Figures for Küchemann, D. & Hoyles, C. (2009) From empirical to structural reasoning in mathematics: tracking changes over time In Stylianou, D.A, Blanton, M. L. & Knuth, E.J. (Eds) *Teaching and Learning Proof Across the Grades K-16 Perspective* Lawrence Erlbaum Associates pp171- 191

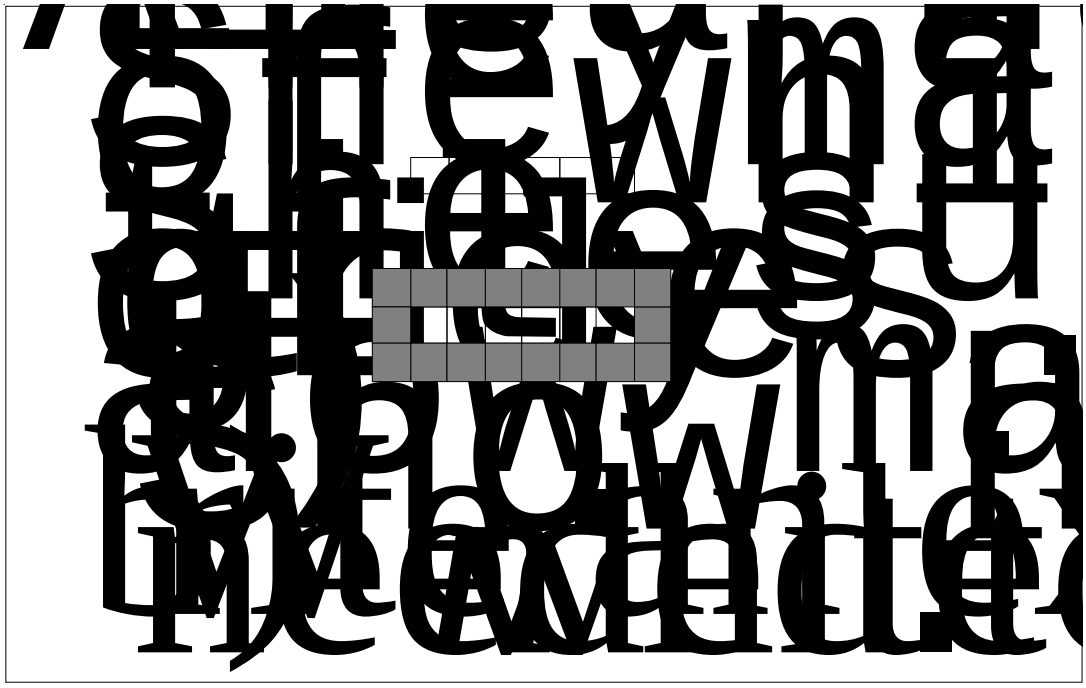

Figure 1: Yr 8 version (part *a* only) and Y10 version of question A1

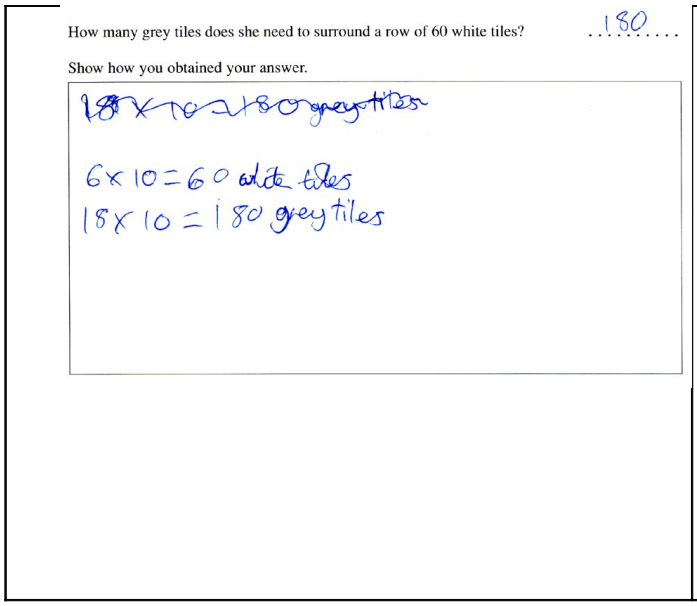

Figure 2a: Student JG's response to A1 in Yr 8

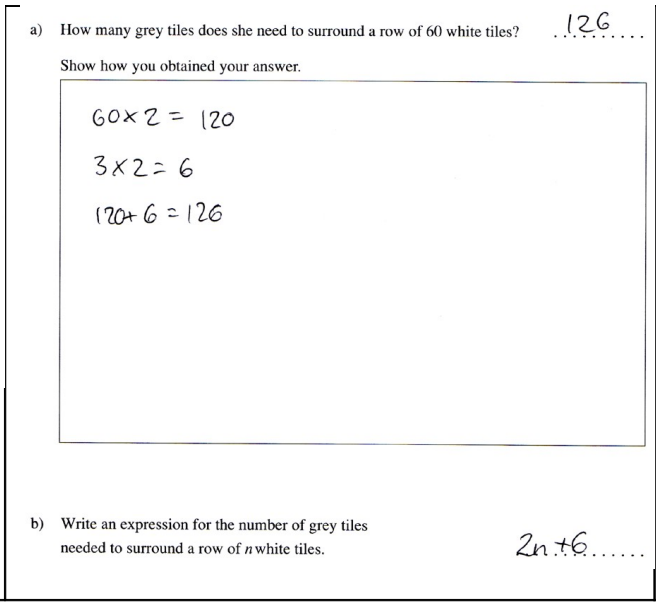

Figure 2b: Student JG's responses to A1 in Yr 10

