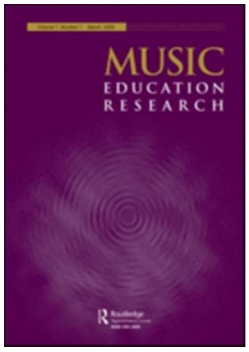

The integration of music and mathematics education in Catalonia and England: Perspectives on theory and practice

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The integration of music and mathematics education in Catalonia and England: Perspectives on theory and practice

The relationship between music and mathematics has often been the subject of discussion, both inside and outside the field of education. As part of an exciting project on a European scale, the paper explores the changing contexts in Catalonia (Spain) and England (UK) in relation to the integrated approach to the teaching of music and mathematics. We analyse three areas: academic literature, the curriculum frameworks, and publications and resources prepared by and for teachers. Our findings suggest that due to the more favourable attitude towards cross-curricular approaches in education, more progress has been made in England, in terms of developing resources to support an integrated approach to the teaching of music and mathematics, than in Catalonia. Nonetheless, teachers in both locations are very interested in developing these approaches. Although there is a need for further teacher training and support, there is evidence of progress already being made in schools.

Keywords: Integrated approach; mathematics; music

Introduction

Mathematics education is a major priority across Europe and in many other parts of the world. Attitudes towards music education, however, are more varied. Music education takes many different forms in European schools, albeit on account of the different musical traditions in each country/region, differences in school syllabi, and because initial teacher education (with the option of specialising in music or not) varies from one country to another. Within this mixed landscape, there is an emerging interest in the promotion of a joint, unifying vision of an integrated approach to the teaching and learning of music and mathematics that is of mutual benefit.

In Catalonia and England, there have been repeated curriculum changes over the last 30 years, many of which have been politically driven. Each of these changes has

resulted in quite different curricula and educational intentions. Also during this period, the issue of integrated approaches has often been discussed. In music and mathematics, integration has usually taken the form of music being used to support children to rote-learn number facts, but does this represent an integrated approach that benefits both areas of knowledge?

For the purpose of this article we will be defining integrated approaches as those that seek to develop understanding, knowledge and appreciation of both music and mathematics at the same time (in the sense suggested by Viladot and Cslovjecsek, 2014).

Within the framework of the *Comenius European Music Portfolio-Maths: Sounding Ways into Mathematics* project (2013-2016), we present a range of educational issues (academic literature, curriculum requirements, publications and resources for practitioners) in relation to an integrated approach to the teaching and learning of music and mathematics in Catalonia and England. We hope that this discussion will help to identify the complex nature of the task, and also the challenges and opportunities that have arisen in seeking to address this within the Comenius project.

General overview of the academic literature in Spanish and English

Existing literature from a Spanish perspective

In a Spanish-speaking context, the literature that relates to music and mathematics in the field of education is rather limited. This can be inferred from an initial analysis of articles on this topic published in the main Spanish refereed journals on both music and mathematics education, as shown by Casals et al. (2014). Among these journals, *Suma*

deserves special mention, as this is a mathematics journal review that includes an important number of the articles published on this topic in a Spanish-speaking context, most of them written by the same author, Vicente Liern. Liern's articles provide a theoretical perspective on questions relating mathematics to music and are aimed primarily at teachers in secondary and higher education (see, e.g., Liern 1994, 2009).

Most of the available literature is to be found in publications specialising in the field of mathematics education and taking a mathematical perspective. The reasons for this are unclear. It is possible that the scientific and theoretical profile of mathematics, as compared to the artistic idea that we have of music, is driving this bias. Perhaps also, a widespread perception that mathematics is more important than music in the curriculum also has had a decisive influence in this respect.

A second level of analysis of the literature surrounding this issue identifies two main types of publication, namely those adopting a theoretical approach (without teaching experiences and contributions) and others that provide an applied or pedagogical approach (with example strategies, materials and classroom activities). In this first group are found informative articles on specific aspects (e.g., Liern 2008, who describes the meaning that number seven has in the field of music/ the meaning of number seven in the field of music), educational research papers (e.g., Villasmil de Vásquez and Palomares 2007, who examine the influence of music in the development of logical-mathematical thinking) and studies taking an analytical approach (e.g., Lopez and Gustems 2007 or Navarra and Cian 1994, who reflect on the connection between music and mathematics as a basis for interdisciplinary approaches in the classroom). Although such a pedagogical application is not their main objective, these articles often propose exercises, especially on mathematical topics, for both mathematics and music teachers in secondary and higher education.

