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ELASTIC AND QUASI-ELASTIC NEUTRINO SCATTERING OFF NUCLEONS

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1 Foreword

This graduation project report is an attempt to catch in a few words and formulae my learning experience of the past few months on some specialized subject. Unfortunately, the time spent on and the skills developed in reading the more general literature (like Pilkuhn [1], Halzen & Martin [2], Mulders [3, 4]), using the necessary computer software, and above all communicating with the scientists in the theory group of NIKHEF-K, cannot be adequately conveyed in a report like this.

The project mainly consisted of the actual calculation of the neutrino scattering cross sections. Also some preliminary work was done on the hypothesis of "simulating neutrinos with polarized electrons" using the density matrix formalism. I will not discuss the last subject in this report. Still I have added a small appendix on the density matrix formalism.

2 Introduction

The area of physics I am concerned with here is the scattering of neutrinos off bound hadrons, i.e. strongly interacting particles. Neutrinos are massless particles (to be more correct: at present the experimental upper limit on the mass is about 10 eV). Hadrons are the massive particles our world's mass mainly consists of: this category includes baryons (e.g. protons and neutrons) and mesons (e.g. pions). Nuclei are composed of hadrons. Hadrons are considered to be composed of quarks, which come in six different "flavors": up, down, charm, strange, bottom, and top. Only the last flavor (top) has not yet been "seen" experimentally, but positive results are expected soon. The neutrino, on the other hand, belongs to the family of leptons, of which the electron is the most well-known example. Leptons interact only via electromagnetic and weak interaction, and not via strong interaction. Analogous to the one-photon exchange approximation for electron-hadron scattering, we describe neutrino-hadron scattering as the exchange of a massive vector boson (if both the in- and outgoing leptons are neutrinos, this vector boson is the Z^0 boson).

In Section 3 I will introduce neutrino scattering as an example of the more general class of weak neutral-current reactions. Section 4 will focus on the effects of the quarks inside the nucleons probed by neutrinos. Then, in Sections 5 and 6 I calculate the elastic and quasi-elastic cross sections, respectively. And finally, in Section 7 the subject of final state interactions is briefly touched upon.

The appendices summarize the conventions of three papers relevant for this report and introduce the density matrix formalism.

3 Weak neutral-current reactions

Weak neutral-current reactions have been studied for several reasons. For example, to test the standard model, to investigate the axial structure of hadrons, or to test the conserved vector current hypothesis. In this report the reason for investigating the weak neutral-current is the possibility to probe for s-quarks in nucleons.

The simplest possible neutral-current reactions are the purely leptonic reactions like elastic scattering of neutrinos off electrons:

$$\nu_\mu e^- \quad \rightarrow \quad \nu_\mu e^-.$$

Also very simple are the semi-leptonic elastic reactions, like

$$\begin{aligned} \nu_\mu p &\rightarrow \nu_\mu p, \\ \overline{\nu}_\mu p &\rightarrow \overline{\nu}_\mu p, \\ \nu_\mu n &\rightarrow \nu_\mu n, \\ \overline{\nu}_\mu n &\rightarrow \overline{\nu}_\mu n. \end{aligned}$$

The first experimental results on neutrino-proton scattering were obtained in 1976 at Brookhaven. The most recent and more accurate measurements, done using a 170-ton high-resolution target detector at Brookhaven, were published in 1986 and 1987 [5].

Because in this report the energy of the neutrino is low compared to the vector boson masses, the current-current lagrangian

$$\mathcal{L}_{\rm int} = -\frac{4G_{\rm F}}{\sqrt{2}} J_\mu^{{\rm Z/W}^\pm}(x) \frac{g^{\mu\nu} M^2_{{\rm Z/W}^\pm} - q^\mu q^\nu}{Q^2 + M^2_{{\rm Z/W}^\pm}} J_\nu^{\dagger\,{\rm Z/W}^\pm}(x) \tag{1}$$

reduces to the effective lagrangian

$$\mathcal{L}_{\rm int,eff} = -\frac{4G_{\rm F}}{\sqrt{2}} J^{\mu\,{\rm Z/W}^\pm}(x) J_\mu^{\dagger\,{\rm Z/W}^\pm}(x). \tag{2}$$

In these equations the $J_\mu^\dagger$ is the charge-lowering weak current. Since the weak neutral-current is hermitian, we will use $J_\mu^\dagger = J_\mu$ from now on. The relevant interaction term when studying neutrino-hadron scattering, is

$$\mathcal{L}_{\rm int,eff} = -\sqrt{2} G_{\rm F} \overline{\nu}(x) \frac{\gamma^\mu(1 \mp \gamma_5)}{2} \nu(x) J_\mu^{\rm H,Z}(x), \tag{3}$$

where H denotes "hadronic". For conventional reasons the hadronic current used in equation (3)—and in the rest of this report—differs a factor two from the currents occuring in equations (1) and (2) (i.e. we have defined $J_\mu^{\rm H,Z} \equiv 2J_\mu^{\dagger\,\rm Z}$). The minus-sign (plus-sign) corresponds to neutrino (antineutrino) scattering.

For weak neutral-current reactions the standard model gives the hadronic current

$$J_\mu^{\rm H,Z}(x) \quad = \sum_{a=0,3,8} (z_{\rm V}^a V_\mu^a(x) + z_{\rm A}^a A_\mu^a(x)), \tag{4}$$

where

$$\begin{aligned} z_{\rm V}^0 &= -\tfrac{1}{2} & z_{\rm A}^0 &= \tfrac{1}{2} \\ z_{\rm V}^3 &= 1 - 2\sin^2\theta_W & z_{\rm A}^3 &= -1 \\ z_{\rm V}^8 &= \tfrac{1}{\sqrt{3}}(1 - 2\sin^2\theta_W) & z_{\rm A}^8 &= -\tfrac{1}{\sqrt{3}}. \end{aligned} \tag{5}$$

The zs are determined by the weak quantum numbers of the quarks in the standard model. Small correction terms for z_V^0 ($\leq 10^{-4}$) and z_A^0 (≈ -0.02) have been calculated (see [7] for the details. The definitions of [7] are used in this report). The vector and axial vector currents occuring in equation (4) can be written in terms of the quark fields,

$$V_\mu^a(x) = \overline{q}(x)\gamma_\mu(\frac{1}{2}\lambda^a)q(x) \quad (6)$$

$$A_\mu^a(x) = \overline{q}(x)\gamma_\mu\gamma_5(\frac{1}{2}\lambda^a)q(x), \quad (7)$$

with $1 \leq a \leq 8$ and λ^a the eight Gell-Mann matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (8)$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Assuming flavor SU(3) symmetry all components of the vector current are conserved. Furthermore, it will be useful to define the U(1) vector and axial vector currents

$$V_\mu^0(x) = \overline{q}(x)\gamma_\mu(\frac{1}{3}I)q(x), \quad (9)$$

$$A_\mu^0(x) = \overline{q}(x)\gamma_\mu\gamma_5(\frac{1}{3}I)q(x). \quad (10)$$

In all these currents $q(x)$ is defined as

$$q(x) \equiv \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix}. \quad (11)$$

In fact also charmed quark fields should be included, because of quark mixing effects. However, at the scale applicable to this report all matrix elements involving charmed quarks are assumed to vanish.

In electromagnetic scattering we can write the invariant amplitude as

$$\mathcal{M} = -\left(\frac{e^2}{q^2}\right) J^{\mu\gamma}(x)J_\mu^{\mathrm{H},\gamma}(x). \quad (12)$$

In the hadronic current $J_\mu^{\rm H,\gamma}$ we only deal with $V_\mu^3(x)$ and $V_\mu^8(x)$, in the following combination:

$$J_\mu^{\rm H,\gamma}(x) = V_\mu^3(x) + \frac{1}{\sqrt{3}} V_\mu^8(x) = \overline{q}(x)\gamma_\mu \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} q(x), \tag{13}$$

where the matrix is the representation of the charge operator. This current is conserved, even if the SU(3) symmetry is not exact, i.e. if the masses of the u, d and s quarks are not equal.

4 Strange (and other) quark matrix elements

To see why we can "observe" the effects of the strange quarks, notice that from I and λ_8 we can construct

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{14}$$

The quantities that can be measured in experiments are linear combinations of the matrix elements

$$\langle {\rm P}'|V_\mu^{0,8}(0)|{\rm P}\rangle = \overline{\mathcal{U}}(p',s')\left(F_1^{0,8}(Q^2)\gamma_\mu + F_2^{0,8}(Q^2)\frac{i\sigma_{\mu\nu}q^\nu}{2M_{\rm N}}\right)\mathcal{U}(p,s), \tag{15}$$

$$\langle {\rm P}'|V_\mu^{3}(0)|{\rm P}\rangle = \overline{\mathcal{U}}(p',s')\left(F_1^{3}(Q^2)\gamma_\mu + F_2^{3}(Q^2)\frac{i\sigma_{\mu\nu}q^\nu}{2M_{\rm N}}\right)\mathcal{U}(p,s), \tag{16}$$

$$\langle {\rm P}'|A_\mu^{0,8}(0)|{\rm P}\rangle = \overline{\mathcal{U}}(p',s')\left(G_1^{0,8}(Q^2)\gamma_\mu\gamma_5\right)\mathcal{U}(p,s), \tag{17}$$

$$\langle {\rm P}'|A_\mu^{3}(0)|{\rm P}\rangle = \overline{\mathcal{U}}(p',s')\left(G_1^{3}(Q^2)\gamma_\mu\gamma_5\right)\mathcal{U}(p,s), \tag{18}$$

where P stands for the proton. If isospin SU(2) symmetry would be exact, then the matrix elements for the neutron would be the same, except that $\langle {\rm N}'|V_\mu^3|{\rm N}\rangle$ and $\langle {\rm N}'|A_\mu^3|{\rm N}\rangle$ have the opposite sign compared to the corresponding proton matrix elements. We have used the invariant $Q^2 \equiv -q_\mu q^\mu$. Note that we have omitted the second axial form factor $H_{\rm A}$, since inserting $H_{\rm A} q_\mu \gamma_5$ terms in equations (17) and (18) would not give contributions to the cross section (see Section 5).

