BAYESIAN INFERENCE IN REPEATED ENGLISH AUCTIONS *

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Abstract

We propose a Bayesian approach to inference in repeated English auctions. The model tests the dynamic behaviour of a sequence of prices in a repeated English auction. We apply the technique to a sale of antiques and find that the sequence of selling prices displays a dynamic pattern.

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1 Introduction

Auctions are among the most widely used market mechanisms. They also provide one of the few examples in which game theory predictions can be tested on real datasets. A (non-cooperative) auction game can be described as follows. A set of participants (say firms interested in a procurement contract, individuals interested in antiques) submit bids for an object put for sale. Each participant is assumed to have some private information concerning the object. In our setting, each participant to a sale of antiques privately knows his willingness to pay for the object. Submitting a bid has a strategic feature since each bidder has to formulate an offer without knowing how much the other participants are willing to pay. In general, an equilibrium analysis of such a game leads to sharp predictions about individual behavior. Thus by observing all (or a subset of) bids submitted by the participants, it is possible to infer the distribution of participants' willingness to pay.

This paper adopts a Bayesian approach to the inference problem in repeated oral auctions. We claim that the Bayesian framework is more suitable for evaluating game theoretic models. Indeed the asymptotic approach has rather weak structural foundations when the underlying economic model involves strategic behaviour (Jouneau-Sion (2000)). The problem is that the structural interpretation of the econometric model may no longer be valid when the number of observations tends to infinity. This point deserves some explanations.

In empirical models of auctions there exist two feasible asymptotic dimensions: the number of participants and the number of objects put for sale. Changing the number of players affects the outcome of an auction game and consequently the data generating process itself. This route has nevertheless been followed by Florens and Protopopescu (1997) and Florens, Protopopescu and Richard (1998).[1] It is also is well-known that the relevance of strategic behavior disappears when the number of players tends to infinity.

Another approach is to use longitudinal data as in Laffont *et al.* (1995) and Paarsch (1992). This approach creates other difficulties. Laffont *et al.* (1995) correctly note that increasing the number of repetitions may not be harmless. More precisely, as the number of repetitions goes to infinity, the set of equilibria does not reduce to the simple repetition of the optimal

[1]This approach only appears in the mimeo versions of theses two papers (see also, in the context of collusion, Pesendorfer (1995)).

one-shot strategy. This multiplicity of equilibria undermines the structural feature of the econometric model. The coincidence between the econometric and the economic model is no longer guaranteed owing to the multiplicity of outcomes. One way to get around this problem is to assume that the repetitions of the game are (conditionally) independent from each other. Then the equilibrium strategy of the repeated game is simply the repetition of the equilibrium strategy of the one-shot game. The "independence" assumption precludes *a priori* any dynamic feature of the sequence of prices. This assumption is particularly unsatisfactory if one deals with art auctions as in our case. In theses auctions, several items are offered for sale in a short period of time. Insofar as the items are not completely identical, the sequence of prices might display a dynamic pattern which would not be captured by a "static" econometric model. We propose a Bayesian model which allows us to test whether the observed sequence of prices displays any dynamic feature. Finally, a Bayesian approach seems appropriate whenever, as in our case, the data set contains only a few observations and the econometrician cannot use more detailed information than the selling prices.

Other authors have already proposed a Bayesian framework for empirical models of auctions. Bajari (1997) estimates a first-price sealed-bid asymmetric auction game. In Bajari (1997), because of the sealed-bid nature of the auction mechanism, in principle all bids can be used for inference purposes. Moreover the numbers of participants is directly observed, since it equals the number of submitted bids.

Sareen (1998) treats the case of Dutch (i.e., descending first price) auctions. As noticed by Laffont *et al.* (1995), the evaluation of first-price auctions raises two kinds of difficulties. First, the players' optimal strategy involves the unknown distribution of private signals. The main consequence being that the observed outcome of the game is a complicated function of the underlying statistical model. Second, the number of players is not observed by the econometrician. This is a crucial issue since the number of players affects the strength of competition.

