

Students' intentions to study non-compulsory mathematics: the importance of how good you think you are

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Increasing the number of students who study mathematics once it is no longer compulsory remains a priority for England. A longitudinal cohort from England (1085 students) was surveyed at Years 10 and 12. Students' self-beliefs of ability influenced their GCSE mathematics grades and their intended and actual mathematics subject-choices; the degree of under-confidence or over-confidence related to these self-beliefs was also influential. Additional factors that significantly influenced students' intentions at Year 10 to study mathematics in Year 12 were the advice or pressure to do so, the extrinsic motivation associated with mathematics, their gender and the emotional response associated with doing mathematics. These same factors were also significant influences on students' intentions at Year 12 to study mathematics at university, with the addition of their intrinsic motivation associated with mathematics. Although gender was not a significant influence on GCSE mathematics grades or whether students actually studied A-Level mathematics, boys were associated with higher intentions to study mathematics into Year 12, 13 and university. Additionally, girls were generally more under-confident than boys in their self-beliefs.

Introduction

Self-beliefs and subject-choices

Increasing the numbers of mathematics students and graduates remains a priority for England; mathematics helps solve problems throughout the physical sciences, computer sciences, engineering, medicine and many other areas, and more students and graduates are hoped by policy-makers and stake-holders to ultimately benefit the wider economy (The Royal Society, 2011). Mathematics A-Level entries have only recently recovered following a decline owing to the introduction of Curriculum 2000 (Department for Education, 2011), and it remains important to explore why students decide to study A-Level mathematics or not, especially as fewer students in England study non-compulsory mathematics compared with many other countries (Hodgen *et al.*, 2013).

Students' attainment in GCSE mathematics has a major effect on whether they continue with the subject once it becomes non-compulsory. Students' GCSE grades in 2008 and their A-Level choices in 2009 and 2010, for example, highlighted that 79% of students with GCSE mathematics grade A* and 48% of students with grade

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A continued onto A-Level mathematics, but only 15% of students with grade B and 1% with grade C did so (Department for Education, 2012). Other subjects such as the sciences, history and languages, are less critically dependent on GCSE grades for progression to A-Level. Additionally, students' self-beliefs of their own attainment, ability, or success, such as their academic subject-specific self-concept beliefs, are fundamental to both attainment (e.g. Huang, 2011) and subject-choices (e.g. Blenkinsop *et al.*, 2006). A systematic literature review (Tripney *et al.*, 2010) has highlighted that these self-beliefs, together with the perceived usefulness of subjects, enjoyment, and the complementary nature of some subjects, are commonly reported reasons for A-Level choices. Mathematics subject-choices in England have also been influenced by the perceived difficulty of A-Level study and (low) confidence, enjoyment and perceptions of the personal utility of mathematics (Brown *et al.*, 2008; Cann, 2009); perceptions of the utility of a subject or extrinsic motivation have also been found to be more influential on some subject choices than intrinsic motivation (Mujtaba & Reiss, 2013). Outside of England, self-concept and self-efficacy for mathematics, past attainment and the intrinsic value associated with mathematics have been found to influence mathematics subject-choices in general (Watt, 2006).

Examinations results for England from 2001 to 2012 show that on average fewer girls than boys sat A-Level mathematics examinations, although girls and boys generally performed equally (JCQ, 2012). Girls have been found to be more concerned than boys with being able to cope with A-Level mathematics, while boys were more concerned than girls with the utility of the qualification (QCA, 2007). Girls also have reported mathematics as being more difficult, which reduced their mathematics self-beliefs and the intrinsic value they associated with mathematics, and subsequently influenced their subject-choices (Watt, 2006).

In work largely undertaken outside of England, boys have generally reported higher mathematics self-concepts than girls (e.g. Fredricks & Eccles, 2002), even though girls often perform equally or even slightly higher than boys (e.g. Skaalvik & Skaalvik, 2004). Boys have also reported higher mathematics self-efficacy and intrinsic motivation associated with mathematics compared with girls (e.g. Chen, 2003; Nagy *et al.*, 2006). These self-beliefs may form in different ways: mathematics grades have been found to have a larger positive influence on perceived mathematics ability for girls compared with boys (Correll, 2001), for example. Other differences in influences across genders have also become apparent. For boys and girls, prior attainment was found to influence mathematics subject-choices indirectly, mediated through the perceived utility of the subject; girls' intentions were also directly influenced by their ability beliefs, while boys' intentions were also directly influenced by prior attainment (Crombie *et al.*, 2005). Additionally, when students (both boys and girls) believed a gender stereotype to be associated with subject attainment (i.e. stereotypes such as girls performing lower in mathematics or boys performing lower in the arts) they tended accordingly to under-report their own attainment (Chatard *et al.*, 2007).

The accuracy of self-beliefs

While students' self-beliefs are influential to attainment and subject-choices, an often-overlooked point is the extent to which such self-beliefs reflect students' actual

abilities, attainment, or success. When considering the accuracy of students' self-beliefs, a student's justification for holding the belief is not necessarily explored, and the reported belief is compared with an external indicator (such as examination grades); however, a student might feel justified in holding a belief that coheres with their other beliefs, for example, even when comparison against an external indicator might suggest that the belief is too high or too low. Measures of the accuracy of self-beliefs are artificially created indicators, and cannot necessarily confirm whether students undertake similar self-evaluation of their beliefs; nevertheless, such measures have been considered to reflect self-awareness of cognitive processes and related areas (Zimmerman, 2000), and the area has revealed significant and informative findings.

Students often, but not universally, over-evaluate their abilities. This has been seen at various ages, including at primary school (e.g. Bouffard *et al.*, 1998; Bouffard *et al.*, 2011) and at university (e.g. Gramzow *et al.*, 2003) in Canada, Europe and North America. Social-cognitive theories of behaviour (Bandura, 1997) propose that such over-confidence is normal and can facilitate increased motivation and persistence when difficulties arise and provide protection from negative affect; the associated implication, that under-confidence can be limiting and associated with affective or other costs, has been supported through further research in Canada and Europe (e.g. Bouffard *et al.*, 2003; Narciss *et al.*, 2011). Alternately, self-regulated learning models (Butler & Winne, 1995) promote accurate beliefs as integral to personal well-being and functioning, and this has important implications to students' studying approaches and motivation; students may study less if they believe that they already master an area, for example, which becomes problematic when the belief is inaccurate (Winne, 1995).

Accurate self-beliefs have frequently been associated with higher performance or attainment. For example, higher-achieving primary students in Canada had more accurate beliefs of their reading ability than lower-achieving students (Bouffard *et al.*, 1998); primary students in Germany with accurate self-evaluations had greater increases in satisfaction with their performance after a four-week arithmetic training programme compared with students who over-evaluated (Narciss *et al.*, 2011); higher-attaining secondary students in North America had higher self-efficacy, self-concept and more accurate beliefs (Pajares & Graham, 1999). Accurate evaluation also predicted mathematics performance in secondary students in North America (Chen, 2003) and in students surveyed by the Programme for International Student Assessment (PISA) in 2000 (Chiu & Klassen, 2010); accurate evaluation has been additionally associated with more interest in mathematics in Greece (Gonida & Leondari, 2011). Higher-performing undergraduate and graduate students in North America have generally been found to be more accurate but slightly under-confident in their predictions and retrospective evaluations of their performance, while lower-performing students were much less accurate and largely over-confident (e.g. Ackerman & Wolman, 2007; Bol *et al.*, 2005).

The present study

This study draws on data from the Understanding Participation rates in post-16 Mathematics and Physics (UPMAP) project, a longitudinal mixed-methods project

exploring mathematics and physics choices in non-compulsory education (Reiss *et al.*, 2011).

While earlier research has explored factors influencing subject-choices, attainment or the accuracy of self-beliefs, less work has explored these areas together. In addition, research into the accuracy of self-beliefs has mainly occurred in Canada, North America and Europe (but to no great extent in England), and fewer studies have considered secondary school students. The accuracy of self-beliefs may be relevant to students' choices and eventual progression: under-confident students may not select subjects that they might otherwise succeed in and enjoy, for example, while over-confident students may select subjects that they are subsequently unable to continue in.

