Competitive Nonlinear Pricing and Bundling*

Mark Armstrong
Department of Economics
University College London

John Vickers
All Soul's College
University of Oxford

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Abstract

We examine competitive nonlinear pricing in a model in which consumers have heterogeneous and elastic demands and can buy from more than one supplier. It is an equilibrium for firms to offer a menu of efficient two-part tariffs. Compared with linear pricing, nonlinear pricing tends to raise profit but harm consumers when: (i) demand is elastic, (ii) there is substantial heterogeneity in consumer demand, (iii) consumers face substantial shopping costs when buying from more than one firm, and (iv) a consumer's brand preference for one product is correlated with her brand preference for another product. Nonlinear pricing is more likely to lead to welfare gains when (iii) and (iv) hold, but (ii) does not.

1 Introduction

Most economic analysis of imperfect competition is based on the assumption of linear pricing, where the price of a combination of purchases from a firm, whether of one or more products, is equal to the sum of the prices of the component parts. While many markets operate on that basis, an increasing number feature nonlinear pricing—for example, discounts for purchases of larger volumes or of more products. In the absence of easy arbitrage, which would undermine it, such pricing can be observed in more or less competitive markets as well as in some with market power.

Examples include energy markets, which traditionally were monopolies but are now open to varying degrees of competition. Consumers are often able to buy their gas and electricity from a single supplier or from two suppliers. In each case, consumers typically

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face nonlinear tariffs, and they will often enjoy an additional discount if they purchase both services from a single supplier. Similarly, consumers nowadays can often source telecommunications, cable television and internet services from a single supplier or from several, and nonlinear pricing and bundling are commonplace. Nonlinear pricing and (sometimes inter-temporal) bundling is also increasingly to be seen in sectors such as air travel and supermarkets. The economic importance of nonlinear pricing goes wider still. Thus labour contracts may include extra pay for longer tenure, so that a two-period worker gets a better deal than two one-period workers.[1] While the provision of incentives is doubtless a prime reason for such nonlinear wages, the nature of multi-period competition by firms for labour may also be relevant, and there may be parallels with multi-product competition among firms for consumers.

There is an extensive literature on nonlinear pricing and bundling.[2] In the context of monopoly supply, analyses include Adams and Yellen (1976), McAfee, McMillan, and Whinston (1989), Armstrong (1996), and Rochet and Choné (1998). In particular, the latter two papers demonstrate the complexity of optimal multiproduct nonlinear pricing for a monopolist, and the profit-maximizing tariff can be derived only in a few isolated examples. There is also a growing literature on (more or less) competitive nonlinear pricing. For example Spulber (1979), Stole (1995), Armstrong and Vickers (2001), Rochet and Stole (2002), Yin (2004), Thanassoulis (2007) and Yang and Ye (2008) examine competitive nonlinear pricing in situations in which each consumer purchases all products from a single supplier. The papers by Armstrong and Vickers (2001), Rochet and Stole (2002), Yin (2004) and Thanassoulis (2007, section 3) suggest that when the market is fully covered, marginal-cost pricing often emerges as a nonlinear pricing equilibrium, in which case welfare is boosted when firms offer such tariffs compared with linear pricing. Moreover, in this one-stop shopping framework nonlinear pricing tends to increase profit but harm consumers.

But how reasonable is this one-stop shopping framework? It seems implausible in various settings, including dynamic ones where we expect to see some switching between firms by consumers. There are two strands of work which examine competitive nonlinear pricing when consumers can buy from several suppliers. First, there is the situation where there are several products, which consumers may wish to purchase simultaneously (such as

[1] See, for example, Stevens (2004).

[2] Armstrong (2006) and Stole (2007) are recent surveys.

gas and electricity), and there is a sole supplier for each product.[3] This case is somewhat similar to the monopoly analysis and, especially when there is multi-dimensional private information, it can be hard to derive the equilibrium tariffs.

Closer to our approach is the competitive bundling literature, which assumes there is intra-product competition. Some of this literature has focussed on anti-competitive behaviour by a multiproduct incumbent facing a potential single-product entrant.[4] More relevant for our purposes, however, are contributions which assume a more symmetric market structure, including Matutes and Regibeau (1992), Anderson and Leruth (1993), Reisinger (2006) and Thanassoulis (2007). This work tends to proceed by way of specific examples (e.g., that consumers are uniformly distributed on the unit square) and assumes that each consumer wishes to buy only one unit of a given product. Moreover, the analysis is based on the polar opposite assumption from the one-stop shopping models, that consumers face *no* intrinsic extra "shopping cost" if they buy from more than one firm. The analysis of such models suggests, contrary to that of one-stop shopping models, that nonlinear pricing tends to harm profit and welfare but to be pro-consumer. So different strands of the literature on competitive nonlinear pricing and bundling have yielded conflicting results about the pros and cons of nonlinear pricing. Our analysis seeks to reconcile and extend these findings within a unifying framework.

The rest of the paper, and the main results, can be summarized as follows. Our framework for analysis is set out in section 2. There are two symmetrically placed firms, each supplying two products. Consumers, who may have elastic and heterogeneous demands, choose on the basis of prices and brand preferences whether to buy from both firms or just one, and how much of each product to buy. This potentially complex model has a remarkably simple equilibrium when nonlinear tariffs are used: firms offer efficient two-part tariffs—i.e., with marginal prices equal to marginal costs—with the fixed elements of the tariffs precisely the same equilibrium prices as in the very special case of the model in which identical consumers have inelastic (unit) demands. In particular, there is the same discount for one-stop shopping, which is obtained from a neat elasticity condition. As well as generalizing to endogenous multi-stop shopping the finding that efficient two-part tariffs are an equilibrium, our central equilibrium result provides a framework for comparing linear with nonlinear pricing.

[3]For instance, see Martimort and Stole (2005) as well as the discussion in Stole (2007, section 6.4).

[4]See Whinston (1990) and Nalebuff (2004), for instance.

This comparison, which section 3 provides, allows a synthesis of the contrasting results from earlier work referred to above, as well as yielding new results. Two economic issues are central to the synthesis. The first is the familiar *excessive marginal price* problem with linear pricing in imperfect competition that marginal price exceeds marginal cost, with resulting inefficiency unless demand is inelastic. Second, with discounts for one-stop shopping, which are a feature of nonlinear pricing provided that exogenous shopping costs are not too high, there is the problem of *excessive loyalty*, that there is more one-stop shopping than at the efficient optimum. The efficient two-part tariff equilibrium has only the latter problem; equilibrium with linear pricing has only the former. Nonlinear pricing discounts for one-stop shopping, when they are significant, tend to make competition keener, because they increase the number of relatively lucrative one-stop shoppers for whose (double) custom the firms compete strongly. This competition can at the same time benefit two-stop shoppers. So there is a tendency for nonlinear pricing, via the discounts and extra intensity of competition, to benefit consumers and diminish profits.

	Welfare	Profit	Consumers
(i) demand elasticity	?	+	−
(ii) consumer heterogeneity	−	+	−
(iii) shopping costs	+	+	−
(iv) brand preference correlation	+	+	−

Table 1: Effects on the relative merits of nonlinear pricing

Section 3 pursues the comparison between linear and nonlinear pricing in terms of four underlying economic factors: (i) demand elasticity, (ii) consumer heterogeneity, (iii) shopping costs of buying from more than one supplier, and (iv) correlation in brand preferences. Table 1 indicates whether an increase in each factor tends to add to or subtract from the merits of nonlinear pricing relative to linear pricing for welfare, profit and consumer surplus. The excessive marginal prices problem becomes more important relative to the excessive loyalty problem as consumer homogeneity, shopping costs and brand preference correlation increase. For example, the excessive loyalty effect increases in importance as shopping costs fall and as brand correlation decreases (in each case because there are more two-stop shoppers). However, the impact of demand elasticity on the welfare comparison is ambiguous because elastic demand intensifies competition under linear pricing, and this can reduce the equilibrium dead-weight losses associated with linear pricing. The direction of effects on profit tends to be the opposite of that on consumer surplus. In respect of

(iii) and (iv), the profit effect has the same sign as the welfare effect. But with greater consumer heterogeneity, the relative merits of nonlinear pricing for welfare and consumers tend to be lower. That is because heterogeneity sharpens linear price competition, which is good for consumers and welfare (as the excessive marginal price effect diminishes) but bad for profit.

Section 4 concludes by considering why the duopoly problem analyzed in this paper is in some ways simpler than the corresponding monopoly problem, and by suggesting directions for future research on competitive nonlinear pricing and bundling.

2 A Model of Nonlinear Pricing and Bundling

2.1 Description of the model

Two firms, A and B, each offer their own brand of two products, 1 and 2. Suppose that each firm incurs a constant marginal cost c_i for serving a consumer with a unit of product i.[5] In the proposed model, consumers differ in two kinds of ways: they differ in their preference for the supplier of a given product ("horizontal" brand preferences), and they differ in the quantity of a given product they wish to buy ("vertical" preferences).[6] Brand preferences are captured within a two-dimensional Hotelling model. A consumer's brand preferences are denoted $(x_1, x_2) \in [0,1]^2$, where x_1 represents the consumer's distance from firm A's brand of product 1 and x_2 represents the consumer's distance from the same firm's brand of product 2. This consumer's distance to firm B's brand of product i is $1 - x_i$. The density of (x_1, x_2) is $f(x_1, x_2)$ and firms are symmetrically placed in terms of brand preferences, so that

$$f(x_1, x_2) \equiv f(1 - x_1, 1 - x_2) \ . \qquad (1)$$

We assume that this density is continuous and strictly positive on the unit square.

The transport cost (or product differentiation) parameter is t_1 for product 1 and t_2 for product 2. In addition to these transport costs, consumers face an exogenous "shopping

[5] Note that we do not allow for the possibilities (i) of having fixed costs of supplying an individual consumer, or (ii) of it being cheaper to supply a product to a consumer to whom the same firm already supplies the other product. Both of these possibilities may be relevant for energy and communications markets. Allowing for more ornate cost structures does not significantly alter our analysis of equilibrium nonlinear pricing, but it does significantly affect the comparison between nonlinear and linear pricing in section 3. Indeed, with these more ornate cost functions it is not obvious that linear pricing is a natural benchmark.

[6] Clearly, we could re-label "quantity" as "quality", and little would change in our analysis of nonlinear tariffs. However, linear pricing has no natural meaning with this alternative interpretation.

cost" $z \geq 0$ when they source supplies from two firms rather than one. This shopping cost might represent the time or cost involved in visiting two shops rather than one, or it might measure a consumer's perceived cost of dealing with two firms (paying two bills rather than one, for instance). In order to have some two-stop shoppers in equilibrium the shopping cost cannot be too large, and the (necessary and sufficient) condition for some two-stop shoppers to be present in equilibrium is

$$z < \min\{t_1, t_2\} \ . \tag{2}$$

This condition applies both when nonlinear and linear pricing is employed. Unless otherwise stated, we assume that this condition is satisfied. (The situation where (2) is violated is discussed in section 3.4.)

