PRE-PRINT

ERME book

Chapter 11: Technology and resources in mathematics education

Jana Trgalová, Alison Clark-Wilson & Hans-Georg Weigand

1. The technology thematic working group at CERME: history, background, aims and scope

The thematic working group on technology at CERME was established at the very first CERME in 1999, as one of only seven themes, which highlights the importance that the mathematics education community had placed on research on technology in mathematics teaching and learning at this time. Over the subsequent conferences, the group continued to grow, from nine contributions at CERME1 to around forty at CERME6 and 7, which has led to a sub-division to two groups addressing the theme from the perspectives of students and teachers respectively.

From the outset, the group has considered mostly digital tools and technology encompassing mathematical software and applications, programming languages, communication platforms, and mobile devices. Recently, more elaborated concepts of resources have led to tools and technology being considered more systematically as a component of the full range of resources available for students, teachers or teacher educators. Thus, since CERME6, the group welcomes contributions not only on digital tools, but also on more traditional mathematical tools, textbooks and other resources.

Since that very first conference, the work of the technology group has been framed by the following "three embedded levels" that can be considered "when analyzing the use of tools in mathematics education":

- the level of the interactions between tool and knowledge;
- the level of interactions between knowledge, tool and the learner;
- the level of integration of a tool in a mathematics curriculum and in the classroom (Laborde, Gutiérrez, Noss, & Rakov, 1999, pp. 183-184).

These three levels highlight the four components that can be distinguished in a didactic system involving any technological tool, namely the tool, some knowledge, student(s) and a teacher, and the inevitable relationships between these poles. Such a system can be represented by a *didactic tetrahedron*, as shown in Figure 1, inspired by Tall (1986, p. 25). The didactic tetrahedron introduces a fourth component (vertex), a technology or a

cite as: Trgalová, J., Clark-Wilson, A. & Weigand, H-G. (for publication 2018), Technology and resources in mathematics education. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven, *Development of European Research in Mathematics Education: Twenty Years of Communication, Cooperation and Collaboration*.

resource, into the traditional representation of a didactic system as a triangle: teacher – learner – knowledge; and the impact of this introduction on the other three vertices.

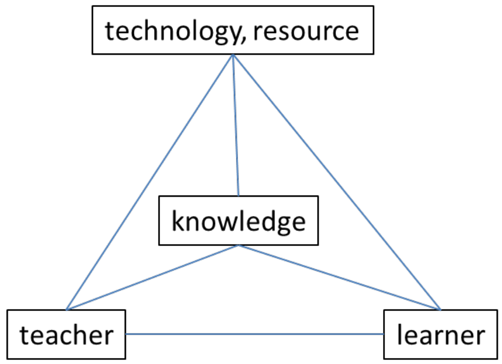

Figure 1: Didactic tetrahedron

The chapter aims to capture the work of the technology group community over time with respect to three sub-themes that have permeated this collaborative work: deep articulation of the nature of technological tools and resources and related interactions (Section 2); explanations of the principles and theories relating specifically to task design in technology-mediated environments (Section 3); and an expansion of our knowledge of theories and approaches that underpin and/or explain research (Section 4).

We conclude the chapter with a summary of the significant research by the group over the past 20 years, its potential and limitations; and the impact of this work within and beyond the CERME community. We close by offering a prospective vision for the possible trajectories for future research. This is set within the context of a world where the rapid growth in both access to, and design of, new technologies within and beyond mathematics education is increasingly hard to understand.

2. Tools and resources

A *resource* or *medium* (lat.: medium = middle, midpoint) is something that is positioned between two domains. Language, gestures, paper, pencil, books, videos, ruler and compass, computer or interactive whiteboards are media and, in mathematics, they are positioned between mathematical objects (concepts, statements, algorithms) and human thinking, they mediate between mathematics and understanding. *Tools* are a special form of media. Monaghan, Trouche and Borwein (2016) give a quite general and "somewhat crude" definition of a "tool" as "something you use to do something" (p. 5). This already shows the fundamental and global aspects of a tool in human activities, which makes it difficult to give a meaningful and satisfactory definition. *Mathematical tools* enable us to create, to operate with and to transform mathematical objects. Computers and especially programs such as spreadsheets, dynamic geometry systems (DGS) and computer algebra systems (CAS) are *digital mathematical tools*. Mathematics education questions the meaning of media and tools for the teaching and learning of mathematics, which is the primary interest of the CERME technology group. In what follows, we aim to not only reflect these discussions from within the CERME technology group(s), but also connect the group's work with the global discourse on digital technologies in mathematics and mathematics education.

Some common threads include:

- the recurring discussions on the relationships between the theoretical and practical aspects of tool use;
- the construction and development of theories that have been modified for (digital) tools and resources;
- the interrelationships between digital tools and other (non-digital) resources.

2.1 From suggestions of classroom use to more general reflections on technology-enhanced teaching and learning

Prior to CERME1 in 1999, an intensive discussion about *new* – nowadays digital – technologies in mathematics education had already begun. At that time, the main goal was to develop and evaluate strategies for the integration of digital tools in mathematics curricula and classrooms. For example, as one of the first DGS, Cabri-Géomètre had appeared in 1988, by CERME1, the group was able to draw upon experiences of over 10 years of its use. Consequently, the most important features of this class of tools such as variation of objects by dragging, visualisations of loci, and authoring macro-constructions had begun to be widely discussed. In addition, the didactical implications of these features created opportunities for new problem solving strategies, the discovery of geometrical theorems as invariant features, developing conjectures and making or discovering mathematical proofs.

CERME1 built on this earlier work by adopting a research-oriented view with questions such as: What are students' views and interpretations of dynamic tools? What is the relationship between drawings and symbols? How can technologies support the learning of the concepts of variables and functions? The contributions to this conference concentrated mainly on particular uses of tools in the classroom with many examples of the dynamic affordances of these tools.

