

# Herd Behavior and Contagion in Financial Markets

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## Abstract

Imitative behavior and contagion are well-documented regularities of financial markets. We study whether they can occur in a two-asset economy where rational agents trade sequentially. When traders have gains from trade, informational cascades arise and prices fail to aggregate information dispersed among traders. During a cascade all informed traders with the same preferences choose the same action, i.e., they herd. Moreover, herd behavior can generate financial contagion. Informational cascades and herds can spill over from one asset to the other, pushing the price of the other asset far from its fundamental value.

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# 1 Introduction

The 1990s witnessed a series of major international financial crises, e.g., in Mexico in 1995, Southeast Asia in 1997-8, Russia in 1998 and Brazil in 1998-9. These episodes have revived interest among economists in the study of financial system fragility. Two main perspectives have emerged. One part of the literature has emphasized the relation between financial crises and weak fundamentals of the economy (see, for instance, Allen and Gale, 1998, Corsetti et al., 1999, and Burnside et al., 2000). Another has stressed that crises may just be due to random events and self-fulfilling prophecies, with variables unrelated to the real economy acting as “sunspots” (see, among others, Chang and Velasco, 1999 and Masson, 1999). Rather than alternative explanations, these two views are now considered complementary: financial crises are not always due to weak fundamentals, but weak fundamentals can make the financial system fragile and increase the likelihood that economic agents will co-ordinate on a bad equilibrium. Indeed, this is consistent with a common finding in much of the empirical work on financial crises (see, e.g., Kaminski, 1999): fundamentals do help to predict when a crisis will occur; nonetheless, crises may occur when the fundamentals seem sound or may not occur when fundamentals are weak.

A possible explanation of why sound fundamentals may not be reflected in asset prices is that information about these fundamentals is spread among investors and prices may fail to aggregate it. In particular, this would happen if investors, instead of acting according to their own private information, simply decided to herd. Herd behavior may, therefore, be a reason why we can observe a misalignment between prices and asset values.

The imitative, herd-like behavior of market participants is often linked to another widely recognized feature of financial markets, i.e., the strong co-movements among seemingly unrelated financial assets. During 1997, for instance, financial asset prices plunged in most emerging markets, following the financial crisis that hit some Asian economies. This high degree of co-movement across markets that are very different in size and structure and are located in different regions of the world is not a peculiarity of the Asian crisis. Indeed, it is a very common and well documented regularity of financial markets. Since falling asset prices are associated with recession and reduction in growth, it is important to try to understand the source of such co-movement.

There are two main theories explaining why prices in different markets

are strongly correlated. The first is based on common aggregate shocks, such as a change in the level of international interest rates or in the price of commodities.

The second theory is based on contagion: co-movements are said to be driven by contagion whenever they cannot be explained by common aggregate shocks. For instance, in Masson (1999) a financial shock in one region can create a self-fulfilling expectation of a crisis in another region. A different mechanism relies on real or informational linkages. In Allen and Gale (2000) liquidity shocks in one region can spread, through the banking sector, to the whole economy. In Calvo (1999) asymmetric information among agents causes a shock to one asset to affect the price of another. In Kodres and Pritsker (1999) idiosyncratic shocks spill over from one market to the other because of cross-market hedging of macroeconomic risk.

In this paper we present an alternative explanation of financial contagion: herd behavior. We will discuss how herd behavior, by spreading from one market to the other, can be a reason why crises or booms transmit themselves across markets.

## 1.1 The Theoretical Problem

Herd behavior seems a plausible explanation for the misalignment of prices and fundamentals. Furthermore, there is anecdotal, empirical and experimental evidence of herding in financial markets.<sup>1</sup>

From a theoretical standpoint, however, it is difficult to justify herd behavior in financial markets. Although several models of social learning have shown that herding is not necessarily an irrational phenomenon, their explanation of herding cannot be directly applied to financial markets. In this Section, we briefly discuss why.

In the last decade, many papers have tried to explain imitative behaviors in a world of rational agents. Focusing on the role of knowledge in markets, these papers (see, among others, Banerjee, 1992; Bikhchandani et al., 1992; Chamley and Gale, 1996; Chari and Kehoe, 2000) have studied the social learning effects of actions taken by agents who act sequentially. When deci-

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<sup>1</sup>For empirical evidence, see Lakonishok et al. (1992) and the other references in Bikhchandani and Sharma (2000). For experimental evidence, see Cipriani and Guarino (2002).

sions are sequential, the earliest actions may have a disproportionate effect on the choices of the following agents and herd behavior may arise.

With few exceptions (e.g., Avery and Zemsky, 1998), however, this literature studies the decision to buy or to sell a good the price of which is fixed. This feature makes these models unsuitable to analyze financial markets, where asset prices are certainly flexible. Avery and Zemsky study whether informational herding can arise in a sequential trading model à la Glosten and Milgrom (1985), in which the market maker modifies the price on the basis of the order flow. By allowing the price to react to the traders' decisions, they limit the possibility of informational cascades and herding<sup>2</sup> since agents will always find it convenient to trade on the difference between their own information (the history of trades and the private signal) and the commonly available information (the history only). Therefore, it is never the case that agents neglect their information and imitate previous traders' decisions.

Avery and Zemski show that when there are multiple sources of uncertainty, herd behavior can arise even in their framework. Their definition of herding, however, is not the standard one in the literature. Even with multidimensional uncertainty, informational cascades and herds (as usually defined) cannot arise in their study (see their Proposition 2 and their comments at page 733<sup>3</sup>).

Let us illustrate Avery and Zemski's point with a simple example. Consider an economy in which agents can trade a financial asset that can take two values, \$0 or \$100, with equal probability. Agents do not trade among themselves, but with a market maker, who sets the price at which traders can buy or sell the asset. Let us assume that the market maker sets a price equal to \$50, the expected value of the asset. This price is kept fixed, i.e., the market maker does not change it after observing a buy or a sell. Each agent, before making his trading decision, receives some private information (a binary signal) on the value of the asset. This signal has a 70% chance of being correct. Suppose that the value of the asset is \$100, but the first two agents arriving in the market receive the wrong (i.e., low) signal and therefore sell the good. What will the third agent do? Even if his signal is

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<sup>2</sup>We will formally introduce the concepts of herding and cascade in Section 3. In the literature, a cascade is defined as a situation where agents disregard their own private information. In contrast, herd behavior arises when all agents with the same preferences choose the same action. For a general discussion of the concepts of herds and cascades and their distinction see Smith and Sørensen (2000) and Chamley (2002).

<sup>3</sup>See also the considerations contained in Brunnermeier(2001).

high, he realizes that the two previous agents (who sold) had low signals. The negative information contained in the first two sell orders overwhelms his private information. Therefore, he will also sell and will not reveal his positive information on the asset value. All the following agents will be in the same position as the third, since they realize that the third agent's action did not depend on his private information. Therefore they will all join the sell herd. Although the value of the asset was \$100, everyone will sell and the true state of the world will never be revealed<sup>4</sup>. The actions of the first two agents have a disproportionate and pathological effect on the history of trades.

In the previous example the price does not adjust to the order flow. Indeed, we have assumed that even after a series of buy orders the price is fixed at the level of \$50. This is a perfectly reasonable assumption in many economic contexts. For instance, Bikhchandani et al (1992) refer to the choice of adoption of a new technology whose cost is fixed. In financial markets, however, prices are certainly not fixed. Avery and Zemsky (1998) have shown that, in this case, the argument of Banerjee (1992) and Bikhchandani et al (1992) no longer holds. The presence of a flexible price induces people to follow their own private information since the price adjusts in order to factor in the information contained in the past history of trades. Let us repeat the example in the previous paragraph, but assume that the price, instead of being fixed, is set equal to the expected value of the asset given the past history of trades. After the first two traders sell, the market maker will lower the price from \$50 to \$15.50<sup>5</sup> to take into account that the first two sells came from agents with a low signal. The third agent knows that the two previous traders had a negative signal. If his signal is high, his expected value of the asset will be \$30. Given that he faces a price of \$15.50, he will buy, i.e., he will follow his own private information. By the same argument all traders will always follow their private information. Since the signal that they receive is correct 70% of the time, over time the price will converge to the fundamental value of the asset, thus aggregating the private information dispersed among traders. Therefore, when prices are set efficiently, agents will follow their own private information and the price will aggregate the information spread among traders. Consequently, we should not observe misalignments of the

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<sup>4</sup>As it would be, by the law of large numbers, if all agents traded according to their own private information.