We assume that consumers make all their purchases of a given product from one firm or the other. We also assume that a firm's tariff depends only on quantities purchased from that firm, and not on a consumer's dealings with the rival firm. (In section 2.3, we argue that the nonlinear tariff we derive remains an equilibrium within the more general class of tariffs.) Firm i's tariff then consists of three options: $T_1^i(q_1)$ is the charge for q_1 units of product 1 if the consumer does not buy any product 2 from the firm; $T_2^i(q_2)$ is the corresponding stand-alone tariff for product 2, and $T_{12}^i(q_1, q_2)$ is the tariff if the consumer buys all her supplies from the firm. Suppose a type-θ consumer has gross utility $u(\theta, q_1, q_2)$ if quantity q_i of product i is consumed. (The parameter θ will most naturally be multi-dimensional.) If the type-(θ, x_1, x_2) consumer buys quantities (q_1, q_2) from firm A, her net utility is

$$u(\theta, q_1, q_2) - T_{12}^A(q_1, q_2) - t_1 x_1 - t_2 x_2 \ ;$$

if she buys quantities (q_1, q_2) from firm B, her net utility is

$$u(\theta, q_1, q_2) - T_{12}^B(q_1, q_2) - t_1(1 - x_1) - t_2(1 - x_2) \ ;$$

and if she buys quantity q_i of product i from A and quantity q_j of product j from B, her utility is

$$u(\theta, q_1, q_2) - T_i^A(q_i) - T_j^B(q_j) - t_i x_i - t_j(1 - x_j) - z \ .$$

The consumer will choose quantities and the mix of suppliers to maximize this utility.

There are two important assumptions needed to derive the equilibrium nonlinear tariff in this context, and these are assumed throughout the paper. First, the vertical taste parameter θ is distributed independently from the horizontal brand preference parameters (x_1, x_2) in the population of consumers, so that the observation that a consumer prefers firm A's brand of product i is not informative about the size of that consumer's demand for units of product i or product j. Second, we assume that parameters are such that all consumers wish to buy some of both products in equilibrium.[7]

2.2 Equilibrium nonlinear tariffs with unit demands

Before analyzing equilibrium tariffs in this general framework, we focus first on the special case where all consumers wish to buy exactly one unit of each of the two products. (Thus, there is neither heterogeneity in demands nor sensitivity of demand to marginal price.) This case is of interest in its own right (see further section 3.2); it will also provide the key to the more general elastic demand analysis in the next section.

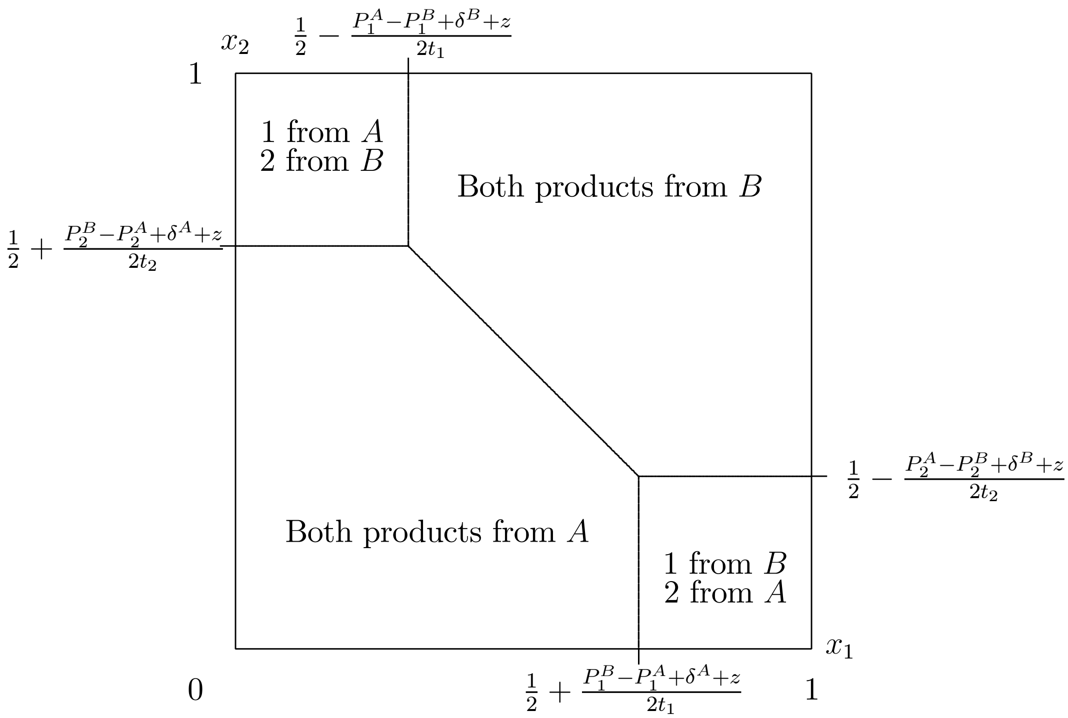

Figure 1: The Pattern of Consumer Demand

[7] In the monopoly context of Armstrong (1996), there is no plausible condition which ensures that all consumers participate in the market. In technical terms, it is a significant advantage of a duopoly framework that we can make relatively natural assumptions—such as requiring that willingness-to-pay is sufficiently large relative to costs—to ensure full coverage. With nonlinear pricing, the natural assumption is simply that, for all consumer types, consumer surplus at marginal-cost prices exceeds the fixed fees derived below in Proposition 2 plus $(t_1 + t_2)/2$, the latter being the maximum "transport cost" paid by any consumer.

Suppose in this sub-section that production is costless.[8] Since consumers have unit demand for each product, a firm's tariff consists just of three prices: P_1^i is firm i's stand-alone price for its product 1; P_2^i is its stand-alone price for its product 2, and δ^i is its discount if a consumer buys both products, so the total charge for buying both products from firm i is $P_1^i + P_2^i - \delta^i$. The resulting pattern of demand is shown in Figure 1.

It is useful to introduce some further notation. For $\delta + z \leq \min\{t_1, t_2\}$, define

$$\Phi(\delta) \equiv 2 \int_0^{\frac{1}{2}-\frac{\delta+z}{2t_1}} \int_{\frac{1}{2}+\frac{\delta+z}{2t_2}}^1 f(x_1, x_2)\ dx_2 dx_1$$

to be the proportion of consumers who are two-stop shoppers when firms set the same tariff which involves discount δ. The discount δ can be viewed as the "price" for two-stop versus one-stop shopping, and $\Phi(\delta)$ is the downward-sloping demand for two-stop shopping.[9] This demand function summarizes many of the relavant features of the market, and it is depicted on Figure 2. (The symmetry assumption (1) implies that the two rectangles of two-stop shoppers each contain the same proportion $\Phi/2$ of consumers.) When the two firms offer the same tariff with stand-alone prices P_1 and P_2 and discount δ, industry profit is

$$P_1 + P_2 - \delta(1 - \Phi(\delta))\ . \tag{3}$$

In addition, write

$$\alpha_1(\delta) = \int_{\frac{1}{2}+\frac{\delta+z}{2t_2}}^1 f(\tfrac{1}{2} - \frac{\delta+z}{2t_1}, x_2)\ dx_2\ ;\ \alpha_2(\delta) = \int_0^{\frac{1}{2}-\frac{\delta+z}{2t_1}} f(x_1, \tfrac{1}{2} + \frac{\delta+z}{2t_2})\ dx_1 \tag{4}$$

for the line integrals depicted on Figure 2. (From (1), the two integrals marked α_1 have the same value, as do those marked α_2.) Then

$$\Phi'(\delta) = -\left(\frac{\alpha_1(\delta)}{t_1} + \frac{\alpha_2(\delta)}{t_2}\right)\ . \tag{5}$$

Finally, write

$$\beta_1(\delta) = \int_{\frac{1}{2}-\frac{\delta+z}{2t_2}}^{\frac{1}{2}+\frac{\delta+z}{2t_2}} f(\tfrac{1}{2} + \frac{t_2}{t_1}(\tfrac{1}{2} - x_2), x_2)\ dx_2\ ;\ \beta_2(\delta) = \int_{\frac{1}{2}-\frac{\delta+z}{2t_1}}^{\frac{1}{2}+\frac{\delta+z}{2t_1}} f(x_1, \tfrac{1}{2} + \frac{t_1}{t_2}(\tfrac{1}{2} - x_1))\ dx_1 \tag{6}$$

[8] This is without loss of generality if marginal costs are constant, when the prices P_i derived below can be considered to be prices net of marginal costs. The reason why it is convenient to assume costless production in this section is that the bundling tariff derived here will define the fixed charges in the two-part tariffs derived in the more general model in section 2.3. If we had costly production in this section, this would be equivalent to having fixed costs of supplying consumers, in addition to the marginal per-unit costs, which we have ruled out.

[9] The "price" of two-stop shopping δ is additional to the exogenous shopping cost z, so the overall extra cost to the consumer of buying from two firms is $\delta + z$.

for the line integrals along the diagonal segment depicted on Figure 2. Here, β_1 is the integral in the vertical direction, β_2 is the integral in the horizontal direction, and they are related by $t_2\beta_1 = t_1\beta_2$.

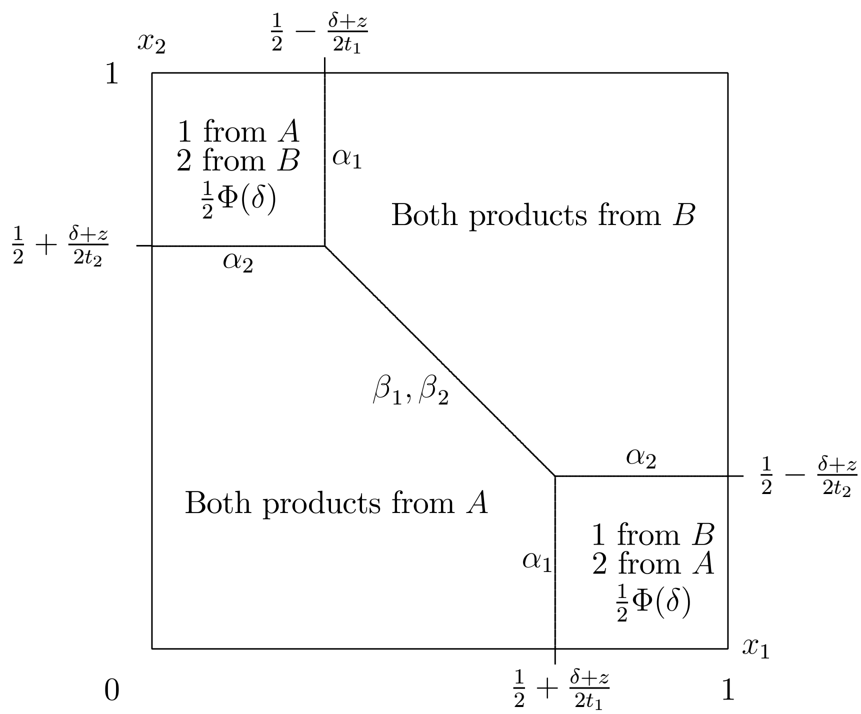

Figure 2: Notation Used in the Analysis

It turns out that there is a simple and general formula for the equilibrium discount:

Proposition 1 *In a symmetric equilibrium the bundling discount δ satisfies*

$$\Phi(\delta) + \tfrac{1}{2}\Phi'(\delta)\delta = 0 \ . \tag{7}$$

Proof. Suppose that the symmetric equilibrium nonlinear tariff involves the stand-alone price P_1 for product 1, the stand-alone price P_2 for product 2, and the bundling discount δ. (In fact, the following argument does not depend on P_1 and P_2 being *equilibrium* prices, only that both firms offer this pair of stand-alone prices.) Consider the effect on firm A's profit if it increases its discount by a small amount 2ε and simultaneously increases each of its stand-alone prices by ε. The result is that its total charge to its one-stop shoppers is unchanged, but the total charge for each two-stop shopping option rises by ε.