From the static electromagnetic properties of the nucleon we can deduce

$$\begin{aligned} F_1^3(0) &= \tfrac{1}{2} & F_1^8(0) &= \tfrac{1}{2}\sqrt{3} \\ F_2^3(0) &= \tfrac{1}{2}(\kappa_{\rm p} - \kappa_{\rm n}) & F_2^8(0) &= \tfrac{1}{2}\sqrt{3}(\kappa_{\rm p} + \kappa_{\rm n}), \end{aligned} \tag{19}$$

in which $\kappa_{\rm p} = 1.793$ and $\kappa_{\rm n} = -1.913$ are the anomalous magnetic moments of the proton and neutron, repectively. Furthermore we know that $F_1^0(0) = 1$, because it is the baryon number of the nucleon, and from weak charged-current processes it is possible to determine

$$G_1^3(0) \equiv \frac{1}{2} g_A(0) \simeq 0.63 \tag{20}$$

and

$$G_1^8(0) \simeq \frac{0.28}{\sqrt{3}}. \tag{21}$$

As we show in Section 5, where the neutrino scattering cross section is expressed in terms of all forementioned form factors, the quantities $F_2^0(0)$ and $G_1^0(0)$ can be determined by doing this weak neutral-current experiment. These two form factors enable one to obtain information on the s-quark matrix elements

$$\begin{aligned}\langle \mathrm{P}'|\overline{s}(x)\gamma_\mu s(x)|\mathrm{P}\rangle = \langle \mathrm{N}'|\overline{s}(x)\gamma_\mu s(x)|\mathrm{N}\rangle = \\ \overline{\mathcal{U}}(p',s')\left(F_1^s(Q^2)\gamma_\mu + F_2^s(Q^2)\frac{i\sigma_{\mu\nu}q^\nu}{2M_\mathrm{N}}\right)\mathcal{U}(p,s)e^{iq\cdot x},\end{aligned} \tag{22}$$

$$\begin{aligned}\langle \mathrm{P}'|\overline{s}(x)\gamma_\mu\gamma_5 s(x)|\mathrm{P}\rangle = \langle \mathrm{N}'|\overline{s}(x)\gamma_\mu\gamma_5 s(x)|\mathrm{N}\rangle = \\ \overline{\mathcal{U}}(p',s')\left(G_1^s(Q^2)\gamma_\mu\gamma_5\right)\mathcal{U}(p,s)e^{iq\cdot x},\end{aligned} \tag{23}$$

where

$$F_2^s(Q^2) \equiv F_2^0(Q^2) - \frac{2}{\sqrt{3}}F_2^8(Q^2), \tag{24}$$

$$G_1^s(Q^2) \equiv G_1^0(Q^2) - \frac{2}{\sqrt{3}}G_1^8(Q^2). \tag{25}$$

In equation (22) for the first matrix element we will assume $F_1^s(Q^2)$ to be zero, which is true only at $Q^2 = 0$. Evaluated at $Q^2 = 0$ equations (24) and (25) give

$$F_2^s(0) = F_2^0(0) - (\kappa_\mathrm{p} + \kappa_\mathrm{n}), \tag{26}$$

$$G_1^s(0) = G_1^0(0) - \frac{0.55}{3}. \tag{27}$$

We adopt the following dipole forms for the form factors (see [7]):

$$F_2^{0,3,8}(Q^2) = \frac{F_2^{0,3,8}(0)}{\left(1+\frac{Q^2}{4M_\mathrm{N}^2}\right)\left(1+\frac{Q^2}{M_\mathrm{V}^2}\right)^2}, \tag{28}$$

$$F_1^{3,8}(Q^2) = \frac{F_1^{3,8}(0)}{\left(1+\frac{Q^2}{M_\mathrm{V}^2}\right)^2} + \frac{Q^2}{4M_\mathrm{N}^2}F_2^{3,8}(Q^2), \tag{29}$$

$$G_1^{0,3,8}(Q^2) = \frac{G_1^{0,3,8}(0)}{\left(1+\frac{Q^2}{M_\mathrm{A}^2}\right)^2}. \tag{30}$$

We will use $M_{\rm V} = 840$ MeV and $M_{\rm A} = 1030$ MeV.

It is also possible to single out other than strange quark matrix elements. For completeness we will write for the u and d quarks, using expressions analogous to equations (22) and (23),

$$\begin{aligned} F_i^{\rm u}(Q^2) &= F_i^0(Q^2) + F_i^3(Q^2) + \frac{1}{\sqrt{3}}F_i^8(Q^2), & (31)\\ G_1^{\rm u}(Q^2) &= G_1^0(Q^2) + G_1^3(Q^2) + \frac{1}{\sqrt{3}}G_1^8(Q^2), & (32)\\ F_1^{\rm u}(0) &= 2, & (33)\\ F_2^{\rm u}(0) &= F_2^0(0) + \kappa_{\rm p}, & (34)\\ G_1^{\rm u}(0) &= G_1^0(0) + \frac{1}{2}g_{\rm A}(0) + \frac{0.28}{3}, & (35) \end{aligned}$$

and

$$\begin{aligned} F_i^{\rm d}(Q^2) &= F_i^0(Q^2) - F_i^3(Q^2) + \frac{1}{\sqrt{3}}F_i^8(Q^2), & (36)\\ G_1^{\rm d}(Q^2) &= G_1^0(Q^2) - G_1^3(Q^2) + \frac{1}{\sqrt{3}}G_1^8(Q^2), & (37)\\ F_1^{\rm d}(0) &= 1, & (38)\\ F_2^{\rm d}(0) &= F_2^0(0) + \kappa_{\rm n}, & (39)\\ G_1^{\rm d}(0) &= G_1^0(0) - \frac{1}{2}g_{\rm A}(0) + \frac{0.28}{3}. & (40) \end{aligned}$$

This shows the importance of obtaining $F_2^0(0)$ and $G_1^0(0)$ for measuring nonstrange quark matrix elements too.

5 Elastic neutrino scattering

In order to obtain information about the form factors $G_1^{\rm s}$ and $F_2^{\rm s}$ relevant for the strange quark content of the nucleon we express the neutrino scattering cross section in terms of these. First we calculate the unpolarized differential cross section for the elastic scattering of neutrinos and antineutrinos off nucleons using the one boson exchange diagram of Figure 1. The cross section can be written as

$$d\sigma = \frac{1}{F}|\mathcal{M}|^2 d\mathcal{R} \tag{41}$$

with flux factor $F = 4M_{\rm N}E_\nu$ and final state phase space

$$d\mathcal{R} = \frac{d^3\boldsymbol{k}'}{(2\pi)^3 2E_\nu'}\frac{d^3\boldsymbol{p}_{\rm N}'}{(2\pi)^3 2E_{\rm N}'}(2\pi)^4\delta^4(k + p_{\rm N} - k' - p_{\rm N}'). \tag{42}$$

The invariant amplitude squared for this process can be written as

$$\mathcal{M}(\nu{\rm N} \to \nu'{\rm N}')\,\mathcal{M}^*(\nu{\rm N} \to \nu'{\rm N}') = 2G_{\rm F}^2 L_{\mu\nu}(k_\nu, k_\nu')H^{\mu\nu}(p_{\rm N}, p_{\rm N}'), \tag{43}$$

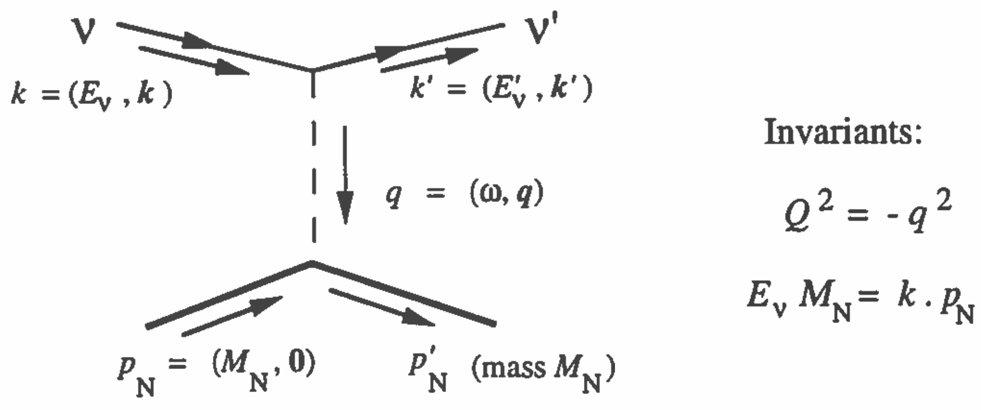

Figure 1: Feynman diagram for neutrino scattering off (fixed) nucleon.

