optimal breaching strategy: if breach of agreement takes place, the player will deviate to her weakly dominant strategy, $s_1 = 0$ and $s_2 = 1$ for players 1 and 2 respectively. Moreover, constant benefit and harm allow us to derive simple comparative statics.

3.2.4 Discounting

Let us call $0 < \delta_i \leq 1$ the discount factor of player i. This parameter has two interpretations in the literature: either the time preference of player i; or, if $\delta_i = \delta_{-i}$, the probability that the negotiation continues at the following period. We focus on the former interpretation: if the outcome of the underlying game is s after t rounds of negotiation, player i's payoff is $\delta_i^t u_i(s)$.

If players discount the future, the agreed payoff is directly affected by the timing of the agreement. Suppose that the discount factor of player i is δ_i and the guilt cost takes the form of (BD). The agreed payoff of player i for an agreement m reached at time t is $u_i^t(m; \delta) = \delta^t u_i(m)$. Thus the longer it takes to reach an agreement $m \in S$ the lower the agreed payoff of each player. From here on we suppose that players discount the future using discount factors δ_i, $i = 1, 2$ and we suppress the notation $u_i^t(m) \doteq u_i^t(m; \delta) = \delta_i^t u_i(m)$. Thus the player's guilt cost for an agreement m reached at t and for a deviation s_i equals $\theta_i(\max\{\delta_i^t u_i(m), 0\})^{\varphi}(\max\{\delta_i^t h_{-i}(m, s_i), 0\})^{\gamma}$ and for an individually rational agreement $m_1 = m_2$, player 2's incentive to breach to $s_2 = 1$ reads

$$B_2^t(m, 1; \theta_2) \equiv \delta_2^t \kappa - \theta_2(\delta_2^t m_2)^{\varphi}(\delta_1^t \eta)^{\gamma} \tag{3.4}$$

3.3 The main result

Let us now show that a player may benefit from being marginally more impatient. Suppose that $\gamma = \varphi = 2$ and $\eta = \sqrt{5}$, $\kappa = 1 = \theta_i$, for $i = 1, 2$ and suppose further that $\sqrt[3]{\frac{4}{5}} < \delta_i = \delta_{-i} < \sqrt[6]{\frac{4}{5}}$. It is easy to check that the symmetric efficient agreement $m_2 = \frac{1}{2}$ is incentive compatible ((3.4) is non-positive for player 2) in period one but not in period two. Hence, since the players are identical and the game is practically symmetric, at period two, the agreeable set is nothing but the UG equilibrium $\{(0, 1)\}$. Yet, there are many other agreements in the agreeable set in period one.

In period one, player 1 proposes

$$m_2 = \sqrt{\frac{1}{5\delta_2\delta_1^2}} < \frac{1}{2} \tag{3.5}$$

to make player 2 indifferent between breaching and respecting. Since (1) players are identical, (2) incentive to breach is decreasing in the agreed payoff and (3) the situation is symmetric around $\frac{1}{2}$, this is incentive compatible for player 1.

A standard subgame perfection argument suggests that player 2's equilibrium proposal at period zero is $u_1(m) = \delta_1(1 - \sqrt{\frac{1}{5\delta_2\delta_1^2}})$ and her equilibrium payoff is

$$u_2(m) = 1 - \delta_1 + \sqrt{\frac{1}{5\delta_2}}. \tag{3.6}$$

Now notice that player 2's equilibrium payoff, (3.6), is decreasing in her patience. Indeed, in this example player 2's equilibrium payoff is not affected by the standard discounting effect since the negotiations end at period one. Impatience improves player 2's equilibrium payoff and the actions are adjusted in her favour unlike in standard bargaining (Rubinstein, 1982) where patience benefits a player.[1]

Yet, it is easy to see that guilt and patience may also jointly have a positive effect on a player's equilibrium payoff. Looking at player 1's proposal in period one, player 1's share of the period one surplus is increasing in her patience δ_1. This is not because of the standard direct effect of patience which is present in bargaining but because the harm that player 2 inflicts on player 1 by breaching in period one increases in δ_1.

To sum up, there are three effects of patience on player 1's equilibrium payoff: i) the standard direct effect identified in Rubinstein (1982), iia) the advantageous guilt effect through harm, iib) the disadvantageous guilt effect through agreed payoff. Effect (iib) was identified above. Effect (i) appears in the multiplier on the left in player 1's

[1]Bargaining is trivial since only the underlying game Nash equilibrium is argeeable.

equilibrium payoff, $\delta_1(1-\frac{1}{\sqrt{5\delta_2\delta_1^2}})$, and effect (iia) appears in the denominator on the right.

3.4 General case

Let us now relax the specific assumptions on the parameters of the model. Let us first consider player 2's incentive to breach, (3.4). Notice that if for player 2, (IC_i) holds as an equality, the expression is increasing in t if and only if $\delta_2^{1-\varphi} > \delta_1^{\gamma}$.

Thus if there is player i for whom

$$\delta_i^{1-\varphi} > \delta_j^{\gamma}, \tag{3.7}$$

holds, eventually that player will not be able to commit to any other agreement but the equilibrium of the underlying game. We say that there is *scope for negotiations* if the set of agreeable profiles is a non-singleton set and it includes some UG non-equilibrium profiles. If (3.7) holds for at least one of the players, there is scope for negotiation for a finite length of time only.

Any agreeable action profile which is not an UG-equilibrium satisfies $m_1 = m_2$ by lemma 5. Thus the agreed action of player 2, m_2, fully characterizes an agreement. From here on a proposal or an agreement m_2 means an agreement (m_1, m_2) where $m_1 = m_2$.

The least upper bound for there being scope for negotiation is given by the largest t such that there exists an m_2 such that both incentive compatibility conditions are satisfied

$$\delta_2^t\kappa - \theta_2(\delta_2^t m_2)^{\varphi}(\delta_1^t\eta)^{\gamma} \leq 0$$
$$\delta_1^t\kappa - \theta_1(\delta_1^t(1-m_2))^{\varphi}(\delta_2^t\eta)^{\gamma} \leq 0$$

or

$$T(\theta,\delta) \doteq \max_{t\in\mathcal{N}}\{t|(\frac{\delta_2^{1-\varphi}}{\delta_1^{\gamma}})^{\frac{t}{\varphi}}(\frac{\kappa}{\theta_2\eta^{\gamma}})^{\frac{1}{\varphi}} \leq m_2 \leq 1-(\frac{\delta_1^{1-\varphi}}{\delta_2^{\gamma}})^{\frac{t}{\varphi}}(\frac{\kappa}{\theta_1\eta^{\gamma}})^{\frac{1}{\varphi}}$$
$$0 \leq (\frac{\delta_2^{1-\varphi}}{\delta_1^{\gamma}})^{\frac{t}{\varphi}}(\frac{\kappa}{\theta_2\eta^{\gamma}})^{\frac{1}{\varphi}} and (\frac{\delta_1^{1-\varphi}}{\delta_2^{\gamma}})^{\frac{t}{\varphi}}(\frac{\kappa}{\theta_1\eta^{\gamma}})^{\frac{1}{\varphi}} \leq 1\} \tag{3.8}$$

The first condition in (3.8) requires that the incentive compatibility conditions are not mutually exclusive and the two latter conditions require that underlying game payoffs are feasible and individually rational. We call time $T(\theta,\delta)$ *the end of the scope of negotiation.* Any agreement on a non-equilibrium profile made at time later than $T(\theta,\delta)$ is not agreeable.

Lemma 6. *If there is i such that $\delta_i^{1-\varphi} > \delta_j^{\gamma}$, there is a finite scope for negotiation, $T(\theta,\delta) < \infty$.*

Proof. At the boundary of the agreeable set 3.4 equals zero. When time increases by one, the positive term is multiplied by δ_i whereas the negative term is multiplied by $\delta_i^{\varphi}\delta_j^{\gamma}$. Thus the incentive to breach is increasing iff $\delta_i^{1-\varphi} > \delta_j^{\gamma}$. □

Let $m(\theta, \delta, t)$ be the equilibrium proposal of actions $m_1 = m_2 = m(\theta, \delta, t)$ made if the negotiation reaches t. Thus $m(\theta, \delta, T(\theta, \delta))$ is the proposal made at the end of the scope of the negotiation. If it is player 1's turn to propose when the negotiation reaches the end of its scope, it is clear that player 1 proposes the efficient agreement which player 2 is indifferent between breaching and respecting, $m_2(\theta, \delta, T(\theta, \delta)) = (\frac{\delta_2^{1-\varphi}}{\delta_1^{\gamma}})^{\frac{T(\theta,\delta)}{\varphi}}(\frac{\kappa}{\theta_2\eta^{\gamma}})^{\frac{1}{\varphi}}$. The backward induction logic that we are familiar with from Rubinstein (1982) and Stahl (1972) suggests that, in the preceding round $T(\theta, \delta) - 1$, player 2 proposes the agreement which player 1 is indifferent between accepting and rejecting. Formally, player 2 would propose the agreement $1 - \delta_1(1 - m_2(\theta, \delta, T(\theta, \delta)))$. Yet, for this to hold, we must ensure that this agreement is incentive compatible. We simplify here and derive a condition to guarantee that this is the case. Specifically, we impose a sufficient condition for δm to be incentive compatible for both at $t - 1$ if m is incentive compatible at t.

Lemma 7. *If $\delta_i^{1-\varphi} > \delta_j^{\gamma}$ and m is agreeable at t, then δm is agreeable at $t - 1$.*

Proof. In the appendix. □

Suppose now that for both players, $\delta_i^{1-\varphi} > \delta_j^{\gamma}$. By lemma 6, this implies that there is a finite scope for negotiations. But, by lemma 7, this also implies that the backward induction logic initiated from the proposal at the end of the scope of the negotiations delivers the first round proposal and we need not to worry about the incentive compatibility of other agreements on the equilibrium path than the agreement proposed at the end of the scope of the negotiations.

Suppose further that it is player 2 who receives the proposal at the end of the scope of negotiations, $T(\theta, \delta)$. She is proposed the efficient agreement at which she is indifferent between breaching and respecting, $m_2 = (\frac{\delta_2^{\frac{1}{\varphi}-1}}{\delta_1^{\frac{\gamma}{\varphi}}})^t(\frac{\kappa}{\theta_2\eta^{\gamma}})^{\frac{1}{\varphi}}$. If $\varphi > 1$, this expression is decreasing in δ_2. Thus there is a beneficial effect of impatience for player 2 on the last round proposal.

Yet, this is not sufficient to show that impatience is good overall for two reasons: firstly, more impatient players discount future payoffs more heavily and the standard effect of patience may dominate; secondly, decreasing δ_2 may alter the end of the scope of negotiation to $T(\theta, \delta) - 1$ (see equation 3.8). The latter happens if and only if

$$(\frac{\delta_2^{1-\varphi}}{\delta_1^{\gamma}})^{\frac{T}{\varphi}}(\frac{\kappa}{\theta_2\eta^{\gamma}})^{\frac{1}{\varphi}} = 1 - (\frac{\delta_1^{1-\varphi}}{\delta_2^{\gamma}})^{\frac{T}{\varphi}}(\frac{\kappa}{\theta_1\eta^{\gamma}})^{\frac{1}{\varphi}}. \tag{3.9}$$

We will rule out the latter by assuming that (3.8) but not (3.9) holds. Moreover, we formally disentangle the standard effect of paticence (former) from the effect of patience on the extent of guilt. In order to do so let us write down the equilibrium payoffs of the two players. We assume now again that φ and γ can be arbitrary as long as condition 3.7 is satisfied and thus the equilibrium agreement is derived from $m(\theta, \delta, T(\theta, \delta))$ using the backward induction principle that keeps the receiver at t indifferent between the proposal at t and the equilibrium path proposal at $t + 1$.

Proposition 6. *Let $\delta_1 = \delta_2$, $\gamma \geq 1$ and $n \in \mathcal{N}$.*

- *If $T(\theta, \delta) \in 2n$*

$$U_2^*(m, t, \delta) = \frac{(1-\delta_1)(1-(\delta_1\delta_2)^{T(\theta,\delta)/2})}{1-\delta_1\delta_2} + (\delta_1\delta_2)^{T(\theta,\delta)/2} m(\theta, \delta, T(\theta, \delta)) \tag{3.10}$$

- *If $T(\theta, \delta) \in 2n$*

$$U_2^*(m, t, \delta) = \frac{(1-\delta_1)(1-(\delta_1\delta_2)^{(T(\theta,\delta)+1)/2})}{1-\delta_1\delta_2} + \delta_1(\delta_1\delta_2)^{(T(\theta,\delta)-1)/2} m(\theta, \delta, T(\theta, \delta)) \tag{3.11}$$

- $U_1^* = 1 - U_2^*$

Proof. In the appendix. (Special case of Binmore (1987, p.93)) □

We define *the guilt effect* of patience as $\frac{\partial U_i^*(m,t,\delta)}{\partial m} \frac{\partial m(\theta,\delta,T)}{\partial \delta_i}$ and *the direct effect* of patience as $\frac{\partial U_i^*(m,t,\delta)}{\partial \delta_i}$. As identified by Rubinstein (1982), the direct effect of player's patience on her equilibrium payoff is positive. The effect on the opponent's equilibrium payoff is negative.

Lemma 8. $\frac{\partial U_i^*(m,t,\delta)}{\partial \delta_i} > 0$, $\frac{\partial U_i^*(m,t,\delta)}{\partial \delta_j} < 0$

Proof. In the appendix. □

Being more patient benefits the player, since a patient player suffers less from postponing the negotiation than an impatient player and thus a threat of postponing the agreement is more credible.

Yet, in section 3.3, we show that guilt may have the opposite effect on player 1's payoff. The following proposition identifies when in general this opposite effect is present.

Proposition 7. *Let condition 3.7 hold. Let (3.8) but not (3.9) hold for $i = 1, 2$.*

Let i make a proposal to j at $T(\theta, \delta)$

- i's equilibrium payoff increases in δ_i
- $j's$ equilibrium payoff decreases in δ_i
- $i's$ guilt effect is positive on i and negative on j.
  Formally, $\frac{\partial U_i^*}{\partial m} \frac{\partial m(\theta,\delta,T(\theta,\delta))}{\partial \delta_i} > 0$, $\frac{\partial U_j^*}{\partial m} \frac{\partial m(\theta,\delta,T(\theta,\delta))}{\partial \delta_i} < 0$
- $j's$ guilt effect is negative on j and positive on i iff $\varphi > 1$
  Formally, $\frac{\partial U_j^*}{\partial m} \frac{\partial m(\theta,\delta,T(\theta,\delta))}{\partial \delta_j} < 0$ and $\frac{\partial U_i^*}{\partial m} \frac{\partial m(\theta,\delta,T(\theta,\delta))}{\partial \delta_j} > 0$ iff $\varphi > 1$

Proof. WLOG, let $T(\theta, \delta) \in 2n - 1$, where $n \in \mathcal{N}$ so that player 1 is the proposer. Thus, $m(\theta, \delta, T(\theta, \delta)) = (\frac{\delta_2^{1-\varphi}}{\delta_1^{\gamma}})^{\frac{T(\theta,\delta)}{\varphi}} (\frac{\kappa}{\theta_2 \eta^{\gamma}})^{\frac{1}{\varphi}}$. This expression is decreasing in δ_1 and from proposition 6 the effect $\frac{\partial U_1^*}{\partial m}$ is negative. Thus, the third and the fourth claim.

By lemma 8, the direct effect of player 1's patience is positive. Thus both the guilt and the direct effects are positive. The first and the second claim follow.

On the other hand, if $\varphi > 1$, then $m(\theta, \delta, T(\theta, \delta))$ decreases in δ_2. Thus the guilt effect of player 2's patience on player 2's equilibrium payoff is negative and on player 1's equilibrium payoff it is positive. □

The third and the fourth result show that the beneficial effect of impatience is present if and only if *guilt is convex* in the agreed payoff and the player *receives a proposal* at the end of the scope of negotiations. Yet, three remarks should be made. The first, the standard effect of patience may dominate. The second, the *proposer* at $T(\theta, \delta)$ always benefits from being more patient. Finally, if guilt is concave in the agreed payoff, *also the player who receives* the proposal at $T(\theta, \delta)$ benefits from being more patient.

Why does convexity, on the one hand, and the position of being the receiver of the proposal, on the other hand, play such an important role? A receiver is in a weak bargaining position at the end of the scope of the negotiations. He cannot affect the agreement directly but only threaten the proposer by implicit claims of rejecting an offer or breaching an agreement when the underlying game will be played. Such threats are credible only if there is sufficient motivation to carry them out. An impatient player has suffered more by the fact that negotiations have been postponed to their ultimate relevant endpoint. Thus an impatient player with δ_j considers the opponent's strategy leading to a given agreement $m \in S$ and a payoff $\delta_j^{T(\theta,\delta)} u_j(m)$ not as nice as a more patient player $\delta_j' > \delta_j$. Thus, being motivated by reciprocity, an impatient player's threat of breaching the agreement is more credible.

As far as the relevance of convexity of guilt in the agreed payoff is concerned, notice that player's patience does not only affect player's agreed payoff but also her benefit from breaching $\delta_i^{T(\theta,\delta)}\kappa$. Due to discounting, the benefit from breaching is smaller if the agreement is postponed. Moreover, the rate of decay of the breaching benefit crucially depends on player's patience. Whether the negative effect of a lower breaching benefit on the incentive to breach dominates the positive effect of a lower agreed payoff depends on whether guilt is convex in the agreed payoff. When convexity holds, the agreed payoff effect dominates and the incentive to breach an agreement increases as the timing of the agreement is postponed.

The intuition why the proposer at the end of the scope of the negotiation always benefits from patience is the follwing. In addition to the direct effect of patience, there is an indirect guilt effect: the receiver of the proposal at $T(\theta, \delta)$ will inflict more harm on a patient proposer at $T(\theta, \delta)$ than on an impatient proposer. Thus the receiver feels more guilty about breaching a more patient proposer at $T(\theta, \delta)$ and her threath of breaching is less credible. Thereby, there is an added positive effect of patience on the proposer's equilibrium payoff when players pre-play negotiate rather than merely bargain on how to share a pie.

3.5 Discussion

We have shown that impatience may be beneficial in alternating offer pre-play negotiations when players are prone to guilt. This should be contrasted with bargaining where being more patient always benefits the player (Rubinstein, 1982). That impatience may improve player's equilibrium payoff is due the reciprocity of guilt: there is less guilt about breaching a worse agreement. An impatient player suffers more if the negotiation is prolonged. Thereby, an impatient player feels less guilty about breaching a postponed agreement than a patient player. Thus an impatient player's threat about breaching a late agreement is more credible than that of a patient player. Such a threat forces an impatient player's opponent to agree on worse terms of an agreement and thus impatience benefits the player. This effect may dominate the standard effect of patience identified by Rubinstein (1982) in bargaining with full commitment.

Yet, we also identify emotional triggers that may strengthen the prediction that being more patient increases a player's equilibrium payoff. A patient player is harmed more than an impatient player if her opponent breaches a late agreement. Since the opponent feels more guilty about breaching if she inflicts more harm on the player, the threat of an opponent of a patient player is less credible than that of an opponent of an impatient player.

Notice that some of the emotional drivers identified here are also present in reciprocally motivated alternating offer bargaining. Also there a more impatient player suffers more if negotiations are prolonged. Thus being motivated by reciprocity, a more impatient player can make credible threats in follow-up subgames which are not credible for more patient players. Thereby, impatience may be beneficial for a player even in bargaining if bargainers are reciprocally motivated.

Furthermore, an agreement cannot be breached in bargaining. Thereby, whereas in the present framework the benefit and the harm of breaching a late agreement increase in patience thereby improving a patient players position, a patient bargainer does retain these advantages of being more patient in bargaining with full commitment power. Thus the effect beneficial impatience may be even more pronounced in reciprocally motivated bargaining than in pre-play negotiations.

Chapter 4

Analogy-Based Expectations Equilibrium and Steady States of Learning

4.1 Introduction

In this chapter we consider learning as a justification of the Nash equilibrium. We review existing insights and conclude with new ones into why the Nash equilibrium may be too restrictive a prediction.

The Nash and the Bayesian-Nash equilibrium suppose that players have correct conjectures about each others' strategy choices; and if exogenous randomness is involved, they have correct probability estimates about it, too. In an equilibrium, given what others do, no player has an incentive to deviate. A researcher does not need to worry that conjectures about others' and nature's choices might be incorrect and yet plausible. Technically speaking, the Nash equilibrium derives its strength from being the fixed point of the best reply correspondence.

But we lose a lot in terms of realism in assuming correct conjectures. One reason why conjectures may end up being correct is learning: if each player plays the game repeatedly, she may acquire experience about the uncertainty she faces and about how others play. Formal models of learning allow for this and relax the correct conjecture hypothesis and suppose that players update their conjectures about others over time. The evolution of conjectures and play becomes a stochastic process. In a steady state of the process best replies to conjectures generate behaviour which does not contradict the conjectures. This is the core of the *conjectural equilibrium* (Battigalli 1987). It is easy to see that Nash equilibria with correct conjectures satisfy this steady state condition. The question then is whether there are non-Nash steady states.

The answer to this question is affirmative: Depending on specific assumptions on the learning process various other strategy profiles than Nash equilibria may be steady states of such processes. Firstly, expectations about off-equilibrium-path behaviour may be incorrect (Fudenberg and Levine, 1993). Secondly, players may not observe the equilibrium path but rather a less informative signal (Battigalli, 1987; Dekel Fudenberg and Levine, 2004). Thirdly, even if players observe decisions at each node along the path, cognitive limitations may necessitate simplifications of opponents' behaviour by bundling several decision nodes of the opponents together and keeping track of opponents' average behaviour in each such class (Jehiel, 2005). Thus depending on the assumptions on the learning environment - what players know in the beginning of the process, what players observe, how many other people are involved in the process, how they handle information, etc. - various equilibrium concepts emerge.

In this chapter we review such equilibrium concepts and some of the literature on learning. In section 4.2, we introduce the underlying game, the context of learning. In section 4.3, we study learning when the set of players remains fixed over the entire learning process. We start from two benchmark approaches: the first, complete information and correct conjectures; the second, incomplete information and Bayesian learning with a common prior. In these approaches strategies constitute a Nash and a Bayesian-Nash equilibrium of the game, respectively. Section 4.3.2 relaxes the assumption of mutually consistent initial conjectures and studies learning with a fixed set of players who start with possibly mutually inconsistent initial conjectures about the behaviour of others and update their conjectures as the game continues (Kalai and Lehrer (1993), Jordan (1995), Nahcbar (1997) Foster and Young (2001)).

Section 4.4 is the core of our contribution. It studies learning in large populations with random matching of players before each stage game is played - subsection 4.4.1 focuses on static games of incomplete information and section 4.4.2 on extensive form games. We suppose throughout that opponents' actions are observed and kept track of in the learning process. In addition, opponents' types profiles are also observed but possibly not as precisely as opponents' themselves observe these when choosing their strategies. Our first contribution is to illustrate how we can reach various equilibrium concepts in the anonymous learning literature (Bayesian-Nash equilibrium, Harsanyi (1967-68); self-confirming equilibrium, Fudenberg and Levine (1993); self-confirming equilibrium, Dekel et al (2004); cursed equilibrium, Eyster and Rabin (2005)) by changing the complexity of beliefs about the strategies of others in the analogy-based expectation equilibrium. This complexity is driven by the precision with which players observe the opponents' type profile after each round of play. (ABEE; Jehiel (2005), Jehiel and Koessler (2006)). Thus the ABEE provides a way to analyse the complexity of various other equilibrium concepts.