Accordingly, this study aimed to explore the influence of self-beliefs and their associated degree of overall accuracy and bias (towards over-confidence/over-evaluation or under-confidence/under-evaluation),¹ on both mathematics attainment and subject-choices; the work also aimed to identify any further influential factors on mathematics attainment and subject-choices. This required a focus on self-reported beliefs rather than wider factors (whether personal, contextual, social or other factors; the influences of such factors are explored through further work developing from the wider project). Under-confidence/under-evaluation was hypothesised to link with less inclination to study mathematics, following the association of affective costs with under-confidence/under-evaluation (e.g. Narciss *et al.*, 2011).

Some studies have additionally found that boys tend to over-estimate/over-evaluate their performance (e.g. Gonida & Leondari, 2011), while others have found no gender differences within overall accuracy or bias (e.g. Bouffard *et al.*, 2011), so exploring any gender differences was a supplementary aim; girls were hypothesised generally to under-evaluate their performance compared with boys.

The main research question was to determine the influence of the accuracy and bias of self-beliefs of attainment and confidence in mathematics on both mathematics attainment and subject-choices. Highlighting any further beliefs that may influence attainment and subject-choices, and exploring any gender differences within the accuracy and bias of these self-beliefs, were supplementary areas.

Methods

Participants

Schools across England were identified within categorised levels of mathematics/physics attainment and progression into A-Level (see Table A1), using data provided by the Department for Children, Schools and Families (now known as the Department for Education), and those with above-average mathematics/physics attainment and/or progression were over-sampled. Such selection was necessary to provide insight into those students with the likely potential to continue to study A-Level mathematics/physics and their actual subject choices (although this selection influenced wider generalisation). Within the main categories, schools were also selected to cover sub-categories of students' socio-cultural statuses (represented by categories of students' eligibility for free school meals in schools, and by grammar schools), ethnically diverse or non-diverse schools, and single sex schools.

Within schools, data collection targeted students predicted to attain A* to D grades at GCSE mathematics/physics (approximately the top two-thirds of students, again to provide insight into those students with the likely potential to continue to study A-Level mathematics/physics) and involved participants completing subject-specific questionnaires on two occasions; students were free to decline to participate or to omit responses to any particular question at any time. Students were followed where possible across any changes of school or college between the two phases.

From the wider data collected by the project, this specific work focused on a longitudinal cohort of students from England (1085 students, 434 male and 651 female) who responded at both Year 10 (age 15) in the first phase and Year 12 (age 17) in the second phase. This cohort covered 80 schools at Year 10 and 92 schools at Year 12. The schools were broadly spread across England with (at Year 12) 13% in the East, 4% in the East Midlands, 21% in London, 6% in the North East, 9% in the North West, 22% in the South East, 8% in the South West, 12% in the West Midlands and 5% in Yorkshire and the Humber.

Questionnaires

The student questionnaires allowed participants to report their agreement or disagreement (on Likert-type scales) with statements covering affective responses to academic subjects, lessons, teachers and wider views and subject-choice intentions (e.g. 'I intend to continue to study maths after my GCSEs'). Validated measures were used to inform the questionnaire design. For example, the self-concept measure focused on personally orientated perceptions of ability or mastery experiences (e.g. 'I do well in maths tests'), and perceived peer-comparison (or frame of reference effects, e.g. 'Thinking about your maths lessons, how do you feel you compare with the others in your group?'; see Bong and Skaalvik (2003), for a detailed discussion of self-concept and potential measurement differences). Further measures were developed to cover perceptions of mathematics lessons and teachers, and other potential influences on subject-choices. The questionnaires were developed through five rounds of iterative piloting and refinement.² The final item structures were also confirmed (using the main project data) by principal component (and confirmatory factor) analysis. The scoring of items was reversed when necessary to ensure parity of the overall measures, which were calculated as the mean of the relevant individual items.

Self-belief and other measures

The measures were: self-concept (or belief of current/retrospective ability, attainment, or success; five items, e.g. 'I am good at maths'; $\alpha = 0.848$ at Year 10 and $\alpha = 0.804$ at Year 12); mathematics intrinsic motivation (or interest in or value of mathematics in itself; seven items, e.g. 'Maths is interesting'; $\alpha = 0.768$ and $\alpha = 0.795$); mathematics extrinsic motivation (utility of mathematics; five items, e.g. 'I think maths will help me in the job I want to do in the future'; $\alpha = 0.706$ and $\alpha = 0.743$); perceptions of mathematics lessons (six items, e.g. 'I enjoy my maths lessons'; $\alpha = 0.800$ and $\alpha = 0.865$); emotional responses to mathematics (four items, e.g. 'When I am doing maths, I am bored'; $\alpha = 0.610$ and $\alpha = 0.701$); perceptions of

mathematics teachers (11 items, e.g. 'My maths teacher believes that all students can learn maths'; $\alpha = 0.880$ and $\alpha = 0.918$); advice or pressure to study mathematics (five items, including the influence of friends, teachers and family, e.g. 'My teacher thinks that I should continue with maths beyond my GCSEs'; $\alpha = 0.833$ and $\alpha = 0.841$); and home support for mathematics achievement (five items, e.g. 'Someone in my family wants me to be successful at school in maths'; $\alpha = 0.727$ and $\alpha = 0.646$).

At Year 10, the students reported their agreement or disagreement to a questionnaire item stating 'I intend to continue to study maths after my GCSEs'. At Year 12, the students listed the subjects that they were currently studying (which were subsequently coded to indicate whether mathematics was being studied or not by each student), and their agreement or disagreement towards the statements 'I intend to continue to study maths after this year' (i.e. in Year 13) and 'I intend to study maths at university'.

Selected tasks were also included at the end of the questionnaires, assessing students' skills in algebra and interpreting graphs, developed from earlier studies of mathematical proof and from PISA (Kuchemann, 2008; OECD, 2009). Further items allowed the students to report their confidence in their answers (e.g. 'How confident are you that your answers to the racing car questions are correct'), providing a retrospective self-evaluative assessment of their task performance, together with task-specific measures of enjoyment, ease and interest. Students' task scores were calculated as the mean proportion of correct answers across the task questions. Students' Key Stage 3 (KS3) scores and GCSE grades were provided by the Department for Education.

Calibration measures

The degree to which self-beliefs or judgments (such as self-concept) reflect an actual situation (such as attainment evidenced through examinations) has been called 'calibration'; this can measure the overall accuracy of beliefs and the bias or direction of any discrepancy (Hacker *et al.*, 2008). Methodologically, calibration compares self-beliefs against an external measure, such as test results, in various ways (e.g. Boekaerts & Rozendaal, 2010; Schraw *et al.*, 2013).

Calibration measures for each student were created through the 'difference score' method (Pajares & Graham, 1999). The measures of self-belief (mathematics self-concept and task confidence) and the measures of attainment (mathematics task performance, KS3 score and GCSE grade) were equalised to the same scales. At both phases, mathematics task confidence calibration bias was calculated by subtracting the students' task performance from the students' task confidence; mathematics self-concept calibration bias was calculated at the first phase by subtracting the students' KS3 score from the students' self-concept reported at Year 10, and calculated at the second phase by subtracting the students' GCSE grade from the students' self-concept reported at Year 12. A positive calibration bias value denoted over-evaluation or over-confidence (the terms are used synonymously), a negative value denoted under-evaluation or under-confidence and a value of zero denoted perfect accuracy in calibration. The calibration bias measures were converted to -1 to $+1$ scales and calibration accuracy measures on 0 to $+1$ scales were created by subtracting the absolute bias values from 1.