The following two conferences, CERME2 and 3, led to a shift from the fascination of the new technological possibilities and novel examples of tool use in particular content-oriented environments, to theoretical reflections concerning comparisons, relationships and connections between tools. Moreover, other considerations of technology design and use emerged, such as the use of technology for distance education and the potential impact of technology on the nature of examinations. In this period increasing attention was being paid to teachers' roles in technology-mediated mathematical activities and their associated knowledge base.

Simultaneously, the focus of the scientific discussion concerning digital technologies *outside CERME* was CAS. In 1995 the first calculator with CAS became available, the *Texas Instruments' TI-92*, followed in 1999 by the *Casio FX 2.0*. These calculators generated a high level of expectation within the community. The possibility for students to have a readily available tool, which would do (nearly) all symbolic transformations of high school mathematics at the press of a button, was predicted to lead to deep, far-reaching changes for the content of the curriculum and its examination. These particular tools promoted the research community to rethink existing theories and develop new interpretations (see Section 4).

By the time of CERME4, the questions concerning tool classification and design were more specified. It was evident that the word *tool* meant a variety of objects with different characteristics. There were special *pedagogy-free environments* like CAS, DGS, graphing and programming tools. Other tools, often called applets, microworlds or special learning programs could be considered as local *dedicated* environments. This raised the question concerning the characteristics of each of these tools: What kind of technological tools do we need in our teaching? Moreover, it was also apparent that the reflective use of tools in the learning process needed theoretical frameworks specific to the tool and mathematical content, for which the *instrumental approach* emerged as a central theoretical framework. CERME5 drew on the concepts introduced by the instrumental approach and opened questions about the design and appearance of tools to support successful appropriation, integration and institutionalization for both students and teachers. These ideas are described further in Section 4.

2.2 The move to technologies as tools within a resource system

In the beginning of the new century, there were two demands concerning the use of technology in the teaching and learning process. On the one hand, there was a request for more sustained and longitudinal projects to obtain significant and convincing research findings in "real" classroom situations. A number of empirical studies had taken place over longer period that had revealed results that were common to many studies. e.g. e-CoLab[1] project in France, RITEMATHS[2] project in Australia, or the 'M^3-**M**odel Project New **M**edia in **M**athematics Education' (Weigand, 2008).

These increasingly robust findings established that using technology provides opportunities to:

- work within dynamically-linked multiple mathematical representations;
- construct new problem solving environments;
- design more personalised learning;
- integrate more realistic modelling problems into the mathematics classes;

whilst highlighting that technology use demanded:

- new types of learners' knowledge, e.g. to move between representational forms with understanding;
- new types of teachers' knowledge in relation to design, implementation and assessment;
- some rethinking of the content and hierarchies of the mathematics curriculum and its assessment.

On the other hand, questions around connectivity were emerging, e.g. how to connect students and mathematics through technology, students and teachers, and technology to other resources for teaching and learning (Monaghan et al., 2016, p. 433). On the tool level, CERME6 reacted to this aspect by adding the word "resources" in the name of the TWG, which was previously called "Tools and technologies in mathematical didactics". This expressed the need for considering technologies within the full range of resources available for students, teachers and teacher educators. Resources might be software, computers, interactive whiteboards, online resources, but also traditional geometry tools and textbooks. This demanded a deeper understanding of the relationship between technologies and the traditional tools and resources. How can these *old* and *new* resources interact with each other? For example, how can digital features be incorporated in new forms of textbooks? However, in the subsequent period there have been only a few contributions to the CERME technology group that have addressed this demand, mainly with reference to textbooks, e-textbooks and some online courses.

[1] e-CoLab = Expérimentation Collaborative de Laboratoires mathématiques, see http://educmath.ens-lyon.fr/Educmath/ressources/lecture/dossier_mutualisation/ecolab.pdf

[2] RITEMATHS = The project is about the use of real problems (R) and information technology (IT) to enhance (E) students' commitment to, and achievement in, mathematics (MATHS). http://extranet.edfac.unimelb.edu.au/DSME/RITEMATHS

2.3 From the students' uses of tools to that of teachers

The introduction of the term *resources* also was in line with the development of a *documentational approach to didactics* (elaborated in Section 4). This places a greater emphasis on the roles and actions of teachers, a theme that was first introduced at CERME3 through the following questions:

- How can we understand how mathematics teachers integrate technology in their teaching?
- How might we encourage more mathematics teachers to use technology?
- How does using technology change the ways mathematics teachers think about teaching and learning? (Jones & Lagrange, 2003)

These questions, supplemented by others, have remained an important focus for the group, and have sowed the seeds for the more recent sub-division of the group (see Section 1). However, with respect to tools, this has meant that, in many cases the user is no longer the student alone. The teacher perspective is now not only considered alongside, but specific functionalities and environments have been developed for the primary purpose to support the teacher.

Adopting a more holistic perspective of tools that includes teachers, we should consider all of the processes inherent in the design of teaching: looking for resources, integrating these in a personal resource system, implementing resources in practice, sharing resources with colleagues, revising to take account of feedback, etc. This wider discussion concerning teachers' integration of technology continued at CERME8 through particular examples from research: for example, using interactive whiteboards in geometry, creating tests and examinations in a CAS-environment and exploring the potential of technology for the teaching and learning of functions.

In addition, the group again emphasized the need to focus research more intensively towards longer-term studies involving practising teachers within "real" classrooms settings. Another request was to concentrate more on emerging research themes present within the general technology literature, which at that time had been underrepresented at ERME conferences. Examples included the design and use of innovative technologies such as Web 2.0, mobile technologies, the development of e-textbooks or the design and use of technologies and resources for learners with special educational needs. CERME9 and CERME10 reflected the great variety of digital books or e-books for classroom use. Participants also explored the meaning and the impact on mathematical learning of: free and widely available *online courses*, such as the Khan Academy[3], that offer free tools to allow teachers to monitor students' activity and provide them with feedback and guidance; tablets that emphasize the meaning of gestures, e.g. zooming with finger movements, drawing graphs by using the finger; and particular digital learning environments, which also included different kinds of computer games.

