Running Head: DECISION BY SAMPLING

Decision by Sampling

Neil Stewart

Nick Chater

Gordon D. A. Brown

University of Warwick, England

neil.stewart@warwick.ac.uk

Tel: +44 (0) 24 7657 3127

Fax: +44 (0) 24 7652 4225

Department of Psychology

University of Warwick

Coventry

CV4 7AL

England

Stewart, N., Chater, N., & Brown, G. D. A. (in press). Decision by sampling. *Cognitive Psychology.*

Abstract

We present a theory of decision by sampling (DbS) in which, in contrast with traditional models, there are no underlying psychoeconomic scales. Instead, we assume that an attribute's subjective value is constructed from a series of binary, ordinal comparisons to a sample of attribute values drawn from memory and is its rank within the sample. We assume that the sample reflects both the immediate distribution of attribute values from the current decision's context and also the background, real-world distribution of attribute values. DbS accounts for concave utility functions; losses looming larger than gains; hyperbolic temporal discounting; and the overestimation of small probabilities and the underestimation of large probabilities.

Keywords: judgment; decision making; sampling; memory; utility; gains and losses; temporal discounting; subjective probability.

Decision by Sampling

Here we offer an account of why the descriptive psychoeconomic functions - concave utility functions for money, hyperbolic temporal discounting functions, and inverse-S-shaped subjective probability functions - take the forms that they do. The essence of our decision by sampling (DbS) account is that attribute values (e.g., monetary amounts, probabilities, delays) are evaluated against a sample of other attribute values using simple cognitive tools. The *decision sample* is assumed to comprise both attribute values from both the immediate context in which a decision is made (e.g., the attribute values of other options on offer) and values from memories of previously encountered attribute values (e.g., those values encountered in previous decisions). In this article, we focus upon the effect of previously encountered attribute values.

Theories of decision making often take economic theory as a starting point: expected utility theory for decision under risk; exponential discounting for decisions with delayed outcomes. The next step is to assess the degree to which people make decisions as they should (e.g., Allais, 1953; Kahneman & Tversky, 1979, 2000). The normative theory is then modified to create a descriptive theory of observed behavior by including additional psychological insight (e.g., prospect theory, Kahneman & Tversky, 1979, Tversky & Kahneman, 1992, regret theory, Loomes & Sugden, 1982, and rank dependent utility theory, Quiggin, 1983, 1992, in decision under risk; hyperbolic discounting, Rachlin, 1989, for intertemporal choice; support theory, Tversky & Koehler, 1994; for probability judgment). In beginning with a limited set of simple cognitive tools we are taking psychology as a starting point. We then consider how economic decisions might be made using these simple tools.

A key difference between the approach we develop here and those derived from normative economic accounts is that we do not assume that people have stable, long-term internal scales along which they represent value, probability, temporal duration, or any other magnitudes. Instead, we assume that people can only sample items from memory and then

judge whether a target value is larger or smaller than these items. This approach is inspired by and builds on a series of successful accounts of key aspects of judgment and decision making based on psychological assumptions concerning sampling from, and comparison with, items from memory. In norm theory (Kahneman & Miller, 1986) the normality of a stimulus is derived by comparing it to the norm (counterfactual examples and a set of exemplars retrieved from memory) that it evokes. In decision field theory (Busemeyer & Townsend, 1993), and its multialternative generalization (Roe, Busemeyer, & Townsend, 2001), the time course of decision making is accounted for by the sequential sampling of information from the decision context, with outcome valances constructed relative to one another. In support theory (Tversky & Koehler, 1994), the subjective probability of a focal hypothesis depends on the sample of alternative hypotheses considered by the subject, and is given by the ratio of the support for the focal hypothesis and the sum of the support for all hypotheses under consideration (see Windschitl & Well's, 1998, comparison heuristic for a similar mechanism). Dougherty, Gettys, and Oden's (1999) decision making model MINERVA-DM (based on Hintzman's, 1984, 1988 memory model) gives a mechanism by which the support for hypotheses depends on the similarity to traces stored in memory, providing an account of many heuristics and biases (see also Juslin & Perrson, 2002). In the stochastic difference model (González-Vallejo, 2002), the differences between the target attribute value and other attribute values in the sample of items in the decision context determines the preference for one prospect over another. In summary, in all of these models, judgments and decisions result from comparison of an attribute's value to a sample of other values, either from the decision context or from memory. For a review of memory processes in judgment and decision making see Weber, Goldstein, and Barlas, (1995) and Weber & Johnson (in press).

In DbS, we assume that only the most simple cognitive processes - ordinal comparison and frequency accumulation - are involved in evaluating a target attribute against

a decision sample. Our assumption that, to a first approximation, comparisons are only ordinal (i.e., only "greater than", "equal to", or "less than") is motivated by evidence from psychophysics which suggests that people are rather good at discriminating stimuli from one another, but rather bad at identifying or estimating the magnitude of the same stimuli (see Garner, 1962; Miller, 1956; Laming, 1984, 1997; Shiffrin & Nosofsky, 1994; Stewart, Brown, & Chater, in press). Our assumption that people are good at keeping track of and manipulating frequencies is well established (e.g., Gigerenzer & Hoffrage, 1995; see Sedlmeier & Betsch, 2002, for a recent review). By keeping a frequency count of the number of comparison outcomes that favor the target, one can derive the rank of the target attribute value within the decision sample (see, e.g., Kornienko, 2004, for a demonstration that a cardinal utility function may be derived by keeping a frequency count of binary, ordinal comparisons.) It is this rank that we assume is the subjective value of an attribute. When normalized to lie between 0 (the worst attribute value) and 1 (the best attribute value), the subjective value or *relative rank* of an attribute value is given by $r = (R - 1) / (N - 1)$ where R is the rank within the sample of N items. The relative rank is effectively the proportion of attribute values in the sample that are less than the target attribute value or, equivalently, the probability that a randomly selected attribute value will be less than the target attribute value.

In assuming that the subjective value of an item is its rank within a sample, DbS embodies the frequency principle of range-frequency theory (Parducci, 1965, 1995). In range-frequency theory, the subjective value of an item is a weighted sum of its rank within the immediate context and its position within the range set by the immediate context. We consider the range principle further in the General Discussion.

So far we have said little about the sample of attribute values against which an item is compared. The basic idea is that, when considering a target attribute value, there will typically already be some other attribute values from the context of the decision in the sample. The target attribute value will also evoke other values from long-term memory, and it

is the effect of these attributes that we focus upon in this article (cf., Kahneman and Miller, 1986). Thus, the subjective value of an option is constructed online whenever it is considered (cf., Bettman, M. F. Luce, & Payne, 1998; Payne, Bettman, & Johnson, 1992; Slovic, 1995) and will vary from occasion to occasion with (a) the distribution of attribute values from the immediate decision context, (b) the distribution of attribute values in memory, and (c) stochasticity in the sample of attribute values from both the immediate decision context and also from memory.

As a starting point, we assume that the contents of memory reflects the structure of the world, and represents a subset of the attribute values that people typically encounter. There is good evidence that memory is adaptively reflects the structure of the environment (e.g., Anderson, 1990; Anderson & Milson, 1989; Anderson & Schooler, 1991; Chater & Brown, 1999; Oaksford & Chater, 1998; Shepard 1987). In the following sections we will examine the distributions of gains, losses, time delays, and probabilities that people encounter. We focus on these attributes because they are the psychological primitives of economic decisions: Many decisions involve evaluating the value of some risky, uncertain, or delayed gain or loss. We will use these distributions to make predictions about the subjective value functions that will be revealed when people make decisions in the context of these real-world distributions.

Gains

First we consider gains. Following Kahneman and Tversky (1979) we consider gains and losses separately. Key questions are: (a) What is the distribution of gains in people's memories? (b) What effect will this distribution have on the subjective valuation of gains?

We assume that the decision sample, to which a target gain is compared, is a small, random sample of gains from memory. Of course this random sampling assumption is likely to be incorrect: other factors, such as recency, similarity, and background knowledge will surely play a role. However, in what follows we pursue this random sampling hypothesis as a

first approximation.

An approximation to the distribution of gains that people encounter can be revealed by examining credits to people's current (in the UK; checking in the US) bank accounts. Figure 1A shows the frequency with which credits of different amounts are made. These data are a random sample of one year of credits to current accounts held by a leading UK bank. Automatic credits were omitted, but all manual payments including direct debits, standing orders, and salary payments were included. The distribution of credits approximately follows a power law, with many small gains and relatively fewer larger gains (the data roughly follow a straight line on the plot of log frequency against log credit). The observation of this power-law relation between event magnitude and event probability is unsurprising, as it is seen in many aspects of the world (see Bak, 1997, for a review). For example, natural phenomena such as earthquake energies follow this pattern (Gutenberg & Richter, 1949; Johnstone & Nava, 1985), as do social phenomena like the size of corporations (Ijiri & Simon, 1977), city sizes, and the frequencies of words within natural language (Zipf, 1949).

Supposing that the decision sample can contain an unlimited number of exemplars, the subjective value of a target credit within our larger sample of credits is given by its relative rank within this large sample. Figure 2A plots the relative rank of each credit. Because of the equivalence between the relative rank of a target attribute value and the proportion of attribute values that are smaller than the target, Figure 2A can also be described as a plot of the cumulative probability of obtaining a gain at least as big as that on the abscissa.

As a direct consequence of the distribution of credits, relative rank is an increasing but negatively accelerating function of the size of the credit. Thus, additional incremental wealth has a diminishing impact on the relative rank of the credits. For example, a credit of £1000 has less than twice the psychological value of a credit of £500. In summary, from only the assumption that people make ordinal comparisons with a sample of values reflecting the

positively-skewed real-world distribution, DbS predicts that the marginal subjective value of an extra unit of wealth diminishes as wealth increases (i.e., concave utility functions for gains).

If the distribution f(g) of gains g in the world follows a power-law distribution with power γ (i.e., $f(g) = c\, g^{\gamma}$ where c is a normalizing constant) then DbS predicts a power-law revealed utility function, as the relative rank of g is given by the cumulative distribution function $r(g) = c / (1 + \gamma)\, g^{\gamma+1}$.

The assumption that gains are fully sampled is unlikely to be true given the well-established finding of a severely limited capacity of short-term memory (Miller, 1956). However, if a small, randomly drawn sample of gains is considered, then similar predictions follow. The relative rank of an attribute value is determined by the probability that a randomly sampled credit will be less than or equal to that value. Thus, the distribution of relative ranks for a given target and given sample size will be binomial. Figure 3A illustrates the binomial distribution of relative ranks obtained for a target value of £250 if 5 items are sampled randomly from the distribution of credits. Figure 3B illustrates how this binomial distribution will change as a function of the target credit. (Every plane perpendicular to the attribute value axis is a binomial distribution.) As the mean of a binomial distribution is its probability parameter, then Figure 2A represents the mean relative rank for a target credit, independent of sample size.