The impact on the firm's demand is depicted in Figure 3, where the two-stop shopping regions shrink to the dashed regions: the deviation moves the upper boundary marked α_1

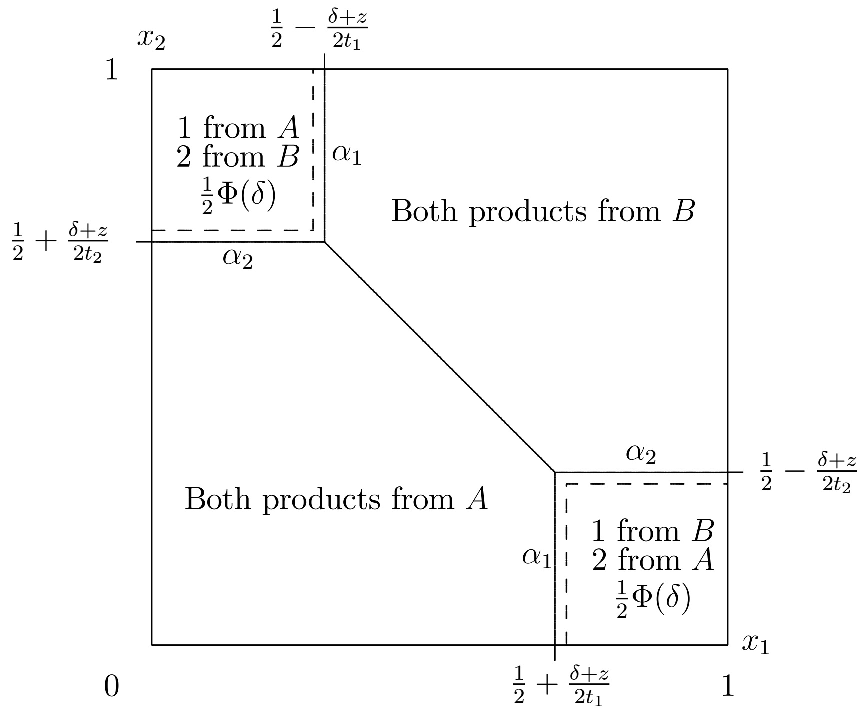

Figure 3: Effect of Changing the Bundle Discount

to the left by $\varepsilon/(2t_1)$ and lower boundary α_1 to the right by the same amount; the upper boundary α_2 is moved up by $\varepsilon/(2t_2)$ and the lower boundary α_2 down by the same amount. The net effect on firm A's profit to first order in ε is approximately

$$(\varepsilon\Phi(\delta)) + \frac{\varepsilon}{2t_1}\alpha_1(P_1 - \delta) - \frac{\varepsilon}{2t_1}\alpha_1 P_1 + \frac{\varepsilon}{2t_2}\alpha_2(P_2 - \delta) - \frac{\varepsilon}{2t_2}\alpha_2 P_2 = \varepsilon\left(\Phi(\delta) + \tfrac{1}{2}\Phi'(\delta)\delta\right) ,$$

where the equality follows from (5). For this deviation to be unprofitable, equality (7) must hold in equilibrium. ■

Thus, viewing $\Phi(\delta)$ as the demand for two-stop shopping as a function of the price δ, this result shows that in equilibrium the elasticity of this demand, $-\delta\Phi'/\Phi$, is equal to 2.

Note that the left-hand side of (7) is positive when $\delta = 0$. In particular, linear pricing cannot be an equilibrium when firms have the ability to offer bundling discounts. Moreover, when δ is close to its upper limit of $\min\{t_1, t_2\} - z$, the left-hand side of (7) is negative.[10] Thus, there is at least one $\delta \in (0, \min\{t_1, t_2\} - z)$ which satisfies (7). If the left-hand side of (7) decreases with δ, or if the elasticity $-\delta\Phi'/\Phi$ increases with δ (e.g., if Φ is log-concave),

[10] When $t_1 = t_2$, the left-hand side of (7) equals zero when $\delta = t - z$, since both Φ and Φ' are zero there, but for slightly smaller δ the expression is certainly negative.

there is then a unique solution with $\delta \in (0, \min\{t_1, t_2\} - z)$ to expression (7).[11] If Φ is log-concave in δ, a simple corollary to Proposition 1 is that the equilibrium discount is decreasing in z. (Increasing z then causes the elasticity $-\delta\Phi'/\Phi$ to rise for given δ.) Indeed, the discount falls to zero as the exogenous shopping cost z approaches $\min\{t_1, t_2\}$. That is to say, if almost all consumers are anyway one-stop shoppers, there is little benefit to firms in inducing still more consumers to become one-stop shoppers by means of a bundle discount. In this sense, raising the exogenous shopping cost reduces the incentive to engage in bundling.

An immediate corollary to Proposition 1 is the following:

Corollary 1 *In a symmetric equilibrium the bundling discount is positive.*

This result is in the same spirit as that derived in the (primarily) monopoly context by McAfee, McMillan, and Whinston (1989). However, in McAfee *et al.* it was sometimes possible for a bundling premium, as opposed to a discount, to be optimal. Our duopoly setting, in which there is full market coverage and firms are symmetric, ensures that a discount is always optimal.

An important point is that welfare—measured by the sum of consumer surplus and profit—is reduced when firms offer discounts for joint purchase: whenever $\delta > 0$ there is *excessive loyalty*, as more consumers than is efficient buy both products from the same firm. (For instance, a consumer participating in a frequent flyer programme might choose flights departing at less preferred times in order to enjoy bundling discounts.) The efficient pattern of consumption requires there to be no bundling discount. In more detail, by examining Figure 3, one sees that when the discount δ is increased by ε, the number of extra consumers who buy product 1 from their less preferred firm is equal to $\frac{\alpha_1}{t_1}\varepsilon$, and each of these consumers incurs the extra travel cost $(\delta + z)$ compared to buying the product from the closer firm, although these consumers now save the shopping cost z. Thus, their net cost is δ. Similarly, $\frac{\alpha_2}{t_2}\varepsilon$ extra consumers buy product 2 from the less preferred firm,

[11] There is always another, less interesting, *pure bundling* equilibrium. Suppose one firm offers a tariff involving extremely high stand-alone prices. Then no consumer will ever be a two-stop shopper, and so the rival might as well offer a similar tariff. However, there are good reasons to believe that this second equilibrium is non-robust. For instance, it involves firms playing weakly dominated strategies. Moreover, Thanassoulis (2007) shows that when there are some consumers who wish to buy just one item, pure bundling ceases to be an equilibrium.

and these each incur a net cost δ. In sum, the extra welfare loss caused by increasing δ by ε is

$$\varepsilon\delta \times \left(\frac{\alpha_1}{t_1} + \frac{\alpha_2}{t_2}\right) = -\varepsilon\delta\Phi'(\delta) \ .$$

Denote by $w(\delta)$ the level of welfare corresponding to the bundling discount δ relative to the first-best welfare level (i.e., the welfare level corresponding to the case $\delta = 0$). It follows that

$$w'(\delta) = \delta\Phi'(\delta) \ . \tag{8}$$

This condition together with $w(0) = 0$ yields $w(\delta)$.

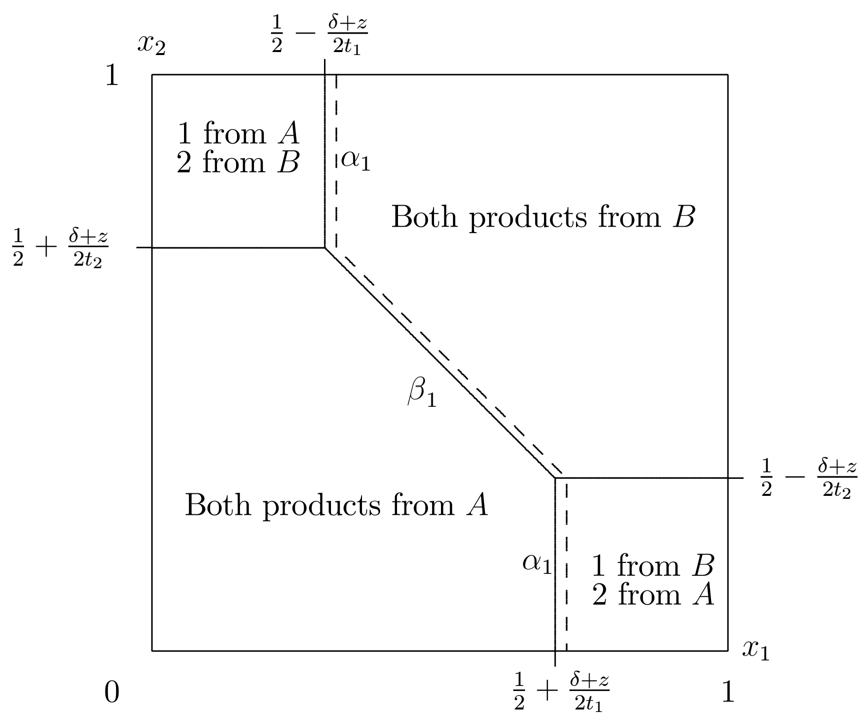

Figure 4: Incentive to Reduce Stand-Alone Price for Product 1

Having found the equilibrium bundling discount in (7), we now derive the equilibrium stand-alone prices, P_1 and P_2. For brevity, given the equilibrium discount δ, write $\alpha_i = \alpha_i(\delta)$ and $\beta_i = \beta_i(\delta)$. The pattern of demand is as shown on Figure 4. Consider firm A's incentive to reduce its stand-alone price P_1 by ε, keeping its discount unchanged at δ and its stand-alone price for product 2 unchanged at P_2. At a symmetric equilibrium, half the consumers buy product 1 from firm A, and the firm loses revenue ε from each of these infra-marginal consumers. The price reduction shifts the boundary of the set of consumers who buy product 1 from the firm uniformly to the right by $\varepsilon/(2t_1)$. Those consumers on the upper boundary α_1 on the figure generate profit P_1 to the firm, those on the diagonal boundary β_1 generate "double" profit $(P_1 + P_2 - \delta)$, and those on the lower boundary α_1 generate incremental profit $(P_1 - \delta)$.

Since the profit gained from the marginal consumers must equal the profit lost from the infra-marginal consumers, it follows that in equilibrium

$$\alpha_1 P_1 + \beta_1(P_1 + P_2 - \delta) + \alpha_1(P_1 - \delta) = t_1 \ . \tag{9}$$

Similarly, the first-order condition for the stand-alone price P_2 is

$$\alpha_2 P_2 + \beta_2(P_1 + P_2 - \delta) + \alpha_2(P_2 - \delta) = t_2 \ . \tag{10}$$

Solving these linear simultaneous equations in (P_1, P_2) yields explicit formulae for the stand-alone prices, as reported in the next result.[12]

Proposition 2 *At a symmetric equilibrium, the discount δ satisfies (7) and the stand-alone prices are*

$$P_1 = \frac{\delta}{2} + \frac{\alpha_2 t_1}{\alpha_1\beta_2 + \beta_1\alpha_2 + 2\alpha_1\alpha_2} \ ; \ P_2 = \frac{\delta}{2} + \frac{\alpha_1 t_2}{\alpha_1\beta_2 + \beta_1\alpha_2 + 2\alpha_1\alpha_2} \ . \tag{11}$$

Notice that the stand-alone prices in (11) are uniquely determined given the discount δ. Provided there is a unique solution to the first-order condition for the discount in (7), which is guaranteed if Φ is log-concave, there is then only one possible symmetric equilibrium with some two-stop shoppers.[13]

For instance, consider the following example:

Uniform Example: $t_1 = t_2 = t > z$ and $f(x_1, x_2) \equiv 1$.

