Competition in Two-Sided Markets*

Mark Armstrong
Department of Economics
University College London

August 2002: revised May 2005

Abstract

There are many examples of markets involving two groups of agents who need to interact via "platforms", and where one group's benefit from joining a platform depends on the number of agents from the other group who join the same platform. This paper presents theoretical models for three variants of such markets: a monopoly platform; a model of competing platforms where each agent must choose to join a single platform; and a model of "competing bottlenecks", where one group wishes to join all platforms. The main determinants of equilibrium prices are (i) the relative sizes of the cross-group externalities, (ii) whether fees are levied on a lump-sum or per-transaction basis, and (iii) whether a group joins just one platform or joins all platforms.

1 Introduction and Summary

There are many examples of markets where two or more groups of agents interact via intermediaries or "platforms". Surplus is created—or perhaps destroyed in the case of negative externalities—when the groups interact. Of course, there are countless examples where firms compete to deal with two or more groups. Any firm is likely to do better if its products appeal to both men and women, for instance. However, in a set of interesting cases, cross-group network effects are present, and the benefit enjoyed by a member of one group depends upon how well the platform does in attracting custom from the other group. For instance, a heterosexual dating agency or nightclub can *only* do well if it succeeds in attracting business from both men and women. This paper is about such markets.

*An early version of this paper was presented at the ESEM meeting in Venice, August 2002. I am grateful to the editor and two referees, to the audiences at many seminar presentations, to Simon Anderson, Carli Coetzee, Jacques Crémer, Xavier Vives and especially to Julian Wright for discussion, correction and information.

A brief list of other such markets includes: credit cards (for a given set of charges, a consumer is more likely to use a credit card which is accepted widely by retailers, while a retailer is more likely to accept a card which is carried by more consumers); television channels (where viewers generally prefer to watch a channel with fewer adverts, while an advertiser is prepared to pay more to place an advert on a channel with more viewers); and shopping malls (where a consumer is more likely to visit a mall with a greater range of retailers, while a retailer is willing to pay more to locate in a mall with a greater number of consumers passing through). See Rochet and Tirole (2003) for further examples of such markets.

As this paper will argue in more detail, there are three main factors that determine the pattern of relative prices offered to the two groups in equilibrium.

Relative sizes of cross-group externalities: If a member of group 1 exerts a large positive externality on each member of group 2, then group 1 will be targeted aggressively by platforms. In broad terms, and especially in competitive markets, it is group 1's benefit to the other group that determines group 1's price, not how much group 1 benefits from the presence of group 2 (see Proposition 2 below). In a nightclub, if men gain more from interacting with women than *vice versa* then we expect there to be a tendency for nightclubs to offer women lower entry fees than men.

Unless they act to tip the industry to monopoly, positive cross-group network externalities act to intensify competition and reduce platform profits—see expression (13) below. In order to be able to compete effectively on one side of the market, a platform needs to perform well on the other side (and *vice versa*). This creates a downward pressure on the prices to both sides compared to the case where no cross-group network effects exist. This implies that platforms would like to find ways to mitigate network effects. One method of doing this is discussed next.

Fixed fees or per-transaction charges: Platforms might charge for their services on a lump-sum basis, so that an agent's payment does not explicitly depend on how well the platform performs on the other side of the market. Alternatively, if it is technologically feasible, the payment might be an explicit function of the platform's performance on the other side. One example of this latter practice is where a TV channel or a newspaper makes its advertising charge an increasing function of the audience or readership it obtains. Similarly, a credit card network levies (most of) its charges on a per-transaction basis, while the bulk of a real estate agent's fees are levied only in the event of a sale. The crucial difference between the two charging bases is that cross-group externalities are weaker with per-transaction charges, since a fraction of the benefit of interacting with an extra agent on the other side is eroded by the extra payment incurred. If an agent has to pay a platform only in the event of a successful interaction, then that agent does not need to worry about how well that platform will do in its dealings with the other side. That is to say, to attract one side of the market, it is not so important that the platform first gets the other side "on board". Because network effects are lessened with per-transactions charges, it is plausible that platform profits are higher when this form of charging is used.[1] (See Propositions 3 and 5 for illustrations of this effect.) Finally, the distinction between the two forms of tariff only matters when there are competing platforms. When there is a monopoly platform (see section 3 below), it makes no difference to outcomes if tariffs are levied on a lump-sum or per-transaction basis.

Single-homing or multi-homing: When an agent chooses to use only one platform it has become common to say that the agent is "single-homing". When an agent uses several platforms he is said to "multi-home". It makes a significant difference to outcomes whether groups single-home or multi-home. In the broadest terms, there are three main cases to consider: (i) both groups single-home; (ii) one group single-homes while the other multi-homes, and (iii) both groups multi-home. If interacting with the other side is the primary reason for an agent to join a platform, then we might not expect case (iii) to be very common—if each member of group 2 joins all platforms, there is no need for any member of group 1 to join more than one platform—and so this configuration is not analyzed in the paper. Configuration (i) is discussed in section 4. While the analysis of this case provides many useful insights about two-sided markets, it is hard to think of many markets that fit this configuration precisely.

By contrast, there are several important markets that look like configuration (iii), and in section 5 these cases are termed "competitive bottlenecks". Here, if it wishes to interact with an agent on the single-homing side, the multi-homing side has no choice except to deal with that agent's chosen platform. Thus, platforms have monopoly power over providing access to their single-homing customers for the multi-homing side. This monopoly power naturally leads to high prices being charged to the multi-homing side, and there will be too few agents on this side being served from a social point of view (Proposition 4).[2] By contrast, platforms do have to compete for the single-homing agents, and high profits generated from the multi-homing side are to a large extent passed on to the single-homing side in the form of low prices (or even zero prices).

Before embarking on this analysis in more detail, in the next section there is a selective literature review, followed in section 3 by an analysis of the monopoly platform case.

[1] An exception to this occurs when the market tips to monopoly. In that case the incumbent's profits typically *increase* with the size of the network effects since entrants find it hard to gain a toehold even when the incumbent sets high prices. This partly explains one conclusion of Caillaud and Jullien (2003), which is that equilibrium profits typically rise when platforms cannot use transaction charges.

[2] This tendency towards high prices for the multi-homing side is tempered somewhat when the single-homing side benefits from having many agents from the other side on the platform, for then high prices to the multi-homing side will drive away that side and thus disadvantage the platform when it tries to attract the single-homing side. However, this point is never sufficient to undermine the basic result that the price charged to the multi-homing side is too high.

2 Related Literature

I discuss some of the related literature later, as it becomes most relevant in the paper (especially in section 5 below). However, it is useful to discuss two pioneering papers up front.

Caillaud and Jullien (2003) discuss the case of competing matchmakers, such as dating agencies, real estate agents, and internet "business-to-business" websites.[3] There is potentially a rich set of contracting possibilities. For instance, a platform might have a subscription charge in combination with a charge in the event of a successful match. In addition, Caillaud and Jullien allow platforms to set negative subscription charges, and to make their profit from taxing transactions on the platform. Caillaud and Jullien first examine the case where all agents must single-home. (I provide a parallel analysis in section 4 below.) In this case, there is essentially perfect competition, and agents have no intrinsic preference for one platform over another except insofar as one platform has more agents from the other side or charges lower prices. Therefore, the efficient outcome is for all agents to use the same platform. Caillaud and Jullien's Proposition 1 shows that the only equilibria in this case involve one platform signing up all agents (as is efficient) and that platform making zero profits. The equilibrium structure of prices involves negative subscription fees and maximal transactions charges, since this is the most profitable way to prevent entry. Caillaud and Jullien go on to analyze the more complicated case where agents can multi-home. They analyze several possibilities, but the cases most relevant for this paper are what they term "mixed equilibria" (see their Propositions 8 and 11). These correspond to the "competitive bottleneck" situations in this paper, and involve one side multi-homing and the other side single-homing. They find that the single-homing side is treated favourably (indeed, its price is necessarily no higher than its cost) while the multi-homing side has all its surplus extracted. I discuss the relationship between the two approaches in more detail in section 5.5 below.

The second closely related paper is Rochet and Tirole (2003). The flavour of their analysis can be understood in the context of the credit card market (although the analysis applies more widely). On one side of the market are consumers and on the other side is the set of retailers, and facilitating the interaction between these two groups are two competing credit card networks. For much of their analysis, the credit card platforms levy charges purely on a per-transaction basis, and there are no lump-sum fees for either side. Suppose that one credit card offers a lower transaction fee to retailers than its rival. A retailer choosing between accepting just the cheaper card or accepting both cards faces a trade-off. If it accepts just the cheaper card then its consumers have a stark choice between paying by this card or not using a card at all. Alternatively, if it accepts both cards then (i) more consumers will choose to pay by some card but (ii) fewer consumers will use the retailer's preferred lower-cost card. If a credit card reduces its charge to retailers relative to its rival, this will "steer" some retailers which previously accepted both cards to accept only the lower-cost card. In a symmetric equilibrium, all retailers accept both credit cards (or neither), while consumers always use

[3] See also van Raalte and Webers (1998).

their preferred credit card. The share of the charges that are borne by the two sides depends on how closely consumers view the two cards as substitutes. If few consumers switch cards in response to a price cut on their side, then consumers should pay a large share of the total transaction charge; if consumers view the cards as close substitutes, then retailers will bear most of the charges in equilibrium. Rochet and Tirole also consider the case where there are fixed fees as well as per-transaction fees, under the assumption that consumers use a single card. This is essentially the same model as the competitive bottleneck model in this paper, and I discuss this part of their paper in more detail in section 5.5 below.

There are a number of modelling differences between the current paper and Rochet and Tirole (2003) which concern the specification of agents' utility, the structure of platforms' fees, and the structure of platforms' costs.[4] In both papers agent j has gross utility from using platform i of the form

$$u_j^i = \alpha_j^i n^i + \zeta_j^i \ .$$

Here n^i is the number of agents from the other side who are present on the platform, α_j^i is the benefit that agent j enjoys from each agent on the other side, and ζ_j^i is the fixed benefit the agent obtains from using that platform. Rochet and Tirole assume that ζ_j^i does not depend on i or j (and can be set equal to zero), but that α_j^i varies both with agent j and platform i. In section 3 and 4 of the current paper, by contrast, I assume that α_j^i does not depend on i or j but only on which side of the market the agent is on, while ζ_j^i depends on the agent and on the platform. (In section 5 I suppose that the interaction term for one side does vary.) The decision whether to make agents' heterogeneity to do with the interaction term α or the fixed effect ζ has major implications for the structure of prices to the two sides in equilibrium. For instance, with a monopoly platform the formulas for profit-maximizing prices look very different in the two papers. Moreover, when α_j^i depends on the platform i, an agent cares about on which platform the transaction takes place (if there is a choice): this effect plays a major role in Rochet and Tirole's analysis but is absent in the present paper.[5]

Turning to the structure of the platforms' fees, for the most part Rochet and Tirole assume that agents pay a per-transaction fee for each agent on the platform from the other side. If this fee is denoted γ^i then agent j's net utility on platform i is $u_j^i = (\alpha_j^i - \gamma^i)n^i$ (when ζ is set equal to zero). This confirms the discussion in section 1 above that per-transactions charges act to reduce the size of network effects. In the monopoly platform case, an agent's incentive to join the platform does not depend on the platform's performance on the other side, and she will join if and only if $\alpha_j^i \geq \gamma^i$. The present paper, especially in section 4,

[4] The assumptions in Caillaud and Jullien (2003) to do with utility and costs are closer to the current paper than to Rochet and Tirole. Caillaud and Jullien do not have any intrinsic product differentiation between the platforms. However, there is a benefit to join two platforms rather than one since they assume that there is a better chance of a match between buyers and sellers when two platforms are involved.