In the second group of publications there is a larger number of works focusing on music and mathematics in early childhood education (e.g., Ayala et al 2003; Lázaro and Riaño 2009) and also on secondary education (e.g., Arenzana and Arenzana 1998), rather than on primary education. This biasing may be because early years practice is characterised by blurred subject boundaries, whereas subjects are usually much more distinct at secondary school level. Certainly, the type of music and mathematics content found in publications relating to secondary education are quite specific (e.g. the Pythagorean scale, or the connection between fractions and musical notes).

Lastly, the literature published in a Spanish-speaking context reveals a stronger tradition in combining music and mathematics in education in the Latin America region compared to Spain. An example of this are the projects carried out by Venegas et al. (2013) in Chile, who developed a computer program with activities relating Maths to Music, and Bustos (2007) in Argentina, who presented two applied research projects in the classroom that demonstrate the potential to connect these two fields.

Existing literature from an English perspective

Whilst there is much literature published in English which focuses on issues linking music and mathematics, we will focus here on a few particularly pertinent examples.

In terms of actual classroom practices, there have been a number of studies exploring the integration of mathematics and music teaching (e.g. An et al., 2013; Jones and Pearson, 2013; Still and Bobis, 2005). However, these seem to have been mostly short-term interventions, so it is not possible to identify significant impact on overall curriculum development and practice. The NCTM (National Council of Teachers of Mathematics, USA) is intending to fund a number of classroom-based studies to

investigate the use of music to teach mathematics during the period from June 2016 to May 2017 (https://www.nctm.org/Grants-and-Awards/Grants/Using-Music-to-Teach-Mathematics-Grants/, last accessed 21 January 2016).

In a broader sense, there have been a wide range of studies which have tried to identify the positive effects of music on children's achievement in other areas of the curriculum, including mathematics. For example, the visual representation of music is reported to help to develop children's sensory integration and memory (Aizenman et al., 2013). In addition, playing music from notation, whether formal or informal, has been reported to help to develop visuo-motor coordination (Brown et al., 1981). Taking such findings together, alongside the proposal that there is a positive link between children's visuo-spatial skills and their mathematical attainment (Tosto et al., 2014), implies that both playing music and notating music can contribute to the development of children's competence in mathematics.

In terms of mathematics education, there is much literature (e.g. Burton, 1984; Fauvel et al., 2003; Hofstadter, 1979) that has tried to demonstrate the link between mathematics and music by exploring what is meant by 'mathematical thinking'. By describing mathematical thinking as relying on pattern recognition, iteration and repetition, it is easy to see that these processes are found not only in mathematics, but also in music (*cf* Ockelford, 2005). Moreover, recent work in neuroscience (e.g. Zeki et al. 2014) suggests that our brains experience beauty in music in the same way as we experience beauty in mathematics.

Finally, the evidence that mathematics is best learned in a context that is meaningful and engaging for children (Van den Heuvel-Panhuizen, 2001) suggests that, if mathematics and music are to be approached in an integrated way, teachers will require appropriate subject knowledge, confidence and creativity within and across

these two knowledge areas.

Integration of music and mathematics in education

Curriculum perspectives in Catalonia

Catalonia, as an autonomous community in Spain, holds delegated powers in the field of education. Since 2007, the school curriculum in Catalonia has been aimed at achieving eight basic competences. These include mathematical as well as artistic and cultural competences,[1] although – at present – not all are defined to the same degree of detail and the artistic and cultural curriculum remains to be developed.

The current primary school level curriculum (Departament d'Educació, 2015) is divided into six fields, which include mathematics and arts education (where music enters). In relation to arts education, the following goal was established:

> *Arts education in primary education aims to develop the competences of perception and expression that allow students to understand their environment and the artistic and cultural worlds, and acquire tools to communicate through artistic languages*.
> (Departament d'Educació, 2015: 103).