where the hadronic tensor $H_{\mu\nu}$ is

$$\begin{aligned} H_{\mu\nu} = & \\ & \frac{1}{2}\sum_{s,s'}\langle p'_N, s'|J_\mu(0)|p_N, s\rangle\langle p_N, s|J^\dagger_\nu(0)|p'_N, s'\rangle = \\ & \frac{1}{2}\mathrm{Tr}\sum_{s'}\left[\mathcal{U}(p'_N, s')\overline{\mathcal{U}}(p'_N, s')\Gamma_\mu(\not{p}_N + M_N)\gamma_0\Gamma^\dagger_\nu\gamma_0\right] = \\ & \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)Q^2\left[(F_1+F_2)^2 + G_1^2\right] - 4g_{\mu\nu}M_N^2 G_1^2 + \\ & 4\bar{p}_\mu\bar{p}_\nu(F_1^2 + \frac{Q^2}{4M_N^2}F_2^2 + G_1^2) + 4i\epsilon_{\mu\nu\rho\sigma}\bar{p}^\rho q^\sigma G_1(F_1+F_2). \end{aligned} \tag{44}$$

In the last step the quantity $\bar{p}^\mu \equiv p^\mu_N - \frac{p_N \cdot q}{q^2}q^\mu$ has been introduced. In the calculation of the hadronic tensor the following relations have been used:

$$\Gamma_\mu = F_1\gamma_\mu + F_2\frac{i\sigma_{\mu\nu}q^\nu}{M_N} + G_1\gamma_\mu\gamma_5, \tag{45}$$

$$\gamma_0\Gamma^\dagger_\nu\gamma_0 = F_1\gamma_\nu - F_2\frac{i\sigma_{\nu\tau}q^\tau}{M_N} + G_1\gamma_\nu\gamma_5, \tag{46}$$

$$F_1 \equiv F_1(Q^2) = z^0_V F^0_1 + z^3_V F^3_1\tau^3 + z^8_V F^8_1, \tag{47}$$

$$F_2 \equiv F_2(Q^2) = z^0_V F^0_2 + z^3_V F^3_2\tau^3 + z^8_V F^8_2, \tag{48}$$

$$G_1 \equiv G_1(Q^2) = z^0_A G^0_1 + z^3_A G^3_1\tau^3 + z^8_A G^8_1, \tag{49}$$

$$\mathcal{U}(p'_N, s')\overline{\mathcal{U}}(p'_N, s') = (\not{p}'_N + M_N)\frac{1 \pm \gamma_5 \not{n}}{2}. \tag{50}$$

In the last relation n denotes the polarization vector, boosted from the rest-frame; it obeys $n \cdot p'_N = 0$ and n is spacelike. The plus-sign corresponds to the spin $+\frac{1}{2}$ state and the minus-sign to the spin $-\frac{1}{2}$ state. The factor τ^3 equals $+1$ for protons and -1 for neutrons.

The leptonic tensor is (without initial spin averaging!)

$$\begin{aligned} L_{\mu\nu} &= \mathrm{Tr}\left[\not{k}'\frac{\gamma_\mu(1 \mp \gamma_5)}{2}\not{k}\frac{\gamma_\nu(1 \mp \gamma_5)}{2}\right] \\ &= \mathrm{Tr}\left[\not{k}'\gamma_\mu\not{k}\frac{\gamma_\nu(1 \mp \gamma_5)}{2}\right] \\ &= 2k_\mu k'_\nu + 2k'_\mu k_\nu - Q^2 g_{\mu\nu} \mp 2i\epsilon_{\mu\nu\rho\sigma}k^\rho k'^\sigma. \end{aligned} \tag{51}$$

Here the minus-sign (plus-sign) corresponds again to the neutrino (antineutrino).

As we already remarked in Section 4, we have omitted the term $H_A q_\mu \gamma_5$ in equation (45), since all terms in the hadronic tensor proportional to $q_\mu q_\nu$ give zero when they are contracted with the leptonic tensor. However, in order to obtain a gauge invariant hadronic tensor it is necessary to take into account H_A.

The outgoing neutrino is not observed; its phase space is integrated over and the three-fold differential cross section becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega'_N \mathrm{d}E'_N} = \int \frac{\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3 2E'_\nu}\frac{1}{4\pi M_N}\frac{\sqrt{E_N'^2 - M_N^2}}{E_\nu}\left|\frac{\mathcal{M}}{4\pi}\right|^2 (2\pi)^4\delta^4(k - q - k'). \tag{52}$$

The integration over $\boldsymbol{k}'$ can be performed using

$$\begin{aligned} &\frac{1}{2\pi}\int \frac{\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3 2E'_\nu}(2\pi)^4\delta^4(k - q - k') = \\ &= \delta\left((k-q)^2\right) \\ &= \delta(2k \cdot q + Q^2) \\ &= \delta\left(2E_\nu(E'_N - M_N) - 2E_\nu\sqrt{E_N'^2 - M_N^2}\cos\theta'_\nu - (E'_N - M_N)^2 + E_N'^2 - M_N^2\right) \\ &= \delta\left(2E_\nu T'_N - 2E_\nu\sqrt{T_N'^2 + 2T'_N M_N}\cos\theta'_\nu + 2T'_N M_N\right) \\ &= \frac{1}{2E_\nu\sqrt{T_N'^2 + 2T'_N M_N}}\delta\left(\frac{T'_N E_\nu + T'_N M_N}{E_\nu\sqrt{T_N'^2 + 2T'_N M_N}} - \cos\theta'_\nu\right), \end{aligned} \tag{53}$$

where $T'_N \equiv E'_N - M_N$ is the kinetic energy of the detected nucleon. The z-axis has here been chosen parallel to $\boldsymbol{q}$. Suppose $E_\nu = 200$ MeV. Then for $\theta'_\nu = 0$, $T'_N = 0$ or $T'_N \simeq 60$ MeV. The latter value for the kinetic energy of the outgoing nucleon is also the maximum kinetic energy over all angles. The angles are not measured and after integration over Ω'_N we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E'_N} = \frac{2\pi \cdot 2\pi}{4\pi \cdot 2E_\nu^2 M_N}\left|\frac{\mathcal{M}}{4\pi}\right|^2 = \frac{\pi}{2E_\nu^2 M_N}\left|\frac{\mathcal{M}}{4\pi}\right|^2 = \frac{|\mathcal{M}|^2}{32\pi E_\nu^2 M_N}. \tag{54}$$

When we alternatively choose to integrate over the total phase space of the nucleon and all energies of the outgoing neutrino—this does not give an experimentally useful quantity since in practice we cannot measure the angle of the outgoing neutrino: this neutrino is not detected at all—we get

$$\frac{d\sigma}{d\Omega'_\nu} = \left(1 - \frac{Q^2}{2M_N E_\nu}\right) \frac{|\mathcal{M}|^2}{64\pi^2 M_N^2}. \tag{55}$$

One can write down for Q^2 the following relation

$$Q^2 \equiv -(k-k')^2 = 4E_\nu \left(E_\nu - \frac{Q^2}{2M_N}\right) \sin^2(\frac{\theta'_\nu}{2}). \tag{56}$$

Using equation (56) as a relation between Q^2 and θ'_ν and assuming that there is no ϕ'_ν dependence in $\mathcal{M}$, we can write down the cross section

$$\frac{d\sigma}{dQ^2} = \frac{|\mathcal{M}|^2}{64\pi E_\nu^2 M_N^2}. \tag{57}$$

Because $E'_N = M_N + \frac{Q^2}{2M_N}$ we would indeed expect that equations (54) and (57) are related by

$$\frac{d\sigma}{dE'_N} = 2M_N \frac{d\sigma}{dQ^2}. \tag{58}$$

Using the relation

$$k \cdot q = -k' \cdot q \;=\; -\frac{Q^2}{2}, \tag{59}$$

and (in the laboratory frame) the relations

$$k \cdot \bar{p} = k' \cdot \bar{p} \;=\; M_N \left(E_\nu + \frac{M_N}{2} - \frac{E'_N}{2}\right), \tag{60}$$

$$\bar{p} \cdot \bar{p} \;=\; M_N^2 \left(1 + \frac{(E'_N - M_N)^2}{Q^2}\right), \tag{61}$$

$$E'_N \;=\; M_N + \frac{Q^2}{2M_N}, \tag{62}$$

we finally obtain for the elastic cross section

$$\begin{aligned}\frac{d\sigma}{dQ^2} = \frac{2G_F^2}{16\pi} \frac{L_{\mu\nu} H^{\mu\nu}}{4E_\nu^2 M_N^2} = \frac{G_F^2}{2\pi} &\left[G_1^2 \left(1 + \frac{Q^2}{4E_\nu^2}\right) + F_1^2 \left(1 - \frac{Q^2}{4E_\nu^2}\right) \right. \\ &- \left[\pm G_1(F_1 + F_2) + \frac{1}{2}(G_1^2 + F_1^2)\right] \frac{Q^2}{E_\nu M_N} \\ &\left. + \frac{1}{8}\left[(F_1 + F_2 \pm G_1)^2 - \frac{1}{2}F_2^2\right] \frac{Q^4}{E_\nu^2 M_N^2} - \frac{1}{8} F_2^2 \frac{Q^4}{E_\nu M_N^3} + \frac{1}{4} F_2^2 \frac{Q^2}{M_N^2} \right].\end{aligned} \tag{63}$$