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Figure 3: Yr 8 and Yr 9 versions of question A4

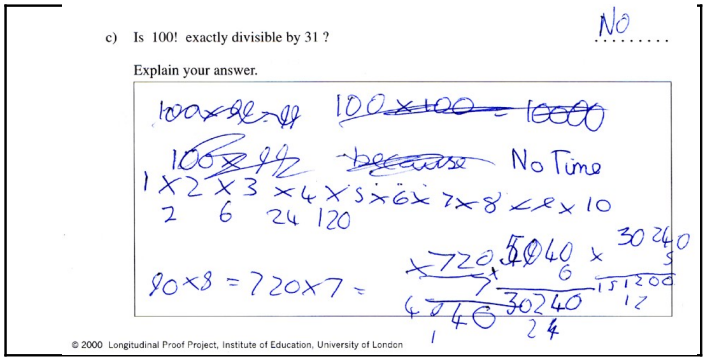
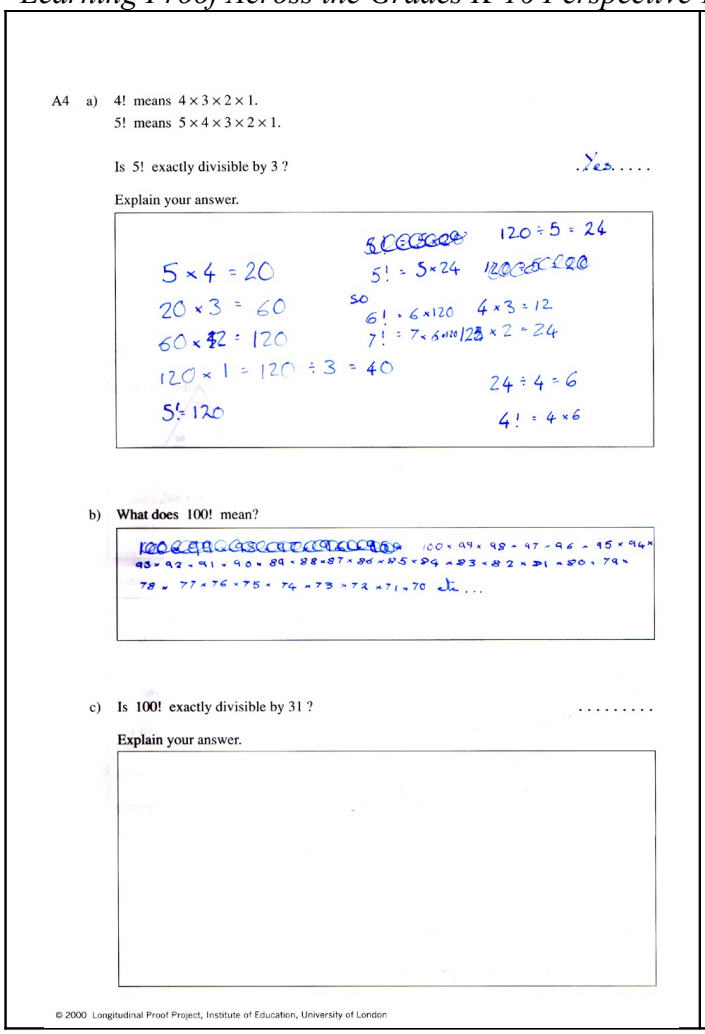


Figure 4: A student response to A4*c* in Yr 8

Figures for Küchemann, D. & Hoyles, C. (2009) From empirical to structural reasoning in mathematics: tracking changes over time In Stylianou, D.A, Blanton, M. L. & Knuth, E.J. (Eds) *Teaching and Learning Proof Across the Grades K-16 Perspective* Lawrence Erlbaum Associates pp171- 191



A4 a) 4! means $4 \times 3 \times 2 \times 1$.
5! means $5 \times 4 \times 3 \times 2 \times 1$.

