Inefficiencies in Bargaining: Departing from Akerlof and Myerson-Satterthwaite

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Abstract

We consider bargaining problems in which parties have access to outside options. The size of the pie is commonly known and each party privately knows the realization of her outside option. Parties are assumed to have a veto right, which allows them to obtain at least their outside option payoff in any event. Besides, agents can receive no subsidy ex post. We show that inefficiencies are inevitable for virtually all distributions of outside options, as long as the size of the surplus generated by the agreement is uncertain and may be arbitrarily small for all realizations of either party's outside option. Our inefficiency result holds true whatever the degree of correlation between the distributions of outside options, and even if it is known for sure that an agreement is beneficial. The same insights apply to the bargaining between a buyer and a seller privately informed of their valuations and to public good problems among agents privately informed of their willingness to pay.

1 Introduction

Private information is known to induce inefficiencies in a number of important economic applications. A well known illustration follows from the

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celebrated impossibility theorem of Myerson and Satterthwaite (1983): Bargaining between a seller and a buyer must result in inefficiencies when each agent privately knows the valuation for herself of the item for sale, there is some uncertainty as to which agent values the good more, and, most importantly, valuations are independently distributed between the seller and the buyer. Similar conclusions arise for other applications like the provision of public goods or bargaining situations in which private information bears on the outside option. But, a key feature of all such results is that private information should be independently distributed between the various agents.

In many applications, the assumption that private information is independently distributed across agents seems very demanding. For example in the seller/buyer problem the seller and the buyer may know that they have similar tastes, resulting in positive correlations of the valuations. In bargaining with outside options, if the environment is competitive, a signal that a party has a good outside option may indicate that the other party has a poor outside option, resulting in negatively correlated distributions of outside options. It is thus of practical importance to understand the effect of private information when correlations are allowed.

The main contribution of this paper is to show that whenever parties can exert a veto right and get their reservation value at any point in time, inefficiencies are inevitable, even if the distributions of private signals are correlated between agents, as long as agents can receive no outside subsidy.

The idea that parties can exert their veto right at any point in time is novel in the mechanism design literature in which it is generally assumed that once an agent has agreed to participate in the mechanism he has no further right to quit.[1] Under the usual interim participation constraints, efficiency can be obtained in the correlated case even without subsidy (from an ex ante viewpoint). This follows from the work of Crémer and McLean (1986) (see also Myerson (1981) and Johnson et al. (1990)). But, when agents keep their right to quit at any point in time (as is assumed with the

[1] Ex post participation constraints are sometimes examined. Note however that such constraints are usually combined with a dominant strategy implementation requirement. Besides, as we will later emphasize, the veto right idea is not equivalent to imposing ex post participation constraints.

veto right idea) inefficiencies are inevitable even in the correlated case.

The veto right idea is well suited to deal with those applications in which parties never make binding decisions until a complete agreement is ratified by all interested parties. In the seller/buyer bargaining problem this means that in any event the seller must get at least her valuation and the buyer must get a non-negative payoff. In the bargaining with outside option application, this means that in any event parties must get at least their outside option payoffs. Our inefficiency result applies to the equilibria of any game (whether one-shot or multi-stage) in which each party keeps the right to quit and gets her reservation value (the one that she can obtain on her own without the consent of other parties) before the final agreement is implemented.

The veto rights obviously limit the set of transfers that can possibly be implemented. It is thus not surprising that when the distributions of private information are almost independent between agents inefficiencies must arise as in the case of independent distributions.[2]

But, our inefficiency result does not solely arise for small degrees of correlation. We prove that inefficiencies are inevitable for virtually all distributions of private information whatever their degree of correlation. This should be thought of as a surprising result given that the veto right constraints a priori leave significant room for complex transfer schemes between agents.

Our paper can thus be viewed as providing a strong argument as to why private information, even if correlated among agents, is a source of inefficiency. We note that inefficiencies may arise even in those cases in which it is known for sure what the best alternative is.[3] This observation is reminiscent of another celebrated result due to Akerlof (1970), the lemon's problem.

[2]The bounds on transfers implied by ex post veto constraints would immediately deliver an impossibility result in the almost independent case, by application of continuity arguments, as in Robert (1991) who considers the case of limited liability and risk-aversion. See also Laffont-Martimort (2000) for a different approach based on collusion among agents.

[3]In our private value setup, such an impossibility result may only arise when the distributions of private information are correlated among agents. If the distributions are independent, inefficiencies may only arise when there is some uncertainty about what the best alternative is (see Myerson and Satterthwaite (1983)).

But, unlike Akerlof' s lemon problem our setup is one with private values and the logics between our and Akerlof's results are completely different.

2 Some illustrative applications

We consider three classes of situations: the seller/buyer problem, the public good problem and a multi-person bargaining setup. In each situation, we will show that under incomplete information if parties keep their right to withdraw from the interaction until an explicit and complete deal is being made - this will be referred to as a veto right - then inefficiencies are inevitable. Remarkably, the result holds true even if the private information held by the various agents is correlated and whatever the degree of correlation.

The seller/buyer problem: Agent 1, the seller, owns an object which he considers selling to agent 2, the buyer. The seller's valuation for the object is given by v_S; the buyer's valuation for the object is given by v_B. The seller knows his valuation v_S but not that of the buyer v_B. Symmetrically, the buyer knows her valuation v_B, but not that of the seller v_S. Agents also know that (v_B, v_S) is drawn from a joint distribution with support on $(0, \overline{v})^2$. Correlations between v_S and v_B are *a priori* allowed.

We are interested in whether bargaining between the seller, the buyer and possibly an intermediary might lead the good to be efficiently allocated, i.e. to agent 1 (the seller) whenever $v_S > v_B$ and to agent 2 (the buyer) whenever $v_B > v_S$. We will show under fairly general conditions that if the seller and the buyer can receive no subsidy ex post (i.e. the sum of side-payments received by the two agents can never exceed 0) efficiency cannot be achieved whenever each agent must get at least his reservation utility in any event (that is, in any event the seller must get at least v_S and the buyer must get at least 0).

This inefficiency result is reminiscent of that of Myerson and Satterthwaite (1983), but our setup differs from theirs in two fundamental respects. First, we allow for correlations between the distributions of the seller's and the buyer's valuations, and the analysis of Myerson and Satterthwaite does not apply to the correlated case.[4] Second, we require that the agents should in any event approve the deal after the deal has been proposed. This is not the usual assumption made in mechanism design; generally agents are asked to decide whether or not to participate before knowing the terms of the trade. Our assumption that agents must get their reservation utility in any event is reminiscent of the idea of ex post participation constraint, but it is not equivalent. In fact, ex post participations constraints are implied by our veto right assumption, but the veto right constraints also affect the nature of the incentive constraints, since when considering a deviation an agent should anticipate that he will always keep the option of getting his reservation utility. We will elaborate on this point when we develop the formalism in the next section.

The public good problem: A representative must decide whether or not to build a public good. There are n agents $i = 1, ...n$. The cost of the public good is C. Agent i values the public good at $\theta_i \in (\underline{\theta}, \overline{\theta})$. Each agent i knows the value of θ_i, but not of θ_j, $j \neq i$. Everybody knows that $(\theta_1, ..., \theta_n)$ is distributed according to a joint distribution on $(\underline{\theta}, \overline{\theta})^n$, and we assume that $n\bar{\theta} - C \leq \overline{\theta} - \underline{\theta}$. That is, the maximum surplus from the public good does not exceed the uncertainty about any agent's valuation for the public good.[5] Here, again, we allow for any correlations between the willingness to pay of the various agents. Efficiency would require to build the public project whenever $\sum_i \theta_i > C$, and we assume that the community cannot receive ex post subsidies (that is, the sum of financial payments made by the agents must be at least equal to the cost C of the public good).[6]

Our analysis will show that efficiency cannot be achieved whenever agents

[4] Virtually the whole mechanism design literature relying on Bayes-Nash implementation assumes that signals are independently distributed accross agents. Besides, the results of Crémer and McLean or McAfee and Reny all suggest that in the correlated case the first-best can be achieved.

[5] This reflects the idea that a single agent's lack of enthusiasm for the public project may undermine the desirability of making the public project.

[6] We also assume that building the public good requires the consent of every agent.

have the right to veto the public project (thereby enjoying a reservation utility of 0). As in the previous application, inefficiency is inevitable even if the distributions of willingness to pay are correlated and whatever the degree of correlation.

Bargaining with Outside Options: There are n parties $i = 1, 2, ...n$ bargaining over the division of a pie of size V. Each party i has an outside option w_i where $w_i \in [0, V]$. That is, if the parties do not reach an agreement, party i gets w_i. The values of $w = (w_1, w_2, ...w_n)$ are not commonly known. Party i (but not party j, $j \neq i$) knows the realization of w_i. We let $g(w)$ denote the joint density of w on $[0, V]^n$. Efficiency requires that an agreement be reached when $\sum_i w_i < V$ but not when $\sum_i w_i > V$.

