Beginner mathematics teachers assessing advanced problem solving: what do they bring, what do they need, and how can the gap be bridged?

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Secondary mathematics teachers in England are expected to develop teaching and assessment of problem solving in ways which have not been common in schools or initial teacher education in recent years. This study asked what beginner teachers bring to this process, including knowledge, skills and beliefs, and what should be further developed, in the specific context of problem solving for advanced school mathematics. It suggests that the intentions of the change are well-aligned with the professional beliefs of many both beginner and experienced teachers, and that carefully structured workshops can enable beginner teachers to re-envision practice, as well as to acquire specific assessment-related skills. However, acquiring deep expertise in the area is demanding, and perceived to be daunting for both beginner and experienced teachers.

Keywords: Beginner teachers; problem solving; advanced mathematics; assessment values

Background

Post-16 'Advanced Level' (A Level) mathematics curricula in England, in common with pre-16 curricula in England and elsewhere, are being re-developed with a renewed emphasis on problem solving. The scale of participation and success in A Level Mathematics is thought to be of national importance (Noyes, Wake, & Drake, 2011). For students it is a high stakes year 12/13 qualification, often a gatekeeper to university entrance, and its summative assessments embody (de facto) rules for the discipline. Indeed, it is common practice in English classrooms and textbooks (Ofsted, 2012) for examination type questions to be used as a proxy for formative methods in teaching and learning.

Teachers also represent and embody the discipline for their students (Fordham, 2012), in terms of its values and its rules, yet the renewed emphasis on problem solving is an aspect of mathematics that beginner teachers may not have experienced as school students. We use 'beginner teachers' to refer to secondary teachers engaged in one- year preparation courses for teaching, either school-led or Higher Education-led, with substantial, but variable, lengths of classroom experience.

Classroom studies of early career teachers' beliefs about teaching for problem solving are sparse, though Cooney's (1985) study suggests such teaching can be very demanding, and its valuing fragile. Little appears to be known, either, about how beginner teachers learn to engage with authentic classroom assessment. Grainger and Adie (2014) asked how key assessment areas develop when beginner teachers engage in assessment of peers' coursework, and highlight the profound influence on beliefs and understandings of teachers' background experiences. Lavi and Shriki (2014) identify the emotional impact of feedback when beginner teachers engage in peer assessment of proof – and the increase in mathematics knowledge for teaching gained from reflection on alternative solutions. Webb (2009) reports on middle grade

experienced teachers engaging in written assessment design, highlighting the interpretive skills needed, especially regarding student strategies and evidence of their understanding, the extensive demands made on teachers' mathematical knowledge for teaching, and the role of informal representations and strategies to support such understanding. Such expertise is clearly demanding for beginner teachers.

In relation to both formative and summative assessment, then, we asked

- *What knowledge and values inform beginner teachers' approaches to the teaching and assessment of problem solving at an advanced school level?*
- *What are their needs in relation to developing assessment of such problem solving as part of their preparation for effective teaching? (and how does that compare with the needs of more experienced teachers adapting to this change?)*
- *How could initial teacher education and early professional development be strengthened to include education for better assessment of problem solving?*

Beginner teachers vs experts:

We adopt Livingston and Borko's (1990) conceptualisation of teaching as a complex cognitive skill which includes improvisational performance. They suggest link-making and strategic thinking (with implicit metacognition) characterise core differences between expert and novice teacher functioning. However, Cochran-Smith and Lytle (1999) show that relationships between 'knowing more' and 'teaching better' are anything but straightforward. They distinguish between knowledge for/in/of practice, claiming that effective teachers need all of these. Further, Berliner (2004) argues the teaching expertise is both domain-specific and a long-term goal, taking 5-7+ years to develop, if it ever does. Horn (2005) highlights curriculum resources and professional interactions as key resources for teacher learning, so teacher learning is both social and situated. These social learning interactions are re-presented in Horn (2010, 243) as 'replays, rehearsals, and re-envisioning of practice'.

Problem solving

Mathematics 'problem solving' is a contested term in the literature (Schoenfeld, 2007). Clearly, what is a problem to one student might well be anything but to another: for example, summing an arithmetic series might comprise a problem to an 14 year old who, two years later, has learnt a standard algorithm which requires little thinking. We take 'problems' to be those tasks to which there is no approach or algorithm familiar to the student that (s)he knows is almost certain to result in a closed solution. Instead, the task might contain unfamiliar, unstructured or complex aspects to which the student might bring one or more approaches, but without high initial confidence of success. This is an interpretation which appears to be consistent with the early specimen materials produced by A Level examination bodies in response to the English policy change. These questions require problem solving behaviours such as: familiarising oneself with notations and definitions by specialising, interpreting constraints, recognising analogous reasoning. We sought existing evidence of the differential demands of teaching mathematics, including problem solving, at this advanced school level, but found little.

