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Augmenting mathematics with mobile technology

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Abstract

This chapter describes two case examples of the use of mobile technology for mathematics. Building on the assumption that mobile learning has a positive effect on student attitudes and academic outcomes, including STEM subjects (Hsi, 2007; Wu et al., 2012) we develop a theoretical lens for future studies for 'mobile mathematics'. The two case examples describe how mobile technology could provide opportunities for 'mathematics outside the classroom'. The first example describes a dynamic Ferris wheel, the second a static cathedral. Both examples demonstrate how 'geo-location' and 'augmented reality' features allow mobile technologies to bridge formal and informal mathematics learning (Lai, Khaddage and Knezek, 2013).

Key words: augmented reality, mathematics education, mobile learning

1 Introduction

This chapter capitalizes on the potential of, and synergy with, informal learning using mobile devices (Laurillard, 2009). Whilst research is limited, evidence suggests that mobile learning has a positive effect on student attitudes and academic outcomes including STEM subjects (Hsi, 2007; Price, Davies, Farr, Jewitt, Roussos, & Sin, 2014; Wu, Wu, Chen, Kao, Lin, & Huang, 2012). Lowrie (2005) argues that the technology-rich contexts that are used at school are often different than the technology that children regularly use at home, and so it is important for educational research to consider the impact that technology not commonly found in school, can have on children's meaningful mathematics learning. In a study that explored opportunities for engaging children in mathematical activities through the use of a location-based game with mobile handheld technology, Wijers, Jonker and Drijvers (2010) collected data from observations, online storage game data, an online survey, and interviews and report findings that indicate enhanced student engagement. Ludwig and Jesberg (2015) explored the potential of mobile technology by provided 'geo located' modelling tasks, that is, 'maths trails' that were guided by the GPS options of mobile phones. Another development concerns the use of 'augmented reality' in informal learning environments: a field experiment in a mathematics exhibition showed that visitors performed significantly better on knowledge acquisition (Sommerauer & Müller, 2014). Despite a lack of mobile learning research in informal contexts (Wright & Parchoma, 2011), we aim to better understand the concept of mobile learning and how mobile technologies can be used to bridge formal and informal mathematics learning (Lai, Khaddage, & Knezek, 2013). We hypothesize that the planned activities for the different geolocations will provoke curiosity (Arnone, Small, Chauncey, & McKenna, 2011) and improve learning and interest in mathematics because:

- Visitors to these locations are likely to attend in a friendship or family group. In this sociocultural context, visitors become learners and learning takes place through social interaction with others in a sociocultural context in which they act and interact in shared experiences (Vygotsky, 1978);
- The locations, and their planned activities within them, are intriguing and this physical factor may influence how visitors feel about learning in this context. Learning outcomes can be a result of the ease with which the activity can be accomplished and how well it demonstrates scientific and mathematical concepts;
- Visitors are engaged in multiple ways e.g., physically, socially, emotionally and cognitively;
- Visitors have control over whether to engage in the activities or not;
- The mathematical ideas are experienced in an authentic and dynamic fashion.

In the following we further elaborate on the key dimensions of our study.

2 Theoretical perspectives

What is distinct in 'mobile learning'?

Mobile technologies, such as portable and handheld devices, with powerful social networking, communication and geo-location capabilities, have become ubiquitous worldwide and offer immense opportunities and new potential in education (Dhir, Gahwaji, & Nyman, 2013; Domingo & Gargante, 2016; Larkin & Calder, 2016). Mobile technology devices have become widely available, convenient and less expensive, with each successive generation being equipped with new features and sophisticated applications (Wu et al., 2012). The immense power of mobile technology is underlined by the fact that society and mobile technology interact with, and shape each other. Despite its ubiquitous nature, increased affordability and functionality, the integration of mobile technology devices in education is considerably limited and the effectiveness of mobile learning needs to be evidenced in a more systematic way.