In this case $\Phi(\delta) = \frac{1}{2}(1 - \frac{\delta+z}{t})^2$, and so expression (7) implies that $\delta = \frac{1}{2}(t - z)$. It follows that $\alpha_1 = \alpha_2 = \frac{1}{4} - \frac{z}{4t}$ and $\beta_1 = \beta_2 = \frac{1}{2} + \frac{z}{2t}$. From (11) the equilibrium tariff is

[12] Observe that, although our presentation may give the impression that the discount is chosen first and stand-alone prices chosen subsequently, in fact all three elements of a firm's tariff are chosen simultaneously. By contrast, Gans and King (2006) analyze a model where the discount is chosen in a first stage, and there is subsequent competition in stand-alone prices. In their model, the two products which enjoy the bundling discount are supplied by separate firms who choose their stand-alone prices non-cooperatively. They find that bundling has no impact on profits (in contrast to our results), but that it harms welfare.

[13] We have examined the second-order conditions for the *Uniform Example* described in the text and verified that the tariff (12) is a global best response for one firm if the rival offers the same tariff. However, we have not investigated second-order conditions in general for the tariff in Proposition 2.

then[14]

$$P_1 = P_2 = \tfrac{1}{4}(t-z) + \frac{2t^2}{3t+z} \ ; \ \delta = \tfrac{1}{2}(t-z) \ . \tag{12}$$

Notice that the shopping cost z acts to lower all prices. The shopping cost expands the margin of "doubly profitable" one-stop shoppers, which intensifies competition. Expression (8) implies that the equilibrium welfare loss relative to the first-best outcome is

$$\frac{(t-z)^3}{12t^2} \ . \tag{13}$$

For instance, when $z = 0$ only one-eighth of consumers are two-stop shoppers with the equilibrium nonlinear tariff, whereas the efficient choice of supplier would involve half of the consumers buying from two suppliers. As expected, when the shopping cost is large, the welfare loss (13) falls to zero since the bundling discount also falls to zero.

2.3 Equilibrium nonlinear tariffs with general demands

Having analyzed equilibrium nonlinear tariffs with unit demands, we now turn to the more general case in which (i) there is heterogeneity among consumers and (ii) demands are sensitive to price. The results from the unit demand case are the key to understanding equilibrium nonlinear tariffs in the general case, and they allow us immediately to state our central result—that it is an equilibrium for firms to offer two-part tariffs with marginal prices equal to marginal costs and with fixed charges corresponding to the bundling prices derived in section 2.2.[15]

Proposition 3 *It is an equilibrium for each firm to offer the following nonlinear tariff:*

$$T_1(q_1) = P_1 + c_1 q_1 \ ; \ T_2(q_2) = P_2 + c_2 q_2 \ ; \ T_{12}(q_1, q_2) = T_1(q_1) + T_2(q_2) - \delta \ , \tag{14}$$

where P_1, P_2 and δ comprise the mixed bundling tariff described in Proposition 2.

Proof. See appendix. ■

[14] The model of Matutes and Regibeau (1992) corresponds to the case $z = 0$, when $P_i = \frac{11}{12}t$ and $\delta = \frac{1}{2}t$. That example was extended in Armstrong (2006, section 4.2) to situations where $t_1 \neq t_2$ and where $z > 0$, and in Thanassoulis (2007, section 4) to situations where some consumers only want a single item. These analyses took a "brute force" approach by simply calculating the areas of the various regions in Figure 1. This method is only practical when the distribution of (x_1, x_2) is uniform, and it cannot be used to derive more general results, such as Propositions 1, 2 and 4 in this paper.

[15] We do not claim that this is the *unique* symmetric equilibrium. For instance, there is at least one other, pure bundling equilibrium (see footnote 11).

Despite the generality of the demand structure and consumer heterogeneity, this equilibrium is remarkably simple. The shape and heterogeneity of consumer utility $u(\theta, q_1, q_2)$ has no impact on the form of these tariffs, and hence on industry profit. In particular, and perhaps surprisingly, whether or not the two products are complements or substitutes in consumer demand makes no difference to the equilibrium incentive to offer bundling discounts. Moreover, since there is marginal-cost pricing for both products, the only welfare loss relative to the first best is the *excessive loyalty* effect described in section 2.2. This welfare loss, $w(\delta)$ in (8), also does not depend on the form or variety of consumer utility functions.

If measurement issues do not preclude them, more ornate nonlinear tariffs could be considered in which firm i's charge for product 1, say, depends on how much of product 2 the consumer buys from B. That is to say, firm i's charge for q_1 stand-alone units of product 1 is $T_1^i(q_1, q_2^j)$, where q_2^j is the consumer's verifiable demand for product 2 from the rival. One can check that the nonlinear tariffs we derive as an equilibrium in Proposition 3 (which do not have this cross-dependence) remain an equilibrium even when firms can employ these more complicated tariffs.

Proposition 3 generalizes Armstrong and Vickers (2001, Proposition 5) to the situation in which consumers are able, and sometimes willing, to buy products from several suppliers. In our earlier result, which assumed exogenous one-stop shopping by consumers, the equilibrium nonlinear tariff was a (single) two-part tariff involving marginal-cost pricing and a fixed charge which depended only on brand preferences (not the form or variety of consumer preferences for the product). The same is true in Proposition 3, except that there are several two-part tariffs, all of which involve marginal-cost pricing and where the fixed charges depend only on brand preference data.

An important assumption for this result is that all consumers choose to buy both products at relevant tariffs. While this might be a reasonable assumption in some markets (e.g., with gas and elasticity), in other markets it is a drastic simplification. The greatest challenge to extend our analysis to situations with partial consumer coverage will be to develop a technical apparatus able to characterize equilibrium nonlinear tariffs in this setting. We expect that our results will be sensitive to the full coverage assumption. For instance, marginal-cost prices will no longer be an equilibrium nonlinear tariff when market participation is elastic. (This is so even in the simpler one-stop shopping framework when there is partial market coverage—see Rochet and Stole (2002).)

3 Comparison with Linear Pricing

We now compare the outcomes corresponding to linear pricing and the nonlinear tariff derived in Proposition 3.[16] After describing the equilibrium linear prices in the next section, we show the importance of four kinds of economic effect: (i) demand elasticity, (ii) consumer heterogeneity, (iii) shopping costs, and (iv) correlation in brand preferences. The impact of these effects was summarised in Table 1 in the introduction.

3.1 Linear prices

Write $v(\theta, p_1, p_2) = \max_{q_1,q_2} u(\theta, q_1, q_2) - p_1 q_1 - p_2 q_2$ for the consumer surplus, excluding transport and shopping costs, of the type-θ consumer when she pays linear prices p_1 and p_2. If both firms offer the same pair of linear prices, the surplus $v(\theta, p_1, p_2)$ is unaffected by whether the consumer buys both products from firm A, from firm B, or one product from each firm. Therefore, the pattern of consumer demand is as depicted on Figure 5. (This is the same as Figure 2 above, except that $\delta = 0$. The θ parameter has no impact on a consumer's choice of suppliers when firms offer the same linear prices.) In particular, in a symmetric equilibrium there will be no inefficiency due to excessive loyalty, although there will be inefficiency due to above-cost marginal prices. In sum, in our model nonlinear pricing causes one welfare loss (excessive loyalty) but solves another (marginal prices are equal to cost), whereas the reverse occurs with linear pricing, where there is no excessive loyalty but consumers pay excessive prices. The welfare comparison between nonlinear and linear pricing, as well as their relative impact on profit and consumer surplus, is ambiguous in this general model, as we will explore in the coming sections.

We next derive the equilibrium linear prices, p_1 and p_2. For brevity, write $\alpha_i^0 = \alpha_i(0)$, $\beta_i^0 = \beta_i(0)$ and $\Phi^0 = \Phi(0)$. If firm A undercuts B's product 1 price by ε, this shifts the

[16] One could also consider an intermediate regime in which firms offer quantity discounts for a particular product, but cannot offer bundling discounts across products. In welfare (and profit) terms, this pricing regime delivers the best of both worlds, since the excessive loyalty effect and the excessive marginal price effect are both avoided. However, it would be hard for public policy to enforce this intermediate regime, since in practice it is not always clear what constitutes a distinct product. (For instance, should a season ticket to a concert series count as an inter-product or intra-product discount?) For this reason, we focus on the transparent distinction between linear pricing and unrestricted nonlinear tariffs.

boundary of the set of type-θ consumers who buy the product from A uniformly to the right by $\frac{\varepsilon}{2t_1}q_1(\theta,p_1,p_2)$. Each of the marginal consumers on the vertical boundaries α_1^0 brings extra profit $(p_1-c_1)q_1(\theta,p_1,p_2)$, while each consumer on the diagonal boundary β_1^0 brings "double" profit $(p_1-c_1)q_1(\theta,p_1,p_2)+(p_2-c_2)q_2(\theta,p_1,p_2)$. Set against this is the impact of the price cut on the profit from its infra-marginal consumers. Each of A's existing consumers of product 1 contribute product 1 profit which is reduced by $\varepsilon[q_1(\theta,p_1,p_2)+(p_1-c_1)\frac{\partial}{\partial p_1}q_1(\theta,p_1,p_2)]$, and in a symmetric equilibrium the proportion of such consumers equals a half. Finally, there is the effect of changing p_1 on demand for the firm's product 2. This is only relevant for the firm's one-stop shoppers, who number $\frac{1}{2}(1-\Phi^0)$ in equilibrium as shown on Figure 5. Each of these one-stop shoppers generates product 2 profit which is reduced by $\varepsilon(p_2-c_2)\frac{\partial}{\partial p_1}q_2(\theta,p_1,p_2)$.

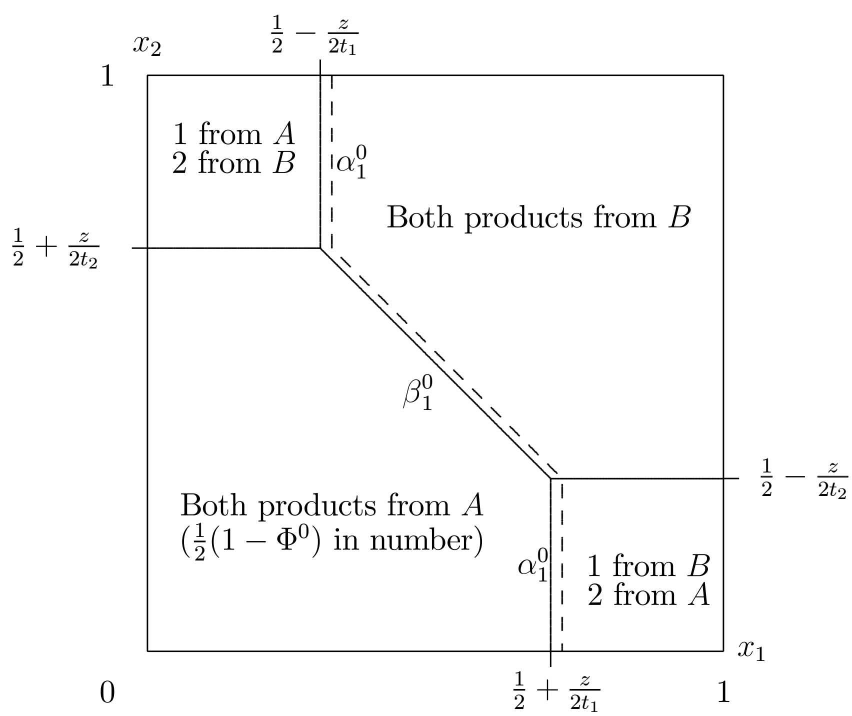

Figure 5: Incentive to Reduce Linear Price for Product 1

Putting all this together and taking expectations over θ implies that the first-order condition for p_1 to be the equilibrium price for product 1 is:

$$E_\theta\left[\left(2\alpha_1^0(p_1-c_1)q_1+\beta_1^0((p_1-c_1)q_1+(p_2-c_2)q_2)\right)q_1\right]$$
$$=t_1\times E_\theta\left[q_1+(p_1-c_1)\frac{\partial q_1}{\partial p_1}+(1-\Phi^0)(p_2-c_2)\frac{\partial q_2}{\partial p_1}\right]\ . \tag{15}$$

(Here, the dependence of demands q_i on p_1, p_2 and θ has been suppressed.) A similar expression holds for product 2.