In this paper, we stress that the oral nature of English oral auctions implies that the number of active participants is in most cases unobservable. This phenomenon is likely to be a major point in art auctions since the heterogeneity among similar but not identical objects may raise the interest of a different number of bidders across sales. In oral auctions the estimation procedures have to rely only on one observation, namely the winning bid.

The paper is organized as follows. Section 2 sets forth the econometric model. In section 3 we explain the estimation techniques and apply our approach to a real data set. In particular, we discuss some empirical points related to the dynamic structure of the auction. Section 4 concludes.

2 The econometric model

2.1 Modelling art auction prices

We outline the essential feature of a model of English auction with independent private values. An object has to be auctioned to a set of risk-neutral bidders. Each bidder $i, i = 1, \dots, N$, is assumed to know exactly how much the item is worth to him, say S_i. He does not know anyone else's valuation of the object; instead, he perceives any other bidder's valuation as a draw from some probability distribution, $F(\cdot)$, which is assumed to be common knowledge. Similarly, he knows that the other bidders regard his own valuation as being drawn from the same probability distribution. Bidders' valuations are assumed to be independent.

To summarize, each bidder knows his own evaluation S_i, the distribution $F(\cdot)$, the number of potential buyers N. It is assumed that $F(\cdot)$ is absolutely continuous with respect to the Lebesgue measure on the (positive) real line.

We are mainly interested in art auctions. Our dataset consists in 106 chinese gold ingots found in wrecked ship. The art/antiques auction market has some particular features that are worth describing. First, several items are typically sold in a short period of time, say, from some minutes to an hour. These items are usually grouped: works by the same artist, antiques from the same period, etc. Sometimes the items appear – at least to the layman – as almost perfect substitutes. This is the case in our data set. We thus deal with highly homogenous items though not completely identical. Second, it is very difficult to observe the intermediate bids and the identity of active bidders even if one is attending the sale. Finally, the number of bidders interested in each item is likely to be small. In brief, the set of observations consists in the number of people attending the sale and the winning bids. Considering the limited informational content of our data set, a Bayesian approach seems particularly adequate.

We want to keep the framework as simple as possible. This implies several choices for our model. We have stressed that using the number of repetitions as an asymptotic dimension may create some difficulties in defining participants' optimal strategies. We are not aware of closed form equilibrium strategies in the context of repeated English auctions[2]. However, a general result in repeated game theory is that individual behaviors may

[2]Bikhchandani (1988) analyzes a special model of repeated second-price auction.

display dynamic patterns. This seems particularly relevant for our case where the items are sold in a very short time period, since then the time preference parameter is likely to be close to one. We thus propose to take into account a possible dynamic pattern in the sequence of selling prices.

We also limit our analysis to the case of a parametric distribution for the signals. In our evaluation we restrict to the class of log-normal distributions. A Bayesian non-parametric approach would be much more challenging.

The above discussion leads us to model the signals of the l-th ($l \geq 1$) auction as follows:

$$\log(S_{i,l}) = \alpha + \beta_1 p_{l-1} + \beta_2 p_{l-2} + \gamma_0 g_l + \gamma_1 g_{l-1} + \gamma_2 g_{l-2} + \sigma \epsilon_{i,l}, \\ \forall\, i = 1, \ldots, N_l$$

where N_l is the number of active participants for the l-th auction, $\epsilon_{1,1}, \ldots, \epsilon_{N_L,L}$ is a standard Gaussian vector, $p_l = \log P_l$ is the logarithm of the price and g_l is the logarithm of the weight of the l-th ingot. We also use the notation θ for the parameters $\alpha, \beta_1, \beta_2, \gamma_0, \gamma_1, \gamma_2$. Accordingly, $x_l = (1, p_{l-1}, p_{l-2}, g_l, g_{l-1}, g_{l-2})'$ denote the explanatory variables for auction l.