These comparisons were appropriate as the task confidence measure explicitly considered the students' assessment of their task performance and the self-concept measure focused on personally orientated perceptions of ability or mastery experiences which include attainment; the self-concept measure did not include affect, importance, quickness of learning, or other dimensions less focused on ability or attainment, which would have reduced the validity of such a comparison.

Analytical approach

Initial descriptive statistics summarised the measures at Years 10 and 12, including any gender differences, in order to contextualise the sample. A series of linear regression models were then created in order to explore how the measures predicted students' GCSE mathematics grades and their reported intentions to study mathematics further. Logistic regression was similarly used to predict whether students reported that they were actually studying mathematics in Year 12 or not. The students were also grouped by their Year 10 and 12 self-concept calibration bias measures: values of -1 to -0.17 were classified as 'under-confident'; just above -0.17 to $+0.17$ were classified as 'accurate'; and just above $+0.17$ to $+1$ were classified as 'over-confident'; these boundaries allowed a divergence of ± 0.5 of an original scale point away from absolute calibration to still be considered as accurate. The regression models were then repeated separately for each group to explore any potentially varying predictors.

Regression analysis assumes that residuals are normally distributed, with constant variance, and that errors are uncorrelated; residual plots were produced for the models and were satisfactory. Separating the analysis by Years 10 and 12 avoided issues of potentially correlated errors caused by the longitudinal sample.

Results

Summary statistics

Table 1 summarises the sample, and differences between the measures at Years 10 (first year of GCSE) and 12 (first year of A-Level); Table A2 also provides a full correlation table. On the mathematics task level, confidence was higher at Year 12; the sample was generally slightly under-confident of their task ability at Years 10 and 12. On the subject level, mathematics self-concept beliefs were similar at both Years; the sample was slightly over-confident in their self-concepts at Year 10, but under-confident at Year 12.

Table 1 also summarises mean gender differences. Girls had, on average, lower task scores and confidence, and lower self-concept beliefs, at Years 10 and 12, although there was no significant difference in boys' and girls' GCSE mathematics grades. A Pearson chi-square test highlighted that the boys and girls had similar distributions of GCSE mathematics grades ($\chi^2(6) = 6.709, p = 0.349$), and most gained grade B and above (grade A*: attained by 28% of boys and 23% of girls; A: 41% of both boys and girls; B: 21% of boys, 26% of girls; C: 9% of boys, 10% of girls; D: 1% of both boys and girls). Boys and girls differed in their calibration of task confidence and self-concept beliefs at both Year 10 and 12: girls were more under-confident than

Table 1. Summary of measures

Measure	Year 10			Year 12			Year 10 to 12 difference					
	All	Boys	Girls	All	Boys	Girls	All	Boys	Girls			
	Mathematics task score (1-4)	3.09	3.20	3.01	0.226	***	2.86	2.97	2.78	0.147	*	***
Mathematics task confidence (1-4)	2.90	3.21	2.69	0.826	***	3.13	3.41	2.93	0.599	***	***	***
Mathematics task confidence calibration bias (-1 to +1)	-0.09	-0.03	-0.12	0.369	***	-0.05	0.03	-0.11	0.402	***	**	-
Mathematics task confidence calibration accuracy (0-1)	0.79	0.81	0.77	0.245	***	0.75	0.79	0.72	0.313	***	***	***
Mathematics task enjoyment	3.43	3.71	3.25	0.375	***	3.71	3.90	3.57	0.210	**	***	*
Mathematics task ease	3.72	4.25	3.39	0.795	***	4.08	4.54	3.77	0.549	***	***	***
Mathematics task interest	3.40	3.62	3.26	0.283	***	3.49	3.57	3.43	0.088	-	-	**
Mathematics KS3 score (0-100)	65.71	67.65	64.42	0.244	***					.	.	.
Mathematics GCSE grade (1-9, 9 = A*)						7.79	7.86	7.74	0.115	-	.	.
Mathematics self-concept	4.14	4.52	3.89	0.685	***	4.12	4.37	3.95	0.418	***	-	***
Mathematics self-concept calibration bias (-1 to +1)	0.04	0.09	0.01	0.434	***	-0.14	-0.10	-0.16	0.286	***	***	***
Mathematics self-concept calibration accuracy (0-1)	0.84	0.83	0.84	-0.099	-	0.81	0.84	0.79	0.340	***	***	-
Mathematics intrinsic motivation	3.93	4.01	3.88	0.156	*	4.15	4.23	4.10	0.141	*	***	***
Mathematics extrinsic motivation	4.73	4.84	4.66	0.241	***	4.65	4.74	4.59	0.168	**	***	*
Perception of mathematics lessons	4.12	4.16	4.10	0.075	-	4.22	4.27	4.19	0.072	-	***	**
Emotional response to mathematics	3.92	4.00	3.87	0.142	*	3.86	3.92	3.82	0.091	-	-	-
Perception of mathematics teachers	4.63	4.63	4.64	-0.003	-	4.64	4.60	4.67	-0.072	-	-	-
Advice/pressure to study mathematics	4.35	4.50	4.25	0.213	***	4.31	4.57	4.14	0.350	***	-	*
Home support for mathematics achievement	4.47	4.58	4.40	0.187	**	4.13	4.32	4.01	0.346	***	***	***

Notes: Overall sample: 1085 students, 434 male and 651 female (the exact number of students for each result may vary slightly due to missing data on the item level). Measures use 1-6 scales (1 = strongly disagree, 6 = strongly agree) unless specified otherwise. Key: All (mean for the whole sample); Boys (mean for boys); Girls (mean for girls); D (Cohen's d, a measure of effect-size from independent-sample t-tests between boys and girls). Year 10 to 12 difference shows the significance of paired-sample t-test results (note that the paired-sample means can slightly differ from the other means shown in the table due to missing data on the item level). The significance of the t-tests results is shown by: ***p < 0.001; **p < 0.01; *p < 0.05; . p > 0.05, considered non-significant; - an inapplicable entry or comparison.

boys for task confidence; boys were more over-confident in their self-concept beliefs at Year 10, while girls were relatively accurate; both boys and girls were generally under-confident at Year 12, with girls on average showing a higher degree of under-confidence.

Predictive models

The regression models could not include together the self-beliefs (task confidence and self-concept), their associated calibration accuracy and bias measures, and the attainment measures (task performance and KS3/GCSE grade) without indicators of multicollinearity, given that all these measures were related through the calibration calculation process. The models therefore included only the self-beliefs and calibration measures, omitting attainment, as these directly related to the research questions. Including interaction terms between the self-beliefs and their calibration bias measures produced low R^2 changes (predicting GCSE mathematics grade: R^2 change = 0.004, F change = 3.817, p = 0.022; predicting intentions reported at Year 10 to study mathematics in Year 12: R^2 change = 0.005, F change = 5.652, p = 0.004; predicting intention reported at Year 12 to study mathematics in Year 13: no significant change, p = 0.561; predicting intention reported at Year 12 to study mathematics in university: R^2 change = 0.008, F change = 4.992, p = 0.007), so calibration interaction effects were not explored further.

Tables 2 and 3 highlight the significant predictors across the main models. The students' mathematics self-concept beliefs and the reported advice and pressure on students (i.e. suggestions from teachers and parents and their friends' choices) were significant in all models. At Year 10, mathematics self-concept and self-concept calibration bias had the largest relative influences on students' subsequently-attained GCSE mathematics grades. At Year 10, the reported advice or pressure to study mathematics, self-concept and extrinsic motivation associated with mathematics had the largest relative influences on students' reported intentions at Year 10 to study mathematics into Year 12. Extrinsic motivation reported at Year 10, however, was not significantly predictive of whether the students reported that they were actually studying mathematics in Year 12. Instead, self-concept, advice or pressure and the students' emotional response to doing mathematics were the three significant influences with the largest changes in odds of the students studying mathematics in Year 12. At Year 12, extrinsic motivation and intrinsic motivation reported at Year 12 were both significantly predictive of students' intentions reported at Year 12 to study mathematics into Year 13 and into university.