Formula (15) is complex, and reflects the effects of own- and cross-price elasticities, consumer heterogeneity (via the quadratic terms q_1^2 and q_1q_2), the extent of product differentiation, the shopping cost (via the terms α_i^0 and β_i^0), and correlation in brand preferences (via the size of Φ^0). An extension to the *Uniform Example* illustrates some of these effects.

Linear Uniform Example: $t_i = t > z;\ f \equiv 1;\ q_i(\theta_1, \theta_2, p_1, p_2) = \theta_i(1 - bp_i);\ c_i = 0.$

Here, demand functions are linear and exhibit no cross-price effects, and consumer heterogeneity is represented by a multiplicative term θ_i for each product. Suppose that each θ_i has mean 1 and variance σ^2, and let the covariance of θ_1 and θ_2 be $\kappa\sigma^2$ for $-1 \leq \kappa \leq 1$. It is natural to suppose that there is positive correlation in the scale of demands for the two products across consumers (i.e., $\kappa > 0$), since it is likely that a consumer's income will be positively correlated with her demand for each product. The parameter b represents the sensitivity of demand to marginal price. With a linear price p for each product, industry profit is $2p(1-bp)$ and welfare relative to the first best is $-bp^2$. In this example $\alpha_i^0 = \frac{1}{2} - \frac{z}{2t}$ and $\beta_i^0 = \frac{z}{t}$, and so (15) implies that the equilibrium linear price for a unit of either product, p, satisfies

$$\frac{p(1-bp)^2}{1-2bp} = \frac{t^2}{t + z + \sigma^2(t + \kappa z)}\ . \tag{16}$$

This equilibrium price increases with t and falls with z, b, σ^2 and κ.[17] (By contrast, the equilibrium nonlinear tariff in Proposition 3 does not depend at all on b, σ^2 and κ.) The impact of these comparative statics, together with their intuition, will be explored in the following sections.

3.2 Impact of demand elasticity

First consider the extreme case where each consumer wishes to consume one unit of each product, so that demand is completely inelastic and there is no consumer heterogeneity in the size of demand. (This is the situation examined in section 2.2.) In this limit case, we wish to compare the welfare, profit and consumer surplus obtained with nonlinear pricing with those obtained with linear pricing. First, it is immediate that welfare is higher with

[17] The left-hand side of (16) is increasing in p and b over the relevant range $0 \leq p \leq 1/(2b)$. The right-hand side is increasing in t and decreasing in z, κ and σ^2. (Recall that $z < t$ and $\kappa > -1$ so $t + \kappa z > 0$.)

linear pricing, since the excessive loyalty problem is eliminated. (With unit demands, that is the only possible welfare problem.) To understand the impact of nonlinear pricing on profit and consumer surplus requires further analysis.

In the unit demand model, equilibrium linear prices P_1 and P_2 satisfy the pair of equations (9)–(10) in the situation where the bundling discount δ is set equal to zero. That is, from (11) the equilibrium linear prices are

$$P_1 = \frac{\alpha_2^0 t_1}{\alpha_1^0\beta_2^0 + \beta_1^0\alpha_2^0 + 2\alpha_1^0\alpha_2^0} \; ; \; P_2 = \frac{\alpha_1^0 t_2}{\alpha_1^0\beta_2^0 + \beta_1^0\alpha_2^0 + 2\alpha_1^0\alpha_2^0} \; . \tag{17}$$

In the *Uniform Example* discussed in section 2.2, for instance, it follows that[18]

$$P_1 = P_2 = \frac{t^2}{t+z} \; . \tag{18}$$

Note that when z is small, the linear prices in (18) are higher than the stand-alone prices (let alone the discounted price for joint consumption) in (12). Thus, *all* consumers then pay lower prices with nonlinear pricing, with the result that consumer surplus is higher, and profit is lower, with nonlinear pricing. For sufficiently larger z, the two-stop shoppers face higher prices with nonlinear pricing than they would with linear prices. Nevertheless, as is implied by the next result, aggregate consumer surplus rises and profit falls in these cases too.

Proposition 4 *Suppose that all consumers have unit demands for the two products. Suppose that x_1 and x_2 are independently distributed, with respective density functions $f_1(x_1)$ and $f_2(x_2)$ and respective distribution functions $F_1(x_1)$ and $F_2(x_2)$. (The densities satisfy $f_i(x) \equiv f_i(1-x)$.) Suppose the distributions satisfy the hazard rate conditions*

$$\frac{d}{dx}\frac{F_1(x)}{f_1(x)} \geq \tfrac{1}{4} \; ; \; \frac{d}{dx}\frac{F_2(x)}{f_2(x)} \geq \tfrac{1}{4} \tag{19}$$

for $x \leq \frac{1}{2}$. Then compared to the outcome with linear pricing, profit and welfare fall, but consumer surplus rises, when nonlinear pricing is used.

Proof. See appendix. ■

This result is in the same spirit as Matutes and Regibeau (1992), Anderson and Leruth (1993) and Thanassoulis (2007, section 4), who find in particular examples involving unit

[18] This can also be derived from (16) by setting $b = \sigma^2 = 0$.

demands that competitive mixed bundling leads to lower profit and welfare, but higher consumer surplus, compared to linear pricing.[19,20]

Proposition 4 gives a relatively mild condition on the density of brand preferences to ensure that consumers overall benefit from nonlinear pricing for the case when brand preferences are independent, so that the fact that a consumer prefers firm A's brand of product 1 is not informative about the consumer's preferred supplier for product 2. The result appears to be robust to having positive correlation in brand preferences, at least in simple examples. (We discuss this point further in section 3.5.)

Under the conditions of Proposition 4, why do firms do worse, and consumers do better, with nonlinear pricing than with linear pricing? With nonlinear pricing, Proposition 1 establishes that firms will choose to offer discounts to their one-stop shoppers. Such discounts can intensify price competition generally so that stand-alone purchases might become cheaper too (or at least they do not rise by enough to overturn the consumer benefits of the discount). When there are discounts for joint purchase, a wider margin of competition for one-stop shoppers opens up—i.e., consumers for whom the operative choice is to buy both products from firm A or both from B. Such consumers are "doubly profitable"; they bring two profit margins (less the discount for joint purchase) so their existence often intensifies price competition generally. Thus larger bundling discounts, by creating more of these doubly profitable consumers, may strengthen incentives to reduce stand-alone prices as well. Bundling discounts do not necessarily do so, because the discount itself reduces the profit obtained from the doubly profitable consumers. Nonetheless, even when bundling acts to raise the stand-alone prices, the price rise is not (under the conditions of Proposition 4) sufficient to outweigh the consumer benefits of the discount. (Essentially, condition (19) ensures there are enough people on the diagonal frontier so that bundling discounts lead to lower average prices for consumers.) Therefore, with inelastic

[19] The same is also often true when comparing regimes of linear pricing and *pure* bundling—see Matutes and Regibeau (1988), Economides (1989) and Nalebuff (2000), for instance.

[20] Proposition 4 holds whether or not the two products are complements, thanks to our assumption of full market coverage. When there is partial market coverage, it becomes relevant whether the two products are complements or not. Assuming perfect complementarity, Matutes and Regibeau (1992) allow for both complete and incomplete market coverage. When transport costs are very high relative to willingness-to-pay, only those consumers near the four corners of the square will participate. Nevertheless, profit is still lower with nonlinear pricing than with linear pricing. On the other hand, if the products enter additively into consumer utility, when transport costs are very high the two firms are local monopolists and the analysis of Adams and Yellen (1976) and subsequent papers applies. In such cases, nonlinear pricing clearly boosts profit (since a monopolist benefits from more pricing instruments), while the impact on welfare and consumer surplus is ambiguous.

demand consumers usually do better with nonlinear pricing than with linear pricing. Yet discounts induce the excessive loyalty inefficiency, so depress welfare. It follows that profit is higher with linear pricing.

What happens when demand is more price sensitive? When nonlinear pricing is used, the shape of the demand functions has no effect on profit or welfare (see Proposition 3). With linear pricing, on the other hand, it is natural that profit is reduced when demand becomes more elastic. For instance, in the limit as demand becomes very elastic, equilibrium linear prices are close to marginal costs and profit is close to zero. This implies that profit with nonlinear pricing is greater than that with linear pricing whenever demand is sufficiently elastic. (In section 3.4 we will come back to this "elasticity effect" and how it depresses the profit obtained with linear prices.) Similar arguments apply to consumer surplus. When demand is highly elastic, there is (approximate) marginal-cost pricing in both regimes but consumers also pay fixed charges in the nonlinear pricing regime. Therefore, consumers are worse off with nonlinear pricing when demand is sufficiently elastic.

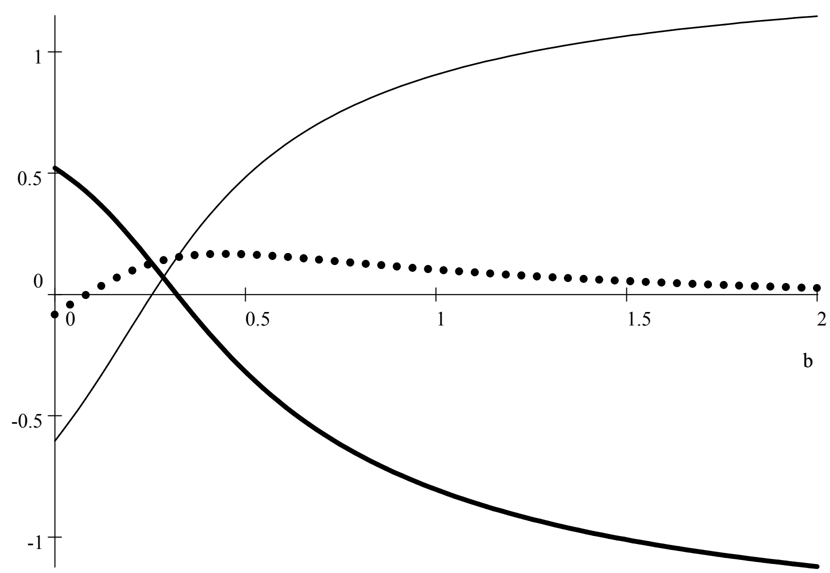

Figure 6: The Effect of Elasticity on Relative Profit, Consumer Surplus and Welfare

The impact of elasticity on the welfare comparison between linear and nonlinear pricing is not so clear cut, however. Linear pricing has the advantage that there is no excessive loyalty inefficiency, but there is the inefficiency due to excessive marginal prices. The latter problem may be expected to become more prominent as demand becomes more elastic, because for a given price the welfare loss is larger if demand is more elastic. On the other hand, greater demand elasticity lowers the equilibrium linear price, which may mitigate or overturn these increased deadweight losses.

To illustrate these elasticity effects more precisely consider the *Linear Uniform Example*, where the linear price is given by expression (16). With nonlinear pricing, Proposition 3 shows that profit does not depend on the price-sensitivity parameter b. However, profit with linear pricing is decreasing in b. When demand is sufficiently sensitive to price, profit with nonlinear pricing exceeds that with linear pricing. Thus, the impact of nonlinear pricing on profit is ambiguous.