[5] A recent paper that encompasses these two approaches with a monopoly platform is Rochet and Tirole (2004), where simultaneous heterogeneity in both α and ζ is allowed. However, a full analysis of this case is technically challenging in the case of competing platforms.

assumes that platform charges are levied as a lump-sum fee, p^i say, in which case the agent's net utility is $u_j^i = \alpha n^i + \zeta_j^i - p^i$. The final modelling difference between the two papers is with the specification of costs: Rochet and Tirole assume mainly that a platform's costs are incurred on a per-transaction basis, so that if a platform has n_1 group 1 agents and n_2 group 2 agents its total cost is cn_1n_2 for some per-transaction cost c. In the current paper costs are often modelled as being incurred when agents join a network, so that a platform's total cost is $f_1n_1 + f_2n_2$ for some per-agent costs f_1 and f_2. Which assumptions concerning tariffs and costs best reflects reality depends on the context. Rochet and Tirole's model is well suited to the credit card context, for instance, whereas the assumptions in the current paper are intended to better reflect markets such as nightclubs, shopping malls and newspapers.

3 Model I: A Monopoly Platform

This section presents the analysis for a monopoly platform. This framework does not apply to most of the examples of two-sided markets that come to mind, although there are a few applications. For instance, yellow pages directories are often a monopoly of the incumbent telephone company, shopping malls or nightclubs are sometimes far enough away from others that the monopoly paradigm might be appropriate, and sometimes there is only one newspaper or magazine in the relevant market.

Suppose there are two groups of agents, denoted 1 and 2. A member of one group cares about the number of the other group who use the platform. (For simplicity, I ignore the possibility that agents care also about the number of the same group who join the platform.) Suppose that the utility of an agent is determined in the following way: if the platform attracts n_1 and n_2 members of the two groups, the utilities of a group 1 and a group 2 agent are respectively

$$u_1 = \alpha_1 n_2 - p_1 \ ; \ u_2 = \alpha_2 n_1 - p_2 \ , \tag{1}$$

where p_1 and p_2 are the platform's prices to the two groups. The parameter α_1 measures the benefit that a group 1 agent enjoys from interacting with each group 2 agent, and α_2 measures the benefit a group 2 agent obtains from each group 1 agent. Expression (1) describes how utility is determined as a function of the numbers of agents who participate. To close the demand model I specify the numbers who participate as a function of the utilities: if the utilities offered to the two groups are u_1 and u_2, suppose that the numbers of each group who join the platform are

$$n_1 = \phi_1(u_1) \ ; \ n_2 = \phi_2(u_2)$$

for some increasing functions ϕ_1 and ϕ_2.

Turning to the cost side, suppose that the platform incurs a per-agent cost f_1 for serving group 1 and per-agent cost f_2 for group 2. Therefore, the firm's profit is $\pi = n_1(p_1 - f_1) + n_2(p_2 - f_2)$. If we consider the platform to be offering utilities $\{u_1, u_2\}$ rather than prices $\{p_1, p_2\}$, then the implicit price for group 1 is $p_1 = \alpha_1 n_2 - u_1$ (and similarly for group 2).

Therefore, expressed in terms of utilities, the platform's profit is

$$\pi(u_1, u_2) = [\alpha_1 \phi_2(u_2) - u_1 - f_1]\, \phi_1(u_1) + [\alpha_2 \phi_1(u_1) - u_2 - f_2]\, \phi_2(u_2)\ . \tag{2}$$

Let the aggregate consumer surplus of group 1 be $v_1(u_1)$, where v_1 satisfies the envelope condition $v_1'(u_1) \equiv \phi_1(u_1)$, and similarly for group 2. Then total welfare, as measured by the unweighted sum of profit and consumer surplus, is

$$w = \pi(u_1, u_2) + v_1(u_1) + v_2(u_2)\ .$$

It is easily verified that the first-best welfare maximizing outcome involves utilities satisfying:

$$u_1 = (\alpha_1 + \alpha_2)n_2 - f_1\ ;\ u_2 = (\alpha_1 + \alpha_2)n_1 - f_2\ .$$

From expression (1) the socially optimal prices satisfy

$$p_1 = f_1 - \alpha_2 n_2\ ;\ p_2 = f_2 - \alpha_1 n_1\ .$$

As one would expect, the optimal price for group 1, say, equals the cost of supplying service to a type 1 agent adjusted downwards by the external benefit that an extra group 1 agent brings to the group 2 agents on the platform. (There are n_2 group 2 agents on the platform, and each one benefits by α_2 when an extra group 1 agent joins.) In particular, prices should ideally be below cost if $\alpha_1, \alpha_2 > 0$.

From expression (2), the profit-maximizing prices satisfy

$$p_1 = f_1 - \alpha_2 n_2 + \frac{\phi_1(u_1)}{\phi_1'(u_1)}\ ;\ p_2 = f_2 - \alpha_1 n_1 + \frac{\phi_2(u_2)}{\phi_2'(u_2)}\ . \tag{3}$$

Thus, the profit-maximizing price for group 1, say, is equal to the cost of providing service (f_1), adjusted downwards by the external benefit ($\alpha_2 n_2$), and adjusted upwards by a factor related to the elasticity of the group's participation. The profit-maximizing prices can be obtained in the more familiar form of Lerner indices and elasticities, as recorded in the following result:

Proposition 1 *Write*

$$\eta_1(p_1 \mid n_2) = \frac{p_1 \phi_1'(\alpha_1 n_2 - p_1)}{\phi_1(\alpha_1 n_2 - p_1)}\ ;\ \eta_2(p_2 \mid n_1) = \frac{p_2 \phi_2'(\alpha_2 n_1 - p_2)}{\phi_2(\alpha_2 n_1 - p_2)}$$

for a group's price elasticity of demand for a given level of participation by the other group. Then the profit-maximizing pair of prices satisfy

$$\frac{p_1 - (f_1 - \alpha_2 n_2)}{p_1} = \frac{1}{\eta_1(p_1 \mid n_2)}\ ;\ \frac{p_2 - (f_2 - \alpha_1 n_1)}{p_2} = \frac{1}{\eta_2(p_2 \mid n_1)}\ . \tag{4}$$

It is possible that the profit-maximizing outcome might involve group 1, say, being offered a subsidised service, i.e., $p_1 < f_1$. From (4), this happens if the group's elasticity of demand is high and/or if the external benefit enjoyed by group 2 is large. Indeed, the subsidy might be so large that the resulting price is negative (or zero, if negative prices are not feasible). This analysis applies, in a stylized way, to a market with a monopoly yellow page directory. Such directories typically are given to telephone subscribers for free, and profits are made from charges to advertisers. The analysis might also apply to software markets where one type of software is required to create files in a certain format and another type of software is required to read such files. (For the analysis to apply accurately, though, there needs to be two disjoint groups of agents: those who wish to read files and those who wish to create files. It does not readily apply when most people wish to perform both tasks.)

4 Model II: Two-Sided Single-Homing

This model involves competing platforms, but assumes that, for exogenous reasons, each agent from either group chooses to join a single platform.

4.1 Basic Model

The model extends the previous monopoly model in a natural way. There are two groups of agents, 1 and 2, and there are two platforms, A and B, which enable the two groups to interact. Groups 1 and 2 obtain the respective utilities $\{u_1^i, u_2^i\}$ from platform i. These utilities $\{u_1^i, u_2^i\}$ are determined in a similar manner to the monopoly model expressed in (1): if platform i attracts n_1^i and n_2^i members of the two groups, the utilities on this platform are

$$u_1^i = \alpha_1 n_2^i - p_1^i \ ; \ u_2^i = \alpha_2 n_1^i - p_2^i \ , \tag{5}$$

where $\{p_1^i, p_2^i\}$ are the respective prices charged by the platform to the two groups.

When group 1 is offered a choice of utilities u_1^A and u_1^B from the two platforms, and group 2 is offered the choice u_2^A and u_2^B, suppose that the number of each group who go to platform i is given by the Hotelling specification:

$$n_1^i = \frac{1}{2} + \frac{u_1^i - u_1^j}{2t_1} \ ; \ n_2^i = \frac{1}{2} + \frac{u_2^i - u_2^j}{2t_2} \ . \tag{6}$$

Here, agents in a group are assumed to be uniformly located along a unit interval with the two platforms located at the two end-points, and t_1, $t_2 > 0$ are the product differentiation (or transport cost) parameters for the two groups that determine the competitiveness of the two markets.

Putting (6) together with (5), and using the fact that $n_1^j = 1 - n_1^i$, gives the following implicit expressions for market shares:

$$n_1^i = \frac{1}{2} + \frac{\alpha_1(2n_2^i - 1) - (p_1^i - p_1^j)}{2t_1} \; ; \; n_2^i = \frac{1}{2} + \frac{\alpha_2(2n_1^i - 1) - (p_2^i - p_2^j)}{2t_2} \; . \tag{7}$$

Expressions (7) show that, keeping its group 2 price fixed, an extra group 1 agent on a platform attracts a further $\frac{\alpha_2}{t_2}$ group 2 agents to that platform.

Suppose that the network externality parameters $\{\alpha_1, \alpha_2\}$ are small compared to the differentiation parameters $\{t_1, t_2\}$ so that I can focus on market-sharing equilibria. (If network effects were large compared to brand preferences then there could only be equilibria where one platform corners both sides of the market.) It turns out that the necessary and sufficient condition for a market-sharing equilibrium to exist is the following:

$$4t_1t_2 > (\alpha_1 + \alpha_2)^2 \tag{8}$$

and this inequality is assumed to hold in the following analysis.