<sup>5</sup>This can be easily calculated using Bayes's rule.

price with respect to the fundamentals.

## 1.2 An Overview of the Paper

In this paper, we will show that, despite the presence of a price mechanism, informational cascades and herds can arise. We will study an economy where agents are heterogenous, e.g., because of differences in endowments or in intertemporal preferences. In this economy, trading can be mutually beneficial, i.e., there are gains from trade. Agents trade for two reasons: they have an informational advantage over the market maker and they have a gain to trading. Eventually, the gain from trade overwhelms the importance of the informational advantage and therefore agents choose their action independently of their information on the asset value. During periods of informational cascades all informed traders will choose the same action, i.e., they herd. Given that agents do not use their own information, private information is not aggregated and prices may not reflect the true value of the assets.

After illustrating our argument for herd behavior, we will discuss how herding can lead to financial contagion. The history of trades in one market can permanently affect the price path of another and make it converge to a different value from what it would have been otherwise. Informational spillovers are to be expected between correlated asset markets. With gains from trade, however, these informational spillovers can have pathological outcomes. Informational cascades in one market generate cascades in another, pushing the prices, even in the long run, far from the fundamentals. This long lasting spillover represents a form of contagion: a crisis or a boom in one market transmits itself to the other without regard to the fundamentals.

Finally, we show that the unconditional correlation between market prices is greater than the correlation of fundamentals. Excess correlation is not necessarily the result of irrational behavior, but may be the result of the learning process of rational agents.

The structure of the paper is as follows. Section 2 describes the model. Section 3 presents the main results on herding. Section 4 discusses financial contagion. Section 5 discusses our results and Section 6 concludes.

## 2 The model

We base our analysis on Glosten and Milgrom (1985), extending their framework to study a two-asset economy.

Consider two assets,  $A$  and  $B$ , with true values  $V^A$  and  $V^B$  distributed according to a joint probability distribution  $p(V^A, V^B)$ , with support belonging to  $[m, M]^2$ ,  $0 \leq m < M$ . The two assets are traded in two markets, market A and market B. At any time  $t$ , a trader is randomly chosen from a continuum of traders to act in either market. He can buy, sell or decide not to trade. Each trade consists of the exchange of one unit of the asset for cash with a competitive market maker. The market maker sets the prices for the asset. At each time  $t$ , market makers and traders know the history of trades ( $H_t$ ) until time  $t - 1$  in both markets.

There are two types of traders: informed and uninformed traders. In both markets the proportion of informed traders is  $\mu$ . Uninformed (or noise) traders<sup>6</sup> trade for unmodeled (e.g., liquidity) reasons: they buy, sell or do not trade the asset with exogenously given probabilities. Informed traders receive a private signal on the asset that they are going to trade and maximize their expected utility based on that signal. Informed traders on asset  $J = A, B$  observe a signal  $x^J$  distributed on a support belonging to  $[m, M]$  according to the conditional probability distribution  $q_J(x^J|V^J)$ . Since the two assets are not independent, a signal on one asset gives some information also on the value of the other. Nevertheless, we say, for instance, that a trader is informed on asset  $A$  because the distribution of  $x^A$  conditional on  $V^A$  does not depend on the value of asset  $B$ , i.e.,  $q_A(x^A|V^A) = q_A(x^A|V^A, V^B)$  for any  $x^A$ ,  $V^A$ , and  $V^B$ .

An informed agent uses his private information and the history of past trades to form his belief on the value of the asset. We denote the expected value of the asset for an informed trader called to trade on market  $J$  at time  $t$  by

$$V_t^J(x^J) = E(V^J|H_t, x^J). \quad (1)$$

Informed agents are heterogeneous. In particular, there are traders who enjoy an extra utility and agents who suffer a disutility from holding the

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<sup>6</sup>Noise traders are a common feature of financial market models. They are necessary to avoid a market breakdown, due to asymmetric information. We will comment more on this later in this section.

asset. Therefore, in our economy, there are gains from trade. Formally, each trader can be of type  $\rho = \{l, g\}$ . Agents with an extra utility  $g > 1$  from asset  $J = A, B$  find it convenient to buy whenever

$$gE(V^J|H_t, x^J) > A_t^J, \quad (2)$$

and find it convenient to sell whenever

$$gE(V^J|H_t, x^J) < B_t^J. \quad (3)$$

On the other hand, agents with a disutility  $l$  ( $0 < l < 1$ ) from the asset find it convenient to buy whenever

$$lE(V^J|H_t, x^J) > A_t^J, \quad (4)$$

and to sell whenever

$$lE(V^J|H_t, x^J) < B_t^J. \quad (5)$$

The first type of trader may buy even when his expected value of the asset is lower than the ask, because he enjoys extra utility from the asset, which is not enjoyed by the market makers. The second type, by contrast, may sell even when his expected value is higher than the bid, because he receives a disutility from holding the asset. The proportion of agents with a gain  $g$  is  $\mu_g \geq 0$  and the proportion of those with a loss  $l$  is  $\mu_l \geq 0$ , with  $\mu_g + \mu_l = \mu$ .

We assume that the type is private information. Moreover, we assume that the type does not convey any information neither on the asset value nor on the trader's private signal, i.e.,  $E(V^J|H_t, x^J, \rho) = E(V^J|H_t, x^J)$  and  $E(E(V^J|H_t, x^J)|\rho, H_t) = E(V^J|H_t)$ .

Gains from trade can be present for several reasons. For instance, they can stem from different preferences over present and future consumption. Suppose that agents maximize a utility function as  $u(c_1, c_2) = c_1 + \rho c_2$ . In this case,  $g$  and  $l$  can be interpreted as two different values of the intertemporal discount factor (the discount factor of the market maker would be normalized to 1). Agents of type  $g$  value future consumption more and are inclined to invest in the asset to substitute present consumption with future consumption. Type  $l$  agents are more myopic and have a lower desire to invest. Another source of heterogeneity among traders, can be risk aversion. For instance, a risk averse agent who holds a third asset negatively correlated



with  $A$  or  $B$ , will be more willing to buy the asset for hedging reasons. In their seminal paper, Glosten and Milgrom (1985) study a market with a similar kind of asymmetric valuation of the asset. As they do in their paper, we can also interpret the parameters  $g$  and  $l$  as “the result of imperfect asses to capital markets or [...] differential subjective assessments of the distribution of the random variable [...]” Decamps and Lovo (2003) study the case in which heterogeneity arises from shocks to the wealth of risk averse traders. For the rest of the paper, we do not restrict to any of these different interpretations, but will just use the reduced form of gains from trade presented above.

Without gains from trade noise traders are necessary for trade to take place.<sup>7</sup> The presence of gains from trade makes trade possible even without noise traders by allowing the market maker to set a bid and ask spread at which he makes zero profit. At a given ask those traders with a negative signal and whose gains from trade are high enough to offset the expected loss would buy. Therefore, it is not true, in general, that the market makers suffer an expected loss each time they buy or sell to an informed trader. Although we could build a model without noise traders, we keep them to maintain a setup as close as possible to that used in the literature in order to compare our results to the previous ones.

Let us consider now the role of the market makers. The expected value for the market makers will be conditioned only on the public information available at time  $t$ , i.e., it will be

$$P_t^J = E(V^J | H_t). \quad (6)$$

We refer to the market makers’ expectations as the prices of the assets. It is important to note that, since the action in the other market reveals some information on the value of the asset, each market maker will revise his expectations at time  $t$  even when the trade did not occur in his market at time  $t - 1$ .

The market makers must take into account the possibility that they may trade with agents more informed than they are. Therefore, they will set a bid-ask spread between the prices at which they are willing to sell and to buy (see Glosten and Milgrom, 1985). Given that the market makers behave competitively, they will make zero expected profits. Hence, the bid and ask prices for asset  $J = A, B$  will be

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<sup>7</sup>Indeed, in the absence of noise traders the market would break down (as shown by the “no-trade” theorem of Milgrom and Stokey, 1982).

$$B_t^J = E(V^J | H_t, h_t^J = \text{sell}^J) \text{ and } A_t^J = E(V^J | H_t, h_t^J = \text{buy}^J), \quad (7)$$

where  $h_t^J$  is the action in market  $J$  taken by the trader who arrives at time  $t$ . Note that each market maker computes the expected value conditional on past trades in both markets, since they are public information, and on the action taken in his own market. Each action taken by a trader may reveal some private information, since the actions of the informed traders depend on the signals.