Bordley and LiCalzi (2000) present an argument that is similar to the DbS account above. In their account, the value of a gain is the probability that it will meet an uncertain target. Thus, the value of the gain depends on its location within the distribution of target values. Bordley and LiCalzi do not give a detailed psychological account of the origin of the distribution of the target values, but do suggest that they result from uncertainty over which targets are necessary to achieve higher superordinate goals. They assume that people select an outcome to maximize the probability of meeting this target and show that this approach

makes the same predictions as expected utility. This approach is similar to DbS in that the subjective value of a target attribute depends upon its ordinal position in some reference distribution, but differs from DbS in assuming that the reference distribution reflects an uncertain aspiration level rather than the real-world distribution of gains.

In summary, DbS predicts a power-law utility function modulated by binomially distributed noise. The power-law function is a result of memory reflecting the scale-free distribution of credits observed in the environment, and the binomial noise is the result of a sampling process. This motivation of this prediction stands in contrast to descriptive models, which simply assume a curvature of the utility function, rather than explaining it. For example, in prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) the curvature of the utility function describes risk aversion. Here we have independently motivated the curvature, and risk aversion will follow as a consequence of this curvature.

Losses

We carried out a parallel analysis for losses. Figure 1B shows how the frequency with which debits are made from current bank accounts depends on the magnitude of the debit. Like gains, the distribution of losses also follows a power law, with many small losses and relatively few large losses. The mean relative rank of losses (for any sample size) can be derived as for gains, and is illustrated in Figure 2B. As before, incremental loss produces a diminishing rise in relative rank. A comparison with gains reveals an interesting prediction. There are relatively more small losses compared to small gains (as reflected in the differing best fitting powers of -0.93 for gains and -0.96 for losses). This makes intuitive sense: One is paid in a lump sum (e.g., a monthly salary) which one spends on many things (e.g., mortgage, grocery bills, etc.). Because of this asymmetry, a loss of a given magnitude will have a higher relative rank than that of a gain of the same monetary amount. Of course, this conclusion, that losses loom larger than gains, is exactly that embodied in Kahneman and Tversky's (1979) prospect theory. DbS predicts this asymmetry in behavior because there is an asymmetry in

the natural real-world distribution of gains and losses.

Friedman (1989) gives an argument related to DbS. Friedman assumes that there are more small gains and losses than large gains and losses, consistent with our data on credits and debits. He further assumes that we have a limited capacity for sensitivity to these gains and losses (because of time, memory, and other cognitive constraints) which we distribute over the most likely outcomes: We are assumed to be more sensitive to small gains and losses because there are more of them. This is consistent with our DbS account according to which people are sensitive to small gains and losses because they are more numerous and hence more frequently sampled. Friedman proves that these two assumptions are sufficient to produce an S-shaped approximation to the true, conventional, concave utility function with the point of inflection at current wealth.

One might wonder whether the positively skewed distributions of gains and losses will be found else where or whether they are specific to bank accounts. One reason to expect that these positively skewed distributions will occur in many contexts is the ubiquity of power law distributions. Another is that we found positively skewed distributions in other domains. For example, we have also examined the distribution of prices in UK supermarkets. Figure 4A shows the relative ranks for a large number of prices in the supermarket. Figures 4B and C show two examples of the relative ranks calculated for bread and chocolate products. In almost all of the cases we have examined, we have found positively skewed distribution of prices, which leads to a concave function for relative ranks.

Time

We seek a uniform account of behavior across a wide variety of domains. There is evidence that the processing of number and time may rely upon a common cortical resource (Walsh, 2003). Thus, the treatment of temporal delays that we offer here is the same as that outlined above for gains and losses. More specifically, the subjective value of a target temporal delay will be determined in the context of a decision sample of other temporal

delays.

We argue that DbS explains of some of the key temporal anomalies reviewed by Loewenstein and Thaler (1989). As before, we assume that the distribution of delays in memory reflects the distribution in the real world. To obtain a crude approximation, our colleague, Stian Reimers, collected the number of hits produced by an internet search engine (http://www.google.com) when prompted with various temporal delays. We accumulated hits over different search strings representing the same period (e.g., "a day", "one day", "1 day", "24 hours") for intervals between 1 day and 1 year. Figure 5 plots the frequency of different temporal intervals as a function of their magnitude. As for gains and losses, the distribution approximately follows a power law (replicating the findings of Pollmann, 1998; and Pollmann & Baayen, 2001, who used different sources of data and time periods). The best fitting slope for this distribution, and those obtained by Pollmann from other corpora with other ranges, are listed in Table 1. (Power laws also describe the time intervals between repetitions of words in New York Times headlines, words in parental utterances to children, and e-mails from particular correspondents in Anderson's mail box, Anderson & Schooler, 1991).

Hyperbolic Temporal Discounting

Figure 6 shows the mean relative rank assigned to each delay as a function of delay magnitude assuming random sampling from the distribution in Figure 5. Incremental delay has a diminishing effect, just as for gains and losses. DbS predicts a specific form for the mean relative rank of a delay as a function of its magnitude. A straight line provides a better fit to a log-log plot of the distribution of temporal intervals (Figure 5) than it does to a linear-log plot, indicating that a power law function describes the distribution of intervals better than an exponential function. Approximating the distribution of times t with a power law $f(t) = c\, t^{-\tau}$ gives the cumulative distribution function, which is the mean relative rank function, of $r(t) = c\, t^{-\tau+1} / (1 - \tau)$. Thus DbS predicts power-law temporal discounting, in which the

discount rate decreases over time, rather than the normative exponential discounting, where the discount rate is constant. It is experimentally well established that people's discount rate does indeed decrease over time (Kirby, 1997; Benzion, Rapoport, & Yagil, 1989; Thaler, 1981).

As estimates of τ range from -1.7 to -1.4 (see Table 1), estimates of the power of the discounting function will range from -0.7 to -0.4. A power of -1.0 gives hyperbolic discounting and therefore DbS predicts sub-hyperbolic discounting. This differs from hyperbolic discounting in that it predicts that people will not discount long delays as much as is predicted either by hyperbolic or exponential discounting. Just such a finding is reported by Myerson and Green (1995) and Simpson and Vuchinich (2000).

Discount Rate Depends on the Magnitude of the Gain

Discount rate decreases with the magnitude of the gain on offer (e.g., Benzion, et al., 1989; Green, Myerson, & McFadden, 1997; Holcomb & Nelson, 1989, as cited in Loewenstein & Thaler, 1989; Thaler, 1981). If magnitudes are sampled at random from memory then DbS does not account for this phenomenon. If it is assumed that similarity plays a role in the sampling process, DbS can offer an account. There must be a positive correlation between the delay until a gain and the size of the gain in the world: As large gains are less frequent than small gains, the average delay between large gains must be larger than the delay between small gains. Assume that people sample large delays when considering large gains, because large gains and large delays were associated in the past. In this context of large delays, the target delay will receive a low relative rank compared to the case when the sample comprises small delays. In other words, in the context of delays evoked by the large gain, the given target delay will seem less bad, and thus be discounted less. We return to the issue of similarity sampling in the General Discussion.

Discount Rate is Greater for Gains than Losses

Thaler (1981) found that discount rates were higher for gains than for losses of

equivalent magnitude. In any account where losses loom larger than gains, including DbS, the discount rate for gains will appear higher. This is because the discount rate depends not only on the discounting function but also upon the curvature of the utility function. Consider the discount rate implied by an indifference between £x now and the larger amount £y delayed by time t. With a power law utility function $u(x) = x^{\gamma}$ and any discount function $f(t)$,

$$x^{\gamma} = y^{\gamma} f(t)$$

gives a discount rate of

$$\frac{x}{y} = (f(t))^{\frac{1}{\gamma}}$$

The discount rate incorporates the curvature of the utility function (e.g., Benzion et al. 1989; Mazur, 1987; Thaler, 1981, but see Chapman, 1996, for a separation, and also Kirby & Santiesteban, 2003, though this example does not involve gains and losses). Thus, if the curvature γ is larger for losses, the discount rate will be smaller. Note that, even if the free parameter(s) of a utility function are fitted at the same time as the free parameter(s) of the discounting function and differences in the discounting parameters are found, one cannot be sure that the difference in discounting parameters reflects different discounting of gains and losses. Johnson and Bickel (2005) found that, when fitting a hyperbolic-like discounting function of the form $x / y = 1 / (1 + k t)^{s}$, the k and s free parameters were correlated. The equivalence of s in this form with γ in the above form means that if γ is different for gains and losses, k will also differ for gains and losses even if gains and losses are discounted in exactly the same way.

DbS and Working Memory Load

The DbS explanation of the shape of the temporal discounting function is that the subjective value of a target delay is derived from comparisons with a sample of delays from memory. In support of this, a working-memory load has been found to affect temporal discounting. With a larger working-memory load, discounting of delayed gains is greater

(Hinson, Jameson, & Whitney, 2003). According to DbS, a larger working memory load should reduce the number of items in the decision sample. In turn, this means that, in the absence of other larger delays from memory, the delay associated with a delayed outcome will seem particularly bad in comparison to only the zero delay of an immediate outcome. Thus, DbS correctly predicts the finding of greater discounting when working memory is loaded.

Summary

We do not suggest that DbS can offer an account of all of the intertemporal choice phenomena reported in the literature. There are surely other important psychological factors at play, such as savoring and dread (e.g., Loewenstein, 1987) and mental accounting (e.g., Shefrin & Thaler, 1988; Prelec & Loewenstein, 1998). However, DbS can explain why discounting is (sub)hyperbolic and, with a plausible modification (assuming that similarity sampling rather than random sampling), can explain why the discount rate is reduced for larger amounts of money. Finally, because the curvature of the utility function is often combined within the measure of discount rate, DbS (and presumably other models) can explain why gains are discounted more heavily than losses.

Risk

We treat probability in the same way as we have treated gains, losses, and delays. We will argue that the distribution of probabilities that people experience is such that small probabilities will be over weighted and large probabilities will be under weighted. In other words, subjective probability is an inverse S-shaped function of actual probability (e.g., Abdellaoui, 2000; Bleichrodt & Pinto, 2000; Gonzalez & Wu, 1999; Kahneman & Tversky, 1979; Prelec, 1998; Tversky & Kahneman, 1992; Wu & Gonzalez, 1996, 1999; but see Hertwig, Barron, Weber, & Erev, 2004, for the opposite pattern for small probabilities when probabilities are experienced as the number of successful outcomes over a series of trials).

There is some evidence that probabilities (or frequencies) are compared with attribute

values retrieved from memory. Dougherty and Hunter (2003a, b) found correlation between working memory span and probability judgments. Larger working memory spans coincided with less subadditivity. (Subadditivity is the extent to which the judged probabilities of a set of mutually exclusive, exhaustive events sum to greater than 1.) Further, time constraints increased subadditivity. They argued that these data are consistent with a model where larger working memory and longer time allows target probabilities to be compared to a larger pool of sampled probabilities. Together with the finding that the particular frequencies with which the items were experienced affected the probability judgments, this is strong evidence that probabilities are judged in comparison to a decision sample.

There is one striking difference between the distributions of gains, losses and delays, and the distribution of probabilities: Probabilities are bounded to be between 0 and 1, and thus cannot follow a power-law distribution. Here we shall argue that there are more cognitively relevant events with small and large probabilities than with mid-range probabilities. Specifically, we shall present four arguments. Each leads to the same conclusion: that small probabilities will be overestimated and large probabilities underestimated in a DbS framework.

