Researching teachers' epistemological development concerning mathematical variance and invariance in a multi-representational technological setting – An expansion of Verillon and Rabardel's triad of instrumented activity.

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1 Abstract

This paper reports one of the outcomes of a doctoral study which sought to apply and expand Verillon and Rabardel's 1995 triad of instrumented activity as a means to understand the longitudinal epistemological development of a group of secondary mathematics teachers as they began to integrate a complex multi-representational technology (MRT), the TI-Nspire handheld and software, into their classroom practices (Clark-Wilson, 2010b). The research was carried out in two phases. The initial phase involved fifteen teachers who contributed a total of sixty-six technology mediated classroom activities to the study. The second phase adopted a case study methodology during which the two selected teachers contributed a further fourteen activities. The subsequent analysis of this data, supported by questionnaires and interviews, led to a number of conclusions relating to the nature of the teachers' own technology-mediated learning.

2 Acknowledgements

The data collection carried out during Phase One of the study (and part of the data collection in Phase Two) was funded by Texas Instruments within two evaluation research projects, which have been published in Clark-Wilson (2008a) and (2010a).

3 Introduction

The research that is reported in this paper had the broad aim to articulate the nature of secondary mathematics teachers' epistemological development as they began to use a new multi-representational technological tool (MRT) with students in their classrooms. The specific mathematical focus of the study centred on the activities that privileged explorations of variance and invariance. The research was carried out in two phases, July 2007 - Nov 2008 and April 2009 – December 2009, during which time groups of teachers were selected, and a series of methodological tools developed, to capture rich evidence of the teachers' uses of the technology in their classrooms to enable the aims of the study to be realised. The first phase of the project was located within a professional development setting, which blended opportunities for the teachers to learn about the affordances of the technology alongside time for the teachers to design

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activities and give subsequent feedback about the outcomes of their lessons. The second phase of the study was situated within the participating teachers' mathematics classrooms.

4 Theoretical background

The theoretical foundations for the study concerned three domains: coming to know new technologies and the role of technology in developing subject and pedagogic knowledge; the concept of variance and invariance in a MRT setting; and making sense of the process of teacher learning. In this article, for reasons of space, the emphasis within this section of the paper is on the first of these three themes as it is most relevant to the ATATEMLO Symposium.

The theoretical framework that was developed for the study was rooted in Verillon and Rabardel's theory of instrumented activity systems as a model to describe the processes involved in human-instrument interactions (Verillon and Rabardel, 1995). In their original model, the Subject-Instrument-Object triad assumed that the subject's primary consideration was to evolve uses of the technology for some clear purpose. This module has been applied to a number of situations within mathematics education research where the lens has been trained on students of mathematics who were beginning to use chosen technologies for the purpose of solving mathematical problems (Guin and Trouche, 1999, Artigue, 2001, Ruthven, 2002). However, the context for my own study brought another consideration to the fore. As the subjects within my study were teachers, there were two facets to the object for their subsequent use for the technology. It was obviously necessary for them to become familiar with the affordances of the technology but also, a simultaneous consideration for them was whether and how these affordances could be integrated into educationally legitimate classroom activities for mathematics.