Note that our assumption of unit trade is formally equivalent to assuming that the trade size does not convey any information on the asset value and, therefore, that the market maker does not post bid and ask prices contingent on the trade size. This would hold, for instance, if traders are risk neutral and the size of each trade is determined only by liquidity constraints. The few empirical analyses available on the topic show, consistently with our assumption, that “trade size provides no informational content beyond that conveyed by the underlying transactions” (Easley et al, 1997, p 830; the same conclusion is reached by Jones et al., 1994).

Before proceeding to the main analysis, we must provide some results about the behavior of the market prices. These results are an immediate generalization of those obtained by Avery and Zemsky (1998) for the one-asset economy. In our model, at each time  $t$ , only one market is open and the history of trades in the other market is public information. Therefore, the computation of bid and ask prices is identical to that of a one-asset economy: at any time  $t$ , there exists a unique bid and ask price for the asset  $J = A, B$  traded in that period, which satisfies  $B_t^J \leq P_t^J \leq A_t^J$ . The market maker takes into account that buying or selling orders contain private information and sets a spread between the price at which he is willing to sell and to buy. Equilibrium prices always exist because noise traders are willing to accept any loss and, therefore, markets will never shut down.

Moreover, given that market prices  $P_t^A$  and  $P_t^B$  are expectations based on all public information, they are martingales. Formally,

$$E(P_{t+1}^J | H_t) = E(E(V^J | H_{t+1}) | H_t) = E(V^J | H_t) = P_t^J, \quad (8)$$

where the second equality comes from the law of iterated expectation. Given that prices are martingales they will converge almost surely to a random variable. In the absence of gains from trade, Avery and Zemsky (1998) prove

that prices converge almost surely to the true values  $V^J$ . With gains from trade we will see that herding can arise, prices will be unable to aggregate correctly private information and this convergence result is destroyed.

### 3 Informational Cascades and Herd Behavior

In Section 1 we illustrated through a simple example how the price mechanism is able to aggregate private information. Now we will show that, when there are gains from trade, prices are unable to aggregate private information correctly. Indeed, there will be a time when information stops flowing to the market and the price may remain stuck at a level far from the fundamental value of the asset. This blockage of information is called informational cascade. During a cascade, agents disregard their private information, i.e., they herd.

In order to present our results, we first introduce the formal definitions of informational cascade and herd behavior.

**Definition 1** *An informational cascade on asset  $J$  arises in period  $t$  when  $\Pr(h_t^J|x^J, H_t) = \Pr(h_t^J|H_t)$  for all  $x^J$  and for all  $h_t^J$ .*

An informational cascade requires that the actions be independent of the signal on the asset values<sup>8</sup>. In a situation of informational cascade, the market maker will be unable to infer anything from the action of the traders and will be unable to update his beliefs on the asset value. In other words, in an informational cascade trades do not convey any information on the value of the asset. The concept of informational cascade is closely related to that of herd behavior:

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<sup>8</sup>The definition that we use here is standard in the literature. Avery and Zemski use a definition in which actions are independent of the asset value. In our setup, the two definitions are equivalent. If a trader is not making use of his signal, his action does not depend on the asset value (given that the signal itself depends on it). On the other hand, if the action is independent of the asset value, then the trader is not making use of the signal that he received.

**Definition 2** *There is herd behavior in market  $J$  at time  $t$  if, in market  $J$ , informed traders of the same type choose the same action with probability 1.*

When agents are homogeneous herd is usually defined as a situation where all agents act alike. Here, since agents are heterogeneous, we characterize herding in a different way. Indeed, what is relevant is not the traders' decisions per se, rather the use that they do of their own private information. Therefore, following the existing literature (see, e.g., Smith and Sørensen, 2000), we say that agents herd when, *conditional on their types*, they all choose the same action.<sup>9</sup>

We can show that any history of trades will go through periods of informational cascades. After a sufficiently large number of trades, the valuation of the traders and of the market makers will be so close that all informed traders with an extra utility will decide to buy, independently of their signal, in order to enjoy the extra utility from the asset. On the other hand, all the traders with a disutility from holding the asset will sell. The probability of an action will be independent of the signal that the trader receives and, thus, of the realized value of the assets. The market maker will be unable to update his beliefs on the asset value and, as a consequence, prices will not respond to the trade action.

**Proposition 1** *If  $m > 0$  and if there does not exist a realization of the signal  $\hat{x}^J$  and of the asset values  $\hat{V}^J$  such that  $\Pr(\hat{x}^J|\hat{V}^J) = 0$ , in equilibrium an informational cascade occurs with probability 1.*

**Proof.** See the Appendix.

The condition  $m > 0$  is needed only to avoid the case in which the gain and loss from trade- $gE(V^J|H_t, x^J)$  and  $lE(V^J|H_t, x^J)$ -vanish because the expectations of the traders converge to zero. The condition  $\Pr(\hat{x}^J|\hat{V}^J) \neq 0$  rules out too informative signals, that is, signals that tell the traders that

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<sup>9</sup>Our notion of herd behavior is the one usually adopted in the literature. It differs from Avery and Zemsky's (1998), according to which an informed trader engages in herd behavior at time  $t$  if he buys when  $E(V^J|x^J) < P_0^J < P_t^J$  or if he sells when  $E(V^J|x^J) > P_0^J > P_t^J$ . E.g., there is herd buying when a trader upon receiving his signal sells, but after seeing the price rise changes his mind and buys.

some realizations of the asset value cannot have occurred. Upon receiving one of these signals a trader would disregard even a very long history of trades and his expectation would diverge from the market maker's. Instead, condition  $\Pr(\hat{x}^J | \hat{V}^J) \neq 0$  implies, that traders' beliefs are bounded, so that there are some history of trades that cannot be offset by any realization of the signal.

Note that, during an informational cascade, informed agents herd, i.e., all agents of the same type act alike. We formalize our result in the following corollary:

**Corollary 1** During an informational cascade, there is herd behavior, i.e., all informed agents with a gains from trade  $g$  buy and all informed agents with a loss from trade  $l$  sell.

**Proof.** See the Appendix.

An informational cascade can be misdirected, that is, it can occur when the price is far away from the realized value of the asset. In the following example, the asset can take values 1 and 2 and the realized value of the asset is 2. However, a history of sells triggers an informational cascade and the price remains stuck at a value close to 1.

**Example 1** For simplicity's sake, we consider an economy with only one asset  $J$  that takes values 1 and 2 with equal probability. The proportion of informed traders is  $\frac{9}{10}$ . Informed traders receive a signal distributed as follows:  $\Pr(x^J = k | V^J = k) = \frac{9}{10}$  and  $\Pr(x^J = k | V^J \neq k) = \frac{1}{10}$ , for  $k = 1, 2$ . Noise traders buy, sell or do not trade with equal probability. Finally, 50% of the informed traders have a gain from trade  $g = 1.1$  and 50% have a loss from trade  $l = 0.90$ . Suppose that the realized value of the asset is 2 and that in the first three periods the market maker receives three sell orders. At time 0, the price is equal to the unconditional expectation, i.e.,  $P_0^J = 1.5$ . The bid and ask prices are  $B_0^J = 1.15$  and  $A_0^J = 1.85$ .<sup>10</sup> At time 1, after

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<sup>10</sup>The bid and ask prices are computed by using Bayes's rule. It is important to note, however, that -for the parameters of this example- if at time 0 the market maker set the bid by assuming that all traders with a negative signal sold the asset, traders with a negative signal and a gain from trade would indeed not trade, which clearly is not an equilibrium. Therefore, the market maker sets the bid assuming that only traders with a negative signal and a loss from trade sell the asset while the other traders with a negative signal and a gain do not trade, which occurs in equilibrium. Similarly, the ask is computed by assuming that only traders with a positive signal and a gain from trade buy the asset, while the other traders with a positive signal do not trade, which is true in equilibrium.

the first sell order, the price goes down to  $P_1^J = 1.15$  and the market maker posts the following bid and ask prices, 1.5 and 1.03.<sup>11</sup> At time 2, after the second sell, the price, the bid and the ask are updated down to  $P_2^J = 1.03$ ,  $A_2^J = 1.05$  and  $B_2^J = 1.006$ .<sup>12</sup> At time 3, the price becomes  $P_3^J = 1.006$ . At this price, all informed traders with  $g = 1.1$  find it convenient to buy. This happens because even a trader with a negative signal but a gain from trade has a utility from the asset equal to  $gE(V^J|H_3, x^J = 1) = (1.1)(1.0006) = 1.10$ , which is greater than the price. On the other hand, all agents with a loss from trade find it convenient to sell the asset, since  $lE(V^J|H_3, x^J = 2) = (0.9)(1.409) = 0.9441 < P_3^J$ . Therefore, at time 3 an informational cascade occurs and the market maker sets the bid and the ask equal to the price.