Within this focus, the arts subject seeks to develop the three following competent dimensions: a) Perception, comprehension and valuation; b) Performance and production; c) Imagination and creativity. A part from these dimensions, the curriculum divides the contents into two parts: exploring and perceiving; and creating and

[1] The eight competences are as follows: communicative, linguistic and audiovisual; mathematical; knowledge and interaction with the physical world; artistic and cultural; digital; social and civic; learning to learn; and autonomy, personal initiative and entrepreneurship.

interpreting. On a musical level, this structure coincides in good measure with the leading musical-didactic tendencies of the present day, based on three areas of competence: listening, performance and composition (e.g. Swanwick and França, 1999; Zaragozá, 2009; Malagarriga et al., 2010). In short, the curriculum aims to promote music education through musical practice and experiences, but barely specifies the levels of competence to be achieved. Therefore, it allows teachers in Catalonia a great deal of flexibility when having to adapt the curriculum to their own circumstances.

The arts education curriculum focuses on two different subjects: music and dance on the one hand, and the visual and plastic arts on the other. For all these areas, it is recommended that teaching should aim at developing communication competences, which include the artistic and cultural competences, as well as others such as mathematics. Regarding this latter example, the only recommendation is found in the previous curriculum (2009) and states that the field of arts should "tackle geometric concepts and shapes, and work on rhythm and the musical scales" (Departament d'Educació, 2009: 96).

Meanwhile, the mathematics curriculum is focused on "learning mathematics for everyday life, to help interpret the world around us" (Departament d'Educació, 2015: 61). The main purpose of mathematics education is:

> *To help train citizens to know the world which they live in and make sure they are able to base their criteria and their decisions on solid foundations and adapt to changes in the different areas of their lives.*
>
> (Department d'Educació, 2015: 61).

Therefore, mathematics is conceived as an instrument of knowledge, reasoning,

and critical analysis of reality and the problems of the environment itself.

The mathematics curriculum is inspired by the NCTM Standards for School Mathematics (National Council of Teachers of Mathematics, 2000) and is organised into five blocks: numbers and operations, relations/connections and change, space and form, measurement, and data analysis and probability.[2] In any case, what guides the teaching-learning process in this area are the mathematical processes to be developed, which correspond to the aspects that structure mathematical competence: problem solving, reasoning and proof, connections, communication and representation.

Regarding the didactic meeting between music and mathematics in the curriculum and other official documents in relation to mathematics education at school, we only find vague references. For example:

> *It is important to offer different presentations of problems: oral, written, in images, with ICTs, etc. And in different contexts: real, using a story, mathematical and in other areas (science, music, arts, etc.)*
> (Departament d'Educació, 2013: 11).

In contrast, the very focus of the curriculum on competences and the emphasis on interdisciplinary projects encourages schools to work increasingly with inter- and cross-disciplinary proposals (González-Martín, 2013).

Curriculum perspectives in England

[2] The blocks of content 'relations and change' and 'space and form' are equivalent to 'algebra' and 'geometry' in the NCTM Standards.

What does music education look like in English primary schools? As in many other countries, attempts have been made to design a music curriculum that encompasses broader musical learning, includes playing an instrument, singing, creating music (through improvisation and composition) as well as developing understandings of musical culture and context.

The stated aims of music education in England are described as:

> *'Music is a universal language that embodies one of the highest forms of creativity. A high-quality music education should engage and inspire pupils to develop a love of music and their talent as musicians, and so increase their self-confidence, creativity and sense of achievement. As pupils progress, they should develop a critical engagement with music, allowing them to compose, and to listen with discrimination to the best in the musical canon'* (DfE, 2013).

The National Curriculum for Music in England has been revised a number of times since its inception in 1992 and is now in its fifth iteration. The amount of specification as to what children should study has recently been reduced, giving schools a framework in which to develop their own curricula for music. In the previous version of the curriculum, attainment targets were outlined that gave teachers an indication of musical progression through the curriculum. The 2013 revision for 2014 implementation removed these attainment targets. This amendment was designed to encourage schools to develop context specific understandings of musical progress, based on the particular musical strengths of the school. However, this has resulted in a national curriculum briefing that gives no guidance to teachers as to how to implement

the curriculum, nor how to approach the activities, nor what the teacher's role should be and what the relationship is between and across activities.

What does mathematics education look like in English primary schools? Mathematics education has seen similar changes to music education within the framework of the 2014 revised national curriculum (DfE, 2014). The mathematics programmes of study have been reduced for Key Stages 1 and 2 (for children aged 5 to 11 years). There is less detail in terms of how mathematics should be taught, so teachers have the freedom to make decisions about how and when to teach particular mathematical ideas and concepts. The emphasis is on depth of understanding, with children being moved on only when they have securely grasped the mathematical ideas and concepts being covered. There is also now an emphasis on age-related expectations, as opposed to levelling (as in older versions of the national curriculum). The intention is that the majority of the children in each year group will achieve a minimum set of outcomes in mathematics, as specified in the programmes of study.