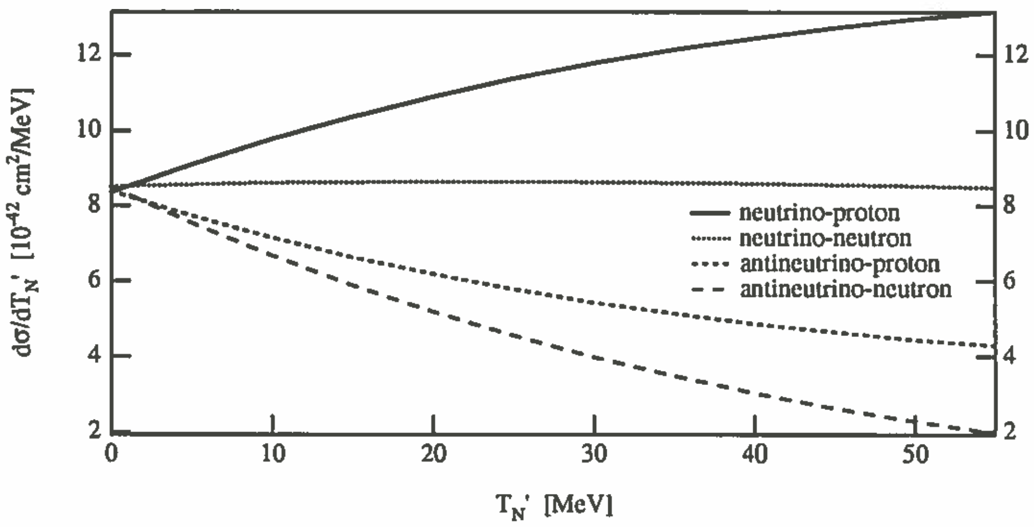

Figure 2: Elastic scattering cross sections for neutrino-nucleon scattering. The initial nucleon is fixed. For this plot we have taken $F_2^s(0) = -0.42$ and $G_1^s(0) = -0.19$. The neutrino energy used is 200 MeV.

This result is identical to the cross section given in [5] (be aware about their conventions, see Appendix!). In Figure 2 these cross sections are plotted.

In the analysis of experiments with relatively low neutrino energies, such as the one proposed at LAMPF (see Section 6), equation (63) is often approximated to first order in $E_\nu/M_{\rm N}$ and $Q/M_{\rm N}$ by

$$\frac{{\rm d}\sigma}{{\rm d}Q^2} = \frac{G_{\rm F}^2}{2\pi}\left[G_1^2\left(1+\frac{Q^2}{4E_\nu^2}\right)+F_1^2\left(1-\frac{Q^2}{4E_\nu^2}\right)\right.$$
$$\left. -\left[\pm G_1(F_1+F_2)+\frac{1}{2}(G_1^2+F_1^2)\right]\frac{Q^2}{E_\nu M_{\rm N}}\right]. \qquad (64)$$

In the rest of this report, however, we will use equation (63) for the cross section.

6 Quasi-elastic neutrino scattering

At LAMPF an experiment is planned with the new LSND (Liquid Scintillator Neutrino Detector). This detector contains 200 tons of mineral oil (CH_2). The presence of carbon nuclei gives a possibility to measure proton to neutron ratios quite accurately and also enhances the cross sections compared to single proton (or neutron) scattering. A neutrino beam, originating from a decaying pion beam, goes through this detector. The quantity that will be measured is the energy deposition of recoiling neutrons and protons, which have been hit by a neutrino. The maximum energy of the neutrino beam will be slightly more than 200 MeV, and hence the maximum kinetic energy of elastically hit hydrogen nuclei will be about 60

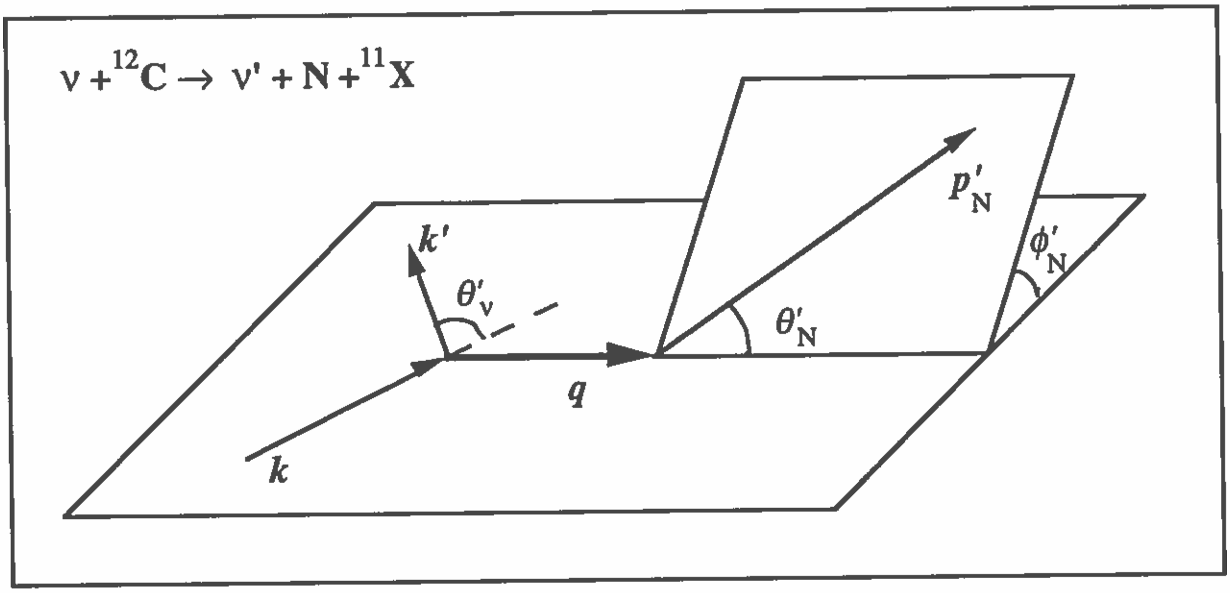

Figure 3: Definition of scattering planes and variables.

MeV (see Section 5). The idea now is to keep only those events that have $T'_{\rm N} > 60$ MeV; these must come from the ^{12}C$(\nu,\nu'$N$)$X or ^{12}C$(\overline{\nu},\overline{\nu}'N)$X reactions. In the laboratory frame we have a fixed ^{12}C target and an incoming neutrino-beam. A Z^0 boson is exchanged and a nucleon is knocked out of the nucleus. We can describe this process in the formalism for semi-inclusive experiments (see [3] for more details on the formalism). The neutrino and nucleon variables are defined in Figure 3. We work in *impulse approximation*, in which the total hadronic current is the sum of the currents of the individual nucleons treated as free particles (see Figure 4).

The final state phase space is now

$$d\mathcal{R} = \frac{d^3\boldsymbol{k}'}{(2\pi)^3 2E'_\nu}\frac{d^3\boldsymbol{p}'_{\rm N}}{(2\pi)^3 2E'_{\rm N}}\frac{d^3\boldsymbol{p}'_{\rm R}}{(2\pi)^3 2E'_{\rm R}}(2\pi)^4\delta^4(k + p_{\rm T} - k' - p'_{\rm N} - p'_{\rm R}). \tag{65}$$

Using

$$\begin{aligned} d^3\boldsymbol{p}'_{\rm N} &= |\boldsymbol{p}'_{\rm N}|^2 d|\boldsymbol{p}'_{\rm N}| d\Omega'_{\rm N} = |\boldsymbol{p}'_{\rm N}| E'_{\rm N} dE'_{\rm N} d\Omega'_{\rm N} = \frac{E'_{\rm N} dE'_{\rm N} d^2\boldsymbol{p}'_{\rm N\perp}}{|\boldsymbol{p}'_{\rm N\|}|} \\ &= \frac{E'_{\rm N} dE'_{\rm N} |\boldsymbol{p}'_{\rm N\perp}| d|\boldsymbol{p}'_{\rm N\perp}| d\phi'_{\rm N}}{|\boldsymbol{p}'_{\rm N\|}|} \end{aligned} \tag{66}$$

and

$$d^3\boldsymbol{k}' = |\boldsymbol{k}'|^2 d|\boldsymbol{k}'| d\Omega'_\nu = {E'_\nu}^2 dE'_\nu d\Omega'_\nu, \tag{67}$$

we have for the six-fold differential cross section

$$\frac{d\sigma}{d\Omega'_\nu dE'_\nu d\Omega'_{\rm N} dE'_{\rm N}} =$$

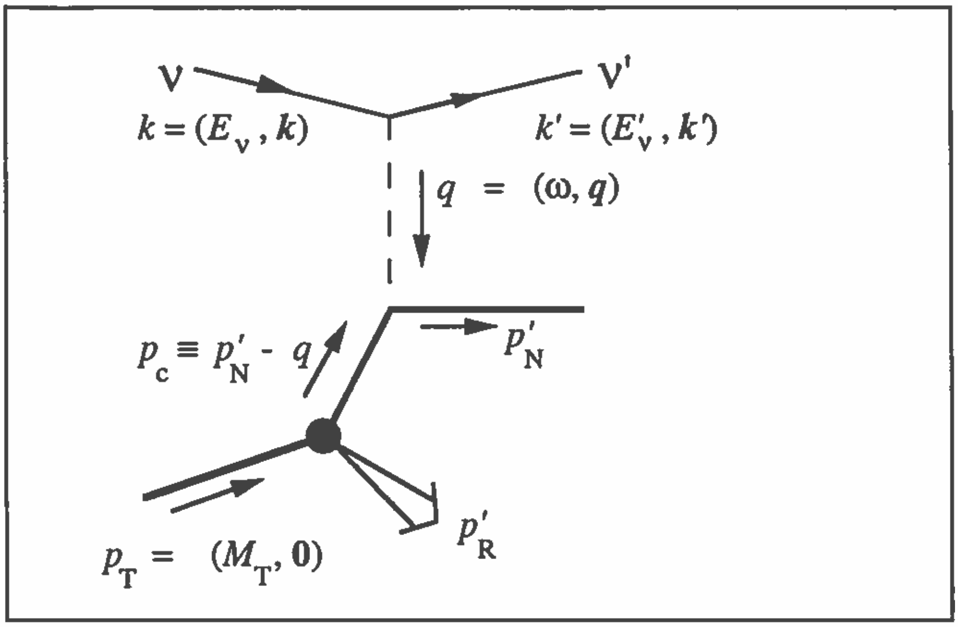

Figure 4: The impulse approximation.