Consequently, Verillon and Rabardel's triad was adapted as shown in Figure 1

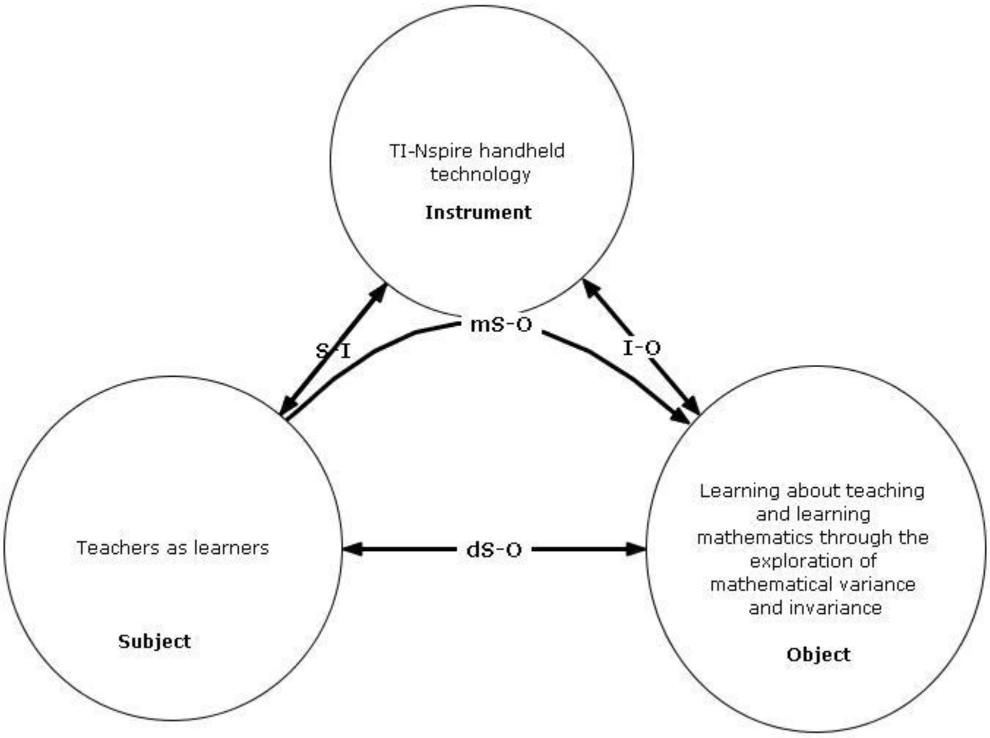


Figure 1 The adaption of Verillon and Rabardel's triad characteristic of Instrumented Activity.

This refined triad defined the subject as 'teachers as learners', with the object of their activity concerning their epistemological development within the context of designing, teaching and evaluating explorations of mathematical variance and invariance. The instrument incorporated the mediating artefact, that is the TI-Nspire handheld and software alongside the emergent utilisation schemes developed individually by each teacher or socially, where collaboration was involved. Hence the study sought to gain a deeper insight into the mediating role of the technology, as indicated by the pathway mS-O within Figure 1.

The mathematical focus for the study concerned activities that privileged the students' 'explorations of variance and invariance', the justification for which was partly a constraint of the project's methodology. The multi-representational features of TI-Nspire prompted a review of key texts and research that had considered both the mediating role of technology in supporting such explorations alongside a review of literature on the nature of a mathematical variable (Bednarz et al., 1996, Moreno-Armella et al., 2008, Sutherland and Mason, 1995, Kaput, 1986, Kaput, 1998, Kieran and Wagner, 1989). This review led me to define mathematical learning as being predominantly concerned with the privileging of students' opportunities to generalise and specialise as a means to constructing their own mathematical meanings. Within the context of this study, the teacher's role was to design and orchestrate classroom activities and approaches, using the various functionality of the MRT to achieve this. However, as teachers' individual belief systems about mathematical learning (and the role of technology within this) would undoubtedly influence their decisions and actions, the trajectory of teacher development to which I refer also revealed evidence of these preconceptions.

Finally, as the study was concerned with the nature and processes of mathematics teachers' epistemological development, two areas of related literature were reviewed. The first area concerned definitions and interpretations of mathematics teachers' personal knowledge, subject knowledge for teaching and pedagogic knowledge (Shulman, 1986, Rowland et al., 2005, Zodik and Zaslavsky, 2008, Polanyi, 1962). The second area examined constructs concerning the process of teacher learning (Schön, 1984, Thompson, 1992, Mason, 2002, Jaworski, 1994, Ahmed and Williams, 1997)[i]. The review of literature referring to the content, nature and process of teacher learning led me adopt a broad interpretation of knowledge as proposed by Shulman's 'knowledge for teaching'. It also highlighted the complexities of the process of teacher learning and supported the development of methodological tools that would capture the evidence of this learning in line with my desire to describe teachers' trajectories of epistemological development.