To a large extent mobile learning or learning with mobile technology builds on the same foundations as that of technology-enhanced learning. It is not our intention to review the complete literature on the topic; we refer to the large body of literature available (e.g. Voogt & Knezek, 2008). We will focus specifically on the 'mobile' aspect, 'mobile learning' (m-learning), which differs from the broader technology topics by its ability to obtain and supply information at any time, resulting from its built-in wireless connectivity (Kukulska-Hulme & Traxler, 2005). However, it is problematic to conclude a concise definition of m-learning due to the ambiguity of the concept of m-learning itself (Kukuska-Hulme, 2009). As so often in technology-oriented literature it revolves around the question whether m-learning is about the mobility of the learning technology or the mobility of the learner him or herself? The same question is noted by Traxler (2009). To illustrate this with an extreme example, imagine a student brings his or her desktop computer outside, or a student places his or her mobile phone on the desk in a classroom and types in an essay. In other sources there seems to be a distinct emphasis on one, the other or both, without really concluding a clear definition of m-learning. Consequently, we focus on m-learning from the perspective of the mobility of the learner, which resonates with the views of O'Malley, Vavoula, Glew, Taylor, Sharples, and Lefrere (2005) suggesting that m-learning happens when the learner is not at a fixed, predetermined location and takes advantage of the learning opportunities offered by mobile technologies. Kukulska-Hulme and Traxler (2005) approached m-learning as learners' engagement in educational activities and communications with others via wireless technologies in mobile devices, without any specific location. Mobile learning takes place whenever and wherever the learners desire (Keengwe & Bhargava, 2014; Traxler, 2009). In addition, the affordances of mobile technologies offer to learners different levels of engagement and may provide inquiry-based learning activities inside the school, but also in out-of-school environments (Churchill & Churchill, 2008). What these perspectives have in common is that the learner is central and mobile.

In a review synthesis of 164 studies on m-learning from 2003 to 2010, Wu et al. (2012) revealed two major research strands. The first strand concerns the effectiveness of mobile learning and the second one the design of mobile learning systems. A significant number of studies revealed positive, neutral and negative findings regarding the effectiveness of mobile learning. From a methodological perspective, surveys and experiments were used as the primary research methods. As the review is already somewhat older, mobile phones and PDAs were the most widely used devices for mobile learning. The authors suggest that these findings might be displaced by emerging technologies, which has become evident in the case of tablet

computers. Crompton and Burke (2014) concluded similar findings to Wu et al (2012) from a mathematics specific review. They found that: (a) most of the studies focus on effectiveness, followed by learning design, (b) mobile phones were the most widely used device, and (c) the use of mobile devices for mathematics learning was most common in elementary (5-11 years old) school settings.
There is an ongoing need to examine the pedagogies that are suitable for mobile learning from the perspective of learners' needs and not based only on the affordances of the new technological features (Traxler, 2009). Mobile learning devices have been considered as a new type of computing platform that can be used to push beyond the restrictions of traditional pedagogies, provided they are designed and implemented in a way that takes into consideration the social and cultural context of learning (Crompton & Traxler, 2015).

What are the advantages of Augmented Reality (AR)?
Another development concerns the use of 'augmented reality' in informal learning environments. When reality is augmented, technology adds an additional 'layer' to reality. It combines real and virtual objects, has real-time interaction and three-dimensional affordances (Azuma, 1997). With AR devices users can actually see 3D objects, work with complex spatial problems and involve spatial relationships. In addition, AR technologies help learners engage in authentic exploration in the real world and conduct investigations of the real-world surroundings. As Sommerauer and Müller (2014) indicate, advances in mobile technologies (especially smartphones and tablets with built-in cameras, location options and internet access) have made augmented reality (AR) applications available for the broad public. Their pretest–posttest crossover field experiment with 101 participants at a mathematics exhibition aimed to measure the effect of AR on acquiring and retaining mathematical knowledge in an informal learning environment. The study was based on principles from the cognitive theory of multimedia learning (CTML), suggesting that people learn better from words and pictures than from words alone (Mayer, 2010). AR might, when designed correctly, address several design principles for effective multimedia instruction: firstly, the multimedia principle by overlaying pictorial content with text; secondly, the spatial and temporal contiguity principles by aligning virtual and physical information, for example, in three dimensions; thirdly, the modality principle by integrating auditory elements. Finally, the signaling principle could be obtained by highlighting essential information in a learning environment through cues, for example geographic location information and triggers (Sommerauer & Müller, 2014).

