Robust Secrecy Beamforming With Energy-Harvesting Eavesdroppers

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***Abstract*—This letter considers simultaneous wireless information and power transfer (SWIPT) in multiple-input–single-output downlink systems in which a multiantenna transmitter sends a secret message to a single-antenna information receiver (IR) with multiple single-antenna energy receivers (ERs). We aim to maximize the harvested energy by the ERs while maintaining the signal-to-interference-and-noise ratio (SINR) threshold at the IR and keeping the message secure from possible eavesdropping by the ERs by suppressing their SINRs. Both scenarios of perfect and imperfect channel state information at the transmitter are studied. Using semidefinite relaxation techniques, we show that there always exists a rank-one optimal solution for the IR, i.e., transmit beamforming is optimal for the IR.**

***Index Terms*—Robust, secrecy, SWIPT, energy harvesting.**

I. Introduction

ENERGY harvesting from radio frequency (RF) signals that exist in the surrounding environment is being treated as an auspicious technique to power battery-constrained wireless devices. Since RF signals that carry energy can transport information at the same time, simultaneous wireless information and power transfer (SWIPT) has great prospects [1]–[5].

Conventional transceivers designed for information transfer is no longer optimal for SWIPT since information and power transfer operate at different power sensitivity (e.g., -10 dBm for energy receivers (ERs) versus -60 dBm for information receivers (IRs)). Based on this setup, a receiver-location based scheduling for information and energy transmissions has been proposed [1] in which only those receivers in closer vicinity to the transmitter are scheduled for transmitting energy.

However, SWIPT gives rise to a security vulnerability since ERs mostly have better fading channels than IRs and thus have higher probability to overhear the information sent to the IRs [6]. SWIPT systems need to be designed to guarantee information secrecy such that the legitimate user (IR) can correctly decode the confidential information, but the eavesdroppers (ERs) can retrieve almost nothing from their observations.

To make this feasible, we need the IR's channel condition to be better than the eavesdroppers', which seems a *conflict-of-interest* in line of energy harvesting. As a remedy, multi-antenna technologies exploiting spatial multiplexing technique can be used. Recently, there has been growing interest in using multiple antennas to achieve physical-layer secrecy [7], [8].

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To ensure that the message is delivered secretly to the IR in SWIPT even in the presence of possible eavesdropping by the individual ERs, multiple-input single-output (MISO) secrecy communication schemes were investigated in [6] assuming that perfect channel state information (CSI) of the IR and ERs is available at the transmitter side. A resource allocation based robust secure beamforming scheme has also been considered in [9] to minimize the total transmit power with imperfect eavesdroppers' CSI at the transmitter. Note that robust designs for SWIPT have also been investigated in [3] and [10] but *without* taking the information secrecy into account.

In this letter, we investigate a secret MISO SWIPT system. Unlike [9], our objective is to jointly design the information and energy transmit beamforming for maximizing the minimum of the harvested energy of the ERs while achieving secrecy according to the individual signal-to-interference and noise ratio (SINR) constraints of the IR and the ERs. Also, we consider robust beamforming in the worst-case sense with imperfect CSI of both IR and ERs available at the transmitter.[1] Applying semidefinite relaxation (SDR) techniques, we show that there always exists a rank-one optimal solution.

II. System Model

Consider a MISO downlink system for SWIPT with $K+1$ receivers. The transmitter, or base station (BS), has $N_{\rm T}$ antennas and each receiver has only one antenna. One of the receivers is an IR while the rest are ERs. The BS performs transmit beamforming to send secret information to the IR. It is assumed that the BS trusts the ERs only to harvest energy. By letting $\mathbf{x}$ be the transmit signal vector, the received signals at the IR and the kth ER can be modeled, respectively, as

$$y_{\rm I} = \mathbf{h}_{\rm I}^H \mathbf{x} + n_{\rm I}, \tag{1}$$

$$y_{{\rm E},k} = \mathbf{h}_{{\rm E},k}^H \mathbf{x} + n_{{\rm E},k}, \text{ for } k = 1, \ldots, K, \tag{2}$$

where $\mathbf{h}_{\rm I}$ and $\mathbf{h}_{{\rm E},k}$ are the conjugated complex channel vectors between the BS and the IR and between the BS and the kth ER, respectively, $n_{\rm I} \sim \mathcal{CN}(0, \sigma_{\rm I}^2)$ and $n_{{\rm E},k} \sim \mathcal{CN}(0, \sigma_{\rm E}^2)$ are the additive Gaussian noises at the IR and the kth ER, respectively. The BS chooses $\mathbf{x}$ as the sum of information beamforming vector $\mathbf{b}_{\rm I} s_{\rm I}$ and the energy-carrying artificial noise (AN) vector $\mathbf{b}_{\rm E}$ such that the baseband transmit signal vector is

$$\mathbf{x} = \mathbf{b}_{\rm I} s_{\rm I} + \mathbf{b}_{\rm E}, \tag{3}$$

where $s_{\rm I} \sim \mathcal{CN}(0, 1)$ is the confidential information-bearing signal for the IR and $\mathbf{b}_{\rm E} = \sum_{i=1}^{d} \mathbf{b}_{{\rm E},i} s_{{\rm E},i}$ is the sum of d energy beams, in which $\mathbf{b}_{{\rm E},i}$ and $s_{{\rm E},i} \sim \mathcal{CN}(0, 1)$ denote the ith energy beamforming vector and the ith energy-carrying noise signal, respectively. By denoting $\mathbf{Q}_{\rm I} \triangleq \mathbf{b}_{\rm I} \mathbf{b}_{\rm I}^H$ as the transmit