Our second contribution is to propose a refinement of the payoff-confirming ABEE (PCABEE), where each player observes and keeps track of her private payoffs. If we wish to model learning where each player plays repeatedly, it is plausible to assume that each player observes and keeps track of her own payoffs. This is because the payoff is what players ultimately care for: if they do not, why should they strive to best-reply in the first place? Successes and failures, which are measured in payoffs, are what players vividly experience and tend to remember. This provides further information about opponents' equilibrium strategies. This is what motivates our refinement of the ABEE.

We show that incorrect conjectures may survive and non-Bayesian-Nash PCABEE may exist. We provide a sufficient condition for an ABEE to be PCABEE. This condition is also necessary in an interesting class of games.

In section 4.4.3 we briefly discuss experimentation, learning about responses of others to changes of one's own behaviour. Section 4.5 discusses.

4.2 The Game

There are N players $i = 1, ..., N$. The outcomes of the game are its *terminal histories* $z \in Z$. Players' *preferences* over terminal nodes are captured by von Neumann-Morgernstern payoff functions, $u_i(z) : Z \to \mathcal{R}$, $i = 1, ..., N$.

The set of nodes or histories is denoted by H. The histories are sequences which satisfy the following properties:

- $\varnothing \in H$
- If $(a^k)_{k=1}^K \in H$ (where K may be infinite) then $(a^k)_{k=1}^L \in H$ for all $L < K$
- If an infinite sequence $(a^k)_{k=1}^\infty$ satisfies $(a^k)_{k=1}^L \in H$ for every $L \geq 0$, then $(a^k)_{k=1}^\infty \in H$

Let h be a history of length k. Then we denote (h, a) a history of length $k+1$ consisting of h followed by a. We call such a history an *immediate successor* of h. On the other hand, h is an *immediate precedessor* of (h, a). Any history for which a sequence of immediate successors from h to that history exists is a successor of h. And h is a precedessor of such a history.

If $(h, a^k)_{k=1}^K$ is the set of immediate successors of h, we call $A(h) = \{a_1, ..., a_K\}$ the *action set* at h. Here K may be infinite. If the set of histories is finite then the game is *finite* otherwise the game is *infinite*. Even an infinite game may have a *finite horizon*, namely the maximum length of any sequence is finite.

The player function maps nonterminal histories $H \backslash Z$ into the set of players, $\varphi(h)$: $H\backslash Z \to \{0, 1, ..., N\}$, where player 0 is the nature describing random events independent of players choices. The set of histories where player i moves is denoted H_i and the edges with root $h_i \in H_i$ are the actions of player i at h_i. Thus the set of histories H is partitioned into sets $H_0, ..., H_N, Z$.

Furthermore, the set of histories of each player i is partitioned into information sets I_i where $\iota_i(h_i)$ is the *information set* where history h_i belongs to. The set $\iota(h_i)$ describes what i knows when h_i is reached. Alternatively the set describes what she does not know: she does not know which of the histories in $\iota(h_i)$ is reached.

A *pure strategy* of player i maps the set of histories of player i into actions at those histories, $s(.) : H_i \to A(h_i)_{h_i \in H_i}$. Denote by Σ_i the set of such strategies. A *mixed strategy* $\pi_i : H_i \to \Delta(\Sigma_i)$ is a probability distribution on the set of pure strategies. We suppose that the game has perfect recall and thus by Kuhn's theorem (Kuhn, 1953) the mixed strategies are equivalent to *behavior strategies*, mappings from histories of i to probability distributions over the action sets at those histories, $\sigma_i(.) = H_i \to \times_{h_i \in H_i} \Delta(A_i(h_i))$. Strategy of player i is a I_i-measurable function.

The *probability of terminal history* z given strategy profile σ is denoted by $P^\sigma(z)$ where for $z = a^1, ..., a^K$, we have $P^\sigma(z) = \Pi_{k=0}^K \sigma_{\varphi(a^1,...,a^K)}(a^{k+1}|a^1, ..., a^k)$. In a similar vein we define $P^{s_i,\sigma_{-i}}(z)$. From the probability theory perspective, the set of elementary states is the set of terminal histories, Z, and each history h is an event containing all terminal histories which have h as a precedessor. Each strategy profile σ induces a probability measure on the set of terminal histories.

Savage (1954) shows that a rational decision maker's decisions under uncertainty allow for *an expected utility representation*, $\int u(z)dP(z)$ where $P(z)$ captures player's subjective probability distribution over terminal nodes and $u(z)$ is a von Neumann-Morgernstern payoff function. When a rational decision maker makes decisions in an environment involving uncertainty, she puts positive probability only on actions that maximize her expected payoffs given expectations. In an environment of strategic interaction, the uncertainty is about natures move and opponents' strategies. Let us denote by $\widehat{\sigma}_{-i} = (\widehat{\sigma}_0^i, ..., \widehat{\sigma}_{i-1}^i, \widehat{\sigma}_{i+1}^i, ..., \widehat{\sigma}_N^i)$ the conjecture of player i about this uncertainty. Sometimes it is relevant to think of a conjecture as a probability distribution over opponents' behaviour strategies. We denote this $\widehat{\mu}^i$. If $\widehat{\mu}^i$ is generate, the choice of player i is characterized by $a_i \in$supp$[\sigma_i(h_i)$=supp$[\sigma_i(\iota(h_i))]$ iff

$$a_i \in \arg\max \sum_z \sum_{h_i \in \iota(h_i)} P^{\sigma_i, \widehat{\sigma}_{-i}}(h_i) P^{\widehat{\sigma}}(z|h_i, a_i) u_i(z)$$

Alternatively if $\widehat{\mu}^i$ is generate, we can think of the conjecture as a probability distribution over opponents' behaviour strategies, and thus $a_i \in$supp$[\sigma_i(h_i)$=supp$[\sigma_i(\iota(h_i))]$ iff

$$a_i \in \arg\max \int \sum_z \sum_{h_i \in \iota(h_i)} P^{\sigma_i, \sigma_{-i}}(h_i) P^{\sigma}(z|h_i, a_i) u_i(z) \widehat{\mu}^i(\sigma_{-i}) d\sigma_{-i}.$$

If the game is one of perfect information, then the information sets are singletons and $P^{\sigma_i, \sigma_{-i}}(h_i)$ in the expressions above is degenerate. In this case, all probability mass is on one history. If there are information sets which are not singletons, then the player only knows $\iota_i(h)$ when she moves at h and she has a subjective probability distribution over the histories h' in $\iota_i(h)$, each h' has probability $P^{\sigma_i, \widehat{\sigma}_{-i}}(h')$.

4.2.1 Static games of incomplete information

A static game of incomplete information with N players is a special case of an extensive form game described above. Exogenous uncertainty is modelled by letting nature draw randomly a payoff type profile, a type for each of the players. In terms of the extensive form game, the empty history is followed by a type profile. Player's information sets then determine how much each player knows about the realization. Then players choose their actions. An action of player i is $a_i \in A_i$ and the action sets coincide in all states. The actions of players other than i are denoted by $a_{-i} \in \times_{j \neq i} A_j$. An action profile is $a \in A = \times_{i=1}^N A_i$. The terminal histories are type-action profile combinations, (a, θ).

The type vector is denoted by $\theta \in \Theta = \times_{i=0}^N \Theta_i$ where $\theta_i \in \Theta_i$ is the type of player i. The vector of types of players other than i is $\theta_{-i} \in \Theta_{-i} = \times_{j \neq i} \Theta_j$. Thus, if nature draws θ', player i knows $\theta \in \iota_i(\theta') = \{\theta | \theta_i = \theta_i'\}$ when she chooses her action. The payoff depends on the actions and the type profile: $u_i(a; \theta) : A \times \Theta \to R$ for $i = 1, ..., N$.

A strategy of player i is a function of her type, $\sigma_i(\theta_i) : \Theta_i \to \Delta(A_i)$ and the probability that i chooses action $a_k \in A_i$ is denoted by $\sigma_i(a_k|\theta_i)$. The strategies of players other than i are denoted by $\sigma_{-i}(\theta_{-i}) : \Theta_{-i} \to \Delta(A_{-i})$ and a strategy profile is $\sigma(\theta) : \Theta \to \Delta(A)$. The degenerate conjecture of i about the strategy of the opponents is denoted by $\widehat{\sigma}_{-i}(\theta_{-i}) : \Theta_{-i} \to \Delta(A_{-i})$ and i's generate conjecture, $\widehat{\mu}^i$, is a probability distribution over the (mixed) strategies of players other than i.

Notice that the way a static game of incomplete information is put forward above is not exactly a special case of the extensive form game above: in a static game all players move simultaneously when the type profile is drawn whereas in an extensive form game model only one player moves at each history. Yet, we could easily modify the static game model by letting players move sequentially once the type profile is drawn but keeping the action choice private information to the player.

4.3 Non-anonymous learning

In this section we consider *non-anonymous* learning: the set of players is fixed over the entire learning process and it coincides with the actual set of players $\{1, ..., N\}$. We start by introducing the Nash equilibrium and the Bayesian-Nash equilibrium in games of complete information and incomplete information respectively in section 4.3.1. These approaches are characterized by strong consistency requirements: players have correct and common conjectures about how others and nature choose. Section 4.3.2 considers Bayesian learning without a common prior distribution. Players probability estimates about others' behaviour may originally differ and be inconsistent. Players update their estimates as the game evolves. Even if the common prior assumption is dropped, these players are still rather sophisticated in that they use Bayes's rule to update their estimates and they are forward looking in that they maximize a discounted sum of payoffs.

4.3.1 Consistent conjectures and Nash equilibrium

Let us begin with the standard complete and incomplete information approaches. Games of complete information make an assumption that not only all payoffs but also the structure of the game is common knowledge: the way each individual perceives the interaction that players are facing is identical for every player; furthermore, every player knows that the way how the interaction is perceived is identical, every player knows that every player knows and so and so forth.

Games of incomplete information allow for asymmetries in these perceptions and in the relevant information. If there is private information or various perceptions to the game, players other than the privately informed must perceive the private information uncertain. Referring to Savage (1954), this can be modelled as a player assigning a probability distribution over all privately known events that the player perceives possible.

Notice that Savage's approach does not imply that two players should have identical and correct perceptions about the likelihood of various events that are private information to a third. Yet, in most of the games of incomplete information, this is assumed. This consistency requirement implies that the game can be modelled as if nature first draws players' private information from a prior distribution which is common knowledge.

Moreover in a Bayesian-Nash equilibrium, the conjectures about opponents' strategies must be correct and thus $\widehat{\sigma}_{-i} = \sigma_{-i}$. Since in games of perfect recall mixed strategies are equivalent to behaviour strategies (Kuhn, 1953), we can define the Nash equilibrium as follows:

Definition 2. *σ is a Bayesian-Nash equilibrium if for every i and for every σ_i'*

$$P^{\sigma_i,\sigma_{-i}}(z)u(z) \geq P^{\sigma_i',\sigma_{-i}}(z)u(z)$$

where for $z = a^1, ..., a^K$, we have $P^{\sigma}(z) = \Pi_{k=0}^{K}\sigma_{\varphi(a^1,...,a^K)}(a^{k+1}|a^1, ..., a^k)$.

As already stated above, there are two consistency ideas implicit in the Nash equilibrium.

1. Bayesian updating along the equilibrium path
2. Common and correct prior beliefs about nature's move and common and correct conjectures about opponents' strategies.

Three questions emerge: 1) Why should players have correct conjectures about the move of the nature? 2) Why should players have correct conjectures about each other's strategies? 3) Is Bayesian updating a plausible model of learning? The literature on learning in games provides an avenue which allows us to relax one or several of these assumptions and nevertheless reach plausible and meaningful predictions on outcomes as steady states of these learning processes. The learning literature is interested in several questions: Will learning will eventually lead to the play of a Nash equilibrium? If so, under which circumstances? May a learning process lead to non-Nash equilibrium strategies and conjectures or even to a non-Nash equilibrium outcomes? Which of the Nash equilibria will learning lead to if any? As we will see, often non-Nash steady states may exist: a price for higher plausibility is lower precision.

4.3.2 Inconsistent prior beliefs

The assumptions about players' rationality and knowledge in the standard model are heavy, even unrealistic. As a first step in throwing away the heavy artillery we consider a model where the assumption that players have correct conjectures about each other (and nature) is relaxed. Players start with a prior conjecture about the behaviour of others and learn within the game.

Perfect monitoring, (Kalai-Lehrer, 1993)

Kalai and Lehrer (1993) study an *infinitely repeated game of complete information* with N players. They suppose that players perfectly observe each others' choices. Thus the model is one of *perfect monitoring* and *observed deviators* - the preceding history is perfectly observed and deviations by two different players cannot lead to the same information set.

Each player starts with a prior conjecture about other players' strategies. Yet unlike above, players' prior conjectures do not necessarily satisfy the consistency requirement - there is no common prior distribution. Yet, each distribution must contain a grain of truth: there must be a strategy profile of other players which together with player's own strategy generates the same path of play as the true strategies. More technically conjectures must be *absolutely continuous* with respect to the true outcome path: if the conjectures put a zero probability weight on a path of play, then that play path must have a zero probability according to the true strategies.

Definition 3. *Absolute continuity. If P^{σ} is the true probability distribution of the play paths, then for each player i and each z the conjecture about the play path, $P^{\widehat{\sigma}^i}$, must satisfy, $P^{\widehat{\sigma}^i}(z) = 0 \Rightarrow P^{\sigma}(z) = 0$.*

A further restriction on the beliefs is that the *conjectures* about opponents' strategies *must be a product measure*: no player can have a mistaken belief that opponents' behaviour strategies are correlated (they are assumed to be chosen independently).

At each stage of the game players observe their opponents' actions and update their strategies using the Bayes's rule. Kalai and Lehrer (1993) show that, eventually players conjectures about future play will be $\varepsilon-$close to correct. The conjectures may remain incorrect off the path. The concept of being ε-close to correct is defined as follows:

Definition 4. *Let $\varepsilon > 0$ and let μ and $\widetilde{\mu}$ be two probability measures defined on the same probability space. We say that μ is ε-close to $\widetilde{\mu}$ if there is a measurable set Q satisfying*

- *$\mu(Q)$ and $\widetilde{\mu}(Q)$ are greater than $1 - \varepsilon$ and*
- *for every measurable set $A \subset Q$*

$$(1-\varepsilon)\widetilde{\mu}(A) \leq \mu(A) \leq (1+\varepsilon)\widetilde{\mu}(A)$$

This definition of closeness is strong in the sense that it requires the two measures to be relatively close even for events of small probability.

Definition 5. *Let $\varepsilon \geq 0$. We say that σ' plays ε-like σ'' if $P^{\sigma'}$ is ε-close to $P^{\sigma''}$*

Formally, the main results of Kalai-Lehrer (1993a) are the following.

Theorem 3. *(Theorem 1, Kalai-Lehrer (1993a)) Let σ and $\widehat{\sigma}^i$ be two $N-$vectors of strategies, representing the ones actually chosen and the beliefs of player i respectively. Assume that σ is absolutely continuous w.r.t. $\widehat{\sigma}^i$. Then for every $\varepsilon > 0$ and for almost every every path z, there is a time $T(\varepsilon, z)$ such that that for all $t \geq T$, $\sigma_{z(t)}$ plays ε-like $\widehat{\sigma}^i_{z(t)}$ where $\sigma_{z(t)}$ is the truncation of σ at $z(t)$.*

Theorem 3 above does not tell us anything how the strategies are chosen. Thus it does not use the optimizing and equilibrium tools which characterize game theoretic argumentation. Kalai and Lehrer take a step closer to game theory by defining the concept of *subjective equilibrium* where players maximize a discounted sum of stage game payoffs.

Definition 6. *An N-vector of strategies, σ, is a subjective equilibrium if there is a matrix of strategies $(\widehat{\sigma}_i)_{1\leq i,j\leq n}$ with $\widehat{\sigma}^i_i = \sigma_i$ such that*

- *σ_i is a best response to $\widehat{\sigma}^i_{-i}$, $i = 1, ..., n$*
- *σ plays ε-like $\widehat{\sigma}^i$, $i = 1, ..., n$.*

As a corollary to theorem 3, Kalai and Lehrer reach the conclusion that for every ε there is a time T when the play will have converged into an ε-subjective equilibrium.

Corollary 2. *(Corollary 1, Kalai-Lehrer (1993a)) Let for $i = 1, ..., N$, σ and $\widehat{\sigma}^i$ be two $N-$vectors of strategies, representing the ones actually chosen and the beliefs of player i respectively. Assume that σ is a best reply and absolutely continuous w.r.t. $\widehat{\sigma}^i$. Then for every $\varepsilon > 0$ and for almost every every path z, there is a time $T(\varepsilon, z)$ such that that for all $t \geq T$, $\widehat{\sigma}_{z(t)}$ is a subjective ε-equilibrium.*

Eventually Kalai and Lehrer show that there is an outcome-equivalent Nash-equilibrium.

Proposition 8. *(Corollary 1, Kalai-Lehrer (1993a)) For every $\varepsilon > 0$, there is $\eta > 0$ such that if σ is a subjective equilibrium then there exists $\overline{\sigma}$ such that*

Theorem 4.
- *σ plays ε-like $\overline{\sigma}$ and*
- *$\overline{\sigma}$ is an $\varepsilon-$Nash equilibrium*

Imperfect monitoring (Kalai, Lehrer, GEB 1995)

Kalai and Lehrer (1995) extend their (1993) approach to the case of imperfect monitoring in repeated games. Now the history of past play - the actions of the opponents - is not perfectly observed. Rather players only observe an informative signal about past play. behaviour strategies are now functions of private signal history rather than past play. The past signal history can be used as a correlation device since private signals are correlated in general.

Kalai and Lehrer (1995) find closely related results to those in the (1993a) paper but now the set of predictions coincides with the set of correlated equilibria rather than Nash equilibria. In a correlated equilibrium nature sends stochastic and correlated messages $m_1, ..., m_N$ to each player prior to the play of the game. In a subjective correlated equilibrium, players have subjective conjectures about how an outcome in the game, c, depends on her private message and her action, $\widehat{e}_i(c|m_i, a_i)$, or given a message m_i, how the outcome depends on player's own action a_i, $\widehat{e}_{m_i}(c|a_i)$. The true mapping is given by $e_{m_i}(c|a_i)$. These obviously depend on opponents' and nature's choices and thus $\widehat{e}_i$ implicitly embodies conjectures about opponents' strategies and nature's moves.

Definition 7. *A subjective correlated ε-equilibrium in a repeated game G^∞ consists of a correlation device (M, p), a vector of correlated strategies σ, a vector of correlated subjective environment functions $\widehat{e}^i$ such that for each $\varepsilon > 0$*

- *(Subjective optimization) for every player i and signal m_i, $\sigma_i(a_i|m_i)$ is optimal with respect to $\widehat{e}^i_{m_i}$.*
- *(Correlated uncontradicted beliefs) with probability greater than $1 - \varepsilon$, a message vector m will be chosen with $P^{\sigma_i, \widehat{e}_i}(z)$ being ε-close to $P^{\sigma_i, e_i}(z)$ where $P^{\sigma_i, e_i}(z)$ is the probability of terminal history z given strategy σ_i and mapping from player's own actions and private messages to outcomes.*

In a repeated game with private monitoring the message of player i is her private history of play which we also call here m_i for simplicity. Each player has a subjective conjecture how the unobserved outcome is determined in a stage game (implicitly depending on nature's and opponents' moves) given the private history and a private action $\widehat{e}^i(c|m_i, a_i)$. The true outcome rule is denoted by e_i. Each player chooses an optimal action given these conjectures. In a subjective correlated equilibrium, each player maximizes given their (correlated) beliefs and the beliefs are compatible with observations. Since signals are correlated, the steady state of the learning process ends up being ε-close to a subjective correlated equilibrium.

Theorem 5. *(Kalai, Lehrer, 1995, theorem 4.4.1.) Let σ be a vector of strategies and e be a vector of subjective environment response functions. Suppose σ and $\widehat{e}_i$ satisfy the following two conditions for every player i :*

- *(Subjective optimization) σ_i, is optimal relative to $\widehat{e}_i$*
- *(Beliefs are compatible with the truth) $(\sigma_i, \widehat{e}_i)$ is compatible with (σ_i, e_i)*

Then for every $\varepsilon > 0$ there is a time T such that for all times t, with $t \geq T$, from time t on, the players play a subjective correlated ε-equilibrium.

Unlike in the case of perfect monitoring the play no longer ends up ε-close to a prediction that one would obtain were the prior distributions correct (here the objective equilibrium is not a Nash equilibrium but a correlated equilibrium since private histories can be used as a correlation device). Yet, Kalai and Lehrer (1995) show that if initial expectations are sufficiently optimistic, then the play will eventually converge to an outcome which is ε-close to an (objective) correlated equilibrium.

Proposition 9. *(Proposition 5.1. Kalai-Lehrer, 1995) Let $(\sigma, \widehat{e}_i)$ be a subjective correlated equilibrium with each $\widehat{e}_i$ holding optimistic conjectures relative to σ, that is for every σ_i, $u_i(\sigma_i, e_i) \leq u_i(\sigma_i, \widehat{e}_i)$. Then σ is a correlated equilibrium.*

Kalai-Lehrer's critiques

Kalai and Lehrer work can be interpreted as identifying conditions where eventual predictions of play coincide with Nash predictions. Fudenberg and Levine (1993 a,b) show that the Nash-equivalence is due to the assumption that players have *unitary* and *independent* conjectures and the game satisfies the condition of *observed deviators* (see section 4.4.2). Yet, in some circumstances these assumptions in addition to Bayesian updating and maximizing of an infinite sequence of payoffs with time consistent preferences, are rather restrictive. Also it may take quite a long time before the play is ε-close to Nash. How long it takes to get there is a question which is not addressed.

Moreover, Jordan (1995), Nachbar (1997) and Foster and Young (2001) show that the absolute continuity assumption is by no means innocuous. Jordan (1995) shows that even if players' expectations may converge to a mixed strategy profile, the actual strategy choices will not. Nachbar (1997) shows that a very natural restriction on players' initial beliefs about opponents' strategies implies a violation of absolute continuity: each player will choose a strategy that his opponent is certain that it will not be played. Jordan's and Nachbar's results assume that players are at least to some extent myopic. Foster and Young (2001) allow for players to be even completely patient. They consider games where the payoffs are incomplete information, players have a prior distribution about this payoff uncertainty and they learn about this distribution as the play continues. They show that some of the players never learn to predict the strategies of others and that the realized play fails to come close to a Nash equilibrium outcome path. Notice that this result is stronger than Jordan's result also in the sense that neither expectations nor strategies constitute a Nash equilibrium whereas Jordan only shows that the strategies may not constitute a Nash equilibrium.

4.4 Anonymous Myopic Learning

In this section we study learning processes where players are repeatedly drawn from large populations to play a game. Prior to play and prior to the draw of types even, they observe signals about outcomes of previous rounds of play and form conjectures about average play in other player populations which are consistent with these observations. We call such a learning process *anonymous.* More precisely in an anonymous learning process, every time the game is played, N players are drawn from N large populations and each player maximizes her myopic stage game payoff given her conjectures about others. Notice that the same player may play repeatedly against varying opponents or there may be a sequence of player populations where information about previous play is transmitted across generations.

The drawing from large populations dampens incentives for punishing, rewarding and building up a reputation (Kreps et al, 1982) since the probability of facing the same opponent again is vanishingly small. The assumption of myopia in payoff maximization implies that no experimentation is carried out to learn more about the behaviour of others. In static games myopia is a natural assumption since all (the only one) decision nodes are reached with a positive probability in every round and nothing is left to learn about off-path behaviour. In section 4.4.1 we consider static games only. In section 4.4.2 we extend the analysis to extensive form games where experimentation potentially plays a role.