Multiple representations in task design
Building on the dynamic nature of mobile learning and the affordances of AR technologies, we can also position AR in relation to prior research on task design in mathematics education, with a particular emphasis on the potential role of multiple representations. This was a prominent focus within the technology chapter of the 22nd ICMI study on task design (Watson & Ohtani, 2015). This study argued that

often “abstract generalizations come about when critical aspects from multiple mathematical representations and discourses fuse and blend together” (p. 216). In addition, Whiteley and Mamolo (2013) used a framework of conceptual blending. It was found that teachers and students had multiple ways of reasoning about the task and created different conceptual blends for these representations. Earlier, Kaput (1986) had already argued that a multiple representational environment supported by technology might enhance high-level engagement with mathematics. So although AR might realize the potential of doing exactly that, it is important that the bridging and moving between tools and representations are key task design considerations. In moving between different representations we can also think about the distinction between a real situation, a real model as overlay on the real situation and an abstract mode, as per the modelling cycle by Blum and Leiss (2007). This cycle takes as starting point the ‘real situation’ from which a ‘situation’ or ‘real’ model is inferred. The process of ‘mathematising’ then transports it to a mathematical model, which is used to get mathematical results. Finally, the interpretation of these results leads to the final, real results, perhaps leading to an adjustment of the situation model.

Bridging formal and informal learning

Mobile learning has the potential to bring out-of-school contexts and problems into the classroom for learning mathematics, and take school mathematics into out-of-school contexts because mobile technologies have the ability to work within the specific context and environment of the learning (Khaddage, Muller, & Flintoff, 2016). The importance of informal learning has been stressed in research (Cox, 2013). Children can learn anywhere and anytime outside a formal learning environment resulting to an increased desire to continue interacting, playing and exploring from different perspectives. Informal learning is self-directed, has an intentional-interest, is non-assessment driven and spins-off mainly from leisure activities (Lai, Khaddage, & Knezek, 2013). Sawaya and Putnam (2015) suggested that this can be achieved by utilizing the affordances of mobile devices, such as computing input, consuming content, capturing surrounding context, communicating and collaborating with others and creating content. Thus, a suggestion could be to investigate in depth the ways in which mobile technologies can be used to bridge formal and informal mathematics learning (Lai, Khaddage, & Knezek, 2013; Wright & Parchoma, 2011). Along with Sawaya and Putmans’ (2015) framework regarding mobile devices affordances, Lai, Khaddage and Knezek (2013) described a Mobile-Blended Collaborative Learning model that only describes three categories of mobile application tools, namely tools for collaboration, tools for coordination and tools for communication. We suggest that a category, let’s call it ‘tools for augmentation’, given the affordances described previously, also might facilitate formal and informal learning, simply because they augment ‘reality’ (which we see as informal) with a virtual layer (which can be the formal content, for example, provided by curriculum content). In addition, it could be suggested that children’s out of school experiences might be utilized effectively to bridge the gap between home and school (primary and secondary) or home and university. Jay and Xolocotzin (2015), based on the results of an intervention program, asserted that there is enough content and motivation in children's out of school mathematics activities to be explored in ways that may help students' build their own mathematical structures. They suggest that this can be achieved by making connections between the abstract content of mathematics lessons and the multiple ways in which mathematical concepts are involved in out-of-school activities.

Students' mathematical learning processes and activities

Mobile learning could provide immense pedagogical benefits when mobile technologies are used as educational tools (Keengwe & Bhargava, 2014). Research findings suggested that mobile learning is associated with autonomous learning, students' active engagement and easy-access to information through internet resources (Spector, 2015). Mobile technology devices allow students to become contributors of knowledge and co-designers of activities by posing their own real-world scenarios and utilizing the affordances of the handheld devices, such as gathering measurement data, building structures, conducting virtual/augmented experiments or creating multimedia videos. In addition, such devices encourage pupils to take control of their own learning and manage their self-directed learning and individual development (Spector, 2015). Individual development refers to the enhancement of inquiry exploration and self-regulation strategies. The virtual and augmented affordances of the devices facilitate students move from passive-reproducers of information to content creators and thus the further development of reasoning skills, such as analysis, synthesis, evaluation, decision-making, modeling, explanation and problem solving. In addition, mobile learning encourages collaborative learning and promotes social interaction and collaborative feedback.