Across these models, self-concept calibration bias was significantly predictive while both the task confidence calibration accuracy and self-concept calibration accuracy were not; task confidence calibration bias was only significantly predictive of GCSE mathematics grades and whether the student was studying mathematics in Year 12. The negative regression coefficients associated under-confidence with higher attainment and intentions to continue studying mathematics (and the opposite case for over-confidence).

In Table 3, the influence of Year 12 mathematics self-concept calibration bias was also (perhaps surprisingly) stronger than the self-concept itself when predicting the

Table 2. Predictive models using Year 10 measures

Year 10 measures (1–6 unless specified)	Model 1 (linear)		Model 2 (linear)		Model 3 (logistic)	
	β		β		Exp(<i>B</i>)	
		Predicting the students' attained GCSE mathematics grades (1–9, 9 = A*)		Predicting the students' reported intention at Year 10 to study mathematics in Year 12 (1–6, 1 = strongly disagree, 6 = strongly agree)		Predicting if the students listed at Year 12 that mathematics was a studied subject
Gender (0 female, 1 male)	–0.046	–	0.083	***	1.358	–
Mathematics task confidence (1–4)	0.167	***	–0.037	–	1.345	–
Mathematics task confidence calibration bias (–1 to +1)	–0.183	***	0.025	–	(–) 0.210	***
Mathematics task confidence calibration accuracy (0–1)	0.005	–	–0.006	–	1.053	–
Mathematics task enjoyment	–0.091	–	0.084	–	(–) 0.843	–
Mathematics task ease	–0.075	*	–0.024	–	1.097	–
Mathematics task interest	0.117	*	0.059	–	1.271	–
Mathematics self-concept	0.701	***	0.203	***	3.660	***
Mathematics self-concept calibration bias (–1 to +1)	–0.575	***	–0.094	**	(–) 0.018	***
Mathematics self-concept calibration accuracy (0–1)	0.018	–	0.023	–	1.118	–
Mathematics intrinsic motivation	0.026	–	–0.008	–	1.126	–
Mathematics extrinsic motivation	0.002	–	0.153	***	1.135	–
Perception of mathematics lessons	0.012	–	0.066	–	(–) 0.873	–
Emotional response to mathematics	0.025	–	0.069	*	1.307	*
Perception of mathematics teachers	0.041	–	–0.025	–	(–) 0.766	*
Advice/pressure to study mathematics	0.074	*	0.420	***	1.395	***
Home support for mathematics achievement	0.000	–	0.042	–	1.064	–

Notes: β , standardised coefficient in linear regression; Exp(*B*), in logistic regression, the change in the odds of the student listing mathematics, given a unit change in the predicting measure, when all other predictors are held constant; the sign of the original *B* coefficient in the logistic regression model has also been included for information in brackets when negative; gender in the logistic regression model represents the change in odds associated with a change from female to male; ****p* < 0.001, ***p* < 0.01, **p* < 0.05, '–' *p* > 0.05, considered non-significant. Significant predictors are also highlighted in bold. Model 1 (*n* = 925 due to listwise handling of missing data) notes: adjusted *R*² = 0.468; model SE = 0.699; *F*(17, 907) = 48.752, *p* < 0.001 (significance represents that the model was sound). Model 2 (*n* = 915) notes: adjusted *R*² = 0.558; model SE = 1.043; *F*(17, 897) = 69.001, *p* < 0.001. Model 3 (*n* = 918) notes: Hosmer and Lemeshow Test χ^2 (8) = 10.364, *p* = 0.240 (non-significance represents that the model was sound).

Table 3. Predictive models using Year 12 measures

Year 12 measure (1–6 unless specified)	Model 4 (linear)		Model 5 (linear)	
	Predicting the students' reported intention at Year 12 to study mathematics in Year 13 (1–6, 1 = strongly disagree, 6 = strongly agree)		Predicting the students' reported intention at Year 12 to study mathematics in university (1–6, 1 = strongly disagree, 6 = strongly agree)	
	β		β	
Gender (0 female, 1 male)	0.093	***	0.142	***
Mathematics task confidence (1–4)	0.066	–	–0.012	–
Mathematics task confidence calibration bias (–1 to +1)	–0.027	–	0.043	–
Mathematics task confidence calibration accuracy (0–1)	–0.007	–	0.034	–
Mathematics task enjoyment	0.039	–	0.094	–
Mathematics task ease	0.013	–	–0.009	–
Mathematics task interest	–0.052	–	–0.045	–
Mathematics self-concept	0.198	***	0.144	**
Mathematics self-concept calibration bias (–1 to +1)	–0.214	***	–0.148	***
Mathematics self-concept calibration accuracy (0–1)	0.007	–	0.016	–
Mathematics intrinsic motivation	0.141	**	0.114	*
Mathematics extrinsic motivation	0.167	***	0.187	***
Perception of mathematics lessons	0.060	–	0.077	–
Emotional response to mathematics	0.087	**	0.158	***
Perception of mathematics teachers	–0.077	*	–0.063	–
Advice/pressure to study mathematics	0.265	***	0.093	*
Home support for mathematics achievement	0.028	–	0.040	–

Notes: β , standardised coefficient in linear regression; *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$, '–' $p > 0.05$, considered non-significant. Significant predictors are also highlighted in bold. Model 4 ($n = 748$) notes: adjusted $R^2 = 0.556$; model SE = 1.446; $F(17, 730) = 55.994$, $p < 0.001$. Model 5 ($n = 705$) notes: adjusted $R^2 = 0.412$; model SE = 1.268; $F(17, 687) = 30.007$, $p < 0.001$.

students' intentions to study mathematics into Year 13 and at university. To explore this further, the Year 12 regression models were repeated with the self-belief calibration measures replaced by the attainment measures (i.e. with the self-concept calibration measures replaced by the GCSE mathematics grade and with the task confidence calibration measures replaced by the mathematics task score); these models had similar fit and significant predictors to the original models, excepting that self-concept was no longer significant while the GCSE mathematics grade was. (Predicting the intention to study mathematics in Year 13: adjusted $R^2 = 0.556$; model SE = 1.446; $F(15, 732) = 63.327$, $p < 0.001$; GCSE mathematics grade $\beta = 0.211$, $p < 0.001$; mathematics self-concept $\beta = -0.024$, $p = 0.544$; other variables were significant or

not as per the original model and task score was non-significant. Predicting the intention to study mathematics at university: adjusted $R^2 = 0.412$; model SE = 1.268; $F(5, 689) = 33.909$, $p < 0.001$; GCSE mathematics grade $\beta = 0.139$, $p < 0.001$; mathematics self-concept $\beta = 0.004$, $p = 0.935$; other variables were significant or not as per the original model, and task score was non-significant.) The Year 10 models were also repeated for comparison, with the calibration measures replaced by the KS3 mathematics attainment; however, these highlighted the significance of both the self-concept belief and the KS3 attainment on GCSE attainment, choice intentions and whether the student actually studied mathematics in Year 12. These results may highlight the importance of the tangible GCSE grade on students' upper-secondary and university choices; the explanatory power of the calibration bias measure at Year 12 may then also partially result from being calculated in conjunction with the GCSE grade (but it can also be assumed that students' self-concept beliefs are formed, to some degree, by reference to their GCSE attainment).

Predictive models by calibration groups

Since the calibration bias measures use a continuous scale of under-confidence through accuracy through to over-confidence, their influence within predictive models is harder to interpret. Further models were then produced to highlight any differences between under-confident, accurate and over-confident groups of students; these aimed to produce illustrative models as groups can be defined according to various boundary values or different methods. As the groups were defined from students' self-concept calibration bias, this was not included as a predictor within the models. Tables 4–8 show the grouped models. Some differences between the models for each group can be seen, although no clear patterns emerged.

Discussion

The results confirm the influence of self-beliefs on attainment and subject-choices, and highlight that the associated degree of under-confidence or over-confidence is also influential.