Following the well-known result by Vickrey (1961), the price given N_l is:

$$P_l = S_{(N_l-1:N_l)}\,,$$

where $S_{(N_l-1:N_l)}$ corresponds to the second order statistic of the i.i.d. sample $S_{1,l}, \ldots, S_{N_l,l}$. The distribution of the logarithm of the price is given by

$$\pi(p_l \mid \theta, \sigma, x_l, N_l) \propto \\ N_l(N_l-1)\,\sigma^{-1}\,\phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]\,\Phi\left[\sigma^{-1}(p_l - \theta' x_l\right]^{N_l-2}\,\Phi\left[-\sigma^{-1}(p_l - \theta' x_l]\right)\right],$$

where ϕ stands for the p.d.f. of the standard normal distribution and Φ for its c.d.f. counterpart.

2.2 Discussion

Several remarks are immediate. First, notice that, contrarily to Sareen (1998), the number of players may change across auctions. The idea is that in a given auction one specific item may be much more interesting for some unobserved reasons (quality, collector behavior...). This is a major source of heterogeneity across prices. Indeed the distribution of the selling price for the l-th auction is that of the second highest signal among N_l. Increasing the number of players shifts the expectation of the price to the right (it also affects the variance since the signals are log-normally distributed).

Second, past prices and weights may affect the current signals. The reason why we consider lag effects of the weights appears in the following formula, we have:

$$E[\log(S_{1,l}/G_l) \mid p_{l-1}, p_{l-2}, g_l, g_{l-1}, g_{l-2}] =$$
$$\alpha + \beta_1 log(P_{l-1}/G_{l-1}) + \beta_2 \log(P_{l-2}/G_{l-2}) + (\gamma_0 - 1)g_l + (\gamma_1 + \beta_1)g_{l-1} + (\gamma_2 + \beta_2)g_{l-2}.$$

(where G_l stands for the weight of the ingot).

Indeed, if we impose $\gamma_0 = \gamma_1 = \gamma_2 = 0$, then if the β parameters appears to be significative, the price per gram of gold would be influenced by past weights. The presence of the γ parameters allow us to capture a possible effect of past weight independently of that past prices.

When the β and/or γ parameters are significantly different zero, the prices are not independent. However, our model does not incorporate an explicit structural dynamic strategy. Bidders' behavior is entirely myopic. Indeed the mean of the signals depends on the previous outcome but none of the bidders takes into account the influence of his current bid on future prices. A more precise theoretical picture of the underlying dynamic game is needed for a genuine structural interpretation of the dynamic part of the model when it appears to be significative. It is clear that this form of dynamic is somewhat *ad hoc.* However a totally static behavior – as usually assumed – is also *ad hoc.* Our model could be interpreted as a reduced form for some –unknown– dynamic behavior.

¿From an econometric viewpoint, the presence of past prices and weights in the distribution of the current signals may also be interpreted as a proxy for some unobserved exogenous effects. We have stressed that highly homogenous items are often sold in art auctions. Nevertheless, the differences across selling prices may be large. Were the model entirely static, an increase in the prices could only be explained by a sudden strenghtening of the competition (*i.e.* the number of players). The dynamic part of the model may be a proxy for some trended unobserved exogenous variable. For instance, the pictures provided in Christie's catalog clearly show that some ingots are shallowed by encrustrations due to the very long stay in the sea. The first ingot sold has almost no encrustration whereas the 42-nd one has much more. These are the only clear pictures we have. We suspect that the first ingots sold have fewer encrustrations than the last ones. However we have no further information concerning the dynamic of the qualities. The dynamic of the model could cope for a possible unobserved trend in the quality of the ingots.

2.3 Prior distributions of the parameters

To complete the Bayesian framework we now describe the prior distributions for θ, σ and $N_1, \ldots, N_L$. We first consider the case for N_l. To rule out any possible confusion we will call "sale" the set of auctions. At each auction only one item is sold. Moreover, we will call "players" or "active participants" those who are really engaged in the bidding process.

Getting information about the number of active participants before the sale starts is usually a difficult task for auctions of antiques. It may be argued that the number of people attending the sale is a good proxy for the number of *active* participants. This is in general a misleading argument. On the one hand, participants to a sale might be interested only in a subset of the items to be sold. In this respect, the number of people attending the sale is larger than the number of active participants to a given auction. On the other hand, some active participants may be absent (see Ginsburgh (1998) for such a case in the context of wine auctions). In this last case, the number of active participants may be larger than the number of people attending the sale.

