Retaliatory Equilibria in a Japanese Ascending Auction for Multiple Objects

Gian Luigi Albano
Department of Economics and ELSE
University College London
WC1E 6BT London, United Kingdom

Fabrizio Germano
Departament d'Economia i Empresa,
Universitat Pompeu Fabra
08005 Barcelona, Spain

Stefano Lovo
HEC, Finance and Economics Department
78351 Jouy-en-Josas, France

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Abstract

We construct a family of retaliatory equilibria for the Japanese ascending auction for multiple objects, thus showing that while it is immune to many of the tacitly collusive equilibria studied in the literature, it is not entirely immune when some bidders are commonly known to be interested in a specific object.

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1 Introduction

Since 1994 the United Stated Federal Communication Commission has been using simultaneous ascending auctions for the sale of spectrum licenses. While they ensure a transparent bidding process that enables extensive information revelation of bidders' valuations, they also allow bidders to use their bids to effectively communicate among themselves and tacitly collude.[1] When few bidders compete for few objects, bidders may have important incentives to reduce demands and hence prices. Engelbrecht-Wiggans and Kahn (1999) and Brusco and Lopomo (2002) show that, for auction formats close to the FCC's, there are equilibria where bidders can coordinate a division of the available objects at very low prices. On the other hand, Albano *et al.* (2001, 2004) study a variant of the simultaneous ascending auction, namely the Japanese auction for multiple objects (JAMO), which differs from the FCC auction mechanisms in that (i) prices are exogenously raised by the auctioneer, and (ii) closing is object-by-object. Among other things, they show that because of these two features all the collusive "low-revenue" equilibria constructed by Engelbrecht-Wiggans and Kahn (1999) and Brusco and Lopomo (2002) are not possible in the JAMO.

This paper shows that collusive equilibria of a retaliatory type, (related to ones reported in Cramton and Schwartz (2000, 2002)), nonetheless exist in the JAMO. Their logic is as follows. Suppose two objects are put for sale to two bidders, a bundle bidder interested in both objects, and a unit bidder interested only in object 1. Assume this to be common knowledge. The two bidders have overlapping interests on object 1; in particular, the unit bidder wants the bundle bidder to exit early from object 1. In order to achieve this, the unit bidder actively bids on object 2, though he has zero value for it. Such a strategy is potentially costly to both the unit and the bundle bidder; we refer to the unit bidder's behavior as *retaliatory strategy*. The extent to which the unit bidder is successful in inducing the bundle bidder to drop early from object 1 depends on whether he succeeds in making his threat credible. We show that the JAMO admits equilibria with such strategies.

2 A Japanese Ascending Auction

2.1 Framework

Two objects are auctioned to a set of participants of two types: $M \geq 1$ bundle bidders who are interested in both objects and one unit bidder interested in only one of the two objects, $k = 1, 2$. Bundle and unit bidders draw their values independently from some smooth distribution F with

[1] Cramton and Schwartz (2000, 2002) have a detailed analysis of the signaling that took place during the FCC's DEF auction; Salmon (2004) contains a survey of collusive equilibria in ascending auctions.

positive density f, both defined over $[0, 1]$. Let v_k and u_k denote the value of object $k = 1, 2$ to a bundle and to a unit bidder respectively. The value of the bundle v_B to a bundle bidder is $v_B = v_1 + v_2$. The nature of bidders, bundle and unit, is also commonly known.

2.2 The Auction Mechanism

We consider a Japanese (or English clock) auction for multiple objects. Prices start from zero on all objects and are simultaneously and continuously increased until only one agent is left on a given object, in which case prices on that object stop and continue to rise on the remaining ones. Once an agent has dropped from a given object, the exit is irrevocable. The last agent receives the object at the price at which bidding on that object stopped. The number and the identity of agents active on any object is publicly known at any given time. The overall auction ends when all agents but one have dropped out from all objects. We refer to this mechanism as the Japanese auction for multiple objects (JAMO); this auction is also studied in Albano *et al.* (2001, 2004).

3 Retaliatory Equilibria

Our results are summarized as follows.

Proposition 1 *There exist Perfect Bayesian Equilibria of the JAMO where bidders use retaliatory strategies effectively.*

We show this by means of the following three examples, presented in order of generality.

Example 1. Consider the JAMO with two objects and two bidders; one unit bidder interested in object 1 and one bundle bidder interested in both objects 1 and 2, with the same value for the two objects, $v_1 = v_2 = x$; assume also all values are drawn according to the uniform distribution on $[0, 1]$. It is easy to see that the following is a PBE of the JAMO:

— all types of the unit bidder bid on *both* objects and stay on object 1 until u_1 and on object 2 until $\min(u_1, t_1^2)$, where t_1^2 is the bundle bidder's exiting time from object 1;

— all types of the bundle exit from object 1 at t if at t the unit bidder is on object 2; otherwise all types of the bundle bidder stay on both objects until x.