Under-confident/under-evaluated self-beliefs (for both mathematics self-concept and task confidence) were associated through linear regression with increased GCSE mathematics attainment, and over-confident beliefs with decreased attainment; the magnitude of the standardised coefficients highlighted that the self-beliefs still had a larger influence than their associated calibration bias measures. These results are similar to those from undergraduate and graduate students in North America, where higher performance was associated with slight under-confidence and lower performance was associated with over-confidence (e.g. Ackerman & Wolman, 2007; Bol *et al.*, 2005). While under-confidence has been associated with affective costs (e.g. Narciss *et al.*, 2011, who considered students in Germany), especially in studies with younger students, the results here also suggest that over-confidence may require attention in older students.

Contrary to the earlier hypothesis, under-confidence of self-beliefs of ability was associated through the regression models with higher intentions to study mathematics

Table 4. Predictive models using Year 10 measures by self-concept calibration bias groups

Year 10 measures	Predicting the students' attained KS4 mathematics grades					
	Under-confident		Accurate		Over-confident	
	β	p	β	p	B	p
Gender (0 female, 1 male)	-0.011	-	-0.063	-	-0.025	-
Mathematics task confidence	0.310	**	0.235	***	0.112	-
Mathematics task confidence calibration bias	-0.272	**	-0.162	***	-0.200	**
Mathematics task confidence calibration accuracy	-0.053	-	-0.067	-	0.091	-
Mathematics task enjoyment	-0.306	*	-0.019	-	-0.186	-
Mathematics task ease	-0.145	-	-0.051	-	-0.033	-
Mathematics task interest	0.156	-	0.059	-	0.227	-
Mathematics self-concept	0.643	***	0.438	***	0.450	***
Mathematics self-concept calibration accuracy	-0.179	*	0.014	-	0.320	***
Mathematics intrinsic motivation	0.167	-	0.036	-	-0.046	-
Mathematics extrinsic motivation	-0.084	-	-0.047	-	0.093	-
Perception of mathematics lessons	0.032	-	0.038	-	-0.045	-
Emotional response to mathematics	0.022	-	0.031	-	0.042	-
Perception of mathematics teachers	0.089	-	0.010	-	0.046	-
Advice/pressure to study mathematics	-0.001	-	0.135	**	0.063	-
Home support for mathematics achievement	0.082	-	0.012	-	-0.035	-

Notes: β , standardised coefficient in linear regression; *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$, '-' $p > 0.05$, considered non-significant. Under-confident model ($n = 125$): adjusted $R^2 = 0.532$; model SE = 0.658; $F(16, 108) = 9.822$, $p < 0.001$. Accurate model ($n = 565$): adjusted $R^2 = 0.504$; model SE = 0.663; $F(17, 547) = 34.712$, $p < 0.001$. Over-confident model ($n = 235$): $R^2 = 0.332$; model SE = 0.789; $F(16, 218) = 8.282$, $p < 0.001$.

further. However, using calibration on a continuous scale from under-confidence through accuracy through to over-confidence means that such results are harder to interpret; while the separate group models suggest small differences in influential factors across the calibration groups (but with no clear patterns), further research is needed to explore the influence of the magnitude of under-confidence or over-confidence in isolation, or through other improved methodologies. In contrast to some earlier studies of secondary students (Chen, 2003; Chiu & Klassen, 2010), the measures of calibration accuracy were (surprisingly) not significantly predictive of mathematics attainment. Further work is also needed to explore this result, perhaps using varied calibration calculation methodologies; however, in an applied comparison of methodologies, Boekaerts and Rozendaal (2010) also found that a calibration accuracy measure was less predictive when compared with a bias measure. The potential problem of generalisation across varying student ages, numbers and calibration calculation methodologies may also be highlighted here.

Intrinsic motivation for mathematics at Year 10 was not predictive of students' GCSE mathematics grades or their reported intentions to study mathematics into Year 12, while intrinsic motivation at Year 12 did predict the students' reported intentions at Year 12 to study mathematics into Year 13 and into university. The extrinsic motivation associated with mathematics was also not predictive of GCSE

Table 5. Predictive models using Year 10 measures by Year 10 self-concept calibration bias groups

Year 10 measures	Predicting the students' reported intention at Year 10 to study mathematics in Year 12					
	Under-confident		Accurate		Over-confident	
	β		β		β	
Gender (0 female, 1 male)	0.162	–	0.052	–	0.118	*
Mathematics task confidence	0.078	–	–0.014	–	–0.091	–
Mathematics task confidence calibration bias	–0.079	–	0.044	–	0.032	–
Mathematics task confidence calibration accuracy	0.062	–	–0.044	–	–0.009	–
Mathematics task enjoyment	0.341	*	0.085	–	–0.116	–
Mathematics task ease	–0.046	–	–0.029	–	–0.026	–
Mathematics task interest	–0.200	–	0.058	–	0.223	*
Mathematics self-concept	0.073	–	0.175	***	0.123	*
Mathematics self-concept calibration accuracy	0.082	–	–0.017	–	–0.020	–
Mathematics intrinsic motivation	–0.078	–	–0.034	–	0.161	*
Mathematics extrinsic motivation	0.169	–	0.144	***	0.150	**
Perception of mathematics lessons	0.029	–	0.088	–	–0.003	–
Emotional response to mathematics	0.188	*	0.038	–	0.095	–
Perception of mathematics teachers	0.080	–	–0.056	–	–0.042	–
Advice/pressure to study mathematics	0.351	***	0.464	***	0.364	***
Home support for mathematics achievement	0.058	–	0.037	–	.071	–

Notes: β , standardised coefficient in linear regression; *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$, ‘–’ $p > 0.05$, considered non-significant. Significant predictors are also highlighted in bold. Under-confident ($n = 123$) model: adjusted $R^2 = 0.443$; model SE = 1.227; $F(16, 107) = 7.108$, $p < 0.001$. Accurate ($n = 558$) model: adjusted $R^2 = 0.559$; model SE = 1.016; $F(16, 541) = 45.194$, $p < 0.001$. Over-confident ($n = 233$) model: $R^2 = 0.545$; model SE = 0.998; $F(16, 216) = 18.349$, $p < 0.001$.

mathematics grades, but predicted students' reported intentions given at Years 10 and 12 to study mathematics further. These different and changing influences of intrinsic and extrinsic motivation on attainment and intentions over time match earlier research findings (Bong, 2001; Köller *et al.*, 2001).

Girls were on average, as hypothesised, more under-confident than boys for both their task-level confidence and their subject-level self-concepts for mathematics, which is consonant with earlier studies with slightly younger students (Boekaerts & Rozendaal, 2010; Gonida & Leondari, 2011). Girls also reported lower self-concept beliefs, again similar to general findings (e.g. Fredricks & Eccles, 2002). Interestingly, gender was not a significant predictor of GCSE mathematics attainment or of whether students were actually studying mathematics at Year 12, although gender did predict students' intentions reported at both Years 10 and 12 to study mathematics further, with boys expressing greater intentions to continue to study mathematics into the future. Gender was also highlighted when considering the calibration bias groups: for the under-confident and accurate groups, boys were associated with

Table 6. Predictive models using Year 10 measures by self-concept calibration bias groups

Year 10 measures	Predicting if the students listed at Year 12 that mathematics was a studied subject					
	Under-confident		Accurate		Over-confident	
	Exp(<i>B</i>)	p	Exp(<i>B</i>)	p	Exp(<i>B</i>)	p
Gender (male)	(-) 0.537	-	1.341	-	1.690	-
Mathematics task confidence	1.980	-	1.641	*	1.013	-
Mathematics task confidence calibration bias	(-) 0.287	-	(-) 0.245	*	(-) 0.152	*
Mathematics task confidence calibration accuracy	(-) 0.125	-	(-) 0.569	-	1.979	-
Mathematics task enjoyment	(-) 0.654	-	(-) 0.837	-	1.012	-
Mathematics task ease	1.258	-	1.183	-	1.058	-
Mathematics task interest	(-) 0.954	-	1.436	*	0.968	-
Mathematics self-concept	3.119	*	3.039	***	(-) 3.937	***
Mathematics self-concept calibration accuracy	41.033	-	2.963	-	11.261	-
Mathematics intrinsic motivation	2.622	-	1.068	-	(-) 0.782	-
Mathematics extrinsic motivation	1.117	-	1.045	-	1.086	-
Perception of mathematics lessons	(-) 0.539	-	(-) 0.987	-	1.208	-
Emotional response to mathematics	1.739	-	1.182	-	1.512	-
Perception of mathematics teachers	(-) 0.467	*	(-) 0.831	-	(-) 0.838	-
Advice/pressure to study mathematics	1.307	-	1.321	*	1.786	**
Home support for mathematics achievement	2.123	*	1.010	-	(-) 0.977	-