As we already explained, we treat $N_1, \ldots, N_L$ as parameters. Choosing a prior distribution for N_l should take into account the characteristics of real auctions. For art auctions, people are typically interested in a few items. Suppose for the moment that L items are to be sold in L auctions and that M people are attending the sale. Moreover, assume that the probability that a potential buyer is interested in a given item is equal to $1/L$. This corresponds to the case where most potential buyers are interested in exactly one item, but the econometrician ignores which participant is interested in which item. Thus the econometrician is bound to assign *a priori* equal probabilities to the event that a given potential buyer is willing to buy a specific object.

We assume that potential buyers behave "independently".[3] Then the number of active participants to each auction is identically distributed as a Bernoulli distribution $\mathcal{B}(M, 1/L)$. When both M and L are large, we then have the Poisson approximation $\mathcal{B}(M, 1/L) \simeq \mathcal{P}(M/L)$. Now recall that the auction model requires the number of participants to be larger than 2. Then, we are naturally led to choose a Poisson distribution as a prior for N_l with a lower truncation at 2. More formally we set :

[3]We rule out the case where a potential buyer is interested in a given item for the mere reason that another participant is willing to buy it.

$$\pi(N_1, \ldots, N_L) \propto \prod_{l=1}^{L} \frac{\mu^{N_l}}{N_l!} \mathbb{1}_{N_l \geq 2},$$

with $\mu = M/L > 1$. Note that this distribution is written w.r.t. the count measure on $\mathbb{N}^L$. Furthermore N_l is assumed to be *ex ante* independent of the other parameters of the model.

We choose $\mu = 1.2$ which gives the following prior distribution:

N_l	p.d.f	c.d.f
2	64.28%	64.28%
3	25.71%	89.99%
4	7.71%	97.70%
≥ 5	2.3%	100%

The justification of this value for μ is the following. The sale concerns 105 ingots. As μ may be interpreted as M/L our choice for the prior is consistent with less than 130 people attending the sale.

The total number of parameters is $L + 7$. We thus face an incidental parameter problem. Clearly, it would be difficult to handle such a problem in the classical framework (see, however the finite sample technique proposed by Jouneau-Sion and Torrès (2000)). Our modelling choice raises two different questions. First, there may be some identification problems if we use improper distributions as priors (see below). Second, the sequence $(N_1, \ldots, N_L)$ may be considered as a nuisance parameter. This last interpretation calls for a reference prior approach (see for instance Bernardo and Smith (1994)). Note however that one could be interested in estimating the strength of competition, thus the number of players in a specific auction may be of economic interest. Moreover, the reference approach seems difficult to apply in our setup. Indeed, it relies on asymptotic invariance of the choice of prior, but, as we argued above, any reference to the asymptotic approach is inadequate in our problem (both for econometric and economic reasons). Note finally that our choice of prior for the numbers of players is not completely *ad hoc* as the above discussion shows that it is related to some priors on the participation behavior of the bidders.

For the θ parameters, we only avoid the case where the dynamic process is explosive. Thus the θ parameters are assumed to fullfil the conditions implying the stationarity of the underlying process. As we did not specify any dynamic for the process of ingots' weights we only impose the usual restriction for an AR(2) process to be stationary, precisely $\beta_2 \leq \min\{1+\beta_1, 1-\beta_1\}$ and $\beta_2 \geq -1$ (see, for instance, Hamilton (1994)). We shall denote Θ the subset of $I\!R^6$ such that the stationarity conditions are fulfilled.

We choose the uniform prior on $I\!R^{++}$ for the logarithm of σ.