Gains from trade cause a blockage of the information flow because they introduce a wedge between the utility that the market makers and the traders obtain and forgo when they exchange the assets. In Ho Lee (1998) shows that a similar wedge would exist if one introduced trade costs. With trade costs, however, when the traders' and the market makers' expectations converge, traders stop trading. Therefore, information ceases to flow to the market only because the market shuts down. In our model, on the contrary, information stops flowing to the market despite the fact that agents keep trading. In our model, the price does not aggregate information not because traders abstain from trading but because they herd.

In a one-asset economy an informational cascade never ends. After the cascade has started, no information arrives in the market and agents face the same decision problem in each period. In contrast, in a two-asset economy, even when there is an informational cascade on one asset, the history of trades of the other reveals some information. When there is a cascade only in market  $A$ , traders in market  $B$  will still act according to their own signals. If the two assets are not independent, the market learns not only on asset  $B$ , but also on asset  $A$ . The price of asset  $A$  will move despite the informational cascade. Moreover, the trades of asset  $B$ , by moving the price of asset  $A$ , can make the valuation of traders and market maker diverge, thus breaking the cascade.

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<sup>11</sup>The computations are done as indicated in the previous footnote.

<sup>12</sup>The bid is computed as before. The equilibrium ask is computed assuming that agents with a negative signal and a gain from trade also buy.

**Proposition 2** *The history of trades on one asset can break an informational cascade on the other.*

**Proof.** We prove this through an example. Let us consider an economy where the unconditional distribution of asset values is the following:

	$V^B = 1$	$V^B = 2$	
$V^A = 1$	$\frac{9}{20}$	$\frac{1}{20}$	$\frac{1}{2}$
$V^A = 2$	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	1

The proportion of informed traders is  $\frac{9}{10}$ . Informed traders receive a signal on asset  $A$  distributed as follows:  $\Pr(x^A = k|V^A = k) = \frac{65}{100}$  and  $\Pr(x^A = k|V^A \neq k) = \frac{35}{100}$ . Informed traders on asset  $B$  received a signal distributed as  $\Pr(x^B = k|V^B = k) = \frac{95}{100}$  and  $\Pr(x^B = k|V^B \neq k) = \frac{5}{100}$ . Noise traders buy, sell or do not trade with equal probability. Finally, 50% of informed traders have a gain from trade  $g = 1.1$  and the other 50% have a loss from trade  $l = 0.90$ . Suppose that the realized values of both assets is 2 and that, in the first three periods, there are three sell orders in market  $A$ . At time 0, the price of the assets is equal to their unconditional expectations, i.e.,  $P_0^A = P_0^B = 1.5$ . The bid and ask prices in market  $A$  are  $A_0^A = 1.63$  and  $B_0^A = 1.485$ .<sup>13</sup> At time 1, after the first sell order, prices decrease to  $P_1^A = 1.485$  and  $P_1^B = 1.49$ . The ask and the bid in market  $A$  become  $A_1^A = 1.62$  and  $B_1^A = 1.36$ .<sup>14</sup> At time 2, after the second sell, the prices are updated down to  $P_2^A = 1.36$ ,  $P_2^B = 1.39$ . In market  $A$ , at a price equal to 1.36 all agents with a gain from trade find it convenient to buy: even a trader with a negative signal has a utility from the asset ( $gE(V^J|H_2, x^J = 1) = (1.1)(1.15) = 1.265$ ), which is greater than the price. On the other hand, all agents with a loss from trade will sell the asset since  $lE(V^J|H_2, x^J = 2) = (0.9)(1.375) = 1.2375 < P_2^A$ . Therefore, at time 2 an informational cascade occurs in market  $A$  and the market maker sets the bid and the ask equal to the price. Without the other market, this

<sup>13</sup>The equilibrium ask is such that only traders with a positive signal and a gain from trade buy the asset. The equilibrium bid is in mixed strategies. Indeed, it is equal to the expected utility of an agent with a low signal and a gain from trade. At that bid, agents with a low signal and a loss from trade sell the asset, while agents with a low signal and a gain from trade mix between the strategy of selling and not trading.

<sup>14</sup>The equilibrium ask is computed as before. At the equilibrium bid only traders with a negative signal and a loss from trade sell the asset while the other traders with a negative signal and a gain do not trade.

informational cascade would last forever and the price would be unable to recover and reflect the fundamental value of 2. Let us see, now what happens if at time 3 there is a buy order in market  $B$ . The market maker sets the following bid-ask spread:  $A_3^B = 1.77$  and  $B_3^B = 1.05$ .<sup>15</sup> After the buy order, the price of asset  $B$  moves to  $P_4^B = 1.77$ . Following this buy order, also the price in market  $A$  is updated. Indeed, it goes to  $P_4^A = 1.62$ . This movement is sufficient to break the informational cascade in market  $A$ . Indeed, the expected utility for an informed trader with a low signal and a gain from trade is now  $gE(V^J|H_4, x^J = 1) = (1.1)(1.47) = 1.617 < P_4^A$ . Therefore, now a buy order does reveal some information and the informational cascade is broken.

Our model gives some insights on how financial markets may recover. After a crisis, gains from trade in a market can make trading completely uninformative. Without observing trading in the other market, the price of the asset would remain undervalued even though traders receive new positive information. Trading in the other market, however, by revealing some new information, can help the market to recover. A positive history of trades in the other market leads to an increase in the price of the other asset. After the price starts to rally, gains from trade cease to be binding and the normal flow of information to the market resumes.

Given the previous result, it is important to distinguish the case of an informational cascade in only one market from the case of an informational cascade in both markets. In the latter case, no new information will reach the markets and the cascades will last for good. We refer to the case of informational cascades in both markets as an “informational breakdown.”

**Definition 3** *An informational breakdown arises at time  $t$  when there is an informational cascade at that time in both markets.*

We can show that an informational breakdown will happen with probability one and that it will be misdirected with positive probability. The argument is similar to that for the informational cascade. The implication, however, is much more far-reaching. Whereas an informational cascade may block the flow of information only temporarily, the informational breakdown,

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<sup>15</sup>The equilibrium ask is such that only traders with a positive signal and a gain from trade buy the asset. The equilibrium bid is such that only traders with a low signal and a loss from trade sell.



once arisen, never ends. Therefore, the prices can remain stuck forever at levels far from the realized values. If the informational breakdown is misdirected, the markets will never correct their valuations and will never learn the true values of the assets.

**Proposition 3** *If  $m > 0$  and if for  $J = A, B$  there does not exist a realization of either signal  $\hat{x}^J$  and of either asset value  $\hat{V}^J$  such that  $\Pr(\hat{x}^J | \hat{V}^J) = 0$ , an informational breakdown occurs with probability 1.*

**Proof.** See the Appendix.

Above we have shown that informational cascades can be misdirected. By the same argument an informational breakdown can happen when prices are far away from the fundamentals. In this case, prices will remain stuck forever at those levels and the market will never learn the true values of the assets. Avery and Zemsky (1998) argue that, when prices are efficiently set, informational cascades cannot occur. This result crucially depends on the fact that in their economy agents trade for informational reasons only. When one takes into account that agents in the market are heterogeneous, so that trade can be mutually beneficial, this result no longer holds. With gains to trading, informational cascades arise despite the fact that prices are efficiently set by competitive market makers.

In Example 1 we showed an example in which the price of the asset converges to the wrong value. We have simulated the model to see how frequently this misalignment occurs. In the simulation we study a one-asset economy in which the asset takes values 1 and 2 with equal probability. The proportion of informed traders is 70%. Half of the informed traders have a gain from trade of 10% and half a loss from trade of 10% (i.e.,  $g = 1.1$ ,  $l = 0.9$ ). Finally, informed agents receive a binary signal with a precision of 70%. We considered the case in which the realized value of the asset was 2. Figure 1 shows the frequency of the asset price after 300 trade periods. After these periods, an informational cascade had occurred in almost all the 1,000,000 runs of the simulation. The price distribution is bimodal: sometimes it converges close to the realized fundamental (2), sometimes it remains stuck near 1. In particular, the price misalignment occurred in 20% of the cases. In the Introduction we mentioned that, according to empirical analyses, fundamentals do help in predicting financial crises, but that crises may still occur even though the fundamentals are good. Similarly, in our model, fundamentals do help, since when the asset value is high the probability of a crisis is lower.