Notes: Exp(*B*), in logistic regression, the change in the odds of the student listing mathematics, given a unit change in the predicting measure, when all other predictors are held constant; the sign of the *B* coefficient has also been included for information in brackets when negative; ****p* < 0.001, ***p* < 0.01, **p* < 0.05, ‘-’ *p* > 0.05, considered non-significant. Under-confident model (*n* = 123): Hosmer and Lemeshow Test χ^2 (8) = 3.187, *p* = 0.922. Accurate model (*n* = 559): Hosmer and Lemeshow Test χ^2 (8) = 11.620, *p* = 0.169. Over-confident model (*n* = 236): Hosmer and Lemeshow Test χ^2 (8) = 8.733, *p* = 0.365.

higher reported intentions; this means that for students with accurate self-concept beliefs, girls were still less likely to intend to study mathematics at university, for example, and especially so for under-confident students. It is possible that the non-significance of gender on whether students were actually studying mathematics at Year 12 or not relates somewhat to the specific longitudinal sample used; assuming that the sample generalises to students with above-average attainment, then a nationally representative sample may potentially produce differing results. Mathematics grades may have a larger positive influence on perceived mathematics ability for girls (Correll, 2001), and perhaps also on their other self-beliefs or even their actions (i.e. actually selecting a subject) compared with their intentions (but see also Crombie *et al.*, 2005). This area (and the alternate regression models using attainment grades instead of calibration bias) highlights that further comprehensive research, including attainment measures, would be necessary to clarify the factors that influence subject-choices; the influence of GCSE grades compared with self-concepts beliefs may vary over time, in certain situations or for particular groups of students.

A number of methodological points are also relevant. The calibrations of students’ self-beliefs were produced through comparisons with single measures of attainment as extensive data was not available. The time between the attainment measures and

Table 7. Predictive models using Year 12 measures by Year 12 self-concept calibration bias groups

Year 12 measures	Predicting the students' reported intention at Year 12 to study mathematics in Year 13					
	Under-confident		Accurate		Over-confident	
	β		β		β	
Gender (0 female, 1 male)	0.075	–	0.095	*	0.132	–
Mathematics task confidence	0.086	–	0.064	–	–0.026	–
Mathematics task confidence calibration bias	–0.075	–	–0.009	–	0.137	–
Mathematics task confidence calibration accuracy	0.024	–	–0.047	–	0.035	–
Mathematics task enjoyment	0.140	–	0.015	–	–0.551	*
Mathematics task ease	–0.039	–	0.060	–	–0.302	–
Mathematics task interest	–0.090	–	–0.080	–	0.673	–
Mathematics self-concept	0.160	*	0.168	**	0.260	–
Mathematics self-concept calibration accuracy	–0.052	–	–0.030	–	0.200	–
Mathematics intrinsic motivation	0.145	*	0.177	**	–0.431	–
Mathematics extrinsic motivation	0.110	–	0.197	***	0.420	–
Perception of mathematics lessons	0.048	–	0.007	–	0.401	–
Emotional response to mathematics	0.079	–	0.086	–	0.271	–
Perception of mathematics teachers	–0.084	–	–0.035	–	–0.426	*
Advice/pressure to study mathematics	0.315	***	0.250	***	0.475	*
Home support for mathematics achievement	0.036	–	0.039	–	–0.183	–

Notes: β standardised coefficient in linear regression; *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$, '–' $p > 0.05$, considered non-significant. Significant predictors are also highlighted in bold. Under-confident ($n = 324$) model: adjusted $R^2 = 0.531$; model SE = 1.470; $F(16, 307) = 23.877$, $p < 0.001$. Accurate ($n = 389$) model: adjusted $R^2 = 0.572$; model SE = 1.419; $F(16, 372) = 33.464$, $p < 0.001$. Over-confident ($n = 35$) model: $R^2 = 0.536$; model SE = 1.376; $F(16, 18) = 30.456$, $p = 0.007$ (note the smaller group size and lower significance of model soundness).

the reported self-concepts (i.e. between the KS3 tests and Year 10 self-concept, and between the GCSE grade and Year 12 self-concept) may also have introduced unknown variability: a student's KS3 score may have been low, for example, but subsequent effort could have resulted in higher school test results or homework marks in the meantime; a high reported self-concept at Year 10 may then have been more accurate than the measure used here would show. The proposed reciprocal relation between self-concept and attainment (Huang, 2011) also highlights the importance of considering when measures are collected in time. Considering the relative influence of attainment (and other factors) on self-beliefs would also help clarify suitable indicators for use in calibration measures; the mastery experiences that influence self-concept beliefs (Bong & Skaalvik, 2003) may include GCSE grades, for example, but also classroom and homework performance.

Prior attainment itself has strong influences on GCSE grades and subject-choice intentions, but the calibration calculation method ensured that attainment could not also be included in the regression models. As this work shows, considering the accuracy of self-beliefs is informative, but researchers would need to decide whether the loss of any other information is an acceptable trade, which also depends on research questions, calibration calculation methods, and analytical approaches. Further work

Table 8. Predictive models using Year 12 measures by self-concept calibration bias groups

Year 12 measures	Predicting the students' reported intention at Year 12 to study mathematics in university					
	Under-confident		Accurate		Over-confident	
	β		β		β	
Gender (0 female, 1 male)	0.198	***	0.108	*	0.140	–
Mathematics task confidence	–0.057	–	–0.020	–	0.051	–
Mathematics task confidence calibration bias	0.085	–	0.012	–	0.226	–
Mathematics task confidence calibration accuracy	0.070	–	0.002	–	0.069	–
Mathematics task enjoyment	0.172	–	0.069	–	–0.326	–
Mathematics task ease	–0.098	–	0.085	–	–0.508	*
Mathematics task interest	0.024	–	–0.139	–	0.544	–
Mathematics self-concept	0.173	*	0.080	–	0.399	–
Mathematics self-concept calibration accuracy	–0.060	–	–0.045	–	0.232	–
Mathematics intrinsic motivation	0.088	–	0.146	–	–0.299	–
Mathematics extrinsic motivation	0.138	–	0.248	***	0.355	–
Perception of mathematics lessons	0.054	–	0.086	–	0.332	–
Emotional response to mathematics	0.107	–	0.155	**	0.334	–
Perception of mathematics teachers	–0.079	–	–0.034	–	–0.439	*
Advice/pressure to study mathematics	0.111	–	0.077	–	0.218	–
Home support for mathematics achievement	0.041	–	0.037	–	0.024	–

Notes: β , standardised coefficient in linear regression; *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$, '–' $p > 0.05$, considered non-significant. Significant predictors are also highlighted in bold. Under-confident ($n = 308$) model: adjusted $R^2 = 0.372$; model SE = 1.228; $F(16, 291) = 12.367$, $p < 0.001$. Accurate ($n = 360$) model: adjusted $R^2 = 0.430$; model SE = 1.303; $F(16, 343) = 17.933$, $p < 0.001$. Over-confident ($n = 37$) model: $R^2 = 0.469$; model SE = 1.118; $F(16, 20) = 2.989$, $p = 0.011$ (note the smaller group size and lower significance of model soundness).

from the wider project (using additional data) aims to explore the reasons for subject choices, including attainment, wider student factors such as ethnicity, and wider home and school factors, which may all have varying degrees of influence on attainment and subjects choices.