2.4 From local empirical studies to scaling up good practices with digital resources

In 2010 the 17th ICMI study "Mathematics Education and Technology – Rethinking the Terrain" (Hoyles & Lagrange, 2010) was published, revisiting the theme of the very first ICMI study "The Influence of Computers and Informatics on Mathematics and its Teaching" (Churchhouse 1986). Given the great enthusiasm for the new possibilities that computers and technology might open to mathematics and mathematics education 20 years previous, the 2010 study gave a disappointing account of the current situation concerning the dissemination of technology. Despite a high number of research studies and accounts of classroom practices, the use of technologies in mathematics education and the impact on curriculum and assessment change was still limited.

The group at CERME5 had also first highlighted the predominance of small empirical studies, calling for longer-term larger-scale research. More recent ERME conferences have begun to include contributions on this theme (Clark-Wilson, Hoyles & Noss, 2015; Lavicza et al., 2015), which explore ways to implement or transfer ideas or consequences of empirical investigations concentrated in the word *scaling* or *scaling-up*, i.e., researching how to realise the results of research into the reality of daily teaching.

Since an effective use of technological tools requires the thoughtful development of relevant tasks, we move now to consider in more detail the nature of mathematical digital tasks and aspects of their design and use for mathematical learning.

3. The design and implementation of digital mathematical tasks

Mathematical tasks are an integral element of mathematics education and its associated research agenda. The design of *digital* mathematical tasks has recently received particular attention, as a sub-theme of the 22nd ICMI Study "Task Design in Mathematics Education" (Watson & Ohtani, 2015) and within a dedicated volume of the "Mathematics in the Digital Era" book series (Leung & Baccaglini-Frank, 2016).

[3] www.khanacademy.org/

This section focuses on technology-mediated tasks with an emphasis on the explicit design decisions that influence how tasks are subsequently used in and for mathematical learning. It considers how such tasks are combined or developed to produce learning sequences or *courses* and finishes by addressing an important emerging theme - the design of tasks for prospective and practising teachers/lecturers for *their professional development to introduce and use technology in mathematics classrooms*.

3.1 Tasks and task designers

At the very first meeting of the TWG, it was highlighted that:

> One of the key issues for teachers is how to design tasks based on tools or technologies in which real questions for the learner emerge from the use of the tool, in which the tool is relevant and gives a new dimension to the task. (Laborde et al., 1999, p. 187)

Indeed, elements of task design have featured within many contributions to the early conferences, varying from the individual design decisions for tasks within classroom or research laboratory settings to those concerning whole courses within large-scale university courses. However, more usually the tasks were offered as a *given*, often subsumed within the notion of the tool or activity. Consequently, unless the research was specifically reporting aspects of task design, the constraints of the length and format of a CERME research paper/presentation often limited the opportunity for the task design to be explicitly described or theorised about. Early collaborations within the technology group at CERME highlighted the possible gap between a task designer's intended learning goals for a digital task and the mathematical meaning that learners ultimately construct. Whilst it has always proved challenging to separate aspects of the design of the tool from that of the task itself, in this section we try to distil the contribution of ERME research to the community knowledge concerning task design.

The term "task designer" has always held a broad definition to include teachers, researchers, teacher educators and technology developers – with many of the CERME participants representing one or more of these roles – and, as a result, offering enriched perspectives. Task designers appear to have been motivated by two broad approaches:

1. Development of innovative technological tasks that provide access to traditional mathematical knowledge and activity – often attempting to create a technology-mediated version of the equivalent paper and pencil task. For example, an early paper at CERME2 by Gélis and Lenne (2001) described the design of tasks using CAS-based technology to support upper secondary French students to learn about arithmetic sequences.

2. Development of innovative technological tasks that may lead to new forms of mathematical knowledge and activity. For example, the advent of dynamic geometry software and the dragging affordance led to a re-examination of the role of empirical measurement and 'checking by dragging' within the processes of justification and proof (e.g., Olivero & Robutti, 2001). By contrast researchers, who have been involved in the design of digital environment that have aimed to disrupt mathematics education norms by offering new ways of mediating existing mathematical knowledge or suggesting new epistemologies, have concluded that resulting tasks lack educational legitimacy at a system level. From the perspective of teachers and schools, tasks may not align with institutional constraints such as the prevailing classroom norms or assessment regimes, which may not have kept pace.

However, it has been common for research that began with more pragmatic motivations to integrate technology to improve the learning experiences in relation to the traditional curriculum to report findings in relation to new epistemologies and learning hierarchies.

3.2 Contexts and theories for task design

Reviewing the earlier contributions to the technology group, the vast majority of papers included examples of tasks that were used in the context of the research or study. However, it was rare for authors to describe explicitly their motivations for the design of the task, choice of representational forms and intended mathematical progression of potential pathways through the task. An exception to this is the example by Jones (1999), who described an empirical study in which pairs of students worked through a sequence of "specially designed tasks" involving the construction of quadrilaterals using Cabri-géomètre over a nine-month period (see Figure 2).