The study

Our planned provision was informed by an online questionnaire (n=50). We drew on the ideas above to design a two-hour face-to-face development session with beginner

teachers (n=32 over two sessions), followed up with interviews (n=4) of a purposive sample of those teachers, chosen to represent a range of teacher preparation routes and mathematical backgrounds. Teachers used selected year 12 A Level assessment questions to develop their understanding of teaching for such problems, evaluating and assessing authentic solutions (written by peers and school students) including through the use of formal mark schemes, and giving formative feedback. We also compared beginner teachers' responses to those of more experienced A-Level teachers to the same session (total n=73, in 3 sessions, opting in at teacher conferences) seeking to understand commonalities and differences between groups.

Data comprised web questionnaire responses, written group and individual responses solicited during workshop activities, and transcribed recordings of workshop activity and of individual post-workshop interviews. We used open, axial and selective coding to analyse data (Charmaz, 2006).

Findings and discussion

Web questionnaire

We targeted about 200 beginner teachers across a range of routes into secondary teaching linked with our own institution, which is well-respected among Education departments in England; the response rate at about 25% was disappointingly low. We asked about teacher education route, teachers' own A level and degree background, confidence for teaching year 12 or year 13 A Level mathematics, experience of formal mark schemes, response to student solutions that differed from the mark scheme, and familiarity with 'new' examination papers at 16, the foundation for A Level Mathematics.

Of those who responded, about two-thirds felt confident to teach at least one year of A Level Mathematics and wanted to do so (though in workshops we saw some very superficial understandings). Only about a quarter had at that stage (May/June 2015) used a formal examination mark scheme, and over 50%, all of them on school-led routes, had not seen a specimen assessment for examinations at 16, despite the fact that typically, all might expect to be teaching for that assessment from September 2015. Most with a non-mathematics-rich degree said 'they'd accept any solution which gave the right answer'.

Although we do not claim the response is representative of all entrants into English secondary mathematics teaching we would highlight several aspects of these responses as being of concern. First, many of these beginner teachers appear to have inflated ideas of their own preparedness for teaching mathematics at a higher level. Second, the beliefs of those from non-mathematics specialist backgrounds and those from mathematics specialist backgrounds appear to attach different importance to *how* students arrive at a solution. Thirdly, many respondents had at that stage experienced little education in the use of formal mark schemes or of new assessments for courses they were about to teach. We would argue that in the context of English education, these are important facets of a teacher's work if beginner teachers are to be well-prepared to enter teaching and their students to have confidence in them.

Workshops

Opt-in workshops, offered during a conference-style day for beginner teachers, focussed on the policy-driven shift from fairly structured to semi-structured questions on year 12 mathematics content. We used past and sample assessment materials to

demonstrate intended changes, educate participants in the use of standard mark schemes, and probe their response to changes, both in principle as representatives of the discipline and for their implications for teaching and learning. Participants attempted questions under time pressure, peer-marked resulting solutions and reflected on both the technical challenges of answering and marking questions, and their emotional responses to less structure and to formal assessment of their work.

Participant beginner teachers claimed to value the intended changes highly, but perceived them as demanding and daunting for both teaching and learning. They problematized the associated single, timed assessment window and long examinations while simultaneously recognising the inappropriateness of short examinations for genuine problem solving. In feedback, they claimed a wide range of learning from the workshops, especially related to the 'live' solutions elicited. These related especially to a range of pedagogical gains: skills in 'unpicking' written solutions (as in Webb, 2009), ways to use assessment of solutions formatively, development of written and oral feedback (as in Grainger & Adie, 2014), specific teaching strategies such as peer marking using formal mark schemes, and techniques for teaching for mathematical communication (Webb, 2009). They suggested an increased appreciation of peer discussion, of student resilience, of teacher modelling, and of teachers working questions themselves. They also identified significant emotional responses to getting stuck, to attempting unstructured questions under time pressure, and to peer feedback, as in Lavi and Shriki (2014), and also implications for their practice at all levels of teaching.

These beginner teachers generally concluded the intended policy changes are exciting, promote aspects of mathematical learning they value deeply, and could be enjoyable. They will also be challenging for both teachers and students.

