Diverse cities or the systematic paradox of Urban Scaling Laws

Clémentine Cottineau [a,*], Erez Hatna [b,a], Elsa Arcaute [a], Michael Batty [a]

[a] Centre for Advanced Spatial Analysis, University College London, UK
[b] Center for Advanced Modeling in the Social, Behavioral and Health Sciences, Johns Hopkins University, USA

ARTICLE INFO

ABSTRACT

Scaling laws are powerful summaries of the variations of urban attributes with city size. However, the validity of their universal meaning for cities is hampered by the observation that different scaling regimes can be encountered for the same territory, time and attribute, depending on the criteria used to delineate cities. The aim of this paper is to present new insights concerning this variation, coupled with a sensitivity analysis of urban scaling in France, for several socio-economic and infrastructural attributes from data collected exhaustively at the local level. The sensitivity analysis considers different aggregations of local units for which data are given by the Population Census. We produce a large variety of definitions of cities (approximatively 5000) by aggregating local Census units corresponding to the systematic combination of three definitional criteria: density, commuting flows and population cutoffs. We then measure the magnitude of scaling estimations and their sensitivity to city definitions for several urban indicators, showing for example that simple population cutoffs impact dramatically on the results obtained for a given system and attribute. Variations are interpreted with respect to the meaning of the attributes (socio-economic descriptors as well as infrastructure) and the urban definitions used (understood as the combination of the three criteria). Because of the Modifiable Areal Unit Problem (MAUP) and of the heterogeneous morphologies and social landscapes in the cities' internal space, scaling estimations are subject to large variations, distorting many of the conclusions on which generative models are based. We conclude that examining scaling variations might be an opportunity to understand better the inner composition of cities with regard to their size, i.e. to link the scales of the city-system with the system of cities.

© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

1. Introduction

At the age of big data, working with decennial Census data may seem out-dated. Shouldn't we use the profusion of new data sources and the capacity of computation newly available to produce new research and solve new questions? This debate is on-going, unresolved and potentially irrelevant. First because there might be complementary research to be done at the intersection of small and big data Batty (2015). Second, because today's (Census) small data are also yesterday's big data[1] and so there might be no radical shift in paradigm involved Barnes and Wilson (2014), Taylor, Schroeder, and Meyer (2014). Third, because one could very well admit that cutting-edge research is not a direct function of cutting-edge data, and that the quality of the questions asked and the adequacy of data used to answer them is the important subject – so that Census data can still be the relevant data for some contemporary research design. Our final point is that, in the same way that urban data are big with interactions Batty (2015), Census data can become "big" for combinatorial reasons.

Indeed, because Census data systems – and the geographies at which the information is collected – are a legacy of the past and because they take a long time to adapt to the moving socioeconomic geographies, there are few cases in which Census data are readily usable for spatial analysis at the scale of interest. Aggregations of local areal units are the rule rather than the exception, especially in the field of urban studies. However, in order to preserve the social, economic, and spatial patterns of the data and match meaningful definitions of cities, no single aggregation is optimal, and we propose as an alternative to build systematic aggregations for which we explore the outcomes with respect to the combination of definitional parameter values. The choice of one of the multiple possible aggregations determines the spatial extents of the cities considered, the measurement of their population size, and most probably the way we observe the urban system's response to size West (2014). The systemic property related to size is known as scaling and is used to study the quantitative variation of cities' characteristics (for instance the number of people of a certain economic category, or the quantity of a certain infrastructure) with respect to their size (population for example). The exhaustivity of Census data clearly is a

---

[1] "The Population census was in fact one of the only systematic catalogues of data produced on a continuing basis until national accounts and related economic data began to be collected seriously and routinely in the 1920s (Bos, 2011). But right from the start, data were always big with respect to the available means by which it could be manipulated" [Batty (2015), p. 2]

---

strong advantage for relating one variable to another, and the combinatorial increase of possible representations of the system does challenge computing, analytic and visualisation capacities.

Urban scaling laws have fostered urban researchers' interest over the last decade because they provide powerful summaries of the variations of urban attributes with city size.[2] Indeed, when considering the variation of an absolute urban quantity Y against total population P in a city i, there is almost always a covariation between the two Shalizi (2001), frequently in the mathematical form of a power law

$$Y_i = a * P_i^{\beta}$$

where a represents a time dependent normalisation constant, and β the scaling exponent under enquiry. Superlinear relationships (i.e.: $\beta > 1$) indicate positive returns to scale, i.e. larger amounts of Y per capita in larger cities; whereas sublinearity ($\beta < 1$) is associated with economies of scale, i.e. smaller amounts of Y per capita in larger cities. Linear scaling ($\beta \approx 1$) means that the quantity per capita is constant across city size. Scaling exponents β estimated from empirical data have been interpreted as static or evolutionary properties, respectively by Bettencourt, Lobo, and Strumsky (2007) and Pumain, Paulus, Vacchiani-Marcuzzo, and Lobo (2006). Bettencourt (2012) developed models of network interactions predicting an exponent of 5/6 for infrastructural variables and of 7/6 for socioeconomic variables. However, even though most estimations lay in a range commensurable with these values, they are subject to variations, as attested by the meta-analysis of estimates for CO_2 emissions by Rybski et al. (2013) or the sensitivity analysis of a large pool of variables with city definitions by Arcaute et al. (2015). These two studies question the validity of a universal interpretation.

For example, despite the existence of theoretical models to predict the value of urban scaling from local interactions Bettencourt (2012), Lobo, Bettencourt, Strumsky, and West (2013), Ortman, Cabaniss, Sturm, and Bettencourt (2014), an easy way to argue against the universality of scaling exponent values is to look at their variation with city definition Fragkias, Lobo, Strumsky, and Seto (2013), Rybski et al. (2013), Arcaute et al. (2015). For instance, in France, there are two definitions of cities defined by the statistical office INSEE (cf. Table 2 and Fig. A1 in Appendix A):

- Urban Units or *Unités Urbaines* (UU), which correspond to the aggregation of local units (*communes*) sharing a continuous built-up area of less than 200 m between buildings, and
- Metropolitan areas or *Aires Urbaines* (AU), defined as the aggregation of a central Urban Unit and all the *communes* with more than 40% of active commuters to the centre.

Comparing scaling results from those two official definitions, we find not only marginal discrepancies between expected values and estimated exponents, but evidence of different scaling regimes when we compare morphological and functional city delineations (Table 1) with similar goodness of fits (i.e. quite low for manufacturing jobs and relatively high for the other attributes). In one case, say employment in the manufacturing sector, the number of jobs grows more than proportionally with the population of density-defined Urban Units, whereas the number of such jobs per capita decreases with the size of functionally-defined Metropolitan Areas. The paradox obtained from the comparison of city definitions can question the very motivation for using urban scaling and its empirical analysis. However, even though

Table 1
Scaling exponents for two city definitions in France.

Urban Attribute	City Definition	β	CI* (95%)	R^2	N
Manufacturing	UU	1.175	[1.13; 1.22]	0.543	2226
	AU	0.849	[0.81; 0.89]	0.691	771
Vacant Dwellings	UU	1.051	[1.03; 1.07]	0.797	2233
	AU	0.902	[0.88; 0.92]	0.928	771
Basic Services	UU	1.086	[1.07; 1.10]	0.892	2233
	AU	0.956	[0.94; 0.97]	0.965	771
Education	UU	1.215	[1.19; 1.24]	0.778	2230
	AU	0.981	[0.96; 1.00]	0.922	771

Source of the data: French Census, 2011. UU: density-based Urban Units. AU: functionally defined Urban Areas. *N*: Number of cities in the regression. *CI: confidence interval.

there seems to be no point in trying to fit absolute scaling parameters, the variations in scaling estimation are of theoretical interest because of what they say about the relation between intra-urban spaces (micro-scale), city definitions (meso-scale) and urban scaling (macro-scale).

Indeed, we suggest that the variations in scaling estimations between dense cities definitions and metropolitan areas are not a failure of a robustness test, but the expression of the different nature of urban spaces implied by the two definitions: the former describes the population within a dense environment of social interactions and infrastructural elements; the latter refers to a much larger functional space of economic interactions. Both can be called cities but they are not equivalent. For example, if one was interested in modelling the development of industry locations, one would consider different strategies in the central and suburban parts of the city, because of differentiated opportunities to locate certain types of buildings, because of housing rent gradients or because of the different urban atmospheres available in the different parts of the city. Therefore, where the boundary is set to observe cities with respect to scaling is of crucial importance, because it defines the level of morphological and socioeconomic diversity included in the concept of city under enquiry. The boundary concept applies to the spatial extent as well as to the minimum population required to call a population aggregate urban: there might be differences of nature (and quality) between small towns and large metropolises with respect to certain indicators.

An additional motivation to explore multiple city definitions comes from the fact that official definitions rely on the choice of unique thresholds (e.g. distance between buildings, the percentage of commuters or a minimum population). Those have proven useful to describe urbanisation over time, but their precise value contains a share of arbitrariness that we want to evaluate in order to strengthen or question conclusions based on these definitions. Finally, varying definitional criteria will eventually produce a picture of scaling estimates that lies in between the two official definitions for France and this will help us understand better the discrepancies observed empirically, as well as to compare studies performed on a large number of cities with studies that look at the upper part of the urban hierarchy only.

In this paper, we analyse the observed transitions from one scaling regime to another when varying city delineations. We do so by generating a whole range of city delineations; in other words, by aggregating local Census units in multiple ways following the systematic variation of definitional parameter values (Section 2). We analyse the variation of urban scaling estimates with respect to the parameter values used to delineate such cities, and argue that variations are not random (Section 3.1). Instead, they can inform our knowledge of cities and of the different areas they are composed of. We suggest a way to describe these discrepancies and provide potential explanations (Section 3.2). Section 4 concludes by stressing the importance of using urban scaling along with complementary explanations of the genesis of city systems (regional integration, path-dependent processes, etc.) to better understand the socioeconomic and morphological complexity of cities and systems of cities.

---

[2] Although some authors focus on intra-urban scaling (by investigating the fractal distribution of transportation networks or the scaling of the height of buildings within a city Longley and Mesev (2002), Kim, Benguigui, and Marinov (2003), Carvalho and Penn (2004), Batty et al. (2008), Niedzielski, Horner, and Xiao (2013), Masucci et al. (2015), our interest here lies at the inter-urban scale only. We only consider the variation of an aggregated quantity with city population at the scale of a country or region.