Figure 6 plots (as the thin solid line) the difference between the profit with nonlinear pricing and with linear pricing as a function of the elasticity term b (here $t = 1$, $z = 0$ and $\sigma^2 = 0$). The welfare difference between nonlinear and linear pricing is plotted as the dotted line on the figure, while the difference in consumer surplus is the thick solid line. When $b = 0$ we return to the unit demand setting described by Proposition 4, where profit and welfare are reduced with bundling, while consumers benefit. With sufficiently elastic demand, the reverse holds. In this example, welfare is improved by nonlinear pricing for moderate elasticities, but for very elastic demand (outside the range of Figure 6) welfare is again higher with linear pricing. Therefore, the impact of elasticity on relative welfare is non-monotonic, and this explains the "?" in Table 1.

3.3 Impact of consumer heterogeneity

Our general framework is more general than unit demand models in that it allows consumers heterogeneity as well as sensitivity of demands to prices, and in this section we explore this effect in more detail. Consumer heterogeneity acts to depress the equilibrium linear prices, as can be seen from expression (15). Speaking loosely, a "mean-preserving spread" of the demand functions has no impact on the right-hand side of (15), but it raises the left-hand side via the quadratic terms. The impact of this is similar to a reduction in the product differentiation parameter t_1, which will typically cause prices to fall. The economic intuition is that with heterogeneity a price cut attracts proportionally more high demand (hence high profit) consumers from the rival firm, and so improves, at the margin, the *mix* of consumers (an effect absent with homogeneity). Since prices are then closer to marginal costs, this acts to boost the welfare and consumer surplus associated with linear pricing, but to depress profits. We deduce that welfare and consumer surplus are more likely to be higher with linear pricing when there is substantial consumer heterogeneity, while profit is then more likely to be higher with nonlinear pricing. However, the reason is not that heterogeneity boosts the profits from engaging in price discrimination (as one might perhaps have expected), but rather that it harms profit when linear prices are used.

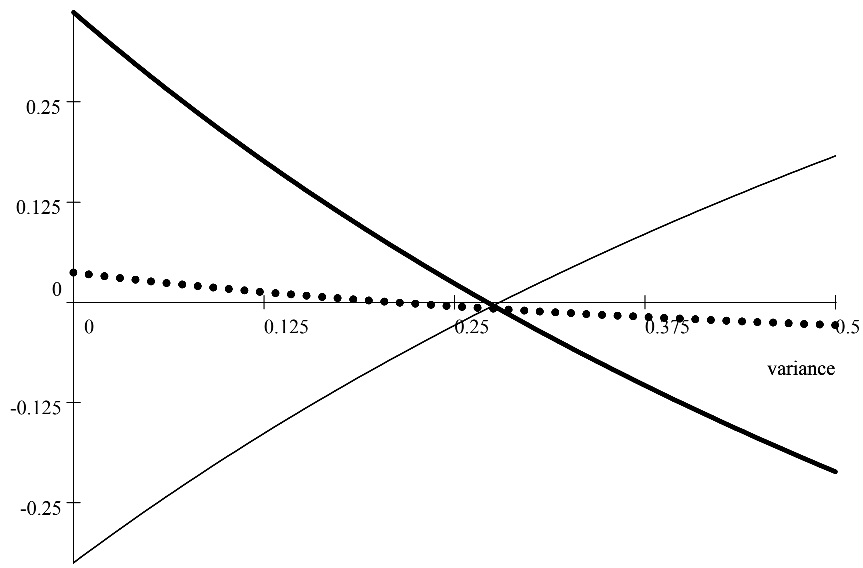

Figure 7: The Effect of Demand Heterogeneity

The effect is illustrated once more by the *Linear Uniform Example*. Expression (16) shows that the equilibrium linear price for each product is decreasing in σ^2, which confirms the intuition that heterogeneity in consumer demand pushes down linear prices. In addition, when $z > 0$ expression (16) shows that the linear price is decreasing in the correlation in demands for the two products, κ. When $z > 0$, there is a set of consumers for whom the relevant margin is whether to buy both products from either firm A or firm B. When there is more correlation in the scale of demand for the two products (κ increases), the variance of the total profit from both products rises, and this intensifies competition for these one-stop shoppers yet further.

By contrast, when firms use nonlinear tariffs, Proposition 3 shows that profit, welfare and consumer surplus do not depend on the variance of, or correlation between, consumer demands for the two products. Thus, the relative profitability of using nonlinear pricing increases with σ^2 and κ. The reductions in linear prices caused by increased heterogeneity result in relative welfare with nonlinear pricing decreasing with σ^2 and κ. Consumers may be better off with nonlinear pricing when σ^2 is relatively small, but when their demands are more varied, consumers (in aggregate) prefer linear pricing. This is illustrated in Figure 7, which shows relative profit (the thin line), consumer surplus (the thick line) and welfare (the dotted line) associated with nonlinear versus linear pricing as σ^2 varies. (Here, $b = \frac{1}{8}$, $t = 1$ and $z = 0$.)

3.4 Impact of shopping costs

In the final two sub-sections we focus on the impact of two-stop shoppers. There are two natural reasons why the number of such consumers might be reduced: first, substantial shopping costs make it unattractive to source from multiple suppliers, and second, correlation in brand preferences means that the number of consumers who prefer one firm's profit 1 and the other's product 2 is small. Here we consider the impact of the size of the shopping cost on the performance of nonlinear pricing relative to linear pricing, while in the next sub-section we analyze brand preference correlation.

Consider first the extreme case where z is so large that (2) is violated and no consumer chooses to buy from two suppliers.[21] In this case, since all consumers are one-stop shoppers regardless of whether or not nonlinear pricing is employed, there is no scope for welfare losses associated with excessive loyalty. Without loss of generality, suppose that $t_1 \geq t_2$. One can show that the optimal nonlinear tariff in this case is the (single) two-part tariff

$$T(q_1, q_2) = \frac{t_1}{\hat{\beta}_1} + c_1 q_1 + c_2 q_2 \ ,$$

where

$$\hat{\beta}_1 = \int_0^1 f(\tfrac{1}{2} + \frac{t_2}{t_1}(\tfrac{1}{2} - x_2), x_2) \ dx_2 \ .$$

(Here, $\hat{\beta}_1$ is β_1 in expression (6) when z is so large there are no two-stop shopping regions in Figure 2.) Thus, industry profit with nonlinear pricing is $\pi_{NL} = t_1/\hat{\beta}_1$.

Consider next the outcome when linear prices are used. With linear prices, firms will set prices different from marginal costs since that is the only way they can generate a profit. It is immediate that welfare is lower relative to the situation with nonlinear pricing. To understand the impact of nonlinear pricing on profit and consumer surplus requires

[21] For instance, not many people will have one course of food at one restaurant and then move to another restaurant for a second course.

further analysis. Expression (15) implies in this one-stop shopping case that the first-order conditions for p_1 and p_2 to be the equilibrium linear prices are, respectively:

$$E_\theta[((p_1-c_1)q_1+(p_2-c_2)q_2)q_1]=\frac{t_1}{\hat{\beta}_1}\times E_\theta\left[q_1+(p_1-c_1)\frac{\partial q_1}{\partial p_1}+(p_2-c_2)\frac{\partial q_2}{\partial p_1}\right] \qquad (20)$$

$$E_\theta[((p_1-c_1)q_1+(p_2-c_2)q_2)q_2]=\frac{t_1}{\hat{\beta}_1}\times E_\theta\left[q_2+(p_2-c_2)\frac{\partial q_2}{\partial p_2}+(p_1-c_1)\frac{\partial q_1}{\partial p_2}\right] \ . \qquad (21)$$

If we write $\pi_i(\theta,p_1,p_2)\equiv q_i(\theta,p_1,p_2)(p_i-c_i)$ to be profit from the type-θ consumers at the equilibrium linear prices, expressions (20)–(21) imply that industry profit with linear pricing, denoted π_L, satisfies

$$\pi_L=E_\theta[\pi_1+\pi_2]=E_\theta\left[\frac{\hat{\beta}_1}{t_1}(\pi_1+\pi_2)^2-\sum_{i,j=1}^{2}(p_i-c_i)(p_j-c_j)\frac{\partial q_i}{\partial p_j}\right]$$

$$\geq E_\theta\left[\frac{\hat{\beta}_1}{t_1}(\pi_1+\pi_2)^2\right]\geq\frac{\hat{\beta}_1}{t_1}\left(E_\theta[\pi_1+\pi_2]\right)^2=\frac{\hat{\beta}_1}{t_1}\pi_L^2\ . \qquad (22)$$

Here, the first inequality follows from the observation that the matrix of derivatives of demand is negative semi-definite, and this inequality is strict if demands are not perfectly inelastic. This is the "elasticity" effect mentioned in section 3.2. The second inequality in (22) follows from Jensen's inequality. It is strict if there is some consumer heterogeneity (i.e., if the variance of $\pi_i(\theta,p_1,p_2)$ is positive): this is the heterogeneity effect discussed in more detail in section 3.3. In sum, expression (22) shows that $\pi_L\leq t_1/\hat{\beta}_1=\pi_{NL}$. Hence profits are lower with linear pricing than with nonlinear pricing, with strict inequality if demand is elastic or if there is consumer heterogeneity.[22] The reason for this is a combination of the elasticity and heterogeneity effects, which act in the same direction.

Turn next to the impact on consumer surplus. Suppose that total welfare, which is

$$W(p_1,p_2)\equiv E_\theta[v(\theta,p_1,p_2)+\pi_1(\theta,p_1,p_2)+\pi_2(\theta,p_1,p_2)]\ ,$$

is concave in linear prices. In particular, W then lies below its tangent plane at the equilibrium linear prices p_1 and p_2, and so the difference in welfare with nonlinear pricing

[22] It is obvious that the argument here works just as well with n products as with two products. Moreover, the same profit comparison often holds in an alternative framework in which consumers incur transport costs on a per-unit rather than a lump-sum basis, although the analysis is considerably more complicated. For instance, see Proposition 5 in Yin (2004) for analysis with linear demand. One major difference is that the equilibrium two-part tariffs are not efficient, and marginal price is above marginal cost.

and linear pricing, denoted $\Delta W = W(c_1, c_2) - W(p_1, p_2)$, satisfies

$$\begin{aligned}
\Delta W &\leq -(p_1 - c_1)\frac{\partial W(p_1, p_2)}{\partial p_1} - (p_2 - c_2)\frac{\partial W(p_1, p_2)}{\partial p_2} \\
&= E_\theta\left[-\sum_{i,j=1}^{2}(p_i - c_i)(p_j - c_j)\frac{\partial q_i}{\partial p_j}\right] \\
&= \pi_L - E_\theta\left[\frac{\hat{\beta}_1}{t_1}(\pi_1 + \pi_2)^2\right] \\
&\leq \pi_L - \frac{\hat{\beta}_1}{t_1}\pi_L^2 = \frac{\hat{\beta}_1}{t_1}\pi_L\left[\frac{t_1}{\hat{\beta}_1} - \pi_L\right] \leq \frac{t_1}{\hat{\beta}_1} - \pi_L = \pi_{NL} - \pi_L \ .
\end{aligned}$$

Here, the second equality follows from (22) and the final inequality follows from our previous result that $\pi_L \leq t_1/\hat{\beta}_1$. Therefore, the welfare gain is less than the gain in industry profit, and so consumers in aggregate are worse off when nonlinear tariffs are used.