The Distribution of Probability Phrases

As with time and money, here we attempt to find a proxy for the distribution of probabilities in long-term memory from which people sample when they evaluate a target probability. Because people prefer to give verbal rather than numerical descriptions of probabilities (Beyth-Marom, 1982; Brun & Teigen, 1988; Budescu & Wallsten, 1985; Erev & Cohen, 1990; Olson & Budescu, 1997; Wallsten, Budescu, Zwick, & Kemp, 1993), use many different verbal labels (Budescu, Weinberg, & Wallsten, 1988; Karelitz & Budescu, 2004), and find it about as easy to reason with verbal or numerical descriptions of probabilities (see Budescu & Wallsten, 1995, for a review) we chose to analyze the frequency with which verbal phrases occurred in natural language. As before, we assume that the availability of

probabilities in memory reflects this real world distribution.

Karelitz and Budescu (2004) asked 20 participants to generate phrases to "select phrases that spanned the whole probability range and that they also used in their everyday lives" (p. 29). We used the 71 different phrases that their participants generated in our analysis. For each phrase, we attempted to determine two things: (a) the numerical probability equivalent of the phrase, and (b) the frequency with the phrase is used to describe probabilities in natural language.

There is already a literature that attempts to relate numerical probabilities and verbal phrases (see Budescu & Wallsten, 1995, for a review). Here, we simply asked 40 participants to imagine that a truthful person had used each phrase to describe the probability of winning an urn draw by drawing a red ball from 100 balls in total. For each phrase, participants were asked to say how many red balls (between 0 and 100 inclusive) the phrases suggested were in the urn. For each participant, phrases were presented in a different random order. Table 2 shows the mean and standard deviation of the probability attached to each phrases. Out of a total of 2,840 responses, 121 lay two interquartile ranges outside the upper and lower quartiles and were deleted as outliers. Their deletion does not affect the qualitative pattern of the results. Where our phrases overlap with those of other researchers (Beyth-Marom, 1982; Budescu & Wallsten, 1985; Clarke, Ruffin, Hill, & Beaman, 1992; Regan, Mosteller, & Youtz, 1989) there is reasonable agreement on the numerical equivalents.

To estimate the frequency of the phrases in natural language, we searched the British National Corpus (BNC) World Edition (http://www.natcorp.ox.ac.uk/index.html). There are about 100 million words in the BNC, which was designed to be representative of spoken and written English. The frequency with which each phrase occurred is listed in Table 2. Where one phrases is a sub-phrase of another (e.g., 'certain' is a sub-phrase of 'fairly certain'), then the frequency of the sub-phrase was counted ignoring occurrences of the subsuming phrase. Because some of the phrases also occur in natural language outside the context of probability

description, a random sample of twenty occurrences was analyzed for each phrase to estimate the proportion of the time that the phrases was used to describe a probability. The product of the frequency of occurrence and the proportion of times a phrase is used to describe a probability was calculated to give the frequency with which each phrase was used to describe a probability. (Omitting this weighting does not alter qualitative pattern described below.)

Figure 7 plots the relative rank of each phrase against the probability that best represents it. Because very small (or zero) and very large (or certain) probabilities are more frequent than for midrange probabilities, the function has an inverse S-shape. Because large probabilities are more frequent than small probabilities the point at which probability judgments would be accurately calibrated (i.e., at which the subjective probability function crosses the line y = x) is less than p = .5. When the function $w(p) = p^{\beta} / ((p^{\beta} + (1 - p)^{\beta})^{1/\beta})$ is fitted to these data, the best estimate for β is .59 (r^2 = .92). The range of β values for which 90% of the variance is captured is .46 to .67. This range coincides reasonably well with β values found by Camerer and Ho (1994, β = .56), Tversky & Kahneman (1994, β = .61) and Wu and Gonzalez (1996, β = .71). In other words, there is good agreement between the function we have derived here using the distribution of probability phrases in natural language and those that best describe choices between gambles.

Table 2 shows that the numerical values assigned to many probability phrases are quite variable. This finding is well established in the literature (see Budescu & Wallsten,1995). Thus, the positioning of each probability phrase on the abscissa of Figure 7 is subject to some noise. However, if one instead smears out the contribution to the increase in relative rank due to each phrase over the full distribution of numerical probability equivalents for each phrase, rather than just using the mean equivalent, a very similar inverse S-shape function is found.

The Distribution of Probabilities in Experiments

Brown and Qian (2004) examined the distribution of probabilities used in experiments designed to elicit the form of the probability weighting function in decision making under both risk and uncertainty. In a majority of studies, smaller and larger probabilities are over-represented compared to mid-range probabilities. Figure 8A illustrates this with the probabilities used by Gonzales and Wu (1999). Figure 8B shows the relative rank that would be assigned to a target probability if the sample people compared it to comprised the experimental probabilities for each experiment. Again, small probabilities are overestimated and large probabilities are underestimated.

Subjective Estimates of Probability Frequency

Brown and Qian (2004) asked participants to estimate the relative frequencies with which different probabilities occur in the environment, and found that (a) low and high probabilities are rated as occurring most frequently, and (b) high probabilities are rated as occurring more often than low probabilities. Assuming the veridicality of participants' ratings, DbS can therefore explain both the S-shape of the probability weighting curve and also its asymmetry.

Sampling of Events

From assuming that there are few frequent events and many rare events (Oaksford & Chater, 1994) we argue that the distribution of probabilities experienced is such that there are many small and large probabilities and relatively few moderate probabilities. Here we illustrate this argument by considering a toy universe, where there are only 100 possible events that can and will ever occur. We begin by assuming that the frequency of these events follows Zipf's power law (see Figure 9A). Many real-world events, such as the frequency of words in natural language, follow just such a distribution (see, e.g., Bak, 1997; Ijiri & Simon, 1977; Mandelbrot, 1982; Zipf, 1949). According to support theory (Tversky & Koehler, 1994), people judge the probability of an event by comparing it to possible alternative events. Thus, here we do not assume that people have access to the raw frequencies of each event.

Instead they judge how likely each event is compared to a subset of possible alternatives. Continuing the word frequency example, the raw frequencies themselves are not of communicative importance. Instead, what matters and what is experienced is co-occurrence (indeed many computational models of the lexical semantics are constructed from just such co-occurrence relations, e.g., the hyperspace analogue to language, Lund & Burgess, 1996, and latent semantic analysis, Landauer and Dumais, 1997). That is, they experience the relative frequencies of words in a particular context. For example, the raw frequencies of "hedge" and "fence" are not experienced directly. Instead, we experience their relative frequencies in contexts like "the horse jumped over the...".

Figure 9B shows the probability with which various relative probabilities are experienced when pairs of events are drawn from the universe of events. Specifically, consider sampling two events E_1 and E_2. Call the absolute probability of these events p_1 and p_2. Thus, the probability of randomly sampling the pair from the universe is given by the pair probability $p_1\, p_2$. The relative probability of event E_1 is $p_1 / (p_1 + p_2)$ and the relative probability of event E_2 is $p_2 / (p_1 + p_2)$. The probability with which each relative probability can be experienced can be calculated by averaging over all possible event pairs, and it is this distribution that is plotted in Figure 9B. We suggest that it is these relative probabilities that people encode, and thus sample from memory. Figure 9C plots the mean relative rank of a probability within a sample from all of the relative probabilities (effectively the cumulative density function, exactly as for gains, losses, and delays). There are two important features of this resulting function. First, there are more small and large relative probabilities than intermediate values: The cumulative density function is steepest initially and finally. Second, certain round fractions (e.g., 1/2, 1/3) occur frequently. Note that most of the density of the fractal-like pattern is at the edges despite the central spikes.

The immediately preceding argument assumes people are sensitive to the relative probability of one event compared to another $p_1 / (p_1 + p_2)$. An alternative assumption is that

people are sensitive to the odds p_1 / p_2. Because odds are a simple monotonic transform of relative probability - specifically *odds* = *probability* / (1 - *probability*) - the distribution and cumulative distribution of odds can be derived directly from those for relative probability. Crucially, for a given pair of events, the relative rank of the relative probability is the same as the relative rank for the corresponding odds. Thus, according to DbS the relative rank of an event will be the same whether people are sensitive to odds or relative probability (though presenting the chances of an event happening as odds rather than probability might well evoke a different sample of chances from long-term memory).

General Discussion

The shapes of the descriptive functions for the utility of gains and losses, temporal discounting, and the subjective value of probabilities are well established in the literature. Here, we have offered an account of why these functions might take the forms that they do. DbS makes two key claims about the psychology of decision making. First, people can make only binary, ordinal comparisons between attribute values. Second, attribute values are compared with a decision sample comprising a sample of values from memory. The distribution of values in memory is assumed to reflect the distribution of attribute values in the world. Thus, according to DbS, these functions take the forms they do because of the real-world distribution of gains, losses, delays, and probabilities. These assumptions are sufficient to account for incremental wealth having diminishing incremental utility (i.e., risk aversion); losses looming larger than gains; sub-hyperbolic temporal discounting, with a dependency of magnitude and nature of the outcome; and overestimation of small probabilities and underestimation of large probabilities.

DbS and Economic Theory

The assumption that people do not directly utilize internal scales for value constitutes a break from Bentham's (1789/1970) notion that utility is calibrated on an internal psychological scale and thus a break from psychological theories derived from economics

that make a similar assumption. Interestingly, mainstream economic theory has not assumed the existence of such scales. Indeed the revealed preference interpretation (Samuelson, 1937), which has become standard in economics, takes utility to be revealed by observable preferences. For one item to have higher utility than another for a particular person is taken to mean no more than the first item would be chosen over the second by that person. Savage (1954) generalized this result to utilities and probabilities, showing that, given certain normatively reasonable constituency conditions on people's preferences over gambles, these preferences could be used to reveal utility and probability information simultaneously. From the revealed preference perspective, the utility and probability scales are derived from dispositions concerning preferences, rather than amounting to psychological claims. The approach developed in this paper has intriguing similarities to and differences from this view. The similarity is that, in our approach, people have access only to their binary preferences (or more generally binary, ordinal comparison of perceptual magnitudes) and hence, to the extent that people have a broader grasp of their own more global preferences, these must be constructed from their own binary preferences (Kornienko, 2004), just as the economist constructs probability and utility scales from a person's binary choices. This account also has a striking dissimilarity from the economists' conception. This is because we assume that sampling from memory is limited and stochastic. People's judgments of a particular attribute will be strongly influenced by the particular comparison items that they happen to sample. Hence, people's assessments of payoffs, probabilities, time intervals, and other attributes, will be highly malleable, rather than conforming to a stable ordering as in standard economic theory.

Prospect Relativity

In this article we have focused upon the effect of the attribute values that people sample from memory. However, as we suggested above, we also think that attribute values from the immediate context in which a decision is made are also likely to be sampled and

thus influence judgment and decision making. Two existing experiments have examined the effect of the context in which a decision is made on judged certainty equivalents of risky prospects (Birnbaum, 1992; Stewart, Chater, Stott, & Reimers, 2003) and in decision under risk (Stewart et al., 2003; see Benartzi & Thaler, 2001, for a real-world example). In both of these experiments the distribution of options (either values from which a participant had to draw a certainty equivalent, or the range of prospects from which a participant could select one to play) was manipulated. Birnbaum and Stewart et al. both found strong effects of these manipulations which were consistent with attribute values being judged in comparison to other attribute values in the immediate decision context.