2. A multiple representation of the French system of cities

"*The extent of the city is important in a number of respects, not least in relation to the question of city size, an issue of considerable significance in urban and regional analysis*" Parr (2007), p.381. Since our analysis relates to the variation of attributes with city size, knowing the sensitivity of this variation with respect to the spatial criteria for urban delineation is an important aspect. Beyond sensitivity, the different parts of the city (centre and periphery for example) are not expected to behave similarly vis-a-vis economic and infrastructural patterns. Indeed, they are composed of very different populations, built environments and lifestyles, hence representing an internal diversity of physical and social landscapes within the city Guilluy (2013). Any complex relation between the morphological landscape and the socioeconomic landscape might affect the effect of size. Thus inclusion of some or all of these urban components into the urban delineation might affect our estimation of scaling laws. This argument is usually left out of the scope of predictive scaling theories where the city is considered as a given, but more importantly a homogeneous object. We present three criteria used to delineate cities in France (Section 2.1), and the resulting urban clusters generated from combining them in a systematic way (Section 2.2), before comparing results with classical definitions (Section 2.3).

2.1. Systematic criteria for defining cities

Our systematic delineation of cities adapted from Arcaute et al. (2015) follows a similar strategy to the one used in the official identification of cities by the French Census Bureau (INSEE), that is: identifying a centre based on a density criteria to aggregate local units, delineating the periphery functionally associated with that centre, based on travel-to-work patterns of each local unit, and eventually applying a minimum population cutoff. The dense centres are called Urban Units (UU), and group together communes sharing the same built-up area with a maximum distance of 200 m between buildings. The metropolitan areas (AU) correspond to the aggregation of an UU concentrating more than 1500 jobs and the communes from which more than 40% of the working population work in the UU. In order to perform a sensitivity analysis of urban scaling, we consider a variety of values for each criterion:

- Urban density is associated with historical and morphological centres (or "core cities"), and characterises the part of a city in which the concentration of interactions and economic activity is maximised Parr (2007). Densities can be expressed in relation to the number of buildings or persons per unit of surface. We choose the latter option here, and let the minimum number of residents per hectare define dense cities varying from 1 (loose centres) to 20 (very dense city cores). Population densities of official UUs are distributed within this range, with the median half of cities in the interval [0.9; 3.2] and the mean equal to 5.
- Over the last two or three decades though, density alone has failed to reflect the spatial extent of urban labour and housing markets. Therefore, it is common to take into account functional indicators of urban activities taking place beyond dense city cores Parr (2007), Guérois and Paulus (2002). To define such urban aggregates, researchers usually consider commuting patterns to the centres. The criterion for aggregation can refer to the share of income earned in the city core or the share of active residents commuting to the city centre. We choose the latter, and explore the variation of urban extent when this proportion varies from 0 (where a single commuter is sufficient to attach a commune to the urban centre) to 100% (where the functional city basically corresponds to the dense city).
- Finally, the population minimum plays a major role in the definition of cities, and affects many urban measures (most notably Zipf's exponent, cf. Malecki (1980), Guérin (1995), Cristelli, Batty, and Pietronero (2012). This parameter reveals the conceptual trade-off between acknowledging that cities appear above a critical mass of population and using the larger scope of city sizes in the study of systems of cities

(when we use scaling measures). We present results obtained with population cutoffs from 0 (all clusters are considered) to 50,000 inhabitants.

2.2. Resulting "urban" clusters

The clustering algorithms developed and described in Arcaute et al. (2015) first group contiguous local units that each exceed the population density criterion C. A commune is then linked to the core that attracts its largest percentage of commuters if this percentage is above the flow cutoff F. By combining multiple values of C, F and a population minimum P, this process leads us to consider thousands of representations of the system of cities in France.

In the following sensitivity analysis, we consider 4914 such representations based on the combinations of 39 density values (D from 1 to 20 residents per ha in steps of 0.5), 21 commuting cutoffs (F from 0 to 100% in steps of 5) and 6 population cutoffs (P from 0 to 50,000 inhabitants in steps of 10,000). Thirty different combinations are shown in Fig. 1.

2.3. Matching with official definitions

To evaluate the quality of our clusters and their ability to describe the transition between official definitions, we measure the correspondence between each cluster definition and three classical urban delineations: UU in 2010, AU in 2010, and Urbanised Land Use Areas given by CORINE LandCover 2006 raster data (cf. Table 2 and Fig. A1 in Appendix A). This measure consists of the correlation between the urbanised local units in the official and the cluster definitions (1 meaning that each "urban" local unit by official standards belongs to one of the systematic clusters for a given definitional combination). Given the methods used by the Census, we expect definitions to differ on the commuting criteria (F), which should be close to 100% for the UU equivalent, and close to 40% for the AU equivalent, with similar density cutoffs. (See Table 2.)

Indeed, we find a good match between UUs and clusters defined as having a density over 1.5 residents per ha, almost no commuting flows ($F = 100\%$) and no population cutoff (Table 2 and Fig. A1, top left). The correlation coefficient between belonging to such a cluster and belonging to an UU for each commune is 0.66 ($R2 = 44\%$). For AUs, we find the following parameter values for definitional criteria: a density cutoff of 2.0 (close to that of UUs), a commuting cutoff of 35% (close to the official 40%) and a larger population cutoff (10,000 people). However, the match is of weaker quality ($R2 = 0.134$), probably because of the different way in which we and the Census handle local units integrated to multiple centres (they are not attached to a core city in the Census method, and are attached to one of the centre in our method). When computing the correlation coefficient between the belonging to a cluster and the % of land classified urban in the CORINE images for each commune, we find a match of better quality ($R2 = 0.58$) with clusters defined with $D = 4.5$ persons per ha, $F = 100\%$ and no population cutoff.

3. Understanding the variations of scaling behaviours

Building on an almost continuous representation of systems of cities, we are able to estimate scaling laws and their sensitivity to the variation of the urban definition criteria. The variables used to describe urban attributes in the following analysis have been collected at the local level ($\approx 36,000$ local units in continental France). Socioeconomic and travel-to-work data come from the last population Census and surveys in 2011, whereas land-use data are extracted from CORINE LandCover 2006 raster data, road length derive from an Open Street Map dataset computed by C. Quest in 2014, and housing permits come from governmental open data. A detailed description is available in Appendix A (section A.2).
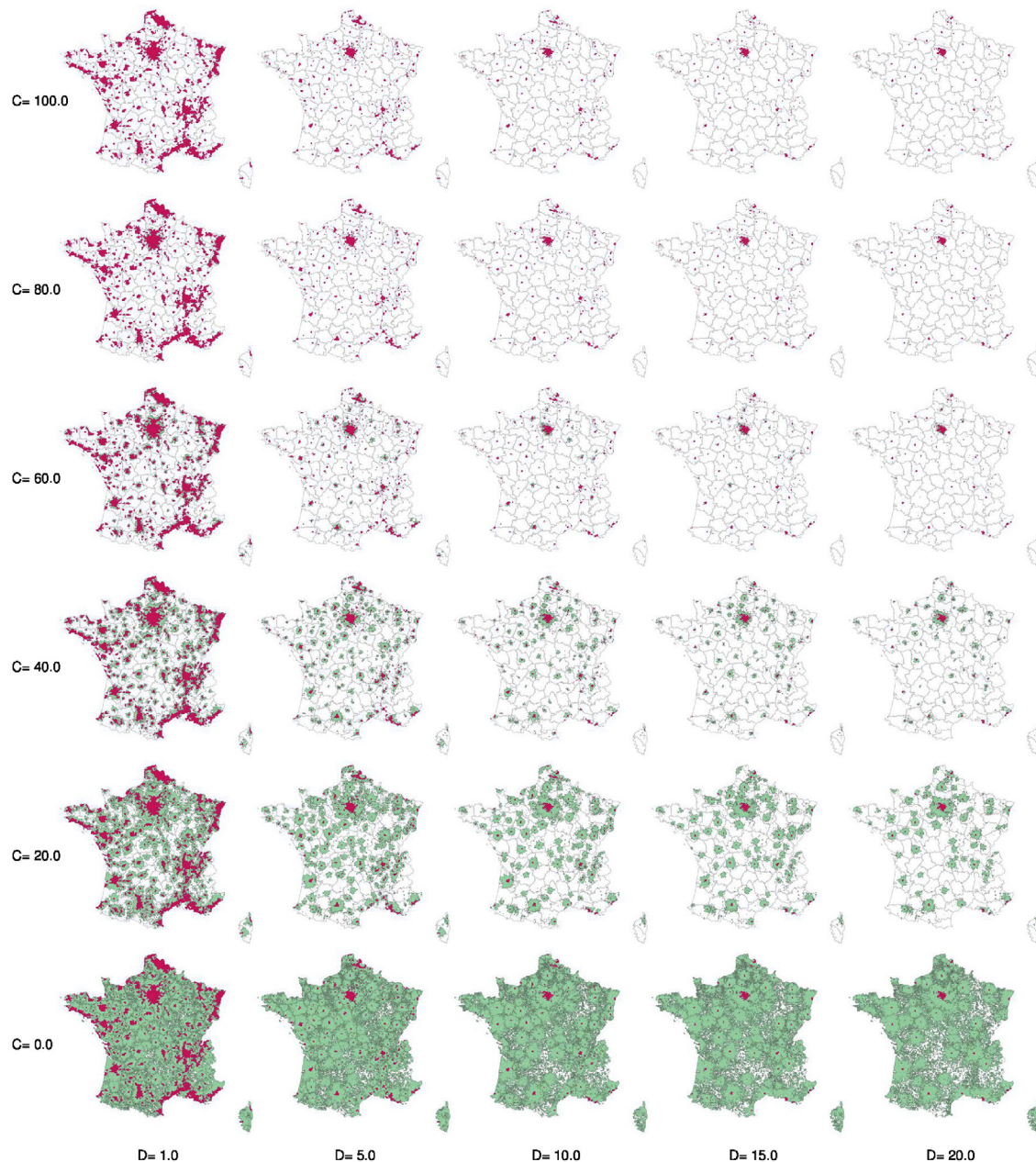
D is the minimum density of residents per ha which defines urban centres (in red). C indicates the minimum share of Commuters (in %) living in the periphery and working in the density-based urban clusters (in green). P, the population minimum, is here set to 0. Aggregation is performed using the 2013 GeoFla geometry of communes. `https://www.data.gouv.fr/fr/datasets/geofla-communes/`

Fig. 1. A wide range of representation of systems of cities in France (P = 0).

3.1. Variations of scaling behaviours

Fig. 2 represents the distribution of the estimated β for the 4914 representations of the French urban system. We used a kernel density estimation[3] to ease visual comparisons across attributes and cluster definitions. This analysis of density estimations can be summarised by two findings.