Suppose that no subsidy can be received ex post. That is, in case of agreement the sum of payments received by all parties cannot exceed the size of the pie V. We will show that efficiency cannot be achieved whenever parties can at any point in time leave the bargaining table thereby enjoying their outside option. Again, our result applies even if the distributions of outside options are correlated and whatever the degree of correlation.

3 The Inefficiency Result

We will state our impossibility result in the bargaining with outside option application. We will later show how the other applications can be dealt with using the analysis of the bargaining with outside option application.

The bargaining protocols. A bargaining protocol is a process that generates a non-binding proposal, as a function of messages or information transmitted between parties and/or to a third party. Specifically, a non-binding proposal consists of a decision whether or not to share the pie, combined with tentative transfers. We assume that (i) final implementation requires ratification by all parties, and (ii) each party may quit bargaining at any stage, including right before ratification. These bargaining protocols capture bargaining situations in which tentative agreements are generated by agents who do not have the power to commit to make transfers in the course of bargaining. We will also assume that no third party can subsidize the bargaining parties, thus leading to a no subsidy constraint.

We will refer to such situations as *non-binding* bargaining protocols, as the parties are assumed to keep their veto right until a complete agreement is ratified by all parties.

In the mechanism design language to be developed next, the possibility of vetoing the proposal will imply (but will not be equivalent to assuming) that ex post participation constraints must be satisfied. We will further illustrate the differences between ex post participation constraints and ex post veto constraints (see subsection 5.3).

The main result. The following result summarizes a striking result that will be proven later on:

> **Theorem 1:** *Let* $\Gamma_v = \{w = (w_1, ...w_n) \mid \sum_i w_i < v, w_i > 0\}$, *fix* M *and* $m > 0$, *and consider the class* $\mathcal{G}_{M,m}$ *of distributions with compact support in* R_+^n, *that are bounded (by* M*), positive (no smaller than* $m > 0$*) and smooth (with derivatives bounded by* M*) on their support.*[7] *There exists* ε *such that for any* $g \in \mathcal{G}_{M,m}$ *and any* $v \geq V - \varepsilon$, *if the support of* g *contains* $\overline{\Gamma_v}$, *then inefficiencies must arise in equilibrium in any non-binding bargaining protocol.*

Observe that our inefficiency result holds if the support of g coincides with Γ_V in which case it is known for sure that an agreement is beneficial. It also holds for all distributions (correlated or not) with full support on $[0, V]^n$.

At this point, it may be worth stressing a few notable differences with the celebrated impossibility results obtained by Akerlof (1969) and Myerson and Satterthwaite (1983).

Akerlof (1969) considered a bargaining problem between a buyer and a seller. The seller is privately informed about the quality of the good, and

[7] By smooth, we mean that g is continuously differentiable with respect to each w_i , $i = 1, ...n$ on the support of g.

the quality affects the valuations of both the seller and the buyer. Moreover, the buyer is assumed to value the good more than the seller whatever the quality. In a beautiful and simple example, Akerlof shows that no trade can take place in equilibrium. Consider the result of Theorem 1 with a support of g that coincides with Γ_V. As in Akerlof's example, there is no uncertainty as to which alternative is best: an agreement is always beneficial. However, while Akerlof's model and logics crucially depend on the common value character of the payoff specification (i.e., the private information held by the seller affects the buyer's valuation), our model is one of private values, that is, each party's private information is irrelevant to determine the payoff of the other party in the various alternatives.[8] Thus, the logics of our result is radically different from that of Akerlof.[9]

Myerson and Satterthwaite (1983) considered a bargaining problem between a seller and a buyer who are assumed to know their valuation of the good. Hence it is a private value setup like our model. But, Myerson and Satterthwaite (1983)'s impossibility result crucially hinges on the facts that (1) the supports of valuations of the seller and of the buyer overlap - hence it is not common knowledge who values the good most, and (2) the distributions of seller and buyer's valuations are independent. This should be contrasted with our setup in which the distributions of outside options are not independent and there may be no uncertainty as to which alternative is best.[10] Our result can be viewed as providing a considerable generaliza-

[8] In the agreement alternative there is no uncertainty. In the outside option alternative, each party i is assumed to know w_i.

[9] Mailath and Postlewaite (1990) provide an interesting private (and correlated) value example in which it is common knowledge that the provision of a public good is efficient, and yet, no mechanism with fixed limited liability permits to implement it when the number of agent is large enough (the probability even tends to 0 as the number of agents tend to infinity). By contrast, our result does not rely on the number of agents being large, and the limited liability constraint is replaced by the veto constraint.

[10] If we assume that the distributions of w_i, $i = 1, 2$ are independent from each other, then we have a result similar to that of Myerson and Satterthwaite. That is, as soon as $\Pr(w_1 + w_2 > V) > 0$ there are inefficiencies, but not otherwise. To see the Myerson-Satterthwaite type of inefficiency, consider the Vickrey-Clarke-Groves mechanism such that the transfers associated with the outside option alternative are set to zero (hence the participation constraints are automatically satisfied). The associated transfer received by

tion (to the case of correlated distributions) of the fundamental insight that private information is a source of inefficiencies in bargaining (non-binding protocols).

It should be mentioned that the veto right that parties can exert at any time is essential for the derivation of our result. If we had allowed parties to surrender their veto rights (after joining the mechanism), then only interim participation constraints would need to be satisfied (as in most mechanism design works using Bayesian Nash implementation). But, ex post veto constraints somehow reduce the transfers that can be made for the various realizations of the outside options. This in turn translates into unavoidable inefficiencies (despite the correlation), as we show. Observe that our impossibility result does not solely arise for distributions of outside options that are nearly independent. It arises for virtually all distributions whatever their degree of correlation. Thus, our result goes far beyond the simple observation that ex post veto constraints impose a continuous transition from the independent distribution case to the correlated distribution case (due to the induced bounds on transfers). It establishes in a strong way that private information even if correlated among agents is an inevitable source of inefficiency in non-binding bargaining protocols.

We have already mentioned that the requirement of ex post veto constraints is different from the more usual one of ex post participation constraints. To illustrate the difference we will note that when the support of the distribution $g(\cdot)$ coincides with $\overline{\Gamma_V}$, ex post participation constraints alone (together with the Bayesian Nash incentive constraints and the ex post no subsidy constraints) is consistent with efficiency (see subsection

party i in the agreement alternative should be set equal to $t_i = V - \widehat{w}_j$ where $\widehat{w}_j$ denotes the announcement of party j's outside option. It is readily verified that if the efficienct allocation is chosen on the basis of the announced types, it is a dominant strategy to report honestly his true type. The problem is about the budget constraint. Whenever the agreement is optimal, i.e. $w_1 + w_2 > V$, the total transfer recievd by parties 1 and 2 should be $t_1 + t_2 = V + (V - w_1 - w_2) > V$. Hence, the budget constraint cannot be met in this mechanism. By the allocation equivalence principle, it is also immediate to check that no mechanism that induces efficiency can satisfy both the participation constraints of the parties and the budget constraint. (See Williams (1999) or Krishna-Perry (2000) for a related point in the original setup of Myerson-Satterthwaite (1983)).

5.3). Contrast this with the result of Theorem 1. The essential reason for this difference is that veto rights can be exerted off the equilibrium path in our setup, which in turn affects the form of the incentive constraints (see below).

4 The Mechanism Design Approach

To analyze our bargaining problem it is convenient to develop a mechanism design approach. We first develop some preliminary definitions, and then develop our main result. Applications are discussed next.

4.1 Preliminaries

In order to prove the Theorem, it is useful to use a mechanism design approach. The revelation principle tells us that there is no loss of generality in looking at direct truthful mechanisms (we will be more explicit about how to apply the revelation principle in the next section). That is, any equilibrium outcome of any game (whether static or dynamic) can be viewed as the (equilibrium) outcome of a static game in which parties are asked to simultaneously reveal their private information and each party finds it optimal to report her true information assuming other parties do. Thus, proving that no direct truthful mechanism allows to induce an efficient outcome is enough to prove that no mechanism whatsoever permits to get an efficient outcome.

Formally, a direct mechanism takes the form that each party i is asked to report a valuation $\widehat{w}_i$. Based on the profile of reports $\widehat{w}$ it is decided whether an agreement should be proposed where the agreement includes the specifications of monetary transfers $t_i(\widehat{w})$ to each party i.

Before an agreement is effectively implemented we assume that each party has the option to quit, thereby enjoying her outside option - such a possibility will be referred to as a veto right option and will in turn give rise to veto right constraints. More precisely, the veto right option can be modelled as resulting in a ratification stage: after the proposal is made, parties sequentially[11] decide whether they accept the agreement or not. If

[11] The sequentiality is only meant to avoid coordination problems that would be caused

all parties accept, the agreement is implemented; otherwise parties get their outside options.

The analysis of the ratification stage is pretty straightforward. Based on the proposal $(t_1(\widehat{w}), ...t_n(\widehat{w}))$ party i with outside option w_i says "yes" if $t_i(\widehat{w}) > w_i$ and "no" if $t_i(\widehat{w}) < w_i$.[12] The key feature of the ratification game is that a party with outside option w_i can always secure a payoff of w_i whatever the profile $\widehat{w}$ of announcements made at the announcement stage by deciding to reject the agreement at the ratification stage. This feature referred to as the veto right constraint will play a major role in our analysis (and will be met in any game in which parties may decide to opt out at any point in time).