$$
\begin{aligned}
& \frac{1}{4M_{\mathrm{T}}E_\nu}\frac{E'_\nu}{2(2\pi)^3}\frac{|\boldsymbol{p}'_{\mathrm{N}}|}{2(2\pi)^3}(2G_{\mathrm{F}}^2) \\
& \times L^{\mu\nu}\int\frac{\mathrm{d}^3\boldsymbol{p}'_{\mathrm{R}}}{(2\pi)^3 2E'_{\mathrm{R}}}H_{\mu\nu}^{(\mathrm{T}\to\mathrm{R'N'})}(2\pi)^4\delta^4(k-k'+p_{\mathrm{T}}-p'_{\mathrm{N}}-p'_{\mathrm{R}}) \\
= \;& |\boldsymbol{p}'_{\mathrm{N}}|E'_{\mathrm{N}}\left(\frac{2G_{\mathrm{F}}^2}{16\pi^2}\right)\left(\frac{E'_\nu}{E_\nu}\right)L_{\mu\nu}\mathcal{W}^{\mu\nu},
\end{aligned} \tag{68}
$$

where

$$
H_{\mu\nu}^{(\mathrm{T}\to\mathrm{R'N'})} = \langle p_{\mathrm{T}}|J_\mu^\dagger(0)|p'_{\mathrm{R}},p'_{\mathrm{N}}\rangle\langle p'_{\mathrm{R}},p'_{\mathrm{N}}|J_\nu(0)|p_T\rangle. \tag{69}
$$

We integrate over the residual nucleus states since the residual nucleus remains unobserved. The tensor $\mathcal{W}^{\mu\nu}$ is defined by equation (68) as

$$
\mathcal{W}^{\mu\nu} \equiv \frac{1}{(2\pi)^4}\frac{1}{4M_{\mathrm{T}}}\frac{1}{E'_{\mathrm{N}}}\int\frac{\mathrm{d}^3\boldsymbol{p}'_{\mathrm{R}}}{(2\pi)^3 2E'_{\mathrm{R}}}H_{\mu\nu}^{(\mathrm{T}\to\mathrm{R'N'})}(2\pi)^4\delta^4(k-k'+p_{\mathrm{T}}-p'_{\mathrm{N}}-p'_{\mathrm{R}}). \tag{70}
$$

Since the outgoing neutrino is also unobserved and we do not measure the angles of the outgoing nucleon we will calculate

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}E'_{\mathrm{N}}} = \int\mathrm{d}E'_\nu\int\mathrm{d}\Omega'_\nu\int\mathrm{d}\phi'_{\mathrm{N}}\int\mathrm{d}|\boldsymbol{p}'_{\mathrm{N}\perp}|\left[\frac{|\boldsymbol{p}'_{\mathrm{N}\perp}|}{|\boldsymbol{p}'_{\mathrm{N}\parallel}|}E'_{\mathrm{N}}\left(\frac{2G_{\mathrm{F}}^2}{16\pi^2}\right)\left(\frac{E'_\nu}{E_\nu}\right)L_{\mu\nu}\mathcal{W}^{\mu\nu}\right]. \tag{71}
$$

As already mentioned we work in impulse approximation; furthermore we neglect the ambiguities arising from the off-shellness of the nucleons. For an unpolarized target the hadronic tensor $\mathcal{W}_{\mu\nu}$ factorizes into a spectral function and the hadronic tensor for unpolarized spin $\frac{1}{2}$ constituents. We have then

$$\mathcal{W}_{\mu\nu} = \int \mathrm{d}W_{\mathrm{R}}\mathrm{d}^3\boldsymbol{p}_{\mathrm{c}}\mathcal{S}(\boldsymbol{p}_{\mathrm{c}}, W_{\mathrm{R}})\frac{H^{(\mathrm{c})}_{\mu\nu}(p_{\mathrm{c}}, q)}{4E'_{\mathrm{N}}E_{\mathrm{c}}}\delta^4(p_{\mathrm{T}} - p'_{\mathrm{R}} - p'_{\mathrm{N}} + q), \tag{72}$$

where the spectral function $\mathcal{S}(\boldsymbol{p}_{\mathrm{c}}, W_{\mathrm{R}})$ gives the chance that the knocked out nucleon has an initial momentum $\boldsymbol{p}_{\mathrm{c}}$ and leaves behind a nuclear state of invariant mass W_{R}, and $H^{(\mathrm{c})}_{\mu\nu}(p_{\mathrm{c}}, q)$ is the hadronic tensor for the constituent nucleon. This tensor is given in equation (44). Assuming that the wave function of the target nucleus can be written as a relative wave function $\phi(\boldsymbol{p}_{\mathrm{rel}})$ between the residual nucleus and the knocked out constituent, it can be shown that

$$S(\boldsymbol{p}_{\mathrm{rel}}) = |\phi(\boldsymbol{p}_{\mathrm{rel}})|^2. \tag{73}$$

Assuming a gaussian form for the relative wave functions one obtains

$$S(\boldsymbol{p}_{\mathrm{c}}) = N\alpha^3 \left(\frac{N}{(N-1)\pi}\right)^{3/2} e^{-\frac{N\alpha^2|\boldsymbol{p}_{\mathrm{c}}|^2}{N-1}}. \tag{74}$$

In this formula N is the number of constituents—in the case of ^{12}C, $N = 6$: proton and neutron knock-out are described separately. We could have added an isospin index to the tensor $\mathcal{W}_{\mu\nu}$ of equation (72). The total hadronic tensor is then written as a summation over both proton and neutron hadronic tensors—and α is a parameter related to the width of the momentum distribution. The normalization factor is chosen in such a way that

$$\int \mathrm{d}^3\boldsymbol{p}_{\mathrm{c}}S(\boldsymbol{p}_{\mathrm{c}}) = N. \tag{75}$$

The W_{R} dependence of $\mathcal{S}(\boldsymbol{p}_{\mathrm{c}}, W_{\mathrm{R}})$ is kept as simple as possible: only the ground state of the residual nucleus is accounted for in the description, so

$$\mathcal{S}(\boldsymbol{p}_{\mathrm{c}}, W_{\mathrm{R}}) = S(\boldsymbol{p}_{\mathrm{c}})\delta(W_{\mathrm{R}} - M_{\mathrm{R}}). \tag{76}$$

If the integral over $\boldsymbol{p}_{\mathrm{c}}$ is performed we get

$$\mathcal{W}_{\mu\nu} = \int \mathrm{d}W_{\mathrm{R}}\mathcal{S}(\boldsymbol{p}_{\mathrm{c}}, W_{\mathrm{R}})\frac{H^{(\mathrm{c})}_{\mu\nu}(p_{\mathrm{c}}, q)}{4E'_{\mathrm{N}}E_{\mathrm{c}}}\delta(M_{\mathrm{T}} + \omega - E'_{\mathrm{R}} - E'_{\mathrm{N}}). \tag{77}$$

Note that because of the performed integration, $\boldsymbol{p}_{\mathrm{c}}$ is now only used as a notation for $\boldsymbol{p}_{\mathrm{T}} - \boldsymbol{p}'_{\mathrm{R}}$ and therefore is not an independent variable anymore. Integration over W_{R} gives

$$\mathcal{W}_{\mu\nu} = S(\boldsymbol{p}_{\mathrm{c}})\frac{H^{(\mathrm{c})}_{\mu\nu}(p_{\mathrm{c}}, q)}{4E'_{\mathrm{N}}E_{\mathrm{c}}}\delta(M_{\mathrm{T}} + \omega - \sqrt{M^2_{\mathrm{R}} + |\boldsymbol{p}_{\mathrm{c}}|^2} - E'_{\mathrm{N}}). \tag{78}$$

Since we are going to integrate this expression analytically over $|\boldsymbol{p}'_{\mathrm{N}\perp}|$ we have to rewrite the argument of the delta function as a function of $|\boldsymbol{p}'_{\mathrm{N}\perp}|$. The only $|\boldsymbol{p}'_{\mathrm{N}\perp}|$-dependence is via