DbS and the Time Course of Decision

Recently, psychologists have begun to consider the time course of decision making (e.g., Roe, Busemeyer, & Townsend, 2001; Diederich, 2003). We can formulate DbS as a sequential sampling model, where pairs of attribute values are subject to ordinal comparison, and frequency counts of favorable comparisons are maintained. This formulation could naturally be extended to model the time course of decision making. We envisage that this accumulation will continue either until a response deadline or until some threshold or difference is reached.

This account differs from that of Roe et al. (2001) and Diederich (2003). In their account, dimensions, rather than attribute values, are sampled in an all or none process, with stochastic switching between dimensions during the course of the decision process. At each step, the valence of each alternative is derived by comparison with every other alternative in the choice set. Valences are integrated over time to produce preferences, with the preferences for each option competing via similarity weighted lateral inhibition. In DbS, valences would simply be incremented by favorable ordinal, binary comparisons. Competition between options in DbS would not come from lateral inhibition, but instead from the fact that comparisons are binary. Because comparisons are assumed to be binary, introducing a new

option that is similar to an existing option would cause the favorable comparisons to be shared between them.

Range-Frequency Theory

In Parducci's (1965, 1995) range-frequency theory, an attribute value is a weighted sum of its ordinal rank within the immediate context and its interval scale position within the range set by the immediate context. In DbS, only rank matters. However, in DbS, effects of the absolute magnitude of an attribute value (i.e., range effects) can arise because items in the decision sample includes not only items from the immediate context but also other values from memory. If the distribution of extra-contextual attribute values is uniform, then the subjective attribute value is that given by range-frequency theory. Thus, we suggest that demonstrations of effects of the position within the range with rank held constant in fact reflect the use of attribute values from outside the immediate context. To the extent that these are fixed from one situation to the next, it will appear as if more than pair-wise ordinal information is available when this is not necessarily the case.

Consistent with this, applications of range-frequency theory to areas such as price perception and wage satisfaction ratings have typically found that the rank/frequency weighting is weighted more highly, and the range/end-point relative position less highly, when the distribution of the decision sample is made salient (e.g., by simultaneous presentation: Brown, Gardner, Owald, & Qian, 2004; Niedrich, Sharma, & Wedell, 2001; cf. also Alba, Mela, Shimp, & Urbany, 1999).

Decision by Similarity Sampling

It is unlikely that attribute values in the decision sample are sampled randomly from memory. It seems likely that other factors such as similarity and recency must play a role. Most models of memory retrieval assign a major role to recency as a factor determining retrieval probability, and hence any complete account must assume that recent items are more likely to be included in the decision sample. For example, Parducci (1996) argues that the

context for evaluation includes both recent exemplars and also remembered extreme exemplars (anchors). However similarity will also determine the probability of inclusion: for example, the price of a car is likely to be judged with reference to a sample of similarly-priced cars, and wage satisfaction is likely to be evaluated in terms of a sample of wages earned by individuals in similar occupations and earning similar wages (e.g., Rablen, Brown, & Oswald, 2004).

In the discussion of how discount rate depends on the magnitude of the outcome, we suggested that the long [shorter] delays experienced in the receipt of large [smaller] monetary values would be sampled when considering the discounting of larger [smaller] values. This suggestion is consistent with the idea that whole exemplars are sampled, rather than isolated attribute values. Many exemplar models of memory offer the potential for independently-motivated accounts of the retrieval processes that might underpin the formation of decision samples. Indeed, some of these accounts have been applied to judgments of probability. In Kahneman and Miller's (1986) norm theory, for example, a stimulus or event is judged and interpreted in relation to an evoked contextual set of relevant stimuli or events that are retrieved in response to the event to be judged. Such retrieval may be similarity-based. Dougherty et al. (1999) develop a similarity-based model of memory, Hintzman's MINERVA2 (Hintzman, 1984, 1988), and apply it to a wide range of likelihood judgment phenomena. Thus, exemplar theories of memory can underpin models of availability, and DbS can be interpreted as an account of processes operating subsequent to availability-stage phenomena. More specifically, the availability heuristic suggests that event frequencies or likelihoods are judged by the ease with which instances come to mind (Tversky & Kahneman, 1973). As Schwarz and Vaughn (2002) note, fluency of recall and content of recall may provide distinct sources of information. DbS, while focussing on content, is distinctive in assuming that only relative magnitude judgments are available to provide the basis for judgment, and that judgments are made purely on the basis of a tally of the number

of retrieved exemplars above and below the target item on the dimension of interest. In some cases (e.g., Brown et al., 2004) this simplistic sampling provides a better fit to the data than when similarity (or dissimilarity) are taken into account.

Unifying Normative and Contextual Models of Decision Making

We see the DbS framework as an important step towards unifying traditional models of decision making, where attribute values are derived from fixed psychoeconomic functions of external values, and contextually driven models, such as range frequency theory and multialternative decision field theory. We have offered an account where the frequently observed psychoeconomic functions arise from the real-world decision making environment which also incorporates an explanation of how variations in that context will influence decisions.

References

Abdellaoui, M. (2000). Parameter free elicitation of utilities and probability weighting functions. *Management Science, 46,* 1497-1512.

Alba, J. W., Mela, C. F., Shimp, T. A., & Urbany, J. E. (1999). The effect of discount frequency and depth on consumer price judgments. *Journal of Consumer Research, 26*, 99-114.

Allais, M. (1953). Le comportement de l'homme rationel devant le risque, critique des postulats et axiomes de l'ecole americaine. *Econometrica, 21,* 503-546.

Anderson, J. R. (1990). *The adaptive character of thought*. Hillsdale, NJ: Erlbaum.

Anderson, J. R., & Milson, R. (1989). Human memory: An adaptive perspective. *Psychological Review, 96,* 703-719.

Anderson, J. R., & Schooler, L. J. (1991). Reflections of the environment in memory. *Psychological Science, 2,* 396-408.

Bak, P. (1997). *How nature works: The science of self-organized criticality.* Oxford, UK: Oxford University Press.

Benartzi, S., & Thaler, R. H. (2001). Naive diversification strategies in defined contribution saving plans. *The American Economic Review, 91,* 79-98.

Benzion, U., Rapoport, A., & Yagil, J. (March, 1989). Discount rates inferred from decisions: An experimental study. *Management Science, 35,* 270-284.

Bentham, J. (1970). *An introduction to the principles of morals and legislation* (J. H. Burns & H. L. A. Hart, Eds.). London: The Athlone Press. (Original work published, 1789)

Bettman, J. R, Luce, M. F., & Payne, J. W. (1998). Constructive consumer choice processes. *Journal of Consumer Research, 25,* 187-217.

Beyth-Marom, R. (1982). How probable is probable? A numerical translation of verbal probability expressions. *Journal of Forecasting, 1,* 257-269.

Birnbaum, M. H. (1992). Violations of the monotonicity and contextual effects in choice-

based certainty equivalents. *Psychological Science, 3,* 310-314.

Bleichrodt, H., & Pinto, J. L. (2000). A parameter-free elicitation of the probability weighting function in medical decision analysis. *Management Science, 46,* 1485-1496.

Bordley, R., & LiCalzi, M. (2000). Decision analysis using targets instead of utility functions. *Decision in Economics and Finance, 23,* 53-74.

Brown, G. D. A., Gardner, J., Oswald, A., & Qian, J. (2004). *Rank dependence in employees' wellbeing.* Manuscript submitted for publication.

Brown, G. D. A., & Qian, J. (2004). *The origin of the probability weighting function: A psychophysical approach.* Manuscript submitted for publication.

Brun, W., & Teigen, K. H. (1988). Verbal probabilities: Ambiguous, context-dependent, or both? *Organizational Behavior and Human Decision Processes, 41,* 390-404.

Budescu, D. V., & Wallsten, T. S. (1985). Consistency in interpretation of probabilistic phrases. *Organizational Behavior and Human Decision Processes, 36,* 391-405.

Budescu, D. V., & Wallsten, T. S. (1995). Processing linguistic probabilities: General principles and empirical evidence. In J. Busemeyer, D. L. Medin, & R. Hastie (Eds.), *Decision making from a cognitive perspective* (pp. 275-318). San Diego, CA: Academic Press.

Budescu, D. V., Weinberg, S., & Wallsten, T. S. (1988). Decisions based on numerically and verbally expressed uncertainties. *Journal of Experimental Psychology: Human Perception and Performance, 14,* 281-294.

Busemeyer, J. R., & Townsend, J. T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review, 100,* 432-459.

Chapman, G. B. (1996). Temporal discounting and utility for health and money. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 22,* 771-791.

Chater, N., & Brown, G. D. A. (1999). Scale-invariance as a unifying psychological

principle. *Cognition, 69,* b17-b24.

Clarke, V. A. Ruffin, C. L., Hill, D. J., & Beamen, A. L. (1992). Ratings of orally presented verbal expressions of probability by a heterogeneous sample. *Journal of Applied Social Psychology, 22,* 638-656.

Diederich, A. (2003). MDFT account of decision making under time pressure. *Psychonomic Bulletin & Review, 10,* 157-166.

Dougherty, M. R. P., Gettys, C. F., & Ogden, E. E. (1999). MINERVA-DM: A memory process model for judgments of likelihood. *Psychological Review, 106,* 180-209.

Dougherty, M. P. R., & Hunter, J. (2003a). Hypothesis generation, probability judgment, and individual differences in working memory capacity. *Acta Psychologica, 113,* 263-282.

Dougherty, M. P. R., & Hunter, J. (2003b). Probability judgment and subadditivity: The role of working memory capacity and constraining retrieval. *Memory & Cognition, 31,* 968-982.

Erev, I., & Cohen, B. L. (1990). Verbal versus numerical probabilities: Efficiency, biases, and the preference paradox. *Organizational Behavior and Human Decision Processes, 45,* 1-18.

Friedman, D. (1989). The S-shaped value function as a constrained optimum. *American Economic Review, 79,* 1243-1248.

Garner, W. R. (1962). *Uncertainty and structure and psychological concepts.* New York: Wiley.

Gigerenzer, G., & Hoffrage, U. (1995). How to improve Bayesian reasoning without instruction: Frequency formats. *Psychological Review, 102,* 684-704.

Green, L., Myerson, J., McFadden, E. (1997). Rate of temporal discounting decreases with amount of reward. *Memory & Cognition, 25,* 715-723.

González-Vallejo, C. (2002). Making trade-offs: A probabilistic and context-sensitive model of choice behavior. *Psychological Review, 109,* 137-154.

Gonzalez, R., & Wu, G. (1999). On the shape of the probability weighting function. *Cognitive Psychology, 38,* 129-166.

Gutenberg, B., & Richter, C. F. (1949). *Seismicity of the earth.* Princeton, NJ: Princeton University Press.

Hertwig, R., Barron, G., Weber, E. U., & Erev, I. (2004). Decisions from experiences and the effect of rare events in risky choice. *Psychological Science, 15,* 534-539.