We also assume that our n parties can receive no subsidy ex post. That is, if they agree on the division of the pie with a transfer $t_i(\widehat{w})$ to party i, we require that

$$\sum_i t_i(\widehat{w}) \leq V. \tag{1}$$

Observe that we allow for situations in which the entire pie V is not fully distributed to the agents, i.e. $\sum_i t_i(\widehat{w}) < V$. This allows us to cover applications in which a third party (say an intermediary) may extract some surplus from offering a division of the pie.[13]

We consider direct mechanisms of the above form in which it is an equilibrium to report the true private information $\widehat{w}_i = w_i$ at the announcement stage. That is, for every party i we let $U_i(\widehat{w}_i; w_i)$ denote the expected payoff obtained by party i in the above game when party i's outside option is w_i, party i's announcement is $\widehat{w}_i$ and party i expects other parties j, $j \neq i$ to report truthfully $\widehat{w}_j = w_j$; and we require that

$$U_i(w_i; w_i) \geq U_i(\widehat{w}_i; w_i). \tag{2}$$

by the simultaneous refusal of several agents.

[12] Cases in which $t_i(\widehat{w}) = w_i$ will play no role in our analysis.

[13] Third parties are often thought of as helping achieving better outcomes in bargaining, and many practical negotiations do include the presence of third parties or mediators or arbitrators. It is thus of importance to be able to cover such applications. Of course, our inefficiency result holds *a fortiori* is we further impose that the surplus should be entirely distributed, i.e. $\sum_i t_i(\widehat{w}) = V$.

We ask ourselves whether there can be a mechanism satisfying the above constraints and at the same time results in an efficient outcome whatever the realizations $w = (w_i)_{i=1}^{i=n}$ of the outside options.

In our bargaining setup efficiency means that an agreement should be reached when $\sum_i w_i < V$ and the outside option should be chosen when $\sum_i w_i > V$. To simplify the exposition (even though this is inessential for the derivation of our result) we will require that an agreement be also reached whenever $\sum_i w_i = V$.[14]

Ex post veto constraints imply ex post participation constraints. Thus assuming parties report truthfully and agreement should be reached, each party should get at least her outside option. Formally, let Γ^g denote the support of g. For efficiency to be possible, it should be that for any $w \in \Gamma^g \cap \Gamma_V$, an agreement is proposed and satisfies:

$$t_i(w) \geq w_i. \tag{3}$$

But, ex post veto constraints also have an effect on the analysis of the incentive constraints (2). Still assuming that efficiency can be achieved, suppose that party i makes a false announcement $\widehat{w}_i$. Assuming other parties j report their true type $\widehat{w}_j = w_j$, an agreement should be proposed and accepted by all $j \neq i$ whenever $(\widehat{w}_i, w_{-i}) \in \Gamma^g \cap \Gamma_V$ (note that for such an announcement profile (3) applies to each $j \neq i$). Party i should agree as well whenever $t_i(\widehat{w}_i, w_{-i}) > w_i$ and say no whenever $t_i(\widehat{w}_i, w_{-i}) < w_i$, thereby resulting in a payoff of $\max(t_i(\widehat{w}_i, w_{-i}), w_i)$. It follows that (noting that party i can secure her outside option w_i in any event):

$$\begin{aligned} U_i(\widehat{w}_i; w_i) \quad \geq \quad & \int_{(\widehat{w}_i, w_{-i}) \in \Gamma^g \cap \Gamma_V} \max(t_i(\widehat{w}_i, w_{-i}), w_i) g_i(w_{-i} \mid w_i) dw_{-i} \\ & + \int_{(\widehat{w}_i, w_{-i}) \notin \Gamma^g \cap \Gamma_V} w_i g_i(w_{-i} \mid w_i) \, dw_{-i} \end{aligned} \tag{4}$$

where $g_i(\cdot \mid w_i)$ denotes the marginal density of w_{-i} given w_i.

On the other hand, making the true announcement $\widehat{w}_i = w_i$ and assuming efficiency can be achieved when everybody reports truthfully, one should

[14] This plays no role in our analysis because events such that $\sum_i w_i = V$ occur with probability 0.

have:

$$U_i(w_i; w_i) = \int_{w \in \Gamma^g \cap \Gamma_V} t_i(w_i, w_{-i}) g_i(w_{-i} \mid w_i) dw_{-i} + \int_{w \notin \Gamma^g \cap \Gamma_V} w_i g_i(w_{-i} \mid w_i) dw_{-i} \tag{5}$$

We will show that the above constraints (1)(2)(3)(4)(5) cannot be simultaneously satisfied, thereby showing our impossibility result.

4.2 Getting to the Inefficiency Result

The veto right constraint, together with the ex post no subsidy constraint, imply the following set of inequalities on transfers:[15]

$$w_i \leq t_i(w_i, w_{-i}) \leq V - \sum_{j \neq i} w_j. \tag{6}$$

Our approach consists in showing that incentive compatibility conditions require that the second inequality binds, i.e.:

$$t_i(w_i, w_{-i}) = V - \sum_{j \neq i} w_j. \tag{7}$$

That is, each party i must always get the residual surplus generated by the agreement assuming that all other parties are set to their reservation utility (their outside option payoff). Of course, this cannot be, as such transfer rules would result in the violation of the ex post no subsidy constraint for quite a range of outside option profiles (think of w_j being close to 0 for every j; all transfers t_i should then be close to V, leading to a violation of the no subsidy constraint). Thus, we will have shown that no mechanism whatsoever can implement the efficient allocation in our setup.

We will now explain why incentive compatibility conditions lead to equality (7).

A preliminary intuition.

To fix ideas, we consider two players, and examine a case where outside options have full support over the finite grid

$$G = \{(k_1 V/N, k_2 V/N), k_i \in \{0, ..., N\}\}.$$

[15] The second inequalities follows from $t_i(w) + \sum_{j \neq i} t_j(w) \leq V$ and $t_j(w) \geq w_j$ for all j.

This case is not covered by our main Theorem, but it will permit us to provide a simple intuition as to why our result holds.

Because the distribution over outside options has full support over $\Gamma_V \cap G$, an agreement should be proposed in any event where $(w_1, w_2) \in \Gamma_V \cap G$, that is, in any event where $k_1 + k_2 \leq N$. We wish to show that in any such event,

$$t_1(w_1, w_2) = V - w_2. \tag{8}$$

When $k_1 = N$ and $k_2 = 0$ (and more generally in any event where $k_1 + k_2 = N$), player 1's outside option w_1 coincides with the residual surplus $V - w_2$, so that there are no other choices than setting the transfer t_1 equal to $V - w_2$.

Now fix $k_1^0 \leq N$, and assume that for all $k_1 \geq k_1^0$ and $k_2 \leq N - k_1$, equality (8) holds. We will show below that equality (8) must also hold for all $k_1 \geq k_1^0 - 1$ and $k_2 \leq N - k_1$, thereby concluding the argument.

Agent 1 with outside option $w_1 = (k_1^0 - 1)V/N$ could consider reporting $\widehat{w}_1 = k_1^0 V/N$. For all realizations of w_2 that fall strictly below $V - w_1$ (that is, for all realizations $k_2 \leq N - k_1^0$), the induction hypothesis tells us that player 1 should get $V - w_2$, which is in any case the maximum payoff player 1 can hope to get. Now for the realizations of w_2 that coincide with or exceed $V - w_1$ (that is, when $k_2 \geq N - k_1^0 + 1$), player 1 cannot hope to get more than w_1, whether an agreement is proposed or not.

It follows that the announcement $\widehat{w}_1$ allows player 1 to extract all the residual surplus, hence the only way to provide player 1 with incentives to report w_1 truthfully is to give him that surplus even when he announces w_1, that is, to set the transfer t_1 equal to $V - w_2$ for all realizations of w_2 below or equal to $V - w_1$.

A general argument in the differentiable case.

The above argument while very simple relies on a specific discretization of the type space, and it does not extend in a straightforward way to other discretizations. We now provide an argument for the continuous type case. We will assume in the main text that the support of g contains Γ_V. To facilitate exposition, we will also assume that transfers are differentiable. In the Appendix, we show how the argument extends to possibly non-differentiable transfer functions, and to the case where g contains Γ_v with $v < V$, v close to V.

Because the distribution over outside options has a support that contains Γ_V, an agreement should be proposed in any event where $w \in \Gamma_V$.