We contend that there are numerous potentialities for m-learning that can be explored in relation to the above themes. Here, we describe two case scenarios, one for using AR for mathematics involving the London Eye attraction, next to the river Thames in central London, the second situated at a cathedral in the ancient capital of England, Winchester. We describe the scenarios from the viewpoint of the learner and other actors around him/her. We hope to show that elements of aforementioned themes, namely mobile learning, augmented reality, a combination of informal and formal learning, and multiple representations, might come together in one m-learning experience.

3 Case one: The London Eye

Many cities in developed countries around the world boast an observation (or Ferris) wheel of some type that sits proudly on the landscape and inevitably captures the curiosity of onlookers. One such wheel is 'The London Eye', developed to mark the new millennium. It dominates the London skyline and, as the most popular paid visitor attraction in London, it attracts over 3.75 million visitors per year. Some mathematics educators have capitalized on it to create classroom-based resources to support both an introduction to mathematical concepts (Knights, 2014) or to consolidate/assess prior learning (Thomas & Gitonga, 2013). Central to both of these approaches was the prominence of the image of The London Eye, alongside the use of technology to support the further analysis of the mathematics represented by its physical features. This case example demonstrates how, by moving the learning outside of the classroom to the venue, and combining potential functionality from mobile technology such as smartphones, new mathematical activity can be proposed and, more importantly, experienced.

On approaching The London Eye on foot, by wheeled vehicle or by boat, its position on London's South Bank and the curvature of the river Thames make it inevitable that the Eye is seen from different angles. A (mathematical) question such as, 'Where does The London Eye look most like a circle?' is far from trivial as one considers the best place to stand for a circular view. Similarly, the other extreme, 'Where can you view The London Eye at its thinnest?' takes you to a place on the Golden Jubilee Bridge (West) (Figure 1), which runs alongside Hungerford Bridge.

Figure 1 Viewing The London Eye from the Golden Jubilee Bridge (West)

However, these static photographic images mask the most striking feature of this, and any other Ferris wheel – *it is moving at a constant speed of rotation*. In the case of The London Eye, it stops very occasionally to enable disabled visitors to embark and disembark from its 'capsules'. So, imagine that the observer, in our case a learner of lower secondary age, is standing with a friend or older family member at a marked location on the Golden Jubilee Brigade (or possibly, their mobile device has sent an alert to inform them that they are in an augmented reality mathematics space). As they look up at the Eye through the lens of their Smartphone, a mathematical question pops up to provoke their curiosity: 'Why do the capsules look like they are closer together at the top of The London Eye when compared to the middle?'

Again, a few moments of thinking time pass before our learner is asked whether she would like a hint - a prompt to touch one of the capsules on the smartphone screen, so as to mark its changing position over time. Simultaneously, a line segment that indicates this distance is displayed – augmenting reality (Figure 2, right side). The actual measurement can also be displayed.

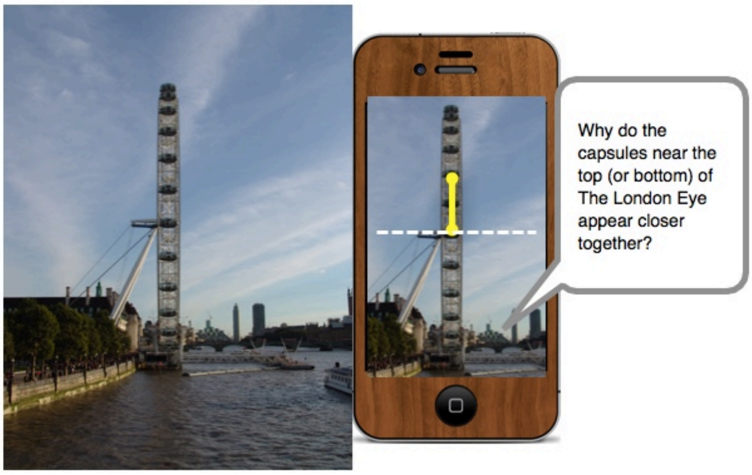

Figure 2 Augmented reality of The London Eye from the perspective of the Golden Jubilee Bridge (West)