Suppose that platforms A and B offer the respective price pairs (p_1^A, p_2^A) and (p_1^B, p_2^B). Solving the simultaneous equations (7) implies that market shares are determined by the four prices as:

$$n_1^i = \frac{1}{2} + \frac{1}{2}\frac{\alpha_1(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{t_1t_2 - \alpha_1\alpha_2} \; ; \; n_2^i = \frac{1}{2} + \frac{1}{2}\frac{\alpha_2(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{t_1t_2 - \alpha_1\alpha_2} \; . \tag{9}$$

(Assumption (8) implies that the above denominators $t_1t_2 - \alpha_1\alpha_2$ are positive.) Thus, assuming $\alpha_1, \alpha_2 > 0$, a platform's market share for one group is decreasing in its price offered to the other group.

As with the monopoly model, suppose that each platform has a per-agent cost f_1 for serving group 1 and f_2 for serving group 2. Therefore, profits for platform i are

$$(p_1^i - f_1)\left[\frac{1}{2} + \frac{1}{2}\frac{\alpha_1(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{t_1t_2 - \alpha_1\alpha_2}\right] + (p_2^i - f_2)\left[\frac{1}{2} + \frac{1}{2}\frac{\alpha_2(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{t_1t_2 - \alpha_1\alpha_2}\right] \; .$$

This expression is quadratic in platform i's prices, and is concave in these prices if and only if assumption (8) holds. Therefore, platform i's best response to j's prices is characterized by the first-order conditions. Given assumption (8), one can check there are no asymmetric equilibria. For the case of a symmetric equilibrium where each platform offers the same price pair (p_1, p_2), the first-order conditions for equilibrium prices are

$$p_1 = f_1 + t_1 - \frac{\alpha_2}{t_2}(\alpha_1 + p_2 - f_2) \; ; \; p_2 = f_2 + t_2 - \frac{\alpha_1}{t_1}(\alpha_2 + p_1 - f_1) \; . \tag{10}$$

Expressions (10) can be interpreted in the following manner. First, note that in a Hotelling model without network effects, the equilibrium price for group 1 would be $p_1 = f_1 + t_1$. In this two-sided setting the price is adjusted downwards by the factor $\frac{\alpha_2}{t_2}(\alpha_1 + p_2 - f_2)$. This adjustment factor can be decomposed into two parts. The term $(\alpha_1 + p_2 - f_2)$ represents the "external" benefit to a platform of having an additional group 2 agent. To see this, note first that the platform makes profit $(p_2 - f_2)$ from each extra group 2 agent. Second, α_1 measures the extra revenue the platform can extract from its group 1 agents (without losing market share) when it has an extra group 2 agent.[6] Thus $(\alpha_1 + p_2 - f_2)$ indeed represents the external benefit to a platform of attracting the marginal group 2 agent. Finally, as shown in expression (7), a platform attracts $\frac{\alpha_2}{t_2}$ extra group 2 agents when it has an extra group 1 agent. In sum, the adjustment factor $\frac{\alpha_2}{t_2}(\alpha_1 + p_2 - f_2)$ measures the external benefit to the platform from attracting an extra group 1 agent; in other words, it measures the opportunity cost of raising the group 1 price enough to cause one group 1 agent to leave. I summarize this discussion by an annotated version of formula (10):

$$p_1 = \underbrace{f_1}_{\text{cost of service}} + \underbrace{t_1}_{\text{market power factor}} - \underbrace{\frac{\alpha_2}{t_2}}_{\text{extra group 2 agents}} \times \underbrace{(\alpha_1 + p_2 - f_2)}_{\text{profit from each extra group 2 agent}} \tag{11}$$

Finally, solving the simultaneous equations (10) implies that $p_1 = f_1 + t_1 - \alpha_2$ and $p_2 = f_2 + t_2 - \alpha_1$. This discussion is summarized as:

Proposition 2 *Suppose that assumption (8) holds. Then the model with two-sided single-homing has a unique equilibrium which is symmetric. Equilibrium prices for group 1 and group 2 are given by*

$$p_1 = f_1 + t_1 - \alpha_2 \ ; \ p_2 = f_2 + t_2 - \alpha_1 \ . \tag{12}$$

Thus, a platform will target one group more aggressively than the other if that group is (i) on the more competitive side of the market and/or (ii) causes larger benefits to the other group than *vice versa*.[7]

While expressions (12) are certainly "simple", they are not intuitive, and this is why I focussed the discussion on (10). The fact that, say, group 1's price does not depend on its own externality parameter α_1 is surely an artifact of the Hotelling specification for consumer demand. In particular, the fact that the total size of each group is fixed, so that when platforms set low prices they only steal business from the rival rather than expand the overall

[6] An extra group 2 agent means that the utility of each group 1 agent on the platform increases by α_1, while the utility of each group 1 agent on the rival platform falls by α_1. Therefore, the relative utility for group 1 agents being on the platform increases by $2\alpha_1$ and each of the agents can bear a price increase equal to this. Since in equilibrium a platform has half the group 1 agents, the extra revenue it can extract from these agents is α_1, as claimed.

[7] It is possible given our assumptions that one price in the above expression is negative. This happens if that side of the market involves a low cost, is competitive, or causes a large external benefit to the other side. In many cases it is not realistic to suppose that negative prices are feasible, in which case the analysis needs to be adapted explicitly to incorporate the non-negativity constraints—see Armstrong and Wright (2004) for this analysis.

market, greatly simplifies the analysis. While the shape of the solution as verbally described in equation (11) seems likely to hold more widely, the neat formulas in (12) will not. One disadvantage of using a framework with fixed group sizes, however, is that there is no scope for meaningful welfare analysis since prices are simply transfers between agents: any (symmetric) pair of prices offered by the two platforms will yield the same level of total surplus.

It is useful to compare the competitive formulas (12) with the monopoly formulas (4). It turns out there is an extra effect with competition. With the Hotelling specification, a platform's elasticities of demand given equal market shares are $\eta_1 = p_1/t_1$ and $\eta_2 = p_2/t_2$ for the two groups. Thus, (12) may be re-written as

$$\frac{p_1 - (f_1 - 2\alpha_2 n_2)}{p_1} = \frac{1}{\eta_1} \; ; \; \frac{p_2 - (f_2 - 2\alpha_1 n_1)}{p_2} = \frac{1}{\eta_2} \; .$$

Comparing these with the monopoly formulas (4) shows that a duopolist puts twice as much emphasis on the external benefit from one group when it sets its price to the other group. The reason for this difference is simple. When a monopoly platform sets a high price which induces an agent from one side to leave, that agent just disappears from the market. When the duopoly platform A sets a high price that induces an agent from group 1, say, to leave, that agent does not disappear but instead joins platform B, and this makes it harder for A to compete for group 2 agents.

In equilibrium each platform makes profit

$$\pi = \frac{t_1 + t_2 - \alpha_1 - \alpha_2}{2} \; . \tag{13}$$

Assumption (8) guarantees that this profit is positive. Positive cross-group network effects act to reduce profits compared to the case where $\alpha_1 = \alpha_2 = 0$, since platforms have an additional reason to compete hard for market share. In section 4.2.2 below I discuss an extension where platforms can choose tariffs that reduce, eliminate or even reverse these cross-group network effects by the use of more complex tariffs.

4.2 Alternative Tariffs

4.2.1 Uniform Prices

In some contexts it is natural to investigate the effect of price discrimination on prices and profits. For instance, are equilibrium profits higher or lower when nightclubs can charge different entry fees for women and men? Does the ability to target one side of the market without sacrificing revenues on the other side raise or lower profits? These issues can be addressed using the framework just discussed.

Suppose that $f_1 = f_2 = f$, say. (It makes little sense to discuss price discrimination if the costs are significantly different for the two groups.) In contrast to the previous analysis, suppose platforms cannot set different prices to the two groups, and that platform i sets the uniform price p_i. (Perhaps sex discrimination laws prevent differential pricing by nightclubs.) Platform i's profits are $(p^i - f)(n_1^i + n_2^i)$ and a platform cares only about the total number of agents it attracts. From expressions (9) this total demand for platform i is

$$n_1^i + n_2^i = 1 + \frac{t_1 + t_2 + \alpha_1 + \alpha_2}{t_1 t_2 - \alpha_1 \alpha_2}(p^j - p^i) \ .$$

Therefore, the equilibrium uniform price in this industry is

$$p = f + 2\frac{t_1 t_2 - \alpha_1 \alpha_2}{t_1 + t_2 + \alpha_1 + \alpha_2} \ . \tag{14}$$

One can show that this uniform price (14) lies between the equilibrium discriminatory prices in (12). Therefore, the consequence of a ban on discrimination is that one group is made better off (the group which used to have the higher price with discriminatory prices) while the other group is made worse off.

Equilibrium profits increase with discrimination if $p - f > \frac{1}{2}\{(p_1 - f) + (p_2 - f)\}$. Using expressions (12) and (14), it follows that this is the case if and only if

$$(t_1 - t_2)^2 > (\alpha_1 - \alpha_2)^2. \tag{15}$$

Condition (15) requires that the differences in the two groups are more to do with differences in the extent of product differentiation (t) than to do with differences in the external benefits (α). Thus, when differences are largely due to differences in α, the ability of platforms to engage in price discrimination is damaging to their profits. Since total welfare is constant in this particular model, it follows that when condition (15) holds, consumers in aggregate are worse off when platforms engage in price discrimination.

4.2.2 Two-Part Tariffs

The analysis so far has assumed that all agents are charged a lump-sum fee to join a platform. There are several other kinds of competition that could be envisaged. For instance, Rochet and Tirole (2003) focus on the case where platforms levy charges on a per-transaction basis, i.e., the total charge to one group is proportional to the platform's realized market share of the other group. Alternatively, platforms could commit to supply agents with fixed *utilities* instead of charging a fixed price. Implicit in such a commitment would be to reduce the charge that group 1 agents pay if it turns out that the market share for group 2 is smaller than expected, assuming that measurement problems do not preclude this. A more general formulation that encompasses these various possibilities is for platforms to offer a "two-part tariff", in which agents pay a fixed charge p together with a marginal price, say γ, for each agent on the other side who joins the platform. That is to say, platform i's tariffs to groups 1 and 2 take the form

$$T_1^i = p_1^i + \gamma_1^i n_2^i \ ; \ T_2^i = p_2^i + \gamma_2^i n_1^i \ . \tag{16}$$

Special cases of this family of tariffs include: (i) $\gamma_1^i = \gamma_2^i = 0$, where platforms compete in fixed fees as in section 4.1, and (ii) $\gamma_1^i = \alpha_1$ and $\gamma_2^i = \alpha_2$, where agents pay exactly the benefit they enjoy from interacting with an additional member of the other group. Thus, in case (ii) a platform commits to deliver a constant utility to customers, irrespective of its success on the other side of the market.