In the following sections, but for section 4.4.3, the learning process is only implicit in the model and characterized by means of the equilibrium concept. The information about opponents' behaviour is assumed to have been transmitted to each player from infinitely many previous rounds. Consequently the player learns about the average behaviour of other populations. An equilibrium is interpreted as a steady state of a learning process.[1] Best replies to the conjectures about opponents' play must generate samples of induced signals which confirm the initial conjectures about opponents' play.

The analogy-based expectation equilibrium (ABEE) is a specific example. In the analogy-based expectation equilibrium players bundle decision nodes into analogy classes (set of classes is called the analogy partition) and they only try to learn the average behaviour in every class. Every type of a player chooses a best reply to these analogy expectations, and expectations correctly represent the average behaviour in every class.

By conjecturing that opponents play average strategies in each analogy class, a player behaves as if she observed (ex-post) previous type profile realizations with a precision given by the player's analogy partition. Alternatively a player behaves as if she believed that opponents condition their action choices on the analogy classes rather than on the information sets. Since she believes that opponents play the same strategy in all nodes of an analogy class, a player believes that opponents use a simpler strategies when her analogy partition is coarser. The analogy grouping allows us to think about complexity of the beliefs about others' strategies. In what follows we will illustrate, by means of the ABEE, how much complexity various other equilibrium concepts require. In other words we ask with which analogy partitions, and with which cardinality of the analogy partitions, an ABEE coincides with a given equilibrium concept.

4.4.1 Static games (SCE, CE, ABEE)

Before moving to extensive form games in section 4.4.2, we start here by considering static *incomplete* information games. Above we only considered static games of complete information where players kept track of opponents' action choices. In static games of incomplete information nature first draws players' payoff *types* and players then simultaneously choose actions conditional on their types. An *outcome* is a vector profile of types and actions, one action and one type for each player.

In incomplete information games what a player has learned depends on what has been observed in the past after each round of play and how these observations are organized. By choice or by physical and environmental constraints each player may not keep track of every detail of the outcome: player's own type, action, payoff, opponents' types, opponents' actions. This is true even if the same player plays the game repeatedly

[1] Whether learning process is fictitious play or Bayesian updating does not really play a role here

against varying opponents.

Rather each player may observe and keep track of only some subset of such scalar *signals*. Or, she may observe and keep track of each scalar more coarsely bundling several scalar realizations into the same set of the scalar's possible values. Moreover, a player may organize the signals into several vector signals and keep track of the marginal sample distribution of each vector rather than the joint distribution of all signals. Each such vector is called a *perception*. Overall we call the signals and the perceptions the *observation structure*.

We introduce the *conjectural equilibrium* (CEMP)[2] as an organizing concept - each of our equilibrium concepts is a special case of this concept with a specific observation structure. Given an observation structure, any CEMP has two main building blocks: The first, each player forms subjective conjectures about her opponents' strategies that are consistent with the objective (sample) marginal distributions of each of the perceptions generated by the equilibrium behaviour. The second, each player's strategy choice is a best reply to these conjectures. Obviously the stringency of the consistency requirement depends on the observation structure, on the one hand, and on how the conjectures are formed based on the observation structure, on the other hand. If players observe the entire outcome, then players get to observe directly the opponents' behaviour at every positive probability history of the game. Thereby conjectures about opponents' behaviour will be correct and every steady state coincides with a Baysian-Nash equilibrium.

When outcomes are not fully observed, alternative equilibrium concepts arise, however. The *self-confirming equilibrium* (Dekel, Fudenberg and Levine (DFL), 2004), the *analogy-based expectation equilibrium* (Jehiel, 2005; Jehiel and Koessler (JK), 2006) and the *cursed equilibrium* (Eyster and Rabin, 2005) each corresponds to an observation structure. As a matter of fact, one of our contributions is to provide a learning steady state interpretation and a corresponding observation structure for the cursed equilibrium.

In the self-confirming equilibrium (SCE) each player keeps track of the joint distribution of all the signals that she observes. Moreover, each player observes at least her own action and her own type. We define the *action-confirming equilibrium (ACE)* as a special case: player observes her action, her type and opponents' actions and nothing else.

In the analogy-based expectation equilibrium (ABEE) players bundle type profiles

[2]MP for multiple perceptions.

into analogy classes and they only try to learn the average behaviour in every class. Every type of a player chooses a best reply to these analogy expectations and expectations correctly represent the average behaviour in every class.

Each analogy-based expectation equilibrium where the analogy partition is finer than the private information partition the ex-post observation of the type profile is at least as precise as the interim observation of types. We call such an ABEE an *analogy-based expectation equilibrium with finer than private information analogy partition* ($ABEE_{An \leq I}$). Each $ABEE_{An \leq I}$ corresponds to an ACE and thus to a SCE.[3]

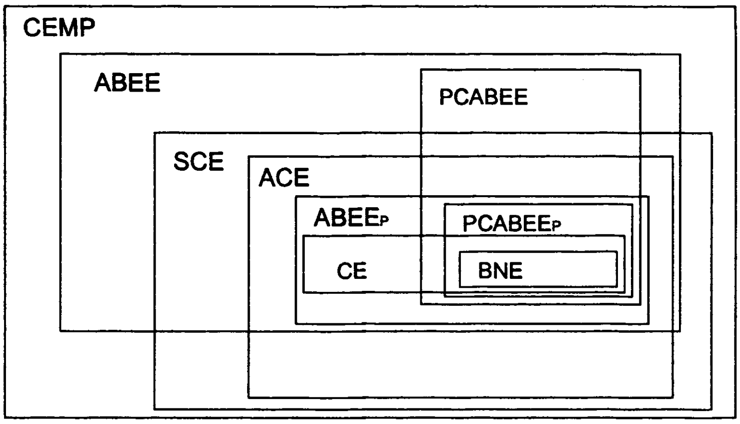
We also define a refinement of the analogy-based expectation equilibrium where the payoff distribution induced by analogy expectations must coincide with the true payoff distribution induced by actual equilibrium strategies. This is a natural assumption in a learning model where each player plays repeatedly, since maximizing payoffs (to the least improving them) is what players strive for and it is likely that successes and failures will be remembered. To save in usage of memory, the payoff is kept track separately of the other signals in a second perception. Therefore correlation between player's own payoffs and other signals will not be paid attention to. This sample of payoffs provides a way to sharpen the inference about the outcome using the fact that payoffs can be mapped inversely to outcomes. We say that *an analogy-based expectation equilibrium* is *payoff-confirming* (PCABEE) if the distribution of payoffs confirms one's conjectures about opponents' behaviour.

In the cursed equilibrium each player fails to correctly conjecture the extent of correlation between opponents' private information and their strategies. Rather each player type expects opponents' to play a pooling strategy at least to some extent.

Our analysis organizes the predictions as indicated in figure 4.1. Moreover, we provide a sufficient condition for an ABEE to be payoff-confirming. This is also a necessary condition in two-player two-action games with binary uncertainty. Along the lines of ER and JK respectively, we illustrate in a bilateral trading game with common value component how the fully cursed equilibrium and the corresponding ABEE alleviate the adverse selection which results in a Bayesian-Nash equilibrium of that game (Akerlof, 1970). Yet, we also illustrate that this ABEE is not payoff-confirming.

Section 4.4.1 reviews the equilibrium concepts and studies the relationships between these concepts. Section 4.4.1 introduces the payoff-confirming ABEE and studies when a PCABEE is non-Bayesian-Nash and when an ABEE is payoff-confirming. Section 4.4.1

[3]Whereas in the ACE the conjectures are arbitrary in an ABEE the opponents are conjectured to choose an action according to its frequency in the corresponding analogy class.



CEMP - conjectural equilibrium with multiple perceptions,
ABEE - analogy-based expectation equilibrium,
PCABEE - payoff-confirming ABEE,
SCE - self-confirming equilibrium,
ACE – action-confirming equilibrium,
ABEEP – ABEE with finer than private information analogy-partition,
PCABEEP – payoff-confirming ABEEP,
CE - cursed equilibrium,
BNE – Bayesian-Nash equilibrium.

Figure 4.1: Equilibrium predictions in static games

considers adverse selection in bilateral trading.

ABEE, Complexity, and Relations to Other Equilibrium Concepts

Complex strategies may require more cognitive effort than simple strategies. Thus a player may prefer a simple strategy to a complex one. Therefore it may more plausbile to conjecture that the opponent prescribes to a simple strategy rather than to a more complex one. Moreover, when choosing a best reply, maintaining beliefs that opponents' are prescribing to complex strategies may require more cognitive effort than best replying to simple beliefs. In all, aversion for complexity sounds plausible. Yet, there is no shared view among game theorists how to model complexity precisely.

One avenue for thinking about complexity is the analogy-based expectation equilibrium. In the ABEE the complexity of beliefs is captured by the cardinality of the analogy partition. In an ABEE each player believes that opponents play pooling strategies at all states of an analogy class, i.e. believes that opponents' condition on analogy classes rather than on their information sets. The ABEE requires that the belief is robust to learning in that the pooling strategy must coincide with the average strategy of the class.

In what follows we will illustrate how much complexity various equilibrium concepts require. In other words we ask with which analogy partition, and with which cardinality, an ABEE coincides with a given equilibrium concept.

We begin with defining the *conjectural equilibrium* as an organizing concept. The self-confirming equilibrium, the action-confirming equilibrium, the analogy-based expectation equilibrium, the payoff-confirming analogy-based expectation equilibrium and even the cursed equilibrium turn out to be special cases of this concept.

Each player plays a static game of incomplete information against $N-1$ opponents drawn randomly from $N-1$ large populations. Before the opponents are drawn each player receives signals about how the play has proceeded in infinitely many preceeding rounds. Opponents are then drawn and private information is revealed to the players. We suppose that the distribution from which nature draws types is known to all[4] and the probability that θ is drawn is denoted $\sigma_0(\theta) = p(\theta)$. Each previous outcome of play (a,θ) induces scalar *signals* $y_k(a,\theta) : A \times \Theta \to Y_k$, $k = 1,...,K$; for instance player's payoff $u_i(a,\theta)$, type θ_i or action a_i may be a signal. A *perception* of player i consists of some (not necessarily all) of such signals. The *perceptions of player* i are $\gamma_i^l(a,\theta)$, $l = 1,...,L_i$ so that each perception $\gamma_i^l(a,\theta) : A \times \Theta \to \Gamma_i^l \subset \times_{k=1}^K Y_k$ consists of signals, $\gamma_i^l(a,\theta) = (y_1^l(a,\theta),...,y_{M_l}^l(a,\theta))$, $M_l \leq K$ and $y_k^l \in \{y_1,...,y_K\}$ and each mapping $\gamma_i^l(a,\theta)$, $l = 1,...,L_i$ is known to player i. The *observation structure* is $\gamma_i = (\gamma_i^1,...,\gamma_i^{L_i})$.

A *conjectural equilibrium (CEMP)* is one where, firstly, each player's conjectures about opponents' behaviour are consistent with the marginal distributions generated by the equilibrium behaviour, one for each perception, and secondly, each player chooses a myopic best reply to her conjectures.

Definition 8. *(CEMP)*

$(\sigma_i, \widehat{\sigma}_{-i}, \gamma_i)_{i=1}^N$ is a CEMP if for all $\theta \in \Theta$, for all i and $a_i^* \in$supp$[\sigma_i(\theta_i)]$

$$a^* \in \arg\max \sum_{\theta_{-i}\in\Theta_{-i}} p(\theta_{-i}|\theta_i) \sum_{a_{-i}\in A_{-i}} \widehat{\sigma}_{-i}(\theta_{-i}) u_i(a;\theta) \tag{4.1}$$

and for all $l = 1,...,L_i$ and for all $\overline{\gamma}_i^l \in \Gamma_i^l$

$$\begin{aligned} &\sum_{\{a\in A,\theta\in\Theta|\gamma_i^l(a_i,a_{-i},\theta_i,\theta_{-i})=\overline{\gamma}_i^l\}} p(\theta)\sigma_i(a_i|\theta_i)\widehat{\sigma}_{-i}(a_{-i}|\theta_{-i}) \\ =& \sum_{\{a\in A_{-i},\theta\in\Theta|\gamma_i^l(a_i,a_{-i},\theta_i,\theta_{-i})=\overline{\gamma}_i^l\}} p(\theta)\sigma_i(a_i|\theta_i)\sigma_{-i}(a_{-i}|\theta_{-i}) \end{aligned} \tag{4.2}$$

Apart from its common prior assumption, this definition is more general than the definition of conjectural equilibrium in Battigalli (1987) or Battigalli and Guaitoli (1997)

[4]But not necessarily common knowledge.

in that it allows for multiple perceptions for each player and a consistency condition, (4.2), for each of these perceptions separately. In Battigalli's and Battigalli and Guaitoli's conjectural equilibrium each player has a unique perception, $L_i = 1$ for $i = 1, ..., N$.

We allow for this more general concept since it better accommodates some characteristics of human learning and decision making where limited cognitive resources have to be allocated to perceiving and handling information. In particular due to memory restrictions, an agent may fail to account for correlation between signals. She may be unable to keep track of samples of multiple signals jointly but rather only keep track of the marginal sample distributions of various perceptions. For instance, if a player keeps track of the joint distribution of types, actions and payoffs each of which allows for say 5 different values, the player has $5^3 = 125$ entries to trace whereas keeping track of the marginals only necessitates 15 entries. Thus multiple perceptions may be a good way to economize in the usage of memory but it may imply a failure to account for correlation between the signals in two different perceptions.

Notice that the observation structure, γ_i, is part of the conjectural equilibrium, $(\sigma_i, \widehat{\sigma}_{-i}, \gamma_i)_{i=1}^N$ description rather than exogenous. This reflects the fact that a player whose conjecture about opponents' behaviour is not confirmed by her observations may seek new information in order to update her conjectures. New information may emerge from choosing to observe new signals or reorganizing one's signals into a new observation structure. It is likely that what can be observed is determined partly by environmental constraints and thus the player may not be able to observe any signal she wishes to.

The self-confirming equilibrium is a special case of the conjectural equilibrium[5]: Firstly, there is a unique perception, $L_i = 1$, and thus a joint sample distribution of the signals in the perception. Secondly, the SCE assumes that the perception contains at least player's own type-action vector and thus, $\gamma_i : A \times \Theta \to A \times \Theta_i \times_{k=1}^M Y_k$ and $\gamma_i(a, \theta) = (a_i, \theta_i, y_3(a, \theta), ..., y_M(a, \theta))$. Action confirming equilibrium is a special case with $\gamma_i(a, \theta) = (a_i, a_{-i}, \theta_i)$.

Definition 9. *(ACE) Action-confirming equilibrium is CEMP such that for each i, $L_i = 1$ and for all i, $\gamma_i : A \times \Theta \to A \times \Theta_i$ and $\gamma_i(a, \theta) = (a_i, a_{-i}, \theta_i)$.*

We refine the action-confirming equilibrium by allowing the players to observe their own payoffs in addition to the opponents' actions, $\gamma_i^1(a, \theta) = (a_i, a_{-i}, \theta_i, u_i(a, \theta))$. We call this equilibrium the *payoff-action-confirming equilibrium* (PACE).

[5]Unlike in DFL (2004), we suppose here that the distributions from which nature draws realizations of exogenous randomness is known to all.

In an action-confirming equilibrium each player has a sample of opponents' actions for each of her own action-type pairs. This latter feature has an appeal if one wishes to model a learning process where each player plays repeatedly: the type and the strategy is known and observed at the interim stage when the strategy decision is made; why should the player have forgotten this information ex-post when the player forms her conjectures based on available observations.

One could argue, firstly, that observations must be recalled over time and jointly with other signals when conjectures are formed. The players' cognitive limitations may make this requirement too demanding. Secondly, the underlying learning model may concern cases where the same player does not play repeatedly, but rather N populations of players appear in generations and information is transmitted from previous to future generations.[6] In this case, not all information available may be transmitted.

In the analogy-based expectation equilibrium each player partitions the support of type profiles into analogy classes. An analogy system characterizes these analogy partitions.

Definition 10. *Analogy System* $(An_1, ..., An_N)$ *where* An_i *is the analogy partition of* Θ *for player* i. *An element of* An_i *is denoted by* α_i, *the element of* An_i *containing* θ *is* $\alpha_i(\theta)$.

Player's analogy partition describes how precisely the player observes and records the type profile realizations. For instance, if type profile θ is drawn, player i observes $\alpha_i(\theta)$. The coarseness of the analogy partition may be determined partly by the environment and partly by the player's cognitive skills, attentiveness and beliefs about complexity of others' strategies. The analogy partition might be coarser or finer than player's private information partition.[7] In the ABEE there is a unique perception which consists of the opponents' actions and the analogy class where the type profile belongs to $\gamma_i(a, \theta) = (a_{-i}, \alpha_i(\theta))$. Thus there is a sample distribution of opponents' actions for each of the analogy classes.[8]

[6] Jackson (1997), for instance, studies this kind of learning model.

[7] Notice that finer then private information analogy partition is possible since a player's signal about type may be more accurate when the game is played than when she is about to choose her action during the game. The analogy partition may be coarser than the information partition since the player may not remember her type when the game has reached its outcome or keep track of it over the entire learning process.

[8] If the lack of observing one's own action and one's own type sounds implausible, one can allow for the observation of these without affecting predictions if these appear in a separate perception, $\gamma_i^2(a, \theta) = (a_i, \theta_i)$.

The analogy-based expectation equilibrium is specific about how the conjectures are formed. Each player conjectures that, at a given type profile, the opponents play their average strategy of the analogy class where that type profile belongs to. This is the simplest theory consistent with the signal. Moreover, this is the only consistent theory where all opponents play a pooling strategy in the states of the class. Thus requiring a pooling strategy in each analogy class allows us to think about how complex each player believes that the opponents' strategies are. To formalize this idea, we define the opponents' average strategy in a set of type profiles $B \subset \Theta$ as follows:

Definition 11. *The opponents' average strategy in $B \subset \Theta$.*

$$\overline{\sigma}_{-i}(B) = \frac{\sum_{\theta \in B} p(\theta)\sigma_{-i}(\theta_{-i})}{\sum_{\theta \in B} p(\theta)} \tag{4.3}$$

The ABEE can now be defined as a special case of CEMP.

Definition 12. *(ABEE) An ABEE is a CEMP where for each i, $L_i = 1$ and*

1. $\gamma_i : A \times \Theta \to A_{-i} \times An_i$ *and* $\gamma_i(a, \theta) = (a_{-i}, \alpha_i(\theta))$

2. $\widehat{\sigma}_{-i}(a_{-i}|\theta) = \overline{\sigma}_{-i}(\alpha_i(\theta))$

As in JK each player conjectures that each opponent uses the same mixed strategy in all states of an analogy class. In particular, at each type profile of an analogy class, each player expects that opponents play their average strategy of the class - a weighted sum of the mixed strategies used in the class, each weight coinciding with the conditional probability of each state, (4.3).

The cardinality of each An_i characterizes, firstly, the precision of opponents' type profile observation, and secondly, the complexity of player's conjecture about others' behaviour. Notice that for each analogy system $An = (An_1, ..., An_N)$, we have a different ABEE. The coarseness of the image of γ_i is part of the equilibrium description. This is where our concept differs from the definition in JK (2006, p. 5).[9] On the one hand, we believe that contradicting observations may induce efforts to track the types and the opponents' behaviour more carefully. On the other hand, complexity of strategies tends to require more congnitive effort (Abreu and Rubinstein, 1988); thus, conjectures with less complexity may be more plausible. If simpler conjectures (more coarse precision)

[9] Indeed Jehiel (2005) acknoledges the need of endogenizing the analogy partitions; the payoff-confirming ABEE defined below can be considered as an approach to such endogenization: players use the payoff signals to rule out implausible analogy partitions. To guarantee existence, the partition is made part of the solution concept.

can explain observations, the player may be prone to resort to such conjectures.[10] Thus the analogy partition is not necessarily determined exogenously but rather endogenously within the learning process.

There are two differences between an ACE and an ABEE: the first, in the ACE players observe their own types and thus an ABEE with coarser than private information analogy partition is generally not an ACE; the second, conjectures are not restricted to the average strategy in the ACE and thus there are generally ACE that differ from ABEE. We will illustrate the former difference in proposition 11. Let us now focus on the latter difference.

In an ABEE the conjecture coincides with the average strategy of the analogy class. This implies that the notion of consistency in an ABEE is much stronger that in an ACE or in a CEMP: the condition in the definition of CEMP, (4.2) now with $\gamma_i = (a_{-i}, \alpha_i(\theta))$ is implied by the average strategy condition (4.3):

$$\begin{aligned}
&\sum_{\theta\in\alpha_i} p(\theta)\widehat{\sigma}_{-i}(a_{-i}|\theta) \\
&= \sum_{\theta\in\alpha_i} p(\theta)\frac{\sum_{\theta'\in\alpha_i} p(\theta')\sigma_{-i}(a_{-i}|\theta'_{-i})}{\sum_{\theta'\in\alpha_i} p(\theta')} && (4.4)\\
&= \sum_{\theta\in\alpha_i} p(\theta)\sigma_{-i}(a_{-i}|\theta_{-i}). && (4.5)
\end{aligned}$$

To illustrate the stronger consistency requirement in the ABEE consider the follwing game.

Example 1. *There are two players: row player, R, and column player, C. Each player has two actions* (-1) *and* $(+1)$*. There are two states* (-1) *and* $(+1)$ *where each state has probability* $\frac{1}{2}$*. The payoff of C is the product of the state and her action. The payoff of R is the product of the state and the actions of R and C.*

(-1)	*(-1)*	*(+1)*
(-1)	$-1, 1$	$1, -1$
(+1)	$1, 1$	$-1, -1$

(+1)	*(-1)*	*(+1)*
(-1)	$1, -1$	$-1, 1$
(+1)	$-1, -1$	$1, 1$

Suppose that the state is private information to C. If C knows the state, then she has a dominant strategy: play (-1) in state (-1) and (+1) in state (+1). Given this

[10] Notice that coarseness of the analogy partitions might be used as a selection criterion among equilibria. Somewhat similar idea is found in Abreu and Rubinstein (1988) where the simplicity of the automaton plays a role.

equilibrium strategy, R's conjecture that two plays -1 in state -1 with probability $\frac{1}{4}$ and +1 in state +1 with probability $\frac{1}{4}$ is consistent in an ACE. However, player one must expect that two plays -1 and +1 with equal probabilities in an ABEE where player one bundles states into the same analogy class, $An_1 = \{\{-1, +1\}\}$. Thus, there are ACE that are not ABEE.

Proposition 10. *There can be ACE which are not ABEE.*

We lose in generality by demanding players to conjecture a simple pooling strategy in each analogy class. But we also gain: the ABEE allows us to discuss the complexity of the conjectures about opponents' strategies which is not possible with arbitrary conjectures. We can ask: how much complexity, at least, do we need in an action-confirming equilibrium or in a Bayesian-Nash equilibrium?

Before answering those questions, let us consider the second difference between the ABEE and the ACE: Let us show that there are ABEE where conjectures are too simple for an ACE or a BNE. If player i's analogy partition is coarser than her private information partition, I_i, then the average strategy in the class does not necessarily coincide with the average strategies in each of the player's information sets (we will see that this would suffice for ACE), let alone with the true strategies (as in BNE).