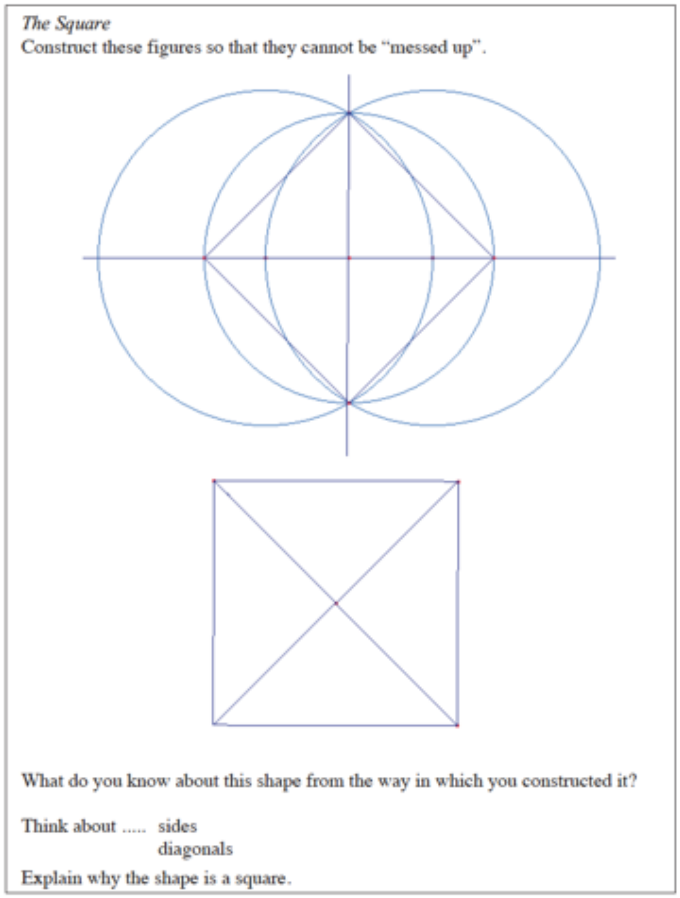

Figure 2: A task asking pupils to construct a square that is invariant under drag (Jones 1999. p. 258)

In addition, although, by reading the task carefully, it may be possible to assume the researcher's mathematical objectives inherent in its design, Jones still holds much "tacit knowledge" (Polanyi, 1966) related to the design, sequencing and classroom implementation of the sequence of tasks that featured in his study. This highlights a challenge for researchers, which is to articulate clearly the epistemological, psychological and sociocultural perspectives that underpin their task design decisions.

Interestingly, the CERME technology group seems to have arrived at some common understandings of what is inherent in such "specially designed tasks" in that they are often constructivist in nature - allowing students to explore and create mathematical knowledge, often working in pairs or small groups. It is only since CERME8 that a number of researchers have begun to share their analyses of aspects of task design using theories such as Brousseau's theory of didactic situations (Lagrange & Psycharis, 2013;

Mackrell, Maschietto, & Soury-Lavergne, 2013) and Variation theory (Attorps, Björk, Radic, & Viirman, 2013).

3.3 From the design of individual tasks to the design of courses

The sequencing of individual digitally mediated tasks to form a distinct course involves complex design decisions. For example, the development of a university-based numerical methods course was described by Belousova and Byelyavtseva (1999) at CERME1, who raised important questions about the balance between: students' empirical and theoretical work; individual work and group work; and a range of mediational roles for the technology. These themes have recurred in many subsequent contributions to the technology group as researchers have responded to the pragmatic decisions by institutions and policy-makers to design digitally-mediated courses, whilst simultaneously seeking to theorise about course designs, implementations and subsequent impacts on mathematical learning.

Task design within digital environments incorporates an articulation of the way in which the task is initiated/mediated. Given that a potential role of technology is to support the communication of both mathematical and meta-cognitive knowledge between the student and 'teacher', there have been many CERME contributions that describe research in which aspects of the teacher's role is outsourced to the technology in the form of scaffolding and feedback to the learner. The notion of *e-learning* can often imply a learning pathway mediated in the absence of an obvious *teacher*. For example, at CERME8, Fredriksen (2013) reported on design decisions concerning prospective teachers' uses of video lectures within an online course in Norway, whilst at CERME9, Jančařík and Novotná (2015) researched how teachers scaffolded student learning in an online course for talented children in the Czech Republic, concluding the importance of *offline discussions*.

There have been many research contributions to CERME that have focused on the design and use of digitally-mediated courses that have been developed over decades. Such examples are the "digital mathematics environment" (Freudenthal Institute, Netherlands) and the "Pepité" course in France. In both cases, an important feature is the provision of formative assessment data for the teacher on the students' mathematical responses to tasks.

More recently, the concept of a mathematics course is blurred with the emergence of the electronic book (e-book), which offers both sequences of tasks and embedded dynamic digital objects. The "creative book" developed by Gerianou and Mavrikis (2017) is such an example. However, there remain many unanswered questions related to their best design, implementation and impact.

3.4. Implementing digital mathematical tasks in research and classroom settings

Whilst it is easy to conceive that the task is an artefact that is offered to learners, a crucial component of task design relates to the many decisions about when, how and with whom a task is implemented in a research or classroom setting. Whilst this detail is a fundamental component of any research methodology, it is often an under reported aspect of task design. An exception to this is the research reported by Gallopin and Zuccheri (2001), which included detailed description of the phases of the "didactical path" adopted for their study that used two contrasting dynamic geometry softwares with Italian secondary school students.

3.5. Designing digital tasks for prospective and practicing teachers

More recently, attention in the technology group is shifting to research that concerns the design, implementation and impact of tasks that are intended for prospective and practicing teachers with the particular aim to develop their professional learning concerning technology use in classrooms. The group has always acknowledged that teachers needed to undertake specific professional learning to achieve this aim, however the nature and complexities of this learning have often been under-defined. Since CERME7, a number of theories have been developed that articulate aspects of teachers' technological knowledge and practices, see for example Ruthven (2009), Drijvers (2011), Haspekian (2011) and Rocha (2015). However, research on the application of these theories within the design of tasks intended for teachers' professional learning initiatives is still in its infancy.

4. Theories and approaches concerning technology and resources

Theoretical frameworks became an explicit theme discussed within the technology group since CERME3. This was a clear milestone in the progress of the group. In what follows, we outline the variety of theories and approaches that have permeated the group and trace the evolution of theories used in research on technologies in mathematics education over the past 20 years.