First, we find substantial variations in the scaling exponents measured for the different representations of the system of cities. The maximum intervals of estimated exponents often range from sub-linear ($\beta < 1$) to superlinear regimes ($\beta > 1$). For example, the area of urban clusters (Fig. 2, top left) scales from sublinearity ($\beta = 0.33$) to superlinearity ($\beta = 1.29$) with population. The length of paths and roads is the second most volatile variable with respect to urban scaling, as the exponents estimated range from $\beta = 0.66$ to $\beta = 1.20$. These results are not so surprising as the two variables are physical and therefore highly dependent on
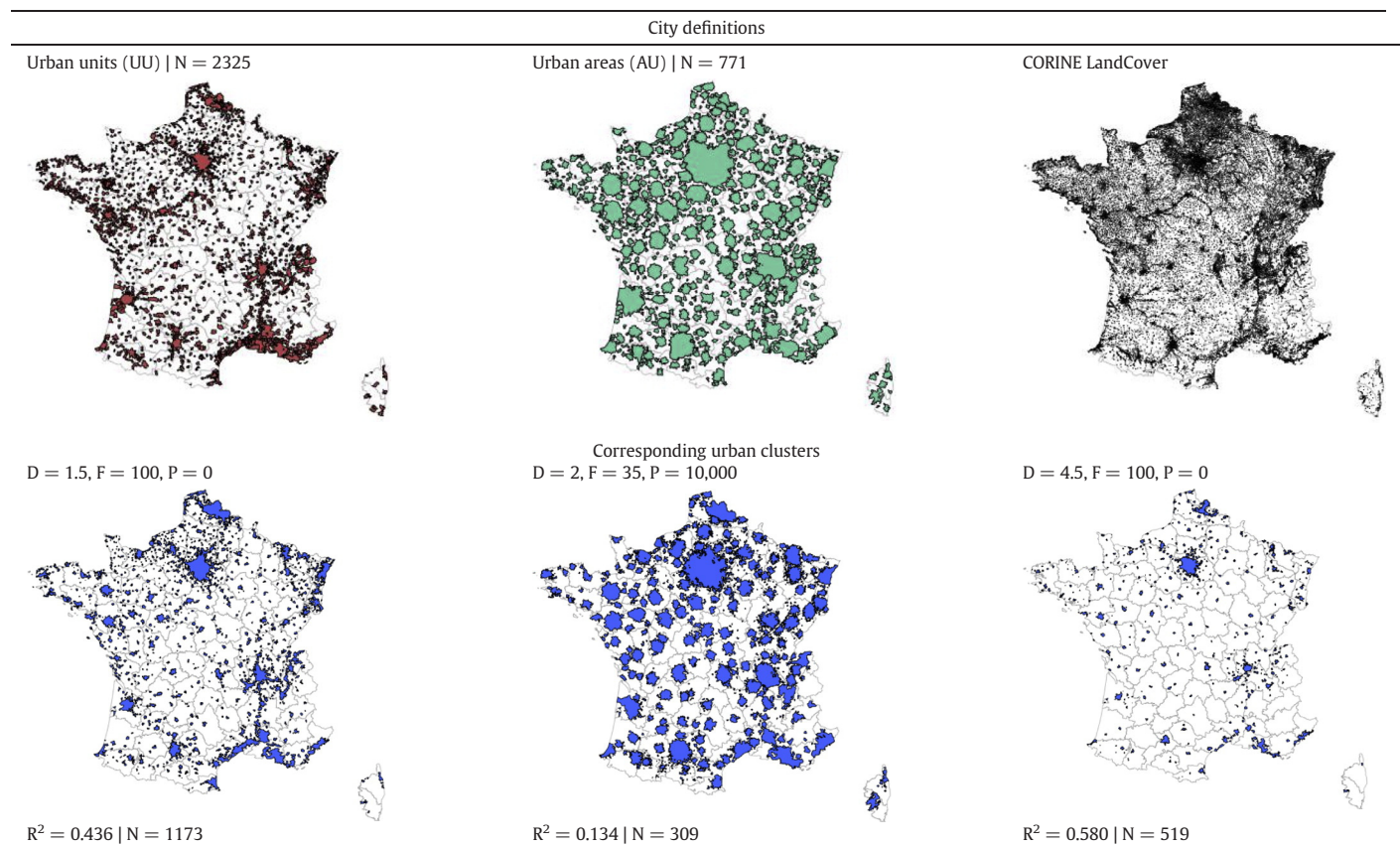
---

[3] http://www.inside-r.org/r-doc/stats/density. We prefer this representation to histograms in this case because it allows us not to fix any bin number or size, which would have taken different values for each distribution. The kernel procedure, on the contrary, makes the representation of the distributions continuous and comparable.

Table 2
Three depictions of city definitions (top) and their corresponding urban clusters (bottom). R2 is the level of correspondence between each pair and N the number of cities.

City definitions		
Urban units (UU) | N = 2325	Urban areas (AU) | N = 771	CORINE LandCover

Corresponding urban clusters		
D = 1.5, F = 100, P = 0	D = 2, F = 35, P = 10,000	D = 4.5, F = 100, P = 0

$R^2 = 0.436$ | N = 1173	$R^2 = 0.134$ | N = 309	$R^2 = 0.580$ | N = 519

Source: INSEE: www.insee.fr, CORINE LandCover 2006 raster data (Categories 111 and 112).
http://www.statistiques.developpement-durable.gouv.fr/donnees-ligne/li/1825.html

the spatial extent of the cities. However, many other variables (such as the number of jobs in the education or the manufacturing sectors, the number of hospitals, etc.) are also affected by the choice of urban criteria. To summarise, with the exception of the number of dwellings which clearly scales linearly with population whatever the delineation ($0.95 < \beta < 1.03$), all the urban attributes considered in the study range across two or more scaling regimes and cannot be described in a definitive way by a single value.

The second finding therefore relates to the different magnitudes of variations observed for the different variables under study. If only the number of dwellings is stable over the complete range of city definitions, some attributes appear stable in relation to population for a majority of definitional criteria combinations (in terms of the scaling regime rather than the specific value of the exponent). For instance, the number of households owning a car ($\beta \in [0.94; 1.03]$[4]) and jobs in proximity services ($\beta \in [0.95; 1.07]$) scale linearly with population in most cases. Similarly, the number of hospitals and of persons employed as 'labourers'[5] together with the urbanised area scale sublinearly in a majority of representations of the system of cities ($\beta \in [0.63; 1.02], [0.87; 1.01]$ & $[0.81; 1.05]$ respectively). The number of people employed as managers and professionals, the jobs in finance or in research are symmetrically mostly superlinear urban attributes ($\beta \in [1.02; 1.27], [0.98; 1.21]$ & $[0.95; 1.5]$ respectively). Such behaviours are consistent with results obtained with ordinary representations of systems of cities Pumain et al. (2006), Bettencourt et al.

(2007). However, given particular criteria for urban definition, the behaviour of these attributes with city size would change to the opposite scaling regime. Finally, bimodal distributions are clearly observed in Fig. 2, where estimated scaling values are distributed on both sides of the linear value $\beta = 1$: e.g. the number of jobs in manufacture ($[0.83; 1.25]$), in health and social services ($[0.9; 1.17]$) and in education ($[0.92; 1.14]$). These features suggest that there might be two sets of urban definitional criteria generating two scaling regimes. This observation calls to investigate further the determinants of variations in urban scaling.

3.2. What makes urban scaling vary?

In this section, we look for systematic variations of β with the definitional criteria, and ways to interpret and explain them. We proceed in four steps:

1. Building a typology of cluster definitions and compare their distributions of β (Section 3.2.1)
2. Using heatmap representations to visualise and compare variations of urban scaling with definitional criteria for the different attributes, producing representations similar to a phase diagram (Section 3.2.2)
3. Grouping the most similar heatmaps with hierarchical clustering (Section 3.2.3)
4. Comparing extreme estimations with extreme observations from the literature (Section 3.2.4).

3.2.1. Clusters typology

Our first approach is to differentiate urban clusters based on their spatial extent and population cutoff. In particular, we want to check

---

[4] N.B. this interval corresponds to the magnitude of estimated βs across definitions, not to a confidence interval.

[5] A category of occupation inherited from the labouring class working in plants but not restricted to this only nowadays.
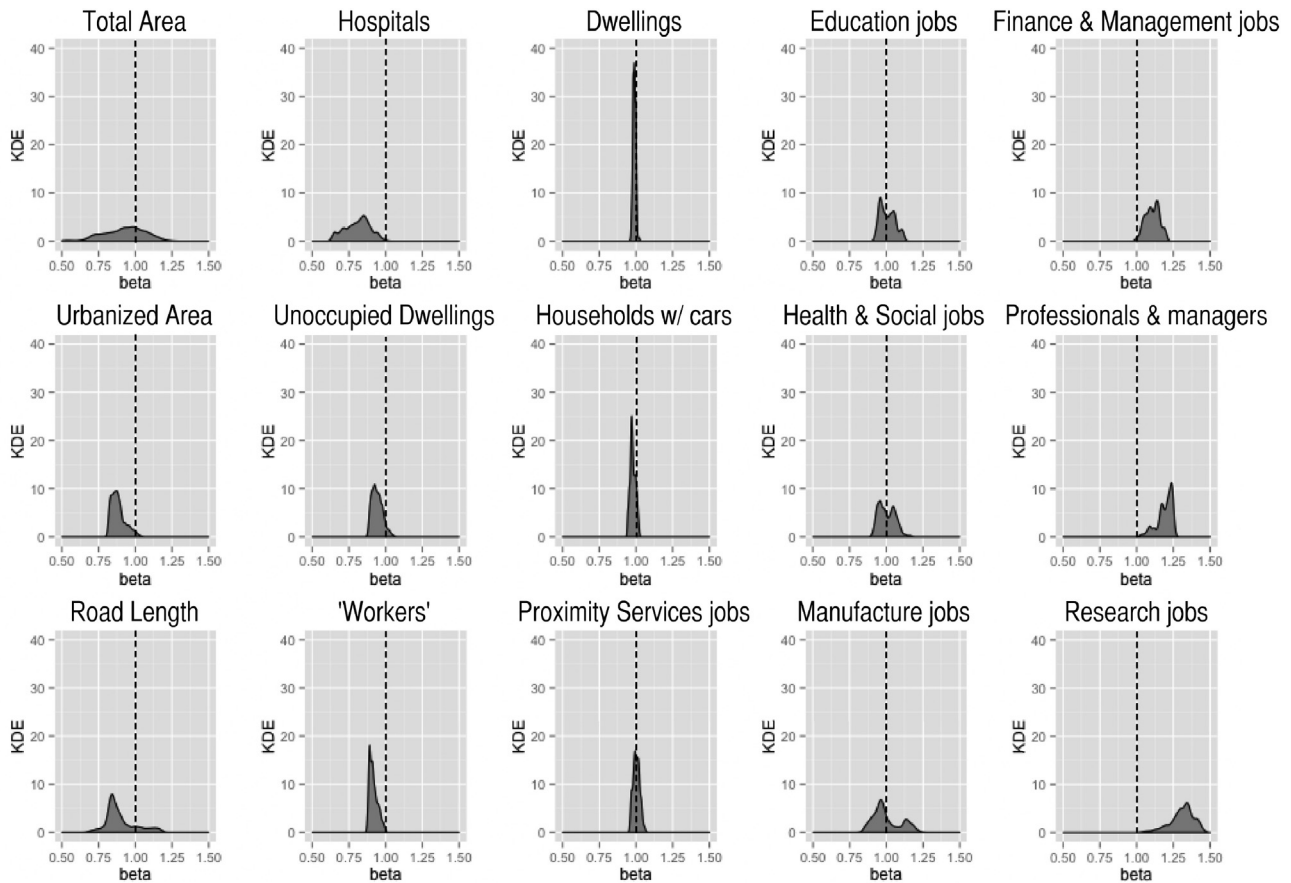
Kernel Density Estimation (KDE) of scaling exponents of 15 urban attributes from 4914 systems of clusters-cities resulting from the combination of three definition criteria. N.B. : the number of clusters in each regression varies, from 1298 to 48, depending on the restrictive character of the definition criteria

Fig. 2. The distribution of scaling exponents for selected attributes over the entire set of city definitions.

if there is a direct relation between the type of cities delineated and the kind of scaling regime they lead to (for bimodal distributions typically).