Nevertheless, a crisis can happen with positive probability even when the realized asset value is high.

## 4 Financial Contagion

### 4.1 Contagious Spillover

In this section we discuss some pathological effects of the informational spillovers from one market to the other. Given that the two assets are jointly distributed, informational spillovers are expected. We will show, however, that gains from trade may generate long lasting spillover effects. The very fact that traders and market makers are able to see the history of trades in another market can cause the price mechanism to fail in aggregating information. The presence of another market can make the flow of information stop early in the market and the price remain stuck at a wrong level. We regard this as a form of contagion. More precisely, we say that there is a *contagious spillover* when an informational cascade occurs in a market and would have not occurred if agents were able to observe only the history of trades in that market. That is, the informational cascade happens only because agents observe the history of the other market.

The contagious spillover can have permanent effects. If the informational cascade in market  $B$ , caused by the spillover effect, happens together with an informational cascade in market  $A$ , an informational breakdown arises. The price will remain stuck at a wrong valuation forever. Gains from trade make it possible for the history of trades in one market to have everlasting effects on the price path of the other.

Let us denote  $H_t^I$  the history of trades on asset  $I$  until time  $t - 1$ . We give the following formal definition of contagious spillover:

**Definition 4** *There is a contagious spillover from market  $I$  to market  $J$  ( $I, J = A, B; I \neq J$ ) at time  $t$  when there is an informational cascade in market  $I$  and there exist  $\widehat{V}^{I'}$ ,  $\widehat{V}^{I''}$  and  $h_t^I$  such that  $\Pr(h_t^I | \widehat{V}^{I'}, H_t^I) \neq \Pr(h_t^I | \widehat{V}^{I''}, H_t^I)$ .*

We now present an example in which a contagious spillover arises. If traders in market  $A$  were not able to observe the history of asset  $B$ , the price of asset  $A$  would converge towards its fundamental value. Given that traders are able to observe the history of both assets, however, the initial sells on  $B$

cause an informational cascade on both assets  $A$  and  $B$ , i.e., an informational breakdown of the market. The price of asset  $A$  is stuck for ever at a level below the fundamental value and its initial fall is not reversed, even in the long run.

**Example 2** Consider two assets with the following distribution:

	$V^B = 1$	$V^B = 2$	
$V^A = 1$	$\frac{9}{20}$	$\frac{1}{20}$	$\frac{1}{2}$
$V^A = 2$	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	1

The proportion of informed traders is  $\frac{9}{10}$ . Informed traders receive a signal distributed as follows:  $\Pr(x^J = k|V^J = k) = \frac{65}{100}$  and  $\Pr(x^J = k|V^J \neq k) = \frac{35}{100}$ . Noise traders buy, sell or do not trade with equal probability. Half of the informed traders have a gain from trade  $g = 1.1$  and half have a loss from trade  $l = 0.90$ . Suppose that the realized value of asset  $A$  is 2. Suppose that at times 0, 1, and 2 there are three sell orders in market  $B$ , folowed by many buy orders in market  $A$ . At time 0, the price of the assets is equal to the unconditional expectations, i.e.,  $P_0^A = P_0^B = 1.5$ . The bid and ask prices in market  $B$  are  $A_0^B = 1.89$  and  $B_0^B = 1.11$ .<sup>16</sup> At time 1, after the first sell order, prices go down to  $P_1^A = 1.18$  and  $P_1^B = 1.11$ . The ask and the bid in market  $B$  become  $A_1^B = 1.53$  and  $B_1^B = 1.01$ .<sup>17</sup> At time 2, after the second sell, the prices, the bid and the ask are updated down to  $P_2^A = 1.05$ ,  $P_2^B = 1.015$ .<sup>18</sup> After the third sell order, the prices of the two assets go to 1.01. At these prices, all agents with a gain from trade find it convenient to buy. A trader with a negative signal on asset  $A$  has a utility from the asset  $gE(V^A|H_2, x^A = 1) = (1.1)(1.0001) = 1.1001$  which is greater than  $P_3^A$ . Similarly, a trader with a negative signal on asset  $B$  has a utility from the asset  $gE(V^B|H_2, x^B = 1) = (1.1)(1.03) = 1.133$  which is greater than  $P_3^B$ . On the other hand, all agents with a loss from trade sell the assets: indeed,  $lE(V^A|H_2, x^A = 2) = (0.9)(1.02) = 0.918 < P_3^A$ , and, similarly,

<sup>16</sup>The equilibrium ask is such that only traders with a positive signal and a gain from trade buy the asset. The equilibrium bid is such that only traders with a low signal and a loss from trade sell.

<sup>17</sup>The computations are done as indicated in the previous footnote.

<sup>18</sup>The market maker posts a bid and ask price. In market  $B$ , the bid is computed as before. The, equilibrium ask is computed assuming that also agents with a negative signal and a gain from trade buy.

$lE(V^B|H_2, x^B = 2) = (0.9)(1.006) = 0.9054 < P_3^B$ . Therefore, at time 3 an informational cascade occurs in both markets. Without seeing the sell orders in market  $B$ , the market maker for asset  $A$  would have updated positively the value of asset  $A$  because after time 2 many buy orders arrive in market  $A$ . However, the market maker will not update the price upward because he believes that those buy could come both from traders with positive and negative information. If the market maker had not been able to observe the history on market  $B$ , he would have updated his price after the buys occurring in market  $A$  and price would have reflected the fundamental of market  $A$ . The informational externality from the other market prevents this from happening.

Figure 2 shows a simulated path of the price of asset  $A$  for two different cases: when the agents in market  $A$  are not able to observe the history of market  $B$  (solid line) and when they are able to do so (dotted line)<sup>19</sup>. When both histories of trades are observed, the fall in the price of asset  $B$  makes the price of  $A$  fall at the level at which an informational cascade arises and remain stuck there (far below the fundamental value of 2). On the other hand, if the market maker does not observe the history in market  $B$ , the price of  $A$  remains above the level at which an informational cascade arises and eventually converges to  $\frac{19}{10}$ , a value close to the fundamental.

Given that spillover effects can have a positive role, breaking a cascade in a market (as shown in the previous section), or a negative one (as just illustrated), it is interesting to see how relevant these two effects are. In general, it is difficult to say whether having information coming from a second market helps the price to converge closer to its fundamental. There are indeed forces that go in opposite directions. On the one hand, given that trades in market  $I$  convey less precise information<sup>20</sup> than trades on asset  $J$  on the realized value of asset  $J$ , at each time  $t$  the price will be on average farther away from the fundamental when we can observe the history on both markets. Therefore, if at time  $t$  there is a cascade, having another market may make it more likely that the cascade arises when the price is far from its fundamental value. On the other hand, with two markets, there is a complete blockage of information only when cascades arise on both markets at the same time.

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<sup>19</sup>The dash-dotted line shows the history of prices in market  $B$ . The parameters of the simulation are the same ones used in Figure 1. The realized fundamental values for  $V^A$  and  $V^B$  were 2 and 1 respectively.

<sup>20</sup>We refer to the case in which the precision of the signals in the two markets is the same and the correlation is not 1 or  $-1$ .

Therefore, the average number of periods before we get a complete blockage of information is not lower and in general will be higher. This means that, on average, the market makers will observe a larger number of trades before trading becomes uninformative. The probability that in the long run the price will remain stuck far away from the fundamental depends on which of these two opposite forces is more important.

One can find examples in which long run (i.e., for  $t \implies \infty$ ) price is on average farther away from the fundamental if the market maker and the traders are able to observe the history of trades in the other market. Figure 3 shows the simulated long run price distribution for a two-asset economy where the assets can take only values 1 and 2 with equal probability and  $Pr(V^J = 1|V^I = 1) = Pr(V^J = 2|V^I = 2) = 0.7$ . We use the same parameters of the simulation that we used for Figure 1: the proportion of informed traders and the precision of the signals are both set equal to 0.7, the probability of having a loss or a gain from trade is equal to 50% and the gain and the loss from trade are 1.1 and 0.9. The simulation was run for 1,000,000 times and the realized value of asset  $A$  was always set equal to 2, i.e., we are studying the long run distribution of  $P^A$  conditional on  $V^A = 2$ . The realized value of asset  $B$  was drawn according to its distribution conditional on  $V^A = 2$ . The bar chart shows the distribution of the price of asset  $A$  after 300 periods, when an informational breakdown had occurred in almost all of the 1,000,000 runs. The chart shows the price distribution under two different scenarios: one in which traders in market  $A$  are able to observe the history of market  $B$  (grey bar), and one in which traders in market  $A$  are not able to observe the history of market  $B$  (black bar). Of course the price distribution for the case in which traders do not observe the history on the other asset is identical to the one shown in Figure 1 (where we had a one asset economy). Note that being able to observe the history in market  $B$ , makes  $P^A$  converge far from the fundamental more often than it does in the other case. This example shows that there are cases for which the history of trades in the other market makes the aggregation of information by prices more difficult in an unconditional sense. Not only are there histories for which contagion spillovers happen, but the effect of being able to observe the history of trades across markets makes markets themselves less informational efficient.