5 Methodology

An extensive data collection period between July 2007 and November 2009 resulted in the participating teachers contributing eighty 'lesson bundles' to the study. During the first phase of the study, a lesson bundle comprised all or some of the following:

- lesson evaluation questionnaire (compulsory) - see (Clark-Wilson, 2008b);
- an activity plan in the form of a school lesson planning proforma or a hand-written set of personal notes;
- a lesson structure for use in the classroom (for example a Smart NoteBook or PowerPoint file);
- a MRT file developed by the teacher for use by the teacher (to introduce the activity or demonstrate an aspect of the activity);
- a MRT file developed by the teacher for use by the students, which would normally need to be transferred to the students' handhelds in advance or at the beginning of the lesson;
- an activity or instruction sheet developed by the teacher for students' use;
- students' written work resulting from the activity;
- students' MRT files captured during and/or at the end of the activity;
- audio or video clips of the activity;
- notes or slides from presentations made by the teachers about the activity.

This raw data was imported to Nvivo8 where it was subsequently scrutinised and coded to elicit three elements: a broad description of the lesson; an inference concerning the teacher's interpretation of variance and invariance within the designed activity; and the suggested instrument utilisation scheme. An example of this for a lesson 'Prime factorisation', submitted by one of the teachers early on in the study is shown in Table 1.

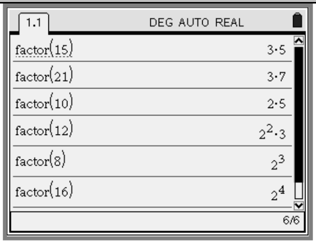
Lesson code, title and activity description	Relevant screenshots and implied evidence of teacher learning	Interpretation of variance and invariance	Implied Instrument Utilisation Scheme
Lesson code: SJK1 Prime factorisation Students created a new file and used the *factor()* command within the Calculator application to explore different inputs and outputs to encourage generalisation. Students recorded their work on a worksheet prepared by the teacher.	 Q. What aspect(s) of the idea would you use again? *Definitely the worksheet idea as this enables pupils to work at their own pace. I needed the worksheet for me as well as for them. I was able to refer to the sheet and that helped my confidence. The sheet also allowed pupils to continue with the work whilst I went round to help students with a problem. BUT - the sheet could have been more structured i.e. not jump around haphazardly but be more systematic. Factor (1), Factor (2), Factor (3), etc... I liked Exercise 1 but questions 1 and 2 were too hard for this group.* *I was nervous to use the device even though I am a very experienced teacher of maths. I needed the worksheet as support. Having done one lesson I would now be confident to try again. The worksheet could have been more interesting. Pupils seemed to enjoy the lesson.* [SJK1(Quest2)]	The teacher had constructed a worksheet (with support from her mentor) that did lead students through a set of suggested input numbers that progressed in their level of complexity. **Variance** = changing the input number (a manual text input to calculator application using factor() syntax) **Invariance** = all prime numbers had only two factors (by definition).	**IUS1**: *Vary a numeric or syntactic input and use the instrument's functionality to observe the resulting output in numeric, syntactic, tabular or graphical form.* **Private utilisation scheme**: Students worked with a numerical input and output within one application responding to the same set of numbers, as provided by the teacher. No use of multiple representations using the technology. **Social utilisation scheme**: Whilst JK developed this lesson in close collaboration with TP, who taught a similar lesson [STP1]. The lessons had very different utilisation schemes. JK provided a structured worksheet for the students that contained a variety of questions that did not appear to follow any conceptual progression, although the factorised forms became increasingly complex.

Table 1 The summary data for the lesson 'Prime factorisation'

These analyses led to the development of nine 'instrument utilisation schemes' which in turn supported the identification of the two teachers, Tim and Eleanor, who became the subjects of more in-depth case studies during the second phase. The second phase of the study still required the teachers to design, teach and evaluate lesson activities using the MRT and, additionally it involved lesson observations, which were all audio-recorded (with key sequences also video-recorded). The two teachers were also interviewed before and after the classroom observations. This more substantive data was initially used to write a detailed description of the lesson, interspersed with mediating screen shots from the teacher's and students' MRT files. This process was greatly supported through the use of the MRT handheld classroom network system TI-Navigator, which facilitated the 'real-time' data collection process without interrupting the flow of the lessons.