We summarise our analysis of the case with large z as:

Proposition 5 *Suppose that (2) does not hold. Then in equilibrium all consumers are one-stop shoppers (both with linear and nonlinear pricing). Compared to the outcome with linear pricing, industry profit and total welfare are higher with nonlinear pricing. If welfare is concave in linear prices, consumer surplus is lower with nonlinear pricing.*

This result is in the same spirit as the one-stop shopping analyses in Armstrong and Vickers (2001, section 3) and Yin (2004), both of which focus on homogeneous consumers, and Thanassoulis (2007, section 3), who focusses on a situation in which consumers have inelastic one-unit or two-unit demands. These papers show (in more specialized settings than is covered by Proposition 5) that a move from linear to nonlinear pricing will enhance profit and welfare but harm consumers.[23,24] As noted in the introduction, it is striking that these are exactly the opposite comparative statics to those obtained in the unit demand situation described in Proposition 4.

[23] Since all consumers have inelastic demand, in Thanassoulis's model welfare is unchanged when nonlinear tariffs are used.

[24] In the one-stop shopping framework, there is now a reasonably good understanding of nonlinear pricing when the market is not fully covered, at least in single-product settings (see Rochet and Stole (2002) and Yang and Ye (2008)). These papers do not discuss the comparison with linear pricing, and so it remains an important avenue for further research to understand the comparative impact of nonlinear pricing on profit and consumer surplus, which could well be more ambiguous than the results we report in Proposition 5. For instance, in the limit where firms are local monopolists for all types of consumer, we already know that nonlinear pricing has ambiguous effects on welfare and consumer surplus relative to linear pricing (though it is obviously beneficial for firms).

In the equilibrium with nonlinear pricing, firms use cost-reflective two-part tariffs. The optimal fixed fee in such a tariff balances (i) the firm's loss of profit on existing consumers against (ii) its gain in profitable consumers from the other firm. Demand per consumer does not change as the fixed fee varies: it is as if consumers had identical inelastic demands in that respect. By contrast, if firms are restricted to linear prices, a firm choosing its price(s) will balance (i) against not only (ii) but also (iii) its gain in average profit per consumer. Profit per consumer can change for two reasons. First, with elastic demands, lower prices expand demand from each type of consumer. Second, and less obviously, if consumers are heterogeneous, lowering prices improves the mix of consumers coming to a firm. This second reason why average profit per consumer may rise as linear prices are reduced occurs even if consumers have inelastic demands (as in Thanassoulis's model). These two reasons are reflected in the two inequalities in (22). They are both reasons why there is more incentive to lower prices—hence why consumers do better—with linear pricing. Yet welfare is higher with nonlinear pricing because there is no marginal price inefficiency. It follows that profits must be higher then too.

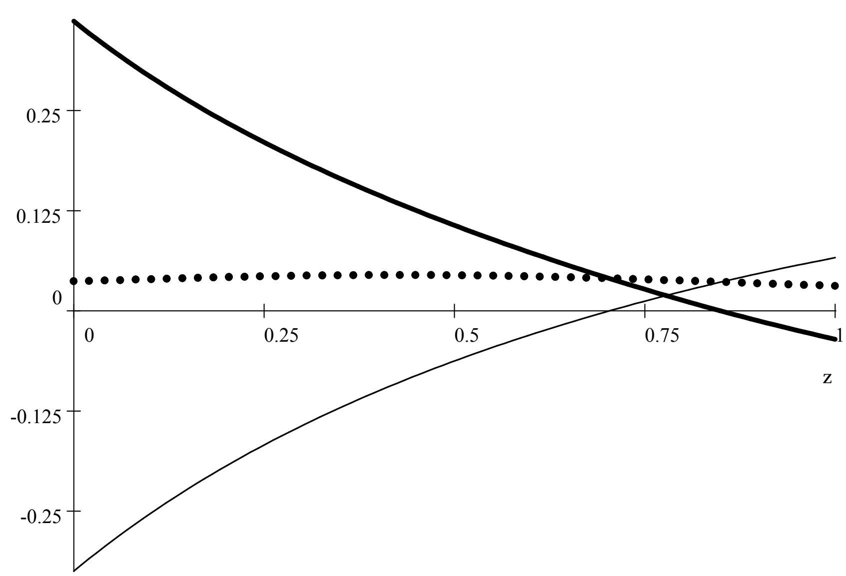

Figure 8: The Effect of the Shopping Cost

What happens with more moderate shopping costs, where some two-stop shoppers are present? When the shopping cost z is relatively large (i.e., near to $\min\{t_1, t_2\}$), the equilibrium bundling discount δ is small and the excessive loyalty welfare loss in expression

(8) is small. Therefore, the only significant factor for welfare is the excessive marginal price problem associated with linear pricing. We deduce that for large z, the welfare effect of nonlinear pricing is always positive. (Of course, the welfare effect of nonlinear pricing may be positive for small z too, depending on demand elasticities and the other factors we discuss.)

We illustrate this, as well as the impact of the shopping cost on profit and consumer surplus, using the *Linear Uniform Example.* (Here, $b = \frac{1}{8}$, $t = 1$ and $\sigma^2 = 0$.) Figure 8 shows relative profit (the thin line), consumer surplus (the thick line) and welfare (the almost flat dotted line) associated with nonlinear versus linear pricing as z varies from zero to its maximal level, $t = 1$. Because demand is relatively inelastic in this example, when the shopping cost is small the impact of nonlinear pricing on profit and consumers is similar to the unit demand situation described in section 3.2, i.e., firms are harmed by nonlinear pricing, while consumers benefit from the tough competition which ensues. As z gets larger, though, the impact eventually reverses and we reach the qualitative outcome described in Proposition 5.

3.5 Impact of correlation in brand preference

Section 3.3 argued that correlation in the scale of demands for the two products, κ, tends to intensify linear pricing competition (while leaving the outcome with nonlinear pricing unchanged). The impact of this form of correlation is to increase effective consumer heterogeneity, and this decreases welfare and consumer surplus associated with nonlinear pricing relative to linear pricing. In this section we consider an alternative form of correlation—involving the brand preferences for the two products—which has distinct economic effects.

In some situations, it is plausible that if a consumer prefers firm A's product 1 then she is more likely to prefer the same firm's product 2, so that x_1 and x_2 are correlated. That is to say, firm-level brand preference may be an important factor in a consumer's brand preferences over individual products. Similarly to the case with substantial shopping costs, the situation with correlation in brand preferences implies that the proportion of two-stop shoppers is smaller, all else equal, than in the uncorrelated case. This implies that the impact of this form of correlation on the relative merits of nonlinear pricing is similar to the analysis in section 3.4.

Consider first the case of unit demands, continuing the analysis presented in section 3.2.

To illustrate the impact of correlation, consider a variant of the *Uniform Example* presented in section 2.2.[25] Suppose product differentiation is symmetric ($t_1 = t_2 = t$). Suppose that a fraction ρ of consumers have perfectly correlated brand preferences, so that $x_1 = x_2$, and this common brand preference is uniform on $[0, 1]$. The remaining $1 - \rho$ consumers have locations (x_1, x_2) uniformly distributed on the unit square. Thus, ρ represents the degree of correlation in brand preferences. In this example, $\alpha_1^0 = \alpha_2^0 = (1-\rho)(\frac{1}{2} - \frac{z}{2t})$ and $\beta_1^0 = \beta_2^0 = \frac{1}{2} - (1-\rho)(\frac{1}{2} - \frac{z}{t})$.[26] Therefore, from (17) the equilibrium linear price for each product is

$$P_1 = P_2 = \frac{t^2}{t + (1-\rho)z} \ . \tag{23}$$

Thus, correlation acts to relax competition in linear prices (unless the shopping cost is zero, in which case correlation has no impact on linear prices).

Turning to the case where bundling is employed, $\Phi(\delta)$ is here equal to $\frac{1}{2}(1-\rho)(1-\frac{\delta+z}{t})^2$, and so expression (7) shows that the equilibrium discount does not depend on ρ and equals $\delta = \frac{1}{2}(t-z)$. From (8), the welfare cost of bundling with this discount is just scaled down by the factor $(1-\rho)$, and so from (13) this welfare loss is $(1-\rho)\frac{(t-z)^3}{12t^2}$. With the discount $\delta = \frac{1}{2}(t-z)$, we have $\alpha_1 = \alpha_2 = \frac{1}{4}(1-\rho)(1-\frac{z}{t})$ and $\beta_1 = \beta_2 = \frac{1}{2} + \frac{1}{2}(1-\rho)\frac{z}{t}$. Expression (11) then shows that the equilibrium bundling tariff is

$$P_1 = P_2 = \frac{1}{4}(t-z) + \frac{2t^2}{2t + (1-\rho)(t+z)} \ ; \ \delta = \frac{1}{2}(t-z) \ . \tag{24}$$

Thus, as with linear prices, the equilibrium nonlinear prices increase with brand preference correlation. By comparing (23) and (24), we see that the outcomes with linear and nonlinear pricing become more similar as correlation becomes increasingly strong. (As ρ approaches 1, almost all consumers are one-stop shoppers, and the sum of linear prices for the two products in (23) converges to the nonlinear price for the two products—that is, $P_1 + P_2 - \delta$—in (24).)

[25] The same method of modelling correlation is used in Nalebuff (2004, section III.C). Clearly, other "copulas" could be used to combine uniform marginal distributions for x_1 and x_2 into correlated variables, although it is likely that numerical methods would then be needed to derive the equilibrium tariffs. See Reisinger (2006) for further analysis of the effect of correlation in a related framework.

[26] Strictly speaking, $\beta_i(\delta)$ in (6) is not defined in this example, since there is no (two-dimensional) "density" on the diagonal $x_2 = x_1$. However, by examining small changes in P_i as depicted on Figure 5, one can verify that this value for β_i^0 is the appropriate value to use in expression (17).

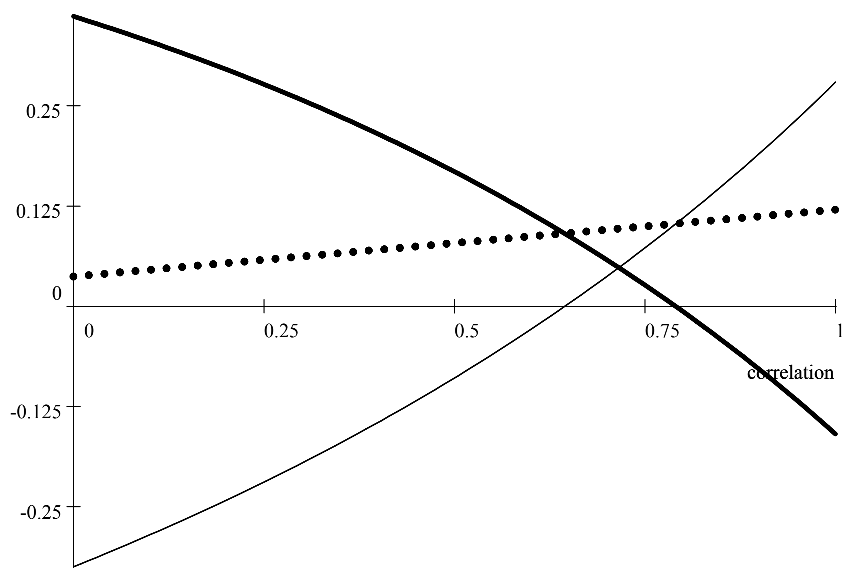

Figure 9: The Effect of Brand Preference Correlation

Finally, consider the more general case with elastic demand. The impact of strong correlation in brand preferences on relative profit, consumer surplus and welfare is qualitatively similar to that of large shopping costs.[27] When this form of correlation is strong, the fraction of consumers who might be two-stop shoppers is small, and the effect of nonlinear pricing mirrors the analysis of one-stop shopping described in Proposition 5: profits and welfare rise, while consumers are harmed. This can be illustrated using the *Linear Uniform Example* modified to allow for a fraction ρ of consumers to have perfectly correlated brand preferences as with the previous unit demand discussion. Figure 9 shows relative profit (the thin line), consumer surplus (the thick line) and welfare (the dotted line) associated with nonlinear versus linear pricing as ρ varies. (Here, $b = \frac{1}{8}$, $t = 1$, $z = 0$ and $\sigma^2 = 0$.)