## 4.2 Excess Correlation

In the empirical literature, contagion is sometimes referred to as an excess correlation between the asset prices relative to the fundamental correlation. The analysis of correlation in our model turns out to be analytically difficult. Through simulation<sup>21</sup>, we can show that, if there is a positive correlation between the fundamental, the unconditional correlation between prices is always greater than the correlation between fundamentals (see Figure 4). Moreover, the correlation between prices is decreasing over time and converges monotonically towards its long run level.

Note that, in the absence of gains from trade, the correlation between prices would converge towards the correlation between fundamentals. This is because without gains from trade the prices themselves converge almost surely to the fundamental values. In contrast, since with gains from trade the true values are never discovered, the correlation is in excess of that of the fundamentals even in the long run. The fact that informational cascades arise and prices do not converge to the fundamentals causes the correlation between prices to be permanently higher than the correlation between the fundamentals. Therefore, with sequential trading and gains from trade contagion -defined as excess correlation- can occur also as a long run phenomenon.

Sequential trading when information is incomplete helps to explain excess correlation. As long as there is uncertainty about the fundamental values of financial assets, the correlation between prices will be more extreme than the fundamentals would imply. Many empirical studies of financial markets show that asset prices are strongly correlated. Our simulation results suggests that this correlation is not necessarily the result of irrational behavior or frictions in the markets, but may be the result of the learning process of rational agents.

## 5 Discussion

Our analysis suggests that financial markets can be informationally inefficient: the price mechanism can be unable to aggregate private information and the price can be misaligned with respect to the asset value. One may wonder how much of our results relies on the specific modelling choices that we have made. In this Section, we address some issues that we consider

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<sup>21</sup>The parameters of the simulation are reported in the Appendix.

particularly important for related or future research:

1. We assumed that each the trader is chosen from a continuum of traders, so that the probability of a trader trading more than once is zero. In real markets, agents may trade many times and this may give them an incentive to manipulate the market. For instance, a trader with a positive signal might decide to ignore his signal and sell in order to buy at a later date (see, e.g., Chakraborty and Yilmaz, 2002). Therefore, the assumption that each trader trades at most once clearly eliminates this possibility and makes the revelation of private information more likely to occur. Despite this, we showed that private signals are not fully revealed.
2. We also assumed that the sequence of traders is exogenous, i.e., traders cannot choose when to go to the market. Chamley and Gale (1994) discussed the case of endogenous timing and proved that even with an endogenous timing of actions informational inefficiencies arise. In their model, there is not a price mechanism, but there is a cost of waiting for making a decision, due to a discount factor. In our model, there is clearly a cost of waiting, due to the fact that the price in expectation moves closer to the fundamental value. Therefore, if an agent waits, this reduces his expected profit from trading. Therefore a similar trade-off as in Chamley and Gale would arise in our framework, leading to an additional source of informational inefficiency<sup>22</sup>.
3. We considered a simple case with two types of traders, some with a loss and some with a gain from trade. One would of course like to analyze gains from trade under different distributional assumptions. A natural extension is the case in which gains from trade are distributed as a continuous random variable. In a companion paper, we study this case, discussing in particular what happens when the gains from trade are uniformly distributed with some agents having gains and others having losses from trade. We show that even in that case there is a measure of traders who herd. Moreover, given that the bid and ask spread converges to 0 over time, this measure of herders converges to

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<sup>22</sup>The case of endogenous timing in a framework with a price mechanism is also studied by Chari and Kehoe (2002).

1. Although the price ultimately converges to the fundamental value, because of the existence of herders, there is still an informational inefficiency similar to the one that we have described. For instance, consider a market where the fundamental value can be 1 or 2, and the realized value is 2. Let us analyze the case in which for some reasons the first trades are sell orders. Figure 5, describes the simulated price path for such an economy under three different distributional assumptions on gains from trade. In a first case there are no gains from trade (solid line); in a second there are discrete gains as described in this model (dash-dotted line); in a third, gains are distributed uniformly between  $\frac{1}{2}$  and 2.(dotted line)<sup>23</sup>. Without gains from trade, after the sell orders, the price drops, but then quickly recovers and converges to the fundamental. With discrete gains from trade, an informational cascade arises soon after the first sells and remains far from the fundamental forever. With continuous gains from trade, the initial sells cause the bid and ask spread to narrow and the measure of herders becomes immediately very high. Most informed traders disregard their private information and the price remains stuck for a long time to a value close to 1, far from the fundamental. We regard the case of continuous gains from trade as fundamentally similar to the one analyzed in this paper: in both cases informational externalities are present and prevent the price from aggregating efficiently the private information dispersed among traders.

## 6 Conclusion

In this paper we have obtained two main results. The first result is on informational cascades. In financial markets, when agents are heterogeneous so that trade can be mutually beneficial, both herd behavior and informational cascades arise. Information stops flowing to the market and, therefore, the market is unable to infer traders' private information and to discover the true values of the assets. The asset prices can remain at levels different from those of the fundamentals. Informational cascades imply that all informed traders will choose the same action, i.e., they will herd. Thus, we can explain the presence of herd behavior in financial markets.

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<sup>23</sup>These bounds of the support guarantee that no agent is a pure noise trader, i.e., no agent buys or sells independently of the asset value and of the price.



The second result is on contagion. The history of trades on one asset can significantly affect the price of the other. Informational cascades and herd behavior on one asset generate cascades and herd behavior on the other asset, pushing the prices far from the fundamental, even in the long run.

## Appendix

*Proof of Proposition 1:* To prove the proposition, one needs to show that there exists a time  $T$  when

$$E(V^J|H_T) - gE(V^J|H_T, x^J) < 0 \quad \text{for all } x^J,$$

and

$$lE(V^J|H_T, x^J) - E(V^J|H_T) < 0 \quad \text{for all } x^J. \quad (\text{A15})$$

In this case, at time  $T$ , a trader enjoying a positive utility from the asset buys independently of his signal. On the other hand, a trader suffering a disutility from the asset always sells. Furthermore, by the assumption  $E(E(V^J|H_t, x^J)|\rho, H_t) = E(V^J|H_t)$ , the market maker cannot update his belief on the asset value by knowing the type of informed trader who would put a buy or sell order.

Let us prove that there exists a time  $T$  when (A14) and (A15) hold. The traders' and the market maker's beliefs,  $E(V^J|H_t, x^J)$  and  $E(V^J|H_t)$ , are martingales. Therefore, they converge to a random variable with probability one. Moreover, in the absence of gains from trade (i.e.,  $\rho = 1$  for all informed traders), they converge to the same random variable (see Glosten and Milgrom, 1985). This means that, without gains from trade, for any  $\varepsilon > 0$  there exists a  $T$  such that, for  $t > T$ ,

$$\Pr(|E(V^J|H_T, x^J) - E(V^J|H_T)| < \varepsilon) = 1 \quad (\text{A16})$$

for all possible  $x^J$  given  $V^J$ . If there does not exist a realization of the signal  $\hat{x}^J$  such that  $\Pr(\hat{x}^J|\hat{V}^J) = 0$ , all realizations of the signals are possible for any  $\hat{V}^J$ . Therefore,

$$\Pr(|E(V^J|H_T, x^J) - E(V^J|H_T)| < \varepsilon) = 1 \quad (\text{A18})$$

for all  $x^J$ . By choosing  $\varepsilon = \min\{m(g-1), m(1-l)\}$ , we know that, in the absence of gains from trade, there is a time  $T$  in which (A14) and (A15) hold.