To illustrate this, consider the game in example 1 above but suppose now that the state is complete information. As before, the column player has a dominant strategy to play -1 at state -1 and +1 at state +1. Suppose that the row player conjectures that the column player's strategy is the simplest possible - she uses the same strategy in both states. In other words, the row player's analogy partition has a unique set containing both states, $\{-1, +1\}$. ABEE requires that the row player expects the column player to choose each action with probability $\frac{1}{2}$, this is the average strategy in the analogy class. From the row player's perspective, each of the two games is a matching of pennies game. Thus, any strategy is a best reply to the conjecture that the opponent mixes with equal probabilities.

It is easy to see that in the unique ACE (which coincides with the unique Bayesian-Nash equilibrium of the complete information game) the row player plays +1 with probability one: in the ACE, the row player observes the joint empirical distribution of the column player's actions and the states. Thus, she must conjecture that the column player plays +1 at +1 and -1 at -1. We see that the conjectures in the simplest ABEE are too simple to be an action-confirming equilibrium or a (Bayesian-)Nash equilibrium.

Proposition 11. *There are ABEE that are*

- *non-BNE (Jehiel, 2005)*
- *non-ACE*

Let us next address the question of minimum complexity of beliefs, i.e. the minimum cardinality of the analogy partition required for an ACE. Notice that if in an ABEE the analogy partition was finer than the private information partition, then $\alpha_i(\theta') \subset \theta_i'$ and learning own action-type signals (as in the ACE) would not add to the player's understanding of opponents' strategies: $(a_i, \theta_i, a_{-i}, \alpha_i(\theta))$ gives the same information about opponents' strategies as $(a_{-i}, \alpha_i(\theta))$.

Definition 13. *($ABEE_{An \leq I}$) If in an ABEE $\theta_i' \neq \theta_i'' \implies \theta' \notin \alpha_i(\theta'')$ then it is an ABEE with finer than private information analogy partition.*

This constitutes the minimum complexity requirement for the ACE: the analogy partitions must be finer than the private information partitions. The corresponding minimum cardinality of the analogy partition of each player is $\#I_i$. Adding players' own type-action signals to an ABEE with analogy partitions coarser than private information partition would actually repartition the analogy classes: keeping track of $(a_i, \theta_i, a_{-i}, \alpha_i(\theta))$ rather than $(a_{-i}, \alpha_i(\theta))$ is as if the analogy partition would correspond to $I_i \cap An_i$ and thus it would be finer than the private information partition I_i. Thus, if each player observes her own type, the ABEE becomes a special case of the action-confirming equilibrium, the average strategy in each class constitutes a specific consistent conjecture.

Proposition 12. *$ABEE_{An \leq I}$ is an ACE.*

However, the Bayesian-Nash equilibrium requires even more complexity. There are $\text{ABEE}_{\text{An} \leq \text{I}}$ that are not Bayesian-Nash equilibria: for instance in example 1, if state is private information to the column player and the row player bundles both states into a single analogy class, the row player expects mixing with equal probabilities by the column player and R may best-reply by mixing with equal probabilities. Clearly, this is not a Bayesian-Nash equilibrium, since in the unique BNE, the row player best-replies to the column player's dominant strategy by choosing $+1$.

Proposition 13. *There can be $ABEE_{An \leq I}$ that are not Bayesian-Nash*

Corollary 3. *There can be ACE that are not Bayesian-Nash. (DFL, 2004, proposition 4)*

The corollary is clearly implied by proposition 12. By allowing for restrictions in beliefs (in a separate perception), this former statement strengthens proposition 4 (i) in DFL which states that there may be ACE that are not Bayesian-Nash.

The immediate question is to study exactly how much complexity the BNE requires. Consider still the incomplete information scenario, but suppose now that the row player believes that the column player's strategy is more complex: the analogy partition consists of two singleton sets, $\{\{-1\},\{+1\}\}$. This implies that the average strategy in each class coincides with the strategy of the column player's unique type in the class: the row player has a correct conjecture about the column player's strategy. More generally, the ABEE is a BNE if the analogy partition coincides with the opponent's information partition. Not surprisingly, when the beliefs about opponents' strategies are exactly as complex as the true strategies, the ABEE coincides with the BNE.

Proposition 14. *(JK, proposition 2) If each player's analogy partition is finer than every opponent's information partition, then the ABEE is Bayesian-Nash equilibrium. Formally, let $(\sigma, \widehat{\sigma}, An)$ be an ABEE such that for all i, $\theta_{-i} \neq \theta'_{-i} \Rightarrow \alpha_i(\theta) \neq \alpha_i(\theta)$, then σ is an BNE.*

Proof. The second claim follows from noticing that is $(\sigma, \widehat{\sigma}, An)$ be an $ABEE$ and for all i, $\theta_{-i} \neq \theta'_{-i} \Rightarrow \alpha_i(\theta) \neq \alpha_i(\theta)$ then

$$\widehat{\sigma}^i_{-i}(a_l|\theta) = \frac{\sigma_{-i}(a_l|\theta)\sum_{\theta_i} p(\theta_i, \theta_{-i})}{\sum_{\theta_i} p(\theta_i, \theta_{-i})} = \sigma_{-i}(a_l|\theta_{-i}).$$

□

Whenever analogy-expectations are correct, we have a BNE. This may hold even if a player's analogy partition is coarser than some opponent's information partition. Namely, it holds if the opponent types in the analogy class really play a pooling strategy as expected by the analogy expectations. This property characterizes the Bayesian-Nash ABEE.

Proposition 15. *(JK, proposition 3) BNE is ABEE iff for each player, her opponents' types play a pooling strategy in each of the player's analogy classes.*

Formally, BNE is ABEE iff $\theta' \in \alpha_i(\theta)$ implies that $\sigma_{-i}(\theta) = \sigma_{-i}(\theta')$.

Let us next define the $\chi-$cursed equilibrium introduced in ER (2005) and study its relationship to the ABEE.

Definition 14. *(The $\chi-$cursed equilibrium) A strategy profile σ is a $\chi-$cursed equilibrium if for each i, $\theta_i \in \Theta_i$, and if $a_i^* \in supp[\sigma_i(\theta_i)]$*

$$a_k^* \in \arg\max \sum_{\theta_{-i}\in\Theta_{-i}} p_k(\theta_{-i}|\theta_i) \sum_{a_{-i}\in A_{-i}} [\chi\overline{\sigma}_{-i}(\theta_i) + (1-\chi)\sigma_{-i}(\theta_{-i})]u_i(a;\theta) \qquad (4.6)$$

In particular, if $\chi = 1$, then the equilibrium is fully cursed.

The cursed equilibrium cannot be defined as a CEMP but it is similar to the $\text{ABEE}_{\text{An}\leq\text{I}}$ in that each player puts a positive weight on an average strategy in her conjectures - here averaging over types in the player's information set. Intuitively, players may fail to take fully into account how other players actions depend on their private information and this failure may alter behaviour vis-à-vis a Bayesian-Nash equilibrium.

It is straightforward to see that a Bayesian-Nash equilibrium is a cursed equilibrium with $\chi = 0$, by definition. Thus, CE generalizes BNE. Moreover, as acknowledged by JK and ER, the fully cursed equilibrium ($\chi = 1$) can be understood from the learning perspective: when the analogy partitions coincide with the private information partitions the fully cursed equilibrium and the ABEE coincide.

Proposition 16. *(JK, proposition 4) The fully cursed equilibrium, $\varkappa = 1$, coincides with the $ABEE_{An=I}$, i.e. ABEE where for all i and all θ, $\iota_i(\theta) = \alpha_i(\theta)$.*

Proposition 16, reveals that the fully cursed equilibrium requires as much complexity from the beliefs as the least complex ACE. Thus, like the simplest ACE, the fully cursed equilibrium is simpler than the Bayesian-Nash equilibrium.

What about the partially cursed equilibrium? It is natural to think that the partially cursed equilibrium would be more complex than the fully cursed one but less complex than the BNE. ER doubt that there is any learning foundation for cursed equilibrium (ER, p. 1633) whenever the equilibrium is not fully cursed ($\varkappa < 1$). If this was true, it would be impossible to to discuss the question of complexity of a partially cursed equilibrium in terms of the ABEE concept. However, here we show that even the non-fully cursed equilibrium has a learning foundation in the $\text{ABEE}_{\text{An}\leq\text{I}}$: there exists a natural extension of the state space where an equivalent ABEE exists. It turns out that the partially cursed equilibrium is not only more complex than the fully cursed one, but in addition, it is more complex than the BNE.

Proposition 17. *Let $(\Omega, \mathcal{B}, q)$ be a probability space where $\Omega = [0,1] \subset \mathcal{R}$, $\mathcal{B}$ is the set of Borel sets on [0,1] and q is the Lebesque measure. Let $\widetilde{\theta} : \Omega \to \Theta$ so that $p(\theta) = q(\widetilde{\theta}^{-1}(\theta))$. Then for each $\chi-$cursed equilibrium there exists an ABEE such that the equilibrium strategies coincide for the $\chi-$cursed equilibrium and the ABEE.*

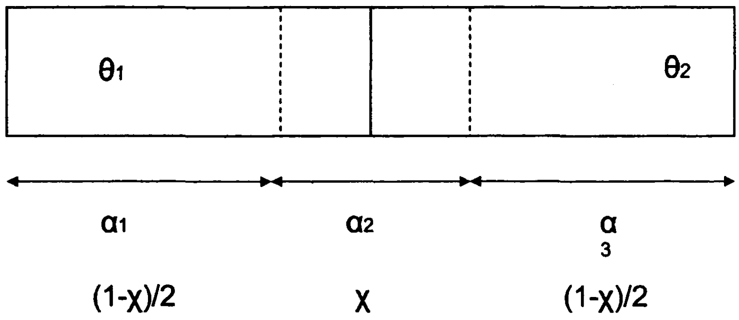


Figure 4.2: Cursed equilibrium as an Analogy-based expectation equilibrium

Proof. In the appendix. □

The idea of the proof is simple. To illustrate (see figure 2), suppose that there are two possible type profiles, the row player has only one type and the column player has two types, each equally likely, as in our example above. Suppose that the underlying elementary states are the real numbers on a unit interval and real numbers in $[0, \frac{1}{2})$ are mapped into type one whereas numbers in $[\frac{1}{2}, 1]$ are mapped into type two. For each $\chi-$cursed equilibrium, consider the analogy partition $\{[0, \frac{1-\chi}{2}), [\frac{1-\chi}{2}, \frac{1+\chi}{2}), (\frac{1+\chi}{2}, 1]\}$. This analogy-based expectation equilibrium corresponds to the $\chi-$cursed equilibrium.

This kind of extension of the state space is rather natural since any model of a real environment could allow the state to capture however large a number (possibly infinite) of characteristics of the actual environment. The payoffs will not depend on the additional characteristics. Moreover, players may not condition thier strategies on these fine characteristics either[11]. Yet, players may well conjecture that opponents are conditioning their strategies on these characteristics. Or, the player's awareness and cognitive skills might differ across players and states (weather conditions, the state of mind, concentration, etc.): two elementary states that are mapped into a given type profile may be bundled into different analogy classes or two states that are mapped into different type profiles may be bundled into the same analogy class. This is closely related to the idea that several belief types may exist for a given payoff type (Mertens and Zamir, 1985). Here belief types emerge in a steady state of a learning process.

Remember that the number of analogy classes reflects a player's belief about the complexity of other players' strategies. This interpretation reveals that the analogy foundation for the fully cursed equilibrium is considerably less complex than that of

[11] Sometimes it is argued that such randomization devices may be used when playing mixed or correlated strategies.

a partially cursed equilibrium. Actually, the analogy foundation for a partially cursed equilibrium is even more complex than that of a Bayesian-Nash equilibrium. The number of each player's analogy classes in a fully cursed equilibrium equals the number of player's information sets, $\#I_i$, so that the opponents' average behaviour can be tracked for each of the player's own types. A Bayesian-Nash equilibrium requires generally at least $\#I_{-i}$ analogy classes of player i so that the choice of each of the opponents' types can be tracked. In a partially cursed equilibrium, there must be at least $\#I_i \times (\#I_{-i}+1)$ analogy classes: for each type of player i there must be $\#I_{-i}+1$ analogy classes - one for each of the opponents' types to track the behaviour of that type profile and one class which only tracks opponents' type profiles on average conditional on player's own type. Thus, if we believe that players tend to stick to simpler theories rather than to more complex ones, the partially cursed equilibrium may seem less appealing.

Above we saw that there are non-cursed ABEE. As a final remark on the relationship between the ABEE and the CE, we show that there are even non-cursed $\text{ABEE}_{\text{An}\leq\text{I}}$. That is, even the set of $\text{ABEE}_{\text{An}\leq\text{I}}$ is a strict superset of the set of CE when we extend the state space to the unit interval of real numbers - even if χ is allowed to take any value $[0,1]$.

Proposition 18. *Let $(\Omega, \mathcal{B}, q)$ be a probability space where $\Omega = [0,1] \subset \mathcal{R}$, $\mathcal{B}$ is the set of Borel sets on [0,1] and q the Lebesque measure. Let $\widetilde{\theta} : \Omega \to \Theta$ so that $p(\theta_i) = q(\widetilde{\theta}^{-1}(\theta_i))$. Then, there may exist $ABEE_{An\leq I}$ that have no $\chi-$cursed equilibrium equivalent for any $\chi \in [0,1]$.*

Proof. Consider the following example. The type is still private infomation to C. There are two players R (row) and C (column). Player C has two types, $(+1)$ and (-1) and the type is private information. The type of C is a random variable on $[0,1]$ with the uniform distribution. $[0, \frac{2}{3})$ is mapped into (-1) and $[\frac{2}{3}, 1]$ is mapped into $(+1)$.

C gets -1 if she mismatches her action with her type and 1 if she matches her action with her type.

The payoff matrixes of R in states (-1) and $(+1)$ respectively are

-1	-1	+1
-1	1+ε,1	$\frac{3}{2}$,-1
+1	1, 1	1,-1

+1	-1	+1
-1	0,-1	1+ε, 1
+1	1,-1	1,1

where $\varepsilon < \frac{1}{10}$.

An $\text{ABEE}_{\text{An}\leq\text{I}}$ is as follows:

The analogy partition of R is $\{[0, \frac{5}{9}), [\frac{5}{9}, \frac{7}{9}), [\frac{7}{9}, 1]\}$.

C plays (-1) at (-1) and $(+1)$ at $(+1)$.

R conjectures that C plays (-1) at $\omega \in [0, \frac{5}{9})$ and $(+1)$ at $\omega \in [\frac{7}{9}, 1]$ and mixes with $\sigma^C_{-1}(\omega) = \sigma^C_{-1}(\omega) = \frac{1}{2}$ at $\in [\frac{5}{9}, \frac{7}{9})$.

R plays $+1$.

Notice that R gets 1 by playing $(+1)$. Thus, $(+1)$ is a best reply to the conjecture if and only if

$$1 \geq (1+\varepsilon)[1 - \frac{2}{9}] + \frac{1}{9}[\frac{1}{2}(1+\varepsilon) + \frac{1}{2}\frac{3}{2}] + \frac{1}{9}[\frac{1}{2}(1+\varepsilon)]$$

or equivalently $\varepsilon \leq \frac{1}{32}$.

Yet, R cannot play $(+1)$ in any cursed equilibrium. The expected virtual game payoff of R for playing -1 in any $\chi-$cursed equilibrium equals $(1-\chi)(1+\varepsilon) + \chi(\frac{2}{3}\frac{3}{2})$ which is strictly greater than 1, the payoff for playing $(+1)$.

Since, in any χ-cursed equilibrium, the average strategy must weight C's strategies using the probabilities of C's types, the implicit expectations of any χ-cursed equilibrium of the original state space cannot correspond to the conjectures generated by the $\mathrm{ABEE}_{\mathrm{An} \leq \mathrm{I}}$ in this example. □

Payoff-confirming ABEE

Consider again example 1 and suppose that state is private information to the column player. Notice that there may be a substantial curse in an ACE. C plays her dominant strategy (-1) at (-1) and $(+1)$ at $(+1)$. Now, in an ACE, a conjecture that C plays $(+1)$ at (-1) and (-1) at $(+1)$ is consistent. The best reply of playing (-1) with probability one leads to a difference of -2 in perceived and actual expected payoffs.

Even in an $\mathrm{ABEE}_{\mathrm{An} \leq \mathrm{I}}$, there may be curse, even if less so that in the ACE above: Suppose still that the state is private information to the column player and the row player bundles both states into a single analogy class. R expects mixing with equal probabilities by the column player. Thus, any action is a best reply to her conjectures, including the strategy (-1) at $(+1)$ and (-1) at (-1). This equilibrium leads to a curse - a difference of -1 in perceived and actual expected payoffs.

Consider the same analogy expectations as above. However, suppose now that the row player best-replies by mixing with equal probabilities. In this equilibrium, despite the incorrect conjectures, there is no curse in the sense that the subjective expected payoff is equal to the objective expected payoff.

If we model a learning process where the same player plays repeatedly against varying opponents, arguably, each player should observe and remember at least her own payoffs. This is because the successes and the failures are what players care for and what they remember: if they do not, why should they best reply in the first place. If there is equilibrium curse, the player should eventually realize this and abandon her incorrect conjectures and the overly simplistic view about opponent's behaviour.

This is what motivates a refinement of the ABEE, the payoff-confirming ABEE (PCABEE). In this section we formally define the concept. We show that there can be $\text{PCABEE}_{\text{An}\leq\text{I}}$ that are not Bayesian-Nash. We characterize the payoff-confirming ABEE in a class of games and present a sufficient conditions for an ABEE to be a PCABEE in static incomplete information games. Finally in the next subsection, we consider a modification of Akerlof's (1970) lemons trading game and we show that, even if in the ABEE that corresponds to the fully cursed equilibrium of the game the selection problem may be alleviated, this equilibrium is not payoff-confirming.

The PCABEE studies a mild robustness check of the ABEE, where each player keeps track of her own payoffs but fails detect how her payoffs are correlated with other signals that she learns from each previous round of play: we suppose that payoffs are perceived in a separate perception.

Definition 15. *(PCABEE) Payoff-Confirming ABEE is CEMP where for each i, $L_i = 2$ and*

1. $\gamma_i^1 : A \times \Theta \to A_{-i} \times An_i$, $\gamma_i^1(a, \theta) = (a_{-i}, \alpha_i(\theta))$
2. $\widehat{\sigma}_{-i}(a_{-i}|\theta) = \overline{\sigma}_{-i}(\alpha_i(\theta))$
3. $\gamma_i^2 : A \times \Theta \to \mathbb{R}$, $\gamma_i^2(a, \theta) = (u_i(a, \theta))$

We also define the PCABEE with finer than private information analogy partition[12].

Definition 16. *($\text{PCABEE}_{An\leq I}$) If in an PCABEE $\theta_i' \neq \theta_i'' \implies \theta' \notin \alpha_i(\theta'')$ then it is an PCABEE with finer than private information analogy partition.*

In a PCABEE, the marginal sample distribution must coincide with the conjectured payoff distribution. In the introductory example of this subsection, the row player will not realize that her conjecture about the other is incorrect even if she observes her

[12]Notice that conceptually the $\text{PCABEE}_{\text{An}\leq\text{I}}$ is not an ACE even if an $\text{ABEE}_{\text{An}\leq\text{I}}$ is. The $\text{PCABEE}_{\text{An}\leq\text{I}}$ allows for players to observe their own payoffs in a separate perception whereas multiple perceptions are not allowed for in the ACE. However, any $\text{PCABEE}_{\text{An}\leq\text{I}}$ prediction is also predicted by some ACE since any $\text{ABEE}_{\text{An}\leq\text{I}}$is an ACE.

payoffs. According to her conjectures, the row player expects payoffs -1 and +1 with equal probabilities which are in fact the probabilities of these payoffs in equilibrium. Thus, we see that there are $\text{PCABEE}_{\text{An}_{\leq I}}$ which differ from Bayesian Nash equilibrium.

Proposition 19. *There can be Payoff-Confirming $ABEE_{An_{\leq I}}$ that are not Bayesian-Nash.*

If payoffs are generic, there is a one to one mapping from payoffs to outcomes. Thus, the payoff realizations reveal the opponents' strategies perfectly and conjectures about opponents' strategies must be correct. Hence, if payoffs are generic and observed, the set of conjectural equilibria collapses to the set of Bayesian-Nash equilibria.

Proposition 20. *If payoffs are generic and observed, then CEMP coincides with the Bayesian-Nash equilibria.*

Proof. Obvious. □

If a payoff-confirming equilibrium differs from a Bayesian-Nash equilibrium, some degenerate payoffs are needed. Notice that if payoffs are generic conditional on player's own type but not unconditionally, then the set of PACE collapses to the set of Bayesian-Nash equilibria but the set of PCABEE does not. This is shown in the following example.

Example 2. *Two players meet to share a snack. Player one decides what the two players will eat: either hot-dogs H or a burger B. Player two decides what they will drink: either pepsi P or coke C. The preferences depend on the state and are are given in the matrixes below. The state is complete information and drawn randomly. The probability of state one is $q \leq \frac{3}{4}$. The decisions are made simultaneously and non-cooperatively.*

The stage game in state 1 is

θ_1	P	C
H	*4,3*	*2,2*
B	*1,1*	*3,4*

(4.7)

and in state 2 it is

θ_1	P	C
H	*1,2*	*3,3*
B	*4,4*	*2,1*

(4.8)

In the (Bayesian) Nash equilibria of the game, players choose either hotdogs and pepsi or burgers and coke in state one and they choose either burgers and pepsi or hotdogs and coke in state two.

The following is a payoff-confirming analogy-based expectation equilibrium: players have coarse analogy partitions; hotdogs and coke are chosen in state one - burgers and pepsi are chosen in state two; player one expects pepsi to be chosen with probability $1 - q$ in both states - player two expects hotdogs to be chosen with probability q in both states. Since hotdogs and coke are chosen in state one, there cannot exist an equivalent Bayesian-Nash equilibrium.

To see that this is indeed an equilibrium, each player plays a separation pure strategy and thus the consistency of beliefs requires that the action chosen in each state is expected to be chosen with the probability of that state. Each player's strategy is a best response to these expectations if and only $q \leq \frac{3}{4}$.

To see that the ABEE is payoff confirming, the probability of payoff 2 is q and the probability of payoff 4 is $1 - q$. Each player expects payoff 2 to result with probability $qq + (1-q)q = q$ and payoff 4 to result with probability $q(1-q) + (1-q)(1-q) = (1-q)$.

So, each of the two players believes that the opponent is choosing the snack randomly not taking into account the state. Yet, each player takes the state into account. Player one observes player two choosing coke in fraction q of rounds and pepsi in fraction $1 - q$ of rounds. She never gets pepsi with a burger but she does not notice that, since she only recalls whether she was happy about the meal or not. On the other hand she expects to be happy about the meal fraction $1 - q$ of times which indeed she is. The payoff sequence drawn from the true distribution supports the false beliefs that the opponent is randomizing.

Proposition 21. *If payoffs are generic conditional on private information, then*

- *the set of PACE coincides with the set of Bayesian-Nash equilibria.*
- *there can be PCABEE that are not Bayesian-Nash equilibria.*

As a corollary to the proposition above, we learn that, even if every ABEE$_P$ is an ACE, every PCABEE is not.[13]

Corollary 4. *There may exist PCABEE that are not ACE.*

When is an ABEE a PCABEE also? When an ABEE is a Bayesian Nash equilibrium, then conjectures about others must be correct by the definition of the Nash equilibrium. Then surely if conjectures are correct, payoff information cannot reveal

[13] In fact there may exist PCABEE that do not correspond to any self-confirming equilibria as defined in DFL (2004).

anything that was not known already. Thus, an ABEE which is Bayesian-Nash must be payoff-confirming also. Propositions 14 and 15 identify two such cases: the first, the ex-post observation of types more precise than the interim information when strategy is chosen; the second, pooling strategies. Thus only if the analogy partition is coarser than some opponents' information partition and the opponent plays a separation strategy, an ABEE might differ from a Bayesian-Nash equilibrium. In this case, according to proposition 20, some payoff genericity is needed in order for a PCABEE to differ from a Bayesian-Nash equilibrium. Otherwise, the payoff information would reveal any mistaken conjectures. In two-player two-action two-state games of incomplete information, we can give a characterization of the set of pure strategy ABEE that are PCABEE.