Additionally, the sequence of data, the marked capsule position from the horizontal mid-line at fixed time intervals, is stored – and can be auto-displayed as either a table or a graph in response to the learners' own curiosity. Of course the same data can be collected and displayed whilst the learner is inside the capsule and experiencing The London Eye first-hand. An in-ride app, if it were to be designed, could be viewable on individual personal devices (or accessed via the many tablets provided in each capsule by the venue), could offer simultaneous screens showing the external views of the London Eye alongside the actual positional data of the individual capsules for the period of the ride. In this case, learners are prompted to make predictions in relation to the magnitude of, and relationships between, key data. By engaging learners with their personal experience of seeing how their own capsule's height varies in relation to those immediately adjacent to them and the ground below, their ride becomes a rich 2-D trigonometric experience as they experience for themselves the journey of a point on a trigonometric graph.

The transition from this early experience of (constant) circular motion as a model of height against time towards more formal trigonometric graphing could follow sequentially by going to stand at another AR mathematics spot that is facing The London Eye (Figure 3).

Figure 3 Viewing The London Eye from the Embankment

The same sequence of questions still applies, but the different perspective allows for alternative approaches that involve optional AR tools. Initially, to justify or explain that the upper and lower capsules indeed *are* close to each other (in the horizontal plane) than those nearest the mid-line, a still image could be augmented as in Figure 4.

Figure 4 An augmented view of The London Eye from the Embankment showing how the height of the capsules vary during the ride.

Working from the moving image, for which you (the reader) need to know that the London Eye moves counter-clockwise when viewed from this perspective, the learner is again invited to mark a capsule, which results in an AR experience whereby the moving image is annotated with a 'mid-line' and an angle measure that shows the marked capsule's position on the wheel – in this case as an angular measure relative to the 'three o'clock' position to fit with the usual mathematical convention (see Figure 5, right side).

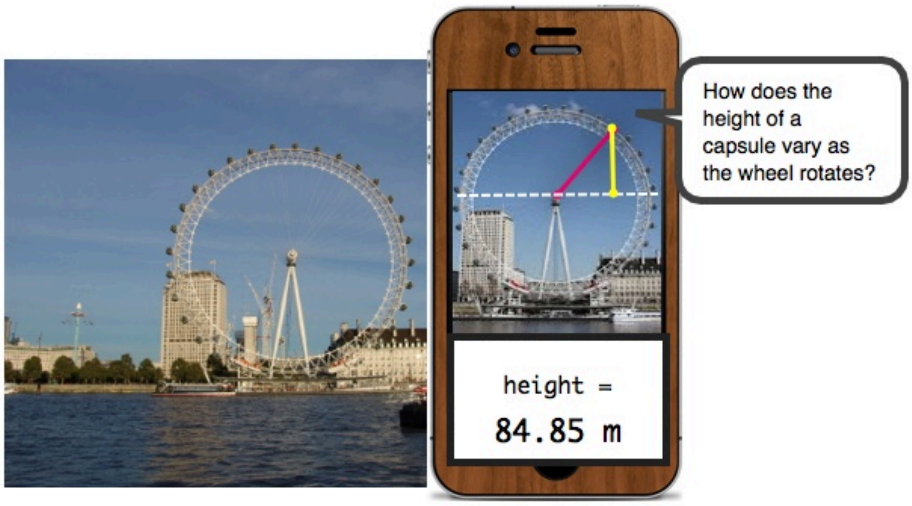

Figure 5 An augmented view of The London Eye from the Embankment – establishing reference points and highlighting changes in position. In the dynamic experience, the measured height would change as capsule moves during its journey.

Automated data collection from the image would then be collected and adjusted to generate a model for the capsule's motion over the journey. This could be made visible to the learner as measurement data that could be viewed and shared both in tabular form and graphically (see Figure 6).