Is 5! exactly divisible by 3 ? Yes

Explain your answer.

$5 \times 4 = 20$
$20 \times 3 = 60$
$60 \times 2 = 120$
$120 \times 1 = 120 \div 3 = 40$
$5! = 120$

$120 \div 5 = 24$
$5! = 5 \times 24$
so
$6! = 6 \times 120$
$7! = 7 \times 6 \times 120$
$4 \times 3 = 12$
$12 \times 2 = 24$
$24 \div 4 = 6$
$4! = 4 \times 6$

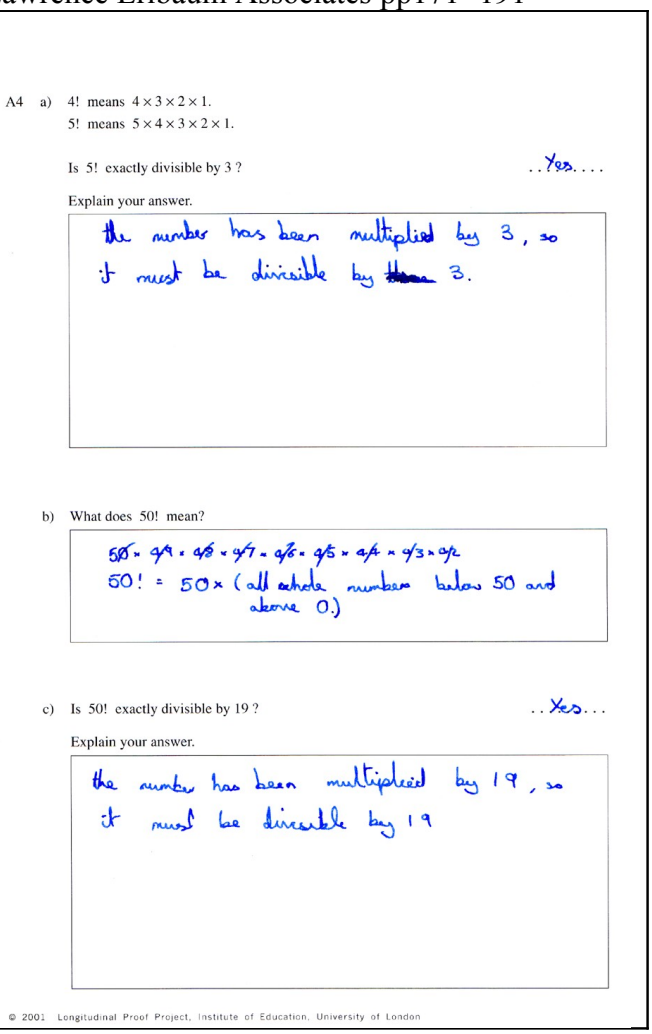
b) What does 100! mean?

$100 \times 99 \times 98 \times 97 \times 96 \times 95 \times 94 \times 93 \times 92 \times 91 \times 90 \times 89 \times 88 \times 87 \times 86 \times 85 \times 84 \times 83 \times 82 \times 81 \times 80 \times 79 \times 78 \times 77 \times 76 \times 75 \times 74 \times 73 \times 72 \times 71 \times 70$ etc...

c) Is 100! exactly divisible by 31 ? .....

Explain your answer.

Figure 5a: Student AM's responses to A4 in Yr 8



A4 a) 4! means $4 \times 3 \times 2 \times 1$.
5! means $5 \times 4 \times 3 \times 2 \times 1$.

Is 5! exactly divisible by 3 ? Yes

Explain your answer.

the number has been multiplied by 3, so it must be divisible by 3.

b) What does 50! mean?

$50 \times 49 \times 48 \times 47 \times 46 \times 45 \times 44 \times 43 \times 42$
50! = 50 × (all whole numbers below 50 and above 0.)

c) Is 50! exactly divisible by 19 ? Yes

Explain your answer.

the number has been multiplied by 19, so it must be divisible by 19

Figure 5b: Student AM's responses to A4 in Yr 9