We first dichotomise our set of urban realisations according to the way cities are defined spatially, using density and commuting cutoffs. The first subset represents urban delineations which are close to existing definitions of cities in France. They are defined as centres with population densities ranging from 1 to 5 residents per ha (57% of UUs belong to this interval) and peripheries made of local units from which 35% to 100% of the active population commutes to the dense centres (the proportion is 40% for the definition of UAs, 35% for their cluster equivalent and virtually 100% for UUs). By contrast, Alternative clusters represent all other combinations of density and commuting parameter values at a given population cutoff (cf. Figs. 1 and A1). Alternative cluster definitions deviate from the official thresholds used to define cities, yet they provide valid representations of metropolitan regions (for commuting cutoffs lower than 35%) or historical centres (for density cutoffs higher than 5 resident per hectare). Population cutoffs do not affect the spatial extension of cities, but drastically change the number of cities considered (the higher the cutoff, the lower their number). For example, with the same criteria of $D = 1.5$ & $F = 100\%$, there are 1172 urban clusters in France, but less than a third of them have more than 10,000 residents (300), and only 98 meet the $P > 50,000$ definition (for $D = 2$ and $F = 30$, the actual numbers are respectively 991, 341 & 138). Therefore, among common and alternative clusters, we distinguish two subsets of definitions: the ones with a population cutoff and the ones without (Table 3).

We find that this first categorisation eases the interpretation of the variations of the scaling exponents, especially in the case of bimodal distributions (Fig. 3). For example, the two modes in the distribution of the scaling exponents for jobs in the manufacturing sector correspond to two types of definition: one with a population cutoff and one without (independently from the spatial extent of cities, common or alternative). In other words, one finds that manufacturing jobs scale superlinearly with city population when all cities are considered, especially very small aggregates. On the contrary, when one sets a minimal size for cities to be considered, they scale sublinearly to linearly with population (fourth column, Fig. 3). This result tells us that manufacturing is neither a specialisation of small nor large cities: the bimodal distribution and the transition in scaling regimes could instead indicate either the importance of medium size cities in the concentration of manufacturing jobs or the high threshold in population for economies of scale to appear. The same pattern holds for the education and the health and social sectors, even though the picture is less clear-cut. In

Table 3
The number of urban realisations within each typological group.

Definitions	Population cutoff	No cutoff	Total
Common clusters	630	126	756
Alternative clusters	3465	693	4158
Total	4095	819	4914

Common clusters: $1 \leq D \leq 5$ & $35 \leq F \leq 100$. Alternative clusters: the rest of the clusters. Population cutoffs: 10,000 or 20,000 or 30,000 or 40,000 or 50,000.
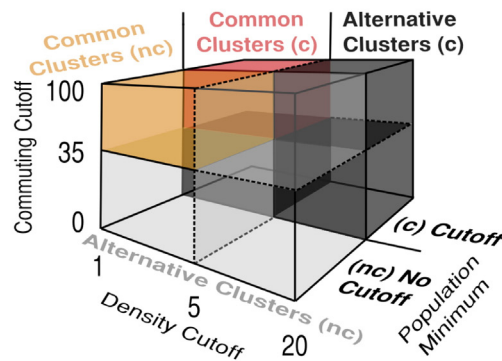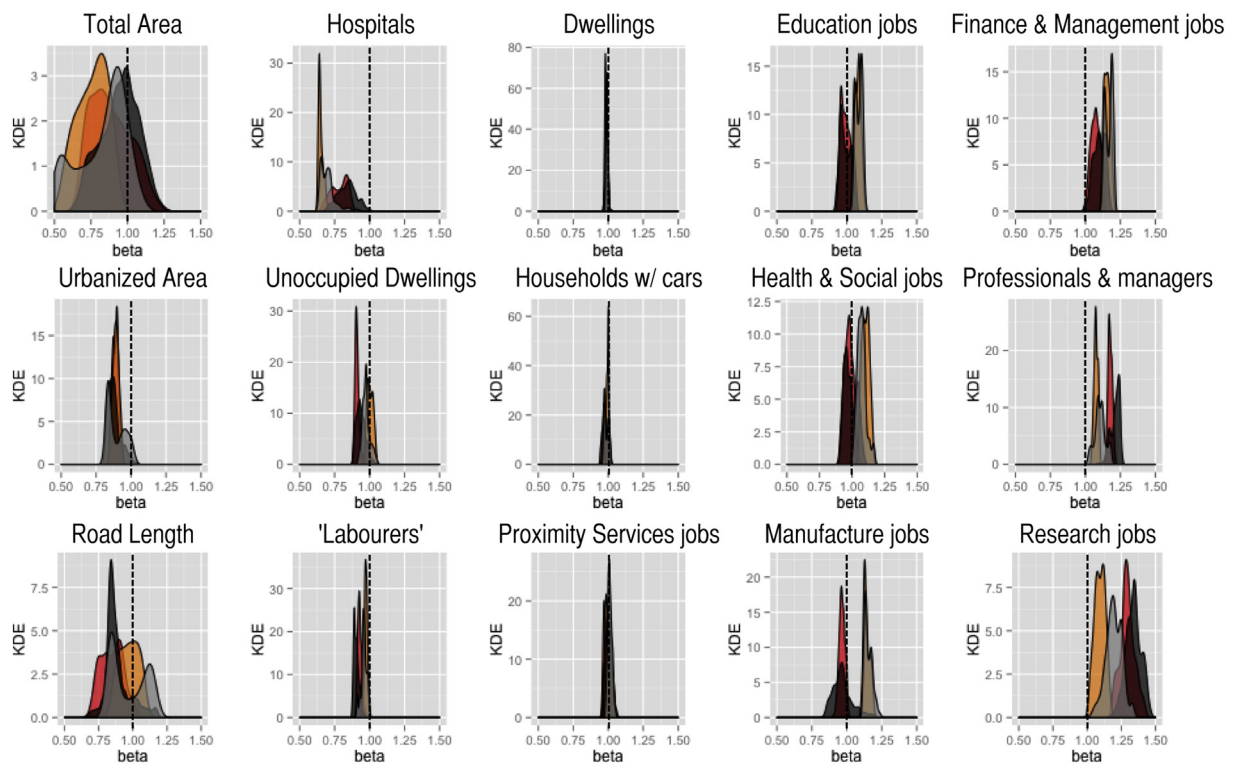
Fig. 3. The distribution of scaling exponents with the clusters typology.

the case of financial and managerial jobs, the application of a population cutoff does not change the scaling regime but clearly lowers the value of scaling exponent β, from around 1.2 to around 1.1. This might begin to explain why mixed evidence appears in the literature, depending only on the minimum size of cities considered in the analysis.

For some urban indicators (the length and surface variables for instance), the scaling behaviour does not respond monotonically to the application of a minimum population threshold (first column, Fig. 3). For the majority of others, however, this definitional criterion appears to be the most important to understand variations in the scaling exponents for most urban indicators, more than the spatial extent of cities.

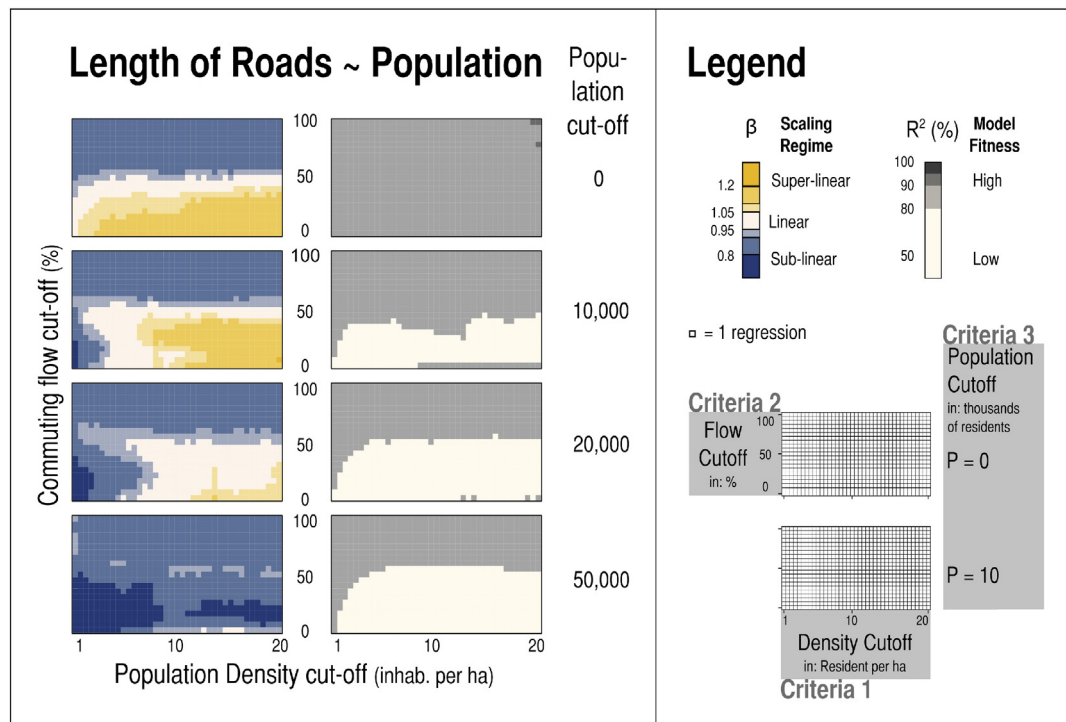### 3.2.2. Heatmaps of scaling exponents

To provide a more detailed example of the sensitivity of attribute scaling with respect to the definitional criteria and their combination, we use multiple representations of 2D heatmaps. Fig. 4 shows the scaling exponents and R^2 values of the regressions of the total road length with population for 3276 definitions. The value of the scaling exponent is represented by the colour scale, and located at the intersection of the Density (x) and Flow (y) criteria (axes) for which we obtain this value. There are as many heatmaps as there are population cutoffs.