Experienced A Level teachers in shorter but similar sessions reported very similar responses, though in some cases more marked in terms of challenge for teachers and students: written comments included 'I feel threatened by this'; 'I'm now panicking: how can I ever get my students to do this?' and 'I'm not sure I have the mathematical knowledge to be confident with this – it makes very different demands on me as a teacher'. Many claimed they would have to make significant changes to their teaching: 'this isn't about learning a new bit of maths, it's about a fundamental change to how I've come to teach', though in contrast, one teacher said 'This validates the way I want to teach: I'm under pressure to produce results by whatever means, but this necessitates genuinely good teaching.' Some readily drew implications for their teaching of younger students: 'this isn't something you can teach in two years, it's got to be developed over a student's lifetime', and these teachers, already immersed in the high-stakes nature of teaching for A Level, appeared more aware of the challenge of devising valid and reliable mark schemes for more genuine problem solving.

The pedagogical implications drawn by experienced teachers focused on specifics of how to build up students' experiences, and where/how to find appropriate support and resources for that. These responses identify sizeable challenges for even experienced teachers, and with final specifications and assessment materials to be accredited less than a year before first teaching, it is hard to see how many will be well prepared. However, strikingly, almost all participant experienced teachers, despite their misgivings about enactment, claimed alignment of the intended changes with their deeply-held mathematics education values: 'this is really what maths is about, isn't it – yes, you need the techniques and the knowledge, but unless you can

use them they're sterile.' Cooney (1985) and Golding (2016) show such beliefs do not necessarily translate into practice if they are avoidable.

Additional findings from interviews

Three beginner teachers interviewed had a predominantly mathematics background, one as an academic mathematician. She, and the fourth teacher, came from a school-based course, and had experienced comparatively little guided reflection or academic input in developing teaching; all, but especially these last two, claimed considerable gains from the comparatively deep reflection elicited in the two-hour workshop. All interviewees valued talking about big issues in teaching, and two identified the importance of Higher Education involvement or of a reflective and knowledgeable school department for their deep and longitudinal development.

Two felt that the workshops had highlighted for them the tensions experienced in school between choices 'as a mathematician' and 'for exam results', and the challenges in following through one's values on a daily basis. They talked about 'you really have to believe it a lot' and the difficulty in standing out against a department, especially given their perception of overcrowded curricula. No interviewees had had in-school discussion about teaching (or assessing) for problem solving – at any level, yet in three months all would be teaching 14-16 year olds for a new curriculum rich in problem solving. All made repeated reference to their own mathematical background, and this appeared to frame their current approach, as in Grainger and Adie (2014). Those with a more mathematical background were able to reason quite deeply about the challenges involved in teaching for problem solving, including a need for developing robust fluency and a long term approach. All centralised challenges of formative assessment for problem solving in their talk, but for two, mathematical awareness of related issues such as communication and rigour still seemed low.

Conclusion

Beginner teachers in our study often appeared ill-equipped to engage with key aspects of teaching for problem solving, or with formal summative assessment. A relatively short, structured workshop session enabled many to enact and critique mark schemes, and to engage quite deeply with a variety of related pedagogical issues, including those of student affect. Teachers used professional interactions with us and with their peers to 'replay' their previous or observed practice and compare and contrast that with possible enactments of the new intentions; they 'rehearsed' specific aspects of pedagogy associated with that, including both formative and summative assessment, and articulated a 're-envisioning of practice', in Horn's (2010) terms.

Although wide claims for beginner teacher learning were made, it would be unrealistic to think that such learning was embedded and it should be further and overtly developed, including in school. Even those with apparently secure and deep subject knowledge identified as problematic the improvisational performance, link-making and strategic thinking required to enact envisaged changes effectively, as Livingstone and Borko suggest, and these take considerable time and effort to develop (Berliner, 2004). However, many experienced A Level teachers also did so: expertise is in part very context-specific, although Berliner (2004) cites evidence that expert teachers more quickly and effectively adapt to new expectations. The envisaged problem-solving focus appears to be well-aligned with the mathematical values of many teachers, both beginner and experienced, but teacher preparation for such changes appears minimal, and for some experienced teachers the changes can appear a

real threat to their effectiveness. If such ambitious change is to be realised across A Level mathematics classrooms, it would appear that substantial targeted teacher support and development is needed: this study offers one way to begin that process.

Acknowledgements:

We are grateful for support from the Further Mathematics Support Programme for this study.

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