In general, each platform now has four degrees of freedom in its tariff choice. The analysis is more complicated than required in the basic model, and the details are left to the appendix. This analysis is summarized in the following result:

Proposition 3 *Suppose that assumption (8) holds. Then when platforms compete in two-part tariffs a continuum of symmetric equilibria exist.*[8] *Let $\gamma_1 \geq 0$ and $\gamma_2 \geq 0$ be marginal prices aimed at group 1 and group 2 respectively. An equilibrium exists where the two platforms offer the same pair of two-part tariffs to group 1 and group 2 of the form*

$$T_1 = p_1 + \gamma_1 n_2 \ ; \ T_2 = p_2 + \gamma_2 n_1$$

provided that the fixed fees satisfy

$$p_1 = f_1 + t_1 - \alpha_2 + \frac{1}{2}(\gamma_2 - \gamma_1) \quad ; \ p_2 = f_2 + t_2 - \alpha_1 + \frac{1}{2}(\gamma_1 - \gamma_2) \ . \tag{17}$$

The expressions (17) generalize (12) above. The profit of each platform in such an equilibrium is modified from (13) to be

$$\pi = \frac{t_1 + t_2 - \alpha_1 - \alpha_2}{2} + \frac{\gamma_1 + \gamma_2}{4} \ .$$

This profit is increasing in the marginal prices γ_1 and γ_2. The reason that high marginal charges yield high profits is that they reduce, or even overturn, the cross-group network effects that make the market so competitive and destroy profits.[9]

Thus, when platforms consider the use of more complicated tariffs that depend on the platform's success on the other side of the market, then a continuum of symmetric equilibria exist, which are ranked by the profit they generate. In technical terms, the source of the multiple equilibria is related to the multiple equilibria that exist in a (deterministic) supply function framework—see section 3 of Klemperer and Meyer (1989). The common issue in the two settings is that, for a given choice of tariff by its rival, a firm has a continuum of best responses.

Finally, the question arises as to which of these equilibria is selected. One suggestion might be that platforms coordinate on an equilibrium that generates high profits, i.e., on a pair of

[8]There are also asymmetric equilibria.

[9]As formulated here, there is no barrier to the platforms setting arbitrarily high prices γ and so generating arbitrarily high profits (and arbitrarily low utilities for agents). A more realistic model would have an outside option for agents, and this would put a ceiling on the platforms' prices.

tariffs with large γ_i. An alternative viewpoint is that the "pure subscription" tariffs analyzed in section 4.1 are robust: if its rival offers a pure subscription tariff ($\gamma_1 = \gamma_2 = 0$), a platform has no incentive to offer a more ornate tariff that depends on its performance on the other side. But more generally, this analysis suggests that, while it is straightforward to analyze the case of pure subscription tariffs (as in section 4.1 of this paper) or the case of pure transaction tariffs (as emphasized in Rochet and Tirole (2003)), blending the two polar kinds of tariffs presents major problems for the predictive power of the model as formulated.

5 Model III: Competitive Bottlenecks

This section presents the third and final model, which is termed a model of "competitive bottlenecks". I modify the model of section 4 and suppose that, while group 1 continues to deal with a single platform (to single-home), group 2 wishes to deal with each platform (to multi-home). Implicitly in this model there is the idea that group 2 puts more weight on the network benefits of being in contact with the widest population of group 1 consumers than it does on the costs of dealing with more than one platform. The crucial difference between this model and that discussed in section 4 is that here group 2 does not make an "either-or" decision to join a platform. Rather, keeping the market shares for group 1 constant, a group 2 agent makes a decision to join one platform independently from its decision to join the other.[10] In this sense, there is no competition between platforms to attract group 2 custom.

There are several examples of markets where this framework seems a stylized representation. Consider the case of competing mobile telecommunications networks (see section 3.1 of Armstrong (2002) and Wright (2002) for formal models of this industry.) Subscribers wish to join at most one mobile network (i.e., they single-home). People on the fixed telephony networks wish to call mobile subscribers. For a specified charge, someone can call any given mobile network, and in this sense the people who call mobile networks multi-home. A mobile subscriber will choose the network with the tariff that leaves him with the most surplus. A network's tariff has two ingredients: the charges for subscription and outbound calls that affect the subscriber's welfare directly, and the charges the network makes to others for delivering calls *to* the subscriber (the so-called call termination charges). Unless the subscriber cares directly about the welfare of people who might call him, the latter charges affect the subscriber's welfare only insofar as they affect the number of calls he receives. (High termination charges will typically act to reduce the number of calls made to mobile networks, and this is detrimental to a subscriber's welfare if he benefits from receiving calls.)

The tariffs that mobile networks set in equilibrium have low charges for subscription and outbound calls and high charges for call termination. In particular, the model predicts that

[10] At least, this is true if group 2 agents are "atomistic". If this was not the case then if a "large" group 2 agent which has already joined platform A decides to join platform B, then this will draw some group 1 agents away from platform A and so cause a negative externality on the surplus from the agent's platform A interaction. Here, though, we ignore this possibility.

high profits made on call termination are passed on to subscribers in the form of subsidized handsets or similar inducements. More precisely, the equilibrium call termination charge is chosen to maximise the welfare of mobile subscribers and mobile networks combined, and the interests of those who call mobile networks are ignored. This feature—that the single-homing side is treated well and the multi-homing side's interests are ignored in equilibrium—is a characteristic of the models presented below. A competitive bottleneck is present: although the market *for* subscribers might be highly competitive, so that mobile networks have low equilibrium profits overall, there is no competition for providing communication services *to* these subscribers.

Other examples of this phenomenon include: most people might read a single newspaper (perhaps due to time constraints) but advertisers might place adverts in all relevant newspapers; consumers might choose to visit a single shopping mall (perhaps because of transport costs) but the same retailer might choose to open a branch in several malls; or a travel agent might choose to use just one computerized airline reservation system, while airlines are forced to deal with all such platforms in order to gain access to each travel agent's customers.

5.1 A General Framework

Suppose there are two, possibly asymmetric, platforms that facilitate interaction between two groups of agents. Suppose that group 2 agents are heterogeneous: if there are n_1^i group 1 agents on platform i, the number of group 2 agents who are prepared to pay a fixed fee p_2^i to join this platform is denoted

$$n_2^i = \phi^i(n_1^i, p_2^i) \ , \tag{18}$$

where the function ϕ^i is decreasing in price p_2^i and increasing in n_1^i. A group 2 agent's decision to join one platform does not depend on whether it chooses to join the rival platform. Let $R^i(n_1^i, n_2^i)$ denote platform i's revenue from group 2 when it has n_1^i group 1 agents and sets its group 2 price such that n_2^i group 2 agents choose to join the platform. Formally, R is defined by the relation

$$R^i(n_1^i, \phi^i(n_1^i, p_2^i)) \equiv p_2^i \phi^i(n_1^i, p_2^i) \ . \tag{19}$$

Similarly to expression (5), platform i's group 1 utility u_1^i is given by

$$u_1^i = U^i(n_2^i) - p_1^i$$

if the platform charges p_1^i to group 1 and n_2^i group 2 agents join the platform. Here U^i is the (possibly nonlinear) function that measures the benefit that a group 1 agent enjoys with greater group 2 participation on the platform. (The function U^i might sometimes be decreasing, for instance when newspaper readers find adverts to be a nuisance.) If a group 1 agent's utility is u_1^i with platform i, suppose that the platform will attract

$$n_1^i = \Phi^i(u_1^i, u_1^j) \tag{20}$$

members of group 1, where Φ^i is increasing in the first argument and decreasing in the second. If platform i's total cost of serving the two sides is denoted $C^i(n_1^i, n_2^i)$, its profit is

$$\pi^i = n_1^i p_1^i + R^i(n_1^i, n_2^i) - C^i(n_1^i, n_2^i) \ . \tag{21}$$

Next, the number of group 2 agents on each platform in equilibrium is derived as a function of the equilibrium market shares for group 1. Suppose that in equilibrium platform i offers utility $\hat{u}_1^i$ to its group 1 agents and attracts a number $\hat{n}_1^i$ of such agents (given by the function Φ^i). Then the platform must be maximizing its profits given this group 1 utility $\hat{u}_1^i$. Therefore, consider varying p_1^i and n_2^i so that utility $\hat{u}_1^i = U^i(n_2^i) - p_1^i$ is constant. Writing $p_1^i = U^i(n_2^i) - \hat{u}_1^i$ in (21) means that profit is

$$\pi^i = \hat{n}_1^i \left[U^i(n_2^i) - \hat{u}_1^i \right] + R^i(\hat{n}_1^i, n_2^i) - C^i(\hat{n}_1^i, n_2^i) \ .$$

Given $\hat{n}_1^i$, platform i will choose to serve a number $\hat{n}_2^i$ of group 2 agents to maximize

$$\hat{n}_1^i U^i(\cdot) + R^i(\hat{n}_1^i, \cdot) - C^i(\hat{n}_1^i, \cdot) \ . \tag{22}$$

The equilibrium lump-sum charge to group 2 is $\hat{p}_2^i$, where this satisfies

$$\hat{n}_2^i = \phi^i(\hat{n}_1^i, \hat{p}_2^i) \ . \tag{23}$$

Notice that, for a given $\hat{n}_1^i$, expression (22) measures the total surplus of platform i and its group 1 agents as the number of group 2 agents is varied. Therefore, the number of group 2 agents is chosen to maximize the joint interests of the platform and its group 1 agents, and the interests of group 2 are completely ignored. In general, this insight implies that there is a market failure present, and for a given allocation of group 1 agents between the platforms, there is a sub-optimal number of group 2 agents on each platform.

In more detail, suppose that the gross group 2 surplus on platform i when that platform has n_1^i group 1 agents and n_2^i group 2 agents is denoted $V^i(n_1^i, n_2^i)$. When there are no externalities *within* the set of group 2 agents, this surplus function differentiates to give the inverse demand function, so that

$$\frac{\partial}{\partial n_2^i} V^i(n_1^i, n_2^i) \equiv \frac{R^i(n_1^i, n_2^i)}{n_2^i} \ . \tag{24}$$

(The right-hand side of this expression is just the price paid by group 2 agents.) In some contexts there are intra-group externalities present, and a group 2 agent might be better off if there were fewer other group 2 agents on the same platform, in which case the formula (24) is not valid. The most obvious examples of this phenomenon are where shops or advertisers are competing amongst themselves to sell to consumers, and I discuss this situation in section 5.4 below. When there are no intra-group 2 externalities present and (24) holds, there is an unambiguous market failure present, in that there are too few group 2 agents on each platform for given numbers of group 1. To see this note that given the group 1 allocation, total surplus on platform i is maximized by choosing n_2^i to maximize

$$\hat{n}_1^i U^i(\cdot) + V^i(\hat{n}_1^i, \cdot) - C^i(\hat{n}_1^i, \cdot) \ . \tag{25}$$

Since $V^i(n_1^i, n_2^i) - R^i(n_1^i, n_2^i)$, group 2's net aggregate surplus, is increasing in n_2^i, the maximizer of (25) is greater than than the maximizer of (22), and so there are too few group 2 agents served in equilibrium. This result is summarized in the following:

Proposition 4 *In the competitive bottleneck model, in any equilibrium the number of group 2 agents on a platform is chosen to maximize the joint surplus of the platform and its group 1 agents, and the interests of group 2 are ignored. Unless there are externalities within the set of group 2 agents, there are too few group 2 agents on each platform given the numbers of group 1 agents on each platform.*

As with the mobile telephony case just discussed, in this model it does not make sense to speak of the competitiveness of "the market". There are two markets: the market for single-homing agents which is, to a greater or lesser extent, competitive, and a market for multi-homing agents, where each platform holds a local monopoly. The excessive prices faced by the multi-homing side do not necessarily result in excess profits for platforms, since platforms might be forced by competitive pressure to transfer their monopoly revenues to the single-homing agents. Rather, the market failure is that there is a sub-optimal balance of prices to the two sides of the market.