Proposition 22. *Let $N = 2$, $\Theta = \{\theta^1, \theta^2\}$ and $A_i = \{a_i^1, a_i^2\}$. Suppose that s is a pure strategy profile of an ABEE.*

The ABEE is payoff-confirming if and only if

- s is a Bayesian-Nash equilibrium

or

- for each i such that $s_i(\theta^m) \neq s_i(\theta^n)$ and $An_j = \{\{\theta^1, \theta^2\}\}$

$$\text{for all } m,\ u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) = u_j(s_j(\theta^m), s_i(\theta^m); \theta^m) \tag{4.9}$$

or

$$\text{for all } m,\ u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) = u_j(s_j(\theta^n), s_i(\theta^n); \theta^n)) \tag{4.10}$$

where $r_i(\theta^m)$ is the action not chosen by i at θ^m.

Proof. In the appendix. □

Each player is trying to detect a correlation between opponents' actions and the opponents' type profile using payoff realizations as evidence. If there is correlation, then clearly, the presumption that the opponents' play the average strategy in each state of the analogy class must be incorrect. The fact that either condition (4.9) or condition (4.10) holds prevents inferring anything about the joint distribution: If condition (4.9) holds and a player's payoff is the same in a given node whatever the opponent chooses, there is no strategic uncertainty about one's own payoffs in each state. Alternatively, if (4.10) holds and the payoff is the same whatever the state, given each action of the opponent, there is no exogenous uncertainty about one's own payoffs given each action of the opponent. In either case, payoffs do not provide any additional information about the joint distribution of actions and types of others and only marginal sample distributions of types profiles and actions profiles of others are known in each analogy class.

In games with more states, more players and more actions, conditions parallel to (4.9) or (4.10) are sufficient but not necessary for an ABEE to be PCABEE.

Proposition 23. *Let in an ABEE σ differ from a Bayesian-Nash equilibrium. If for each j and α_j such that there are $\theta^m, \theta^n \in \alpha_j$ with $\sigma_{-j}(\theta^m) \neq \sigma_{-j}(\theta^n)$,*

- *either for all $\theta \in \alpha_j$ and for all action profiles of players other than j, $a^*_{-j} = (a^*_1, ..., a^*_{j-1}, a^*_{j+1}, ..., a^*_N)$ such that for each $i \neq j$ there is $\theta' \in \alpha_j$ such that $a^*_i \in supp\sigma_i(\theta')$, we have for all $a_j \in supp\sigma_j(\theta)$*

$$u_j(a_j, a_{-j}; \theta) = u_\theta \tag{4.11}$$

- *or for all action profiles of players other than j, $a^*_{-j} = (a^*_1, ..., a^*_{j-1}, a^*_{j+1}, ..., a^*_N)$ such that for each $i \neq j$ there is $\theta' \in \alpha_j$ such that $a^*_i \in supp\sigma_i(\theta')$, we have for all $\theta \in \alpha_j$ and $a_j \in supp\sigma_j(\theta)$*

$$u_j(a_j, a_{-j}; \theta) = u_{a_{-j}}, \tag{4.12}$$

then the ABEE is payoff-confirming.

Proof. In the appendix. □

Now suppose that, for any separating strategy and a coarse analogy partition of the opponent such that an opponent plays different strategies in two nodes of an analogy class, either (4.11) or (4.12) holds. Again, the former condition aggregates over strategic uncertainty and the latter condition aggregates over exogenous uncertainty. Thus, player cannot infer anything about the joint distribution of type profiles and action profiles of other players in each analogy class.

Nevertheless the expected and the sample marginal payoff distributions may coincide even if an ABEE is not a Bayesian-Nash equilibrium and neither (4.11) nor (4.12) holds. This is illustrated in the following example.

Example 3. *There are two players and three states of nature, $\{\theta_1, \theta_2, \theta_3\}$ each drawn with probability $\frac{1}{3}$. This prior is known to both players and the realization of the state is revealed to both. Each state of nature is associated with a simultaneous move two-player game . In each of the games, each player has three actions from which to choose,*

$A_i = \{a_1, a_2, a_3\}$. Player two gets payoff $+1$ if he matches the state and -1 if he does not. Payoffs of player one are indicated in matrixes below.

θ_1	a_1	a_2	a_3
a_1	1	0	-1
a_2	2	-2	-2
a_3	2	-2	-2

θ_2	a_1	a_2	a_3
a_1	-1	0	-1
a_2	0	-1	1
a_3	-1	0	-1

θ_3	a_1	a_2	a_3
a_1	-1	-1	1
a_2	-1	-1	1
a_3	-1	1	0

Consider the following equilibrium: player one has the coarsest analogy partition and player two has the finest. Each player plays a pure separation strategy, each player's choice at state θ_j is a_j. The conjecture of player one is $\widehat{\sigma}_2(a_j|\theta_k) = \frac{1}{3}$ for all $j, k = 1, 2, 3$. Player two matches her action with the state and thus he is best replying. Also, player one is best replying since in state θ_k choosing a_k gives expected payoff zero whereas other actions give negative expected payoffs given that player one expects two to choose each action with probability $\frac{1}{3}$. Thus, this is an ABEE.

Furthermore, only outcomes (a_k, a_k, θ_k), $k = 1, 2, 3$ have a positive probability and each results with probability $\frac{1}{3}$. Thus, the sample distribution of player one's payoff assigns probability $\frac{1}{3}$ to payoffs -1, 0 and 1 respectively. Since this is player one's expectation of payoffs given his equilibrium strategy, we have a PCABEE. Yet, neither is there u_k such that for all l, for all k, $u_1(a_l, a_k, \theta_l) = u_k$, nor is there u_k such that for all l, for all k, $u_1(a_l, a_l, \theta_k) = u_k$. Thus, we have a PCABEE even if neither condition (4.11) nor condition (4.12) in proposition (23) are satisfied.

Adverse Selection

Consider the following *trading game* with one-sided asymmetric information. This is a modification of Akerlof's (1970) trading game. The seller values the object at s while the buyer's valuation is $v = s + x$ where s is the realization of a random variable $\widetilde{s}$ uniformly distributed on [0,1] and x is a constant, $0 < x < \frac{1}{2}$. Seller's value s is private information to her. The buyer knows neither s nor v. The buyer and the seller simultaneously make offers to buy at price p and to sell at price p_S, respectively. If $p_S > p$ the seller keeps the object, and the buyer obtains her reservation utility of zero. If $p_S \leq p$, the buyer gets the object and her payoff is $u(p, v) = v - p$. Throughout this section we suppose that the seller plays her weakly dominant strategy and chooses price equal to her valuation of the object $p_S = s$.

A sophisticated buyer realizes that there is a selection effect. Her expected payoff equals

$$u^{NE}(p, \sigma_S) = \Pr(\widetilde{s} \leq p)[\mathrm{E}(\widetilde{v}|\widetilde{s} \leq p) - p]$$

In a Bayesian-Nash equilibrium of this game, the buyer offers price $p^{BNE} = x$ and her expected equilibrium payoff is $u(x, v) = \frac{1}{2}x^2$. In an $\text{ABEE}_{An_B=I_B}$ where analogy partition of the buyer coincides with her private information partition (equivalently a cursed equilibrium), the buyer does not realize that the offer she chooses selects the average quality of the object. She rather believes that the expected value is given by the unconditional expectation $\mathrm{E}(\widetilde{v})$. The perceived expected payoff is then

$$u^{CE}(p, \sigma_S) = \Pr(\widetilde{s} \leq p)[\mathrm{E}(\widetilde{v}) - p]$$

The buyer's optimal offer is $\frac{1}{2}x + \frac{1}{4}$ which is higher than the Bayesian-Nash equilibrium offer since $x < \frac{1}{2}$. The fact that there may be more trade in an analogy-based expectation equilibrium or in a cursed equilibrium than in a Bayesian-Nash equilibrium in this context is reported both by JK and by ER.

The payoff-confirming $\text{ABEE}_{An_B=I_B}$ requires that the expected payoff is correct. It is easy to verify that in the $\text{ABEE}_{An_B=I_B}$ the perceived expected payoff is higher than the actual expected equilibrium payoff. The buyer suffers from curse due to the fact that she fails to take into account the selection effect. An implication of the equilibrium curse is that this theory about seller behaviour is too simplistic to be sustained when payoffs are observed in the learning process. That is, there is no a payoff-confirming analogy-based expectation equilibrium such that $An_B = I_B$.

4.4.2 Extensive form games (SCE, ABEE)

In static games, if outcomes (terminal histories) are perceived, then CEMP collapses to (Bayesian) Nash equilibria. In extensive form games, observing terminal histories is not sufficient for the steady states to collapse into the set of Nash equilibria. It is natural that if players can track only paths of play and thus have no data about opponents' actions or strategies at histories off the equilibrium path, the conjectures about off-path behaviour may be incorrect. *Self-confirming equilibrium* (Fudenberg, Levine, 1993) is the solution concept where players observe the full terminal history after each round of play. The full history determines the players' choices at each history along the equilibrium path. Yet, off-path behaviour is not observed. Players are myopic or at least not fully patient so that they do not have sufficient incentives to experiment in order to learn about opponents' off-path strategies. More patience would provide more incentives to experiment in order to learn about opponents' actual strategies off the equilibrium path.

Farsightedness and experimentation are discussed in section 4.4.3. In this section we focus on myopic players.

Notice that the Nash equilibrium is not very demanding about off-path behaviour either, unlike the *subgame-perfect equilibrium* (Selten, 1975) or the *sequential equilibrium* (Kreps and Wilson, 1982). The former requires off-path conjectures and strategies to be credible in that the strategies must form a Nash equilibrium in every subgame. The latter requires in addition that beliefs must be updated using Bayes's rule when players tremble in their strategy choices. The Nash equilibrium only requires that players have correct conjectures: that strategies are chosen *independently* (strategy profile is a product measure), that two players have the same conjectures about the play of a third (*consistency*) and that behaviour at each history is supported by the same beliefs (*unitary beliefs*). Indeed, Fudenberg and Levine (1993a) show that these are the only sources how a self-confirming equilibrium may fail to be a Nash equilibrium.

Jehiel (2005) argues that even finite extensive form games can be very complex (eg. chess). Thus player's cognitive limitations may necessitate simplifications of the game. In particular, as in static games of incomplete information, Jehiel (2005) assumes that players bundle opponents' decision nodes into analogy classes. Whereas in static incomplete information games the decision nodes are type profiles, in extensive form games decision nodes are histories. Thus, the probability of reaching an analogy class is endogenous and determined by players strategies.

Keeping track of opponents' behaviour in classes of histories only rather than at each history separately leads to additional sources how an equilibrium may fail to be a Nash equilibrium. Thus, there are ABEE which are not SCE.

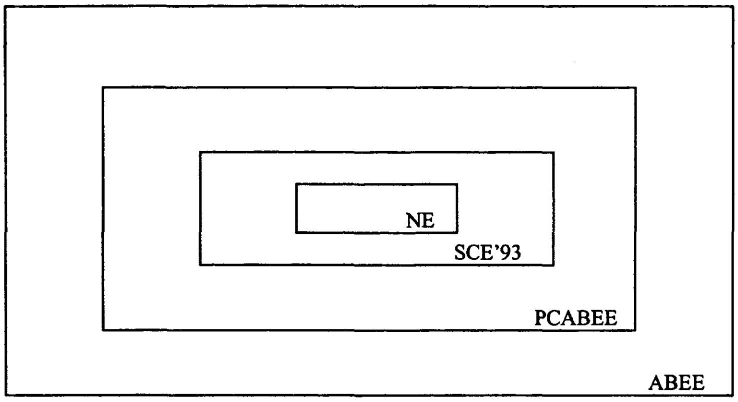
We also consider the payoff-confirming ABEE, the refinement of ABEE, in extensive form games. Each player observing her own payoff takes the ABEE a step closer to the SCE since this observation provides indirect information about the terminal node as an inverse image of the payoff. Yet still, there are PCABEE that are not SCE. (See figure 4.3).

Self-confirming equilibrium

In this section we define and briefly study the self-confirming equilibrium (Fudenberg and Levine, 1993a,b).

Definition 17. *$(\sigma, \widehat{\sigma})$ is a self-confirming equilibrium if and only if $(\sigma, \widehat{\sigma})$ for each i*

σ_i is a best response to $\widehat{\sigma}_i$



ABEE - Analogy-based expectation equilibrium
PCABEE - Payoff-confirming ABEE
SCE'93 - Self-Confirming equilbrium (Fudenberg and Levine, 1993)
NE - Nash equilibrium

Figure 4.3: Equilibrium predictions in extensive form games

$\widehat{\sigma}^i_{-i}(h) = \sigma_{-i}(h)$ *if* $P^\sigma(h) > 0$

In a self-confirming equilibrium, the beliefs about each opponent's behavioural strategy must be correct in nodes along the equilibrium path. Yet, off the equilibrium path, the beliefs may be arbitrary. Each player's strategy must be a best response to such beliefs.

The self-confirming equilibrium is a straightforward generalization of the Nash equilibrium. The Nash equilibrium requires that beliefs about the opponents' behavioural strategies are correct even off the path, for all $h \in H \backslash H^i$, $\widehat{\sigma}_{-i}(h) = \sigma(h)$.

Fudenberg and Levine point out that there are three ways how a self-confirming equilibrium may fail to be a Nash equilibrium: (i) two players have inconsistent conjectures about the play of a third, (ii) two strategies that are played with a positive probability are best-replies to two different conjectures about the opponents' play, (iii) a player's conjectures about the play of her opponents' may be correlated.

In a *consistent* self-confirming equilibrium no two players can disagree about the strategy of a third (and furthermore conjectures must be correct at any history that can be reached given that a player plays his strategy).

Definition 18. *A self-confirming equilibrium* σ *is consistent if for all players* i *and each* $s_i \in supp(\sigma_i)$ *there are beliefs* $\widehat{\sigma}^i$ *such that*

- s_i *is a best response to to* $\widehat{\sigma}^i$ *and*
- $\sigma_j(h_j) = \widehat{\sigma}^i_j(h_j)$

for all $j \neq i$ and h_j such that there is σ_{-i} such that $P^{s_i,\sigma_{-i}}(h_j) > 0$.

In general the belief is a probability distribution, $\widehat{\mu}_i$, over other players' (mixed or behavioural) strategies. In principle it might be the case that two actions that a player plays with a positive probability might be supported by two different conjectured strategies which are drawn according to $\widehat{\mu}_i$. If no player's two equilibrium actions are supported by different conjectures, the beliefs are *unitary.*

Definition 19. *A self-confirming equilibrium σ has unitary beliefs if for all players i the same conjecture $\widehat{\sigma}^i$ can be used to rationalize every $s_i \in \sigma_i$.*

Finally, if conjectures are independent, then correlation is ruled out.

Definition 20. *A self-confirming equilibrium σ has independent beliefs if for all players i and all $s_i \in$supp(σ_i), the associated conjectures satisfy*

$$\widehat{\sigma}^i_{-i}(\times_{j\neq i} \times_{h_j \in H_j} \Delta(A_j(h_j))) = \times_{j\neq i}\widehat{\sigma}^i_j(\times_{h_j \in H_j}\Delta(A_j(h_j)))$$

By using simple examples, Fudenberg and Levine show the sufficiency of each of (i)-(iii) to reach non-Nash self-confirming equilibria. On the other hand, they show the necessity of (i)-(iii) in escaping Nash equilibria as follows

Theorem 6. *(Fudenberg, Levine, 1993a, Theorem 4) Every consistent self-confirming equilibrium with independent, unitary beliefs is equivalent to a Nash equilibrium.*

Analogy-based expectation equilibrium

In the analogy-based expectation equilibrium, each decision node where an opponent moves belongs to one and only one analogy class, and thus, a player's analogy classes are described by her analogy partition An_i. What is specific about extensive form games is that which decision nodes are reached and how often now depends endogenously on players' strategies not just on exogenous uncertainty.

As before in section 4.4.1, an analogy class of player i is denoted α_i and the analogy class where decision node h belongs to is denoted by $\alpha_i(h)$. The action sets at two decision nodes that belong to the same analogy class are assumed to coincide $A(h) = A(h')$ if $h' \in \alpha_i(h)$. The analogy expectation of player i in each analogy class α_i is a probability measure over the action space of that class, $\widehat{\sigma}^i(\alpha_i)$ where the probability that action $a_l \in A(\alpha_i)$ is chosen is denoted $\widehat{\sigma}^i(a_l|\alpha_i)$. This probability distribution captures the average behaviour in the analogy class.

Definition 21. *Player i's analogy-based expectation* $\widehat{\sigma}^i$ *is consistent with the strategy profile* σ *if and only if for all* $\alpha_i \in An_i$

$$\widehat{\sigma}^i(a_l|\alpha_i) = \frac{\sum_{h' \in \alpha_i} \sigma_j(l|h')P^{\sigma}(h')}{\sum_{h' \in \alpha_i} P^{\sigma}(h')}$$

whenever $P^{\sigma}(h) > 0$ *for some* $h \in \alpha_i$.

Thus, we require the analogy expectations to coincide with the average behaviour in the class for each class which is reached with a positive probability. We do not put any requirements on the analogy expectations in the analogy classes that are reached with a zero probability. By construction, if $h' \in \alpha_i(h)$ then $\widehat{\sigma}^i(h') = \widehat{\sigma}^i(\alpha_i(h))$.

Jehiel (2005) provides two alternative consistency requirements for extensive form games. In the strong one players play with trembling hands so that each behavioural strategy must be completely mixed. However, in this chapter, we consider the weaker one where all actions are not necessarily in the support of the behavioural strategy and consistency is required only at analogy classes that are reached with a positive probability.[14]

Any triplet of strategies, beliefs and analogy partitions, $(\sigma, \widehat{\sigma}, An)$, such that, firstly, each player's beliefs about the average behaviour in each analogy class are consistent, secondly, the beliefs of the opponent's behaviour in any node of an analogy class equals the average behaviour in the class, and thirdly, each player is best responding to the beliefs induced by her analogy expectations, is an analogy-based expectation equilibrium.

Definition 22. $(\sigma, \widehat{\sigma}, An)$ *is an analogy-based expectation equilibrium if and only if for each i*

σ_i *is a best response to* $\widehat{\sigma}^i$

for each $h \in H \backslash H^i$, $\widehat{\sigma}^i(h) = \widehat{\sigma}^i(\alpha_i(h))$

$\widehat{\sigma}^i$ *is consistent with* σ

In section 4.4.1 we pointed out that ABEE is a generalization of the Bayesian-Nash equilibrium. In extensive form games, the ABEE is a generalization of the self-confirming equilibrium (FL, 1993) which in turn generalizes BNE. Similar logic holds here between ABEE and SCE as between ABEE and BNE in static games. We can identify two cases

[14]In addition to ruling out trembling hands, the concept differs from Jehiel (2005) in that in the present model the analogy partition is part of the solution concept, not exogenous. Indeed, Jehiel (2005) considers the ABEE concept incomplete to the extent that it does not explain why and which analogy partitions players end up having. Section 6 of Jehiel (2005) presents some avenues for endogenizing the analogy partition.

where a SCE is an ABEE: the first, if players analogy classes are singletons; the second, if opponents play the same strategy (pooling) at every positive probability history of the analogy class. These results are reported in propositions 24 and 25, respectively.

Proposition 24. *Every SCE is ABEE.*

Formally, let $(\sigma, \widehat{\sigma})$ *be a self-confirming equilibrium. If for all* i *and* $h \in H \backslash H^i$, $\alpha_i(h) = \{h\}$, *then* $(\sigma, \widehat{\sigma}, An)$ *is an ABEE.*

Proof. Let $(\sigma, \widehat{\sigma}, An)$ be an $ABEE$ and for each i and $h \in H \backslash H^i$, $\alpha_i(h) = \{h\}$. If $P^\sigma(h) > 0$ then

$$\widehat{\sigma}^i(l|\{h\}) = \frac{\sigma_j(l|h)P^\sigma(h)}{P^\sigma(h')} = \sigma_j(l|h)$$

Thus if $P^\sigma(h) > 0$, $\widehat{\sigma}^i(a_l|\{h\}) = \sigma_j(a_l, h)$. □

Let $(\sigma, \widehat{\sigma}, An)$ be an ABEE such that in every analogy class, if there are two nodes in the class that are reached with a positive probability, then the behavioural strategy is the same in the two nodes. In this case, $(\sigma, \widehat{\sigma})$ is a SCE.

$$\beta(a_l|\alpha_i(h)) = \frac{\sum_{h' \in \alpha_i(h)} P^\sigma(h')\sigma(a_l, h')}{\sum_{h' \in \alpha_i(h)} P^\sigma(h')} = \sigma_{-i}(a_l, h)$$

Here, the expectations along the equilibrium path will be correct since for each node along the path, the equilibrium behavioural strategy is chosen in every positive probability node of the analogy class.

Proposition 25. *If the opponents' behaviour strategies coincide at all probability histories of an analogy class, then the ABEE is SCE.*

Formally, let $P^\sigma(h') > 0$. *If* $h' \in \alpha_i(h)$ *implies that either* $\sigma_j(h) = \sigma_k(h')$ *or* $P(h) = 0$, *then* $ABEE$ *is* SCE.

Along the lines of definition 15 in section 4.4.1, we define *a payoff confirming analogy-based expectation equilibrium* in extensive form games as an ABEE where the subjective and the objective payoff distributions coincide.

Definition 23. *An analogy-based expectation equilibrium is payoff confirming if and only if for all* u_i *and for* $i = 1, 2$, *we have*

$$\sum_{\{z|u_i(z)=u\}} P^\sigma(z) = \sum_{\{z|u_i(z)=u\}} P^{\sigma_i, \widehat{\sigma}^i}(z).$$

In section 4.4.1, proposition 23 establishes two alternative sufficient conditions for an ABEE to be PCABEE in static games of incomplete information when the analogy

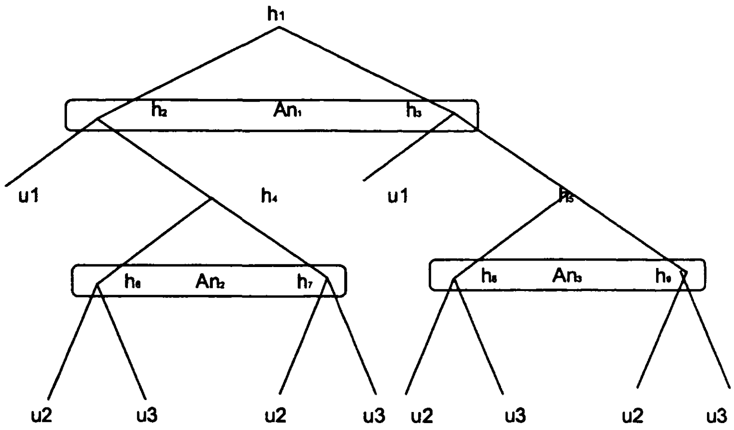


Figure 4.4: Example 1, condition (4.14) is satisfied

partition is coarse and the opponent plays a separation strategy. In proposition 26, we generalize these conditions to extensive form games. We illustrate these by means of examples and we also show that these conditions are not necessary: there are other ways how an ABEE can be a PCABEE.