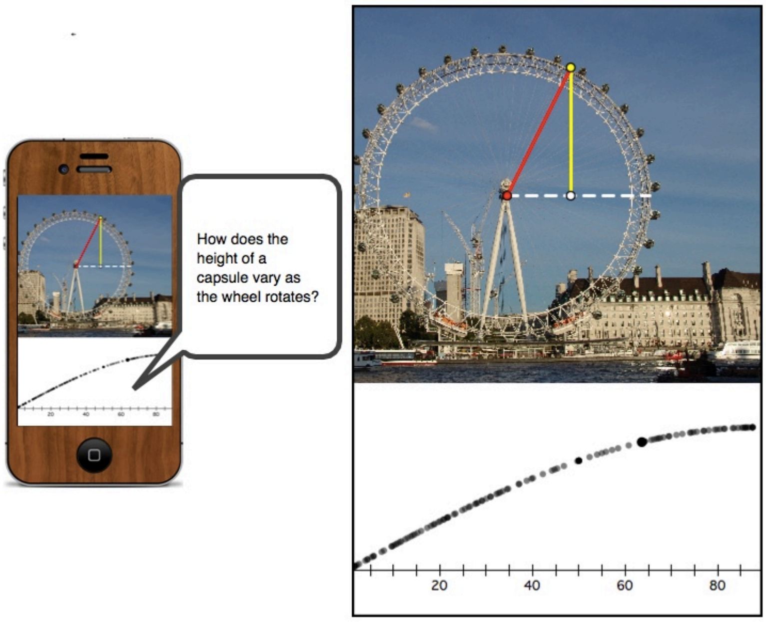

Figure 6 An augmented view of The London Eye from the Embankment – modeling the capsule's relative position during the ride graphically.

Any or all of our learners' explorations could be shared via social platforms – and of course hopefully with her teachers, who could use this real experience as the basis for more formal learning.

4 Case two: augmenting a cathedral

The second case example revolves around Winchester Cathedral, Hampshire, United Kingdom. Winchester used to be the ancient capital of England and its cathedral is one of the largest cathedrals in Europe, with the longest nave and greatest overall length of any Gothic cathedral in Europe. Upon arriving on the scene a student's mobile phone send an alert to indicate that the cathedral has some interactive features. The web-based app shows the student's geolocation and GPS coordinates and indicates that the cathedral is at the starting point of mathematical activities related to proportionality and ratios. The student can point his or her mobile device to the cathedral, after which the installed app recognizes the cathedral and provides some relevant information (see Figure 7). An interface is provided for some further information on the web and custom information for this specific augmented location.

Figure 7 Information of Winchester Cathedral is provided.

There is a feature to download some off-line resources such as task sheets, as part of a broader geo-located Augmented Reality package for the location. The package contains some classroom activities. Next to the information an icon also indicates that there are interactive AR activities for proportionality at this location. Clicking on the toolkit icon provides an additional ‘layer’ with some interactive features. In the case of the cathedral, a Dynamic Geometry System (DGS) can be used to calculate some proportions on the actual view (Figure 8).

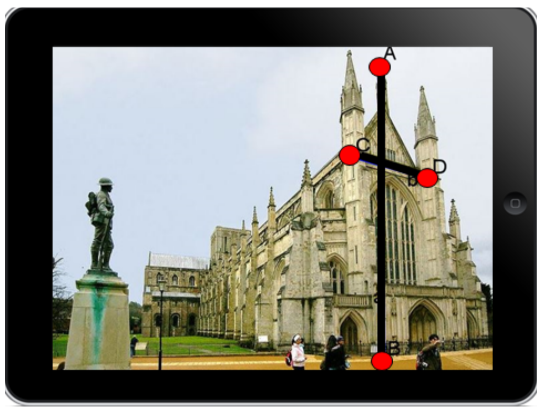

Figure 8 Dynamic Geometry is transposed on the view of Winchester Cathedral.

The platform also shows the lengths of the lines. An in-built clinometer can be used to determine the viewing angle. With some further tools, such as the geometry tool, a sketchpad and an aerial view of the area, the student can further model the situation, hinted by prompts and hints from the platform (Figure 9).