Without putting more structure on this general model it is hard to predict how their joint surplus is shared between platforms and group 1 agents. The price charged to group 1 will depend in part on the strength of competition in the group 1 market for consumers (i.e., on the form of the functions Φ^i). This is investigated further in the two applications that follow.

5.2 Application: Informative Advertising on Media Platforms

Consider a situation where advertisers wish to make contact with potential customers by placing adverts on media platforms such as newspapers or yellow pages directories. Suppose there are two such platforms, A and B, which are assumed to be symmetric. Adverts are placed on the platforms by monopoly retailers ("group 2"). (See section 5.4 for the case where advertisers might compete with each other.) Assume that readers ("group 1") use one or other platform, but not both. A reader will purchase a given product if she sees an advert for the product and the price is no higher than her reservation value. The cost of producing and distributing an individual copy of a newspaper/directory is $c(n_2)$ if it contains n_2 adverts. If there are no other costs, the cost function $C(n_1, n_2)$ in section 5.1 takes the multiplicative form:[11]

$$C(n_1, n_2) = n_1 c(n_2) \ . \tag{26}$$

[11] A richer model would include a per-advertiser cost in addition, since it is plausible that a platform incurs costs in dealing with each advertiser. In this case we would have $C(n_1, n_2) = n_1 c(n_2) + f_2$, say. Similarly, one

Advertisers are differentiated by the parameter α_2: the type-α_2 advertiser has a product that generates profit α_2 from each reader who sees its advert. Thus an advert placed on a platform is worth α_2 for each reader on the platform for the type-α_2 advertiser.[12] Let $F(\alpha_2)$ be the cumulative distribution function for α_2 in the population of advertisers.

If the fixed charge for placing an advert on platform i is p_2^i, then a type-α_2 advertiser will place an advert on the platform if $\alpha_2 n_1^i \geq p_2^i$. That is to say, the function ϕ in (18) is in this context given by

$$\phi(n_1^i, p_2^i) = 1 - F(p_2^i / n_1^i) \ .$$

With an advertiser demand function of this form, the revenue function R in (19) is proportional to the readership:

$$R(n_1, n_2) = n_1 r(n_2)$$

where r is a platform's advertising-revenue-per-reader function.[13] In this case expression (22) is precisely proportional to n_1^i, and the equilibrium advertising volume

$$\hat{n}_2 \text{ maximizes } U(n_2) + r(n_2) - c(n_2) \tag{27}$$

regardless of the platform's performance on the group 1 side. Thus, a media platform's decision on its advertising volume can be made independently of the size of its readership.

Suppose, as in (6), there is a Hotelling functional form for the consumer market share function Φ in (20) given by

$$n_1^i = \frac{1}{2} + \frac{u_1^i - u_1^j}{2t} \ , \tag{28}$$

where t is the parameter that measures the competitiveness of the market for readers. Therefore, the symmetric equilibrium (if it exists) involves the two platforms sharing readers equally: $\hat{n}_1^i = \hat{n}_1^j = \frac{1}{2}$. Expression (23) then shows that the equilibrium charge made to advertisers, $\hat{p}_2$, is given by

$$\hat{n}_2 = 1 - F(2\hat{p}_2) \ , \tag{29}$$

---

could analyze the application to shopping malls using similar techniques. Here, a mall's cost function would plausibly take the additive form

$$C(n_1, n_2) = f_1 n_1 + f_2 n_2$$

instead of the multiplicative form considered in this section. Also, it is plausible to model retailers (and possibly advertisers in the newspaper application as well) as having a fixed cost of joining a platform (the fixed cost associated with starting a shop) in addition to the platform's charge. But the fundamental conclusion—that there will be too few retailers in equilibrium—will continue to hold in this alternative setting, as shown in Proposition 4.

[12] Thus, this model assumes that an advertiser's payoff is *linear* in the number of people who see the advert. If a seller has limited supplies of the product available for sale (or, more generally, if the cost of production is convex), then the seller only obtains benefit from the advert reaching a certain number of potential consumers, and this linearity assumption would not be plausible.

[13] This function $r(\cdot)$ is defined by $r(1 - F(\gamma)) \equiv (1 - F(\gamma))\gamma$.

where $\hat{n}_2$ is given by (27). Proposition 4 then implies that the equilibrium involves *too little* advertising.

Turn next to the outcome on the consumer side of the market. As in section 4.2.2 above, it turns out that the equilibrium price depends on the precise way in which advertising charges are levied. There are two natural bases for charging advertisers: (i) advertising charges are explicitly levied on a per-reader basis, and (ii) advertisers are charged a lump-sum fee for placing an advert.[14] The reason that this makes a difference to the competitiveness of the market for readers is that it affects the profitability of a platform's deviation in the reader price. With case (i), if a platform attracts more readers the number of adverts does not change.[15] With case (ii), by contrast, having more readers acts to attract more advertisers (keeping the lump-sum advertising charge fixed), and this in turn acts to attract still more readers if readers like adverts. (Note that the charging basis for advertising does not affect the equilibrium number of adverts, given by (27) above.) These two cases are next discussed in more detail.

Per-reader advertising charges: Suppose that platform i offers advertising space for a charge γ^i per reader on the platform. Then a type-α_2 advertiser will choose to join the platform if and only if $\alpha_2 > \gamma^i$, i.e., the number of adverts does not depend on the number of readers on the platform. In this case the analysis is extremely simple. Both platforms will choose the number of adverts $\hat{n}_2$ given by (27), and this generates revenue per reader equal to $r(\hat{n}_2)$. Since the two platforms offer the same number of adverts, a reader cares only about the relative price when they decide which platform to use. Given the specification in (28), the profit of platform i if it charges readers p_1^i (while the rival platform charges readers p_1^j) is

$$\pi^i = \left(\frac{1}{2} - \frac{p_1^i - p_1^j}{2t} \right) \left(p_1^i + r(\hat{n}_2) - c(\hat{n}_2) \right) .$$

Therefore, the symmetric equilibrium price for readers is given by

$$p_1 = c(\hat{n}_2) + t - r(\hat{n}_2) . \qquad (30)$$

Thus, a platform's revenue from advertising, r, is passed onto consumers in the form of a low price p_1. Platform profits are given by the product differentiation parameter t.

When competition for readers is intense (t is small) or the advertising revenue r is large, the charge to readers will be below cost. It might even be that the price is negative in (30). If, as seems plausible, there is a non-negativity constraint on prices, the equilibrium will involve

[14] Unlike in section 4.2.2 above, here I do not consider the more general class of two-part tariffs, where advertisers might pay a fixed fee plus a charge per reader. From this earlier analysis, we predict that there will be a severe problem of multiple equilibria if we did so.

[15] Conceptually, competing with per-reader charges is the same as competing in terms of *quantities* of advertising space, and the equilibrium price for adverts is determined to clear the market, given the realized market shares for readers.

group 1 being allowed onto the platform for free. This could be a rationale for: why yellow pages directories and some newspapers are supplied to readers for free; why a shopping mall might not wish to charge consumers for entry even if it were feasible to do so; or why a broadcaster might not wish to charge viewers even when this can be done.

Lump-sum advertising charges: To analyze this more complex case we need to calculate the extra readership a platform attracts when it undercuts its rival's reader price. When a platform undercuts its rival on the reader side it will clearly attract more readers; in consequence the platform attracts more advertisers (given that its lump-sum charge for advertising is unchanged), which thereby attracts further readers if readers like adverts. This "feedback loop" is quite absent when advertising charges are levied per reader. To be more precise, the advertising volume on platform i as a function of the number of readers n_1^i is given by

$$\tilde{n}_2(n_1^i) = 1 - F(\hat{p}_2/n_1) \ ,$$

where the equilibrium lump-sum charge to advertisers $\hat{p}_2$ is given in expression (29). Using this notation, from (28) it follows that platform i's readership, n_1^i, must satisfy the following implicit equation:

$$n_1^i = \frac{1}{2} + \frac{U(\tilde{n}_2(n_1^i)) - p_1^i - \left[U(\tilde{n}_2(1-n_1^i)) - p_1^j\right]}{2t} \ . \tag{31}$$

Suppose that parameters are such that (31) has a unique solution for n_1^i for all relevant prices p_1^i and p_1^j.[16] Implicit differentiation of (31) implies that

$$\left.\frac{\partial n_1^i}{\partial p_1^i}\right|_{p_1^i = p_1^j} = \frac{-1}{2t - 2\tilde{n}_2'(\frac{1}{2})U'(\hat{n}_2)} \ . \tag{32}$$

In particular, when readers like adverts ($U' > 0$) a platform's reader demand is more elastic than was the case with per-reader advertising charges (when $\partial n_1^i/\partial p_1^i = -1/2t$). When readers dislike adverts ($U' < 0$), by contrast, their demand is less elastic. The platform's total profit is given by

$$\pi^i = n_1^i \left[p_1^i - c(\tilde{n}_2(n_1^i)) + r(\tilde{n}_2(n_1^i))\right] \ .$$

[16] If (31) has multiple solutions for n_1^i then there are multiple demand configurations consistent with the prices, and some method of choosing among the possible configurations is needed. To side-step this issue we simply assume that there is a unique solution. In section 4.1 the assumption (8) ruled out this possibility in the single-homing framework. It seems hard to find the precise corresponding assumption needed for the competitive bottleneck model. However, it is clear what is needed for there to be a unique solution to (31): t should be "large" relative to U' and $\tilde{n}_2'$. (This implies that the right-hand side of (31) is relatively flat as a function of n_1^i.) If $U' \leq 0$, so that readers either do not care about adverts or actively dislike adverts, then the right-hand side of (31) is decreasing in n_1^i and there will always be a unique solution to (31). Armstrong and Wright (2004) explore the issue of multiple consistent demand configurations and the existence of equilibrium in a related model where the group 2 agents are homogeneous.