We first derive a condition on transfers implied by incentive compatibility conditions. Party i should prefer reporting he is of type w_i rather than of type $\widehat{w}_i = w_i + \varepsilon$. When he reports $\widehat{w}_i$ (rather than w_i), he gains[16] $t_i(\widehat{w}_i, w_{-i}) - t_i(w_i, w_{-i})$ whenever $(\widehat{w}_i, w_{-i}) \in \Gamma_V$, and he loses no more than $t_i(w_i, w_{-i}) - w_i$ in events where $w \in \Gamma_V$ and $(\widehat{w}_i, w_{-i}) \notin \Gamma_V$. (In other events, there is no loss because he cannot expect more than his outside option payoff.) Incentive compatibility conditions thus require that

$$\int_{(\widehat{w}_i, w_{-i}) \in \Gamma_V} (t_i(\widehat{w}_i, w_{-i}) - t_i(w_i, w_{-i}))g(w)dw_{-i} \leq \int_{\substack{w \in \Gamma_V \\ (\widehat{w}_i, w_{-i}) \notin \Gamma_V}} (t_i(w) - w_i)g(w)dw_{-i} \tag{9}$$

When $(\widehat{w}_i, w_{-i}) \notin \Gamma_V$, the surplus is at most equal to ε. Since $t_i(w) - w_i$ cannot exceed the surplus, the right hand side of (9) is comparable to ε^2. Dividing by ε on both sides and taking the limit of this comparison as ε goes to 0 yields (thanks to the differentiability assumption on t_i):

$$\int_{(w_i, w_{-i}) \in \Gamma_V} \frac{\partial t_i}{\partial w_i}(w_i, w_{-i})g(w_i, w_{-i})dw_{-i} \leq 0. \tag{10}$$

> **Remark**: This inequality already implies that direct mechanisms with monotone transfers cannot achieve efficiency. But, a priori it does not rule out the possibility that more elaborate transfer schemes achieve efficiency.

Define the following function for every $w_i \in (0, V)$.

$$H_i(w_i) \equiv \int_{(w_i, w_{-i}) \in \Gamma_V} (V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}))g(w_i, w_{-i})dw_{-i} \tag{11}$$

We will prove that $H_i(w_i) = 0$ for all $w_i \in (0, V)$. Given that $V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}) \geq 0$ is non-negative (we know from (6) that $V - \sum_{j \neq i} w_j$ is the maximum transfer that party i can hope to get when each

[16] $t_i(\widehat{w}_i, w_{-i}) - t_i(w_i, w_{-i})$ could be negative; so it could be a loss.

party j's outside option is given by w_j), we will deduce that for all $(w_i, w_{-i}) \in \Gamma_V$:

$$t_i(w_i, w_{-i}) \equiv V - \sum_{j \neq i} w_j.$$

To establish that $H_i(\cdot) \equiv 0$, observe that[17]

$$\begin{aligned} \frac{dH_i(w_i)}{dw_i} &= -\int_{(w_i,w_{-i})\in\Gamma_V} \frac{\partial t_i}{\partial w_i}(w_i, w_{-i})g(w_i, w_{-i})dw_{-i} \\ &\quad + \int_{(w_i,w_{-i})\in\Gamma_V} (V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}))\frac{\partial g}{\partial w_i}(w_i, w_{-i})dw_{-i} \end{aligned}$$

Using (10) we get:

$$\frac{dH_i(w_i)}{dw_i} \geq \int_{(w_i,w_{-i})\in\Gamma_V} (V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}))\frac{\partial g}{\partial w_i}(w_i, w_{-i})dw_{-i} \quad (12)$$

But, note that

$$H_i(V) = 0,$$

since when $w_i = V$ the domain of w_{-i} such that $(w_i, w_{-i}) \in \Gamma_V$ has measure 0.

Thus, when $\frac{\partial g}{\partial w_i}(w_i, w_{-i}) \geq 0$ for all $w_i \in (0, V)$, (12) allows us to conclude that $\frac{dH_i(w_i)}{dw_i} \geq 0$ for all $w_i \leq V$. Since $H_i(w_i)$ is non-negative everywhere (by the no ex post subsidy requirement) and since $H_i(V) = 0$, we conclude that $H_i(w_i) = 0$ everywhere, as desired.

In the general case where the variations of g may be arbitrary, observe that the fact that g has a strictly positive lower bound on its support and that g varies smoothly with w_i guarantee that there must exist a constant a (possibly negative) such that for all $(w_i, w_{-i}) \in \Gamma_V$:

$$\frac{\partial g}{\partial w_i}(w_i, w_{-i}) > ag(w_i, w_{-i}).$$

Given the non-negativeness of $V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i})$, we infer from (12) that:

$$\frac{dH_i(w_i)}{dw_i} \geq aH_i(w_i).$$

[17] The term corresponding to the variation of the domain of integration does not appear because at the boundary the veto constraint together with the ex post no subsidy constraint imply that for w such that $\sum_j w_j = V$, $t_i(w_i, w_{-i}) = w_i$ and thus $V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}) = 0$.

Thus, $H_i(V) \geq \exp(a(V - w_i))H_i(w_i)$. Since $H_i(V) = 0$, and $H_i(w_i) \geq 0$, we conclude that $H_i(w_i) = 0$, as desired.

In the above argument we have restricted attention to differentiable transfer functions and we have assumed that the support of g contains Γ_V. In the appendix, we generalize the argument to the case of possibly non-differentiable transfer functions and to the case where the support of g contains Γ_v with v possibly less than V but close to V, thereby providing a complete proof for our Theorem.

4.3 On General Game Forms

The revelation principle:

Let us see now why the analysis presented above applies to any non-binding protocol in which parties may at any point in time get their outside option if they wish. First, observe that the above analysis can be extended to the case in which the transfer function is non-deterministic. This is because if efficiency could be obtained while satisfying the veto constraint, the ex post no subsidy constraint and the incentive compatibility constraint with a non-deterministic transfer scheme $\widetilde{t}_i$, it could a fortiori be obtained with a deterministic transfer scheme t_i defined as the expectation of $\widetilde{t}_i$ over the stochastic element in the transfer scheme.[18]

Next, consider any non-binding bargaining protocol, an equilibrium of the game associated with this protocol, and assume that it involves no inefficiencies. Denote by $\sigma_i(w_i)$ the strategy used by party i in equilibrium, when his outside option is w_i. To each strategy profile $(\sigma_i(w_i), \sigma_{-i}(w_{-i}))$, we may associate a probability that an agreement is proposed in stage k, and distributions of payments $\widetilde{\mathbf{t}}_i \mid_{(w_i, w_{-i})}$ in the agreement scenario. Assum-

[18] More formally, define the deterministic mechanism as follows: make a proposal if and only if $\widehat{w} \in \Gamma_V \cap \Gamma^g$ and a proposal is made with probability 1 under the stochastic mechanism, and let this proposal be defined as $t_i(\widehat{w}) = E[\widetilde{t}_i \mid \widehat{w}]$.

If the stochastic mechanism has the desired properties, then a proposal is made (and accepted) with probability one when $w \in \Gamma_V \cap \Gamma^g$. So t_i is defined on $\Gamma_V \cap \Gamma^g$. It is immediate to check that the ex post participation constraint and the ex post no subsidy constraint are satisfied. For the incentive constraint, note that $E[\max(\widetilde{t}_i, w_i) \mid (\widehat{w}_i, w_{-i})] \geq \max(E(\widetilde{t}_i \mid (\widehat{w}_i, w_{-i})), w_i)$, so if the stochastic mechanism is incentive compatible, the deterministic one is incentive compatible as well.

ing delay is costly, for the equilibrium to involve no efficiency, we should have that an agreement should be reached in stage 1 with probability one whenever $(w_i, w_{-i}) \in \Gamma^g \cap \Gamma_V$, and since the agreement should not be vetoed in equilibrium, we should have for $(w_i, w_{-i}) \in \Gamma^g \cap \Gamma_V$, and all transfer realizations $\widetilde{t}_i$ in the support of $\widetilde{\mathbf{t}}_i \mid_{(w_i, w_{-i})}$,

$$\widetilde{t}_i \geq w_i.$$

Consider now the strategy that consists in following $\sigma_i(\widehat{w}_i)$ during the first stage, and to exercise the outside option if no agreement is proposed by the end of this stage, or if the proposed agreement entails receiving a payment smaller than w_i. The expected payoff associated with that strategy when party i is of type w_i and party j, $j \neq i$ follows $\sigma_j(w_j)$ is denoted $U_i(\widehat{w}_i, w_i)$, and it satisfies:

$$\begin{aligned} U_i(\widehat{w}_i; w_i) \quad \equiv \quad & \int_{(\widehat{w}_i, w_{-i}) \in \Gamma^g \cap \Gamma_V} E[\max(\widetilde{t}_i, w_i) \mid (\widehat{w}_i, w_{-i})] g_i(w_{-i} \mid w_i) dw_{-i} \\ & + \int_{(\widehat{w}_i, w_{-i}) \notin \Gamma^g \cap \Gamma_V} w_i g_i \left(w_{-i} \mid w_i\right) dw_{-i} \end{aligned}$$

Because strategies are in equilibrium, the deviations above must be deterred, which implies that conditions $U_i(w_i; w_i) \geq U_i(\widehat{w}_i; w_i)$ hold for all w_i, $\widehat{w}_i$. It follows that the direct mechanism defined by the transfer rules $\widetilde{t}_i$ must be an efficient direct truthful mechanism with veto rights. But, we have seen that no such mechanism exists, thereby showing that no equilibrium of any non-binding bargaining protocol whatsoever can induce an efficient outcome.