Hinson, J. M, Jameson, T. L., & Whitney, P. (2003). Impulsive decision making and working memory. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 29,* 298-306.

Hintzman, D. L. (1984). MINERVA2: A simulation model of human memory. *Behavior Research Methods, Instruments, and Computers, 16,* 96-101.

Hintzman, D. L. (1988). Judgments of frequency and recognition memory in a multiple-trace memory model. *Psychological Review, 95,* 528-551.

Ijiri, Y., & Simon, H. A. (1977). *Skew distributions and the size of business firms.* Amsterdam: North Holland.

Johnson, M. W., & Bickel, W. K. (2005). *Delay discounting: A review, reanalysis, and critique of models and quantifications.* Manuscript submitted for publication.

Johnstone, A. C., & Nava, S. (1985). Recurrence rates and probability estimates for the New Madrid Seismic Zone. *Journal of Geophysical Research, 90,* 6737.

Juslin, P., & Persson, M. (2002). PROBabilities from EXemplars (PROBEX): A "lazy" algorithm for probabilistic inference from generic knowledge. *Cognitive Science, 26,* 563-607.

Kahneman, D., & Miller, D. T. (1986). Norm theory: Comparing reality to its alternatives. *Psychological Review, 93,* 136-153.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47,* 263-291.

Kahneman, D, & Tversky, A. (Eds.). (2000). *Choices, values and frames.* New York: Cambridge University Press & the Russell Sage Foundation.

Karelitz, T. M., & Budescu, D. V. (2004). You say "probable" and I say "Likely": Improving interpersonal communication with verbal probability phrases. *Journal of Experimental Psychology: Applied, 10,* 25-41.

Kirby, K. N. (1997). Bidding on the future: Evidence against normative discounting of delayed rewards. *Journal of Experimental Psychology: General, 126,* 54-70.

Kirby, K. N., & Santiesteban, M. (2003). Concave utility, transaction costs, and risk in measuring discounting of delayed rewards. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 29,* 66-79.

Kornienko, T. (2004). *A cognitive basis for cardinal utility.* Working paper.

Landauer, T. K., & Dumais, S. T. (1997). A solution to Plato's problem: The latent semantic analysis theory of acquisition, induction, and representation of knowledge. *Psychological Review, 104,* 211-240.

Loewenstein, G. (1987). Anticipation and the valuation of delayed consumption. *Economic Journal, 97,* 666-684.

Loewenstein, G., & Thaler, R. H. (1989). Anomalies: Intertemporal choice. *Journal of Economic Perspectives, 3,* 181-193.

Loomes, G., & Sugden, R. (1982). Regret theory: An alternative theory of rational choice under uncertainty. *Economic Journal, 92,* 805-824.

Lund K., & Burgess, C. (1996). Producing high-dimensional semantic spaces from lexical co-occurrence. *Behavioral Research Methods, Instruments, & Computers, 28,* 203-208.

Mazur, J. E. (1987). An adjusting procedure for studying delayed reinforcement. In M. L. Commons, J. E. Mazur, J. A. Nevin, & H. Rachlin (Eds.), *Quantitative analyses of behavior, Vol. 5: The effect of delay and of intervening events on reinforcement value* (Vol. 5, pp. 55-73). Hillsdale, NJ: Erlbaum.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for information processing. *Psychological Review, 63,* 81-97.

Myerson, J., & Green, L. (1995). Discounting of delayed rewards: Models of individual choice. *Journal of the Experimental Analysis of Behavior, 64,* 263-276.

Niedrich, R. W., Sharma, S., & Wedell, D. H. (2001). Reference price and price perceptions: A comparison of alternative models. *Journal of Consumer Research, 28,* 339-354.

Oaksford, M., & Chater, N. (1998). *Rationality in an uncertain world: Essays on the cognitive science of human reasoning.* Hove, England: Psychology Press/Erlbaum.

Olson, M. J., & Budescu, D. V. (1997). Patterns of preference for numerical and verbal probabilities. *Journal of Behavioral Decision Making, 10,* 117-131.

Parducci, A. (1965). Category judgment: A range-frequency model. *Psychological Review, 72,* 407-418.

Parducci, A. (1995). *Happiness, pleasure, and judgment: The context theory and its applications.* Mahwah, NJ: Erlbaum.

Payne, J. W., Bettman, J. R., & Johnson, E. J. (1992). Behavioral decision research: A constructive processing perspective. *Annual Review of Psychology, 43,* 87-131.

Pollmann, T. (1998). On forgetting the historical past. *Memory & Cognition, 26,* 320-329.

Pollmann, T., & Baayen, R. H. (2001). Computing historical consciousness. A quantitative inquiry into the presence of the past in newspaper texts. *Computers and the Humanities, 35,* 237-253.

Prelec, D. (1998). The probability weighting function. *Econometrica, 66,* 497-527.

Prelec, D., & Loewenstein, G. (1998). The red and the black: Mental accounting of savings and debt. *Marketing Science, 17,* 4-28.

Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organisation, 3,* 323-343.

Quiggin, J. (1993). *Generalized expected utility theory: The rank-dependent model.* Norwell,

MA: Kluwer Academic Publishers.

Rachlin, H. (1989). *Judgment, decision, and choice*. New York: Freeman.

Rablen, M. D., Brown, G. D. A., & Oswald, A. J. (2004). *Social comparison, adaptation, and the utility of income.* Unpublished manuscript.

Reagan, R. T., Mosteller, F., & Youtz, C. (1989). Quantitative meaning of verbal probability expressions. *Journal of Applied Psychology, 74,* 433-442.

Roe, R. M., Busemeyer, J. R., & Townsend, J. T. (2001). Multialternative decision field theory: A dynamic connectionist model of decision making. *Psychological Review, 108,* 370-392.

Samuelson, P. A. (1937). A note of measurement of utility. *The Review of Economic Studies, 4,* 155-161.

Savage, L. J. (1954). *The foundations of statistics.* New York: Wiley.

Schwarz, N., & Vaughn, L. A. (2002). The availability heuristic revisited: Ease of recall and content of recall as distinct sources of information. In T. Gilovich, D. Griffin, & D. Kahneman (Eds.), *Heuristics and biases: The psychology of intuitive judgment* (pp. 534-547). Cambridge, England: Cambridge University Press.

Sedlmeier, P., & Betsch, T. (Eds.). (2002). *Etc. Frequency processing and cognition*. Oxford, England: Oxford University Press.

Shefrin, H., & Thaler, R. H. (October, 1988). The behavioral life-cycle hypothesis. *Economic Inquiry, 26,* 609-643.

Shepard, R. N. (1987). Evolution of a mesh between principles of the mind and regularities of the world. In J. Dupré (Ed.), *The latest on the best: Essays on evolution and optimality*. Cambridge, MA: MIT Press.

Shiffrin, R. M., & Nosofsky, R. M. (1994). Seven plus or minus two: A commentary on capacity limitations. *Psychological Review, 101,* 357-361.

Simpson, C. A., & Vuchinich, R. E. (2000). Reliability of a measure of temporal discounting.

The Psychological Record, 50, 3-16.

Slovic, P. (1995). The construction of preference. *American Psychologist, 50,* 364-371.

Stewart, N., Brown, G. D. A., & Chater, N. (in press). Absolute identification by relative judgment. *Psychological Review*.

Stewart, N., Chater, N., Stott, H. P., & Reimers, S. (2003). Prospect relativity: How choice options influence decision under risk. *Journal of Experimental Psychology: General, 132,* 23-46.

Thaler, R. H. (1981). Some empirical evidence on dynamic inconsistency. *Economic Letters, 8,* 201-207.

Tversky, A., & Kahneman, D. (1973). Availability: A heuristic for judging frequency and probability. *Cognitive Psychology, 5,* 207-232.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5,* 297-323.

Tversky, A., & Koehler, D. J. (1994). Support theory: A nonextensional representation of subjective probability. *Psychological Review, 101,* 547-567.

Wallsten, T. S., Budescu, D. V., Zwick, R., & Kemp, S. M. (1993). Preferences and reasons for communicating probabilistic information in verbal or numerical terms. *Bulletin of the Psychonomic Society, 31,* 135-138.

Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences, 7,* 483-488.

Weber, E. U., Goldstein, W. M., & Barlas, S. (1995). And let us for forget memory: The role of memory processes and techniques in the study of judgment and choice. In J. Busemeyer, D. L. Medin, & R. Hastie (Eds.), *The Psychology of Learning and Motivation (Vol. 32): Decision making from a cognitive perspective* (pp. 33-81). San Diego, CA: Academic Press.

Weber, E. U. & Johnson, E. J. (in press). Constructing preferences from memory. In: Slovic,

P., & Lichtenstein, S. (Eds.), *The construction of preference.* New York: Cambridge University Press.

Windschitl, P. D., & Wells, G. L. (1998). The alternative-outcomes effect. *Journal of Personality & Social Psychology, 75,* 1441-1423.

Wu, G., & Gonzalez, R. (1996). Curvature of the probability weighting function. *Management Science, 42,* 1676-1690.

Wu, G., & Gonzalez, R. (1999). Nonlinear decision weights in choice under uncertainty. *Management Science, 45,* 74-85.

Zipf, G. K. (1949). *Human behavior and the principle of least effort.* Cambridge, MA: Addison-Wesley.

Author Note

Neil Stewart, Department of Psychology University of Warwick; Nick Chater, Department of Psychology University of Warwick; Gordon D. A. Brown, Department of Psychology University of Warwick.

We thank Jerome R. Busemeyer, Dan Friedman, José Quesada, and Stian Reimers with whom we discussed some of the ideas presented in this article. We also thank Rich Lewis for collecting the credit and debit data, Alan Garrec for help collecting the numerical probability equivalents, and Stian Reimers for his idea of using Google as a data source for the distribution of temporal intervals. This research was supported by Economic and Social Research Council grants L328253039 and RES-000-22-0918.

Correspondence concerning this article should be addressed to Neil Stewart Department of Psychology, University of Warwick, Coventry, CV4 7AL, UK. E-mail neil.stewart@warwick.ac.uk.

Table 1

Best Fitting Powers for Power-Law Fits to the Distribution of Delays in Several Corpora

Source	Range	Best-Fitting Power
Telegraph	30 days	-1.7
Google hits	1 year	-1.5
Frankfurter Allgmeine Zeitung NRC/Handelsblad International Herald Tribune (Pollmann & Baayen, 2001)	500 years	-1.4

Table 2

Judged Numerical Equivalents and BNC Frequencies of Probability Phrases.