[1] A similar problem formulation can be found in [11] but with perfect CSI.

covariance and $\mathbf{Q}_{\mathrm{E}} \triangleq \sum_{i=1}^{d} \mathbf{b}_{\mathrm{E},i}\mathbf{b}_{\mathrm{E},i}^{H}$ as the energy covariance, the SINR at the IR is given by

$$\Gamma_{\mathrm{I}} = \frac{\mathbf{h}_{\mathrm{I}}^{H}\mathbf{Q}_{\mathrm{I}}\mathbf{h}_{\mathrm{I}}}{\mathbf{h}_{\mathrm{I}}^{H}\mathbf{Q}_{\mathrm{E}}\mathbf{h}_{\mathrm{I}} + \sigma_{\mathrm{I}}^{2}}, \tag{4}$$

and that at the kth ER (also referred to as Eve) is given by

$$\Gamma_{\mathrm{E},k} = \frac{\mathbf{h}_{\mathrm{E},k}^{H}\mathbf{Q}_{\mathrm{I}}\mathbf{h}_{\mathrm{E},k}}{\mathbf{h}_{\mathrm{E},k}^{H}\mathbf{Q}_{\mathrm{E}}\mathbf{h}_{\mathrm{E},k} + \sigma_{\mathrm{E}}^{2}}, \text{ for } k = 1, \ldots, K. \tag{5}$$

The power harvested by the kth ER is given by

$$\Upsilon_k = \xi_k \left(\mathbf{h}_{\mathrm{E},k}^{H}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}})\mathbf{h}_{\mathrm{E},k}\right), \text{ for } k = 1, \ldots, K, \tag{6}$$

in which $\xi_k \in (0, 1]$ is the energy conversion efficiency of the kth ER, which we assume that $\xi_k = 1, \ \forall\, k$, in this letter.

III. Secrecy Beamforming With Perfect CSI

Assuming perfect CSI at the BS, the objective is the max-min fairness harvested energy beamforming design such that the received SINR at the IR is above a given threshold and that at the ERs is below a certain threshold under the total transmit power constraint. Hence, the problem is written as

$$\begin{aligned}
\max_{\mathbf{Q}_{\mathrm{I}},\mathbf{Q}_{\mathrm{E}}\succeq\mathbf{0}} \quad & \min_{k} \ \mathbf{h}_{\mathrm{E},k}^{H}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}})\mathbf{h}_{\mathrm{E},k}, && \text{(7a)}\\
\text{s.t.} \quad & \frac{\mathbf{h}_{\mathrm{I}}^{H}\mathbf{Q}_{\mathrm{I}}\mathbf{h}_{\mathrm{I}}}{\mathbf{h}_{\mathrm{I}}^{H}\mathbf{Q}_{\mathrm{E}}\mathbf{h}_{\mathrm{I}} + \sigma_{\mathrm{I}}^{2}} \geq \gamma, && \text{(7b)}\\
& \frac{\mathbf{h}_{\mathrm{E},k}^{H}\mathbf{Q}_{\mathrm{I}}\mathbf{h}_{\mathrm{E},k}}{\mathbf{h}_{\mathrm{E},k}^{H}\mathbf{Q}_{\mathrm{E}}\mathbf{h}_{\mathrm{E},k} + \sigma_{\mathrm{E}}^{2}} \leq \eta_k, \ \forall\, k, && \text{(7c)}\\
& \mathrm{tr}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}}) \leq P_{\mathrm{T}}, && \text{(7d)}\\
& \mathrm{rank}(\mathbf{Q}_{\mathrm{I}}) \leq 1. && \text{(7e)}
\end{aligned}$$

Here $\gamma > 0$ and $\eta_k > 0$ are the protection ratios of the IR and kth ER, respectively, and P_{T} is the available power budget at the BS. Moreover, $\mathrm{tr}(\cdot)$ denotes the trace of a matrix and $\mathbf{A} \succeq \mathbf{0}$ indicates that the matrix $\mathbf{A}$ is positive semi-definite (PSD). Throughout the rest of the letter we consider scenarios for which (7) is feasible. Clearly, the problem is non-convex due to the rank constraint (7e). However, by dropping (7e) and introducing a real-valued slack variable t, (7) becomes