$|\boldsymbol{p}_c|^2 \equiv |\boldsymbol{p}'_N - \boldsymbol{q}|^2$. So we write, using the explicit expressions from [3]:

$$\begin{aligned}|\boldsymbol{p}'_N - \boldsymbol{q}|^2 &= |\boldsymbol{p}'_{N\perp}|^2 + (|\boldsymbol{p}'_{N\|}| - |\boldsymbol{q}|)^2 = |\boldsymbol{p}'_N|^2 + |\boldsymbol{q}|^2 - 2|\boldsymbol{p}'_{N\|}||\boldsymbol{q}| \\ &= {E'_N}^2 - M_N^2 + E_\nu^2 + {E'_\nu}^2 - 2E_\nu E'_\nu \cos\theta'_\nu \\ &\quad -2\left[({E'_N}^2 - M_N^2 - |\boldsymbol{p}'_{N\perp}|^2)(E_\nu^2 + {E'_\nu}^2 - 2E_\nu E'_\nu \cos\theta'_\nu)\right]^{1/2}, \end{aligned} \tag{79}$$

in which only our independent variables E'_N, $|\boldsymbol{p}'_{N\perp}|$, E'_ν, and $\cos\theta'_\nu$ have been used—note that $|\boldsymbol{p}'_{N\|}|^2 \equiv {E'_N}^2 - M_N^2 - |\boldsymbol{p}'_{N\perp}|^2$. The quantity $|\boldsymbol{p}'_N - \boldsymbol{q}|$ is independent of the azimuthal angle ϕ'_N. The delta function, taking only positive values for $|\boldsymbol{p}'_{N\perp}|$, can now be rewritten as

$$\delta(M_T + \omega - \sqrt{M_R^2 + |\boldsymbol{p}_c|^2} - E'_N) = \frac{1}{\left|\frac{\partial(-\sqrt{M_R^2 + |\boldsymbol{p}_c|^2})}{\partial|\boldsymbol{p}'_{N\perp}|}\right|_{|\boldsymbol{p}'_{N\perp}|=a}} \delta(|\boldsymbol{p}'_{N\perp}| - a), \tag{80}$$

where a stands for the positive root of the argument of the delta function. The derivative

$$\frac{\partial(\sqrt{M_R^2 + |\boldsymbol{p}_c|^2})}{\partial|\boldsymbol{p}'_{N\perp}|} = \frac{1}{2\sqrt{M_R^2 + |\boldsymbol{p}_c|^2}} \frac{\partial|\boldsymbol{p}_c|^2}{\partial|\boldsymbol{p}'_{N\perp}|} \tag{81}$$

can be calculated from equation (79):

$$\frac{\partial|\boldsymbol{p}_c|^2}{\partial|\boldsymbol{p}'_{N\perp}|} = \frac{-2\sqrt{E_\nu^2 + {E'_\nu}^2 - 2E_\nu E'_\nu\cos\theta'_\nu}}{2\sqrt{{E'_N}^2 - M_N^2 - |\boldsymbol{p}'_{N\perp}|^2}}(-2|\boldsymbol{p}'_{N\perp}|) = 2|\boldsymbol{p}'_{N\perp}|\sqrt{\frac{E_\nu^2 + {E'_\nu}^2 - 2E_\nu E'_\nu\cos\theta'_\nu}{{E'_N}^2 - M_N^2 - |\boldsymbol{p}'_{N\perp}|^2}}. \tag{82}$$

From

$$M_T + \omega - \sqrt{M_R^2 + |\boldsymbol{p}_c|^2} - E'_N = 0 \tag{83}$$

we get, after squaring,

$$|\boldsymbol{p}'_N - \boldsymbol{q}|^2 = (M_T + E_\nu - E'_\nu - E'_N)^2 - M_R^2. \tag{84}$$

Using again the explicit expression for $|\boldsymbol{p}'_N - \boldsymbol{q}|$ in terms of $|\boldsymbol{p}'_{N\perp}|$ it follows that

$$\sqrt{{E'_N}^2 - M_N^2 - |\boldsymbol{p}'_{N\perp}|^2} = \frac{[M_R^2 - M_T^2 - M_N^2 + 2M_T E'_N - 2(E_\nu - E'_\nu)(M_T - E'_N) + 2E_\nu E'_\nu(1-\cos\theta'_\nu)]}{2\sqrt{E_\nu^2 + {E'_\nu}^2 - 2E_\nu E'_\nu\cos\theta'_\nu}}, \tag{85}$$

and

$$|\boldsymbol{p}'_{N\perp}| = \left[{E'_N}^2 - M_N^2 - \frac{[M_R^2 - M_T^2 - M_N^2 + 2M_T E'_N - 2(E_\nu - E'_\nu)(M_T - E'_N) + 2E_\nu E'_\nu(1-\cos\theta'_\nu)]^2}{4(E_\nu^2 + {E'_\nu}^2 - 2E_\nu E'_\nu\cos\theta'_\nu)}\right]^{1/2} \tag{86}$$

The scalar function $L^{\mu\nu}H^{(c)}_{\mu\nu}(p_c, q)$ appearing in our calculation is expressed in terms of the invariants Q^2 and $k \cdot p_c$, which read

$$Q^2 = 2E_\nu E'_\nu(1 - \cos\theta'_\nu) \tag{87}$$

and

$$\begin{aligned} k \cdot p_c \equiv k \cdot p'_{\rm N} - k \cdot q = \\ E_\nu E'_{\rm N} + E_\nu E'_\nu - E_\nu E'_\nu \cos\theta'_\nu \\ - \frac{E_\nu E'_\nu \sqrt{1 - \cos^2\theta'_\nu}\,|\boldsymbol{p}'_{\rm N\perp}| \cos\phi'_{\rm N} + (E_\nu^2 - E_\nu E'_\nu \cos\theta'_\nu)\sqrt{E'^2_{\rm N} - M_{\rm N}^2 - |\boldsymbol{p}'_{\rm N\perp}|^2}}{\sqrt{E_\nu^2 + E'^2_\nu - 2E_\nu E'_\nu \cos\theta'_\nu}}. \end{aligned} \tag{88}$$

The allowed kinematical region for $|\boldsymbol{p}'_{\rm N\perp}|$ is

$$0 \le |\boldsymbol{p}'_{\rm N\perp}| \le \sqrt{E'^2_{\rm N} - M_{\rm N}^2}. \tag{89}$$

The condition for the maximum value of $|\boldsymbol{p}'_{\rm N\perp}|$ will be automatically fulfilled in our numerical integration procedure, since we take $M_{\rm T} + E_\nu - M_{\rm R} - M_{\rm N}$ as an upper limit for the integration over E'_ν. Thus $E'_\nu < E_\nu$ and

$$E_\nu^2 + E'^2_\nu - 2E_\nu E'_\nu \cos\theta'_\nu \ge (E_\nu - E'_\nu)^2 > 0. \tag{90}$$

However, we have to adjust the integration boundaries of $\cos\theta'_\nu$ in such a way that the argument of the square root in equation (86) cannot become negative. In order to do this we express the equation

$$|\boldsymbol{p}'_{\rm N\perp}| = 0 \tag{91}$$

as a second-order polynomial in $\cos\theta'_\nu$:

$$\begin{aligned} 0 = {} & 4(E'^2_{\rm N} - M_{\rm N}^2)(E_\nu^2 + E'^2_\nu - 2E_\nu E'_\nu \cos\theta'_\nu) \\ & - \left[M_{\rm R}^2 - M_{\rm T}^2 - M_{\rm N}^2 + 2M_{\rm T}E'_{\rm N} - 2(E_\nu - E'_\nu)(M_{\rm T} - E'_{\rm N}) + 2E_\nu E'_\nu(1 - \cos\theta'_\nu)\right]^2. \end{aligned} \tag{92}$$

The discriminant of this equation is

$$\begin{aligned} D = E'^2_\nu & \left[64E_\nu^2 \left\{E'^4_{\rm N} - E'^2_{\rm N}(M_{\rm N}^2 + M_{\rm R}^2) + M_{\rm N}^2 M_{\rm R}^2 - 2E'^3_{\rm N} M_{\rm T} + 2E'_{\rm N} M_{\rm N}^2 M_{\rm T} \right.\right. \\ & \left. + E'^2_{\rm N} M_{\rm T}^2 - M_{\rm N}^2 M_{\rm T}^2\right\} + 64E_\nu^3 \left\{-2E'^3_{\rm N} + 2E'_{\rm N} M_{\rm N}^2 + 2E'^2_{\rm N} M_{\rm T} - 2M_{\rm N}^2 M_{\rm T}\right\} \\ & \left. + 64E_\nu^4 \left\{E'^2_{\rm N} - M_{\rm N}^2\right\}\right] \\ & E'^3_\nu \left[128E_\nu^2 \left\{E'^3_{\rm N} - E'_{\rm N} M_{\rm N}^2 - E'^2_{\rm N} M_{\rm T} + M_{\rm N}^2 M_{\rm T}\right\} + 128E_\nu^3 \left\{-E'^2_{\rm N} + M_{\rm N}^2\right\}\right] \\ & E'^4_\nu \left[64E_\nu^2 \left\{E'^2_{\rm N} - M_{\rm N}^2\right\}\right]. \end{aligned} \tag{93}$$

Since the coefficient of the $\cos^2\theta'_\nu$ term is negative, the lhs of equation (92) is positive if $D > 0$ and

$$(\cos\theta'_\nu)_1 \le \cos\theta'_\nu \le (\cos\theta'_\nu)_2, \tag{94}$$

keeping in mind that the domain for $\cos\theta'_\nu$ is [-1,1]. The boundaries are given by

$$(\cos\theta'_\nu)_{1,2} = \frac{A \mp \sqrt{D}}{8E_\nu^2 E_\nu'^2}, \tag{95}$$

in which

$$A = 4E_\nu E'_\nu(2E_\nu E'_\nu + 2E_\nu E'_N - 2E'_\nu E'_N - 2E_N'^2 + M_N^2 + M_R^2 - 2E_\nu M_T + 2E'_\nu M_T + 2E'_N M_T - M_T^2). \tag{96}$$

We now have obtained the integration boundaries of $\cos\theta'_\nu$ explicitly as a function of E'_ν. If for some E'_ν there is no physical region of θ'_ν at all, then the integrand in the numerical integration procedure will be set to zero.