In charting the progress of the group since this time, the considerations of theories presented below are organized with reference to the didactic tetrahedron, attempting to isolate faces or edges according to the research focus to gain a deeper insight into the strengths and limitations of these frames, although we are aware that in technology-supported teaching/learning situations, all four vertices are intertwined and interact with each other.

4.1 Technology and knowledge

Prior to CERME3 most of the contributions investigate "a new epistemology of mathematics created by the use of the technology" (Laborde et al., 1999, pp. 185-186). Referring to the didactic tetrahedron (Figure 1), these issues are related to the *technology – knowledge* edge and address mostly the epistemological dimension of the use of technology in mathematics education. They can be classified in two categories:

1. *Exploring how technology mediates knowledge and the consequences of this mediation on the knowledge itself*. A paradigmatic example is provided by the new behavior of objects in a dynamic geometry environment, which actually gives birth to a new kind of objects (ibid., 1999, p. 185). A consequence of interacting with such new objects on the students' conceptualization of linear algebra notions is explored by Dreyfus, Hillel and Sierpinska (1999).

Knowledge mediation by technology can be addressed in terms of the "computerized transposition" (Balacheff, 1993) bringing to the fore tool-designed constraints (internal, command, interface) introduced by the use of a computer likely to impact upon the mathematical knowledge at stake. Alternatively, knowledge mediation can be interpreted through the notion of the "epistemological triangle" (Figure 3), which represents

> the connection between the mathematical signs, the reference contexts and the mediation between signs and reference contexts which is influenced by the epistemological conditions of mathematical knowledge. (Steinbring, 2006, p. 135)

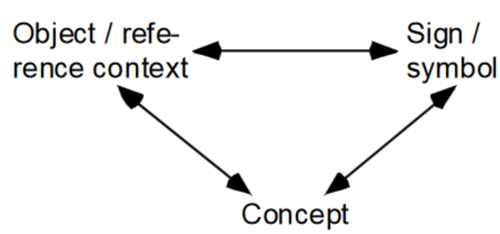

Figure 3: Epistemological triangle

Both theoretical constructs are usually combined with an epistemological content analysis, which defines the essence of the mathematical knowledge at stake.

cite as: Trgalová, J., Clark-Wilson, A. & Weigand, H-G. (for publication 2018), Technology and resources in mathematics education. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven, *Development of European Research in Mathematics Education: Twenty Years of Communication, Cooperation and Collaboration*.

2. *Investigating better ways to learn mathematical concepts with technology*. This category of research focuses on software or task design and on *a priori* analysis of the potential of the task as instrumented by a chosen technology in order to achieve a known learning goal. Among the frameworks mobilized by the researchers are:

- *theory of semiotic mediation* (Bartolini-Bussi & Mariotti, 2008) assuming that, in social contexts, mathematical meaning can be created from specific uses of a tool (e.g., in the *L'Algebrista* microworld (Cerulli, 2001), expressions and commands may be thought as external signs of the algebraic theory and transforming an expression into another using available buttons as proving a theorem);
- *situated abstraction* (Noss & Hoyles, 1996) used to "describe how learners construct mathematical ideas by drawing on the webbing of a particular setting which, in turn, shapes the way the ideas are expressed", the situatedness emphasises the "specificities of the situation, and in particular [...] the linguistic and conceptual resources available for expressing mathematically within them" (p. 122).

4.2 Technology, knowledge and student(s)

From the first ERME conferences, researchers investigated "the complex interplay between the work in a technological environment and the development of mathematical understanding and skills" (Barzel et al., 2005, p. 928), addressing the cognitive and instrumental dimensions of the use of technology.

Several theoretical constructs are used to explore such interplay:

- The concepts of "embodied cognition" and "metaphors" (Lakoff & Nuñes, 2000) viewing mathematical meaning as rooted in our experience of common phenomena such as movement and working as a metaphor. This framework is used for instance to analyze students' cognitive processes while doing activities involving artefacts such as movement sensors and the corresponding development of the function concept.
- The *theory of didactic situations* (Brousseau, 1997) considering digital technology as a component of the 'milieu' with which a learner interacts and allowing for the analysis of the possible learner – milieu interactions alongside the related learning outcomes.
- The notion of "instrumentation" appears at CERME3 as the approach that "distinguishes the instrument (a psychological construction) from the artefact (the material object involved in an instrumented action)" (Jones & Lagrange, 2003). The *instrumental approach* (Rabardel, 2002), which emerged as the most central

theoretical framework at CERME4 (and subsequently), pinpoints that "given a tool, the genesis of a fruitful instrument is far from self-evident, but is the result of a social process, guided by a set of tasks in a given institution" (Barzel et al., 2005, p. 929). This construct has proved particularly helpful when studying the evolution of technology use.

4.3 Technology, knowledge and teacher

As previously alluded to, the awareness of the importance of the teacher dimension in research on technology in mathematics education (the *technology – teacher – knowledge* face of the didactic tetrahedron) emerged slowly. It was first considered explicitly during CERME3 and also featured in a small number of contributions at CERME4, as pointed out by Barzel et al. (2005) who commented:

> On the issue of the second theme, the role of the teacher in technology-rich mathematics education, we observe that in spite of the relevance that is attributed to this theme, little research was reported in this working group. (p. 937)

Subsequently, four aspects of the teacher dimension have been addressed by the technology group community:

- Investigating the role of the teacher in technology-based settings;
- Analyzing teachers' practices involving technology;
- Characterizing the new knowledge and skills required for efficient use of technology, and its evolution;
- Designing and assessing teacher education/teacher training programmes.