$$\begin{aligned}
\max_{\mathbf{Q}_{\mathrm{I}},\mathbf{Q}_{\mathrm{E}}\succeq\mathbf{0},t\geq 0} \quad & t && \text{(8a)}\\
\text{s.t.} \quad & \frac{\mathbf{h}_{\mathrm{E},k}^{H}\mathbf{Q}_{\mathrm{I}}\mathbf{h}_{\mathrm{E},k}}{\mathbf{h}_{\mathrm{E},k}^{H}\mathbf{Q}_{\mathrm{E}}\mathbf{h}_{\mathrm{E},k} + \sigma_{\mathrm{E}}^{2}} \leq \eta_k, \ \forall\, k, && \text{(8b)}\\
& \frac{\mathbf{h}_{\mathrm{I}}^{H}\mathbf{Q}_{\mathrm{I}}\mathbf{h}_{\mathrm{I}}}{\mathbf{h}_{\mathrm{I}}^{H}\mathbf{Q}_{\mathrm{E}}\mathbf{h}_{\mathrm{I}} + \sigma_{\mathrm{I}}^{2}} \geq \gamma, && \text{(8c)}\\
& \mathbf{h}_{\mathrm{E},k}^{H}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}})\mathbf{h}_{\mathrm{E},k} \geq t, && \text{(8d)}\\
& \mathrm{tr}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}}) \leq P_{\mathrm{T}}. && \text{(8e)}
\end{aligned}$$

Now, (8) can be formulated as a semidefinite program (SDP) and efficiently solved by the existing solvers, e.g., CVX [12]. Note that the optimal objective value (t^*) of (7) indicates the minimum power harvested by any individual ER.

Due to the relaxation, the optimal $\mathbf{Q}_{\mathrm{I}}^{*}$ from (8) may not be rank one. However, we will prove that a rank-one optimal $\mathbf{Q}_{\mathrm{I}}$ can always be constructed algorithmically in the next section.

IV. Robust Secrecy Beamforming

The assumption of perfect CSI in Section III is not always practical due to the time-varying nature of wireless communications. In this section, we develop a robust algorithm considering the worst-case design. In particular, we assume that the actual channels $\mathbf{h}_{\mathrm{I}}$ and $\mathbf{h}_{\mathrm{E},k}$ lie in the neighborhood of the estimated channels $\hat{\mathbf{h}}_{\mathrm{I}}$ and $\hat{\mathbf{h}}_{\mathrm{E},k}$, respectively, available at the BS. Hence, the actual channels are modeled as

$$\mathbf{h}_{\mathrm{I}} = \hat{\mathbf{h}}_{\mathrm{I}} + \boldsymbol{\delta}_{\mathrm{I}}, \tag{9}$$

$$\mathbf{h}_{\mathrm{E},k} = \hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_{k}, \ \forall\, k, \tag{10}$$

in which $\boldsymbol{\delta}_{\mathrm{I}}$, and $\boldsymbol{\delta}_{k}, \forall k$, represent the channel uncertainties, which are assumed to be bounded such that

$$\|\boldsymbol{\delta}_{\mathrm{I}}\|_2 = \left\|\mathbf{h}_{\mathrm{I}} - \hat{\mathbf{h}}_{\mathrm{I}}\right\|_2 \leq \varepsilon_{\mathrm{I}}, \text{ for some } \varepsilon_{\mathrm{I}} \geq 0, \tag{11}$$

$$\|\boldsymbol{\delta}_{k}\|_2 = \left\|\mathbf{h}_{\mathrm{E},k} - \hat{\mathbf{h}}_{\mathrm{E},k}\right\|_2 \leq \varepsilon_{k}, \text{ for some } \varepsilon_{k} \geq 0. \tag{12}$$

As such, the robust formulation of (8) becomes

$$\begin{aligned}
& \max_{\substack{\mathbf{Q}_{\mathrm{I}},\mathbf{Q}_{\mathrm{E}}\succeq\mathbf{0}\\ t\geq 0}} \ t \quad \text{s.t.} && \text{(13a)}\\
& \min_{\|\boldsymbol{\delta}_{\mathrm{I}}\|\leq\varepsilon_{\mathrm{I}}} \frac{(\hat{\mathbf{h}}_{\mathrm{I}} + \boldsymbol{\delta}_{\mathrm{I}})^{H}\mathbf{Q}_{\mathrm{I}}(\hat{\mathbf{h}}_{\mathrm{I}} + \boldsymbol{\delta}_{\mathrm{I}})}{(\hat{\mathbf{h}}_{\mathrm{I}} + \boldsymbol{\delta}_{\mathrm{I}})^{H}\mathbf{Q}_{\mathrm{E}}(\hat{\mathbf{h}}_{\mathrm{I}} + \boldsymbol{\delta}_{\mathrm{I}}) + \sigma_{\mathrm{I}}^{2}} \geq \gamma, && \text{(13b)}\\
& \max_{\|\boldsymbol{\delta}_{k}\|\leq\varepsilon_{k}} \frac{(\hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_{k})^{H}\mathbf{Q}_{\mathrm{I}}(\hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_{k})}{(\hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_{k})^{H}\mathbf{Q}_{\mathrm{E}}(\hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_{k}) + \sigma_{\mathrm{E}}^{2}} \leq \eta_k, && \text{(13c)}\\
& \min_{\|\boldsymbol{\delta}_{k}\|\leq\varepsilon_{k}} (\hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_{k})^{H}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}})(\hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_{k}) \geq t, && \text{(13d)}\\
& \mathrm{tr}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}}) \leq P_{\mathrm{T}}. && \text{(13e)}
\end{aligned}$$

In (13b)–(13d), there are infinitely many inequalities which make the worst-case design particularly challenging.