Now suppose that the proposition is false, i.e., there does not exist a time  $T$  when  $lE(V^J|H_t, x^J) - E(V^J|H_t) < 0$  and  $E(V^J|H_t) - gE(V^J|H_t, x^J) < 0$  for all  $x^J$ . This means that gains from trade are never binding, i.e., at least a subset of informed traders follow their own private information. In this

case, the behavior of beliefs is identical to that of the model without gains from trade. Therefore, the beliefs of traders and market maker converge to the same random variable and there exists a time  $T$  when (A14) and (A15) hold, a contradiction. Q.E.D.

*Proof of Corollary 1:* As shown in the proof of Proposition 6, there exists a time  $t$  at which all traders enjoying a gain from trade buy independently of their signal and all traders suffering a loss from trade will sell. Therefore, all informed traders of the same type act alike. Q.E.D.

*Proof of Proposition 3:* By following the same steps in the proof of Proposition 1, we can prove that there exists a time  $t$  when

$$E(V^A|H_t) - gE(V^A|H_t, x^A) < 0, \quad (\text{A20})$$

$$lE(V^A|H_t, x^A) - E(V^A|H_t) < 0 \quad \text{for all } x^A, \quad (\text{A21})$$

and

$$E(V^B|H_t) - gE(V^B|H_t, x^B) < 0 \quad (\text{A22})$$

$$lE(V^B|H_t, x^B) - E(V^B|H_t) < 0 \quad \text{for all } x^B. \quad (\text{A23})$$

When these conditions are satisfied, in both markets a trader enjoying a positive utility from the asset buys independently of his signal. On the other hand, a trader suffering a disutility from the asset always sells. By the assumption  $E(E(V^J|H_t, x^J)|\rho, H_t) = E(V^J|H_t)$ , the market maker cannot update his belief on the asset value by knowing the type of informed trader who would put a buy or sell order. Q.E.D.