Using the expression (32), it follows that the equilibrium consumer price p_1 satisfies

$$p_1 = c(\hat{n}_2) + t - r(\hat{n}_2) - \frac{1}{2}\tilde{n}_2'(\tfrac{1}{2})[2U'(\hat{n}_2) + r'(\hat{n}_2) - \tfrac{1}{2}c'(\hat{n}_2)] \ .$$

The first-order condition for the fact that $\hat{n}_2$ maximizes expression (27) implies that

$$p_1 = c(\hat{n}_2) + t - r(\hat{n}_2) - \tfrac{1}{2}\tilde{n}_2'(\tfrac{1}{2})U'(\hat{n}_2) \ . \tag{33}$$

Thus, comparing this expression with that for the per-reader charging case in (30) we see that when advertising charges are levied on a lump-sum basis, the equilibrium price for readers is lower or higher than when they are levied on a per-reader basis according to whether U' is positive or negative. Platform profits are correspondingly lower or higher with lump-sum charging according to whether U' is positive or negative. These results are akin to those presented in section 4.2.2, where the use of tariffs that depend positively on the platform's success on the other side of the market were seen to relax competition and boost profits. In the knife-edge where readers do not care about adverts at all ($U' = 0$) then there is no difference between the regimes of lump-sum and per-reader charges for advertising. The reason for this is that with lump-sum charging there is no extra incentive for a platform to undercut its rival in the market for readers. While it is true that when platform A gains readership at the expense of platform B, B will still find that its advertising demand shrinks, this effect no longer gives A any advantage in the market for readers.

This discussion is summarized in the following:

Proposition 5 *In the model of informative advertising on media platforms, if advertisers do not compete between themselves for consumers, there will be too few adverts in equilibrium. If readers like (dislike) adverts, when platforms charge advertisers on a lump-sum basis the equilibrium reader price, and platform profit, is lower (higher) than when advertising charges are levied on a per-reader basis.*

In this stark model, advertisers do not gain or lose when the market for readers becomes more competitive. For instance, if two newspapers merge in this model, advertisers will not be made worse off since they will be offered the same monopoly prices in either event.

The model presented here is essentially an extension of Gabszewicz, Laussel, and Sonnac (2001). That paper proposes a model of the newspaper industry where readers single-home and advertisers multi-home. Therefore, although it is not emphasized in the paper, they find there is monopoly pricing for advertising and hence underprovision of advertising. Monopoly revenues from supplying advertising space are passed onto readers in the form of a subsidised cover price. Sometimes these revenues are so large that the newspaper is supplied to readers for free.[17] The main difference is that in the earlier paper, readers were assumed to be indifferent

[17] In contrast to the framework in the current paper, Gabszewicz, Laussel, and Sonnac (2001) have an initial stage in which newspapers choose their "political stance" before competing for readers and dealing with advertisers. When advertising revenues are so high that newspapers are offered for free they show that the newspapers will choose the same middle political stance, whereas if the newspapers compete in prices for readers they will choose to maximally differentiate.

to adverts ($U' = 0$ in the current notation). This implies that there is no difference to the outcome if lump-sum and per-reader advertising charges are used. As is clear from the previous discussion, when platforms use lump-sum advertising charges (which is arguably the more plausible scenario) the analysis is rather complicated when readers do care about advertising intensity.

Another related paper, in the context of the television industry, is Anderson and Coate (2005). For the most part they assume that viewers single-home and advertisers multi-home. It is arguable that single-homing assumption is less natural in the television context than in the newspaper context, especially since the introduction of the remote control, and this is why I focussed my discussion on newspapers and yellow pages. In contrast to Gabszewicz, Laussel, and Sonnac (2001), Anderson and Coate (2005) assume that viewers care (negatively) about advertising levels. They analyze both the case where viewers are not charged for viewing and, more relevant for the current paper, where viewers have to subscribe to a television channel. In the latter case they find that there is too little advertising in equilibrium for exactly the same reasons as outlined above. They assume that advertising charges are levied on a per-viewer basis, which avoids many of the complexities discussed above.

A third paper which deserves mention is Rysman (2004), who provides a structural empirical investigation of markets with two-sided network externalities. This paper estimates the importance of cross-group network effects (on both sides of the market) in the market for yellow pages. He estimates that externalities are significant on both sides of the market: consumers are more likely to use a directory containing more adverts, while an advertiser will pay more to place an advert in a directory that is consulted by more consumers.

5.3 Application: Supermarkets

A second application of the competitive bottleneck model is to supermarkets and other similar kinds of retailer. A commonly-held view about the supermarket sector is that, provided competition is vigorous, consumers are treated well by supermarkets but supermarkets deal aggressively with their suppliers. The model when applied to this industry can generate these stylized features.

Suppose two supermarkets compete to attract consumers. Consumers ("group 1") care both about the prices they pay and the range of products on offer. They will visit either one supermarket or the other (but not both) over the relevant time period. Suppose there is a continuum of monopoly products ("group 2") that could be supplied to either or both supermarkets. For simplicity, suppose that each consumer wishes to buy one unit of each product so long as the price of the product is less than their reservation value, α_1. (Thus, consumers view the various products as being equally valuable and as neither substitutes nor complements in their utility function.) Suppose that supermarkets incur a cost c for selling each unit of any product.[18] Supermarkets are assumed to set retail prices to their consumers

[18]With some extra complexity one could extend this simple framework to allow for per-consumer costs and

and to make take-it-or-leave-it offers to buy from their suppliers. In particular, supermarkets have all the bargaining power when dealing with their suppliers.

Suppose that the unit cost of supply for any product is unknown to a supermarket. Specifically, suppose that for each product the unit cost α_2 is independently and identically drawn from a distribution function $F(\alpha_2)$. From the supermarkets' viewpoint each supplier is *ex ante* identical, and so a given supermarket will make the same offer to all suppliers to buy at the per-unit price p_2. (Note that, unlike other models discussed, this is a payment from the platform *to* the group 2 agents. Note also that this context makes it very natural to assume that platforms make their payments to suppliers on a per-transaction rather than a lump-sum basis.) The number of suppliers who will agree to this level of compensation is $F(p_2)$. If a supermarket sets a retail price p_1 per unit to consumers, a consumer's utility from visiting the supermarket is the number of products multiplied by the net surplus per product:

$$u_1 = F(p_2)(\alpha_1 - p_1) \ , \tag{34}$$

whereas the supermarket's profit per consumer is

$$\pi = F(p_2)(p_1 - c - p_2) \ . \tag{35}$$

Regardless of how competition for consumers affects their utility u_1, a supermarket will choose p_1 and p_2 to maximize its profit per consumer, subject to delivering the required utility u_1. Expressions (34) and (35) then imply that

$$p_2 \text{ maximizes } F(p_2)(\alpha_1 - c - p_2) \ . \tag{36}$$

As with all the competitive bottleneck models, the equilibrium maximizes the joint surplus of the platforms and the single-homing group (supermarkets and consumers in this case, as given in expression (36)), and the interests of the multihoming side (the suppliers) are ignored. This level of payment in (36) will exclude some high-cost suppliers whose presence in the supermarkets is nevertheless socially desirable. (A supplier should supply if $\alpha_2 < \alpha_1 - c$, whereas the equilibrium price p_2 is strictly lower than $\alpha_1 - c$ and so supply is inefficiently restricted.) In other words, payments to suppliers are too low from a social point of view and there are too few products on the shelves.

How well consumers are treated depends on the competitive conditions on their side. If they choose their supermarket according to the Hotelling specification in (28), then one can show that their equilibrium utility u_1 is given by

$$u_1 = F(p_2)(\alpha_1 - c - p_2) - t \ ,$$

so that consumers keep the joint surplus $F(p_2)(\alpha_1 - c - p_2)$ except for the market power element t retained by the supermarkets.[19]

---

for per-product-line costs.

[19] An explict expression for the equilibrium per-unit price to consumers is then

$$p_1 = c + p_2 + \frac{t}{F(p_2)} \ .$$

As with the previous model of informative advertising, the treatment of suppliers is not affected by the strength of competition between supermarkets for consumers. In this model, if two supermarkets merge, consumers would be treated less favourably but suppliers would not be affected. Their payment p_2 in (36) anyway is exactly the payment which would be chosen if there were a single monopoly supermarket.

In this model, supermarkets act to reverse the bargaining asymmetry that consumers might otherwise have in their dealings with the monopoly suppliers.[20] For instance, suppose that without the institution of supermarkets the suppliers sell directly to consumers. (Again, though, there are two "shopping centres" in the same locations as the supermarkets were, and suppliers sell in both of these areas. For consistency with the supermarket analysis, suppose also that each supplier incurs the selling cost c per unit in addition to its cost α_2.) In this case, those suppliers with cost below the consumer reservation price α_1 would choose to supply at the monopoly price $p_1 = \alpha_1$. (Because there is a continuum of suppliers, each supplier sets its price without regard for the effect its high price has on the number of consumers who visit the local shopping area.) This move to direct supply clearly makes consumers worse off, although it does improve efficiency since the competitive bottleneck is overcome and the range of products supplied is efficient.[21]

The same logic suggests that suppliers are likely to be better off when they act independently than when they integrate to form a supermarket. Independent suppliers do not take account of the negative effect on other suppliers in the same shopping area when they set high prices. When suppliers merge to form a supermarket they internalize the effect that each supplier's pricing decision has on the overall attractiveness of the shopping centre for consumers, and this makes competition more intense. Lack of coordination between independent suppliers acts as a kind of commitment to price high in a shopping area, and this boosts profits.[22]

5.4 Intra-Group Competition

An interesting issue is the equilibrium extent of competition between retailers *within* platforms. For instance, a TV channel might charge more for a car advert if it promised not to show a

[20] The competitive bottleneck model assumes that supermarkets hold all the bargaining power with suppliers. Dobson and Waterson (1997) analyze a model where supermarkets bargain with suppliers about supply prices, and where a merger between two supermarkets improves the bargaining position of supermarkets and so drives down the equilibrium supplier prices. In some extreme cases they show a merger between supermarkets might lead to lower retail prices for consumers: the decrease in input prices outweighs the enhanced ability to exploit consumers.

[21] The change from a situation where suppliers sell their products independently and directly to consumers as opposed to selling to a supermarket which then sells on to consumers, corresponds to the distinction made in Smith and Hay (2005) between "streets" and "supermarkets". They propose a slightly different model and find more generally that the comparison of the number of products supplied by a supermarket and by a street is ambiguous.

[22] See Beggs (1994) for further analysis of this point.

rival manufacturer's advert in the same slot. Or a shopping mall might be able to charge a higher rent to a retailer with the promise that it will not let a competing retailer into the same mall. Implementing competition within a platform will often mean that retailers' prices and profits are lower than they would be with monopoly retailers.[23] Thus we expect that if the platform allowed retailing competition it would make less money from the retailing side of the market but more money from the consumer side (if it charged consumers for entry). A plausible hypothesis is: platforms will allow competition within the platform if consumers can be charged for entry, but if consumers have free entry then platforms will restrict competition in order to drive up the revenues obtained from retailers.