4 Conclusions

Our analysis of competitive nonlinear pricing and bundling—and its effects on consumer surplus, profit and welfare—has been general enough to allow for consumer heterogeneity, elastic demands, consumer choice between one- and two-stop shopping, and shopping

[27] Although the impacts of brand preference correlation and shopping costs on the relative performance of nonlinear and linear pricing are similar, their impacts on the *absolute* performance are opposite. For instance, both nonlinear and linear tariffs are raised with stronger brand preference correlation but lowered with a higher shopping cost.

costs. Yet the model was shown to have a remarkably simple equilibrium when firms offer nonlinear tariffs—namely, efficient two-part tariffs with fixed charges equal to the equilibrium prices in the bundling model with identical unit-demand consumers. The model illuminated the importance of keen competition for one-stop shoppers when discounts for one-stop shopping are feasible, and the inefficiencies arising from excessive marginal prices (with linear pricing) and excessive loyalty (with nonlinear pricing). We examined four economic influences on these sources of inefficiency—and hence on the pros and cons of nonlinear pricing relative to linear pricing—which were summarised in the Introduction.

In important respects the analysis of imperfectly competitive equilibrium with nonlinear pricing has turned out to be simpler—both in terms of the complexity of the analysis and in terms of clear-cut nature of some of the results—than for the corresponding monopoly problem. In the monopoly problem, consumers who do not buy from the monopolist do not buy at all, and optimal nonlinear pricing induces some not to buy. In our setting, by contrast, consumers who do not buy a product from one firm buy it from the other, given our assumption that parameters are such as to ensure full participation. So unlike the monopoly problem, where the *margin of participation* is central, our competitive analysis has focused instead on the *competitive margin* between firms. This, together with symmetry and independence between the distributions for brand preference and for other demand characteristics, helps explain the simplicity of equilibrium in section 2, which in turn made possible the comparative analysis and synthesis of section 3.

The economic effects discussed in this paper may operate, along with other influences no doubt, more widely, including in models in which the margin of participation and the competitive margin are both active. Though general in important respects, our framework has been one of static competition between symmetric two-product duopolists in a setting where all consumers buy some of each product. Natural next steps would be to relax these restrictions.

For example, allowing for free entry instead of duopoly would open up the issue of the effect of nonlinear pricing on the equilibrium number of firms.[28] In the unit demand bundling model, nonlinear pricing acts to depress profit and welfare. With free entry, this implies that the equilibrium number of firms will fall, and this could act to mitigate possible "excess entry". Bundling might then have a positive impact on welfare, despite

[28] See Stole (1995) for early work in this direction in the one-stop shopping context.

the excessive loyalty problem.

Another extension would be to make the model dynamic, and to allow for "customer poaching". Existing models of customer poaching, where a firm sets its current price on the basis of whether a buyer is a previous or a new customer of the firm, assume unit demands in each period.[29] In the basic version of these models, firms do not commit to their future prices, and so price low to their rival's existing customer base. Like the (static) bundling analysis in this paper, customer poaching tends to benefit consumers and to harm firms and welfare relative to linear pricing. However, the welfare problem is quite different: since firms offer low prices to existing customers of their rival (rather than discounts to their own loyal customers as in the bundling framework), customer poaching models involve *insufficient* rather than excessive loyalty. It would be interesting to see whether the extension of those models to elastic demand means that nonlinear pricing can benefit firms and welfare, just as we have shown it can benefit firms and welfare in static bundling models.

These are but two lines of possible further analysis. There is much more to be understood about the economics of competitive nonlinear pricing and bundling.

APPENDIX

Proof of Proposition 3: Suppose that firm B offers the menu of two-part tariffs $T_1(\cdot)$, $T_2(\cdot)$ and $T_{12}(\cdot,\cdot)$ described in (14). We establish that firm A's best response is to use the same tariff by means of the following argument.[30]

Suppose firm A can directly observe a consumer's parameter θ (but not x_1 or x_2). We will calculate firm A's best response to B's tariff, given θ. Suppose firm A's tariff is $\{T_1^A(q_1),\, T_2^A(q_2),\, T_{12}^A(q_1,q_2)\}$. Then let

$$\begin{aligned} V_1 &= \max_{q_1,q_2} : u(\theta,q_1,q_2) - T_1^A(q_1) - T_2(q_2) \\ V_2 &= \max_{q_1,q_2} : u(\theta,q_1,q_2) - T_1(q_1) - T_2^A(q_2) \\ V_{12} &= \max_{q_1,q_2} : u(\theta,q_1,q_2) - T_{12}^A(q_1,q_2) \end{aligned}$$

be the type-θ consumer's gross utility (i.e., the utility excluding travel and shopping costs) when she buys only product 1 from A, only product 2 from A or buys all supplies from A,

[29] See Chen (1997) and Fudenberg and Tirole (2000).
[30] A similar argument was used to prove Proposition 5 of Armstrong and Vickers (2001).

respectively. Next, consider firm A's most profitable way to generate the utilities V_1, V_2 and V_{12}. The most profitable way to generate utility V_1 is the solution to the problem

$$\max_{T_1,q_1,q_2} : T_1 - c_1 q_1 \text{ subject to } u(\theta, q_1, q_2) - T_1 - T_2(q_2) = V_1$$

which is the same problem as

$$\max_{q_1,q_2} : u(\theta, q_1, q_2) - c_1 q_1 - T_2(q_2) - V_1 \ .$$

Clearly, the solution to this problem involves marginal-cost pricing for product 1. That is to say, given θ it is a dominant strategy for firm A to choose a tariff of the form $T_1^A(q_1) = P_1^A(\theta) + c_1 q_1$ for some fixed charge $P_1^A(\theta)$. (We write the fixed charge as a function of θ, since in general it will depend on θ.) A similar argument serves to show that $T_2^A(q_2) = P_2^A(\theta) + c_2 q_2$ and $T_{12}^A(q_1, q_2) = P_1^A(\theta) + P_2^A(\theta) - \delta^A(\theta) + c_1 q_1 + c_2 q_2$ for some choice of $P_2^A(\theta)$ and $\delta^A(\theta)$.

Since both firms are setting marginal prices equal to marginal costs, the net utility of the consumer is

$$v(\theta, c_1, c_2) - [P_1 + P_2 - \delta + t_1(1 - x_1) + t_2(1 - x_2)]$$

if both products are purchased from B,

$$v(\theta, c_1, c_2) - [P_1^A(\theta) + P_2^A(\theta) - \delta^A(\theta) + t_1 x_1 + t_2 x_2]$$

if both products are purchased from A, and

$$v(\theta, c_1, c_2) - [P_i^A(\theta) + P_j + t_i x_i + t_j(1 - x_j) + z]$$

if product i is purchased from A and product j is purchased from B. In particular, the consumer's decision over where to buy depends only on her total outlay (the terms in square brackets above). Therefore, given θ, firm A will choose the fixed charges $P_1^A(\theta)$, $P_2^A(\theta)$ and the bundling discount $\delta^A(\theta)$ in order to maximize its profit, given B's tariff (14). But since the firm obtains profit only from the fixed charges, the problem reduces to the unit demand model analyzed in section 2.2, where Proposition 2 established that the optimal response to the bundling tariff (P_1, P_2, δ) in (11) is to use the same bundling tariff.

In sum, we have shown that (i) if firm B sets the tariff (14) and (ii) if firm A can observe a consumer's type θ, then the tariff (14) maximizes A's profit. However, since the tariff (14) does not depend on θ, this tariff must also be the firm's best response in the more constrained problem in which A cannot observe θ, which proves the result.

Proof of Proposition 4: From (11), for any δ the sum of the stand-alone prices is

$$P_1 + P_2 = \delta + \frac{t_1\alpha_2 + t_2\alpha_1}{\alpha_1\beta_2 + \alpha_2\beta_1 + 2\alpha_1\alpha_2} \quad ,$$

or

$$P_1 + P_2 = \delta + \frac{t_1}{\hat{\alpha} + \beta_1}$$

where

$$\hat{\alpha} = \frac{2\alpha_1\alpha_2}{\frac{t_2}{t_1}\alpha_1 + \alpha_2} \ .$$

From (3), for a particular δ industry profit is

$$\pi(\delta) = \frac{t_1}{\hat{\alpha} + \beta_1} + \delta\Phi(\delta) \ . \tag{25}$$

From (25), consumer surplus relative to the case of linear pricing, denoted $v(\delta)$, satisfies

$$v(\delta) = w(\delta) - \left[\delta\Phi(\delta) + \frac{t_1}{\hat{\alpha} + \beta_1}\right]$$

and so

$$v'(\delta) = -\Phi(\delta) - \frac{d}{d\delta}\left[\frac{t_1}{\hat{\alpha} + \beta_1}\right] \ . \tag{26}$$

(Here, $w(\delta)$ is total welfare with discount δ which satisfies expression (8).) Therefore, a sufficient condition for consumer surplus to rise with bundling is that expression (26) be positive. (Consumer surplus with linear pricing corresponds to $\delta = 0$, and we know from Proposition 4 that firms will choose to offer a positive bundling discount.)

To make progress, specialize to the case of independence, so that $f(x_1, x_2) \equiv f_1(x_1)f_2(x_2)$. Write $F_i(\cdot)$ for the distribution function corresponding to $f_i(\cdot)$ and write

$$\eta_i(\delta) \equiv \frac{f_i(\frac{1}{2} - \frac{\delta + z}{2t_i})}{F_i(\frac{1}{2} - \frac{\delta + z}{2t_i})} \ .$$

Then one can show

$$\frac{d}{d\delta}(\hat{\alpha} + \beta_1) = 2t_1 F_1 F_2 \frac{t_1\eta_2^2\eta_1' + t_2\eta_1^2\eta_2'}{(t_1\eta_2 + t_2\eta_1)^2} = t_1\Phi\chi' \ , \tag{27}$$

where

$$\chi(\delta) \equiv \frac{\eta_1(\delta)\eta_2(\delta)}{t_1\eta_2(\delta) + t_2\eta_1(\delta)} \ .$$

Then (26) implies that

$$v'(\delta) = -\Phi + t_1^2 \Phi \frac{\chi'}{(\hat{\alpha} + \beta_1)^2} \ . \tag{28}$$

Assumption (19) implies that $\eta_i'(\delta) \geq 0$, which in turn implies $\chi'(\delta) \geq 0$. Notice that (27) implies

$$\frac{d}{d\delta}(\hat{\alpha} + \beta_1) \leq \tfrac{1}{2} t_1 \chi' \ .$$

And since $\hat{\alpha} + \beta_1 = \frac{1}{2} t_1 \chi$ when $\delta = -z$, we deduce that

$$\hat{\alpha} + \beta_1 \leq \tfrac{1}{2} t_1 \chi \ .$$

From (28) it follows that $v(\delta)$ is increasing if

$$\frac{\chi'}{\chi^2} = \frac{t_1 \eta_1'}{\eta_1^2} + \frac{t_2 \eta_2'}{\eta_2^2} \geq \tfrac{1}{4} \ .$$

A sufficient condition for this inequality to hold is (19), in which case consumer surplus necessarily rises when bundling is used.

Since welfare $w(\delta)$ is deceasing in δ, it follows that industry profit (which equals $w(\delta) - v(\delta)$) is decreasing in δ under the same conditions. This proves Proposition 4.

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