To make (13) more tractable, we transform the constraints into linear matrix inequalities (LMIs) by applying $\mathcal{S}$-procedure given in [13]. According to $\mathcal{S}$-procedure, if there exists $\mu_{\mathrm{I}} \geq 0, \{\mu_{\mathrm{E},k}, \mu_k\} \geq 0, \forall\, k$, we can transform (13b)–(13d) into (14), shown at the bottom of the page. Thus, we have

$$\begin{aligned}
\max_{\substack{\mathbf{Q}_{\mathrm{I}},\mathbf{Q}_{\mathrm{E}}\succeq\mathbf{0}\\ t,\{\mu_{\mathrm{I}}\},\{\mu_{\mathrm{E},k}\},\{\mu_k\}}} \quad & t && \text{(15a)}\\
\text{s.t.} \ & \bar{\boldsymbol{\Gamma}}_{\mathrm{I}}(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, \mu_{\mathrm{I}}) \succeq \mathbf{0}, && \text{(15b)}\\
& \bar{\boldsymbol{\Gamma}}_{\mathrm{E},k}(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, \mu_{\mathrm{E},k}) \succeq \mathbf{0}, \ \forall k, && \text{(15c)}
\end{aligned}$$

$$\bar{\boldsymbol{\Gamma}}_{\mathrm{I}}(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, \mu_{\mathrm{I}}) \triangleq \begin{bmatrix} \mu_{\mathrm{I}}\mathbf{I}_{N_{\mathrm{T}}} + \mathbf{Q}_{\mathrm{I}} - \gamma\mathbf{Q}_{\mathrm{E}} & (\mathbf{Q}_{\mathrm{I}} - \gamma\mathbf{Q}_{\mathrm{E}})\hat{\mathbf{h}}_{\mathrm{I}} \\ \hat{\mathbf{h}}_{\mathrm{I}}^{H}(\mathbf{Q}_{\mathrm{I}} - \gamma\mathbf{Q}_{\mathrm{E}}) & \hat{\mathbf{h}}_{\mathrm{I}}^{H}(\mathbf{Q}_{\mathrm{I}} - \gamma\mathbf{Q}_{\mathrm{E}})\hat{\mathbf{h}}_{\mathrm{I}} - \gamma\sigma_{\mathrm{I}}^{2} - \mu_{\mathrm{I}}\varepsilon_{\mathrm{I}}^{2} \end{bmatrix} \succeq \mathbf{0}, \tag{14a}$$

$$\bar{\boldsymbol{\Gamma}}_{\mathrm{E},k}(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, \mu_{\mathrm{E},k}) \triangleq \begin{bmatrix} \mu_{\mathrm{E},k}\mathbf{I}_{N_{\mathrm{T}}} + \eta_k\mathbf{Q}_{\mathrm{E}} - \mathbf{Q}_{\mathrm{I}} & (\eta_k\mathbf{Q}_{\mathrm{E}} - \mathbf{Q}_{\mathrm{I}})\hat{\mathbf{h}}_{\mathrm{E},k} \\ \hat{\mathbf{h}}_{\mathrm{E},k}^{H}(\eta_k\mathbf{Q}_{\mathrm{E}} - \mathbf{Q}_{\mathrm{I}}) & \hat{\mathbf{h}}_{\mathrm{E},k}^{H}(\eta_k\mathbf{Q}_{\mathrm{E}} - \mathbf{Q}_{\mathrm{I}})\hat{\mathbf{h}}_{\mathrm{E},k} + \eta_k\sigma_{\mathrm{E}}^{2} - \mu_{\mathrm{E},k}\varepsilon_{k}^{2} \end{bmatrix} \succeq \mathbf{0}, \tag{14b}$$

$$\bar{\boldsymbol{\Upsilon}}_{k}(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, t, \mu_{k}) \triangleq \begin{bmatrix} \mu_{k}\mathbf{I}_{N_{\mathrm{T}}} + \mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}} & (\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}})\hat{\mathbf{h}}_{\mathrm{E},k} \\ \hat{\mathbf{h}}_{\mathrm{E},k}^{H}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}}) & \hat{\mathbf{h}}_{\mathrm{E},k}^{H}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}})\hat{\mathbf{h}}_{\mathrm{E},k} - t - \mu_{k}\varepsilon_{k}^{2} \end{bmatrix} \succeq \mathbf{0}, \tag{14c}$$

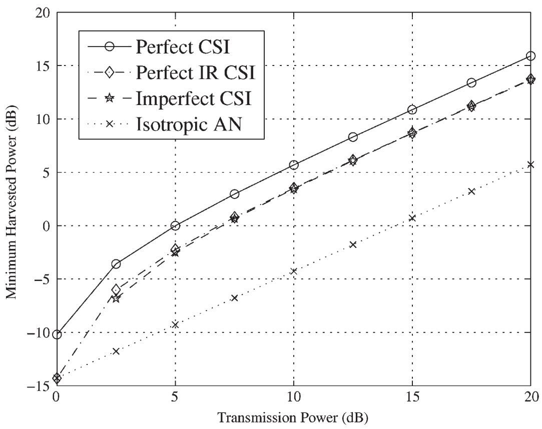

Fig. 1. Harvested power versus total transmit power P_{T} (dB) with $N_{\mathrm{T}} = 10$, $K = 3$, $\gamma = 0$ (dB), and $\eta = -5$ (dB).