This topic deserves a separate paper to itself. Here I merely describe an illustrative, stylized example to show the plausibility of the above hypothesis. Suppose that platforms can serve any number of consumers and retailers costlessly (so $C(n_1, n_2)$ in section 5.1 is equal to zero). If consumers receive utility u_1^i from platform i, the market share of platform i is given by expression (28). Suppose there is a single, homogeneous product supplied by a group of identical retailers. If the retail price for this product is P, each consumer demands a quantity $q(P)$ of the product. Let $v(P)$ be the consumer surplus associated with this demand function. Each retailer has a cost C for supplying a unit of the product (exclusive of payments to platforms). Suppose there is no fixed cost associated with a retailer locating in a given platform (other than the platform charge for entry).

A platform must decide whether to have retailing competition or not (i.e., whether to have more than one retailer on the platform). Suppose that a retailer chooses its price on a platform to reflect competitive conditions on that platform. (That is to say, retailers can price discriminate from one platform to the other.) If there is competition on the platform, then the price of the product on the platform is equal to marginal cost C. If there is no competition on the platform, the product's price will be above cost, and the price-cost markup will depend on the extent of competition *between* the platforms.

Suppose first that platforms can charge consumers for access to the platform. In this case it is optimal for platform i to choose to have competition on the platform. To see this, suppose that the rival platform offers consumers a utility u_1^j. (It does not matter if this utility is achieved by means of competition within the rival platform or not.) If the product costs P^i on platform i and consumers must pay an entry fee p_1^i to the platform, then consumers obtain utility $u_1^i = v(P^i) - p_1^i$ from that platform. Therefore, given (28), the joint profit of platform

[23] Stahl (1982) presents an interesting analysis of a shopping centre where a firm's profits might increase if another firm which supplies (imperfect) substitutes also locates in the shopping centre. The reason is that the increased variety of products available attracts more consumers to the shopping centre, and this market expansion effect can outweigh the more intense competition within the shopping centre. (See also Schulz and Stahl (1996) for related analysis in a search model.) This effect is not possible in the current analysis, where retailing competition, if it is implemented, is assumed to drive retailer profits to zero.

i and the retailer(s) on this platform is

$$\left(\frac{1}{2}+\frac{v(P^i)-p_1^i-u_1^j}{2t}\right)\times\left(q(P^i)(P^i-C)+p_1^i\right) \ .$$

Re-writing this in terms of $u_1^i = v(P) - p_1^i$ this joint profit is

$$\left(\frac{1}{2}+\frac{u_1^i-u_1^j}{2t}\right)\times\left(q(P^i)(P^i-C)+v(P^i)-u_1^i\right) \ .$$

Clearly, for any (u_1^i, u_1^j) this joint profit is maximized by choosing $P^i = C$, and, moreover, this profit is entirely appropriated by the platform. Therefore, when consumers can be charged for access to the platform, the competitive option is the most profitable way for a platform to generate any given level of consumer utility. The platform then makes all its profit from the consumer side, and it does not choose to restrict competition among retailers. This is a "dominant strategy" and does not depend on the particular choice of utility u_1^i that the platform offers its consumers nor on whether the rival platform chooses to have competition. Finally, it is straightforward to show that the equilibrium charge for access by consumers to the platforms is $p_1 = t$ (and the price for the product on the platforms is $P = C$).

By contrast, suppose for some exogenous reason that platforms cannot charge consumers for access, and so they must make their profit from the retailer side of the market. (For instance, television channels historically could not charge viewers for access, and so had to fund their service entirely from the advertising side.) The only way a platform can set a positive charge to a retailer in the present stark framework is if the retailer is a monopoly, and so in this case platforms must restrict competition in order to obtain any revenue at all. The monopoly retailer on platform i will make profits

$$\left(\frac{1}{2}+\frac{v(P^i)-v(P^j)}{2t}\right)\times q(P^i)(P^i-C)$$

if it sets the price P^i while the monopoly on the rival platform sets the price P^j. One can then show that the equilibrium product price P is close to the monopoly price (i.e., the price that maximizes $q(P)(P-C)$) when t is large and is close to cost C when t is small. One can also show that consumers in this framework are always worse off when platforms can charge them for access compared to when they have free access, despite the fact that they pay a lower price for the product once they are on the platform.[24]

One paper that relates to this discussion is Dukes and Gal-Or (2003). They have a model where competing producers place adverts on competing media platforms. They assume that the platforms are broadcasters who do not charge viewers/listeners for access to the platform. Consistently with the above discussion, they find that a media platform (usually) sells advertising space exclusively to a single firm in the producer oligopoly.

[24] See Corollary 1 in Armstrong and Vickers (2001).

5.5 Discussion

In section 2 I discussed two important precursors to this paper. We rejoin this discussion to point out the relationships between the three competitive bottleneck models in the papers.

First, consider Proposition 11 in Caillaud and Jullien (2003). In the notation of section 4 of the current paper, those authors discuss a model where there is no intrinsic product differentiation between the two platforms, where group 1 agents have a benefit α_1 from interacting with each group 2 agent and where group 2 agents each have a benefit α_2 from interacting with each group 1 agent. (Thus there is no variation in α_2 as there was in this section.) Caillaud and Jullien show that there is a symmetric equilibrium of the following form (provided that $\frac{1}{2}\alpha_2 > f_2$). Group 1 single-homes and divides equally between the two platforms, while all group 2 agents join both platforms. The price to group 1 equals their cost, $p_1 = f_1$, while the price to group 2 fully extracts their surplus, so that $p_2 = \frac{1}{2}\alpha_2$. This forms an equilibrium because a platform has no incentive to undercut its rival on either side of the market. If the platform sets a price $p_2 < \frac{1}{2}\alpha_2$ then this has no effect on group 2's choices, and will not boost demand from that side. If the platform sets a price $p_1 < f_1$ then this will attract all group 1 agents but it will not affect demand by group 2 and so this deviation will reduce the platform's profits given that the price is below cost.[25] Thus we see that positive profits can be sustained in equilibrium even when two identical platforms compete. The crucial feature of this model of Caillaud and Jullien is that demands are discontinuous in prices: a small price reduction to group 1 means that the platform attracts the entire set of group 1 agents, and this feature implies that it can never be optimal to set a price $p_1 > f_1$. (Due to the finite cross-elasticities in the present model, there is no reason to rule out above-cost pricing to the single-homing side.) However, a small price reduction to the multi-homing side has no affect on demand, and this provides the source of the profits in this industry.

Second, consider section 6 of Rochet and Tirole (2003). Unlike the rest of their paper, this section allows platforms to compete in two-part tariffs. However, there is a subtle problem with the analysis in this section. Using the notation of section 4.2.2 of the current paper, suppose that platform i offers tariffs given by (16). Rochet and Tirole suppose that platforms can be taken to compete in the "per-transaction" prices P_1^i and P_2^i, defined by

$$P_1^i = \gamma_1^i + \frac{p_1^i - f_1}{n_2^i} \ ; \ P_2^i = \gamma_2^i + \frac{p_2^i - f_2}{n_1^i} \ .$$

However, since these prices depend on the platform's performance on the other side, it is not clear what it means for platforms to "choose" these prices. Specifically, it is true that for a given pair of two-part tariffs offered by platform j, platform i's payoff only depends on its own tariffs via the summary prices P_1^i and P_2^i above. However, for the reasons outlined in section

[25] Here one important issue is not discussed. If the deviating platform simultaneously reduces p_1 and increases p_2 there are multiple consistent demand configurations, and for the stated prices to form an equilibrium a particular choice for the demand configuration needs to be made. See Armstrong and Wright (2004) for further discussion of this issue.

4.2.2 above, platform i's particular choice of two-part tariffs (among those tariffs with the same per-transaction prices P_1^i and P_2^i) *does* matter for platform j, since it affects j's incentive to deviate. Therefore, it seems that it cannot be a valid procedure to collapse a two-part tariff into a scalar "per-transaction price" in this manner.[26]

There are at least three limitations to my analysis of competitive bottlenecks. First, in the applications I made the simplifying assumption that the population of group 1 agents was constant. Thus, the fact that this group tends to be treated favourably in equilibrium has no effect on expanding the number of such agents who choose to participate. If instead there were a market expansion effect then this would make group 2 better off since they then have more group 1 agents to interact with. In principle it is perhaps possible that this effect could imply that the number of group 2 agents served is not too small from a social point of view. However, this turns out not to be possible.[27]

Second, I made the convenient assumption that *no* group 1 agents multi-homed. A richer model would allow for some agents to multi-home (for instance, some people read two newspapers, some people might go to one supermarket for some products and another supermarket for other products, and so on). Platforms then no longer have a monopoly over providing access by group 2 to these multi-homing group 1 agents. So far little progress has been made in extending the analysis to these mixed situations, and this is fruitful topic for future research.[28]

Third and finally, I did not consider a platform's incentive to require an otherwise multi-homing agent to deal with it *exclusively*. It is plausible in the context of the competitive bottleneck model that a platform might try to sign up group 2 agents exclusively, in order to give it an advantage in the market for group 1 agents. Of course, if platforms succeed in forcing group 2 agents to decide to deal with one platform *or* the other, then platforms will find themselves in the two-sided single-homing situation analyzed in section 4. Because network effects are so strong in that situation, it is plausible that platforms find their *equilibrium* profits decrease when they force group 2 to deal exclusively.[29]

APPENDIX

In this appendix I supply the proof of Proposition 3. Suppose that platform i's tariffs take the form (16) above. If a group 1 agent goes to platform i she will obtain utility

$$u_1^i = (\alpha_1 - \gamma_1^i)n_2^i - p_1^i \tag{37}$$

and so the number of group 1 and group 2 agents who join platform i is

$$n_1^i = \frac{1}{2} + \frac{(\alpha_1 - \gamma_1^i)n_2^i - (\alpha_1 - \gamma_1^j)(1 - n_2^i) - (p_1^i - p_1^j)}{2t_1} \tag{38}$$

[26] Note that this issues does not arise with a monopoly platform, as analyzed in Rochet and Tirole (2004).

[27] See section 3.1.3 in Armstrong (2002) for this analysis in the context of telecommunications.

[28] See section 7.1 of Anderson and Coate (2005) for a first step in this direction.