The interest of researchers in the *instrumental approach*, used so far for studying issues related to the *knowledge – technology – learner* face of the didactic tetrahedron, raised the question of its adaptation for teachers. Soon, new concepts and approaches have been developed and shared within mathematics education community through, in particular, the CERME technology group:

- "instrumental orchestration" introduced by Trouche (2004) and further developed by Drijvers et al. (2009) who define it as "the intentional and systematic organisation and use of the various artefacts available in […] computerised – learning environment by the teacher in a given mathematical task situation, in order to guide students' instrumental genesis" (p. 1350);
- "double instrumental genesis" (Haspekian, 2011) highlighting that, in order to efficiently use a digital tool in her teaching, a teacher has to develop not one, but two

instruments: a "personal instrument" for mathematical work and a "professional instrument" for teaching mathematics.

- *documentational approach to didactics* (Gueudet & Trouche, 2009), which, drawing on the instrumental approach, substitutes artefact by resource and instrument by document and takes into account a great variety of resources intervening in teachers' work: textbooks, students' worksheets, Internet resources, discussions with colleagues, etc. The development of this framework has led to the re-conceptualization of tools and technology within the CERME group by considering them within a wider range of resources (see Section 2.2).
- the role of "hiccups" in technology-mediated lessons, the perturbations experienced by the teachers *triggered by the use of the technology* that illuminate discontinuities in their knowledge and offer opportunities for the teachers' epistemological development (Clark-Wilson, 2013).

Several studies aim to analyse teachers' practices with the use of technology in order to get a deeper insight into the complexity of technology integration. Assude (2007) elaborates a theoretical tool that characterises the degree of teachers' integration of technology taking into account both the *instrumental* (i.e. how instrumental integration is taken into account by the teacher, focusing in particular on orchestrations she uses and types of tasks she proposes) and the *praxeological* (i.e., how the pupil's mathematical work is organized by the teacher, focusing in particular on the relationship between paper-pencil tasks and techniques and tasks and techniques within the tool) dimensions.

Ruthven's *Structuring Features of Classroom Practice framework* (2009) introduces five key components that structure teachers' classroom practices with technology: working environment, resource system, activity format, curriculum script, and time economy. These features "shape patterns of technology integration into classroom practice and require teachers to develop their craft knowledge accordingly" (p. 52). Further developments of this approach were envisaged recently at CERME10 by suggesting introducing a sixth component "relating to teacher craft knowledge for managing different types of student behaviors or attitudes" (Gustafsson, 2017 – to appear).

Abboud-Blanchard and Vandebrouck (2013) have developed a model for studying evolutions of teachers' practices in terms of technology uses, combining the *activity theory* (Engeström, 1999), the instrumental approach and the *double approach to teachers' practices* (Robert & Rogalski, 2005). The latter considers a teacher's activity through five components (*personal*; *mediative*; *cognitive institutional* and *social),* a frame also used by Emprin (2007) to analyze training courses aiming at the use of technology.

The *Technology, Pedagogy and Content Knowledge* (TPACK) framework (Mishra & Koehler, 2006) is a dominant frame used to address teachers' professional knowledge and skills, suggesting seven categories of this knowledge: *mathematical content knowledge* (CK), *pedagogical knowledge* (PK), *technological knowledge* (TK) and all possible intersections of these (PCK, TCK, TPK and TPCK). However, to date, only a few studies have been reported to the group on the application of this frame to mathematics education.

At CERME10, Abboud and Rogalski (2017 – to appear) introduced new theoretical concepts of "tensions" and "disturbances", developed within a model of instrumented activities of teacher and students. "Tensions" in a teacher's activity are "manifestations of 'struggles' between maintaining the intended cognitive route and adapting to phenomena linked to the dynamics of the class situation", whereas "disturbances" are "consequences of non-managed or ill-managed tensions that lead to an exit out of the intended cognitive route".

The notion of "community" is another concept widely used, mainly in relation with teacher professional development. "Communities of practice" (Wenger, 1998), or "communities of inquiry" (Jaworski, 2005) are either established purposefully by the researchers or teacher educators to accompany teachers' efforts with integrating technology in their everyday practice (Fuglestad, 2007), or they develop spontaneously around Web2.0 tools enabling sharing resources and practices (Trgalová, Jahn, & Soury-Lavergne, 2009).

This overview of theories used in CERME contributions reveals a wide variety of frames, which can be seen as a wealth of the research field, but there is a risk of "the framework compartmentalization that could hinder the capitalization of knowledge and its practical exploitation" (Artigue, Bosch, & Gascon, 2011, p. 2381). The awareness of this risk appears quite early within the CERME technology group, at CERME4 that concluded:

> a more ecological and systematic approach is needed rather than a unifying theory, which takes into account the existing subsystems, and which combines various theories focusing on each of these subsystems (didactics, instrumental approach, situated and distributed cognition, community of practice) (Barzel et al., 2005, p. 929)

The subsequent conferences call for further development of theories toward a comprehensive and more coherent landscape of articulated frames.

5. Conclusion

In concluding this chapter, a key question to ask is, what do we know now about technology integration in mathematics teaching and learning that we did not know before the technology group was established? The collective knowledge and experience of the community would suggest the following conclusion. It takes a significant amount of time for learners and teachers to become fully instrumentalised, that is to learn to use and apply the technology for their relevant mathematical purpose, which for teachers includes important didactic considerations and the development of their resource systems. The ongoing innovation of technological tools, within and outside of mathematics education, requires us to continue to address the *learner-knowledge-technology* face of the didactic tetrahedron. This might mean that we appear to reinvent the wheel again and again, as new researchers, teachers and technology designers encounter for the first-time *known* issues and challenges concerning technology integration for mathematics education. However, as goals in education have to be continually rethought and evaluated due to new developments in society, science and education, this also suggests it may not be possible for knowledge of theory to short-cut this process and it is a key role for the community of the CERME technology thematic working group to support important connections to be made. In the beginning, the focus of the research was on the effects of using technology on students' learning and teachers' practices. Now, as we know more about these effects, our attention has shifted to be concerned with researching how we can scale 'successful' innovations in mainstream education systems. However, at the most recent CERME10, within the submitted papers, little attention was paid to digital assessment of and for the learning of mathematics, and large-scale experimental studies were not presented.