If we observe L auctions, the joint posterior distribution is given by :

$$\pi(\theta, \sigma, N_1, \ldots, N_L, | \, p_1, \ldots, p_L) \propto$$

$$\frac{\mathbb{I}_{\sigma>0}}{\sigma} \times \prod_{l=1}^{L} \frac{\left(\mu\Phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]\right)^{N_l-2} \mathbb{I}_{N_l \geq 2}}{(N_l-2)!} \frac{\phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]}{\sigma} \Phi\left[-\sigma^{-1}(p_l - \theta' x_l)\right] \mathbb{I}_{\theta \in \Theta}.$$

Note that the parameters of the model are not *ex post* independent. In particular, the numbers of players are independent conditionally on the other parameters but they are not marginally independent.

2.4 Existence of the posterior moments

The ex-post distribution of (θ, σ) is easily computed. Indeed, we have:

$$\begin{aligned}
&\sum_{N_1=2}^{+\infty} \sum_{N_2=2}^{+\infty} \cdots \sum_{N_l=2}^{+\infty} \prod_{l=1}^{L} \left(\mu\Phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]\right)^{N_l-2} \frac{1}{(N_l-2)!} \mathbb{I}_{N_l \geq 2} \\
&= \prod_{l=1}^{L} \sum_{N_l=2}^{+\infty} \left(\mu\Phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]\right)^{N_l-2} \frac{1}{(N_l-2)!} \mathbb{I}_{N_l \geq 2} \\
&= \prod_{l=1}^{L} exp\left(\mu\Phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]\right)
\end{aligned}$$

Thus we deduce

$$\pi(\theta, \sigma \mid p_1, \ldots, p_L) \propto$$

$$\frac{\mathbb{I}_{\sigma>0}}{\sigma} \times \prod_{l=1}^{L} exp\left(\mu\Phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]\right) \frac{\phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]}{\sigma} \Phi\left[-\sigma^{-1}(p_l - \theta' x_l)\right] \mathbb{I}_{\theta \in \Theta}$$

The posterior moments for (θ, σ) exist under usual conditions. Indeed we have the clear upper bound:

$$\begin{aligned}
&exp\left(\mu\Phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]\right) \phi\left[\sigma^{-1}(p_l - \theta' x_l)\right] \Phi\left[-\sigma^{-1}(p_l - \theta' x_l)\right] \leq \\
&\phi\left[\sigma^{-1}(p_l - \theta' x_l)\right] \times \max_{x \in [0,1]} exp(\mu x)(1-x) \qquad (1) \\
&= \phi\left[\sigma^{-1}(p_l - \theta' x_l)\right] exp(\mu - 1)/\mu
\end{aligned}$$

where the last equality takes into account the constraint $\mu > 1$.

Thus the conditions under which the posterior moments for θ and σ exist are the same as for the linear regression model where the endogenous variable is p_l and the explanatory variables are x_l. These conditions are clearly fulfilled in our setting (the matrix of explanatory variables has full column rank).

Finally we have

$$\pi(N_1, \dots, N_L, | \, p_1, \dots, p_L, \theta, \sigma) \propto \prod_{l=1}^{L} (\mu\Phi \left[\sigma^{-1}(p_l - \theta' x_l)\right])^{N_l - 2} \frac{\mathbb{I}_{N_l \geq 2}}{(N_l - 2)!}$$

Thus, conditionally on θ, σ and the data $(N_1 - 2, \dots, N_l - 2)$ are independently distributed. The distribution of $N_l - 2$ given θ, σ and the data is $\mathcal{P}(\mu\Phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]$. Thus all posterior moments of N_l, $l = 1, 2, \dots, L$, exist. Also notice that as the difference $p_l - \theta' x_l$ increases, the forecasted N_l gets larger.

3 Inference technique and application

Equation (1) also provides a way to simulate from the posterior distribution of (θ, σ). Indeed, we propose an acceptance/rejection algorithm in which (θ, σ) is drawn as if the model were the usual gaussian linear regression (stationarity conditions imposed). We accept this draw as soon as

$$\prod_{l=1}^{L} \exp\left(\mu\Phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]\right) \Phi\left[-\sigma^{-1}(p_l - \theta' x_l)\right] exp(\mu - 1)/\mu$$

is larger than an independent draw from the uniform distribution. If μ is not too large, the acceptance rate is reasonable (for instance for $\mu = 1.2$ the acceptance rate is close to 10%.).