6 Discussion

Whilst the study provided data that evidenced the teachers' epistemological development with respect to all four interactions within the adapted triad (S-I, I-O, dS-O and mS-O in Figure 1), this discussion focuses on the development that was mediated by the MRT. This led to the nature of the mediated subject-object (mS-O) interaction being considered in two ways.

The first concerned how the teachers' mathematical ideas about variance and invariance *shaped and were shaped by* their uses of the technology. This was a two-way consideration of the nature of the way in which the mathematical terrain was being reconceptualised with respect to its content and pedagogies. The teachers' mathematical ideas of variance and invariance *shaped* how they used the technology, which was evident from the activities they designed and the representations that they privileged. The converse was also true as their activity designs *were shaped* as a result of their increased knowledge about the affordances of the MRT.

The study revealed a number of ways in which the teachers' mathematical ideas concerning explorations of variance and invariance *shaped* their use of the technology. The data from the first phase revealed teachers' initial tendencies to develop instrument utilisation schemes that relied on single inputs which were examined in a single output representation. Their choice of representations gave an indication of how they initially conceived the variant and invariant properties and it was understandable that the teachers would take it slowly in becoming familiar with different representational forms (and the associated instrumentations) before they were in a position to begin to consider these alongside each other. The significance of Eleanor and Tim's positioning of this initial example space became more apparent as the project progressed and, irrespective of their increased knowledge of the affordances of the MRT, they began to use familiar representations to develop more substantial activities. Tim's activity 'Circles and lines' was a good example of this as it required little more than a geometric figure displayed by the MRT as a stimulus for the activity that followed.

Over time, Eleanor and Tim's increasing knowledge of the technology and its functionalities provided substantial evidence for how the teachers' mathematical ideas concerning explorations of variance and invariance were being *shaped by* their use of the technology. For example, in his final written reflection, Tim wrote,

> *During the original TI-Nspire project I had a very distinct moment when all of a sudden I recognised the importance of variation (in its many forms) to focus student attention. This led to the two-part article that I wrote for ATM. In it I recognised the need to identify the mathematical property (generalisation?) that I am trying to draw attention to... and then identify the variables within the situation. Then by using ICT, vary that which will draw attention to the mathematical property. I mention this because after I had this 'moment'... my thinking processes in task design began to change. There was greater clarity in my head for the need to consider what I wanted to vary as I planned the Mathematics-ICT task. Up until this point I designed tasks that did involve variation... but I don't think that I was as aware of what I was doing in the task design.*

In Tim's case, the focus on lesson design that formed a large part of the professional activity during the first phase of the project had stimulated him to reflect upon the importance of a deep consideration of the variant and invariant properties within his task design. His Phase Two activities clearly evidenced how he was continuing to do this, and still learning about how complex a process this was.

Although Eleanor had not verbally articulated her personal reflections on her epistemological development as deeply as Tim had, the research data was rich with evidence of how her thinking and planning had been significantly influenced by the affordances of the MRT. In her case, several seeds of ideas stayed dormant within her until she saw an opportunity to use them in her own classroom and she was openly influenced by the ideas of others involved in the project. On several occasions Eleanor referred to her lesson activities as having 'a backward approach', in that the students were being given the opportunity to experience a mathematical situation first before being presented with the underlying theory.

A number of activities developed by Eleanor and Tim could be considered to be offering new approaches to the mathematics that are not evident in existing schemes and resources. For example, Tim's 'Pythagoras exploration' took as its starting point an example space that was much broader than that evident within the current legitimate curriculum and Eleanor's 'Generating circles' offered an insight into how the integration of multiple representations might be manifested in accessible activities for secondary students.