$$\bar{\boldsymbol{\Upsilon}}_k(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, t, \mu_k) \succeq \mathbf{0}, \ \forall k, \quad \text{(15d)}$$
$$\mathrm{tr}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}}) \le P_{\mathrm{T}}. \quad \text{(15e)}$$

Let t^* be the optimal objective value of (15). It is evident that the following power minimization problem yields the same optimal solution $(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}})$ as that of (15) [8]:

$$\min_{\substack{\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}} \succeq \mathbf{0} \\ \{\mu_{\mathrm{I}}\}, \{\mu_{\mathrm{E},k}\}, \{\mu_k\}}} \ \mathrm{tr}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}}) \quad \text{(16a)}$$
$$\text{s.t.} \ \bar{\boldsymbol{\Gamma}}_{\mathrm{I}}(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, \mu_{\mathrm{I}}) \succeq \mathbf{0}, \quad \text{(16b)}$$
$$\bar{\boldsymbol{\Gamma}}_{\mathrm{E},k}(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, \mu_{\mathrm{E},k}) \succeq \mathbf{0}, \ \forall k, \quad \text{(16c)}$$
$$\bar{\boldsymbol{\Upsilon}}_k(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, t^*, \mu_k) \succeq \mathbf{0}, \ \forall k. \quad \text{(16d)}$$

The reason for considering the alternative problem formulation in (16) is described in the following theorem.

Theorem 1: Suppose that the SDP problem (16) is feasible for $t^* > 0$. There always exists an optimal solution $(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}})$ to the problem (16) such that $\mathrm{rank}(\mathbf{Q}_{\mathrm{I}}) = 1$.

Proof: See Appendix A. ■

Thus, if $\mathrm{rank}(\mathbf{Q}_{\mathrm{I}}) > 1$ in the optimal solution of (15), then we can alternatively solve (16) to obtain a rank-one transmit covariance matrix $\mathbf{Q}_{\mathrm{I}}$ as the solution to the original problem (7), achieving the same harvested energy. Similarly, a rank-one solution can be obtained for problem (8) as well.

V. Simulation Results

Here, we evaluate the performance of the proposed MISO SWIPT system for Rayleigh flat-fading environments where the channel vectors have entries with variance $1/N_{\mathrm{T}}$. For the case of imperfect CSI, the error vectors were uniformly and randomly generated in a sphere centered at zero with the radius $\varepsilon_{\mathrm{I}} = \varepsilon_k = \varepsilon = 0.2, \forall k$, unless explicitly mentioned. For simplicity, we assume $\eta_k = \eta, \forall k$. All simulation results were averaged over 500 independent channel realizations.

Fig. 1 studies the performance of the proposed algorithms compared to some baseline schemes assuming that the channel gains contain both the path loss and Rayleigh fading factors. We compare the scenarios where only the IR's CSI is perfectly known (ERs' CSI is imperfect) and where the ERs are passive eavesdroppers and their CSI is not available at the BS. For the latter scheme, we use isotropic beamforming to get the energy covariance matrix $\mathbf{Q}_{\mathrm{E}}$ such that the beamforming vector lies in the null space of $\mathbf{h}_{\mathrm{I}}$ [7] and adopts maximum ration transmission (MRT) for delivering the information signal. Results illustrate that the more CSI available at the transmitter, the more efficient the beamforming schemes and the higher power harvested. Note that at $P_{\mathrm{T}} = 10$ (dB), the proposed schemes achieve at least 8 (dB) gain with respect to the conventional isotropic beamforming scheme in terms of harvested power.

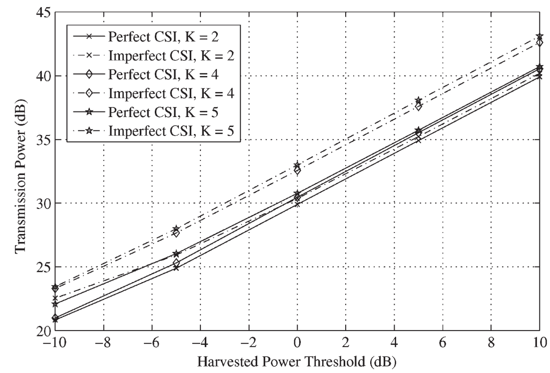

Fig. 2. Total transmission power versus harvested power threshold for the case with $N_{\mathrm{T}} = 10$, $K = 2, 4, 5$, $\gamma = 0$ (dB), and $\eta = -5$ (dB).

Results in Fig. 2 show the minimum transmit power required for given harvested power constraints as well as satisfying the SINR constraints for $N_{\mathrm{T}} = 10$, $K = 2, 4, 5$, $\gamma = 0$ (dB), and $\eta = -5$ (dB), assuming path loss exponent of 2.7 which corresponds to an urban cellular network environment. As can be seen from Fig. 2, with the increasing number of ERs, the required total transmit power increases.