The simulation of the numbers of players is easy. For every accepted drawing of (θ, σ) we simply have to simulate in the Poisson distribution $\mathcal{P}\left(\mu\Phi\left[\sigma^{-1}(p_l - \theta' x_l)\right]\right)$ and to add 2.

3.1 Description of the data set

The data set comes from the sale of 105 Chinese gold ingots that Christie's organized in Amsterdam on April 28, 1986. The Chinese bars of precious metal have the form of the slippers of womenfolk in ancient China. Each rectangular ingot displays a shallowly-domed underside and gently-sloping sides rising to a sharp slightly-raised edge. The top is centrally stamped with Chinese characters. The dimensions of each ingot are 8 cm wide, 2.5 cm deep, and 1.5 cm high at the edge. An essay test on the first ingot (item number 1819 in the Christis's catalogue) of the lot indicated a fineness of 868 parts per 1000 (almost 21-carats gold).

– Insert pictures 1 and 2 about here –

As we already explained, the 106 gold ingots are almost all alike except for small incrustations due to the very long stay in the sea (see picture 1 and 2). Recall we suspect –although we have no clear proof of this statement– that the most well-preserved (i.e. with almost no incrustations) ingots have been sold first.

The following chart shows the time series of the selling prices (unit is a thousand of dutch guilders).

– Insert chart 1 about here –

The data display a clear downward trend. This phenomena is common to many auctions (see, for an explanation, Ginsburg (1998)). This does not automatically induce that the signals are correlated. Indeed, this dynamic pattern may be the outcome of a downward trend in the number of participants.

The following table gives some simple descriptive statistics:

	min.	aver.	max.	std. err.
price ($\times$1000 guilders)	17	21.3	38	4.87
weight (grams)	318	367.05	394	6.06
price/gram (guilders/gram)	46.32	57.99	103.83	13.08

Finally, it is worth noting that the price of gold on the regular market by the time of the selling was 27 guilders per gram. It is clear that the ingots have not been bought as "ordinary" pieces of precious metal. The smallest price per gram is 72% higher than the equivalent amount of gold on the regular market. The ingots are definitively antique collector items.

3.2 Inference results

We have generated 50.000 draws from the posterior distribution of (θ, σ). The results are presented in the following table.

	α	β_1	β_2	γ_0	γ_1	γ_2	σ
Min.	-0.375	0.099	0.211	-0.211	-0.232	0.024	0.073
Q1%	-0.196	0.248	0.306	-0.071	-0.088	0.027	0.081
Q5%	-0.136	0.3	0.333	-0.024	-0.038	0.028	0.083
Q10%	-0.103	0.327	0.348	0.001	-0.012	0.028	0.085
Mean	0.014	0.424	0.397	0.096	0.082	0.03	0.091
Median	0.014	0.425	0.397	0.094	0.081	0.03	0.09
Q90%	0.13	0.52	0.447	0.191	0.179	0.031	0.097
Q95%	0.163	0.547	0.462	0.221	0.208	0.032	0.099
Q99%	0.229	0.596	0.489	0.277	0.263	0.033	0.103
Max.	0.389	0.694	0.561	0.448	0.433	0.035	0.116

The dynamic effects of past prices are clearly present. Larger prices in the past sustain larger current prices. None of the 50.000 simulations sustains the hypothesis of absence of dynamic effect.

The past and current effects of the weight are easier to understand if we use the following model:

$$E[\log(S_{1,l}/G_l) \mid p_{l-1}, p_{l-2}, g_l, g_{l-1}, g_{l-2}] =$$
$$\alpha + \beta_1 log(P_{l-1}/G_{l-1}) + \beta_2 \log(P_{l-2}/G_{l-2}) + (\gamma_0 - 1)g_l + (\gamma_1 + \beta_1)g_{l-1} + (\gamma_2 + \beta_2)g_{l-2}.$$

The table below presents the estimation corresponding to the above parametrization (the last two columns are given to check the stationarity conditions).