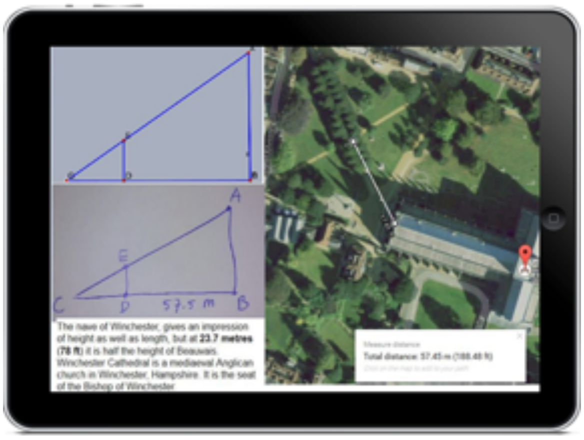

Figure 9 A geographical map of the surroundings of Winchester Cathedral are presented next to abstract diagrams of the situation. The top left is a geometric diagram, bottom left is a learner drawn diagram. Note that the letters in this diagram do not match those in Figure 8; they are, however, related as the vertical AB corresponds with the height of the cathedral.

The model the student has made is followed up by a quick pop quiz on the topic. An extension task, which the student can save for later also appears, emphasising connections between a (real) view of the cathedral in perspective and an abstract diagram, overlaying lines of the cathedral and a point on the horizon. Layers of the view can be turned on and off at will. It provides the student a means to go from reality ('real situation') to an abstract model. The work is shared and commented on via the interactive, social functions of the platform (Figure 10).

Figure 10 A mobile portal site provides information about the geo-location and social media functions.

The scenario in this example can also be extended to a classroom. Students are able to experience the majesty of the cathedral but virtually. The functionality of the tools makes it possible for the teacher to make use of the location-based resources in the classroom. By pointing the device at an image of the cathedral, it can serve as a 'trigger image' whereby the AR app presents a layer over the cathedral with the same functionalities as in the actual location. The location can be seen in a wider geographical map, disclosing that there are several other augmented spots in the area, for example at nearby Stonehenge. In addition to the same resources as the 'real' location, the classroom also provides some other features that are difficult to present on mobile screens. In addition to AR, there is scope to provide a Virtual Reality (VR) experience: using a mobile device to experience the grandeur of the real cathedral, with interactive features added in. The augmented cathedral has provided a way to address proportionality 'in real life' as well as to relate it to the abstract concepts.

5 Towards a theoretical lens for augmented mathematics

Based on a synthesis of the literature we argued that the integration of augmented reality in mathematics teaching might 'augment' learning for mathematics and contribute in developing students' reasoning skills (Spector, 2015). The above case examples made explicit the need to further investigate the role of several key dimensions, or *design decisions*. We propose that these design decisions can be grouped by three heuristics: observe, engage, and create.

Observe mathematics

We propose that the starting point should be the object of interest. Thus, the object of interest (the geo-location) should have interesting characteristics that can contribute in exploring salient mathematical concepts and properties. We should tap into learners' mathematical curiosity by making geo-locations the trigger: 'what are the mathematical questions that might come into the learners' head?' A pertinent question related to this is, who initiates this process? - the learner, the teacher or perhaps the technology. In the theoretical section we had made clear that we see the learner as leading, but acknowledge that teacher and technology could impose constraints on their initiatives. If we indeed take the learner as starting point, this reconceptualizes the 'any time, any place' assumptions of many perspectives of mobile learning, as the mathematical questions that are generated from a leisure activity are the guiding principles of one's self-directed learning process. Rather than focusing on the technology, we suggest that the focus should be on *reality* and *mathematics*. The locations where AR can be meaningfully applied is conditional on the inherent mathematics for any particular location. Luckily, mathematics is quite prevalent in most locations, whether they are *man-made*, like our two case examples or a natural phenomenon like a pattern from nature. If we make mathematics central to AR task design then in our view this also means linking the concrete reality to the mathematical abstract (and back again). AR can then serve as a tool to support the modelling cycle, by providing the mean to create, apply, adapt mathematical models during the processes of interpreting and explaining real-world based problems (e.g. see Blum & Leiss, 2007; Doerr & English, 2003). Real-world based problems might arise from two-types of geo-locations, namely dynamic and static. Dynamic geo-locations are related to situations from the perspective of the user's visual and kinesthetic experience, such as roller-coaster rides or an airplane's take-off or landing. In these types of geo-locations, AR functions as a composer of the viewers' and the experiencers' perspectives. In static-geolocations, such as historic buildings, monuments, bridges and natural spots, AR facilitates mainly the in-depth study of the spot, by providing measurements. For instance, an AR experience may provide data to explore the golden ratio of measurements associated with the Parthenon or to calculate the height of the Eiffel tower based on the measures from a 'selfie' picture.