In equilibrium, the bundle bidder immediately drops out of object 1 inducing the unit bidder to also immediately drop out of object 2. As is often typical in such retaliatory equilibria, the retaliating bidder (here the unit bidder) obtains a higher ex ante payoff than in the "competitive" equilibrium[2] (1/2 versus 1/6), while the other agents (here the bundle bidder and the auctioneer) are both worse off (1/2 versus 2/3 and 0 versus 1/3 respectively). □

The above example relies on the fact that the unit bidder has some extra information about the bundle bidder's valuation of object 1 relative to object 2 (the values are perfectly correlated). Without this information he needs to resort to a more refined threat.

Example 2. Consider the case of a unit bidder competing against an arbitrary number M of bundle bidders with private values distributed according to a general distribution function F defined on $[0, 1]$. Take $l \in (0, 1]$ and let $c < l$ be the unique solution to the equation

$$\int_c^l z \cdot G'(z)dz = c \cdot G(l), \tag{1}$$

where $G = F^M$ is the distribution function for the bundle bidders' highest valuation for object 1 (and also for 2). Then, for any $l \in (0, 1]$, the following is a PBE of the JAMO:

— all types of unit bidder with $u_1 \leq l$ bid only on object 1 and stay until u_1; all types of unit bidder with $u_1 > l$ bid on *both* objects and stay on object 1 until u_1 and on object 2 until c;

— If the unit bidder is active on the two objects, then all types of bundle bidder with $v_1 < l$ bid on both objects and stay on object 1 until c and on object 2 until v_2; all types of bundle bidder with $v_1 \geq l$ bid on both objects always staying until v_1, v_2 respectively;

— If the unit bidder is active only on object 1 then all types of bundle bidder stay until v_1, v_2 on object 1 and 2 respectively.

This characterizes a family of retaliatory equilibria indexed by the parameter l that are PBE of the JAMO. Note that the equilibria are *not* in undominated strategies, since the unit bidder always has a (weakly) dominant strategy to drop from object 2 whenever it is the only object he is bidding on. If the unit bidder is active on both auctions this signals that his valuation is above the threshold l, i.e., $u_1 > l$; if he bids only on object 1, then $u_1 \leq l$, and all bidders bid up to their valuations and only on the objects they value. Unlike the equilibrium of Example 1, here, to ensure incentive compatibility for the unit bidder, he must bid on object 2 up to the threshold c. This is costly for the unit bidder as he wins the object he does not value (object 2) with positive probability. The threshold c is chosen such that only a unit bidder with $u_1 > l$ has an incentive to pay this cost. Thus by remaining active on both objects, the unit bidder provides a credible signal that he actually has a high valuation for object 1. Note that as the

[2]See Brusco and Lopomo (2002) and Albano *et al.* (2001, 2004).

number M of bundle bidder increases, the probability that the unit bidder wins object 2 during the retailatory phase decreases. As a consequence the threshold c must increase with M.

When $l = 1$ we get the competitive equilibrium , since with probability one the unit bidder will not be active on object 1. When $l \to 0$ we almost get the competitive equilibrium, since $c \to 0$, i.e., the unit bidder enters both auctions but almost immediately exits object 2.

These examples are related to the collusive equilibria of Brusco and Lopomo (2002) in the sense that bundle and unit bidders have overlapping interests on object 1; the unit bidder threatens to retaliate (i.e., to be active) on object 2 if bundle bidders do not exit object 1. The signaling is effective since it is common knowledge that the retaliatory bidder is interested in one object only. Thus by not "turning the light off" on object 2 when the price is zero, the unit bidder triggers the beliefs that sustain the retaliatory equilibrium. Unlike Brusco and Lopomo, these equilibria are implementable in the JAMO and thus do not require actual rounds of information exchanges. In particular, bidders do not (and cannot) raise prices on most preferred objects as in Brusco and Lopomo. In the JAMO, in order to signal the retaliatory strategy the unit bidder has to be active on both objects, thus having to adopting a more costly strategy, which is also likely to be overall less costly for the auctioneer.

Proof of Example 2. We first check optimality for any bundle bidder, then we check it for the unit bidder. If the unit bidder is active on both objects, bundle bidders infer that $u_1 > l$. Hence, a bundle bidder with $v_1 \leq l$ knows that she will never pay for object 1 less than what she values this object. Therefore, exiting object 1 at time c and exiting object 2 at v_2 is a (weak) best reply for such a bundle bidder. If, however, $v_1 > l$, then a bundle bidder is better off remaining on each object so long as her expected continuation payoff remains strictly positive. Namely, she will stay on objects 1 and 2 until v_1 and v_2 respectively. This proves that the bundle bidders' strategy is a best reply to the unit bidder's strategy.