Proposition 26. *Let $(\sigma, \widehat{\sigma}, An)$ be an ABEE. If for every i and for every α and for every $h' \in \alpha_h$ such that $P^{\sigma_i, \widehat{\sigma}^i}(h) > 0$ and such that $\sigma(h) \neq \widehat{\sigma}^i(h)$ we have that $P^{\sigma_i, \widehat{\sigma}^i}(h|\alpha) = P^{\sigma}(h|\alpha)$ and*

(1) either for every u for every $a, a' \in A(\alpha)$

$$\sum_{\{z|u_i(z)=u\}} P^{\sigma_i, \widehat{\sigma}^i}(z|h', a) = \sum_{\{z|u_i(z)=u\}} P^{\sigma_i, \widehat{\sigma}^i}(z|h', a') \tag{4.13}$$

(2) or for every u

$$\sum_{\{z|u_i(z)=u\}} P^{\sigma_i, \widehat{\sigma}^i}(z|h', a) = \sum_{\{z|u_i(z)=u\}} P^{\sigma_i, \widehat{\sigma}^i}(z|h, a) \tag{4.14}$$

then $(\sigma, \widehat{\sigma}, An)$ is PCABEE.

Again, the player tries to use payoff realizations to infer whether there is correlation between action and type profiles of other players in each analogy class. Conditions (4.13) and (4.14) make such inferences impossible.

We will now approach the issue by means of three examples. In the first of our examples, depicted in figure 4.4, condition (4.14) is satisfied. Player one chooses at h_1, h_4, h_5. Player two chooses at h_2, h_3 and at h_6 to h_9. We suppose that the equilibrium strategies at h_4 and h_5 are equal, $\sigma(h_4) = \sigma(h_5)$. Also, we suppose that $\sigma(h_6) = \sigma(h_8)$ and $\sigma(h_7) = \sigma(h_9)$. The analogy partition of player one is $\{\{h_2, h_3\}, \{h_6, h_7\}, \{h_8, h_9\}\}$ and

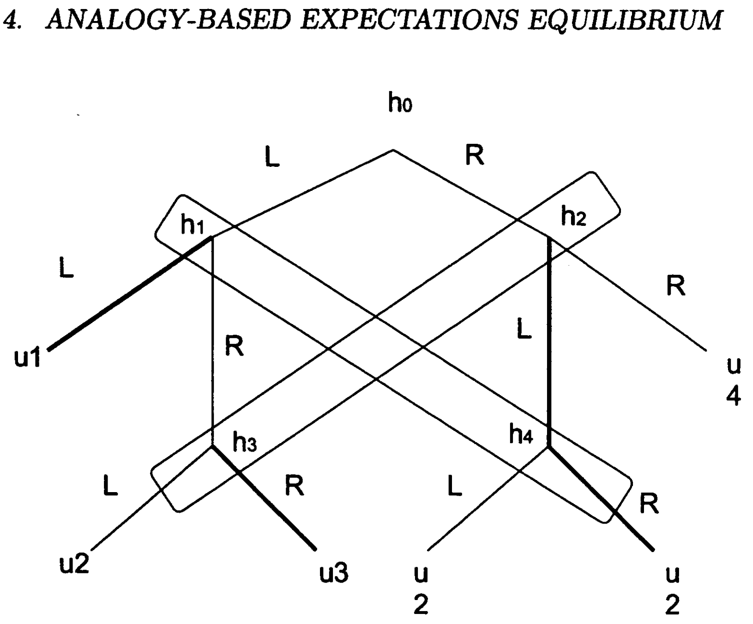


Figure 4.5: Example 2, condition (4.14) is satisfied

player two has the finest analogy partition. Here the analogy classes are dispersed in distinct subgames of the game tree. Thus, when exiting each analogy class from each of its histories, we can truncate away the subtree and associate each action at each history of the class a conjectured payoff probability distribution. Now if, in each analogy class, these distributions are the same for each history and for a given action, then condition (4.14) obviously holds and ABEE is PCABEE.

To see that being able to truncate other analogy classes away is not necessary for an analogy based expectation equilibrium to satisfy (4.14), consider the example in figure 4.5. There are three players in this game. Player one chooses at node h_0. Player two chooses at node h_1 and h_4 and player three chooses at nodes h_2 and h_3. The analogy partition of player one is $\{\{h_1, h_4\}, \{h_2, h_3\}\}$. To fix ideas, let us suppose that the analogy partitions of players two and three are the finest. Suppose that in an analogy expectation equilibrium player two chooses L at h_1 and R at h_4. Player three chooses L at h_2 and R at h_3. Player one chooses L with probability p. This ABEE is a PCABEE since the condition of the proposition is satisfied at both analogy classes. For instance, at each history of the analogy class $\{h_1, h_4\}$, conditional on each of the two actions, the payoff distributions are degenerate and they coincide. For instance for action R, the perceived payoff at both histories $h_1$and h_4 is $u2$ since at the only positive probability history of the analogy class $\{h_2, h_3\}$, L is chosen with probability one.

On the other hand, by means of the example in figure 4.6, we can show that the condition in proposition 26 is by no means necessary and for different reasons than in

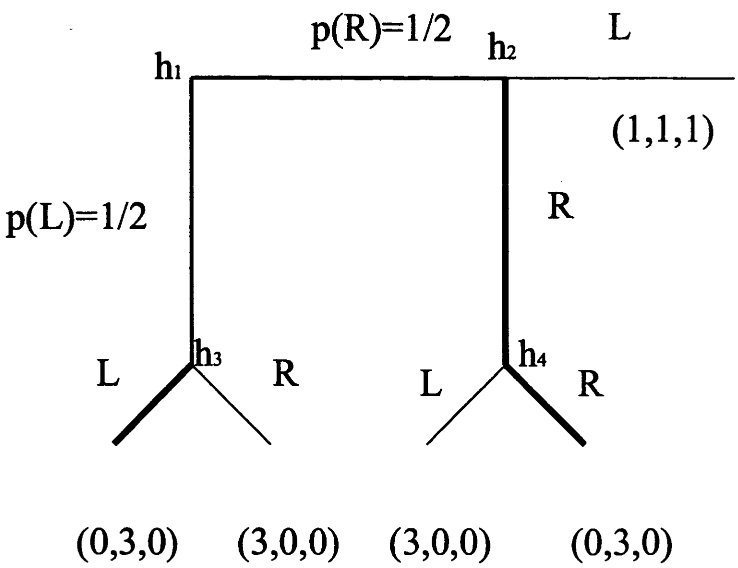


Figure 4.6: PCABEE and the horse

example 3 in section 4.4.1. Consider the following game which is adapted (the labeling of actions is altered, the information partition of player 3 is the finest, and payoffs are slightly modified) from example 2 in Fudenberg and Levine (1993a). In the original game put forward by Fudenberg and Levine (1993a), player three's information set consists of histories h_3 and h_4. With such an information partition, one playing R and player two playing L is a self-confirming equilibrium if player one believes that three chooses R and two believes that three pays L with probability greater than $\frac{2}{3}$. This violates the consistency requirement and points out one source how a SCE may fail to be a Nash equilibrium. This SCE is also an ABEE.

By supposing singleton information sets of player 3 (as in figure 4.6) and assuming suitably chosen analogy partitions we can show that there are ABEE and even PCABEE that are not SCE. Furthermore we will see that, when a PCABEE is not SCE, neither (4.13) nor (4.14) holds. Suppose now that player one has a coarse analogy partition, $\{\{h_2,\ h_3, h_4\}\}$. On the other hand, player two and player three have fine partitions - in particular h_4 is in a singleton analogy class and thus player two's beliefs are correct at that node. The reader can verify that $\sigma_1(L|h_1) = \frac{1}{2}$, $\sigma_1(R|h_2) = 1$, $\sigma_1(L|h_3) = 1$, $\sigma_1(R|h_4) = 1$ is an analogy based expectation equilibrium. However this equilibrium is not payoff-confirming since player one expects payoff 1 to result with positive probability. Yet, if player one's analogy partition is modified to $\{\{h_2\}, \{h_3, h_4\}\}$ the same strategies constitute a payoff-confirming analogy-based expectation equilibrium. Yet, this is not a self-confirming equilibrium since player one's beliefs about the behaviour at nodes h_3 and

h_4 along the equilibrium path are incorrect.

In this example, neither of the conditions in proposition 26 holds: for action L of player three, payoff for player one is 3 in decision node h_3 and 0 in decision node h_4. On the other hand, at node h_3, for instance, payoff for one after L and R differ. Despite failing these conditions, we have a PCABEE. Thus, providing necessary and sufficient conditions for an ABEE to be PCABEE is a more difficult task in extensive form games.

4.4.3 Anonymous Learning with Experimentation

When players preferences are myopic and they only maximize the current payoff, there are no incentives to invest in experimentation to gain information about off path behaviour. But if players maximize a (discounted) sum of payoffs, then experimentation is an issue. Fudenberg and Levine (1993b) study an explicit overlapping generations learning model. There is a large (infinite) population of players for each player role of the finite extensive form game. Each player in each population is randomly matched with other players - one from each other population. Each player lives T periods and for each generation there are $\frac{1}{T}$ of players in each population. Player i maximizes a normalized discounted sum of payoffs $\frac{1-\delta}{1-\delta^T}\sum_{t=1}^{T}\delta^{t-1}u_i^t(z)$ where $u_i^t(z)$ is the stage game payoff in period t.

Each player starts with a conjecture about opponents' strategies. This is a probability distribution over opponents' behaviour strategies. Every player in a given population has the same prior distribution. Players update their priors using Bayes's rule. This model generates a stochastic learning process. We are interested in the steady states of this process.

Fudenberg and Levine show that if the length of life tends to infinity and players become infinitely patient (patiently stable state), then every accumulation point of a sequence of steady states of the system is a Nash equilibrium.

Theorem 7. *(Fudenberg and Levine 1993b, theorem 5.1) For any fixed nondoctrinaire priors $\widehat{\sigma}$ there is a function $T(\delta)$ such that if $\delta_m \to 1$ and $T_m \geq T(\delta_m)$, every sequence of steady states has an accumulation point, and every accumulation point is a Nash equilibrium.*

If patience, δ, is bounded away from 1, then every steady state of the process is a self-confirming equilibrium.

Theorem 8. *(Fudenberg and Levine 1993b, theorem 6.1) For fixed nondoctrinaire priors $\widehat{\sigma}$ and $\delta < 1$ as $T_m \to \infty$ every sequence of steady states has an accumulation point and* *every accumulation point is a Self-confirming equilibrium.*

Fudenberg and Levine (2005) study further the patiently stable states. They show that in *simple games* where each player only moves once and where strategies are *nearly pure*, only nature randomizes, a patiently stable state is *a subgame-confirmed equilibrium* - a Nash equilibrium where in each subgame that lies one deviation off the path the strategies constitute a Self-confirming equilibrium.

Theorem 9. *(Fudenberg and Levine 2005, theorem 6.1) In simple games with no own ties, a subgame-confirmed equilibrium that is nearly pure is path equivalent to a patiently stable state.*

It is easy to see that a subgame-confirmed equilibrium is a refinement of the Nash equilibrium whereas a subgame perfect equilibrium is a refinement of the subgame-confirmed equilibrium.

4.5 Discussion

In this chapter, we have reviewed the literature on equilibrium concepts which have an interpretation as a steady state of a learning process. We have studied how the set equilibria depends on how much consistency is required from players' conjectures and beliefs, what players observe and remember during the learning process, how players organize their observations, and what the underlying game is like: more consistent conjectures, more signals observed and finer organization of signals imply a smaller set of steady states. We have shown that there generally exist steady states that do not correspond to a Nash equilibrium of the game but that any Nash equilibrium is a steady state of the process. Moreover, if players are patient, they may experiment and the set of steady states may be a subset of Nash equilibria.

Our main focus was on anonymous learning. We illustrated how we can reach various equilibrium concepts by changing the precision at which players observe the type profile after each round of play in the analogy-based expectation equilibrium (ABEE). We also proposed a refinement, the payoff-confirming ABEE (PCABEE), where each player observes and keeps track of her private payoffs also. This provides further information about opponents' equilibrium strategies. Yet, incorrect conjectures may survive and non-Bayesian-Nash PCABEE may exist. We provided sufficient conditions for an ABEE to be PCABEE both in static games of incomplete information and in extensive form games. We showed that the sufficient condition is also necessary in two-action two-player games with binary uncertainty. Using examples we showed that generally necessity does not hold, however.

Chapter 5

Appendix A

5.1 A) Repeated games

Results analogous to those in chapter 1 would be obtained, if we suppose that players have zero proneness to guilt and they informally agree on a stationary action profile in an infinitely repeated game with continuous time. The punishment paths are not negotiated, however, but they are exogenously determined (in a commonly known social contract, for instance). If the agreement is breached players revert to mutual minmax strategies and the punishment phase lasts for time interval $k(.)$ and the length of the punishment depends on the agreed payoff and the harm.

If such punishment paths indeed reflect a common sense of justice prevailing in society, then, in one-shot games, the guilt cost might serve as an internalized punishment that reflects society's sense of justice. Psychologists such as Clark and Mills (1979) argue for such origins of guilt.

It is easily verified that to make the incentives to breach identical to that in the single shot model, we must make the following assumptions

- discount rates are equal $\rho_i = 1$ for $i = 1, 2$
- It takes time $w = -\ln(\frac{1}{2})$ to observe that opponent is breaching.
- the punishment function $k(h_j(m, a_i), v_i(m), u_i^P)$ takes the following form

$$k(h_j(m, a_i), v_i(m), u_i^P) = \lim_{\varepsilon \to 0} - \ln(\max\{\varepsilon, 1 - \theta \frac{g(v_i(m), h_j(m, s_i))}{u_i(m) - u_i^P}\})$$

(with u_i^P the mutual minmax payoff for player i). Yet, this formulation, implies that an infinitely long punishment follows a breaching where $(u_i(m) - u_i^P) \leq \theta g(v_i(m), h_j(m, s_i))$.

5.2 B) Exogenous action-norms and moral discourse

Harsanyi (1977) and Binmore (1998) present models where a social contract is agreed upon in a moral discourse which is considered to take place prior to the play of the grand game of life[1]. The social contract can be interpreted as a collection of action-norms that apply in various circumstances in the grand game of life. As indicated by psychological research, violating such norms causes distress, such as guilt, shame and fear of punishment. Another stylized fact of the research in psychology is that guilt (or distress) is proportional to the harm that violation causes on others. Thus, the approach developed here can be applied to general action-norms as outcomes of moral discourse - pre-play negotiations of the grand game of life.

In the game theoretic models of Harsanyi (1977) and Binmore (1998), players have empathetic preferences which are weighted sums of individual preferences and used in moral discourse to derive a shared perception of a fair social contract. The fairness preferences are derived from weighting of the individual preferences in an impartial original position where the player thinks it is equally likely that one ends up playing one's own role or that of the opponent. Empathetic preferences are defined over the set $S \times \{1,2\}$ where S is the set of action profiles of play and $\{1,2\}$ is the set of possible roles. A player has an ordering over the outcomes of the game faced either as oneself or as the opponent. Full empathy says that the ordering of S coincides with that of $u_i(s)$ for each i. This leads to a utility function which is a weighted sum of the preferences of the two players.

If the player uses his fairness preferences when playing the game after communication and considering a deviation that decreases the opponent's payoff, the guilt cost takes the form of example 2.2. The formulation $U_i(m,s) = u_i(s) + \theta_i v_i(m) h_j(m,s)$ is reached by letting the weight depend on the agreed payoff $v_i(m)$. The implication is thus a truncated additive social welfare function where the concern for the opponent depends on how nicely one is treated in the pre-play negotiations and how prone to guilt (empathetic) one is.

[1]Similar philosophical non-game theoretic approaches are provided by Habermas (1990) and Hoppe (1993), for instance.

5.3 C) Proofs

Proof of lemma 1

$m_i \in BR_i(m_j) \Leftrightarrow$ for all s_i, $u_i(m_i, m_j) \geq u_i(s_i, m_j) \Rightarrow$ for all s_i, $u_i(m_i, m_j) \geq u_i(s_i, m_j) - g(v_i(m), h_j(m, s_j)) \Leftrightarrow$ for all s_i, $B_i(m, s_i; \theta_i) \leq 0$. Thus, $m \in A_i(\Gamma, \theta_i)$ iff (IR_i).

For the second claim, $m_i \notin BR_i(m_j) \Rightarrow$ there is s_i' such that $u_i(s_i', m_j) > u_i(m_i, m_j)$. Suppose now that (IC_i) holds. But, for all s_i, $B_i(m, s_i; \theta_i) \leq 0 \Rightarrow B_i(m, s_i'; \theta_i) \leq 0 \Rightarrow g(v_i(m), h_j(m, s_j)) \geq u_i(s_i, m_j) - u_i(m_i, m_j) \Rightarrow g(v_i(m), h_j(m, s_j)) > 0 \Rightarrow v_i(m) > 0$. Thus (IR_i) holds and $m \in A_i(\Gamma, \theta_i)$.

Suppose now that m is agreeable. But, then by definition (IC_i) holds.■

Proof of proposition 2

Since m is an equilibrium $v_i(m) \geq 0$ for $i = 1, 2$. Since m is an equilibrium in Γ, , $m_i \in BR_i(m_j)$ for $i = 1, 2$. Then, by lemma (1), $m \in A_i(\Gamma, \theta_i)$ for $i = 1, 2$ and, by definition, $m \in A(\Gamma, \theta)$.■

Proof of lemma 2

Since both deviate from the agreement the guilt cost is zero for both. Then for all s_i, $U_i(m, s^*) = u_i(s^*) \geq u_i(s_i, s_j^*) \geq U_i(m, s_i, s_j^*)$ where the inequality follows from the fact that s^* is a Nash equilibrium of Γ.■

Proof of lemma 3

Conditions (AC) and (EF) imply that $g_i(v_i(m), h_j(m, s_i)) = 0$ if $h_j(m, s_i) < 0$. But indeed, $h_j(m, s_i) \doteq u_j(m) - u_j(m_j, s_i) \leq 0$. Thus (IC_i) is violated and $m \notin A_i(\Gamma, \theta_i)$ and thus $m \notin A(\Gamma, \theta)$.■

Proof of proposition 3

By assumption, $v_i(m) > 0$ for $i = 1, 2$. Take player i and an arbitrary s_i. First, if $u_i(m_i, m_j) \geq u_i(s_i, m_j)$ then $u_i(m_i, m_j) \geq u_i(s_i, m_j) - g(v_i(m), h_j(m, s_j))$ and $B_i(m, s_i; \theta_i) \leq 0$. Second, if $u_j(m_j, s_i) < u_j(m_j, m_i)$ then $h_j(m, s_i) > 0$. By, (EF) $g(v_i(m), h_j(m, s_i)) > 0$. Thus, since payoffs in Γ are finite, $\lim_{\theta_i \to \infty} \theta_i g(v_i(m), h_j(m, s_i)) \geq u_i(s_i, m_j) - u_i(m_i, m_j)$. Hence, $\lim_{\theta_i \to \infty} B(m, s_i; \theta_i) \leq 0$. Since either $u_i(m_i, m_j) \geq u_i(s_i, m_j)$ or $u_j(m_j, s_i) < u_j(m_j, m_i)$ holds for every s_i, (IC_i) holds. Thus $m \in A_i(\Gamma, \theta_i)$. This is true for both players. Thus, $m \in A(\Gamma, \theta)$.

Let now $m \in A(\Gamma,\theta)$. Suppose to the contrary that there is i and s_i such that neither $u_i(m_i,m_j) \geq u_i(s_i,m_j)$ nor $u_j(m_j,s_i) < u_j(m_j,m_i)$ holds. Then, both are true. But then, by lemma 3, $m \notin A(\Gamma,\theta)$. This is a contradiction.■

Proof of proposition 4

Lemma 9. *Let Γ be finite. Let $m_i \neq \{s_i^1, s_i^n\}$. Let {1} , {2} and {4} hold. Then (IC_i) holds if and only if $\mathbb{B}_i(m,\theta_i) \leq 0$.*

Proof. We will show that (IC_i) does not hold iff $\mathbb{B}_i(m,\theta_i) > 0$.

Let $\mathbb{B}_i(m,\theta_i) > 0$. If $u_i(m_i-1,m_j) - u_i(m_i,m_j) \geq 0$, $B(m,m_i-1;\theta_i) > 0$ and (IC_i) is violated. If $u_i(m_i+1,m_j) - u_i(m_i,m_j) > 0$, then $B(m,m_i+1;\theta_i) > 0$ and (IC_i) is violated.

Let (IC_i) be violated. Thus, there is s_i' such that $B_i(m,s_i';\theta_i) > 0$. Suppose to the contrary that $\mathbb{B}_i(m,\theta_i) \leq 0$. We only need to consider the case $u_i(m_i-1,m_j) - u_i(m_i,m_j) \geq 0$ since if $u_i(m_i+1,m_j) - u_i(m_i,m_j) > 0$, then $\mathbb{B}_i(m,\theta_i) > 0$ by definition.

Let thus $u_i(m_i-1,m_j) - u_i(m_i,m_j) \geq 0$. By assumption $\mathbb{B}_i \leq 0$ and thus

$$u_i(m_i-1,m_j) - u_i(m_i,m_j) \leq g(v_i(m), h_j(m,m_i-1))$$

By assumption $\{1\}$, the harm increases in deviations further downwards. Also by assumption $\{4\}$ guilt cost is convex in h_j and by assumption $\{2\}$ u_j is concave in s_i. Thus the harm is convex in s_i and the guilt cost is also convex in s_i as a composite of two convex functions. Thus the cost is convex in s_i. On the other hand, by assumption $\{2\}$ the payoff u_i is concave in s_i, the benefit from breaching $u_i(s_i,m_j) - u_i(m_i,m_j)$ is concave in s_i. Thus if $\mathbb{B}_i(m,\theta_i) \leq 0$ then $B(m,s;\theta_i) \leq 0$ for all $s_i < m_i$. We have a contradiction. □

Proof of the proposition The result follows directly from lemma 1, lemma 9 and the fact that $A(\Gamma,\theta) = \cap_{i=1,2} A_i(\Gamma,\theta_i)$■

Proof of lemma 4

$\beta(m_i+1,m_j) = u_i(m_i,m_j) - u_i(m_i+1,m_j) - [u_i(m_i-1,m_j) - u_i(m_i,m_j)] = -\delta_i(m)$

$\beta(m_i,m_j+1) = u_i(m_i-1,m_j+1) - u_i(m_i,m_j+1) - [u_i(m_i-1,m_j) - u_i(m_i,m_j)]$
$= -\phi_i(m_i,m_j+1)$

$\eta_j(m_j,m_i+1) = u_j(m_j,m_i+1) - u_j(m_j,m_i) - [u_j(m_j,m_i) - u_j(m_j,m_i-1)] = \sigma_j(m)$

Proof. $\eta_j(m_j+1,m_i) = u_j(m_j+1,m_i)-u_j(m_j+1,m_i-1)-[u_j(m_j,m_i)-u_j(m_j,m_i-1)] = \phi_j(m_j+1,m_i)$■ □

Proof of proposition 5

Since u_i is increasing in s_j, by lemma 3, we need u_i to be decreasing in s_i for (s_i, s_j) to be agreeable. Then, the marginal incentive to breach writes

$$\mathbb{B}_i(m_i, m_j) = \beta_i(m_i, m_j) - \theta_i g(u_i(m_i, m_j), \eta_j(m_i, m_j))$$

But $\beta_i(m)$ is increasing in m_i and $\eta_j(m)$ is decreasing in m_i by lemma 4. Also, $u_i(m_i, m_j) - u_i(m_i - 1, m_j) < 0$ implies that $u_i(m)$ decreases in m_i. But g is increasing in both arguments. Thus, $\mathbb{B}_i(m_i, m_j)$ is indeed increasing in m_i.