Assuming it is common knowledge that an agreement is beneficial:

One important insight of Myerson-Satterthwaite in the uncorrelated case is that inefficiencies arise when and only when it is not common knowledge which alternative is best. In contrast, under the assumption of Theorem 1, inefficiencies arise whether or not it is common knowledge that agreement is beneficial. But, even more is true. Consider any distribution g for which it is not known for sure that an agreement is beneficial, i.e. $\Gamma^g \nsubseteq \Gamma_V$. It is easy to see that if the parties were told whether or not the agreement is beneficial, it would not help them increase expected welfare (in the best non-binding protocol mechanism).[19] Intuitively, the veto constraint and the no subsidy constraint together imply that in any event where the agreement is not beneficial, each party must get his outside option payoff and no more: so it is irrelevant when players learn it.

Relatedly, a simple two-stage procedure can be used to elicit information about whether the agreement is beneficial, as explained below.

We start from a situation in which it is not known for sure that an agreement is beneficial, i.e. $\Gamma^g \not\subseteq \Gamma_V$, and we consider a direct truthful mechanism, defined by a proposal schedule $o(\widehat{w})$ specifying a probability of agreement proposal and transfer functions as a function of the profile of announcement. After a proposal is made, parties sequentially report if they accept the proposal or if they prefer going for their outside option. Remember that we also assume that no subsidy ex post is allowed, which places some constraints on the set of admissible proposals (i.e., in case of agreement, the sum of transfers cannot exceed V).

Consider now the following two-stage procedure. In stage 1, each party i simultaneously announces $\widehat{w}_i^{(1)}$(say, to a third party). If an agreement is found to be beneficial on the basis of these stage 1 announcements (i.e. if $\sum_i \widehat{w}_i^{(1)} \leq V$) one moves to stage 2. Otherwise, parties are requested to go for their outside option. In stage 2, each party i simultaneously announces $\widehat{w}_i^{(2)}$. On the basis of stage 2 announcements, the proposal schedule $o(\widehat{w}^{(2)})$ as defined in the original direct truthful mechanism is made to the parties. Then parties report sequentially if they accept or refuse the proposal.

It is easy to check that in this two-stage mechanism it is an equilibrium for each party i to report truthfully in both stages, i.e. $\widehat{w}_i^{(1)} = \widehat{w}_i^{(2)} = w_i$.[20] Thus, in this equilibrium, stage 1 permits to elicit information about

[19]Indeed, assume by contradiction that there were a direct truthful mechanism generating a strictly higher expected welfare when parties are first told whether the agreement is beneficial (so that players now know that outside options are distributed on $\Gamma^g \cap \Gamma_V$). The mechanism stipulating the same transfers and allocations when $\widehat{w}$ belongs to $\Gamma^g \cap \Gamma_V$, and no agreement and no transfer otherwise remains incentive compatible whether or not parties are told if the agreement is beneficial, and it yields the same expected welfare in both cases.

[20]Intuitively, when $\sum_i w_i > V$ the veto right coupled with the absence of subsidy ex post forces each party i to get exactly her outside option. Thus, separating first the

whether the agreement is beneficial, so that whenever one reaches stage 2 it is common knowledge that an agreement is beneficial. This comment thus gives some appeal to a practice often used in the decisions about whether or not to implement public projects, which generally includes a first stage in which investigations are made solely to determine whether the public project is worthwhile or not.

5 Discussion

5.1 Other Applications

Given that our results have been stated in the bargaining with outside option application, it may be worth explaining how our inefficiency result applies to the seller/buyer problem and to the public good problem set in Section 2.

The seller/buyer problem: Call $p_i(\widehat{v})$ the payment to agent i, $i = S, B$ (it may be negative) in exchange for a trade between the seller and the buyer after the announcements $\widehat{v}_B$ and $\widehat{v}_S$ are made by the buyer and the seller, respectively. Ex post veto rights mean that in any event the

situations in which there is no other choice than getting the outside option has no effect on the overall parties' incentives to report truthfully their private information.

More formally, assume in our two-stage procedure that all parties j, $j \neq i$ report their true outside option in both stages 1 and 2, and let $(\widehat{w}_i^{(1)}, \widehat{w}_i^{(2)})$ denote the reports of party i in stages 1 and 2, respectively. One might worry that party i's stage 1 announcement allows party i to gain extra information on parties j, $j \neq i$ private information in case one moves to stage 2, which party i could exploit in stage 2. We claim however that no reports $(\widehat{w}_i^{(1)}, \widehat{w}_i^{(2)})$ can do strictly better than (w_i, w_i) for party i, given that reporting the truth in the original truthful mechanism is an equilibrium. Suppose by contradiction that $(\widehat{w}_i^{(1)}, \widehat{w}_i^{(2)})$ does strictly better. If $\widehat{w}_i^{(1)} \leq \widehat{w}_i^{(2)}$, party i gets the same payoff as the one he would have obtained by announcing $\widehat{w}_i = \widehat{w}_i^{(2)}$ in the original direct truthful mechanism. So the deviation to $\widehat{w}_i = \widehat{w}_i^{(2)}$ in the original direct truthful mechanism should have been strictly beneficial. If $\widehat{w}_i^{(1)} > \widehat{w}_i^{(2)}$, then player i gets even less than the payoff he would have obtained by announcing $\widehat{w}_i = \widehat{w}_i^{(2)}$ in the original direct truthful mechanism, because he only obtain w_i in events where $\sum_{j \neq i} w_j + \widehat{w}_i^{(1)} > V \geq \sum_{j \neq i} w_j + \widehat{w}_i^{(2)}$. So *a fortiori*, a deviation $\widehat{w}_i = \widehat{w}_i^{(2)}$ in the original direct truthful mechanism should have been strictly beneficial.

seller must receive at least her valuation v_S in case of transaction (that is, $p_S(\widehat{v}) \geq v_S$ in case of trade) and that the buyer must get at least 0 in any event (that is, $v_B + p_B(\widehat{v}) \geq 0$ in case of trade). The ex post no subsidy constraint means that the sum of monetary transfers received by the seller and the buyer cannot exceed 0 ($p_S + p_B \leq 0$).

This trade problem can be cast into a bargaining problem with outside options, where the size of the pie V, outside options and transfers are defined as follows: $V = \overline{v}$, $w_S = v_S$, $w_B = \overline{v} - v_B$, $t_S(w) = p_S(v)$ and $t_B(w) = \overline{v} + p_B(v)$. It is readily verified that the inefficiency result in the seller/buyer problem is equivalent to the inefficiency result in this bargaining with outside option problem.[21]

The public good problem: Let $p_i(\widehat{\theta})$ denote the payment requested from agent i when the profile of announcements is $\widehat{\theta}$. The ex post veto right means that an agent i with type θ_i will refuse to make any payment greater than θ_i. The no subsidy constraint means that for any $\widehat{\theta}$ one should have $\sum_i p_i(\widehat{\theta}) \geq C$. Efficiency means that the public good should be implemented whenever $\sum_i \theta_i > C$.

No mechanism whatsoever permits the implementation of the efficient decision rule whenever $(\theta_1, ...\theta_n)$ is distributed on $(\underline{\theta}, \overline{\theta})^n$ where we assume that $0 < n\overline{\theta} - C < \overline{\theta} - \underline{\theta}$ and the density is assumed to be smooth and bounded by a strictly positive number on its support.

This can be seen as a corollary of Theorem 1 where we define the bargaining problem $V = n\overline{\theta} - C$, with outside options $w_i = \overline{\theta} - \theta_i$.

The transfers in the bargaining problem $t_i(\widehat{w})$ should be identified with $\overline{\theta} - p_i(\widehat{\theta})$, and it is readily verified that the incentive constraints and veto right constraints in the bargaining problem are identical to the incentive constraints and veto right constraints in the public good problem, thereby establishing the inefficiency in the public good decision problem as a corollary of Theorem 1.

[21] Indeed, $w_S + w_B < V$ is equivalent to $v_S < v_B$; the no subsidy constraint $t_S(w) + t_B(w) \leq V$ is equivalent to $p_S(v) + p_B(v) \leq 0$; the ex post participation contraints $t_S(w) \geq w_S$ and $t_B(w) \geq w_B$ are respectively equivalent to $p_S(v) \geq v_S$ and $p_B(v) + v_B \geq 0$.

5.2 Differences with Crémer-McLean

Correlations between the distributions of outside options of the various parties were allowed in our setup. It may be instructive to review how the analysis of Crémer-McLean (which was extended to cover the continuous type case by McAfee and Reny (1992)) would apply to our setup.

To fix ideas, assume that the support of g coincides with Γ_V and that g is bounded from below by a strictly positive number on its support (this ensures that there must be some correlation between the distributions of outside options of the parties).