Phrase	Judged numerical equivalents				BNC frequency		
	M	*SD*	*Mdn*	*IQR*	Raw frequency	Proportion of probability uses	Adjusted frequency
impossible	0.00	0.00	0.0	0.0	6170	1.00	6170
not possible	0.00	0.00	0.0	0.0	1217	1.00	1217
no chance	0.00	0.00	0.0	0.0	534	.60	320
never	0.00	0.00	0.0	0.0	48217	.80	38574
extremely doubtful	3.76	2.81	3.0	3.0	20	.95	19
almost impossible	3.79	3.19	2.5	4.0	486	.90	437
pretty impossible	5.36	5.86	3.0	7.5	2	1.00	2
almost unfeasible	6.33	6.14	5.0	8.0	0	.00	0
highly unlikely	7.11	5.08	5.0	5.0	172	1.00	172
highly improbable	7.31	5.17	5.0	5.0	27	1.00	27
very doubtful	8.08	5.73	5.0	5.0	66	.95	63
very unlikely	8.25	4.58	9.5	5.0	157	1.00	157
little chance	11.75	7.38	10.0	10.0	273	.80	218
faint possibility	11.89	8.71	10.0	15.0	7	1.00	7
pretty doubtful	13.20	8.57	10.0	12.25	1	1.00	1
improbable	13.28	11.22	10.0	15.0	340	1.00	340
small chance	14.43	8.03	10.0	10.0	20	.95	19
not very feasible	14.51	9.63	10.0	12.5	0	.00	0
not likely	15.38	10.23	15.0	15.0	455	1.00	455
slight possibility	16.22	10.98	15.0	12.0	3	1.00	3
doubtful	16.75	10.72	15.0	15.0	1303	.25	326
quite doubtful	17.00	9.51	20.0	10.0	1	1.00	1
pretty unlikely	17.08	10.17	15.0	15.0	6	.83	5
unlikely	17.38	11.55	15.0	17.5	5099	1.00	5099
not very likely	17.72	11.71	15.0	15.0	18	1.00	18
rather unlikely	19.53	12.71	20.0	20.0	17	1.00	17
slight chance	20.93	17.01	15.0	21.3	5	1.00	5
slight probability	21.48	16.22	15.0	20.0	0	.00	0
against the odds	23.46	17.04	20.0	35.0	48	.20	10
a chance	28.49	17.97	25.0	29.0	3093	.22	680
little likely	29.58	18.07	25.0	21.3	2	1.00	2
a possibility	32.85	20.12	30.0	35.0	638	.70	447
uncertain	37.63	16.39	45.0	26.3	4608	.15	691
possible	42.69	16.24	50.0	20.0	31550	.20	6310

(*table continues*)

Phrase	Judged numerical equivalents				BNC frequency		
	M	*SD*	*Mdn*	*IQR*	Raw frequency	Proportion of probability uses	Adjusted frequency
little uncertain	42.87	23.72	40.0	30.0	16	.13	2
maybe	45.66	10.28	50.0	10.0	6064	.20	1213
even odds	50.00	0.00	50.0	0.0	4	.25	1
fifty-fifty chance	50.00	0.00	50.0	0.0	9	1.00	9
toss-up	50.00	0.00	50.0	0.0	13	0.46	6
medium likelihood	50.00	0.00	50.0	0.0	0	.00	0
fair chance	53.22	11.77	50.0	10.0	57	.35	20
fair possibility	53.86	10.17	50.0	10.0	0	.00	0
fair probability	58.98	13.85	50.5	20.0	0	.00	0
quite possible	59.93	16.39	60.0	20.0	234	.60	140
more likely	63.90	9.36	60.0	10.0	3556	.00	0
probable	64.26	9.88	65.0	10.0	1177	.90	1059
good possibility	66.31	14.14	65.0	15.0	2	1.00	2
good chance	69.08	9.07	70.0	10.0	366	.75	275
likely	69.70	13.20	70.0	20.0	16733	.85	14223
good probability	71.90	10.14	70.0	20.0	1	1.00	1
usually	74.15	10.96	75.0	15.0	18619	.85	15826
rather likely	74.25	9.88	75.0	11.3	1	1.00	1
very feasible	74.26	10.15	75.0	10.0	3	.00	0
most of the time	78.74	10.78	80.0	15.0	580	.95	551
high likelihood	79.73	8.50	80.0	16.3	5	1.00	5
fairly certain	79.83	12.16	85.0	20.0	56	1.00	56
great likelihood	80.82	9.64	80.0	12.5	1	1.00	1
high possibility	80.93	7.30	80.0	11.0	1	1.00	1
most likely	81.05	11.86	80.0	15.0	1341	.00	0
very likely	81.53	8.05	80.0	13.5	296	.85	252
great possibility	82.49	8.04	80.0	10.0	1	1.00	1
quite certain	82.85	10.27	85.0	15.0	97	.90	87
pretty certain	85.30	9.19	89.5	10.0	45	1.00	45
very certain	89.78	7.35	90.0	11.3	15	.87	13
almost certain	92.32	5.76	95.0	5.0	1694	1.00	1694
most definitely	95.13	5.32	95.0	7.8	109	.20	22
sure thing	97.53	4.34	100.0	5.0	27	.35	9
always	100.00	0.00	100.0	0.0	41869	.90	37682
absolute certainty	100.00	0.00	100.0	0.0	37	.40	15
certain	100.00	0.00	100.0	0.0	36121	.25	9030
definitely	100.00	0.00	100.0	0.0	3233	.80	2586

Figure Captions

Figure 1. A: The distribution of credits to people's current bank accounts. B: The distribution of debits from people's current bank accounts.

Figure 2. A: The relative rank of credits within the entire population of credits. B: The relative rank of debits within the entire population of debits.

Figure 3. A: The distribution of relative ranks for a target credit of £250. B: The distribution of relative ranks as a function of the target credit. Sample size = 5.

Figure 4. A: The relative ranks of 9756 prices from a UK supermarket. B: The relative ranks of the prices of bread. C: The relative ranks of the prices of chocolate.

Figure 5. The distribution of time delays on the internet.

Figure 6. The distribution of relative ranks of delays within the entire population of delays.

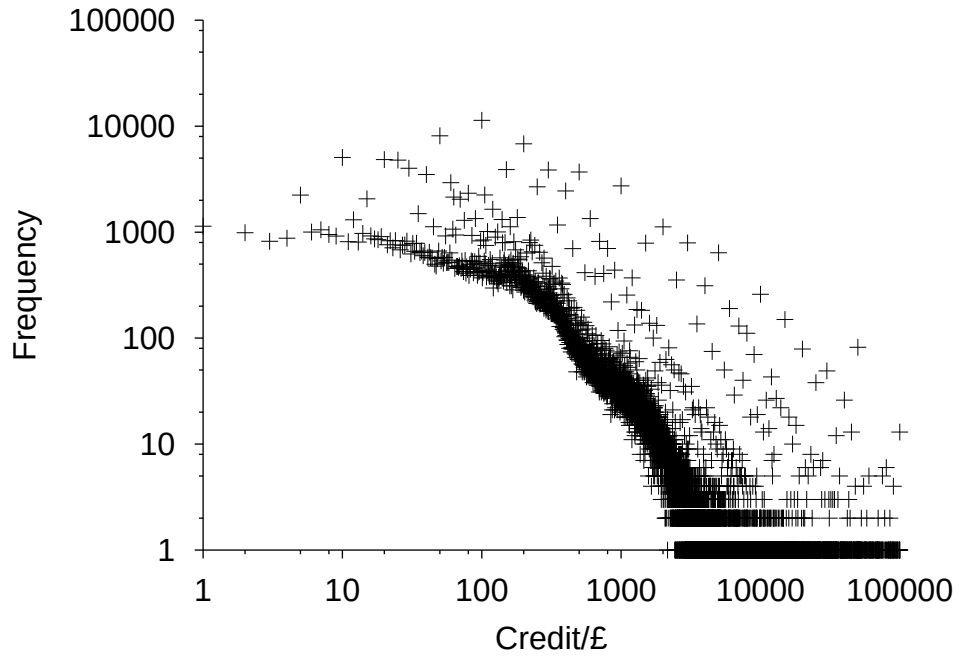
Figure 7. The relative rank of probability phrases.

Figure 8. A: The probabilities used in Gonzalez and Wu's (1999) experiment. B: The relative ranks for probabilities in this experiment.

Figure 9. A: The probability of events in a universe of 100 possible events. B: The probability of the relative probabilities of events in randomly selected pairs of events. C: The cumulative probability of the relative probabilities. See text for details.

Figure 1

A


100000
10000
1000
100
10
1
Frequency
1
10
100
1000
10000
100000
Credit/£

B

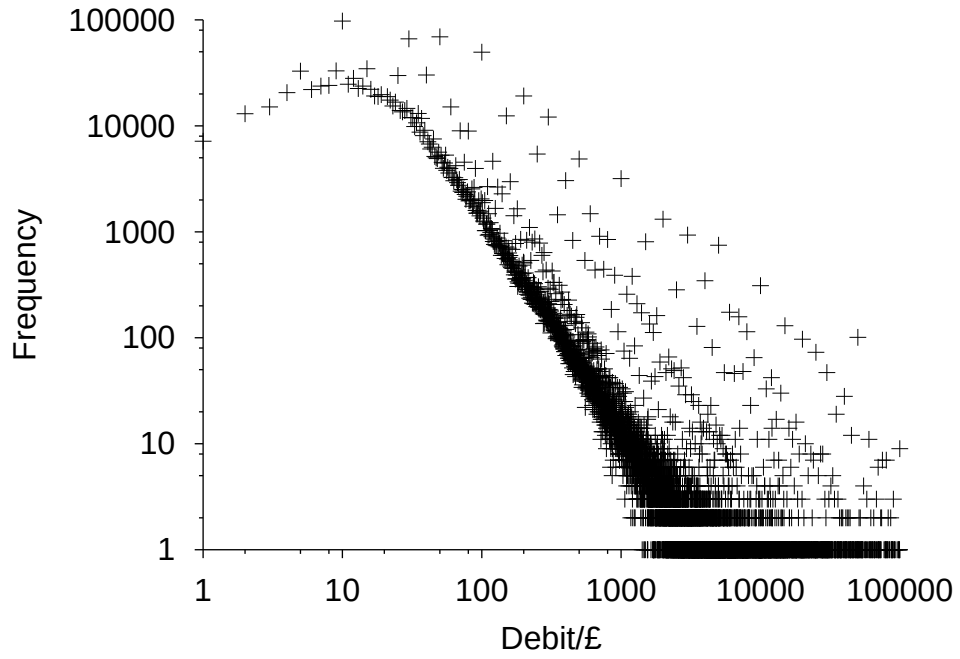
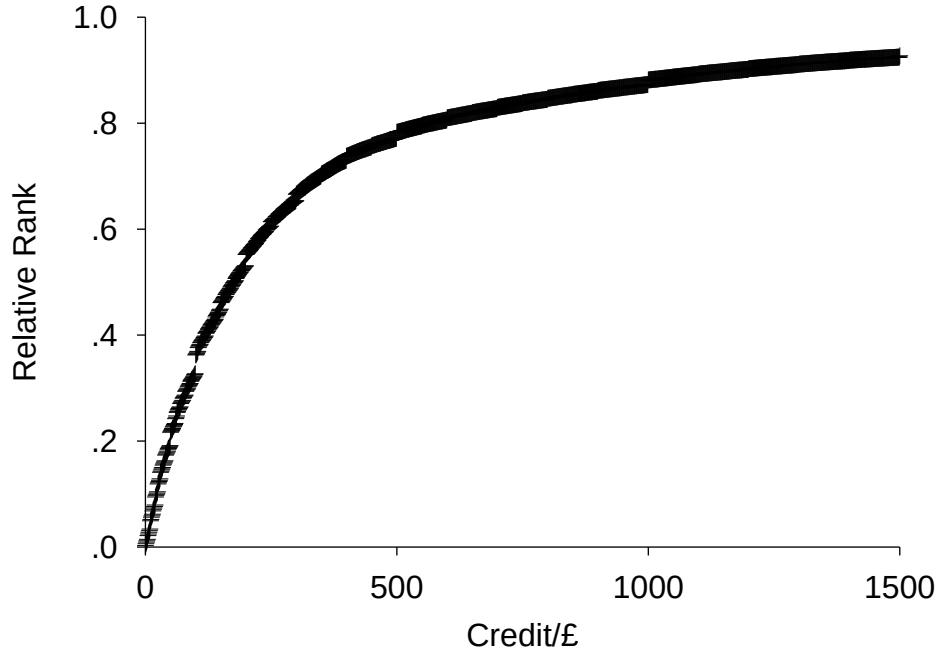