Wider implications

The importance of students studying mathematics at upper-secondary school and at university has been frequently highlighted, together with recommendations that mathematics and science curricula should be engaging to both girls and boys, and that further research into understanding motivations for upper-secondary subject-choices is needed (The Royal Society, 2011).

In order to increase the number of students studying non-compulsory mathematics, the introduction of further types of qualifications or mathematics pathways has been recommended (e.g. ACME, 2012; Hodgen *et al.*, 2013), anticipating or linking with reforms of GCSE and A-Level qualifications, which have been expected since 2010 (Department for Education, 2010). However, such reforms, which include proposed moves away from modular assessment, and any further alternate qualifications and pathways, may not necessarily produce these expected increases in students

studying mathematics at A-Level. It is possible that some alternate pathways might be seen as more realistic or achievable for those with under-confident beliefs who would not have considered A-Level mathematics in itself, but other significantly influential factors on intentions to study mathematics, such as students' extrinsic motivation associated with mathematics and their emotional response to doing mathematics, for example, are dependent on the information available (whether of mathematics qualifications, pathways, university courses and careers, with their associated benefits) and actual teaching practices and curricula.

Differences between girls' and boys' beliefs and attitudes related to mathematics and subject-choices are not always considered within educational policy (e.g. Department for Education, 2010). Given the gender differences in, for example, the formation of beliefs of ability (Correll, 2001), influences on intentions to study mathematics (Crombie *et al.*, 2005), and perceptions of classroom environments (Gherasim *et al.*, 2013), and the results presented from this study, it is important for these to be recognised at least at the level of classroom teaching practices.

Given the influence of self-beliefs and positive attitudes on attainment and subject-choice intentions, it is important that schools consider how these can be improved. Despite this, few interventions have attempted to provide experimental evidence on whether improving attitudes and aspirations would increase students' choices of mathematics in upper-secondary and university education. In isolation, positive encouragement intending to enhance students' self-beliefs of ability, attainment, or success, may result in over-confidence, however. Improving the accuracy of students' beliefs may be a more realistic focus: if this is undertaken simultaneously with the development of students' skills, it can be ensured that increased skills lead to increased (but accurate) self-beliefs, rather than leaving beliefs to remain under-confident or over-confident. This process can be considered through applications of self-regulated learning, the importance of which has been frequently highlighted within mathematics education (e.g. De Corte *et al.*, 2011). Self-regulation is considered to be a process or system including beliefs, feelings and actions which are planned and cyclically adapted in order to achieve personal aims or goals (Zimmerman, 2000); accurate beliefs are necessary to identify discrepancies between a current state and a goal, so that further effort or strategies can then be applied to help achieve the overall goal. Fostering students' self-evaluation, self-reflection and self-monitoring skills can both increase the accuracy of their beliefs and allow effective self-regulation of their studies and perhaps other areas of life.

A meta-review of self-regulated learning interventions (Dignath & Büttner, 2008) highlighted numerous differences between those undertaken at primary and secondary schools. Interventions had higher effect sizes on primary students' mathematics performance than on secondary students, which suggests the benefit of earlier attention. While some attempts to improve calibration or promote metacognition have proven unsuccessful (e.g. Bol *et al.*, 2005), others have produced benefits. For example, providing metacognitive instruction to lower-secondary students gave benefits to mathematics problem solving (Kramarski *et al.*, 2002); secondary students who practiced calibration also had increased accuracy and attainment in biology (Bol *et al.*, 2012). The exact result of any intervention may nevertheless be hard to predict: an experiment to improve calibration accuracy with Grade 5 students in North America,

for example, resulted in over-confidence for those who received the training compared with other students (Huff & Nietfeld, 2009); however, receiving feedback increased the accuracy of mathematics self-evaluations for Grade 5 students in Germany, and in over-confident students the feedback additionally led to slightly increased performance (Labuhn *et al.*, 2010).

Self-regulated learning ultimately requires students to be aware of their own abilities, but the theory recognises that external feedback may be necessary before students are able to develop to a level where self-regulation can be independently controlled by the students themselves (Zimmerman, 2002). Teachers are therefore important in providing direct feedback on students' abilities and progress; teachers' perceptions of students' abilities may additionally influence their teaching approaches and activities, including effectively adapting content to students' learning requirements. As Praetorius and her colleagues suggest (Praetorius *et al.*, 2013), teachers' beliefs of students' abilities and their relative accuracy could be used to optimise learning: for under-confident students, teachers could provide achievable work (potentially but not necessarily easier than usual) to facilitate mastery experiences (influential to the formation of self-concept and self-efficacy beliefs; Bong & Skaalvik, 2003) where the student can attribute success to their own abilities; for accurate students with strong abilities, teachers could provide work slightly above the students' skill level to facilitate progression. Of course, this requires teachers to judge students' abilities and beliefs with reasonable accuracy. Praetorius and her colleagues found that many teachers in Germany were less accurate and were over-confident of their judgements of students' abilities, and teacher confidence was higher for more extreme judgements (i.e. students judged to be of very low or very high ability; Praetorius *et al.*, 2013). The use of calibration measures may be one way for teachers to become more aware of their students' perceived performance.

Conclusion

The degree of under-confidence or over-confidence associated with self-beliefs of ability was a significant predictor of students' GCSE mathematics grades, non-compulsory (post-16) subject-choice intentions and actual subject-choices of A-Level mathematics. In addition to students' self-concept and its associated degree of under-confidence or over-confidence, further factors also significantly influenced students' intentions at Year 10 to study mathematics into Year 12, namely the advice or pressure given to do so, the extrinsic motivation associated with mathematics, the gender of the student, and the emotional response to doing mathematics. These same factors were significant influences on students' intentions at Year 12 to study mathematics into university, with the addition of their intrinsic motivation associated with mathematics. Additionally, girls were generally more under-confident than boys in their self-beliefs. Although gender was not a significant influence (for this longitudinal sample of relatively highly-achieving students) on GCSE mathematics grades or whether students actually studied A-Level mathematics, boys were associated with higher intentions to study mathematics into Year 12 and into university, reported at Year 10 and 12.

These influential factors may be dependent, to varying extents, on teaching practices and curricula. The advice and information provided to students (whether of

mathematics qualifications, pathways, university courses, and careers, and their associated benefits), for example, also become relevant. Improving the accuracy of students' beliefs may be possible through advice, feedback, or applications of self-regulated learning. Of particular importance is the suggestion that teachers should be aware and reflective of their own beliefs of their students' abilities, especially with regard to how this influences their attitudes towards their students, any advice given, or the tailoring of tasks to students.

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NOTES

¹ Over-confidence and over-evaluation, and under-confidence and under-evaluation, are used synonymously; this work does not attempt to propose or explore whether 'over-confidence' and 'over-evaluation' are conceptually, practically or in other ways similar or different.

² The initial iterations of piloting covered 273 students in mathematics and 421 students in physics; the refined questionnaire was checked again with 87 students in mathematics and 73 students in physics. Pilot data are not used or reported in this study.