Finally we write down the full expression for the quasi-elastic cross section

$$\begin{aligned}\frac{d\sigma}{dE'_N} = &\int_0^{M_T+E_\nu-M_R-M_N} dE'_\nu \int_{\max\{-1,(\cos\theta'_\nu)_1\}}^{\min\{1,(\cos\theta'_\nu)_2\}} d\cos\theta'_\nu \int_0^{2\pi} d\phi'_N \\ &\times \left[2\pi \cdot \frac{4(k\cdot p_c)^2}{\pi}\left[\frac{d\sigma}{dQ^2}(Q^2, k\cdot p_c)\right]_{\rm elast} S(p_c)\frac{E'_N}{\sqrt{E_N'^2 - M_N^2 - |p'_{N\perp}|^2}}\frac{E'_\nu}{E_\nu}\right. \\ &\left.\times\frac{1}{4E'_N(E'_N - E_\nu + E'_\nu)}\sqrt{\frac{(M_R^2 + |p'_N - q|^2)(E_N'^2 - M_N^2 - |p'_{N\perp}|^2)}{E_\nu^2 + E_\nu'^2 - 2E_\nu E'_\nu\cos\theta'_\nu}}\right],\end{aligned} \tag{97}$$

where we substituted $E_\nu M_N = k\cdot p_c$ in equation (63). Equation (97) equals

$$\begin{aligned}\frac{d\sigma}{dE'_N} = &\int dE'_\nu \int d\cos\theta'_\nu \int d\phi'_N \left[\frac{d\sigma}{dQ^2}(Q^2, k\cdot p_c)\right]_{\rm elast} \\ &\times\frac{2(k\cdot p_c)^2 S(p_c)E'_\nu(M_T + E_\nu - E'_\nu - E'_N)}{E_\nu(E'_N - E_\nu + E'_\nu)\sqrt{E_\nu^2 + E_\nu'^2 - 2E_\nu E'_\nu\cos\theta'_\nu}}.\end{aligned} \tag{98}$$

The quantity Garvey *et al.* [6] propose to determine from the LAMPF experiment is the integrated ratio

$$R = \frac{\int_{T'_{N\,min}}^{T'_{N\,max}} \frac{d\sigma_p}{dT'_N} dT'_N}{\int_{T'_{N\,min}}^{T'_{N\,max}} \frac{d\sigma_n}{dT'_N} dT'_N}. \tag{99}$$

This ratio depends on $G_1^s(0)$ and $F_2^s(0)$. Since $\frac{d\sigma}{dT'_N} = \frac{d\sigma}{dE'_N}$, the ratio R is easy to calculate once one has calculated $\frac{d\sigma}{dE'_N}$ from equation (98). We will do the integration of equation (98) numerically. Our results are given in Figures 5 to 8. Note that the curves for the cross sections are a factor 4 to 6 higher than those in [6]. The cause of this difference is still unclear. The conventions used for the currents have been checked thoroughly. A possible explanation could be a difference in the normalization of the spectral function that has been used. Fortunately, this factor cancels when we take proton to neutron ratios.

As can be seen in Figures 7 and 8, it is possible to use a measurement of the ratio to contrain the values of $G_1^s(0)$ and $F_2^s(0)$.

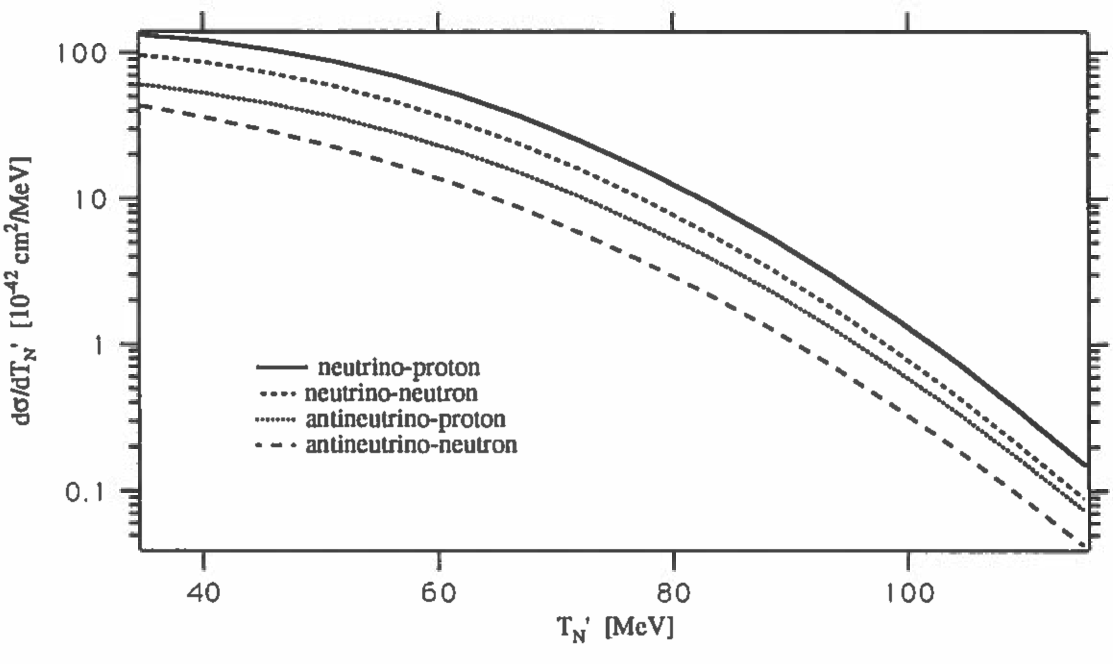

Figure 5: Quasi-elastic scattering cross sections on ^{12}C. For the parameter α we have used 0.009 MeV^{-1}. Furthermore, we have again taken $F_2^s(0) = -0.42$, $G_1^s(0) = -0.19$, and a fixed neutrino energy of 200 MeV.

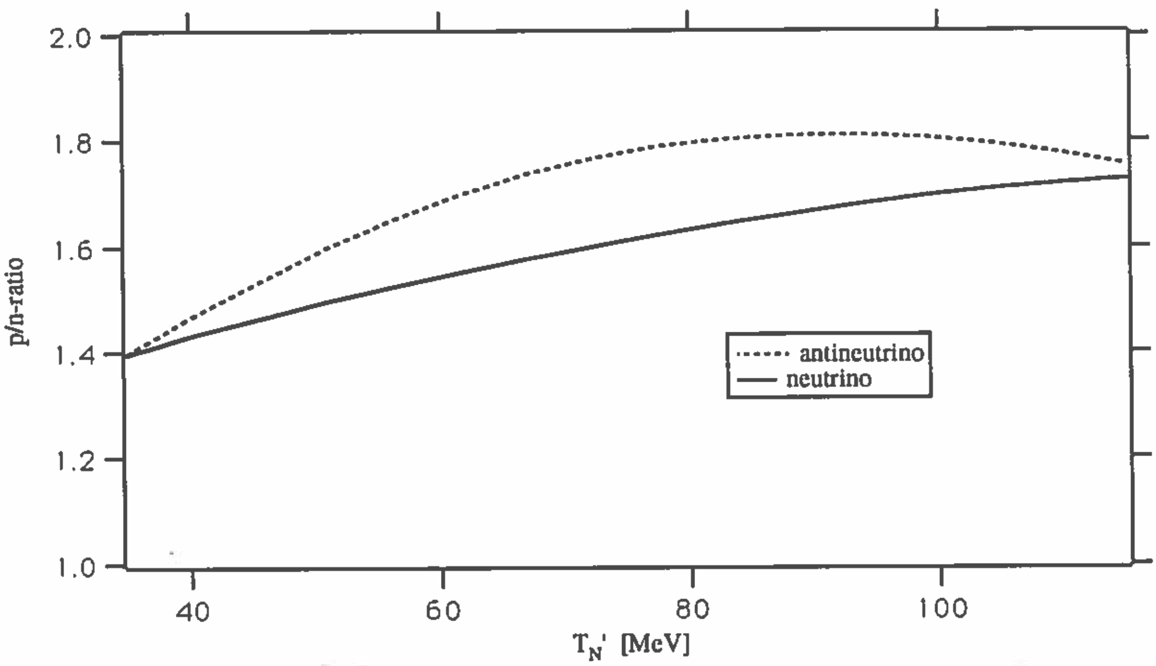

Figure 6: Ratios of proton to neutron cross sections.