100000
10000
1000
100
10
1
Frequency
1
10
100
1000
10000
100000
Debit/£

Figure 2

A


1.0
.8
.6
.4
.2
.0
Relative Rank
0
500
1000
1500
Credit/£

B

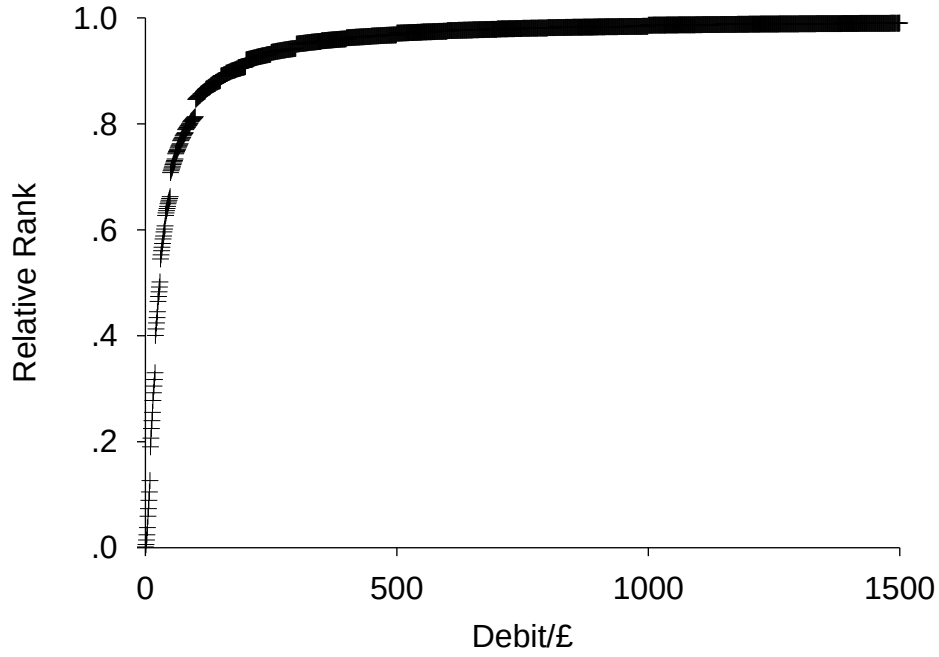
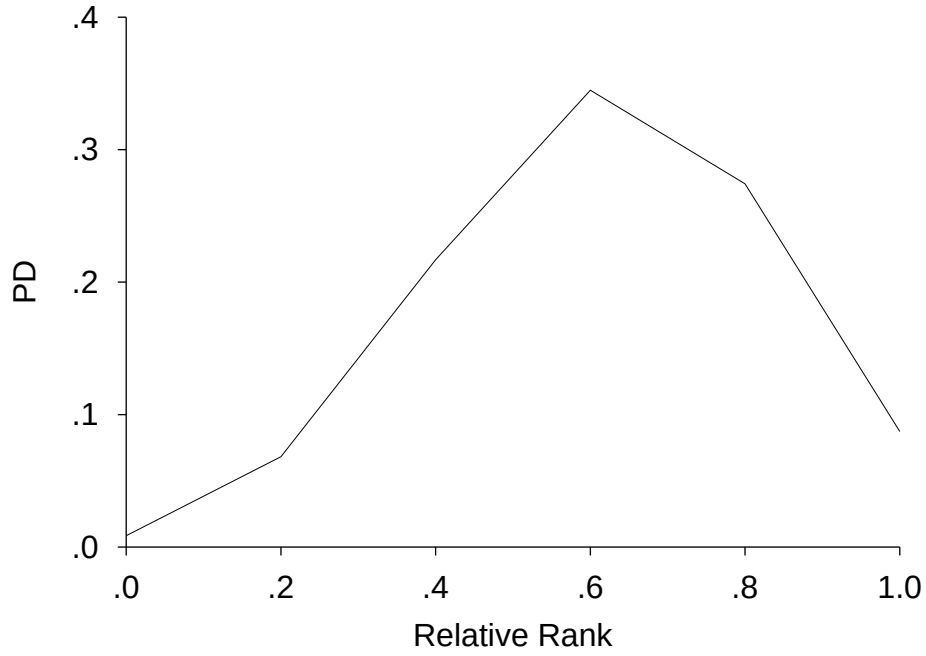

1.0
.8
.6
.4
.2
.0
Relative Rank
0
500
1000
1500
Debit/£

Figure 3

A


.4
.3
.2
.1
.0
PD
.0
.2
.4
.6
.8
1.0
Relative Rank

B

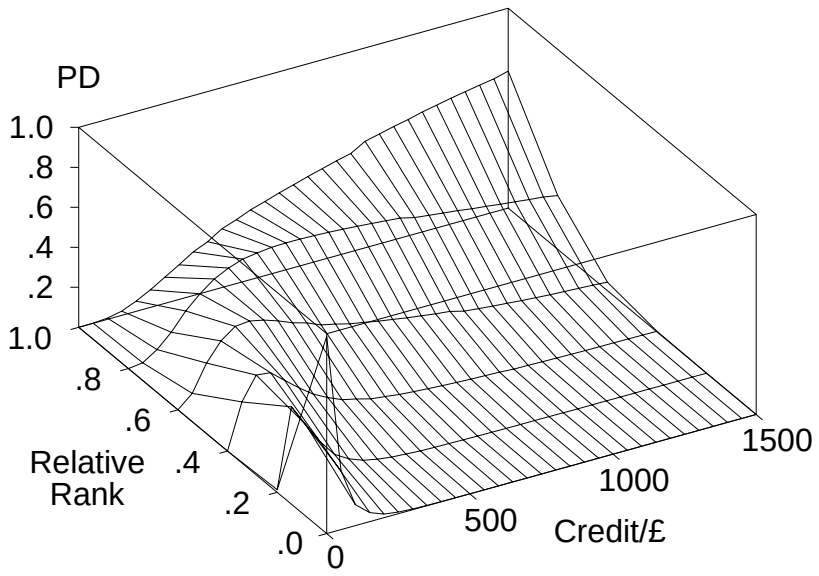
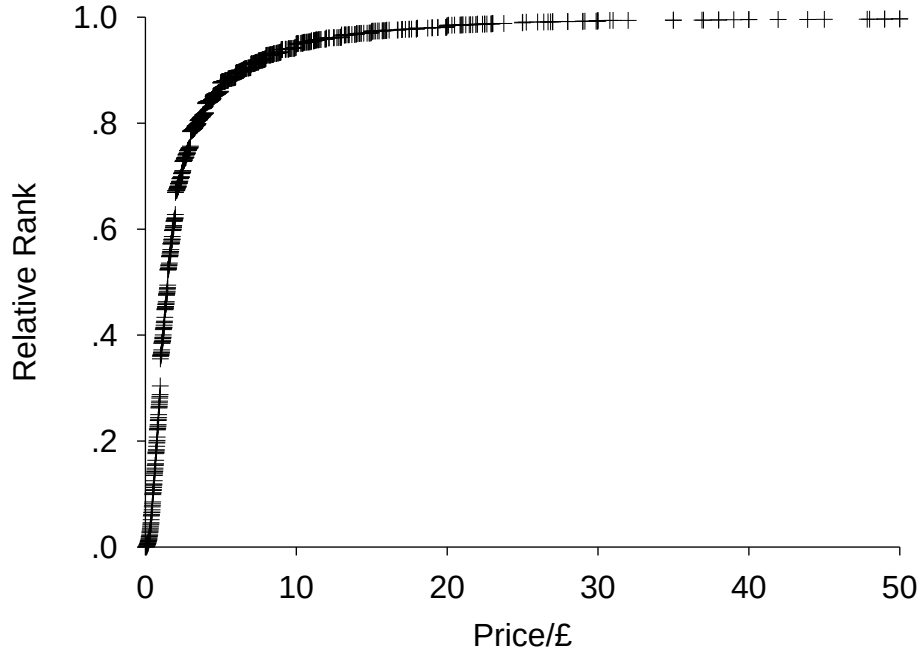
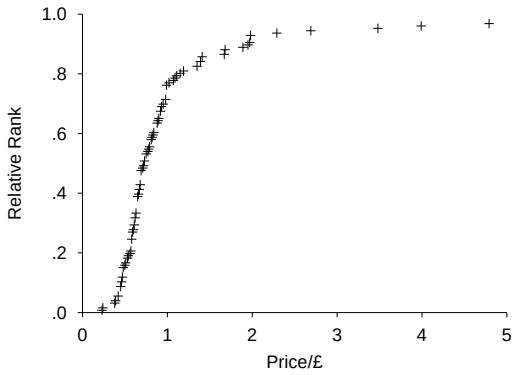