In the case of the road length, the interplay of the three definitional criteria produces differentiated scaling behaviours. Also, not only do the

scaling exponent values vary: the scaling regime (sub- or superlinear) depends on the combination of density, commuting flows and population cutoffs. Indeed, this variable is known to be sublinear from previous reports in the literature (e.g. in Levinson (2012), $\beta = 0.667$, and in Louf and Barthelemy (2014b) $\beta = 0.86$), and we show that this result holds true for the central part of cities (i.e. flow cutoff <50%), and among the largest metropolises (<20,000), although with a worse model fit. In these cases, the larger the city, the most efficient it is with respect to the road infrastructure. However, when one considers cities along with their functional peripheries (where people live while working in the centre), then we find the opposite result: the largest cities become relatively more consumptive of infrastructure per capita. This complete view on cities and networks has strong implications for efficient measurement, for sustainable planning and for metropolitan governance.

3.2.3. Hierarchical clustering

In order to summarise the scaling variations of 20 different urban attributes without showing a hundred heatmaps, we performed a hierarchical clustering of the heatmaps of each attribute for each population cutoff (Fig. 5). The β estimated on the clusters defined by the combination of 39 density cutoffs and 21 commuter cutoffs represent the variables (819 columns) for each urban attribute and for each population cutoff (126 rows). The clustering distinguishes groups of indicators whose scaling

Each square in the heatmaps represent a scaling regression on all urban clusters resulting from the combination of the three definition criteria-coordinates (density, commuting flows and population cutoff).

Fig. 4. Heatmaps of scaling exponents and goodness of fit across city definitions. Ex: Length of roads. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

behaviour is close in value and which responds the same way to the city definitional variations. A typology of 9 classes corresponds to clear cuts in the clustering tree (dendrogram) and covers 47.1% of the total variance.

3.2.3.1. Class 1: monotonically linear attributes. The first class in this typology groups together attributes that scale proportionately with population ($0.95 < \beta < 1.05$) for all values of density and commuting criteria. In the case of the number of dwellings, of firms, of households owning a car and of jobs in the proximity service sector, the scaling exponent measured is linear irrespective of the minimum population cutoff (Fig. 5). By contrast, at high population cutoffs (i.e. when only large cities are considered), jobs in the educational, health and social sectors also belong to this class. At low population cutoffs, the number of "labourers", manufacturing jobs, vacant dwellings and universities also tend to be linear urban attributes. These results relate to the interpretations of Pumain et al. (2006) and Bettencourt (2012) concerning indicators of "mature" industries and "basic needs" proportional to urban populations. Following these researchers, linear attributes describe industries mature enough not to require increasing returns to scale to compensate for innovation costs, or goods and services proportionately distributed among city dwellers, since everyone needs a comparable amount in every city of the system (in terms of dwellings, for example).

3.2.3.2. Class 2, 3 & 6: sub-linear attributes. In the types 2, 3 & 6, we find linear to sublinear attributes such as infrastructure, morphological and physical indicators (number of train stations, road length, urbanised area, number of vacant dwellings and hospitals) as well as "obsolete" Pumain et al. (2006) industries (manufacturing) that scale overall in the way predicted in the formerly cited theories: they grow slower than proportionately with city populations, revealing economies of scale (for physical attributes) and specialisation of small cities (for obsolete industries). However, these three classes do not appear monotonic in their response to changes in city definitions. For example, the

attributes are distributed in various classes depending on the population cutoff applied. Train stations for example scale much more linearly among cities of more than 20,000 inhabitants (class 2) than when cities above 10,000 (class 3) or less (class 6) are taken into account. Indeed, all towns usually contain a local train station in France, leading to a very low exponent associated with no population cutoff (or P0, class 6), especially for dense city cores (i.e. high commuting cutoffs). Among large cities, potentially those with more than one train station, the behaviour becomes more linear.

3.2.3.3. Class 4, 5 & 9: superlinear attributes. As observed in Fig. 5, research, finance and management jobs, along with professional occupations, collective housing and universities generally scale linearly to superlinearly. We also find attributes that belong to the superlinear class 5 on the heatmaps at low population cutoffs (Education, health and social jobs, number of employees), while their scaling behaviour under higher cutoffs is monotonically linear (class 1). For these indicators, the more restrictive the delineation of city centres (i.e. small periphery and large population), the more linearly they grow with population. Therefore, the superlinear scaling observed in class 5 for weak flows and low population cutoffs reveals a higher provision in public services and employment opportunities in urban spaces compared to rural ones, rather than a size differentiation among cities. On the other hand, the increased superlinearity of occupations like professionals and managers at high cutoffs reveals the fact that they concentrate in the largest and most urban (dense and integrated) parts of cities.

3.2.3.4. Class 7 & 8: mixed attributes. Classes 7 & 8 represent non-monotonic regimes of urban scaling and are of particular interest to this paper. Class 7 groups heatmaps for road length, area and individual housing at low population cutoffs, and show a combination of three regimes: sublinear (for commuting flows $F > 50\%$); linear (for $F < 50\%$ &

Fig. 5. Clustering of heatmaps across city definitions.
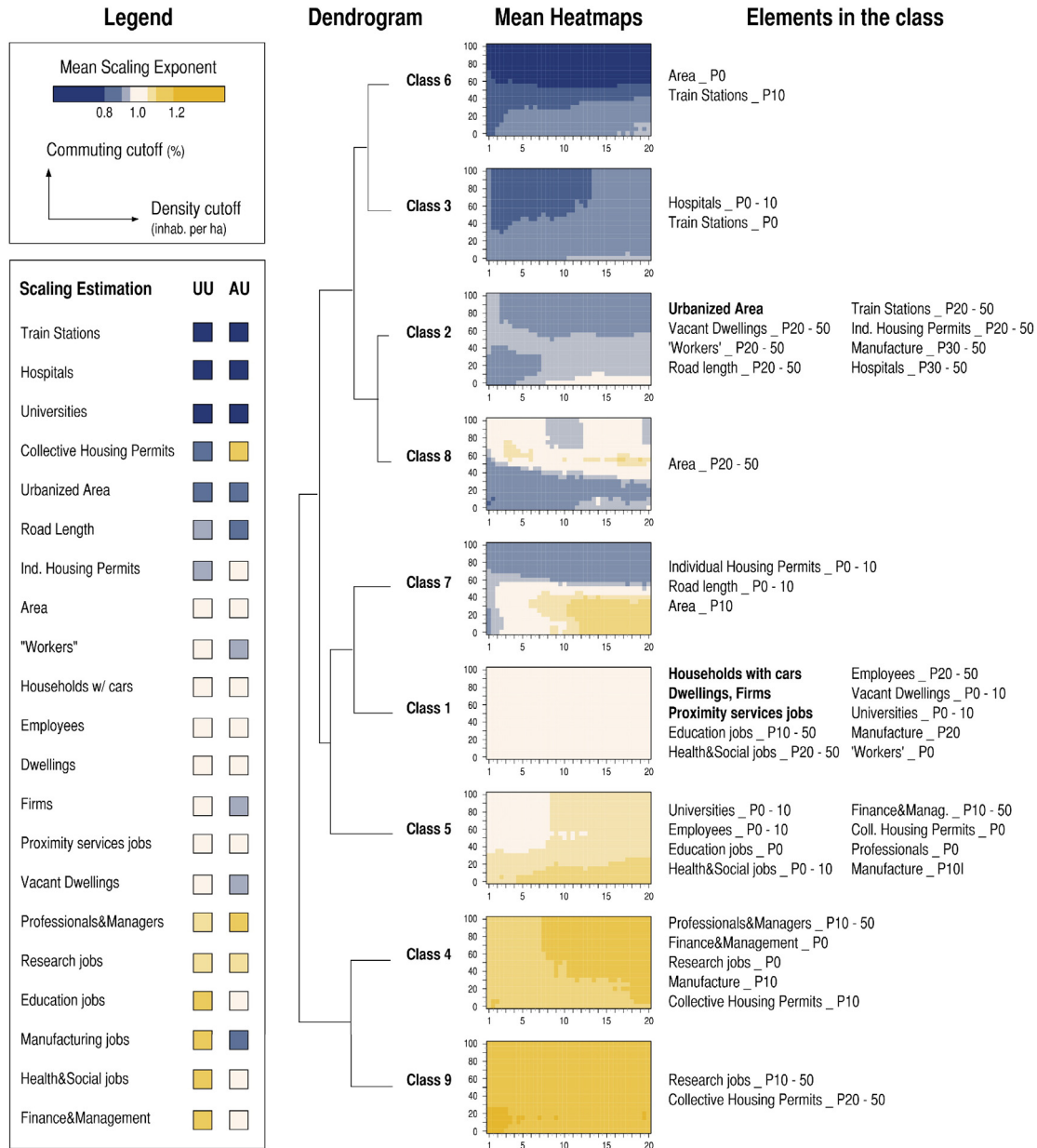
N.B. The "elements in the class" correspond to heatmaps of the scaling exponents for each indicator and for each population cutoff. In bold : all the heatmaps for a given indicator belong to the same class. When a given indicator has heatmaps in different classes of the typology depending on the value of the population cutoff, we use the following notation : $P0$ for "no population cutoff", $P10$ for a population cutoff of 10,000 inhabitants, etc.

$D < 10$ persons per ha); superlinear ($F < 50\%$ & $D > 10$). When we refer to Fig. 1, this means that those indicators of suburban morphology grow less than proportionately with population in city centres (top half of the figure), proportionately in vast configurations of cities that represent most of the French territory (bottom left), and more than proportionately with population when cities are considered as high density kernels surrounded by large peripheries (bottom right). This conclusion highlights the fact that for physical urban attributes, the estimation of scaling exponents is directly related to the choice of city definition. Class 8 comprises only Area heatmaps for high population cutoffs and is to a lesser extent affected by the density criterion to define city centres. The resulting picture is somehow opposite to class 7 where area scales linearly to superlinearly in city centres (high F) and sublinearly in systems with vast peripheries (low F). In other words: large metropolitan areas get denser as they grow, while the surface occupied by city centres is more or less proportionate to population.