VI. Conclusion

This letter studied secret communication in MISO systems for SWIPT and proposed joint design of transmit and energy beamforming algorithms for both perfect and imperfect CSI cases, utilizing SDR techniques. We showed that the relaxation is tight via solving an equivalent power minimization problem. Generalization to the scenario of multiple IRs with multiple antennas as well as deriving the outage probability of the non-robust design can be interesting future works.

Appendix

A. Proof of Theorem 1

We here prove the existence of a rank-one $\mathbf{Q}_{\mathrm{I}}$ based on the Karush–Kuhn–Tucker (KKT) conditions of (16). The Lagrangian of (16) can be expressed as

$$\mathcal{L} \triangleq \mathrm{tr}(\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}}) - \mathrm{tr}\left(\boldsymbol{\Psi}_{\mathrm{I}} \bar{\boldsymbol{\Gamma}}_{\mathrm{I}}(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, \mu_{\mathrm{I}})\right) - \sum_{k=1}^{K} \mathrm{tr}\left(\boldsymbol{\Psi}_{\mathrm{E},k} \bar{\boldsymbol{\Gamma}}_{\mathrm{E},k}(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, \mu_{\mathrm{E},k})\right) - \sum_{k=1}^{K} \mathrm{tr}\left(\boldsymbol{\Phi}_k \bar{\boldsymbol{\Upsilon}}_k(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}}, t^*, \mu_k)\right) - \mathbf{A}_{\mathrm{I}} \mathbf{Q}_{\mathrm{I}} - \mathbf{A}_{\mathrm{E}} \mathbf{Q}_{\mathrm{E}}, \quad (17)$$

where $\boldsymbol{\Psi}_{\mathrm{I}} \succeq \mathbf{0}, \boldsymbol{\Psi}_{\mathrm{E},k} \succeq \mathbf{0}, \forall k, \boldsymbol{\Phi}_k \succeq \mathbf{0}, \forall k$, are the dual variables associated with the constraints (16b)–(16d), respectively, $\mathbf{A}_{\mathrm{I}} \succeq \mathbf{0}$ and $\mathbf{A}_{\mathrm{E}} \succeq \mathbf{0}$ are associated with $\mathbf{Q}_{\mathrm{I}}$ and $\mathbf{Q}_{\mathrm{E}}$, respectively. Now, rewrite $\bar{\boldsymbol{\Gamma}}_{\mathrm{I}}$, $\bar{\boldsymbol{\Gamma}}_{\mathrm{E},k}$, and $\bar{\boldsymbol{\Upsilon}}_k$ as

$$\bar{\boldsymbol{\Gamma}}_{\mathrm{I}} = \boldsymbol{\Lambda}_{\mathrm{I}}(\mu_{\mathrm{I}}) + \bar{\mathbf{H}}_{\mathrm{I}}^H (\mathbf{Q}_{\mathrm{I}} - \gamma \mathbf{Q}_{\mathrm{E}}) \bar{\mathbf{H}}_{\mathrm{I}}, \quad \text{(18a)}$$
$$\bar{\boldsymbol{\Gamma}}_{\mathrm{E},k} = \boldsymbol{\Lambda}_{\mathrm{E},k}(\mu_{\mathrm{E},k}) - \bar{\mathbf{H}}_{\mathrm{E},k}^H (\eta_k \mathbf{Q}_{\mathrm{E}} - \mathbf{Q}_{\mathrm{I}}) \bar{\mathbf{H}}_{\mathrm{E},k}, \quad \text{(18b)}$$
$$\bar{\boldsymbol{\Upsilon}}_k = \boldsymbol{\Sigma}_{\mathrm{E},k}(t^*, \mu_k) + \bar{\mathbf{H}}_{\mathrm{E},k}^H (\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}}) \bar{\mathbf{H}}_{\mathrm{E},k}, \quad \text{(18c)}$$

where

$$\boldsymbol{\Lambda}_{\rm I}(\mu_{\rm I}) \triangleq \begin{bmatrix} \mu_{\rm I}\mathbf{I}_{N_{\rm T}} & \mathbf{0} \\ \mathbf{0} & -\gamma\sigma_{\rm I}^2 - \mu_{\rm I}\varepsilon_{\rm I}^2 \end{bmatrix},$$

$$\bar{\mathbf{H}}_{\rm I} \triangleq \left[\mathbf{I}_{N_{\rm T}}\ \hat{\mathbf{h}}_{\rm I}\right], \bar{\mathbf{H}}_{{\rm E},k} \triangleq \left[\mathbf{I}_{N_{\rm T}}\ \hat{\mathbf{h}}_{{\rm E},k}\right],$$

$$\boldsymbol{\Lambda}_{{\rm E},k}(\mu_{{\rm E},k}) \triangleq \begin{bmatrix} \mu_{{\rm E},k}\mathbf{I}_{N_{\rm T}} & \mathbf{0} \\ \mathbf{0} & \eta_k\sigma_{\rm E}^2 - \mu_{{\rm E},k}\varepsilon_k^2 \end{bmatrix},$$

$$\boldsymbol{\Sigma}_{{\rm E},k}(t^*, \mu_k) \triangleq \begin{bmatrix} \mu_k\mathbf{I}_{N_{\rm T}} & \mathbf{0} \\ \mathbf{0} & -t - \mu_k\varepsilon_k^2 \end{bmatrix}.$$