[29] See Armstrong and Wright (2004) for further analysis of exclusive contracts within this framework.

$$n_2^i = \frac{1}{2} + \frac{(\alpha_2 - \gamma_2^i)n_1^i - (\alpha_2 - \gamma_2^j)(1 - n_1^i) - (p_2^i - p_2^j)}{2t_2} \tag{39}$$

By solving this pair of equations in n_1^i and n_2^i one obtains the following explicit formulas for n_1^i and n_2^i in terms of the eight price parameters:

$$n_1^i = \frac{1}{2} + \frac{1}{2}\frac{\left(2\alpha_1 - \gamma_1^i - \gamma_1^j\right)\left(2p_2^j - 2p_2^i + \gamma_2^j - \gamma_2^i\right) + t_2(4p_1^j - 4p_1^i + 2\gamma_1^j - 2\gamma_1^i)}{4t_1t_2 - (2\alpha_1 - \gamma_1^j - \gamma_1^i)(2\alpha_2 - \gamma_2^j - \gamma_2^i)}$$

$$n_2^j = \frac{1}{2} + \frac{1}{2}\frac{\left(2\alpha_2 - \gamma_2^i - \gamma_2^j\right)\left(2p_1^j - 2p_1^i + \gamma_1^j - \gamma_1^i\right) + t_1(4p_2^j - 4p_2^i + 2\gamma_2^j - 2\gamma_2^i)}{4t_1t_2 - (2\alpha_1 - \gamma_1^j - \gamma_1^i)(2\alpha_2 - \gamma_2^j - \gamma_2^i)}$$

Equilibrium prices are determined by the sensitivities of market shares to changes in the various prices. To determine symmetric equilibria, we need to calculate the derivative of market shares with respect to changes in prices, evaluated when the two platforms set the same quadruple of prices $(p_1, p_2, \gamma_1, \gamma_2)$:

$$\frac{\partial n_1^i}{\partial p_1^i} = -\frac{t_2}{2\Delta} \; ; \; \frac{\partial n_1^i}{\partial \gamma_1^i} = -\frac{t_2}{4\Delta} \tag{40}$$

$$\frac{\partial n_2^i}{\partial p_2^i} = -\frac{t_1}{2\Delta} \; ; \; \frac{\partial n_2^i}{\partial \gamma_2^i} = -\frac{t_1}{4\Delta} \tag{41}$$

$$\frac{\partial n_1^i}{\partial p_2^i} = -\frac{\alpha_1 - \gamma_1}{2\Delta} \; ; \; \frac{\partial n_1^i}{\partial \gamma_2^i} = -\frac{\alpha_1 - \gamma_1}{4\Delta} \tag{42}$$

$$\frac{\partial n_2^i}{\partial p_1^i} = -\frac{\alpha_2 - \gamma_2}{2\Delta} \; ; \; \frac{\partial n_2^i}{\partial \gamma_1^i} = -\frac{\alpha_2 - \gamma_2}{4\Delta} \tag{43}$$

where $\Delta = t_1t_2 - (\alpha_1 - \gamma_1)(\alpha_2 - \gamma_2)$. Notice that in each case, the effect of small changes in γ is exactly half the effect of small changes in the corresponding p. The reason for this is that, with equal market shares, the effect on the total charge an agent must pay with a change in γ is half that with a change in p.

Platform i's profits are

$$\pi^i = (\gamma_1^i n_2^i + p_1^i - f_1)n_1^i + (\gamma_2^i n_1^i + p_2^i - f_2)n_2^i \; . \tag{44}$$

Notice that expressions (40)–(43) imply that at any symmetric set of prices we have

$$\frac{\partial \pi^i}{\partial p_1^i} = 2\frac{\partial \pi^i}{\partial \gamma_1^i} \; ; \; \frac{\partial \pi^i}{\partial p_2^i} = 2\frac{\partial \pi^i}{\partial \gamma_2^i} \; . \tag{45}$$

This feature of the market will generate multiple symmetric equilibria, as we will see.

First, we show that platform i's objective function is concave given its rival's choice $(p_1^j, p_2^j, \gamma_1^j, \gamma_2^j)$, as long as γ_1^j and γ_2^j are non-negative, and that the maintained assumption (8) holds. We need to show this so that we can characterize equilibria in terms of the first-order conditions in the following. Usually, verifying that a function of four variables is concave is a tedious matter. However, in this context, we can easily reduce the number of i's strategic variables to two, which greatly simplifies the calculation. Given the rival prices $(p_1^j, p_2^j, \gamma_1^j, \gamma_2^j)$, it turns out the i's profits are a function only of the utilities u_1^i and u_2^i it offers its consumers. When it offers this pair of utilities then it will attract a certain number n_1^i and n_2^i of each group — we will shortly derive this relationship explicitly — and by combining expressions (37) and (44) we see that its total profit can be written in terms of the utilities as

$$\pi^i = (\alpha_1 n_2^i - u_1^i - f_1) n_1^i + (\alpha_2 n_1^i - u_2^i - f_2) n_2^i \, . \tag{46}$$

That is to say, any choice of (p_1^i, γ_1^i) that leaves u_1^i unchanged in (37) generates the same profits for the platform. One implication of this is that a platform has a continuum of best responses to its rival's choice of prices. If we show that platform i's profits are concave in (u_1^i, u_2^i) then we have done what is required.

First we need to derive platform i's market shares as a function of its offered utilities. Similarly to expressions (38)—(39) we have

$$n_1^i = \frac{1}{2} + \frac{u_1^i - ((\alpha_1 - \gamma_1^j)(1 - n_2^i) - p_1^j)}{2t_1}$$

$$n_2^i = \frac{1}{2} + \frac{u_2^j - ((\alpha_2 - \gamma_2^j)(1 - n_1^i) - p_2^j)}{2t_2} \, .$$

Solving this pair of equations gives the following explicit expressions for market shares:

$$n_1^i = \frac{1}{2} + \frac{(\alpha_1 - \gamma_1^j)(u_2^i - (\frac{\alpha_2 - \gamma_2^j}{2} - p_2^j)) + 2t_2(u_1^i - (\frac{\alpha_1 - \gamma_1^j}{2} - p_1^j))}{4t_1 t_2 - (\alpha_1 - \gamma_1^j)(\alpha_2 - \gamma_2^j)}$$

$$n_2^i = \frac{1}{2} + \frac{(\alpha_2 - \gamma_2^j)(u_1^i - (\frac{\alpha_1 - \gamma_1^j}{2} - p_1^j)) + 2t_1(u_2^i - (\frac{\alpha_2 - \gamma_2^j}{2} - p_2^j))}{4t_1 t_2 - (\alpha_1 - \gamma_1^j)(\alpha_2 - \gamma_2^j)} \, .$$

Notice that these market share functions are linear in utility, and therefore profit in (46) is quadratic in utility. Profit is concave in utility if (i) $\partial^2 \pi^i / \partial (u_1^i)^2 < 0$ and (ii) determinant of the matrix of second derivatives of π is positive. Some tedious calculations then show that (i) holds if

$$4t_1 t_2 > (\alpha_1 + \alpha_2)(\alpha_2 - \gamma_2^j) + (\alpha_1 - \gamma_1^j)(\alpha_2 - \gamma_2^j)$$

which is true when $\gamma^j \geq 0$ and the maintained assumption holds. Also, (ii) holds if

$$16 t_1 t_2 > \left(2\alpha_1 + 2\alpha_2 - \gamma_1^j - \gamma_2^j\right)^2$$

which holds under the same assumptions. We deduce that a platform's choice problem is concave.

Next, we characterize the symmetric equilibria. Suppose that the two platforms choose the same pair of per-user charges (γ_1, γ_2). The first-order condition $\partial\pi^i/\partial p_1^i = 0$ evaluated at the symmetric fixed charges $p_1^j = p_1^i$ and $p_2^j = p_2^i$ yields

$$p_1 = f_1 + t_1 - \frac{\gamma_1 + \gamma_2}{2} - \frac{\alpha_2 - \gamma_2}{t_2}\left(\alpha_1 + p_2 - f_2 + \frac{\gamma_2 - \gamma_1}{2}\right) ,$$

and similarly

$$p_2 = f_2 + t_2 - \frac{\gamma_1 + \gamma_2}{2} - \frac{\alpha_1 - \gamma_1}{t_1}\left(\alpha_2 + p_1 - f_1 + \frac{\gamma_1 - \gamma_2}{2}\right) .$$

Solving this pair of simultaneous equations in p_1 and p_2 implies that expressions (17) are satisfied.

References

Anderson, S., and S. Coate (2005): "Market Provision of Broadcasting: A Welfare Analysis," *Review of Economic Studies*, forthcoming.

Armstrong, M. (2002): "The Theory of Access Pricing and Interconnection," in *Handbook of Telecommunications Economics: Volume I*, ed. by M. Cave, S. Majumdar, and I. Vogelsang. North-Holland, Amsterdam.

Armstrong, M., and J. Vickers (2001): "Competitive Price Discrimination," *Rand Journal of Economics*, 32(4), 579–605.

Armstrong, M., and J. Wright (2004): "Two-Sided Markets, Competitive Bottlenecks and Exclusive Contracts," mimeo.

Beggs, A. (1994): "Mergers and Malls," *Journal of Industrial Economics*, 42(4), 419–428.

Caillaud, B., and B. Jullien (2003): "Chicken and Egg: Competition Among Intermediation Service Providers," *Rand Journal of Economics*, 34(2), 309–328.

Dobson, P., and M. Waterson (1997): "Countervailing Power and Consumer Prices," *Economic Journal*, 107, 418–430.

Dukes, A., and E. Gal-Or (2003): "Negotiations and Exclusivity Contracts for Advertising," *Marketing Science*, 22(2), 222–245.

Gabszewicz, J., D. Laussel, and N. Sonnac (2001): "Press Advertising and the Ascent of the 'Pensée Unique'," *European Economic Review*, 45, 641–651.

KLEMPERER, P., AND M. MEYER (1989): "Supply Function Equilibria in Oligopoly Under Uncertainty," *Econometrica*, 57(6), 1243–1277.

ROCHET, J.-C., AND J. TIROLE (2003): "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 1(4), 990–1029.

——— (2004): "Two-Sided Markets: An Overview," mimeo.

RYSMAN, M. (2004): "Competition Between Networks: A Study of the Market for Yellow Pages," *Review of Economic Studies*, 71(2), 483–512.

SCHULZ, N., AND K. STAHL (1996): "Do Consumers Search for the Highest Price? Oligopoly Equilibrium and Monopoly Optimum in Differentiated Products Markets," *Rand Journal of Economics*, 27(3), 542–562.

SMITH, H., AND D. HAY (2005): "Streets, Malls and Supermarkets," *Journal of Economics and Management Strategy*, 14(1), 29–59.

STAHL, K. (1982): "Location and Spatial Pricing Theory with Nonconvex Transportation Cost Schedules," *Bell Journal of Economics*, 13(2), 575–582.

VAN RAALTE, C., AND H. WEBERS (1998): "Spatial Competition with Intermediated Matching," *Journal of Economic Behavior and Organization*, 34, 477–488.

WRIGHT, J. (2002): "Access Pricing under Competition: an Application to Cellular Networks," *Journal of Industrial Economics*, 50(3), 289–316.