An interesting feature of these mixed classes is the fact that the transition between the two regimes happens around the flow cutoff value of 40%, the one chosen by INSEE to define peripheries of metropolitan areas (AU). This cutoff corresponds mostly to a linear behaviour in the classes, but also to a transition space between two radically different scaling regimes. The linear scaling at this value therefore hides high variability when cutoffs are slightly pertubated. It is thus not robust to city definition, indicating that either 40% is "the true value for cities", or more probably that there is no interpretation of the scaling of these variables independently from the criteria used to define cities.

3.2.3.5. Several types at different population cutoffs: manufacturing jobs. A special case is that of manufacturing. The scaling heatmaps for this

indicator are found in the linear, sub- and superlinear classes according to which population cutoff is considered. Scaling laws are supposed to account for regular variations across several orders of magnitude, and specifically model the distribution of attributes of cities of different population sizes. These results for manufacturing therefore suggest that the power law adjustment might not be the most interesting one to describe the evolution of manufacturing jobs with city population. It also suggests that other factors of explanation are necessary to understand the distribution of manufacturing jobs in French cities, such as resource deposits and path-dependency, regional particularities and economic cycles (cf. Finance (2014), Section 4 and residual analysis from section A.3 in the Appendix A).

3.2.3.6. Clustering summary. The clustering of heatmaps has thus provided a synthetic way to describe the dominant regime and variation of scaling measures of 20 indicators with respect to urban definitional criteria. The magnitude of variations appears marginal for many indicators for which the dominant scaling regime corresponds to the one predicted theoretically or from ordinary definitions of cities. We also identified groups of urban attributes for which the variation of the scaling exponent depends quantitatively on density and flow cutoffs. Physical attributes for instance are not independent from the spatial definitions of cities. There, the scaling behaviour appears impossible to characterise independently from city definitional criteria. On the other hand, we found that social and public services seem to exhibit constant returns to scale over a minimum threshold of population. Finally, manufacturing jobs have been found to be loosely linked to population and are better described by other urban features (especially with respect to their history of early diffusion in north-eastern France).

A last attempt at understanding and explaining variations in scaling comes from the confrontation of extreme values recorded in the literature and by our methodology.

3.2.4. Extreme scaling

This last section examines the extreme cases of scaling measured on systems of cities: in the literature and with a systematic definition. Table 4 gives the maximum intervals of scaling exponents found in the literature and in this study among the 4914 combinations of definitional criteria.

The total area of cities is an interesting example of such extreme variations reported in the literature as well as in our own study. The lowest scaling exponents measured are very low ($\beta = 0.3$) and represent extremely sublinear behaviour compared to the minimum values we found in the literature (0.676 in Batty and Ferguson (2011)). In the case of contemporary urban France, we found this minimum value for a very restrictive definition of cities corresponding to narrow centres without peripheries ($D = 13.5, F = 100, P = 0 \mid N = 202$). On the contrary, a superlinear regime ($\beta = 1.2$) comparable to that found by Veregin and Tobler (1997) for 366 US cities in 1980 corresponds to a relatively common definition of metropolitan areas ($D = 7.5, F = 45, P = 10,000 \mid N = 200$). In this case, the dramatic variation of scaling estimation depends on the way cities are spatially defined, as they mechanistically affect the ratio of surface per inhabitant. For lifestyle

measures such as the urbanised area and the number of households with a car, the maximum variations happen with respect to the way the periphery is taken into account (cf. Appendix A, section A.4). This seems consistent with how morphology and lifestyle interact: suburban spaces are dependent on the use of car, and therefore this variable is the closest to superlinearity in a definition of cities that comprises large peripheries ($\beta = 1.03 \mid D = 20, F = 5, P = 10,000 \mid N = 98$) and the closest to sublinearity in narrow centres with almost no commuting ($\beta = 0.94 \mid D = 17.5, F = 95, P = 50,000 \mid N = 50$). The interesting insight provided here by looking at the scaling behaviour is that the low consumption of cars and housing per capita in city centres is reinforced as they grow in size. On the contrary, peripheries tend to exacerbate suburban lifestyles when they belong to larger cities. In the case of research jobs, the distinction between min and max β is associated with the density dimension (the way city centres are defined). Those urban attributes are known to be strongly linked to the density of interactions and consequently superlinear (β from 1.2 to 1.7 in the literature, cf. Bettencourt et al. (2007), Arbesman and Christakis (2011). What this analysis brings is an insight into extreme behaviours not reported so far in the literature (such as a sublinear scaling for research under extensive definitions probably comprising rural spaces), or rarely (for road length for example, Masucci, Arcaute, Hatna, Stanilov, and Batty (2015) are the only ones to find linear to superlinear scaling for the road length).

The number of jobs in proximity services appears different from the previous category as the minimum and maximum scaling values are opposed in the way the ratio of centre and periphery is defined. Indeed, this variable scales sublinearly when clusters correspond to large dense centres with a restricted periphery ($\beta = 0.95 \mid D = 5, F = 55, P = 20,000 \mid N = 163$) and superlinearly when city cores are very narrow and peripheries extensive ($\beta = 1.07 \mid D = 16, F = 0, P = 10,000 \mid N = 142$). This could mean that, although basic services are subject to economies of scale in the largest cities, they are not equally profitable in suburban spaces and are provided more systematically in the peripheries of large cities rather than in the periphery of smaller ones.

Finally, finance, management, health and social services jobs show extreme scaling behaviour under similar spatial definitions of cities, but at different population cutoffs. This urban attribute is the most sublinear at the top of the urban hierarchy ($\beta = 0.90 \mid D = 4, F = 65, P = 50,000 \mid N = 88$) and most superlinear when the entire spectrum of city size is considered ($\beta = 1.17 \mid D = 1.5, F = 95, P = 0 \mid N = 1094$). This might suggest the existence of a critical size to provide such services[6] which require initial investments in a large infrastructure (such as hospitals for health services) or a sufficiently large network of clients and suppliers (for financial services for example) to exhibit subsequent economies of scale. Evaluating the sensitivity of scaling provides an opportunity to identify where scaling regime transitions might take place and to characterise the types of cities to which it corresponds. This will help build models that could account for the emergence of such scaling (and varying) behaviours, based on a non-uniform internal morphology of the cities considered. It also helps disentangling the variations that are only due to definitional specifications from the ones that truly differentiate city systems in time and space.

4. Conclusion: from quantitative to qualitative changes in the urban hierarchy?

"*Whether a particular class of prosocial behavior scales linearly, superlinearly, or sublinearly might be dependent on two factors: the locality of one's interactions, and feedback of interaction. For locality, this refers to whether or not you are limited to the individuals around you,*

Table 4
Maximum scaling registered in the literature compared to estimated values with a systematic definition of French cities.

Urban attribute	Max. interval for clusters	βmin in literature	βmax in literature	βUU	βAU
Total area	[0.334; 1.291]	0.676	1.163	0.959	0.995
Road length	[0.664; 1.202]	0.667	1.04	0.903	0.888
Research	[0.951; 1.496]	1.174	1.67	1.094	1.079
Health and social	[0.898; 1.171]	0.95	0.98	1.136	1.013

For more values from the literature, the corresponding reference and definitions of cities (territory, date, delineation), cf. Appendix A, section A.5.

---

[6] For example, the largest metropolitan area (AU) which does not host a hospital has 25,040 residents.

or can interact with anyone in the city. [...] The potential for understanding the spectrum of scaling of prosocial behaviours points to the wide variety of aspects of seemingly related behaviours. Their implications for urban growth are intriguing and merit further examination." Arbesman and Christakis (2011), p.2158

Scaling laws were first proposed as an interesting tool to summarise distributions of characteristics in a system of cities with respect to city size over several orders of magnitude. It lies on the conception that cities share common attributes across a wide size spectrum. This proposition finds roots in the long quest to find what makes cities identifiable (specifically urban features). Social scientists agree on the fact that we recognise cities by their function as social interactions maximisers. They do so by concentrating a larger number of heterogeneous social agents in a limited space (hence, dense). Those cities take part in a system of cities at a larger scale, characterised by a regular hierarchy of sizes, competition (in space resulting in regular spatial patterns of settlements) and cooperation (resulting in complementary profiles of economic and functional specialisation) Pumain et al. (2006). Therefore, the analysis of systems of cities should not be restricted to large cities only (a constraint generally imposed by data), as small cities are an essential part of urban systems.

Regimes of sub- and superlinearity indicate quantitative changes which occur with size variations. Moreover, several results from this paper indicate the existence of high variability of scaling exponents with respect to variations in city definition. The transition from one scaling regime to another when population cutoffs vary was found to be the most important criterion in most cases (and the easiest to harmonise in comparative studies). For example, if we go back to the paradoxes implied in Table 1, we find that scaling behaviours with the two official definitions start to match when a population cutoff is applied (cf. Fig. 6). Over 50,000 residents, city centres behave similarly to metropolitan areas with respect to socioeconomic criteria. The paradox appears mostly due to the large dispersion encountered among small cities (cf. Fig. 6). These deviations from the power law adjustment are neither random errors nor systematic biases for particular cities: they reveal factors unrelated to city size that play an important role in the explanation of the attribute location, for example: coastal accessibility for secondary houses, regional diffusion in manufacturing, etc. (cf. residual maps from Fig. A2 in Appendix A).