The relevant KKT conditions can be defined as

$$\nabla_{\mathbf{Q}_{\rm I}}\mathcal{L} = \mathbf{0}, \tag{19a}$$

$$\bar{\boldsymbol{\Gamma}}_{\rm I}\left(\mathbf{Q}_{\rm I}, \mathbf{Q}_{\rm E}, \mu_{\rm I}\right)\boldsymbol{\Psi}_{\rm I} = \mathbf{0}, \tag{19b}$$

$$\mathbf{Q}_{\rm I}\mathbf{A}_{\rm I} = \mathbf{0}, \tag{19c}$$

$$\mathbf{A}_{\rm I} \succeq \mathbf{0}, \boldsymbol{\Psi}_{\rm I} \succeq \mathbf{0}, \boldsymbol{\Psi}_{{\rm E},k} \succeq \mathbf{0}, \forall\, k, \boldsymbol{\Phi}_k \succeq \mathbf{0},\ \forall\, k. \tag{19d}$$

Using (18), the KKT condition (19a) can be expressed as

$$\mathbf{I}_{N_{\rm T}} - \bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H + \sum_{k=1}^{K}\bar{\mathbf{H}}_{{\rm E},k}(\boldsymbol{\Psi}_{{\rm E},k} - \boldsymbol{\Phi}_k)\bar{\mathbf{H}}_{{\rm E},k}^H = \mathbf{A}_{\rm I}. \tag{20}$$

Let $\mathbf{B}$ denote

$$\mathbf{B} \triangleq \mathbf{I}_{N_{\rm T}} + \sum_{k=1}^{K}\bar{\mathbf{H}}_{{\rm E},k}\boldsymbol{\Psi}_{{\rm E},k}\bar{\mathbf{H}}_{{\rm E},k}^H - \sum_{k=1}^{K}\bar{\mathbf{H}}_{{\rm E},k}\boldsymbol{\Phi}_k\bar{\mathbf{H}}_{{\rm E},k}^H, \tag{21}$$

and let $r_{\rm B} \triangleq {\rm rank}(\mathbf{B})$ denote the rank of $\mathbf{B}$. Thus we can express $\mathbf{A}_{\rm I}$ in terms of $\mathbf{B}$ as

$$\mathbf{A}_{\rm I} \triangleq \mathbf{B} - \bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H. \tag{22}$$

Next, we show that ${\rm rank}(\bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H) \leq 1$. Substituting (18a) into the KKT condition (19b), we obtain

$$\boldsymbol{\Lambda}_{\rm I}(\mu_{\rm I})\boldsymbol{\Psi}_{\rm I} + \bar{\mathbf{H}}_{\rm I}^H(\mathbf{Q}_{\rm I} - \gamma\mathbf{Q}_{\rm E})\bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I} = \mathbf{0}. \tag{23}$$

Also, it follows by post-multiplying (23) by $\bar{\mathbf{H}}_{\rm I}^H$ that

$$\boldsymbol{\Lambda}_{\rm I}(\mu_{\rm I})\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H + \bar{\mathbf{H}}_{\rm I}^H(\mathbf{Q}_{\rm I} - \gamma\mathbf{Q}_{\rm E})\bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H = \mathbf{0}. \tag{24}$$

By noting the following facts: $[\mathbf{I}_{N_{\rm T}}\mathbf{0}]\bar{\mathbf{H}}_{\rm I}^H = \mathbf{I}_{N_{\rm T}}$,

$$\left[\mathbf{I}_{N_{\rm T}}\ \mathbf{0}\right]\boldsymbol{\Lambda}_{\rm I}(\mu_{\rm I}) = \mu_{\rm I}\left[\mathbf{I}_{N_{\rm T}}\ \mathbf{0}\right] = \mu_{\rm I}\left(\bar{\mathbf{H}}_{\rm I} - [\mathbf{0}_{N_{\rm T}}\ \hat{\mathbf{h}}_{\rm I}]\right),$$

we premultiply both sides of (24) by $[\mathbf{I}_{N_{\rm T}}\ \mathbf{0}]$ to get

$$\left(\mu_{\rm I}\mathbf{I}_{N_{\rm T}} + \mathbf{Q}_{\rm I} - \gamma\mathbf{Q}_{\rm E}\right)\bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H = \mu_{\rm I}\left[\mathbf{0}_{N_{\rm T}}\ \hat{\mathbf{h}}_{\rm I}\right]\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H. \tag{25}$$

Lemma 1: If a Hermitian matrix $\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \succeq \mathbf{0}$, then it immediately follows that $\mathbf{M}_{11}$ and $\mathbf{M}_{22}$ must be PSD matrices [14].