	$\gamma_0 - 1$	$\gamma_1 + \beta_1$	$\gamma_2 + \beta_2$	$\beta_1 + \beta_2$	$\beta_2 - \beta_1$
Min.	-1.211	0.134	0.241	0.506	-0.431
Q1%	-1.071	0.283	0.336	0.674	-0.263
Q5%	-1.024	0.348	0.364	0.718	-0.194
Q10%	-0.999	0.382	0.378	0.74	-0.157
Mean	-0.904	0.506	0.427	0.821	-0.027
Median	-0.906	0.505	0.427	0.822	-0.028
Q90%	-0.809	0.633	0.476	0.902	0.104
Q95%	-0.779	0.671	0.491	0.925	0.141
Q99%	-0.723	0.743	0.517	0.964	0.212
Max.	-0.552	0.926	0.591	1.000	0.35

We see that heavier ingots appear slightly (althought significatively) under-priced (in terms of guilders per gram of gold). On the other hand, selling the heavier ingots first, helps increase future prices.

Since the stationarity conditions have been imposed, they are always fullfilled. Notice that the only one which appears to be bounded is $\beta_2+\beta_1 \leq 1$. In an unconstrained estimation exercise we evaluate the probability that this conditions is violated at 0.003 whereas all other conditions are always fullfilled.

We now turn to the estimation of the number of players.

price	2 players	3 players	4 players	$\geq$ 5 players
38	99.425%	0.545%	0.026%	0.004%
34	99.182%	0.759%	0.056%	0.003%
32	97.446%	2.218%	0.295%	0.041%
30	99.029%	0.902%	0.064%	0.005%
28	96.366%	2.758%	0.68%	0.196%
27	99.277%	0.691%	0.032%	0%
25	98.997%	0.914%	0.078%	0.01%
24	99.085%	0.858%	0.055%	0.002%
23	97.657%	1.834%	0.424%	0.085%
21	99.895%	0.102%	0.003%	0%
20	99.149%	0.765%	0.075%	0.01%
19	99.395%	0.574%	0.027%	0.003%
18	99.79%	0.202%	0.006%	0%
17	99.955%	0.043%	0%	0%

Clearly, the posterior probability is highly concentrated on the occurence $N_l = 2$ (see also Jouneau-Sion and Torrès (2000) for a similar finding). Notably, the probablity of strictly more than 2 players is not increasing with P_l. This is due to the dynamic effects. Recall indeed that when no dynamic effects are present, any increase in the price is explained by an increase in the number of players. Note also that the posterior distribution differs from the prior. Despite the large number of parameters, the data do help identify the number of players in our model. We may also remark that the largest posterior probabilities for $N_l = 3$ are obtained for $P_l = 32, 28$ and 23. The price 32 correspond to auctions 5,7,9 and 14. In the last three cases the price 32 correspond to an increase in the price (see chart1). The two other cases $P_l = 28$ and $P_l = 23$ correspond either to an increase in the price or to the same price after repeated declining prices. Thus when the prices are much larger than expected (past prices taken into account) the model clearly forecasts a strengthening of the competition.

Finally, we also estimated the probability that $N_l = 2$ for each auction. Not surprisingly, this posterior probability is rather high (81.5%). The heterogeneity introduced by varying the number of active players across auctions cannot explain the dynamic patterns observed in the prices. The variation of the prices cannot solely derive from heterogenous degree of competition. Our dataset definitively shows the presence of dynamic bidding behavior.

We have also assessed the forecasting ability of our model. To this end, the last 20 observations have been withdrawn from the sample. The following chart describes the forecasting errors.

–Insert chart 2 about here–

4 Conclusion

In this paper we propose a Bayesian approach to the estimation of oral English auctions. We describe an inference technique which is applied to the estimation of an antique auction. The results clearly show that the dynamic effects cannot be bypassed. From our viewpoint, this calls for a closer look at possible dynamic aspects in empirical studies of repeated auctions.

Our model is only a first step as the dynamic is introduced in "reduced form" way. A challenging line of research is to formalize a structural econometric model in which individual strategies would exhibit a dynamic pattern. This task is certainly difficult owing to the lack of theoretical results in repeated auctions.

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