The Vickrey-Clarke-Groves mechanism can be described as follows. Each party i is asked to report her outside option $\widehat{w}_i$. If $\sum_i \widehat{w}_i > V$ parties are requested to go for their outside option with no transfer being made. If $\sum_i \widehat{w}_i \leq V$, an agreement is proposed in which party i receives $V - \sum_{j \neq i} \widehat{w}_j$. The payments ensure that party i's interest is aligned with the social interest. Ignoring the participation constraints, it is a weakly dominant strategy for party i to report her true outside $\widehat{w}_i = w_i$. Let us denote by $w_i + \pi_i(w_i)$ the expected payoff obtained by party i that results from the Vickrey-Clarke-Groves mechanism in which participation is assumed to be compulsory. That is, in addition to w_i party i receives $\pi_i(w_i)$ in expectation.

As noted by Clarke and Groves, the above mechanism continues to have its nice truthtelling incentives, even if one subtracts from party i 's payment a transfer function that applies to all possible alternatives (i.e. agreement or outside option) and that depends solely on the reports made by parties j other than i. Let us denote this extra transfer by $z_i(\widehat{w}_{-i})$. So based on the announcements, if the outside option is chosen party i gets $-z_i(\widehat{w}_{-i})$ and if the agreement is chosen party i gets $V - \sum_{j \neq i} \widehat{w}_j - z_i(\widehat{w}_{-i})$.

Following Cremer and McLean the next step is to observe that when the distributions of w_i are correlated it is possible to find z_i functions such that[22]

$$\pi_i(w_i) = E_{w_{-i}}(z_i(w_{-i}) \mid w_i). \tag{13}$$

[22] Under our assumptions, the support of outside options is monotonic and thus the existence of such functions is automatically obtained (see McAfee and Reny 1992 for a general analysis of the continuous type case).

This observation leads to the well known full rent extraction result. Suppose an intermediary were to organize the bargaining between parties $i = 1, ...n$. Suppose further that each party i must decide at the interim stage (when she knows w_i only) whether she agrees or not to participate in the mechanism, but after she agrees party i has no right to leave the mechanism (this is a key difference with our setup in which party i is assumed to keep a right to veto the agreement at any point in time). Then by proposing the Vickrey-Clarke-Groves mechanism augmented by the z_i transfer functions, the intermediary can ensure that each party i exactly gets her outside option payoff by participating. So every party i will choose to participate and the intermediary will keep the entire surplus for himself.

Since the outcome of this mechanism is efficient (the agreement is reached whenever it is efficient), one may wonder how this result relates to our impossibility result (Theorem 1). There are essentially two points of departure. First, as repeatedly emphasized we assume that parties keep the right to veto the agreement. Thus, if the intermediary had to let the parties opt out whenever they wish he could not extract a positive payment $z_i(\widehat{w}_{-i}) > 0$ from party i. This in turn considerably limits the set of admissible transfers available to the intermediary and it makes it impossible to satisfy (13). Second, we assume that parties can receive no subsidy ex post (i.e. in the agreement scenario the sum of transfers received by parties cannot exceed V). In the Cremer-McLean mechanism, the fact that parties are set to their outside option utility ensures that ex ante parties receive no subsidy, but there is no guarantee that there is no subsidy ex post. We will illustrate later how efficiency can sometimes be obtained in our setup if we only require that there is no subsidy ex ante (while maintaining the veto right constraints).

Another notable difference between the Crémer-McLean mechanism and our approach is that Crémer-McLean rely on mechanisms implementable in dominant strategy whereas our notion of incentive constraints is Bayesian (or interim) rather than in dominant strategy.

In some setups it is believed that there is no major difference between Nash-Bayes implementation and dominant strategy implementation. But, this is not so in our setup with veto rights and no subsidy ex post.

Contrasting Nash-Bayes implementation with dominant strategy implementation

To illustrate the claim that Nash-Bayes implementation and dominant strategy implementation do differ, we first establish an impossibility result for mechanisms implementable in dominant strategy. Of course, this is no surprise given the stronger result of impossibility for Bayes-Nash implementation. But, this will allow us in turn to illustrate that sometimes efficiency can be obtained with Nash-Bayes implementation whereas it is impossible with dominant strategy implementation.

Formally, let $U_i(\widehat{w}_i, w_{-i}; w_i)$ denote party i's ex post payoff when party i announces he is of type $\widehat{w}_i$ while every party j, $j = i, -i$ is of type w_j. Implementation in dominant strategy implies that the following constraints hold: for all w_i, $\widehat{w}_i$, w_j,

$$U_i(w_i, w_j; w_i) \geq U_i(\widehat{w}_i, w_j; w_i). \tag{14}$$

Besides these new constraints we still assume that parties have veto rights and that there is no subsidy ex post. We have:

Proposition 1 *Suppose the distribution of* $(w_i)_{i=1}^n$ *contains*

$$\Gamma_v = \left\{ (w_i)_{i=1}^n \mid \sum_i w_i \leq v \right\}.$$

The efficient outcome cannot be implemented in dominant strategy while satisfying the veto right constraints and the (ex post) no subsidy constraint whenever $v > V/n$.

Proof. Suppose $nv > V$ and efficiency can be achieved. This implies that for any $(w_i, w_{-i}) \in \Gamma_v$, an agreement is proposed and $t_i(w_i, w_{-i}) \geq w_i$. This implies

$$U_i(w_i, w_{-i}; w_i) = t_i(w_i, w_{-i})$$

and, for all $\widehat{w}_i < v - \sum_{j \neq i} w_j$,

$$U_i(\widehat{w}_i, w_{-i}; w_i) = \max(t_i(\widehat{w}_i, w_{-i}), w_i)$$

Constraints (14) thus imply that for all $\widehat{w}_i < v - \sum_{j \neq i} w_j$,

$$t_i(w_i, w_{-i}) \geq \max(t_i(\widehat{w}_i, w_{-i}), w_i)$$

hence, since $t_i(\widehat{w}_i, w_{-i}) \geq \widehat{w}_i$ when $\widehat{w}_i < v - \sum_{j \neq i} w_j$,

$$t_i(w_i, w_{-i}) \geq v - \sum_{j \neq i} w_j.$$

It thus follows that

$$\sum_i t_i(0) \geq nv > V,$$

and the no subsidy constraint cannot be satisfied. ■

Remark: Note that this result does not follow from Hagerty-Rogerson (1987), who establish a connection between fixed price mechanisms and mechanisms implementable in dominant strategy in the case of ex post budget balanced transfers. Here, we only require that parties receive no subsidy ex post, not that the entire surplus be split between the two parties.[23]

We now turn to an example in which efficiency can be obtained under Bayes-Nash implementation even if g has full support on Γ_v for some $v > V/n$, thus showing that in our context with veto rights and ex post no subsidy, efficiency may sometimes be obtained with Bayes-Nash implementation while it cannot with dominant strategy implementation.

Assume there are two parties $i = 1, 2$ and consider the bargaining protocol defined as follows (in the rest of the paper, this protocol will be referred to as the *Nash Bargaining protocol*). In the first stage, each party i, $i = 1, 2$ simultaneously announces an outside option $\widehat{w}_i$. If these announcements are compatible, that is, if the sum $\widehat{w}_1 + \widehat{w}_2$ does not exceed V, an agreement is proposed, along with transfers $\tau_1(\widehat{w}_1, \widehat{w}_2)$ and $\tau_2(\widehat{w}_1, \widehat{w}_2)$ chosen so that each party i obtains, in addition to $\widehat{w}_1$, half the surplus $V - \widehat{w}_1 - \widehat{w}_2$, that is

$$\tau_i(\widehat{w}_1, \widehat{w}_2) = \widehat{w}_i + \frac{V - \widehat{w}_1 - \widehat{w}_2}{2} = \frac{V + \widehat{w}_i - \widehat{w}_j}{2}$$

In case the sum $\widehat{w}_1 + \widehat{w}_2$ exceeds V, bargaining stops, and each party gets his outside option. In the second stage, parties sequentially report if they accept the deal. If both parties say "yes", the deal is implemented. Otherwise, the outside option alternative is implemented.

Clearly, in the second stage, it is a dominant strategy for party i with outside option w_i to say "yes" (respectively, "no") if $\tau_i > w_i$ (respectively,

[23] In the no subsidy scenario, allocations other than those corresponding to fixed sharing rules can be implemented in dominant strategy.

$\tau_i < w_i$). The following proposition characterizes the equilibrium of the outside option announcement stage.

Proposition 2 *Suppose* (w_1, w_2) *is uniformly distributed on*

$$\Gamma_v = \{(w_1, w_2) \mid w_1 + w_2 \leq v\}.$$

with $3V/4 \leq v \leq V$*. The following is an equilibrium of the outside option announcement game: party* i *with type* w_i *announces* $\widehat{w}_i = a(w_i)$ *where*

$$a(w_i) = \frac{1}{4}V + \frac{2}{3}w_i.$$

There is agreement when $w_1 + w_2 \leq \frac{3V}{4}$*. The outside option alternative is implemented when* $w_1 + w_2 > \frac{3V}{4}$*.*

So one Corollary of Proposition 2 is that when $v = 3V/4$, full efficiency can be obtained. This shows that the best (i.e. welfare maximizing) mechanism need not in general be implementable in dominant strategy (see Proposition 1 and note that $3V/4 > V/2$).