Thus we claim from (14) that $\mu_{\rm I}\mathbf{I}_{N_{\rm T}} + \mathbf{Q}_{\rm I} - \gamma\mathbf{Q}_{\rm E} \succeq \mathbf{0}$. In addition, $\mu_{\rm I}\mathbf{I}_{N_{\rm T}} + \mathbf{Q}_{\rm I} - \gamma\mathbf{Q}_{\rm E}$ is nonsingular. Since multiplying (left/right) by a nonsingular matrix (of appropriate dimension) does not change the matrix rank [14], we have from (25) that

$$\begin{aligned} {\rm rank}\left(\bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H\right) &= {\rm rank}\left(\left(\mu_{\rm I}\mathbf{I}_{N_{\rm T}} + \mathbf{Q}_{\rm I} - \gamma\mathbf{Q}_{\rm E}\right)\bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H\right) \\ &= {\rm rank}\left(\mu_{\rm I}\left[\mathbf{0}_{N_{\rm T}}\hat{\mathbf{h}}_{\rm I}\right]\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H\right) \\ &\leq {\rm rank}\left(\left[\mathbf{0}_{N_{\rm T}}\ \hat{\mathbf{h}}_{\rm I}\right]\right) \leq 1 \end{aligned} \tag{26}$$

where (26) follows from a basic rank inequality property [14].

Lemma 2: Let $\mathbf{X}$ and $\mathbf{Y}$ be two matrices of same size. Then it holds true that ${\rm rank}(\mathbf{X} - \mathbf{Y}) \geq {\rm rank}(\mathbf{X}) - {\rm rank}(\mathbf{Y})$.

Proof: Clearly, it is known that ${\rm rank}(\mathbf{X}) + {\rm rank}(\mathbf{Y}) \geq {\rm rank}(\mathbf{X} + \mathbf{Y})$. Thus, we have ${\rm rank}(\mathbf{X} + \mathbf{Y}) + {\rm rank}(-\mathbf{Y}) \geq {\rm rank}(\mathbf{X})$. Since ${\rm rank}(\mathbf{Y}) = {\rm rank}(-\mathbf{Y})$, we can conclude that ${\rm rank}(\mathbf{X} + \mathbf{Y}) \geq {\rm rank}(\mathbf{X}) - {\rm rank}(\mathbf{Y})$. Since ${\rm rank}(\mathbf{X} - \mathbf{Y}) = {\rm rank}(\mathbf{X} + \mathbf{Y})$, *Lemma 2* is thus proved. ■

Using *Lemma 2*, we have from (22) that

$${\rm rank}(\mathbf{A}_{\rm I}) \geq r_{\rm B} - 1. \tag{27}$$

If $\mathbf{B}$ in (21) is positive-definite, $r_{\rm B} = N_{\rm T}$ and ${\rm rank}(\mathbf{A}_{\rm I}) \geq N_{\rm T} - 1$. However, if ${\rm rank}(\mathbf{A}_{\rm I}) = N_{\rm T}$, i.e., $\mathbf{A}_{\rm I}$ is of full-rank, then it follows from (19c) that $\mathbf{Q}_{\rm I} = \mathbf{0}$, which cannot be an optimal solution to (16). Therefore, we have ${\rm rank}(\mathbf{A}_{\rm I}) = N_{\rm T} - 1$. According to (19c), we have ${\rm rank}(\mathbf{Q}_{\rm I}) = 1$. That is, $\mathbf{Q}_{\rm I} = \nu\boldsymbol{\varphi}\boldsymbol{\varphi}^H$ such that $\boldsymbol{\varphi}$ spans the null space of $\mathbf{A}_{\rm I}$ and $\nu > 0$. Now the key is to show that $\mathbf{B} \succ \mathbf{0}$. Hence the remaining task is to prove that $\mathbf{B}$ is a positive-definite matrix. Since $\mathbf{A}_{\rm I} \succeq \mathbf{0}$ and $-\bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H \preceq \mathbf{0}$ for $\boldsymbol{\Psi}_{\rm I} \succeq \mathbf{0}$, $\mathbf{B} \succeq \mathbf{0}$. Next, we prove that $\mathbf{B} \succ \mathbf{0}$ must always hold by contradiction. Suppose the minimum eigenvalue of $\mathbf{B}$ is zero. Then, there exists at least a vector $\mathbf{z} \neq \mathbf{0}$ such that $\mathbf{z}^H\mathbf{B}\mathbf{z} = 0$. According to (22), it follows that

$$\mathbf{z}^H\mathbf{A}_{\rm I}\mathbf{z} = -\mathbf{z}^H\left(\bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}\bar{\mathbf{H}}_{\rm I}^H\right)\mathbf{z} = -\left|\mathbf{z}^H\bar{\mathbf{H}}_{\rm I}\boldsymbol{\Psi}_{\rm I}^{\frac{1}{2}}\right|^2 < 0. \tag{28}$$

This means that $\mathbf{A}_{\rm I}$ is not PSD and violates the KKT condition in (19d). Hence, we conclude that $\mathbf{B} \succ \mathbf{0}$ must hold. However, if $\mu_{\rm I} = 0$, one may follow the procedure in [11] in order to obtain a rank-one solution. □

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