On the other hand, $\beta_i(m_i, m_j)$ is decreasing in m_j and $\eta_j(m_i, m_j)$ is increasing in m_j by lemma 4. Also, u_i is increasing in m_j by assumption. But g is increasing in both arguments. Thus, $\mathbb{B}_i(m_i, m_j)$ is indeed decreasing in m_i.■

Proof of theorem 1

Since $\phi_i(s) < 0$ for all s, the best reply curves are downward sloping. Since payoff functions are concave, for $s_i < BR_i(s_j)$, $u_i(s_i+1, s_j) - u_i(s_i, s_j) > 0$. For any symmetric action profile such that $s_i < s_i^*$, $s_i < BR_i(s_j)$. Thus increasing the action of i improves the payoff of both. Thus symmetric profiles such that $s_i < s_i^*$ are not efficient. Thus, If $u_i(s_i^* + k, s_j^* + k) > 0$ for some $k \in \mathbb{Z}$ then $k > 0$. But, since $\phi_i(s) < 0$, by lemma 4, $\beta_i(s^* + k) > \beta_i(s^*)$ and $\eta(s^* + k) < \eta(s^*)$ for $i = 1, 2$.■

Proof of theorem 2

Lemmas 10 to 15

Lemma 10. *If $\delta \neq \phi$ then there is at most one equilibrium s^* where $\beta_i(s^*) = 0$ for $i = 1, 2$*

If $-\delta > \phi$ then $(s_i, s_j) \in M_F$ implies $s_i \geq s^*$ for $i = 1, 2$

If $-\delta < \phi$ then $(s_i, s_j) \in M_F$ implies $s_i \leq s^*$ for $i = 1, 2$

Proof. As a mapping from S_2 to S_1 the best reply curve of player one, $BR_1^{-1}(m_1)$, has slope $-\frac{\delta}{\phi}$ and that of player two, $BR_2(m_1)$, has slope $-\frac{\phi}{\delta}$ which are positive constants. The crossing point of the BR curves is a unique (symmetric) equilibrium, $s^* = (s_1^*, s_2^*)$. $s \in M_F$ implies that $\frac{\partial u_i(s)}{\partial s_i} \leq 0$ for $i = 1, 2$. For player two this is true for $m_2 \geq BR_2(m_1)$ and for player one this is true for $m_2 \geq BR_1^{-1}(m_1)$. Thus the claim. □

Lemma 11. *Let the game be symmetric. If s^c maximizes $max_{k \in Z} u(s+k)$ where $s_i = s_j$ (along the diagonal) then there is no s' such that $u_i(s') > u_i(s^c)$ for $i = 1, 2$.*

Such s^c exists.

Proof. Let WLOG $s_j^{'} < s_i^{'}$ and $s_i^{'} - s_i = k$. Then $u_i(s^c) > u_i(s^c + k) = u_i(s_i^{'}, s_i^{'}) > u_i(s_i^{'}, s_j^{'})$ since the payoff is increasing in the action of the opponent. Thus s^c is efficient.

Since S is finite and $u(s+k)$ is defined for all $k \in \mathbb{Z}$, there must be k that maximizes $u(s+k)$ with $s_i = s_j$ and $s \in S$. □

Lemma 12. *$s \notin M_F \Rightarrow$ there is i such that $u_i(s+1) > u_i(s)$*

Proof. $s \notin M_F \Rightarrow$ there is i such that $u_i(s+1) - u_i(s) = [u_i(s+1, s+1) - u_i(s, s+1)] + [u_i(s, s+1) - u_i(s, s)] > 0$. □

Lemma 13. *Let y be convex and supermodular. Then $y(x+2, z+2) - 2y(x+1, z+1) + y(x, z) \geq 0$*

Proof. Let y be convex and supermodular. Then

$$
\begin{aligned}
& y(x+2,z+2) - y(x+1,z+1) - [y(x+1,z+1) - y(x,z)] \\
= \; & y(x,z) - y(x+1,z) - y(x+1,z) + y(x+2,z) \\
& + y(x+2,z+2) - y(x+2,z+1) - y(x+2,z+1) + y(x+2,z) \\
& + y(x+2,z+1) - y(x+2,z) - y(x+1,z+1) + y(x+1,z) \\
& + y(x+2,z+1) - y(x+2,z) - y(x+1,z+1) + y(x+1,z) \\
\geq \; & 0
\end{aligned}
$$

The first effect on the RHS is the second order effect of the first variable, the second row is the second order effect of the second variable and the remaining two rows are identical and equal to the supermodularity effect. □

Lemma 14. *Let $\sigma + \delta < 0$, $2\phi + \delta + \sigma \geq 0$ and $\phi \geq 0, \delta \leq 0, \sigma \leq 0$. Let $u_i(s) - u_i(s-1) \geq 0$ and $\beta_i(s-1) \geq 0$. Let g satisfy {4} and {5}. Suppose that $\beta_i(s-1) \geq g(u_i(s-1), \eta_j(s-1))$. If $\beta_i(s) \leq g(u_i(s), \eta_j(s))$ then $\beta_i(s+k) \leq g(u_i(s+k), \eta_j(s+k))$ for all $k > 0$.*

Proof. $\delta + 2\phi + \sigma \geq 0$ and $\phi + \delta < 0$ implies that $\phi + \sigma \geq 0$. Then, by lemma 4, $\beta(s+k)$ is increasing and concave in k and $\eta(s+k)$ is increasing and convex in k.

Since $\delta + 2\phi + \sigma \geq 0$ and $u_i(s) - u_i(s-1) \geq 0$, $u(s+k)$ is convex and increasing in k for $k \geq 0$. Thus, $g(u(s+k), \eta(s))$ is convex and increasing in k since g is convex and increasing in u by $\{5\}$. Similarly, $g(u(s), \eta(s+k))$ is convex and increasing in k since g is convex in η for $\eta \geq 0$ by $\{4\}$.

Also since $\beta_i(s-1) \geq g(u_i(s-1), \eta_j(s-1))$ and $\beta_i(s-1) \geq 0$ but $\beta_i(s) \leq g(u_i(s), \eta_j(s))$, we have

$$\begin{aligned} & \beta(s) - \beta(s-1) \\ \leq\ & g(u(s), \eta(s)) - g(u(s-1), \eta(s-1)) \end{aligned}$$

Thus, by lemma 13 and since g is supermodular in its arguments

$$\begin{aligned} 0 &\leq \beta(s+1) - \beta(s) \\ &= -\delta - \phi \\ &= \beta(s) - \beta(s-1) \\ &\leq g(u(s), \eta(s)) - g(u(s-1), \eta(s-1)) \\ &\leq g(u(s+1), \eta(s+1)) - g(u(s), \eta(s)) \end{aligned}$$

We can proceed by induction to show that for every $s+k$ with $k > 0$, we have $\beta(s+k) - g(u(s+k), \eta(s+k)) \leq \beta(s) - g(u(s), \eta(s)) \leq 0$. Above, we showed that $u_i(s+k) > u(s)$ for $k > 0$. Thus every $s+k$ with $k > 0$ is agreeable. □

Lemma 15. *Let $\sigma + \delta < 0$, $\phi + \sigma \geq 0$ and $\phi \geq 0, \delta \leq 0, \sigma \leq 0$. Let $\beta_i(s-1) \geq 0$. Let g satisfy $\{4\}$ and let $g(u', \eta) = g(u, \eta)$ for all u', u, η. Suppose that $\beta_i(s-1) \geq g(u_i(s-1), \eta_j(s-1))$. If $\beta_i(s) \leq g(u_i(s), \eta_j(s))$ then $\beta_i(s+k) \leq g(u_i(s+k), \eta_j(s+k))$ for all $k > 0$.*

Proof. By lemma 4, $\beta(s+k)$ is increasing and concave in k and $\eta(s+k)$ is increasing and convex in k.

Also $g(u(s), \eta(s+k))$ is convex and increasing in k since g is convex in η for $\eta \geq 0$ by $\{4\}$ and for all u, $g(u, \eta(s+k)) = g(u(s+k), \eta(s+k))$ by assumption.

Also since $\beta_i(s-1) \geq g(u_i(s-1), \eta_j(s-1))$ and $\beta_i(s-1) \geq 0$ but $\beta_i(s) \leq g(u_i(s), \eta_j(s))$, we have

$$\begin{aligned} & \beta(s) - \beta(s-1) \\ \leq\ & g(u(s), \eta(s)) - g(u(s-1), \eta(s-1)) \end{aligned}$$

Thus, since $g(u(s), \eta(s+k))$ is convex and increasing in k

$$\begin{aligned}
0 &\leq \beta(s+1) - \beta(s) \\
&= -\delta - \phi \\
&= \beta(s) - \beta(s-1) \\
&\leq g(u(s), \eta(s)) - g(u(s-1), \eta(s-1)) \\
&= g(u(s+1), \eta(s)) - g(u(s), \eta(s-1)) \\
&\leq g(u(s+1), \eta(s+1)) - g(u(s), \eta(s))
\end{aligned}$$

We can proceed by induction to show that for every $s+k$ with $k > 0$, we have $\beta(s+k) - g(u(s+k), \eta(s+k)) \leq \beta(s) - g(u(s), \eta(s)) \leq 0$. Thus every $s+k$ with $k > 0$ is agreeable. □

Proposition 27. *Let $\delta + 2\phi + \sigma \geq 0$. Let $s' - 1 \in M_F$ and $s' - 1 \notin A(\Gamma, \theta)$. Let $u(s') - u(s'-1) \geq 0$. Let s^* be the unique equilibrium of the game and $\beta_i(s^*) = 0$. Suppose that {1}, {2}, and {3} hold. Furthermore, let g satisfy {4}, and {5}. If s' such that $s'_i > s^*_i$ is agreeable then any $s' + k$ such that $k > 0$ is agreeable.*

Proof. $\delta + 2\phi + \sigma \geq 0$ implies that $\phi + \delta \geq 0$ or $\sigma + \phi \geq \phi$. Suppose first that $\delta + \phi > 0$. By lemma 10 $s'_i - 1 < s^*_i$ for $i = 1, 2$. Thus $s'_i \leq s^*_i$ and the claim holds trivially.

Let now, $\phi + \delta = 0$. Then $\beta(s+1) - \beta(s) = 0$ and either there are multiple equilibria or in the unique equilibrium there is i such that $\beta_i(s^*) \neq 0$ both contrary to our assumptions.

Let now, $\phi + \delta < 0$. Then $\sigma + \phi > 0$. The fact that $s' - 1 \in M_F$ implies that $\beta(s'-1) \geq 0$. By lemma 4, $\beta(s+1) - \beta(s) = -\delta - \phi > 0$. Also, since $\phi > -\sigma$, by lemma 4 $\eta(s+1) - \eta(s) = \sigma + \phi > 0$ and thus $\eta(s+k)$ is weakly increasing in k.

On the one hand, $u(s'+1) - u(s') \geq u(s') - u(s'-1) \geq 0$ since $\sigma + 2\phi + \delta > 0$. On the other hand, $u(s') \geq u^*$ since s' is agreeable.

Since $u(s+k)$ is convex in k, then $g(u(s+k), \eta(s))$ is convex in k since g is convex in u. Similarly, $g(u(s), \eta(s+k))$ is convex and increasing in k for $k \geq 0$ since g is convex and increasing in η for $\eta \geq 0$ and $\eta(s+k)$ is convex and increasing in k for $k \geq 0$.

Also $s' - 1 \in M_F$ and since $s' - 1 \notin A(\Gamma, \theta)$ we have $\beta_i(s'-1) > 0$ and $\beta_i(s'-1) - g(u_i(s'-1), \eta_j(s'-1)) > 0$. But $s' \in A(\Gamma, \theta)$, and $\beta_i(s') \geq \beta_i(s'-1) > 0$. Thus, by lemma 14 every, $s' + k$ with $k > 0$ is agreeable. □

Proof of the theorem By proposition 2, s^* is agreeable as a Nash equilibrium of the underlying game. Thus '$\Longrightarrow$' is trivial.

Let us now show that if a symmetric $s \neq s^*$ is agreeable,then an efficient symmetric s is agreeable.

If $\phi + \delta \geq 0$ then since the game is symmetric and there is an inefficient equilibrium such that $\beta_i(s^*) = 0$, (s_1^n, s_2^n) is an equilibrium. To see this consider two subcases, 1) $\phi + \delta > 0$ and 2) $\phi + \delta = 0$. If $\phi + \delta > 0$, since $\phi > 0$, the best reply correspondences are upward sloping and steeper than one and they cross at s^*. By symmetry (s_1^n, s_2^n) is an equilibrium, since $-\beta_i(s^n) > 0$ for $i = 1, 2$. If $\phi + \delta = 0$ and there is s^* such that $\beta_i(s^*) = 0$ for $i = 1, 2$, then both best reply functions have a slope equal to one and they overlap in the entire strategy space. All symmetric profiles are equilibria and thus (s_1^n, s_2^n) is an equilibrium. In both cases, since u_i is increasing in s_j, by theorem 7 in Milgrom, Roberts (1990), (s_1^n, s_2^n) is efficient and by proposition 2, (s_1^n, s_2^n) is agreeable.

Suppose now that $\phi + \delta < 0$. Then if $\sigma + \delta + 2\phi \geq 0$, we must have $\sigma + \phi > 0$. Since $\sigma + \delta + 2\phi \geq 0$ and $\beta_i(s^*) = 0$ for $i = 1, 2$, (s_1^n, s_2^n) is efficient. To see this, first consider profiles $s^* - k$ for $k > 0$. By lemma 10, $s^* - k \notin M_F$. By lemma 12, there is i such that $u_i(s^* - k + 1) - u_i(s^* - k) > 0$. Since the game is symmetric, this holds for both players. But since $\sigma + \delta + 2\phi \geq 0$, $u_i(s + k)$ is convex in k. Thus, $u_i(s + k) > u_i(s)$ for $i = 1, 2$ for all symmetric s and for all $k > 0$. Thus, (s_1^n, s_2^n) maximizes the payoff along the diagonal. Thus, by lemma 11, (s_1^n, s_2^n) is efficient.

Let $\phi + \delta < 0$ still hold and suppose alternatively that $\sigma + \delta + 2\phi < 0$. Then $u_i(s + k)$ is strictly concave in k. By lemma 12 and by symmetry, $u_i(s^*) > u_i(s^* - 1)$ for $i = 1, 2$. Since the strategy set is bounded a maximizer $s^* + k$ along the diagonal exists and it satisfies $k > 0$.

Since asymmetric $s \neq s^*$ is agreeable, by lemma 10, $s = s^* + k$ for some $k > 0$. For each player, consider two subcases, 1) there is $1 < k' < k$ such that $s^* + k' - 1 \notin A_i(\Gamma, \theta_i)$ but $s^* + k' \in A(\Gamma, \theta)$ and 2) $s^* + k'$ where $k' = 1$ is agreeable. It is easy to see that one of the two must hold for each player. In either case the agreeability of $s^* + k'$ implies that $\beta_i(s^* + k') \leq g_i(u(s^* + k'), \eta_j(s^* + k'))$ and in each subcase $\beta_i(s^* + k' - 1) \geq 0$ and $\beta_i(s^* + k' - 1) \geq g_i(u(s^* + k' - 1), \eta_j(s^* + k' - 1))$. Thus, if $\sigma + \delta + 2\phi \geq 0$ and g satisfies $\{5\}$ we can apply lemma 14. On the other hand, if $\phi + \sigma \geq 0$ and $g(u', \eta) = g(u, \eta)$ for all u', u, η, we can apply lemma 15. In either case any $s^* + k$ with $k \geq k'$ is agreeable. Thus an efficient symmetric profile is agreeable.■

Chapter 6

Appendix B

6.1 Proof of lemma 7

Proof. Suppose that $\delta_i^{1-\varphi} > \delta_j^\gamma$. Surely if player i respects an agreement on a profile with payoff $1-m$ at t then she can accept $1-\delta m$ at $t-1$. This holds since $1-\delta m > 1-m > (\frac{\delta_i^{1-\varphi}}{\delta_j^\gamma})^{\frac{t}{\varphi}}(\frac{\kappa}{\theta_2\eta^\gamma})^{\frac{1}{\varphi}} > (\frac{\delta_i^{1-\varphi}}{\delta_j^\gamma})^{\frac{t-1}{\varphi}}(\frac{\kappa}{\theta_2\eta^\gamma})^{\frac{1}{\varphi}}$.

Thus, the relevant condition is that if i respects an agreement on a profile with payoff m at t then she respects an agreement on a profile with payoff δm at $t-1$. WLOG check for player two that

$$\delta_2 m(\delta,\theta,t) > (\frac{\delta_2^{1-\varphi}}{\delta_1^\gamma})^{\frac{t-1}{\varphi}}(\frac{\kappa}{\theta_2\eta^\gamma})^{\frac{1}{\varphi}}.$$

This is equivalent to

$$m(\delta,\theta,t) > \frac{1}{\delta_2}(\frac{\delta_2^{1-\varphi}}{\delta_1^\gamma})^{\frac{t-1}{\varphi}}(\frac{\kappa}{\theta_2\eta^\gamma})^{\frac{1}{\varphi}}$$

But $m(\delta,\theta,t) = (\frac{\delta_2^{1-\varphi}}{\delta_1^\gamma})^{\frac{t}{\varphi}}(\frac{\kappa}{\theta_2\eta^\gamma})^{\frac{1}{\varphi}}$. Thus the inequality above holds iff

$$(\frac{\delta_2^{1-\varphi}}{\delta_1^\gamma})^{\frac{t}{\varphi}}(\frac{\kappa}{\theta_2\eta^\gamma})^{\frac{1}{\varphi}} > \frac{1}{\delta_2}(\frac{\delta_2^{1-\varphi}}{\delta_1^\gamma})^{\frac{t-1}{\varphi}}(\frac{\kappa}{\theta_2\eta^\gamma})^{\frac{1}{\varphi}}$$

or iff

$$\delta_2 > \delta_1^\gamma$$

which is implied by $\delta_i^{1-\varphi} > \delta_j^\gamma$ since $\varphi > 0$. □

6.2 Proof of lemma 8

Proof. From proposition 6, if m is agreed upon in period t, player 2's equilibrium payoff, $U_2^*(m,t,\delta)$, can be written either as $(1-\delta_1)\sum_{k=0}^{\frac{t}{2}-1}(\delta_1\delta_2)^k+(\delta_1\delta_2)^{\frac{t}{2}}m$ (if $t \in 2n$, $n \in \mathbb{N}$ and $t > 0$) or as $(1-\delta_1)\sum_{k=0}^{\frac{t-1}{2}}(\delta_1\delta_2)^k+\delta_1(\delta_1\delta_2)^{\frac{t-1}{2}}m$ (if $t \in 2n+1$, $n \in \mathbb{N}$). In either case, it is easy to see that it is increasing in δ_2. By proposition 6, $U_1^*(m,t,\delta) = 1 - U_2^*(m,t,\delta)$ and thus U_1^* is decreasing in δ_2.

To see the effect of δ_1, suppose first that $t \in 2n$. Then we can write $U_2^*(m,t,\delta)$ as $(1-\delta_1)\sum_{k=0}^{\frac{t}{2}-1}(\delta_1\delta_2)^k + (\delta_1\delta_2)^{\frac{t}{2}}m$. Differentiating with respect to δ_1 gives

$$-\sum_{k=0}^{\frac{t}{2}-1}(\delta_1\delta_2)^k + (1-\delta_1)\frac{1}{\delta_1}\sum_{k=1}^{\frac{t}{2}-1}k(\delta_1\delta_2)^k + \frac{1}{\delta_1}(\frac{t}{2})(\delta_1\delta_2)^{\frac{t}{2}}m$$

Rearranging this gives

$$-(1-\delta_2)\sum_{k=0}^{\frac{t}{2}-2}(1+k)(\delta_1\delta_2)^k - (1-\delta_2 m)\frac{t}{2}(\delta_1\delta_2)^{\frac{t}{2}-1} < 0$$

If $t \in 2n-1$, then then we can write $U_2^*(m,t,\delta)$ as $(1-\delta_1)\sum_{k=0}^{\frac{t-1}{2}}(\delta_1\delta_2)^k + (\delta_1\delta_2)^{\frac{t-1}{2}}m$. Differentiating with respect to δ_1 gives

$$-\sum_{k=0}^{\frac{t-1}{2}}(\delta_1\delta_2)^k + (1-\delta_1)\frac{1}{\delta_1}\sum_{k=1}^{\frac{t-1}{2}}k(\delta_1\delta_2)^k + \frac{1}{\delta_1}\frac{t-1}{2}(\delta_1\delta_2)^{\frac{t-1}{2}}m$$

We can rearrange and get

$$-(1-\delta_2)\sum_{k=0}^{\frac{t-3}{2}}(1+k)(\delta_1\delta_2)^k - (1-\delta_2 m)\frac{t-1}{2}(\delta_1\delta_2)^{\frac{t-3}{2}}m < 0$$

□

6.3 Proof of proposition 6

Proof. Since $\delta_i = \delta_j$ and $\gamma \geq 1$, condition (3.7) holds. Thus, by lemma 6, there is finite scope for negotiations and by lemma 7, if m is agreeable at t then δm is agreeable at $t-1$.

Let $T(\theta,\delta) \in 2n$ where $n > 0$ and $n \in \mathcal{N}$. If play reaches $T(\theta,\delta)$, then player 2 proposes agreement $m(\theta,\delta,T((\theta,\delta))) \doteq 1 - (\frac{\delta_1^{\frac{1}{\varphi}-1}}{\delta_2^{\frac{1}{\varphi}}})^{T(\theta,\delta)}(\frac{\kappa}{\theta_1\eta^\gamma})^{\frac{1}{\varphi}}$. By lemma 7, if play reaches $T(\theta,\delta)-1$, then player 1 proposes agreement $m_2 \doteq \delta_2(1-m(\theta,\delta,T((\theta,\delta))))$. By lemma 7, and by backward induction, player 2 proposes agreement

$$(1-\delta_1)\sum_{k=0}^{T(\theta,\delta)/2-1}(\delta_1\delta_2)^k + (\delta_1\delta_2)^{T(\theta,\delta)/2}m_2$$

at period zero.

Let $T(\theta,\delta) \in 2n-1$. If play reaches $T(\theta,\delta)$, then player 1 proposes $m(\theta,\delta,T((\theta,\delta))) = (\frac{\delta_2^{\frac{1}{\varphi}-1}}{\delta_1^{\frac{\gamma}{\varphi}}})^{T(\theta,\delta)}(\frac{\kappa}{\theta_2\eta^{\gamma}})^{\frac{1}{\varphi}}$. By lemma 7 and backward induction, player 2 proposes

$$(1-\delta_1)\sum_{k=0}^{(T-1)/2}(\delta_1\delta_2)^k + \delta_1(\delta_1\delta_2)^{(T-1)/2}m(\theta,\delta,T((\theta,\delta)))$$

at period zero.