5.3 Ex post participation versus ex post veto constraints.

In this Subsection, we examine the role of ex post veto constraints. To see why these constraints play an important role in our analysis, we now relax them and only impose the standard ex post participation constraints. In a direct truthful mechanism that would be efficient, ex post participation constraints require that

$$t_i(w_i, w_{-i}) \geq w_i \text{ for all } i \text{ and } (w_i, w_{-i}) \in \Gamma_V.$$

In particular, they do *not* impose any constraints on transfers when announcement fall outside the support of g, which, we will assume here, coincides with Γ_V. So parties may be punished when announcements fall outside the support of g.

To illustrate the implication of this observation, assume there are two parties $i = 1, 2$ and consider the Nash bargaining protocol described earlier. This protocol may be amended by assuming that whenever the announcement profile $(\widehat{w}_1, \widehat{w}_2)$ lies outside Γ_V, both players are severely punished, say by an amount equal to P. Then it is easy to check that when P is large enough, each party has incentives to report his own outside option truthfully, and efficiency results. Ex post participation constraints are satisfied, because for any possible realization of (w_1, w_2), party i of type w_i gets at least w_i. Hence ex post participation constraints alone are not sufficient to undermine efficiency.

More generally, consider an arbitrary number n of parties and assume that the support of outside options coincides with Γ_V where g is bounded from below by a strictly positive number on its support. Consider any profile of differentiable transfers $t_i(w_i, w_{-i})$ satisfying

$$\begin{aligned} t_i(w_i, w_{-i}) &\geq w_i \text{ for all } i \text{ and all } (w_i, w_{-i}) \in \Gamma_V, \text{ and} \\ w_i &\rightarrow t_i(w_i, w_{-i}) \text{ is increasing in } w_i \text{ for all } (w_i, w_{-i}) \text{ in } \Gamma_V \end{aligned}$$

We are going to show that it is possible to implement the efficient outcome (i.e agreement iff $w \in \Gamma_V$). Indeed, choose P large, and set

$$t_i(w_i, w_{-i}) = -P \text{ if } \ (w_i, w_{-i}) \notin \Gamma_V$$

Since $w_i \rightarrow t_i(w_i, w_{-i})$ is increasing in w_i, if all parties $j \neq i$ report their type truthfully, party i of type w_i has no incentives to understate his outside option. He has no incentives to overstate his outside option when the following inequalities hold for all $\widehat{w}_i > w_i$

$$\begin{aligned} \int_{(w_i, w_{-i}) \in \Gamma_V} t_i(w_i, w_{-i}) g(w_i, w_{-i}) dw_{-i} &\geq \int_{(\widehat{w}_i, w_{-i}) \in \Gamma_V} t_i(\widehat{w}_i, w_{-i}) g(w_i, w_{-i}) dw_{-i} \\ &\quad + (w_i - P) \int_{(\widehat{w}_i, w_{-i}) \notin \Gamma_V} g(w_i, w_{-i}) dw_{-i} \end{aligned}$$

or equivalently:

$$\int_{(\widehat{w}_i, w_{-i}) \in \Gamma_V} (t_i(\widehat{w}_i, w_{-i}) - t_i(w_i, w_{-i})) g(w) dw_{-i} \leq \int_{(\widehat{w}_i, w_{-i}) \notin \Gamma_V} (P + t_i(w_i, w_{-i}) - w_i) g(w) dw_{-i} \tag{15}$$

Let m be a lower bound on g and M an upper bound on $\frac{\partial}{\partial w_i} t_i$ and on g, then inequalities (15) are satisfied when the following inequality holds:

$$M^2 V(\widehat{w}_i - w_i) \leq Pm(\widehat{w}_i - w_i)$$

Thus, picking $P > \frac{M^2 V}{m}$ ensures that efficiency can be obtained if only ex post participation constraints are required.

This result illustrates that ex post participation and ex post veto constraints are quite different. Ex post participation imposes that in equilibrium, players do not *regret* not having exercised their outside option. In case player i deviates however, he does not have the option of going out, and a penalty may be imposed on him. In contrast, ex post veto constraints capture the possibility that a party would use his outside option *strategically* in the bargaining process, pretending to be another type, and yet keeping the option of going out.

5.4 Ex post no subsidy versus Ex ante no subsidy.

So far we have assumed that parties could receive no subsidy ex post. One may wonder what happens if we only require that the parties receive no subsidy ex ante. We wish to illustrate here that for some distributions over outside options, efficiency can be achieved while satisfying the ex post veto constraints, if only the ex ante no subsidy constraint is required.

To this end, we assume there are two parties $i = 1, 2$, and we consider a distribution over outside options defined as follows.[24] With probability $p > 0$, outside options are distributed according to a density g_0 with full support on Γ_V. With probability $1 - p$, outside options are distributed uniformly on $F = \{(w_1, V - w_1), w_1 \in [0, V]\}$. We construct below transfers that implement the efficient outcome.

Specifically, we set

$$t_i(w_1, w_2) = w_i \text{ when } w_1 + w_2 < V$$

and

$$t_i(w_1, w_2) = w_i + T(w_i) \text{ when } w_1 + w_2 = V$$

Intuitively, the idea is to subsidize agreement ex post by a substantial amount $T(w_i)$ whenever the announcement falls on the frontier. When party

[24] The example falls outside the class of distributions covered in Theorem 1. Yet, we conjecture that a slight modification would allow us to provide an example falling in this class.

i overstates his outside option, and announces $\widehat{w}_i > w_i$, he obtains a transfer equal to $\widehat{w}_i$ instead of w_i with probability $p \Pr_{g_0}\{w_j < V - \widehat{w}_i \mid w_i\}$. However, with probability $(1-p)$, he loses the subsidy. So choosing the subsidy $T(w_i)$ so that

$$(1-p)T(w_i) = p \max_{\widehat{w}_i}(\widehat{w}_i - w_i) \Pr_{g_0}\{w_j < V - \widehat{w}_i \mid w_i\} \tag{16}$$

ensures that party i has incentives to report his outside option truthfully.

Having defined $T(w_i)$ for all w_i, it remains to check whether ex ante, these subsidies remain smaller than the expected surplus generated by the agreement. To do that, it is sufficient to check that conditional on each w_i, the expected subsidy $(1-p)T(w_i)$ is smaller than half the expected surplus, that is,

$$(1-p)T(w_i) \leq \frac{1}{2} p E_{g_0}(V - w_i - w_j \mid w_i). \tag{17}$$

It is easy to check that (16) and (17) are compatible for some distribution g_0. [25]

5.5 What's next?

The above analysis has shown that inefficiencies are inevitable (even if the private informations held by the various parties are correlated) whenever parties can exert their veto right at any point in time and parties can receive no subsidy ex post. The next step would be to analyze the form of the second-best in such situations.

It should be mentioned that unlike in the independent distribution case the mechanism design approach pursued in this paper does not allow for an easy characterization of the second-best in general. (The elegant techniques developed by Myerson and followers do not extend to the case of correlated

[25] For example, if $g_0(w_1, w_2) = p_0 w_1 w_2$, one obtains

$$\begin{aligned}(1-p)T(w_i) &= p p_0 w_1 \max(\widehat{w}_i - w_i)\frac{(V - \widehat{w}_i)^2}{2} \\ &= \frac{2}{27} p p_0 w_1 (V - w_i)^3\end{aligned}$$

and

$$\frac{1}{2} p E_{g_0}(V - w_i - w_j \mid w_i) = \frac{1}{2} p p_0 w_1 \frac{1}{6}(V - w_i)^3.$$

Since $\frac{2}{27} < \frac{1}{12}$, we get the desired inequality.

distributions.) More work will be required to analyze the second-best in general.

But, for some special forms of correlation the classical analysis for independent distributions can be used to characterize the second-best in our setup with veto rights.

As an illustration, assume that there are two parties and outside options are uniformly distributed on Γ_V. Such a distribution induces some form of (negative) correlation. However, because it can be viewed as the restriction to the domain Γ_V of the uniform density over $[0, V]^2$ (for which the second best is known), we can find the second best when outside options are uniformly distributed on Γ_V.[26]

Indeed, from Myerson-Satterthwaite, we know that when outside options are uniformly distributed on $[0, V]^2$ (distributions of outside options are independent), the second-best (requiring only interim participation constraints) leads to having an agreement whenever $w_1 + w_2 < 3V/4$ (see their characterization on pages 276-277). But, this is also the domain of agreement induced by the Nash bargaining protocol when outside options are uniformly distributed on Γ_V. This implies that the allocation resulting from the Nash bargaining protocol induces the second-best in our setup where (w_1, w_2) is uniformly distributed on Γ_V.[27]

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[26] A similar comment applies to the case where outside options are uniformly distributed on Γ_v, with $v > 3V/4$.