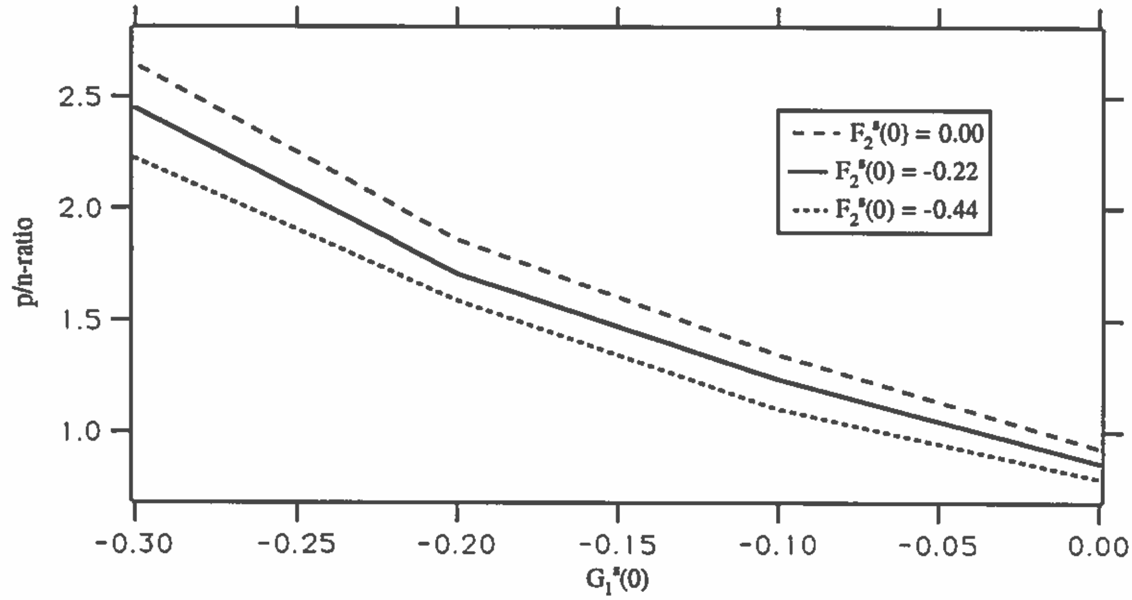

Figure 7: Variation of antineutrino scattering p/n-ratio with form factors.

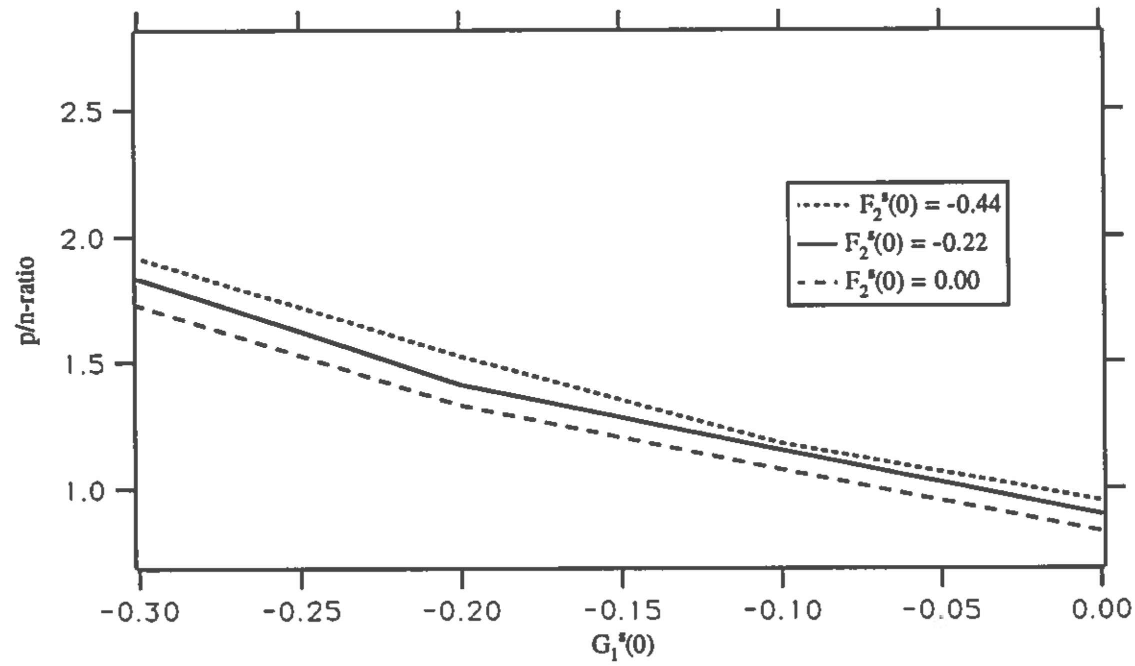

Figure 8: Variation of neutrino scattering p/n-ratio with form factors.

7 Final state interactions

The description should not stop at this simple level, however. Recently, final state interactions are calculated in the model of [8] to be responsible for an increase of both proton and neutron cross sections of about 15% at $T'_{\rm N} = 80$ MeV and 40% at $T'_{\rm N} = 100$ MeV. So these final state interactions seem to be things to worry about. However, in [8] also calculations are presented which are based on models that include both nuclear structure effects and final state interactions. They find that the cross section becomes essentially independent of details of nuclear structure at energies above $T'_{\rm N} = 30$ MeV, corresponding to a nuclear excitation energy of about 45 MeV. And, most important of all, it appears that the p/n-ratio is virtually independent of final state interactions.

A possible means of getting an experimental indication on these final state interactions is by performing a similar kind of experiment with electrons. It will be useful to have electron scattering data in the same kinematical region and with roughly the same kind of spin density matrix for the knocked out nucleons. In this way one can be sure that one is measuring the same contributions of final state interactions as in the neutrino case. It may well be that it is possible by taking combinations of different electron polarizations that the density matrix elements become comparable. For an introduction to the density matrix formalism see Appendix B.

A Conventions

In this Appendix we relate the conventions used in Kaplan [7] (used in this report) to the conventions used in Garvey [6] and in Ahrens [5].

Kaplan (and this report)	Garvey	Ahrens
F_i	$2F_i$	F_i
F_i^3	$F_i^{T=1}$	
F_i^8	$\sqrt{3}F_i^{T=0}$	
F_i^s	$2F_i^s$	
V_μ	$2V_\mu$	
γ_5	γ_5	$-\gamma_5$
G_1	G_1	$-G_A$
$G_1^3 = \frac{1}{2}g_A$	$\frac{1}{2}G_A$	
G_1^s	G_s	
A_μ	A_μ	

B Density matrix formalism

The density matrix formalism is briefly described in relation to the problem of final state interactions of Section 7. We assume that the S-matrix of the total process ('production' and final state interactions) factorizes into one matrix for production ($\gamma^*{\rm N} \rightarrow {\rm N}'$ or ${\rm Z}^{0*}{\rm N} \rightarrow {\rm N}'$)

and another for the final state interactions ($N' \to N''$). If we assume the initial nucleon (N) and the detected nucleon (N'') to be unpolarized we can write the invariant amplitude squared for the total process:

$$|\mathcal{M}|^2_{total} = \mathrm{Tr}\,[R(\mathrm{prod})\, R^*(\mathrm{fsi})]\,, \tag{100}$$

with

$$R(\mathrm{prod})^{\gamma^*}_{m\,m'} = \mathcal{M}(\gamma^* \mathrm{N} \to \mathrm{N}'_m)\,\mathcal{M}^*(\gamma^* \mathrm{N} \to \mathrm{N}'_{m'}), \tag{101}$$

$$R(\mathrm{prod})^{Z^{0*}}_{m\,m'} = \mathcal{M}(\mathrm{Z}^{0*} \mathrm{N} \to \mathrm{N}'_m)\,\mathcal{M}^*(\mathrm{Z}^{0*} \mathrm{N} \to \mathrm{N}'_{m'}), \tag{102}$$

$$R(\mathrm{fsi})_{m\,m'} = \mathcal{M}(\mathrm{N}'_m \to \mathrm{N}'')\,\mathcal{M}^*(\mathrm{N}'_{m'} \to \mathrm{N}''). \tag{103}$$

Note that we have ignored here isospin changing final state interactions. We will now define the production density matrix as

$$\rho_{m\,m'} = \frac{R(\mathrm{prod})_{m\,m'}}{\mathrm{Tr}\,[R(\mathrm{prod})]}. \tag{104}$$

For example, the production matrix for the weak neutral-current reaction can be written as

$$\begin{aligned} R(\mathrm{prod})^{Z^{0*}}_{m\,m'} &= \mathcal{M}(\mathrm{Z}^{0*} \mathrm{N} \to \mathrm{N}'_m)\,\mathcal{M}^*(\mathrm{Z}^{0*} \mathrm{N} \to \mathrm{N}'_{m'}) = \\ &\frac{1}{2}\cdot(2G_F^2)\cdot \epsilon^\mu \left[\overline{\mathcal{U}}(p',m)\Gamma_\mu(\not{p}+M_\mathrm{N})\gamma_0\Gamma^\dagger_\nu\gamma_0\mathcal{U}(p',m')\right]\epsilon^{\nu*} = \\ &\frac{1}{2}\cdot(2G_F^2)\cdot \mathrm{Tr}\left[\mathcal{U}(p',m')\overline{\mathcal{U}}(p',m)\Gamma_\mu(\not{p}+M_\mathrm{N})\gamma_0\Gamma^\dagger_\nu\gamma_0\right]\epsilon^\mu\epsilon^{\nu*}, \end{aligned} \tag{105}$$

where the factor 1/2 is the spin-averaging over the initial spin of the nucleon. Note that we use the normalization $\overline{\mathcal{U}}(p,s)\mathcal{U}(p,s) = 2M_\mathrm{N}$, so that the completeness relation is

$$\sum_{s=+\frac{1}{2},-\frac{1}{2}} \mathcal{U}(p,s)\overline{\mathcal{U}}(p,s) = \not{p} + M_\mathrm{N}. \tag{106}$$

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