PD
1.0
.8
.6
.4
.2
1.0
.8
.6
.4
.2
.0
Relative
Rank
0
500
1000
1500
Credit/£

Figure 4

A


1.0
.8
.6
.4
.2
.0
Relative Rank
0
10
20
30
40
50
Price/£

B


1.0
.8
.6
.4
.2
.0
Relative Rank
0
1
2
3
4
5
Price/£

C

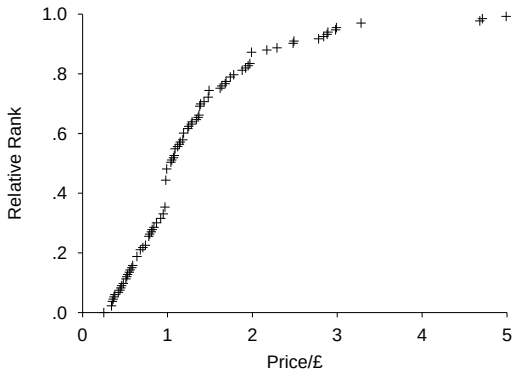

1.0
.8
.6
.4
.2
.0
Relative Rank
0
1
2
3
4
5
Price/£

Figure 5

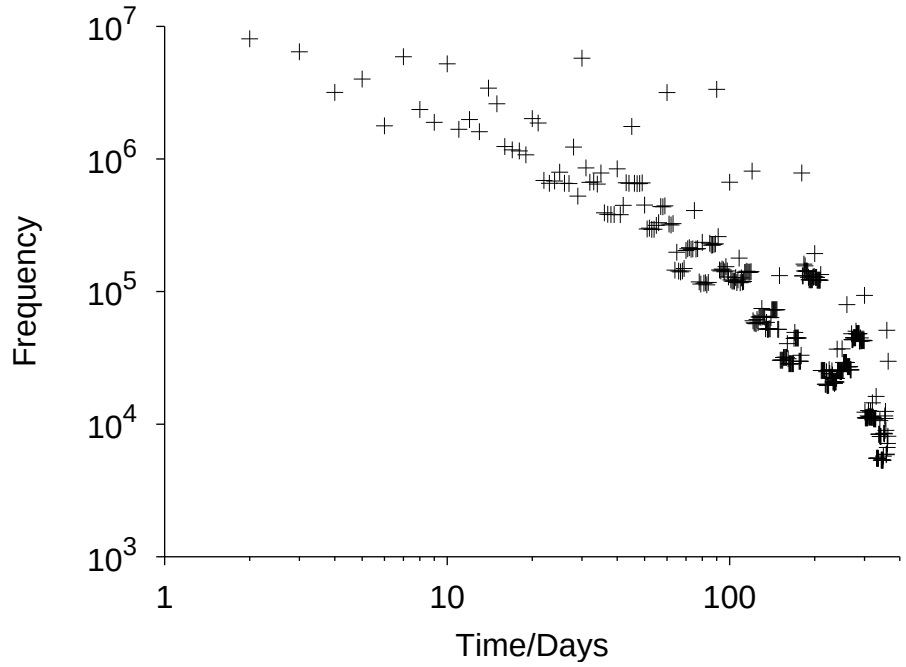


Figure 6

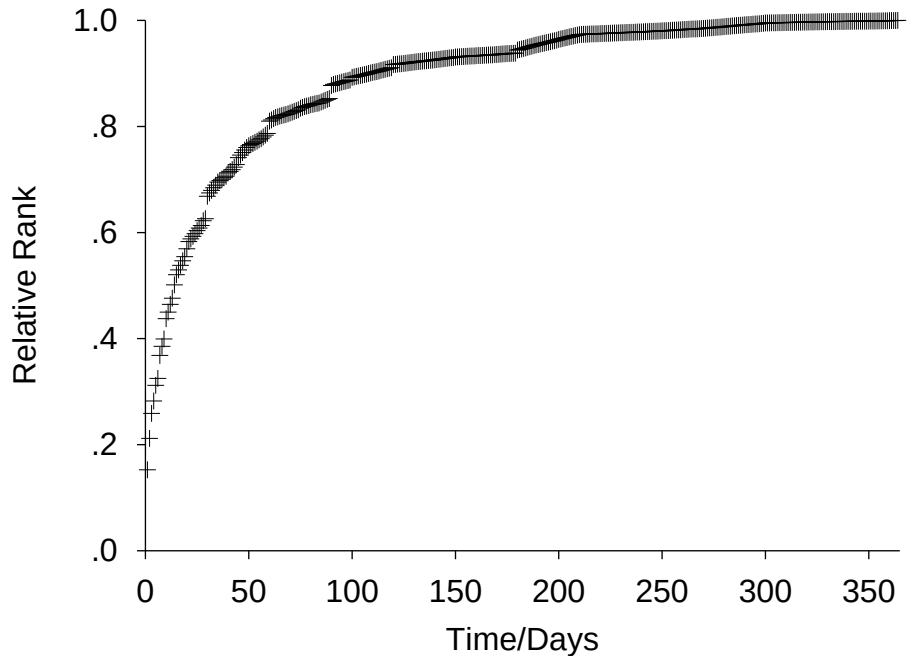


Figure 7

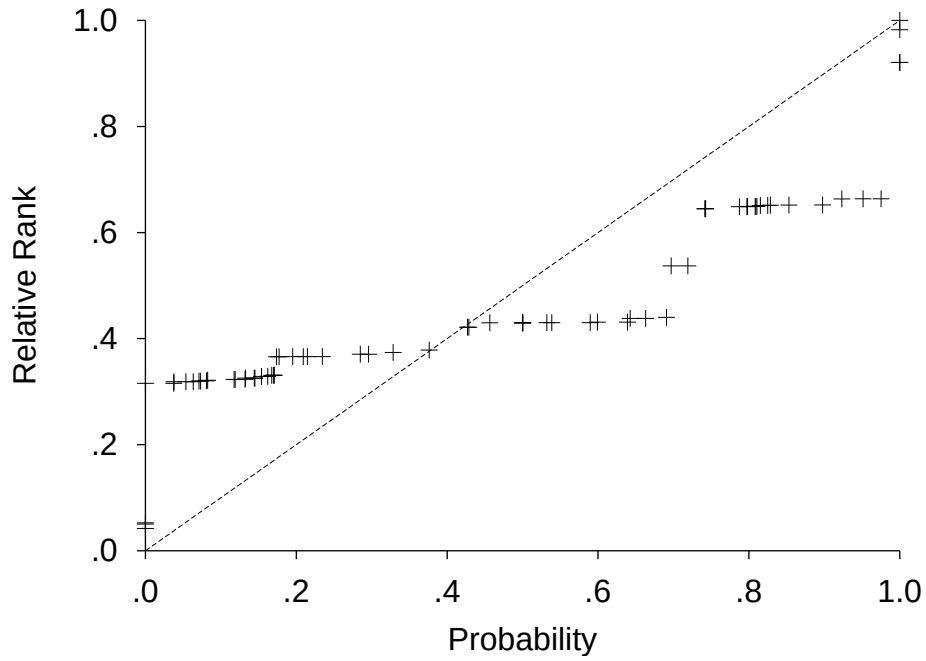
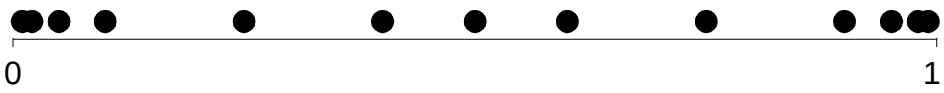


Figure 8

A



B

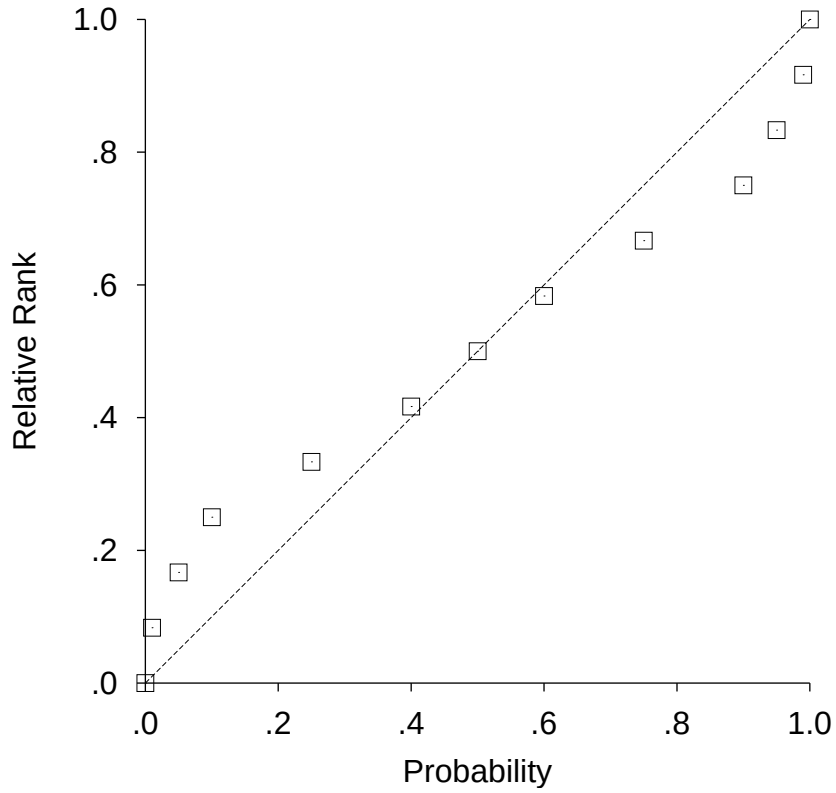
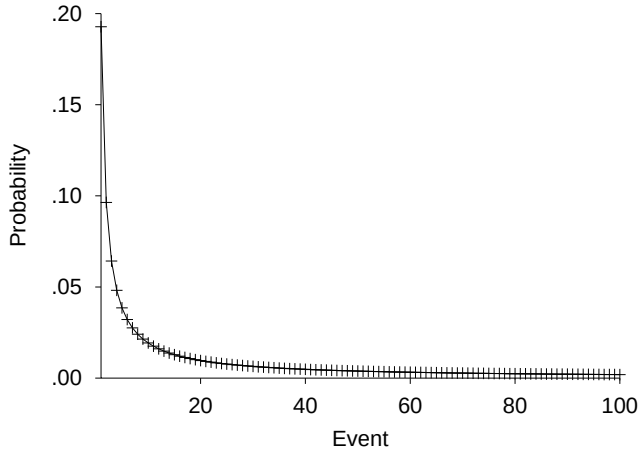
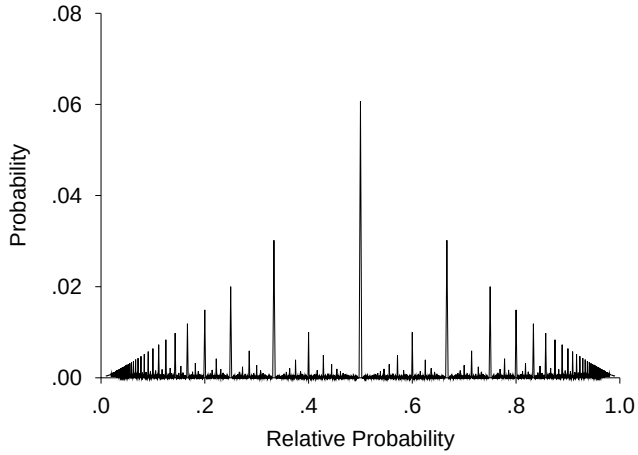


Figure 9

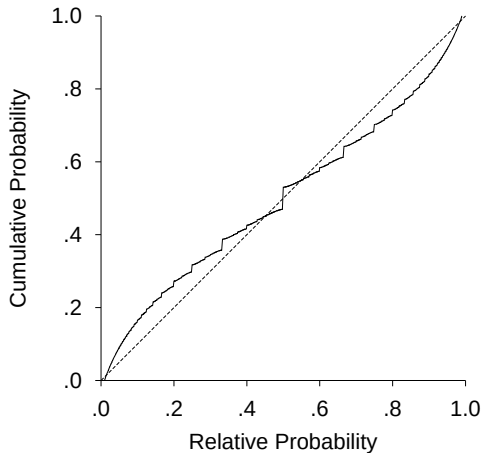
A


.20
.15
.10
.05
.00
Probability
20
40
60
80
100
Event

B


.08
.06
.04
.02
.00
Probability
.0
.2
.4
.6
.8
1.0
Relative Probability

C


1.0
.8
.6
.4
.2
.0
Cumulative Probability
.0
.2
.4
.6
.8
1.0
Relative Probability