The main finding of this paper is that urban scaling is relative to the definition of cities, and most importantly that variations with respect to definitional criteria are neither random (since residuals can be interpreted on a case by case basis) nor universal for all the variables under study. Instead, some attributes are more sensitive to the spatial delineation of centres and peripheries (interaction-based activity, lifestyle attributes), while others respond to changes in population cutoffs (for instance: infrastructures with high fixed costs). Although there seems to be no single "good" definition to study urban scaling, what we found is that while selecting one for comparison in time or between national systems, this choice should depend on the attribute under
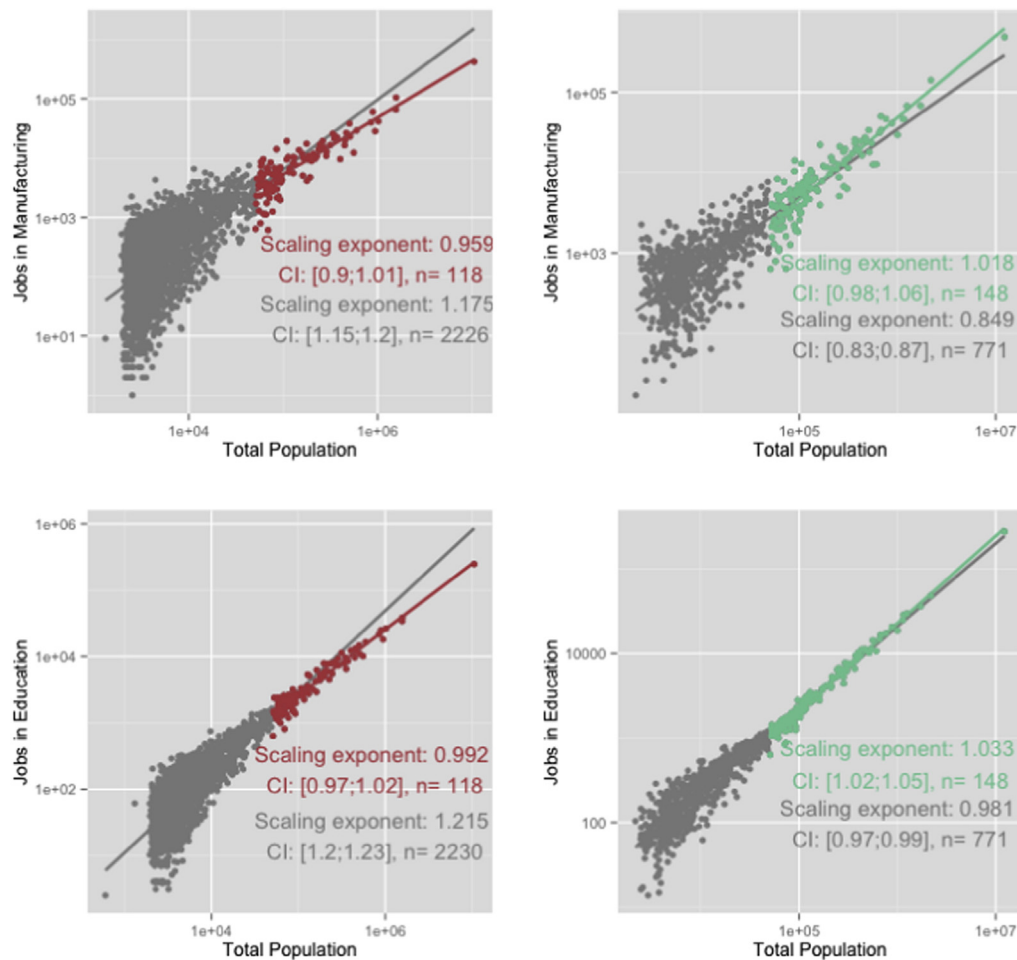
Left : UUs. Right : AUs. In color : Over 50,000 residents.

Fig. 6. Variation of urban scaling with population cutoff for official definitions.

consideration. In particular, the minimum population used to identify cities plays a major role in the scaling variation, and should be integrated fully in the interpretation of results.

As a limitation to this work, we could point to the fact that the assumption that cities are monocentric (by attaching commuters to a single centre) could be refined to provide results more in accordance with the diversity of urban forms (and in particular polycentricity, cf. Le Néchet (2015). Also, according to Guilluy (Guilluy (2013), the "fracture" between central metropolises and peripheral suburbs and rural spaces in France is recent and increasing. It favours the concentration of extremes categories of workers and social classes in globalised metropolises and rejects low and middle social classes to a peripheral France of suburbs and small cities. Provided the collection of temporal data, our method should allow to test these assumptions quantitatively by showing larger variations of urban scaling with city definitions over time for economic and sociologic categories.

With this insight and a deeper search for processes linking intra-urban features to interurban scaling, a logical continuation of this work would be to build models able to simulate empirical scaling patterns. These models (i.e. models that seek to explain the role played by city size in the distribution of functions and infrastructures) should now account for the contrasted behaviours of the different attributes, especially at the intra-urban scale. That is, if absolute single values of urban scaling are meaningless, there is no point in trying to validate explanatory models against them. Instead, the goal in validating generative models should be to reproduce the variations in scaling observed when the definitional criteria are changed. This implies that our models should not consider cities as homogenous objects detached from a sea of non-urban spaces. On the contrary, we should try to simulate differentiated activities and movements between heterogeneous parts of cities, as observed through empirical Census data. The use of Census data and their systematic aggregations thus still prove a promising way to address unresolved urban questions in the future.

Glossary

- AU stands for *Aire Urbaine*, a delineation used by the French Statistical Office (I.N.S.E.E.) to represent metropolitan areas.
- An Urban Delineation is a way to aggregate areal units from the Census into clusters identified as cities, following a specific definition criteria. For example, cities can be considered as political municipalities, morphological aggregates or functional regions.
- UU stands for *Unité Urbaine*, a delineation used by the French Statistical Office (I.N.S.E.E.) to represent built-up areas.

Acknowledgements

Appendix A

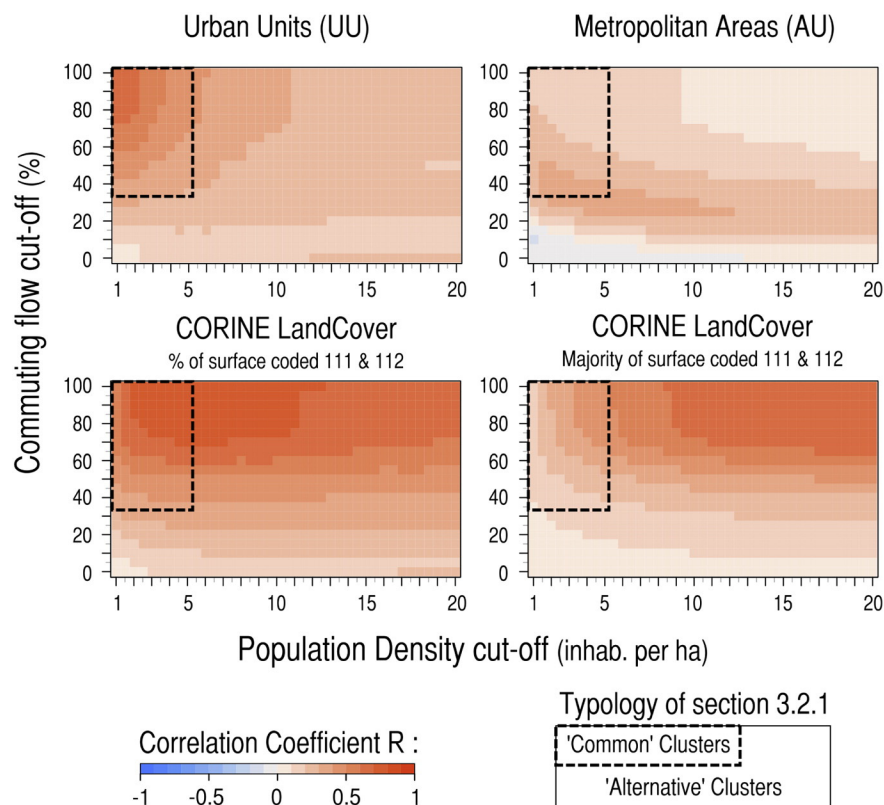### A.1. Correlation between systematic clusters and official definitions

Fig. A1. Correspondence between clusters definitions and official cities.

N.B.: The correlations are made at the level of local units (*Communes*), between the shares of urbanised area according to different city definitions. For UUs, AUs and our systematic definitions, a local unit is 100% urbanised if it belongs to one of the cities delineated. The CORINE raster data are used in two variables: the first one refers to the percentage of surface coded 111 ('Continuous urban fabric') or 112 ('Discontinuous urban fabric'). The second variable is a dichotomy of the first around the value 50%. The correlations thus reflect the similarity between different definitions in considering local units as urban. For a coefficient of 1, all local units considered urban by one definition are considered urban by the second. By contrast, a coefficient of −1 would indicate that all the local units considered urban by one definition are considered non-urban by the other and vice-versa. In the top left corner of the top left heatmap, the colour indicates that there is a strong correlation between the local units composing UUs (defined as built-up areas by the French Census) and local units composing the clusters defined with a density cutoff of 1 resident per hectar, a commuting cutoff of 100% and a population cutoff of 0. On the other hand, the definition using a density cutoff of 1 resident per hectar, a commuting cutoff of 0% and a population cutoff of 0 (bottom left corner of the top left heat map) bears no resemblance to the definition of Urban Units (as indicated by a correlation coefficient of around 0). 'Common' and 'Alternative' cluster definitions are indicated on the figure but are independent from the correlation calculated.

A.2. Description and sources of urban attributes used in the paper

Table A1
Sources of data used in the paper.

Type of Data	Dataset	Description	Source
Geography	GeoFla 2013	Communes Shape Files	www.data.gouv.fr
	CORINE LandCover 2006	Land use	www.statistiques.developpement-durable.gouv.fr
	MOBPRO 2011	Commuting flows	www.insee.fr
Attributes	base-cc-emploi-pop-active-2011	Demographics	www.insee.fr
	CLAP 2011	Jobs by sector	www.insee.fr
	equip-tour-transp 2013	Infrastructures	www.insee.fr
	equip-serv-sante 2013	Hospitals	www.insee.fr
	equip-serv-ens-sup-form-serv 2013	Universities	www.insee.fr
	OpenStreetMap 2014	Length of roads[a]	www.data.gouv.fr
	Sit@del2 2011	Housing permits	www.data.gouv.fr

Note: The term "labourer" here stands for the French category *ouvrier*, that is a social occupation defined by the census, which corresponds partially to a plant or manual worker. Proximity services relate to jobs in everyday services except distribution, transportation, education and health. For example, they include jobs such as hairdressers or laundry services.
Source: http://www.insee.fr/fr/themes/detail.asp?reg_id=99&ref_id=analyse.Housing permits are counted as the ones authorised in 2011. Source: https://www.data.gouv.fr/fr/datasets/permis-de-construire-pc-permis-d-amenager-pa-et-declaration-prealable-dp-sit-del2/Paris, Lyon and Marseille are here considered as single *communes* (i.e. not disaggregated into smaller units known as *arrondissements*).
[a] Primary, secondary, tertiary, motorway and trunk categories added together.

A.3. Residuals from scaling regression
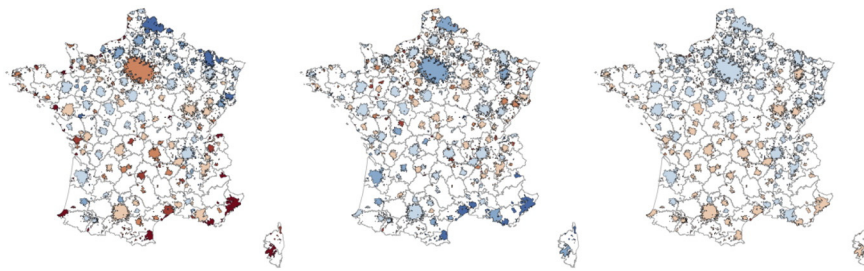
Fig. A2. Residuals of regression with population (in logs).