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Appendix

Table A1. Selected sample information

Category	Longitudinal sample cohort profile at Year 12		Longitudinal sample cohort profile at Year 10		AllDCSF national data provided at project initiation		
	<i>n</i>	(%)	<i>n</i>	(%)	<i>n</i>	(%)	
Total	92	100	80	100	4145	100	
Segmentation by different KS4 attainment/post-16 staying-on rates	Number of schools/colleges/providers						
	High attainment, high staying-on	43	46.7	38	47.5	1169	28.2
	High attainment, average staying-on	12	13.0	14	17.5	636	15.3
	High attainment, low staying-on	7	7.6	7	8.8	162	3.9
Low attainment, high staying-on	Low attainment, high staying-on	5	5.4	5	6.3	117	2.8
	Low attainment, average staying-on	8	8.7	8	10.0	650	15.7
KS4 gender of entry	Low attainment, low staying-on	9	9.8	8	10.0	1124	27.1
	Low attainment, high staying-on	8	8.7	0	0	287	6.9
	Unavailable/missing	11	12.0	10	12.5	306	7.4
	Boys	24	26.1	23	28.7	491	11.8
Girls	Girls	49	53.3	47	58.8	3348	80.8
	Mixed	8	8.7	0	0	0	0
Ethnic category	Unavailable/missing	15	16.3	13	16.3	648	15.6
	No category applied	54	58.7	53	66.3	2097	50.6
	More than 85% white	5	5.4	5	6.3	116	2.8
	High (>50%) in other ethnic groups	8	8.7	7	8.8	210	5.1
Diverse (<50% in any of major ethnic groups)	10	10.9	2	2.5	1074	25.9	
Unavailable/missing							

Table A1. (Continued)

Category	Longitudinal sample cohort profile at Year 12		Longitudinal sample cohort profile at Year 10		AllDCSF national data provided at project initiation	
	<i>n</i>	(%)	<i>n</i>	(%)	<i>n</i>	(%)
Socio-economic status indicators						
0–21% FSM	59	64.1	58	72.5	2104	50.8
21–50% FSM	7	7.6	6	7.5	743	17.9
50+% FSM	1	1.1	1	1.3	94	2.3
Grammar	15	16.3	13	16.3	164	4.0
Unavailable/missing	10	10.9	2	2.5	1040	25.1

Notes: DCSF: Department for Children, Schools and Families (now known as the Department for Education). FSM: students within a school eligible for free school meals. Longitudinal sample cohort: the Year 10 to Year 12 longitudinal sample from England focusing on mathematics (a sub-set of the wider project sample, i.e. not necessarily reflecting the wider project sampling strategy). The DCSF data and categories were provided in 2008, and the table gives 'like for like' data (i.e. the data have not been updated here to reflect any subsequent changes in categorisation, data from more recently-created schools/academies, etc.).

Table A2. Correlations

Measure	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
(1) Intention at Y10 to study mathematics in Y12	1	0.506	0.446	0.303	0.352	0.197	0.241	N/S	0.154	0.321	0.334	0.295	0.401	0.073	0.115	0.398	0.420	0.369	0.328	0.170	0.416	0.321
(2) Intention at Y12 to study mathematics in Y13		1	0.755	0.425	0.548	0.281	0.384	N/S	0.178	0.411	0.426	0.374	0.530	N/S	0.104	0.589	0.586	0.521	0.420	0.226	0.640	0.401
(3) Intention at Y12 to study mathematics in university			1	0.320	0.400	0.196	0.320	N/S	0.163	0.379	0.365	0.342	0.462	0.084	0.143	0.521	0.527	0.497	0.425	0.242	0.494	0.363
(4) Mathematics KS3 score				1	0.616	0.274	0.293	-0.083	0.209	0.175	0.277	0.164	0.430	-0.165	N/S	0.285	0.253	0.248	0.196	N/S	0.409	0.141
(5) Mathematics GCSE grade					1	0.308	0.334	-0.098	0.150	0.273	0.325	0.258	0.542	-0.441	-0.127	0.425	0.364	0.408	0.277	0.207	0.567	0.210
(6) Mathematics task score						1	0.427	-0.687	0.388	0.302	0.420	0.275	0.253	N/S	N/S	0.146	0.200	0.191	0.211	0.107	0.265	0.123
(7) Mathematics task confidence							1	0.443	0.364	0.480	0.653	0.397	0.419	0.089	0.116	0.298	0.290	0.284	0.230	0.124	0.394	0.227
(8) Mathematics task confidence calibration bias								1	-0.146	0.102	0.116	0.068	N/S	0.125	0.147	N/S	N/S	N/S	N/S	N/S	N/S	0.070

Table A2. (Continued)

Measure	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
(9) Mathematics task confidence calibration accuracy	N/S	0.080	0.096	N/S	N/S	N/S	0.254	0.217	0.151	0.171	0.287	0.138	0.257	0.109	0.104	0.109	0.140	0.107	0.146	0.073	0.202	0.114
(10) Mathematics task enjoyment	0.431	0.291	0.290	0.152	0.182	0.196	0.394	0.157	0.097	0.387	0.636	0.828	0.385	0.123	0.114	0.513	0.434	0.492	0.365	0.248	0.380	0.309
(11) Mathematics task ease	0.538	0.330	0.318	0.260	0.212	0.306	0.588	0.224	0.120	0.576	0.432	0.547	0.539	0.235	0.205	0.404	0.383	0.443	0.373	0.228	0.432	0.276
(12) Mathematics task interest	0.436	0.308	0.285	0.156	0.228	0.206	0.354	0.110	0.092	0.857	0.490	0.394	0.355	0.110	N/S	0.520	0.430	0.484	0.369	0.255	0.355	0.302
(13) Mathematics self-concept	0.496	0.447	0.377	0.389	0.405	0.246	0.474	0.125	0.143	0.413	0.525	0.408	0.603	0.512	0.463	0.536	0.505	0.624	0.546	0.364	0.584	0.324
(14) Mathematics self-concept calibration bias	0.233	0.089	0.108	-0.432	-0.118	-0.095	0.181	0.235	0.097	0.293	0.313	0.277	0.663	0.366	0.617	0.136	0.162	0.220	0.278	0.151	N/S	0.110
(15) Mathematics self-concept calibration accuracy	N/S	N/S	N/S	0.227	0.140	0.104	N/S	-0.068	N/S	-0.069	N/S	N/S	N/S	-0.154	0.089	0.181	0.169	0.241	0.269	0.162	0.063	0.123
(16) Mathematics intrinsic motivation	0.505	0.303	0.285	0.167	0.267	0.146	0.203	N/S	0.082	0.543	0.295	0.575	0.406	0.260	N/S	0.494	0.723	0.715	0.466	0.368	0.586	0.431
(17) Mathematics extrinsic motivation	0.550	0.315	0.298	0.139	0.223	0.099	0.198	0.064	N/S	0.415	0.260	0.403	0.356	0.225	N/S	0.656	0.472	0.658	0.421	0.342	0.576	0.422
(18) Perception of mathematics lessons	0.482	0.283	0.261	0.170	0.263	0.138	0.227	N/S	0.072	0.513	0.282	0.530	0.466	0.310	N/S	0.692	0.574	0.438	0.569	0.582	0.540	0.380

Table A2. (Continued)

Measure	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
(19) Emotional response to mathematics	0.388	0.301	0.290	0.185	0.243	0.135	0.217	N/S	0.062	0.401	0.277	0.411	0.472	0.312	N/S	0.469	0.387	0.570	0.404	0.349	0.375	0.250
(20) Perception of mathematics teachers	0.207	0.093	0.095	N/S	0.131	N/S	0.080	N/S	N/S	0.239	0.121	0.265	0.197	0.156	N/S	0.373	0.265	0.552	0.303	0.339	0.297	0.278
(21) Advice/pressure to study mathematics	0.659	0.396	0.329	0.267	0.340	0.233	0.291	N/S	N/S	0.316	0.284	0.341	0.416	0.175	N/S	0.487	0.529	0.421	0.288	0.237	0.427	0.521
(22) Home support for mathematics achievement	0.423	0.285	0.236	0.164	0.181	0.079	0.197	0.079	N/S	0.261	0.205	0.281	0.263	0.125	-0.098	0.364	0.433	0.319	0.183	0.169	0.517	0.434

Notes: For measures labelled 6–22: diagonal results (in bold text and shaded for clarity) show the correlation between the Year 10 and 12 values of the measures; results to the lower left of the diagonal show the correlation between the measures taken at Year 10; results to the upper right of the diagonal show the correlations between the measures taken at Year 12. All results are significant (at least $p < 0.05$ or better) except where highlighted N/S, which indicates $p > 0.05$, considered non-significant.