By the backward induction argument, player 1 accepts player 2's proposal at period zero. Thus, (3.10) and (3.11) directly give player 2's equilibrium payoffs and player 1's equilibrium payoff equals $1-U_2^*$. □

Chapter 7

Appendix C

7.1 Proof of proposition 17

Proof. Consider the following analogy partition. For each θ_i and each θ_{-i}, there is an analogy class $\alpha_i^\theta \subset \widetilde{\theta}^{-1}(\theta)$ and $\frac{q(\alpha_i^\theta)}{p(\theta)} = 1 - \chi$. On the other hand, given θ_i, for every θ_{-i}, the states $\widetilde{\theta}^{-1}(\theta)\backslash\alpha_i^\theta$ belong to $\overline{\alpha}_i^{\theta_i}(\omega)$ so that $\frac{q(\widetilde{\theta}^{-1}(\theta)\backslash\alpha_i^\theta)}{p(\theta)} = \chi$. Notice that this analogy partition is finer than the private information partition of i.

JK (2005, proposition 4) show that when the analogy partition is finer than the private information partition, the analogy based expectation equilibria are equivalent to the Bayesian-Nash equilibria of a virtual game where the payoff of player i when the state is ω and the action profile is a is $\overline{u}_i(a;\omega) = \sum_{\omega\in\Omega} p(\omega'|\alpha_i(\omega))u_i(a;\omega')$. But in fraction $1-\chi$ of the states that are mapped into θ, there are only states that are mapped to the same type vector, and thus $\overline{u}_i(a;\omega) = u_i(a;\theta)$ for these states. On the other hand, in fraction χ of the states that are associated with a given type θ, there are fractions $p(\theta'_{-i}|\theta_i)$ of states associated with types (θ_i, θ'_{-i}) for each $\theta'_{-i} \in \Theta_{-i}$. Thus, the virtual payoff for these states reads $\overline{u}_i(a;\omega) = \sum_{\theta'_{-i}\in\Theta_{-i}} p(\theta'_{-i}|\theta_i)\ u_i(a;(\theta_i,\theta'_{-i}))$. Thus overall, conditional on type θ the virtual payoff for action profile a can be written as $(1-\chi)u_i(a;\theta) + \chi\sum_{\theta'_{-i}\in\Theta_{-i}} p(\theta'_{-i}|\theta_i)\ u_i(a;(\theta_i,\theta'_{-i}))$. But this is exactly the $\chi-$cursed equilibrium virtual game payoff of type θ (Eyster and Rabin, 2005, p. 1631). The Bayesian- Nash equilibria of this game are the $\chi-$cursed equilibria of the original game. □

7.2 Proof of proposition 22

Proposition 28. *Let $N = 2$, $\Theta = \{\theta^1, \theta^2\}$ and $A_i = \{a_i^1, a_i^2\}$. Suppose that s is a pure strategy profile of an ABEE.*

The ABEE is payoff-confirming if and only if

- s is a Bayesian-Nash equilibrium

or

- for each i such that $s_i(\theta^m) \neq s_i(\theta^n)$ and $An_j = \{\{\theta^1, \theta^2\}\}$

$$\begin{aligned} \text{for all } m,\ u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) &= u_j(s_j(\theta^m), s_i(\theta^m); \theta^m) \\ &\text{or} \\ \text{for all } m,\ u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) &= u_j(s_j(\theta^n), s_i(\theta^n); \theta^n)) \end{aligned} \tag{7.1}$$

where $r_i(\theta^m)$ is the action not chosen by i at θ^m.

Proof. If ABEE is BNE, then by lemma 17 the ABEE is PCABEE.

On the other hand, if (7.1) holds for each i such that $s_i(\theta^m) \neq s_i(\theta^n)$ and $An_j = \{\{\theta^1, \theta^2\}\}$, then the ABEE is a PCABEE by lemma 18 below.

Consider now an ABEE which is a PCABEE and suppose to the contrary that the ABEE is not a BNE and there is i such that $s_i(\theta^m) \neq s_i(\theta^n)$ and $An_j = \{\{\theta^1, \theta^2\}\}$ and (7.1) does not hold. Thus, by lemma 19, ABEE is not a PCABEE - a contradiction. □

Lemma 16. *ABEE is BNE iff for each player*

- *the analogy partition is finer than the opponent's information partition*

- *if $An_j = \{\{\theta^1, \theta^2\}\}$, then i plays a pooling strategy.*

Proof. If the analogy is finer than the opponent's information partition, then conjectures about the opponent's behavior at each state are correct. Thus, ABEE is BNE.

If $An_j = \{\{\theta^1, \theta^2\}\}$ and i plays a pooling strategy, then the opponent plays the same action in each state. The average strategy coincides with the pooling strategy. Thus, the conjectures about opponent's behavior at each state are correct. Thus, ABEE is BNE.

Suppose now to the contrary that there is a player with a coarse analogy partition and the opponent plays a separation strategy. Then the average strategy differs from the strategy in each state. The conjectures about opponent's behavior are incorrect. Thus, the ABEE is not a BNE. □

Lemma 17. *BNE is a PCABEE*

Proof. Given a BNE consider an ABEE where each player's analogy partition coincides with opponent's information partition. Then conjectures about opponent's behavior are correct. Hence, expectations about own payoffs must be correct also. □

Lemma 18. *Let $N = 2$, $\Theta = \{\theta^1, \theta^2\}$ and $A_i = \{a_i^1, a_i^2\}$. Suppose that s is a pure strategy profile of an ABEE. Let for each i such that $s_i(\theta^m) \neq s_i(\theta^n)$ and $An_j = \{\{\theta^1, \theta^2\}\}$*

$$\text{for all } m,\ u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) = u_j(s_j(\theta^m), s_i(\theta^m); \theta^m) \tag{7.2}$$

or

$$\text{for all } m,\ u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) = u_j(s_j(\theta^n), s_i(\theta^n); \theta^n)) \tag{7.3}$$

where $r_i(\theta^m)$ is the action not chosen by i at θ^m.

Then the ABEE is PCABEE.

Proof. Define the probability of j getting payoff u given strategy profile σ as $f_\sigma^j(u) \doteq \sum_{\{a,\theta|u=u_j(a,\theta)\}} p(\theta)\sigma_i(a_i|\theta)\sigma_j(a_j|\theta)$.

Let there be i such that $s_i(\theta^m) \neq s_i(\theta^n)$ and $An_j = \{\{\theta_1, \theta_2\}\}$. There are three subcases to consider: first

$$\begin{aligned} u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) &= u_j(s_j(\theta^m), s_i(\theta^m); \theta^m) \\ &= u_j(s_j(\theta^n), r_i(\theta^n); \theta^n) \\ &= u_j(s_j(\theta^n), s_i(\theta^n); \theta^n)) \end{aligned}$$

in which case both conditions hold. In this first subcase trivially $f_{(s_j,\beta^j)}^j(u_j(s_j(\theta^m), s_i(\theta^m); \theta^m)) = 1 = f_s^j(u_j(s_j(\theta^m), s_i(\theta^m); \theta^m))$. Thus, s is a payoff confirming analogy-based expectation equilibrium.

In the second case only (7.3) holds but not (7.2). For each θ^m, the perceived probability that $u_j(s_j(\theta^m), s_i(\theta^m); \theta^m)$ results is

$$\begin{aligned} f_{(s_j,\beta^j)}^j(u_j(s_j(\theta^m), s_i(\theta^m); \theta^m)) &= [f(\theta^m)]\beta^j(s_i(\theta^m)) + [1 - f(\theta^m)]\beta^j(s_i(\theta^m)) \\ &= [f(\theta^m)]^2 + [1 - f(\theta^m)]f(\theta^m) \\ &= f(\theta^m) \\ &= f_s^j(u_j(s_j(\theta^m), s_i(\theta^m); \theta^m)). \end{aligned}$$

Thus, s is a payoff confirming analogy-based expectation equilibrium.

Third, if only (7.2) holds and not (7.3), we have that

$$\begin{aligned} f_{(s_j,\beta^j)}^j(u_j(s_j(\theta^n), s_i(\theta^n); \theta^n)) &= [f(\theta^n)][\beta^j(s_i(\theta^n)) + \beta^j(r_i(\theta^n))] \\ &= [f(\theta^n)] \\ &= f_s^j(u_j(s_j(\theta^n), s_i(\theta^n); \theta^n)). \end{aligned}$$

Thus, s is a payoff confirming analogy-based expectation equilibrium. □

Lemma 19. *Let $N = 2$, $\Theta = \{\theta^1, \theta^2\}$ and $A_i = \{a_i^1, a_i^2\}$. Suppose that s is a pure strategy profile of an ABEE.*

If there is i such that $s_i(\theta^m) \neq s_i(\theta^n)$ and $An_j = \{\{\theta^1, \theta^2\}\}$ and neither

$$\text{for all } m,\ u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) = u_j(s_j(\theta^m), s_i(\theta^m); \theta^m)$$

nor

$$\text{for all } m,\ u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) = u_j(s_j(\theta^n), s_i(\theta^n); \theta^n))$$

where $r_i(\theta^m)$ is the action not chosen by i at θ^m,

then the ABEE is not a PCABEE.

Proof. By lemma 16, since $s_i(\theta^m) \neq s_i(\theta^n)$ and $An_j = \{\{\theta^1, \theta^2\}\}$ the ABEE is not BNE.

Furthermore there is m such that neither (7.2) nor (7.3) holds. We use proof by contradiction. There are two subcases to consider. Suppose first, that there is m such that

$$u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) \notin \{u_j(s_j(\theta^m), s_i(\theta^m); \theta^m),\ u_j(s_j(\theta^n), s_i(\theta^n); \theta^n))\}. \tag{7.4}$$

Define the probability of j getting payoff u given strategy profile σ as

$$f_\sigma^j(u) \doteq \sum_{\{a,\theta | u = u_j(a,\theta)\}} p(\theta)\sigma_i(a_i|\theta)\sigma_j(a_j|\theta). \tag{7.5}$$

Since $s_i(\theta^m) \neq s_i(\theta^n)$, j expects $u_j(s_j(\theta^m), r_i(\theta^m); \theta^m)$ to result with a positive probability,

$$f^j_{s_j, \hat{\sigma}_i^j}(u_j(s_j(\theta^m), r_i(\theta^m); \theta^m)) > 0 \tag{7.6}$$

But since (7.4) holds, $f^j_{s_j, \sigma_i}(u_j(s_j(\theta^m), r_i(\theta^m); \theta^m)) = 0$ which contradicts the consistency condition of PCABEE and thus the ABEE is not PCABEE.

In the second subcase, suppose in addition to $s_i(\theta^m) \neq s_i(\theta^n)$ that there is m and i such that

$$\begin{aligned} u_j(s_j(\theta^m), r_i(\theta^m); \theta^m) &= u_j(s_j(\theta^m), s_i(\theta^m); \theta^m) \\ &= u_j(s_j(\theta^n), r_i(\theta^n); \theta^n)) \qquad (7.7) \\ &\neq u_j(s_j(\theta^n), s_i(\theta^n); \theta^n)) \qquad (7.8) \end{aligned}$$

Then

$$\begin{aligned} f^j_{(s_j, \beta^j)}(u_j(s_j(\theta^m), s_i(\theta^m); \theta^m)) &= f(\theta^m) + f(\theta^n)\beta^j(r_i(\theta^n)) \qquad (7.9) \\ &\neq f(\theta^m) \qquad (7.10) \\ &= f^j_s(u_j(s_j(\theta^m), s_i(\theta^m); \theta^m)) \qquad (7.11) \end{aligned}$$

and thus ABEE is not PCABEE. □

Proposition 29. *Let in an ABEE σ differ from a Bayesian-Nash equilibrium. If for each j and α_j such that there are $\theta^m, \theta^n \in \alpha_j$ with $\sigma_{-j}(\theta^m) \neq \sigma_{-j}(\theta^n)$,*

- *either for all $\theta \in \alpha_j$ and for all action profiles of players other than j, $a^*_{-j} = (a^*_1, ..., a^*_{j-1}, a^*_{j+1}, ..., a^*_N)$ such that for each $i \neq j$ there is $\theta' \in \alpha_j$ such that $a^*_i \in supp\sigma_i(\theta')$, we have for all $a_j \in supp\sigma_j(\theta)$*

$$u_j(a_j, a_{-j}; \theta) = u_\theta$$

- *or for all action profiles of players other than j, $a^*_{-j} = (a^*_1, ..., a^*_{j-1}, a^*_{j+1}, ..., a^*_N)$ such that for each $i \neq j$ there is $\theta' \in \alpha_j$ such that $a^*_i \in supp\sigma_i(\theta')$, we have for all $\theta \in \alpha_j$ and $a_j \in supp\sigma_j(\theta)$*

$$u_j(a_j, a_{-j}; \theta) = u_{a_{-j}},$$

then the ABEE is payoff-confirming.

Proof of proposition 23. Let for each j and α_j such that $\sigma_{-j}(\theta^m) \neq \sigma_{-j}(\theta^n)$ and $\theta^m, \theta^n \in \alpha_j$, for all $\theta \in \alpha_j$, for all action profiles of players other than j, $a^*_{-j} = (a^*_1, ..., a^*_{j-1}, a^*_{j+1}, ..., a^*_N)$ such that for each $i \neq j$ there is $\theta' \in \alpha_j$ such that $a^*_i \in \text{supp}\sigma_i(\theta')$ for all $a_j \in \text{supp}\sigma_j(\theta)$,

$$u_j(a_j, a_{-j}; \theta) = u_\theta.$$

Apply condition (7.1) in definition 8 with $\gamma_j = (a_{-j}, \alpha_j(\theta))$,

$$\begin{aligned}
& \sum_{\{a,\theta | u_j = u_j(a,\theta)\}} p(\theta)\widehat{\sigma}_{-j}(a_{-j}|\theta_{-j})\sigma_j(a_j|\theta_j) \\
= & \sum_{\alpha_j \in An_j} \sum_{\{(a_j,\theta)|u_j=u_j(a,\theta),\ \theta\in\alpha_i\}} p(\theta)\widehat{\sigma}_{-j}(a_{-j}|\theta_{-j})\sigma_j(a_j|\theta_j) \\
= & \sum_{\alpha_j \in An_j} \sum_{\theta^n \in \alpha_j} p(\theta^n)\sigma_j(a_j|\theta^n_j) \sum_{a^*_{-j}} \widehat{\sigma}_{-j}(a^*_{-j}|\theta^n_{-j}) \\
= & \sum_{\alpha_j \in An_j} \sum_{\theta^n \in \alpha_j} p(\theta^n)\sigma_j(a_j|\theta^n_j) \\
= & \sum_{\alpha_j \in An_j} \sum_{\theta^n \in \alpha_j} p(\theta^n)\sigma_j(a_j|\theta^n_j) \sum_{a_{-j}} \sigma_{-j}(a_{-j}|\theta_j) \\
= & \sum_{\alpha_j \in An_j} \sum_{\{a,\theta|u_i=u_i(a,\theta),\theta\in\alpha_j\}} p(\theta)\sigma_{-j}(a_{-i}|\theta_{-j})\sigma_j(a_j|\theta_j) \\
= & \sum_{\{a,\theta|u_i=u_i(a,\theta)\}} p(\theta)\sigma_{-j}(a_{-i}|\theta_{-j})\sigma_j(a_j|\theta_j)
\end{aligned}$$

where the second equality follows from the fact that, in an analogy class, for a state in the class and for an action that is chosen with a positive probability by j in that state the payoff is the same for any action profile of players other than j to which $\widehat{\sigma}_{-j}$ assigns a positive probability. The third and the fourth equality follow because a conjecture and a strategy is a probability distribution and thus sums up to one. ($\sum_{a^*_{-j}} \widehat{\sigma}_{-j}(a_{-j}|\theta_{-j}) = 1 = \sum_{a_{-j}} \sigma_{-j}(a_{-j}|\theta_j)$) and only actions which are assigned a positive probability in the average strategy of the analogy class can be assigned a positive probability in the actual strategy.

Let for each j and α_j such that $\sigma_{-j}(\theta^m) \neq \sigma_{-j}(\theta^n)$ and $\theta^m, \theta^n \in \alpha_j$, for all action profiles of players other than j, $a^*_{-j} = (a^*_1, ..., a^*_{j-1}, a^*_{j+1}, ..., a^*_N)$ such that for each $i \neq j$ there is $\theta' \in \alpha_j$ such that $a^*_i \in \text{supp}\sigma_i(\theta')$, for all $\theta \in \alpha_j$ and $a_j \in \text{supp}\sigma_j(\theta)$

$$u_j(a_j, a_{-j}; \theta) = u_{a_{-j}}.$$

Apply condition 7.1 in definition 8 with $\gamma_i = (a_{-i}, \alpha_i(\theta))$,

$$\begin{aligned}
& \sum_{\{a,\theta|u_j=u_j(a,\theta)\}} p(\theta)\widehat{\sigma}_{-j}(a_{-j}|\theta_j)\sigma_j(a_j|\theta_j) \\
= & \sum_{\alpha\in An_j} \sum_{\{(a,\theta)|u_j=u_j(a,\theta),\ \theta\in\alpha\}} p(\theta)\sigma_j(a_j|\theta_j)\widehat{\sigma}_{-j}(a_{-j}|\theta) \\
= & \sum_{\alpha\in An_j} \sum_{a^*_{-j}} \widehat{\sigma}_{-j}(a^*_{-j}|\alpha) \sum_{\theta\in\alpha} p(\theta)\sigma_j(a_j|\theta_j) \\
= & \sum_{\alpha\in An_j} \sum_{\theta\in\alpha} p(\theta)\sigma_j(a_j|\theta_j) \\
= & \sum_{\alpha\in An_j} \sum_{\{(a_j,\theta)|u_j=u_j(a,\theta),\ \theta\in\alpha\}} p(\theta)\sigma_j(a_j|\theta_j) \sum_{a_{-j}} \sigma_{-j}(a_{-j}|\theta_{-j}) \\
= & \sum_{\alpha\in An_j} \sum_{\{(a,\theta)|u_j=u_j(a,\theta),\ \theta\in\alpha\}} p(\theta)\sigma_j(a_j|\theta_j)\sigma_{-j}(a_{-j}|\theta_{-j}) \\
= & \sum_{\{a,\theta|u_i=u_i(a,\theta)\}} p(\theta)\sigma_{-j}(a_{-i}|\theta_{-j})\sigma_j(a_j|\theta_j)
\end{aligned}$$

where the second equality follows from the fact that for a given action profile a^*_{-j} in $\text{supp}\widehat{\sigma}_{-j}(\alpha)$ the payoff $u_j(a_j, a^*_{-j}, \theta)$ is the same for each θ in α_j and (a_j, θ) such that $a_j \in \text{supp}\sigma_j(\theta)$. The third and the fourth equality follow because a strategy and a conjecture are probability distributions and only actions which are assigned a positive probability in the average strategy of the analogy class can be assigned a positive probability in the actual strategy. □

7.3 Proof of proposition 26

Proposition 30. *If for every i and for every α and for every $h' \in \alpha_h$ such that $P^{\sigma_i,\widehat{\sigma}^i}(h) > 0$ and such that $\sigma(h) \neq \widehat{\sigma}^i(h)$ we have that $P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha) = P^{\sigma}(h|\alpha)$ and*

(1) either for every u for every $a, a' \in A(\alpha)$

$$\sum_{\{z|u_i(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|h',a) = \sum_{\{z|u_i(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|h',a')$$

(2) or for every u

$$\sum_{\{z|u_i(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|h',a) = \sum_{\{z|u_i(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|h,a)$$

then for all u and for all i

$$\sum_{\{z|u_i(z)=u\}} P^{\sigma}(z) = \sum_{\{z|u_i(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z)$$

Proof. If condition 4.13 holds for α, then, then for every u

$$\begin{aligned}
& \sum_{\{z|u(z)=u,\ h\in\alpha\}} P^{\sigma_i,\widehat{\sigma}^i}(z|\alpha) \\
= & \sum_{\{z|u(z)=u,\ h\in\alpha\}} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha)\widehat{\sigma}^i(a|h)P^{\sigma_i,\widehat{\sigma}^i}(z|h,a) \\
= & \sum_{h\in\alpha} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha)\sum_a \widehat{\sigma}^i(a|h) \sum_{\{z|u(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|h,a)
\end{aligned}$$

But for each u, $\sum_{\{z|u(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|h,a)$ is either zero or one and it equals $\sum_{\{z|u(z)=u\}} P^{\sigma}(z|h,a)$. In the former case obviously,

$$\begin{aligned}
& \sum_{h\in\alpha} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha)\sum_a \widehat{\sigma}^i(a|h) \sum_{\{z|u(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|h,a) \\
= & \ 0 = \sum_{h\in\alpha} P^{\sigma}(h|\alpha)\sum_a \sigma(a|h) \sum_{\{z|u(z)=u\}} P^{\sigma}(z|h,a).
\end{aligned}$$

In the latter case,

$$\begin{aligned}
& \sum_{h\in\alpha} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha)\sum_a \widehat{\sigma}^i(a|h) \sum_{\{z|u(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|h,a) \\
= & \sum_{h\in\alpha} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha)\sum_a \widehat{\sigma}^i(a|h) \\
= & \sum_{h\in\alpha} P^{\sigma}(h|\alpha)\sum_a \widehat{\sigma}^i(a|h) \\
= & \sum_{h\in\alpha} P^{\sigma}(h|\alpha)\sum_a \sigma(a|h) \\
= & \sum_{h\in\alpha} P^{\sigma}(h|\alpha)\sum_a \sigma(a|h) \sum_{\{z|u(z)=u\}} P^{\sigma}(z|h,a).
\end{aligned}$$

Thereby,

$$\sum_{\{z|u(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|\alpha) = \sum_{\{z|u(z)=u\}} P^{\sigma}(z|\alpha).$$

Consider now the case that every $h \in \alpha$ such that $P^{\sigma_i,\widehat{\sigma}^i}(h) > 0$ satisfies (4.14). Then, for every u, we can write

$$\begin{aligned}
& \sum_{\{z|u_i(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|\alpha_h) \\
= & \sum_{\{z|u_i(z)=u,\ h\in\alpha\}} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha_h) P^{\sigma_i,\widehat{\sigma}^i}(z|h) \\
= & \sum_{h\in\alpha} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha_h) \sum_{\{z|u_i(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|h) \\
= & \sum_{h\in\alpha} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha_h) \sum_{\{z|u_i(z)=u\}} \sum_{a\in A(\alpha)} \widehat{\sigma}_{-j}(a|\alpha) P^{\sigma_i,\widehat{\sigma}^i}(z|h,a) \\
= & \sum_{a\in A(\alpha)} \widehat{\sigma}^i(a|\alpha) \sum_{\{z|u(z)=u,\ h\in\alpha\}} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha_h) P^{\sigma_i,\widehat{\sigma}^i}(z|h,a)
\end{aligned}$$

But now keeping a fixed and varying h, u is the same for each positive probability terminal history z. Thus $\sum_{\{z|u(z)=u,\ h\in\alpha\}} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha_h) P^{\sigma_i,\widehat{\sigma}^i}(z|h,a)$ is either zero or one. In either case

$$\begin{aligned}
& \sum_{a\in A(\alpha)} \widehat{\sigma}^i(a|\alpha) \sum_{\{z|u(z)=u,\ h\in\alpha\}} P^{\sigma_i,\widehat{\sigma}^i}(h|\alpha_h) P^{\sigma_i,\widehat{\sigma}^i}(z|h,a) \\
= & \sum_{a\in A(\alpha)} \sigma_{-j}(a|\alpha) \sum_{\{z|u(z)=u,\ h\in\alpha\}} P^{\sigma}(h|\alpha_h) P^{\sigma}(z|h,a)
\end{aligned}$$

Thereby,

$$\sum_{\{z|u(z)=u\}} P^{\sigma_i,\widehat{\sigma}^i}(z|\alpha) = \sum_{\{z|u(z)=u\}} P^{\sigma}(z|\alpha).$$

□

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