[27] Indeed by contradiction, assume there is a mechanism that generates a strictly higher expected welfare when outside options are uniformly distributed on Γ_V. We could also improve upon the second-best of Myerson-Satterthwaite when each w_i is independently and uniformly distributed on $(0, V)$ by considering a mechanism stipulating the same transfers and allocations when $(\widehat{w}_1, \widehat{w}_2) \in \Gamma_V$ and no agreement with no transfer when $(\widehat{w}_1, \widehat{w}_2) \notin \Gamma_V$. But, this would contradict the very definition of the second-best, thereby showing that the outcome induced by the Nash bargaining protocol is the second-best when (w_1, w_2) is uniformly distributed on Γ_V and parties have the right to veto the agreement at any point in time.

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6 Appendix

To prove our result, we consider a direct truthful mechanism with veto rights that is efficient and that satisfies the ex post no subsidy constraint, and we establish an upper bound on

$$H_i(w_i) = \int_{(w_i,w_{-i})\in\Gamma_v} g(w_i, w_{-i})(V - \sum_{j\neq i} w_j - t_i(w_i, w_{-i}))dw_{-i}.$$

Let $\varepsilon = V - v$. We will prove that there exists a constant a independent of ε such that

$$H_i(w_i) \leq a\varepsilon. \tag{18}$$

Since $\Gamma_V \cap \Gamma^g$ contains Γ_v, and since $t_i(w_i, w_{-i}) \geq w_i$, this inequality in turn will imply a lower bound on player i's expected utility. Let $S = V - \sum_j w_j$ denote total surplus.

$$\begin{aligned} E(U_i - w_i) &\geq \int_{w\in\Gamma_V\cap\Gamma^g} g(w_i, w_{-i})(t_i(w_i, w_{-i}) - w_i)dw \\ &\geq \int_{w\in\Gamma_v} g(w_i, w_{-i})(t_i(w_i, w_{-i}) - w_i)dw \\ &\geq \int_{w\in\Gamma_v} g(w_i, w_{-i})(V - \sum_j w_j)dw - \int_{w_i\leq V} H(w_i)dw_i \\ &\geq \int_{w\in\Gamma_V} g(w_i, w_{-i})Sdw - \varepsilon \Pr(S \in [0, \varepsilon]) - a\varepsilon V \end{aligned}$$

Adding these inequalities for all players, and since

$$\sum_i E(U_i - w_i) \leq \int_{w\in\Gamma_V} g(w_i, w_{-i})Sdw,$$

we obtain:

$$(N-1)\int_{w\in\Gamma_V} g(w_i, w_{-i})Sdw \leq N\varepsilon(\Pr(S \in [0, \varepsilon]) + aV)$$

which is impossible for ε small.

We now turn to the critical part of the proof, which consists in showing that inequality (18) holds.

First observe that the ex post participation and the no subsidy constraints together imply that for all $(w_i, w_{-i}) \in \Gamma_v$,

$$V - \sum_{j\neq i} w_j \geq t_i(w_i, w_{-i}) \geq w_i, \tag{19}$$

which implies that $H_i(w_i) \geq 0$. We now use incentive compatibility constraints to derive an upper bound on $H_i(w_i)$. Incentive compatibility requires that for all $\widehat{w}_i > w_i$,

$$\begin{aligned}\int_{w\in\Gamma_V\cap\Gamma^g} g(w_i,w_{-i})t_i(w_i,w_{-i})dw_{-i} \quad &\geq \quad \int_{(\widehat{w}_i,w_{-i})\in\Gamma_v} g(w_i,w_{-i})t_i(\widehat{w}_i,w_{-i})dw_{-i} + \\ &+w_i\int_{w\in\Gamma_V\cap\Gamma^g-\Gamma_v} g(w_i,w_{-i})dw_{-i},\end{aligned}$$

Since $V-\sum_{j\neq i} w_j - t_i(w_i,w_{-i}) \geq 0$, we obtain:

$$\begin{aligned}H_i(w_i) \quad &\leq \quad \int_{w\in\Gamma_V\cap\Gamma^g} g(w_i,w_{-i})(V-\sum_{j\neq i} w_j - t_i(w_i,w_{-i}))dw_{-i}. \\ &\leq \quad \int_{(\widehat{w}_i,w_{-i})\in\Gamma_v} g(w_i,w_{-i})(V-\sum_{j\neq i} w_j - t_i(\widehat{w}_i,w_j))dw_j \\ &\quad + \int_{\substack{w\in\Gamma_V\cap\Gamma^g \\ (\widehat{w}_i,w_{-i})\notin\Gamma_v}} g(w_i,w_{-i})(V-\sum_j w_j)dw_{-i}\end{aligned}$$

Let $\Delta = \widehat{w}_i - w_i$. The last term is bounded by $(\varepsilon+\Delta)\Pr\{0 \leq S \leq \varepsilon+\Delta\}$, hence it is below $b(\varepsilon+\Delta)^2$ for some constant b independent of ε and Δ. To bound the first term, observe that there exist $m > 0$ and M such that $m \leq g(w_i,w_{-i}) \leq M$ and $\mid \frac{\partial g}{\partial w_i} \mid \leq M$, $i=1,2$, hence we have:

$$\begin{aligned}g(w_i,w_j) \quad &\leq \quad g(\widehat{w}_i,w_j) + \mid g(w_i,w_{-i}) - g(\widehat{w}_i,w_{-i}) \mid \\ &\leq \quad g(\widehat{w}_i,w_j)(1+M\Delta/m).\end{aligned}$$

We thus obtain:

$$H_i(w_i) \leq H_i(\widehat{w}_i)(1+\frac{M}{m}\Delta) + b(\varepsilon+\Delta)^2$$

Let $\rho = \max(M/m, b)$ and consider $w_i \leq v-\varepsilon$. We choose $\Delta = \frac{v-w_i}{N}$, where N is set so that $\varepsilon/2 \leq \Delta \leq \varepsilon$. We obtain the sequence of inequalities:

$$\begin{aligned}H_i(w_i) \quad &\leq \quad H_i(w_i+\Delta)(1+\rho\Delta) + \rho(\varepsilon+\Delta)^2 \\ &\leq \quad H_i(w_i+2\Delta)(1+\rho\Delta)^2 + \rho(\varepsilon+\Delta)^2(1+(1+\rho\Delta)) \\ ... \quad &\leq \quad H_i(w_i+n\Delta)(1+\rho\Delta)^n + \rho(\varepsilon+\Delta)^2\sum_{k=0}^{n-1}(1+\rho\Delta)^k \\ ... \quad &\leq \quad \rho(\varepsilon+\Delta)^2\sum_{k=0}^{N-1}(1+\rho\Delta)^k \text{ (since } H_i(w_i+N\Delta)=0) \\ &\leq \quad \rho\frac{(\varepsilon+\Delta)^2}{\Delta}\Delta N(1+\rho\Delta)^N\end{aligned}$$

As N gets large, the term $\Delta N(1+\rho\Delta)^N$ remains bounded (by $Ve^{\rho V}$). Since the inequalities hold for all N, $H_i(w_i)$ remains bounded above by $\rho\frac{(2\varepsilon)^2}{\varepsilon/2} = 8\rho\varepsilon$, which concludes the proof.

Proof of Proposition 2 The expected gain of party 1 with type w_1 when announcing $\widehat{w}_1$ is

$$\begin{aligned} G(w_1, \widehat{w}_1) &= \int_{\frac{V-\widehat{w}_1+a(w_2)}{2}>w_2} \max(w_1, \frac{V+\widehat{w}_1-a(w_2)}{2})\frac{dw_2}{V-w_1} \\ &\quad +w_1 \int_{\frac{V-\widehat{w}_1+a(w_2)}{2}<w_2} \frac{dw_2}{V-w_1} \end{aligned}$$

We now check that it is optimal for party 1 to announce $\widehat{w}_1 = a(w_1)$. Given the form of $a(\cdot)$ it is readily verified that whenever the announcements are compatible, i.e. $a(w_1) + a(w_2) < V$, we have that $a(w_i) > w_i$ for $i = 1, 2$, hence the Nash bargaining share of each party i is above w_i. This allows us to simplify the expression of $G(w_1, \widehat{w}_1)$ when $\widehat{w}_1$ lies in a neighborhood of $a(w_1)$ into:

$$\begin{aligned} G(w_1, \widehat{w}_1) &= \int_{a(w_2)<V-\widehat{w}_1} \frac{V+\widehat{w}_1-a(w_2)}{2}\frac{dw_2}{V-w_1} \\ &\quad +w_1 \int_{a(w_2)>V-\widehat{w}_1} \frac{dw_2}{V-w_1} \end{aligned}$$

Differentiating $G(w_1, \widehat{w}_1)$ with respect to $\widehat{w}_1$ yields:

$$\frac{\partial G(w_1, \widehat{w}_1)}{\partial \widehat{w}_1} = \frac{1}{V-w_1}[(1/2)b(V-\widehat{w}_1) - b'(V-\widehat{w}_1)(\widehat{w}_1 - w_1)]$$

where $b(w) = -\frac{3}{8}V + \frac{3}{2}w$ is the inverse of function $a(\cdot)$. Straightforward computations show that

$$\left.\frac{\partial G(w_1, \widehat{w}_1)}{\partial \widehat{w}_1}\right|_{\widehat{w}_1=a(w_1)} = 0.$$