It is notable that technology is now a visible element across all CERME groups. This is an indication of how it now permeates and has wide legitimacy across the mathematics education research landscape. This leads us to question our special role and to justify the important questions or topics that we can make a distinct contribution to. In some ways, this justification comes from the continued disappointment in the lack of widespread uptake of technology, first highlighted at CERME5:

> The use of technologies has simply not scaled up and the changes promised by the case study experiences have not really been noticed beyond the empirical evidence given by the studies themselves. (Kynigos et al., 2007, p. 1541)

It is the technology group that perseveres to confront this issue and, although new, possibly more exciting technologies arrive on the scene with great promise to revolutionise classrooms, we find ourselves expanding our experiential and theoretical knowledge in order to be able to better inform future attempts to scale.

One challenge faced by the group is that it has grown from only nine accepted paper submissions at CERME1 to twenty-seven accepted paper submissions (and 8 posters) at CERME8. The TWG leaders responded by dividing the group to emphasize research that foregrounded students from that which foregrounded teachers. However, given the frame of the didactic tetrahedron this is somewhat dissatisfying as most studies feature both perspectives. An alternative might be to divide the group by educational phase, as in the ICME conference series. However, we question whether it is possible to partition in a wholly satisfying way and this will be an ongoing topic for discussion for the group.

There is still a need to develop more comprehensive theoretical frameworks to address *old* but still topical themes, such as task design and methods for large-scale dissemination of *research-informed* practices with digital technologies. The role of technologies within processes of formative assessment, networked classroom technologies and e-learning (particularly *Massive Open Online Courses*) appear among the emergent issues that require further theoretical and methodological development. It is worth noticing that this issue becomes shared with the CERME group on theoretical perspectives and approaches as its call for contributions at CERME10 includes "Theories for research in technology use in mathematics education"[4].

Finally, with a focus on emergent technologies, we expect that future ERME conferences will feature research on *touch screens and human-computer interaction,* which is already a frequently discussed topic in present classrooms questioning how gestures can help visualising and, hopefully, understanding mathematical concepts; *3D technology*, including the use of 3D printers within mathematics education; *virtual and augmented reality* in mathematics education; *artificial intelligence* features to include intelligent tutoring and support systems that take account of large data sets; *ICT-support and special needs students,* particularly students with physical disabilities; and *digital technologies that support individuality*, for example, the creation of portfolios and personalised e-textbooks.

References

Abboud, M., & Rogalski, J. (2017 - to appear). Real uses of ICT in classrooms: Tensions and disturbances in the mathematics teacher's activity. *CERME10*.

[4] See http://cerme10.org/scientific-activities/twg-teams/

cite as: Trgalová, J., Clark-Wilson, A. & Weigand, H-G. (for publication 2018), Technology and resources in mathematics education. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven, *Development of European Research in Mathematics Education: Twenty Years of Communication, Cooperation and Collaboration*.

Abboud-Blanchard, M., & Vandebrouck, F. (2013). Geneses of technology uses: a theoretical model to study the development of teachers' practices in technology environments. *CERME8* (pp. 2504-2513).

Artigue, M., Bosch, M., & Gascon, J. (2011). Research praxeologies and networking theories. *CERME7* (pp. 2381-2390),

Attorps, I., Björk, K., Radic, M., & Viirman, O. (20013). Teaching inverse functions at tertiary level. *CERME8* (pp. 2524-2533).

Assude, T. (2007). Teacher's practices and degree of ICT integration. *CERME5* (pp. 1339-1348).

Balacheff, N. (1993). La transposition informatique, un nouveau problème pour la didactique. In Artigue M. et al. (Eds.), *Vingt ans de didactique des mathématiques en France* (pp. 364-370). La Pensée Sauvage éditions.

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh, & D. Tirosh (Eds.), *Handbook of International Research in Mathematics Education* (2nd ed.) (pp. 746–783). Mahwah, NJ: Lawrence Erlbaum.

Barzel, B., Drijvers, P., Maschietto, M., & Trouche, L. (2005). Tools and technologies in mathematical didactics. *CERME4* (pp. 927-938).

Belousova, L. & Byelyavtseva, T. (1991). Training explorations on numerical methods course using technology. *CERME1* (pp. 201-208).

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Kluwer Academic Publishers.

Cerulli, M. (2001). Introducing pupils to theoretical reasoning: the case of algebra. *CERME2* (pp. 139-151).

Churchhouse, R. F. (Ed.) (1986). *The Influence of Computers and Informatics on Mathematics and its Teaching*. ICMI Study Series. Cambridge: University Press.

Clark-Wilson, A. (2013). How teachers learn to use complex new technologies in secondary mathematics classrooms: the notion of the hiccup. *CERME8* (pp. 2544-2553).

Clark-Wilson, A., Hoyles, C., & Noss, R. (2015). Scaling mathematics teachers' professional development in relation to technology – probing the fidelity of implementation through landmark activities. *CERME9* (pp. 2333-2339).

Dreyfus, T., Hillel, J. & Sierpinska, A. (1999). Cabri based linear algebra: transformations. *CERME1* (pp. 209-221).

Drijvers, P. (2011). Teachers transforming resources into orchestrations. *CERME7* (pp. 2178-2187).

Drijvers, P., Doorman, M., Boon, P., & van Gisbergen, S. (2009). Instrumental orchestration: theory and practice. *CERME6* (pp. 1349-1358).

Emprin, F. (2007). Analysis of teacher education in mathematics and ICT. *CERME5* (pp. 1399-1408).