From left to right: Residuals in the Number of Unoccupied Secondary Dwellings, Number of Jobs in Manufacturing, Number of Dwellings. Residuals are obtained from a regression in logs. Red values correspond to a value for the attribute higher in reality than expected with respect the size of the city, blue means that scaling over-estimates the value for the attribute based on the size only. Bright colours indicate large deviations from the scaling estimation. Cluster Definition: $D = 4, F = 40, P = 0$.

A.4. Extreme Scaling with Systematic Clusters

Table A2
Extreme urban scaling exponents with systematic definitions.

Variable	βMin/βMax	R2	N	Cluster Definition (Density, Flow, Population)
Area	0.334	0.16	202	$D = 13.5; F = 100; P = 0$
	1.291	0.61	200	$D = 7.5; F = 45; P = 10,000$
Urbanised Area	0.809	0.93	142	$D = 13; F = 85; P = 10,000$
	1.048	0.96	98	$D = 20; F = 0; P = 10,000$

(continued on next page)

Table A2 (*continued*)

Variable	βMin/βMax	R2	N	Cluster Definition (Density, Flow, Population)
Households with cars	0.942	0.99	50	$D = 17.5; F = 85; P = 50,000$
	1.030	0.99	98	$D = 20; F = 5; P = 10,000$
Research jobs	1.021	0.78	880	$D = 1; F = 30; P = 0$
	1.50	0.92	72	$D = 20; F = 20; P = 40,000$
Proximity Services jobs	0.951	0.96	163	$D = 5; F = 55; P = 20,000$
	1.073	0.98	142	$D = 16; F = 0; P = 10,000$
Health and Social Services jobs	0.898	0.95	88	$D = 4; F = 65; P = 50,000$
	1.171	0.85	1094	$D = 1.5; F = 95; P = 0$
Finance and Management jobs	0.984	0.91	90	$D = 3; F = 70; P = 50,000$
	1.210	0.90	1122	$D = 1.5; F = 95; P = 0$

A.5. Extreme Scaling and Allometry form the literature

Table A3
Diversity of urban scaling exponents from the literature.

Variable	β	CI (95%)	R2	Date	Country	Urban Definition	N	Ref
Total area	1.163		0.774	1980	USA	'Cities'	366	[10]
	1.043		0.903	1990	East Anglia	'Cities'	70	[2]
	0.808		0.756	1990	South East UK	'Cities'	801	[2]
	1.014		0.76	2001	Europe	'Cities'	386	[5]
	0.946			2001	UK	'Cities'	67	[5]
	0.765		0.637	2001	UK	Metropolitan local authorities	100	[1]
	0.676		0.309	2005	USA	SMSA	355	[1]
	0.85	[0.84;0.86]	0.93	2010	USA	Urban Areas	3540	[7]
Road length	0.86	[0.84;0.88]	0.92	2011	USA	Urban Areas	441	[7]
	1.00	[0.95;1.05]		2010s	UK	Percolation Clusters	61	[8]
	1.04	[1.03;1.05]		2010s	California	Percolation Clusters	52	[8]
	0.667		0.65	2010	USA	MSA	50	[6]
	0.849	[0.81;0.89]	0.65	2006	USA	Metropolitan Areas		[4]
Research	1.211		0.63	1987	USA	MSA	227	[3]
	1.174		0.67	1997	USA	MSA	266	[3]
	1.185		0.69	2002	USA	MSA	278	[3]
	1.54			2000	USA	SMSA	331	[9]
	1.67	[1.54;1.80]	0.64	1999	France	Aires Urbaines	350	[9]
Health & social	0.95			2000	USA	SMSA	331	[9]
	0.98	[0.94;1.02]	0.89	1999	France	Aires Urbaines	350	[9]

A.6. References

[1] Batty, M., Ferguson, P., 2011, Defining city size. Environment and Planning B: Planning and Design, 38(5), 753–756.

[3] Bettencourt, L. M., Lobo, J., Strumsky, D., 2007, Invention in the city: Increasing returns to patenting as a scaling function of metropolitan size. Research Policy, 36(1), 107:120.

[4] Bettencourt L., 2012, The origins of scaling in cities Science, vol. 340, pp 1438:1441

[5] Fuller, R. A., Gaston, K. J., 2009, The scaling of green space coverage in European cities. Biology Letters, 5(3), 352–355.

[6] Levinson D., 2012, Network Structure and City size. PLoS ONE, vol. 7, No.1.

[7] Louf R., Barthelemy M., 2014, How congestion shapes cities: from mobility patterns to scaling. Scientific reports, Vol.4, No. 5561.

[8] Masucci A. P., Arcaute E., Hatna H., Stanilov K., Batty M., 2015, Logistic Growth and Ergodic Properties of Urban Forms. arXiv pre-print http://arxiv.org/abs/1504.07380v2

[9] Pumain, D., Paulus, F., Vacchiani-Marcuzzo, C., Lobo, J., 2006, An evolutionary theory for interpreting urban scaling laws. Cybergeo: European Journal of Geography, document 343.

[10] Veregin, H., Tobler, W. R., 1997, Allometric relationships in the structure of street-level databases. Computers, environment and urban systems, 21(3), 277–290

References

Arbesman, S., & Christakis, N. A. (2011). Scaling of prosocial behavior in cities. *Physica A, 390*, 2155–2159.

Arcaute, E., Hatna, E., Ferguson, P., Youn, H., Johansson, A., & Batty, M. (2015). Constructing cities, deconstructing scaling laws. *Journal of the Royal Society Interface, 12*(102), 20140745.

Barnes, T. J., & Wilson, M. W. (2014). Big data, social physics, and spatial analysis: The early years. *Big Data & Society, 1*(1). http://dx.doi.org/10.1177/2053951714535365.

Batty, M. (2015). Data about cities: redefining big, recasting small. *Data and the city workshop, the programmable city project at National University of Ireland, Maynooth, August 31-September 1st 2015* http://www.spatialcomplexity.info/files/2015/08/Data-Cities-Maynooth-Paper-BATTY.pdf.

Batty, M., & Ferguson, P. (2011). Defining city size. *Environment and Planning B: Planning and Design, 38*(5), 753–756.

Batty, M., Carvalho, R., Hudson-Smith, A., Milton, R., Smith, D., & Steadman, P. (2008). Scaling and allometry in the building geometries of greater London. *The European Physical Journal B, 63*(3), 303–314.

Bettencourt, L. (2012). The origins of scaling in cities. *Science, 340*, 1438–1441.

Bettencourt, L. M., Lobo, J., & Strumsky, D. (2007). Invention in the city: Increasing returns to patenting as a scaling function of metropolitan size. *Research Policy, 36*(1), 107–120.

Bos, F. (2011). *Three Centuries of Macro-Economic Statistics, Munich Personal, RePEc Archive (MPRA) Paper No. 35391 at* http://mpra.ub.uni-muenchen.de/35391.

Carvalho, R., & Penn, A. (2004). Scaling and universality in the microstructure of urban space. *Physica A: Statistical Mechanics and its Applications, 332*, 539–547.

Cristelli, M., Batty, M., & Pietronero, L. (2012). There is more than a power law in Zipf. *Scientific Reports, 2*.

Finance, O. (2014). Hiérarchie urbaine et ancrage territorial des firmes transnationales étrangères dans le système de villes français: une approche par les lois d'échelle. *Actes de la conférence: Aux Frontières de l'Urbain*, 378–395.

Fragkias, M., Lobo, J., Strumsky, D., & Seto, K. C. (2013). /Does size matter? Scaling of CO 2 emissions and US urban areas. *PloS One, 8*(6), e64727.

Guérin, F. (1995). Rank-size distribution and the process of urban growth. *Urban studies, 32*(3), 551–562.

Guérois, M., & Paulus, F. (2002). Commune Centre, Agglomération, Aire Urbaine: Quelle Pertinence Pour L'étude Des Villes? *Cybergeo, 212*, 15.

Guilluy, C. (2013). *Fractures françaises.* Paris: Flammarion, Champs essais (186p).

Kim, K. S., Benguigui, L., & Marinov, M. (2003). The fractal structure of Seoul's public transportation system. *Cities, 20*(1), 31–39.

Le Néchet, F. (2015). De la forme urbaine à la structure métropolitaine: une typologie de la configuration interne des densités pour les principales métropoles européennes de l'Audit Urbain. *Cybergeo: European Journal of Geography, document, 709.*

Levinson, D. (2012). Network structure and city size. *PloS One, 7*(1).

Lobo, J., Bettencourt, L. M., Strumsky, D., & West, G. B. (2013). Urban scaling and the production function for cities. *PloS One, 8*(3), e58407.

Longley, P. A., & Mesev, V. (2002). Measurement of density gradients and spacefilling in urban systems. *Papers in Regional Science, 81*(1), 1–28.

Louf, R., & Barthelemy, M. (2014b). How congestion shapes cities: from mobility patterns to scaling. *Scientific reports, 4*(5561).

Malecki, E. J. (1980). Growth and change in the analysis of rank-size distributions: empirical findings. *Environment and Planning A, 12*(1), 41–52.

Masucci, A. P., Arcaute, E., Hatna, E., Stanilov, K., & Batty, M. (2015). On the problem of boundaries and scaling for urban street networks. *Journal of the Royal Society Interface, 12*(111), 20150763.

Niedzielski, M. A., Horner, M. W., & Xiao, N. (2013). Analyzing scale independence in jobs-housing and commute efficiency metrics. *Transportation Research Part A: Policy and Practice, 58*, 129–143.

Ortman, S. G., Cabaniss, A. H., Sturm, J. O., & Bettencourt, L. M. (2014). The prehistory of urban scaling. *PloS One, 9*(2), e87902.

Parr, J. B. (2007). Spatial definitions of the city: Four perspectives. *Urban Studies, 44*(2), 381–392.

Pumain, D., Paulus, F., Vacchiani-Marcuzzo, C., & Lobo, J. (2006). An evolutionary theory for interpreting urban scaling laws. *Cybergeo: European Journal of Geography, document, 343.*

Rybski, D., Sterzel, T., Reusser, D. E., Winz, A. -L., Fichtner, C., & Kropp, J. P. (2013). *Cities as nuclei of sustainability.* (arXiv preprint arXiv:1304.4406).

Shalizi, C. R. (2001). *Scaling and hierarchy in urban economies.* (arXiv preprint, arXiv:1102.4101v2).

Taylor, L., Schroeder, R., & Meyer, E. (2014). Emerging practices and perspectives on big data analysis in economics: Bigger and better or more of the same? *Big Data & Society, 1*(2) (2053951714536877).

Veregin, H., & Tobler, W. R. (1997). Allometric relationships in the structure of street-level databases. *Computers, Environment and Urban Systems, 21*(3), 277–290.

West, G. (2014). Scaling: The surprising mathematics of life and civilization. *Santa Fe Institute Bulletin, 28*(2).