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Figure 1

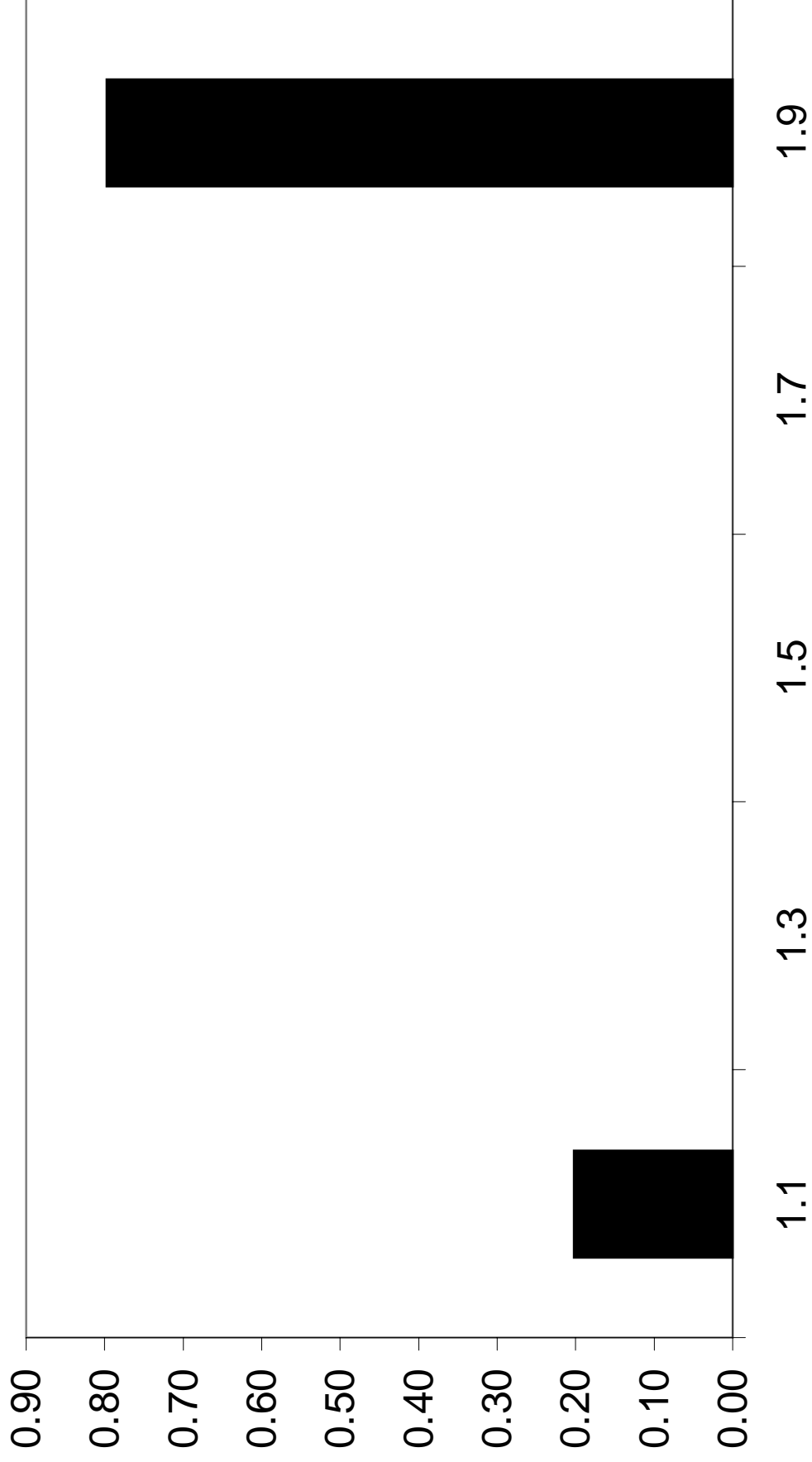


Figure 2

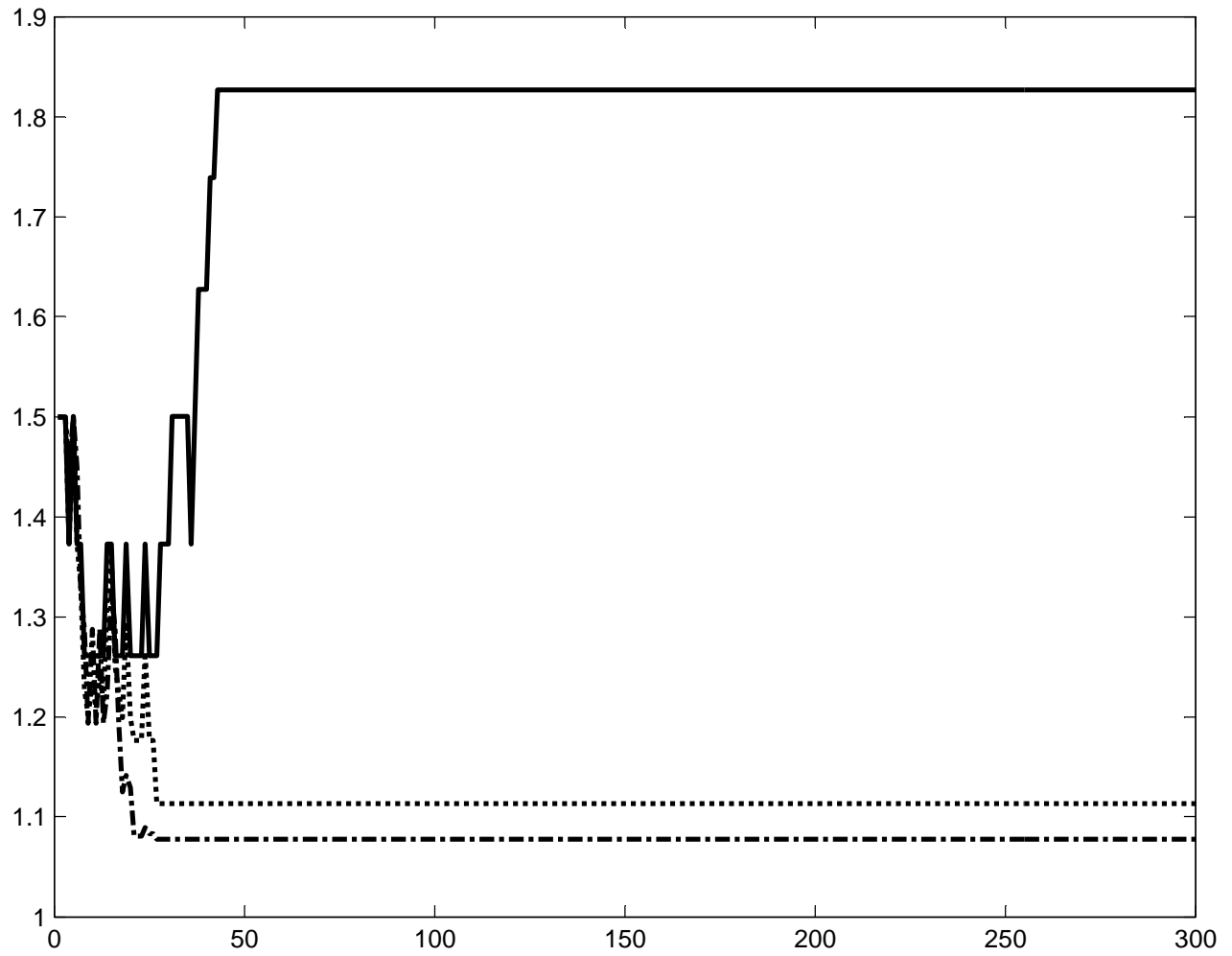




Figure 3

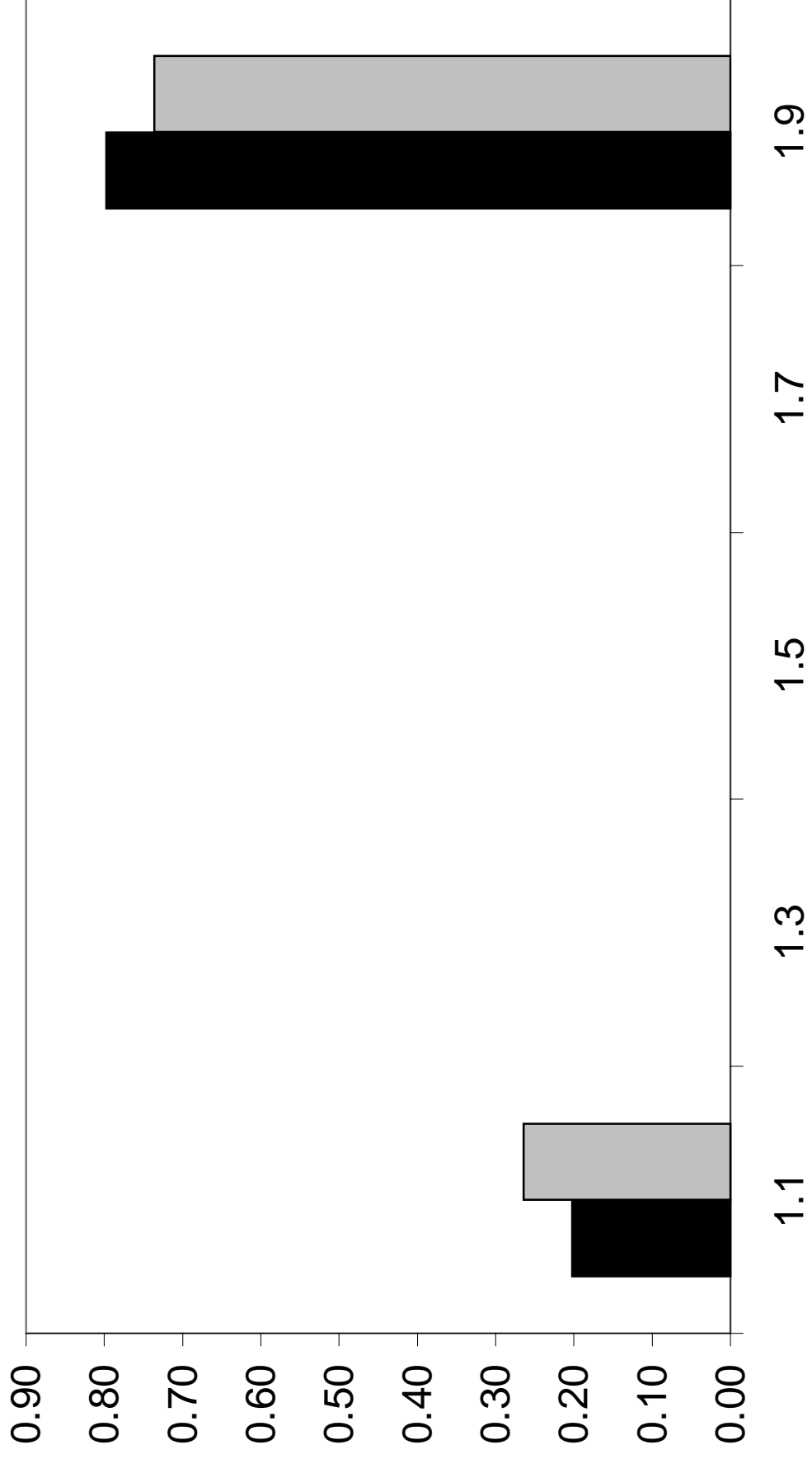


Figure 4

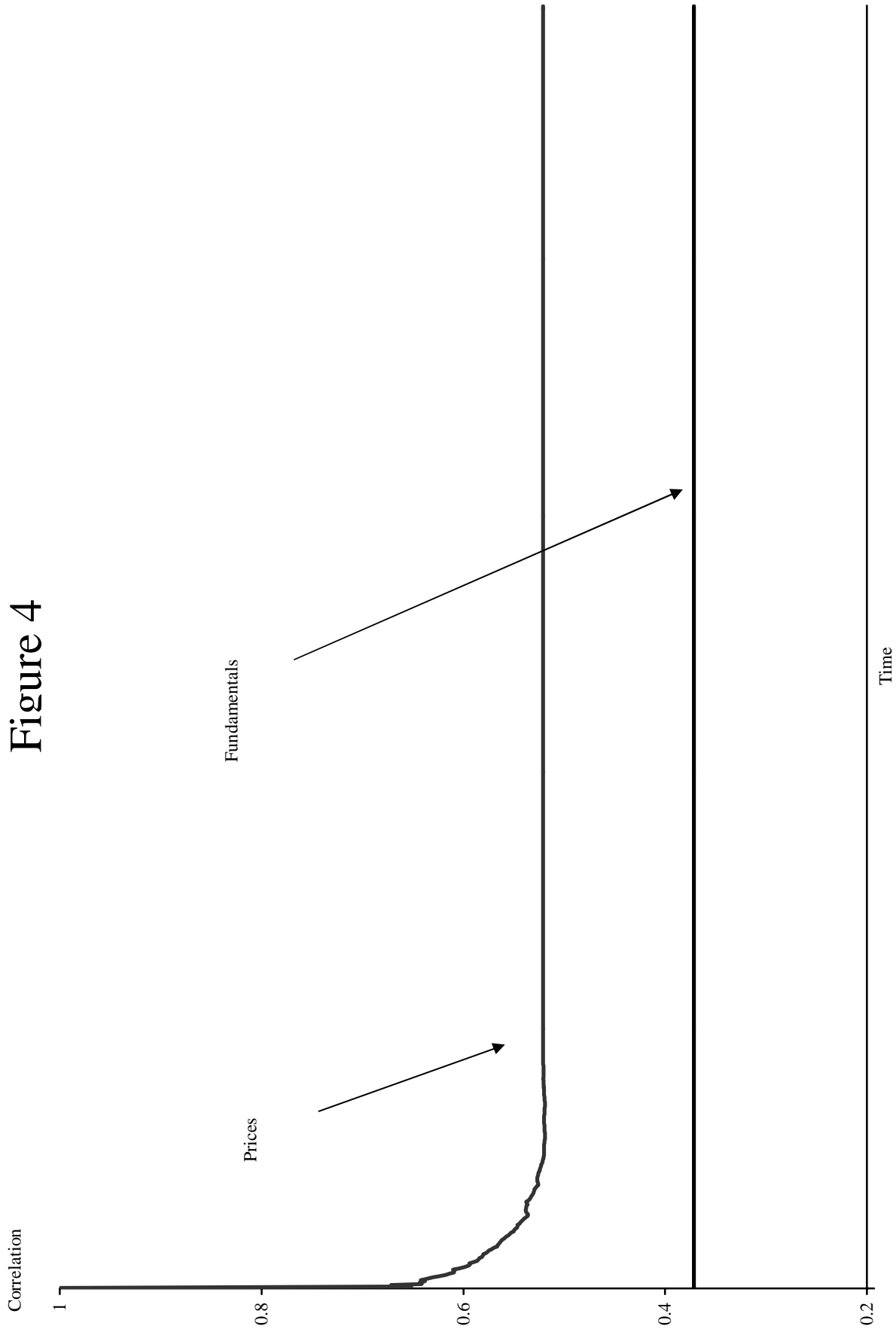


Figure 5