Engeström, Y. (1999) Activity theory and individual social transformation. In Y. Engeström et al. (Eds.), *Perspectives on activity theory* (pp. 19-38), Cambridge University Press.

Fredriksen, H. (2013). Mathematics teaching on the web for student teachers: action research in practice. *CERME8* (pp. 2554-2563).

Fuglestad, A. B. (2011). Challenges teachers face with integrating ICT with an inquiry approach in mathematics. *CERME7* (pp. 2328-2338).

Geraniou, I., & Mavrikis, M. (2017 - to appear) Investigating the integration of a digital resource in the mathematics classroom: the case of a creative electronic book on reflection. *CERME10*.

Gallopin, P. & Zuccheri, L. (2001). A didactical experience carried out using, at the same time, two different tools: a conceptual one and a technological one. *CERME2* (pp. 152-162).

Gélis, J.-M., & Lenne, D. (1999). Integration of learning capabilities into a CAS: The SUITES environment as example. *CERME1* (pp. 222-232).

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, *71*(3), 199-218.

Gustafsson, P. (2017 – to appear). Exploring a framework for technology integration in the mathematics classroom. CERME10.

Haspekian, M. (2011). The co-construction of a mathematical and a didactical instrument. *CERME7* (pp. 2298-2307).

Hoyles, C. & J.-B. Lagrange (Eds.) (2010). *Mathematics Education and Technology – Rethinking the Terrain. The 17th ICMI Study*, Springer: New York

Jančařík, A., & Novotná, J. (2015). Scaffolding in e-learning course for gifted children. *CERME9* (pp. 2354-2360).

Jaworski, B. (2005), Learning communities in mathematics: creating an inquiry community between teachers and didacticians. *Research in Mathematics Education, 7*(1), 101-119.

Jones, K. (1999). Student interpretations of a dynamic geometry environment. *CERME1* (pp. 245-258).

Jones, K., & Lagrange, J.-B. (2003). Tools and technologies in mathematical didactics: research findings and future directions. *CERME3*. http://www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG9/TG9_introduction_cerme3.pdf

Kynigos, C., Bardini, C., Barzel, B., & Maschietto, M. (2007). Tools and technologies in mathematical didactics. *CERME5* (pp. 1499-1508)

Laborde, C., Gutiérrez, A., Noss, R., & Rakov, S. (1999). Tools and Technologies. *CERME1* (pp. 183-188).

Lagrange, J.-B., & Psycharis, G. (2013). Exploring the potential of computer environments for the teaching and learning of functions: a double analysis from two traditions of research. *CERME8* (pp. 2624-2633).

Lakoff, G., & Nuñes, R. (2000): *Where Mathematics Comes From: how the embodied mind brings mathematics into being*. New York: Basic Books.

Lavicza, Z., Juhos, I., Koren, B., Fenyvesi, K., Csapodi, C., et al. (2015). Integrating technology into primary and secondary school teaching to enhance mathematics education in Hungary. *CERME9* (pp. 2430-2431).

Leung, A., & Baccaglini-Frank, A. (2016). *Digital Technologies in Designing Mathematics Education Tasks: Potential and Pitfalls* (Mathematics Education in the Digital Era, Vol. 8). Dordrecht: Springer International Publishing.

Mackrell, K., Maschietto, M., & Soury-Lavergne, S. (2013). Theory of didactical situations and instrumental genesis for the design of a Cabri elem book. *CERME8* (pp. 2654-2663).

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for integrating technology in teacher knowledge. *Teacher College Records, 108*(6), 1017-1054.

Monaghan, J., Trouche, L., & Borwein, J. (2016). *Tools and mathematics*. Dordrecht: Springer International Publishing.

Noss, R., & Hoyles, C. (1996), *Windows on mathematical meaning*. *Learning cultures and computers*. Dordrecht: Kluwer Academic Publisher.

Olivero, F., & Robutti, O. (2001). An exploratory study of students' measurement activity in a dynamic geometry environment. *CERME2* (pp. 215-226).

Polanyi, M. (1966). *The tacit dimension*. Chicago: The University of Chicago Press.

Rabardel, P. (2002). *People and Technology: a cognitive approach to contemporary instruments*. Université Paris 8. https://halshs.archives-ouvertes.fr/file/index/docid/1020705/filename/people _and_technology.pdf. Accessed 18 December 2016.

Robert, A., & Rogalski, J. (2005) A cross-analysis of the mathematics teacher's activity. An example in a French 10th-grade class. *Educational Studies in Mathematics, 59*, 269-298.

Rocha, H. (2015). Knowledge for teaching mathematics with technology and the search for a suitable viewing window to represent function. *CERME9* (pp. 2403-2409).

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Education et Didactique*, 3(1), 131-149.

Steinbring, H. (2006). What makes a sign a mathematical sign?– An epistemological perspective on mathematical interaction. *Educational Studies in Mathematics, 61*, 133–162.

Tall, D. (1986). Using the computer as an environment for building and testing mathematical concepts: A tribute to Richard Skemp, In *Papers in Honour of Richard Skemp* (pp. 21-36), Warwick. http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1986h-computer-skemp.pdf Accessed 15 September 2016.

Trgalová, J., Jahn, A. P., & Soury-Lavergne, S. (2009). Quality process for dynamic geometry resources: the Intergeo project. *CERME6* (pp. 1161-1170).

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematics Learning, 9*, 281-307.

Watson, A., & Ohtani, M. (2015). *Task Design in Mathematics Education: An ICMI Study 22*: Springer International Publishing.

Weigand, H.-G. (2008). Teaching with a symbolic calculator in 10th grade—Evaluation of a one year project. *International Journal for Technology in Mathematics Education, 15*(1), 19–32.

Wenger, E. (1998). *Communities of Practice: Learning, Meaning, Identity*. New York: Cambridge University Press.