There was very clear evidence that Eleanor and Tim had developed 'warranted practice' in their use of the MRT, as described by Ruthven, in which they had articulated explicit rationales for their 'practice in action', analysed the processes in operation, assessed the impact on their students' development and sought to refine their practices accordingly (Ruthven, 1999). In developing this practice, they both developed the language through which they could better articulate the tacit knowledge that Polanyi and Schön describe as an integral part of human thinking and practice (Polanyi, 1966, Schön, 1987).

The second idea related to the teachers' processes of designing activities that privileged the exploration of variance and invariance in a MRT environment, informed by the scrutiny of the substantial data set, in itself offering a response to Stacey's call for further research, 'What are the major considerations when exploiting each of the pedagogical opportunities' within the pedagogical map (Stacey, 2008). An element of the teachers' epistemological development was related to their realisation that expressing generality was a very important aspect of the activities that they went on to design. The teachers' increasing attentions to the way that the MRT environment supported or hindered this, and the design of the associated supporting resources and their role in mediating the associated classroom discourse, was another element of their professional development. Evidence from the study suggests that the process of designing tasks that utilise the MRT to privilege explorations of variance and invariance is a highly complex process, which requires teachers to carefully consider how variance and invariance might manifest itself within any given mathematical topic. The relevance and importance of the initial example space, and how this might be productively expanded to support learners towards the desired generalisation is a crucial aspect of activity design.

The starting point for any classroom activity is its initial design and the following set of questions, generated as a result of this study, offer a research-informed approach:

- What is the generalisable property within the mathematics topic under investigation?
- How might this property manifest itself within the multi-representational technological environment – and which of these manifestations is at an accessible level for the students concerned?
- What forms of interaction with the MRT will reveal the desired manifestation?
- What labelling and referencing notations will support the articulation and communication of the generalisation that is being sought?
- What might the 'flow' of mathematical representations (with and without technology) look like as a means to illuminate and make sense of the generalisation?
- What forms of interaction between the students and teacher will support the generalisation to be more widely communicated?
- How might the original example space be expanded to incorporate broader related generalisations?

There is a degree of resonance between these questions and the principles of lesson design that Pierce and Stacey concluded following an analysis of the use of TI-Nspire handhelds to privilege the use of multiple representations within the teaching of quadratic functions (Pierce and Stacey, 2009). Their principles included:

> *focus on the main goal for that lesson (despite the possibilities offered by having many representations available); identify different purposes for using different representations to maintain engagement; establish naming protocols for variables that are treated differently when working with pen and paper and within a machine; and reduce any sources of cognitive load that are not essential. [ibid, p.228]*

Responses to the questions just posed uncover a very generic 'top level' of thinking which makes little sense in the absence of a clear mathematical context. The next level of thinking becomes closely related to the topic of the mathematics itself for which a number of existing structures and approaches can support teachers to develop further this aspect of their practice (Watson and Mason, 1998, Mason and Johnston-Wilder, 2005, Mason et al., 2005).

7 Conclusion

In conclusion, the study provided a deep insight into teachers' technology-mediated epistemological development over a twenty-four month period as they began to integrate a complex new technology within their classroom practices. Moreover, the adaptation of Verillon and Rabardel's framework provided a useful construct for the research as it focused the research lens onto teachers' classroom practices and demanded a robust set of methodological tools to evidence the different interactions. Finally, whilst this article has focused on the nature of the teachers' epistemological developments, the research also led to the conclusion that it was the contingent moments or ‘hiccups that the teachers experienced when integrating the MRT into their classroom practices that provided both rich contexts for their situated learning and fruitful foci for professional discourse[ii].

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Notes

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[i] The emergence of the TPACK framework occurred too late for consideration within the study.

[ii] A paper articulating the ontological innovation that is the 'hiccup', that is the perturbation experienced by teachers during lessons stimulated by their use of the technology, which illuminates discontinuities within teachers' knowledge, as the major outcome of the doctoral study is in preparation for submission to *Educational Studies in Mathematics*.