Engage in mathematical content: development issues.

A second key decision, following from the mathematical content and context, concerns the appropriateness of using AR. Is it relevant, or deemed beneficial, to experience mathematics in the particular surroundings? If so, then a key factor in this experiential learning is pre-requisite mathematical knowledge and experiences. What prior mathematical knowledge or experience might be desirable? We acknowledge that this is a major prerequisite of what we should refer to as *experiential learning*. If a mathematical topic, according to the teacher or designer, is best learned without context and location-based experience, it might be difficult to make a case for AR. After all, one of the major advantages is that m-learning augmented by AR can make human experience of the surroundings, 'alive' and transpose abstract mathematical concepts on the outside world. Through this AR augmented experience that integrates the real world with abstract mathematic concepts, the learners might formulate and test hypotheses, solve problems and create explanations for what they observe (Bossé, Lee, Swinson, & Faulconer, 2010). We are in no way saying that every topic *should* be experiential. In fact, there are topics where context might impede the acquisition of more abstract mathematical knowledge. Nevertheless, it should be a key consideration while thinking about the adoption of AR. The existence of mathematics in a certain geographical location does not necessarily mean that the location is suitable for augmentation. The decision of augmenting should be made on well-explicit criteria, such as the whether the integration of real-world and digital-augmented learning resources has the potential to engage learners in manipulating virtual manipulatives and the underlying mathematic properties from a variety of perspectives.

Create: depth of experience

A third key decision pertains to the depth of the AR experience and the extent to which the learner might assume ownership of the mathematical activity and create, share and/or communicate their productions. By exploiting different layers of AR users can be engaged in the interesting mathematical features of the geo-located spot and concretely conceptualize the problem to be explored by inspecting the spot from a variety of different perspectives that facilitate their understanding. The different layers and perspectives provided by AR provides learners with data to elaborate their thinking, seek patterns, clarify concepts, synthesize ideas, pose their own questions, and create and own mathematical models. This can be achieved by working collaboratively through the social affordances of mobile technology devices. Users can also extend their understandings to new situations and make connections (connect the characteristics of the location with the collected data and the mathematical models). We suggest that both case examples showed this: the London Eye by linking the wheel to location data and a model of the wheel, the cathedral by linking locations to geometric constructions that could aid calculations of height. The whole scenario could be completed with a reflection regarding the underlying mathematical concepts related to each spot. These considerations all reduce to decisions about how much students can manipulate or interact with the environment, and, for example, whether the technological device responds back (feedback). Mobile technology devices can offer some form of validation and opportunities to further probing and development of students' mathematical thinking.

By imagining what mathematical content students need to observe, how they need to engage with the content and how they can create their own experience, quality AR tasks can be designed more readily.

6 Conclusion

In this chapter we have given an overview of how mobile learning and augmented reality might play a role in learning mathematics. After describing some relevant features of the issues involved in the study, we set out to describe two scenarios in which mobile learning, augmented reality, a combination of informal and formal learning, and multiple representations, came together. We concluded with three core aspects that need to be taken into account when designing such tasks. Firstly, it is important to reflect on the importance of the involved mathematics concepts and more importantly on how the integration of AR and the geo-location can trigger mathematical curiosity. Secondly, how appropriate it is to apply experiential learning to the topic at hand and to what extent the mathematical prerequisites of the activity meets learners' knowledge or experience. Finally, the depth of the learning experience depends on the technical functionalities of the software, and therefore the envisaged technical tool needs to be taken into account. This has less to do with technology per se but more with the learning opportunities that can be offered by the affordances of the technology, and the learning design of the tasks. The above mentioned core aspects provide designers important design parameters that should take into account regarding what students need to observe, need to do to get engaged, and what they need to create. The scenarios presented here are practical examples of its application.

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