To prove optimality for the unit bidder, we need to show that the unit bidder's strategy is a best reply and that it is profitable for the unit bidder to bid on both objects if and only if $u_1 > l$, i.e., that the equilibrium is incentive compatible, so that being active on both objects gives a credible signal that $u_1 > l$. Let $y_k \in [0, 1]$ denote the bundle bidders' highest valuation for object $k \in \{1, 2\}$, and let G denote the corresponding distribution function. Recall that the bundle bidder with the highest valuation for object 2 will exit auction 2 before c only if $y_2 < c$. When the unit bidder is not active on object 2 and still active on object 1, the bundle bidder with the highest valuation for object 1 will exit auction 1 at y_1. When the unit bidder is active on object 2, if $y_1 \leq l$, then the bundle bidder with the highest valuation for object 1 will exit auction 1 at c; whereas if $y_1 > l$ and the unit bidder is still active on object 1, then the bundle bidder with the highest valuation for object 1 will exit auction 1 at y_1.

If $u_1 < c$, then it is clearly not optimal for the unit bidder to bid on both objects since he will have to pay at least c for object 1. Hence we focus on the case $u_1 \geq c$. Suppose that $u_1 \in [c, l]$. If the unit bidder decides to implement the retaliatory strategy, then his expected payoff is $\int_0^l (u_1 - c) \cdot G'(y_1)dy_1 - \int_0^c y_2 \cdot G'(y_2)dy_2$. The first integral is the unit bidder's payoff from object 1: the unit bidder wins object 1 only if $y_1 < l$, (recall that $u_1 \leq l$) and he pays c. The second integral is the expected payoff from object 2: if $y_2 < c$, then he has to buy object 2 at a price y_2. If, however, at time 0 the unit bidder decides to bid only on object 1, his expected payoff is $\int_0^{u_1} (u_1 - y_1)G'(y_1)dy_1$.

At equilibrium we want the unit bidder to bid only on object 1 when $u_1 \leq l$, i.e., the following needs to be satisfied

$$\int_0^l (u_1 - c) \cdot G'(y_1)dy_1 - \int_0^c y_2 \cdot G'(y_2)dy_2 \leq \int_0^{u_1} (u_1 - y_1) \cdot G'(y_1)dy_1,$$

which is always satisfied when c solves Eq. (1); (recall that y_1 and y_2 have the same distribution function G).

Suppose now $u_1 > l$. Then, at $t = 0$, the unit bidder's expected payoff from the retaliatory strategy must be greater or equal than the payoff from bidding only on object 1, i.e.,

$$\begin{aligned} \int_0^l (u_1 - c) \cdot G'(y_1)dy_1 \quad &+ \quad \int_l^{u_1} (u_1 - y_1) \cdot G'(y_1)dy_1 \\ - \int_0^c y_2 \cdot G'(y_2)dy_2 \quad &\geq \quad \int_0^{u_1} (u_1 - y_1) \cdot G'(y_1)dy_1 \end{aligned}$$

It is easy to check that the above inequality is satisfied for any $l \in (0, 1]$. Consider now any time $t < c$, it must be optimal for the unit bidder to insist on his retailatory strategy, i.e., exiting object 2 at c rather than before c. Suppose that the unit bidder deviates and exits object 2 at $t \in (0, c)$. Then a possible out-of-equilibrium-path belief for bundle bidders is that $u_1 > L$, for some $L < 1$. Thus, after observing this deviation, a weakly dominant continuation strategy for a bundle bidder with value v_1 for object 1 is to exit object 1 at $\tau = \max\{L, v_1\}$. Indeed, even if $v_1 < L$, he expects not to have to buy object 1 at L as he think the unit bidder will exit at $u_1 > L$. Therefore the expected payoff for the unit bidder from quitting the retailiatory strategy at $t \in (0, c)$ and continuing optimally on object 1 can be fixed aribtrarily close to 0 by choosing L sufficiently close to 1. This must be smaller than the unit bidder's expected payoff from insisting with the retaliatory strategy, i.e.,

$$\int_0^l (u_1 - c) \cdot G'(y_1)dy_1 + \int_l^{u_1} (u_1 - y_1) \cdot G'(y_1)dy_1 - \int_t^c y_2 \cdot G'(y_2|y_2 > t)dy_2 > 0 \qquad (2)$$

for all $u_1 > l$ and all $t \in (0, c)$. As the left hand side of inequality (2) is increasing in u_1, it is sufficient to check it for $u_1 = l$ i.e., that $(l - c)G(l) - \int_t^c y_2 \cdot G'(y_2|y_2 > t)dy_2 > 0$. Since

$G'(y_2|y_2 > t) = G'(y_2)/(1 - G(t))$ we need to show that

$$(l-c)G(l)(1-G(t)) - \int_t^c z \cdot G'(z)dz > 0. \tag{3}$$

Integrating by parts the right hand side of Eq. (1) and rearranging we have $-cG(c) = (c-l)G(l) + \int_c^l G(z)dz$. Substituting this expression in (3), integrating by parts and simplifying, we have that the left hand side of (3) is equal to

$$\int_t^l G(z)dz - (l-c)G(t)G(l) + tG(t) > \int_c^l G(z)dz - (l-c)G(c) > 0,$$

where inequalities follow from $t < c < l$ and G positive, strictly increasing and bounded by 1. ■

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