The national curriculum makes specific reference to the fact that 'mathematics is a creative and highly interconnected discipline that has been developed over centuries' (DfE, 2014). The overarching aims of the national curriculum for mathematics are that children should:

- *become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately;*
- *reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language;*

- *be able to solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.*

  (DfE, 2014)

The idea of teaching mathematics across the curriculum is not new to the English education system, and has been a topic of discussion for decades. During this time, there have been a number of government-led initiatives, in addition to more general discussions in the world of education and educational research. In the new curriculum, there is an expectation that mathematics will be developed in all subjects areas across the curriculum, in order to enable children to appreciate the relevance and importance of mathematics (DfE, 2014).

In England, there have been a number of government driven initiatives to encourage cross-curricular teaching. The 'Numeracy across the curriculum' initiative (DfES, 2001) was designed to encourage teachers of 11 to 14 year old children in secondary schools to plan for a more cross-curricular approach to the teaching of mathematics. The document provided some useful guidance on where links could be made with music (and other subjects), without outlining any specific activities. Through posing questions, it was hoped that teachers would engage with some of the common features that cross subject boundaries. For example:

> *Music makes significant use of symbolic representation, as does mathematics. Do we use the similarities in the ways symbols are interpreted in both subjects?*
>
> *Can rhythm patterns, represented either symbolically or numerically, be seen to have parallels in mathematical sequences?*

Can pupils' knowledge of time and speed enhance their understanding of musical time, when considering technical issues such as beats per second and the differences between certain types of music, for example music from around the world, pop, techno, and so on?

Is the study of pattern in musical forms such as ABA, AABA, ABAB (leading to fugue, sonata and symphonic form) enhanced by pupils' understanding of repeating patterns in mathematics?

(DfES, 2001, p.93).

In primary education, the benefits of a cross-curricular approach to teaching were highlighted in the Excellence and Enjoyment agenda (DfES, 2004). It was suggested that a cross-curricular approach can deepen children's understanding by providing opportunities to reinforce and enhance learning in all subject areas. This was accompanied by a detailed analysis of the mathematics curriculum and an accompanying document was produced which identified cross-curricular links with a range of subject areas, including music. The proposed links were similar to those identified in the 'Numeracy across the curriculum' initiative, but appropriately adapted for the primary curriculum.

Bearing all this in mind, we still need to ask why with all this encouragement, we do not see more integrated approaches in our classrooms.

Publications and resources for practitioners in Catalonia and England

Catalonia

In the Catalan context, publications that relate to music and mathematics in education are quite scarce. These publications are always by the same authors and deal with

similar topics. These often focus on adapting subject matter to primary education, but taking a more historical than didactic approach. Many of these proposals have a unidirectional nature – making use of music to learn mathematics (Segarra, 2008) – and few of them have a bi-directional character, or promote real interaction between the two subjects (such as Saguer & Saguer, 2004). Lastly, there are some manuals with suggestions for work activities at school to mark special days or festivals (e.g. Liern and Queralt, 2008).

Despite this limited output, it seems that interest in interdisciplinary relationships and connections between subjects has grown in the Catalan educational context in recent years. Evidence of this is the take-up of professional development courses exploring the integration of subjects (Lago, 2007) such as music and mathematics (Casals et al. 2016; Carrillo et al. in press), and the development of undergraduate projects (Frade, 1994; Rué, 2007), and postgraduate research (Andreu, 2012; Viñas 2016) by practising teachers.

In schools there is a growing interest in developing practice to explore the relationship between music and mathematics and, in general, in the interdisciplinary connections that school subjects can generate. Examples of these are the blogs created by educators, such as *Matemusicant* (http://matemusicant.blogspot.com.es/), *Sumado* (http://sumado.blogspot.co.uk/2010/05/matematicas-vs-musica.html) and *Musicomàtics* (https://musicomatics.wordpress.com/) that contain resources and ideas that have been tried and tested by teachers. In addition, the authors of this paper are aware of other resources such as workshops and exhibitions, where pupils and teachers can experiment

with materials and activities that they can later make use of in the classroom (see examples below[3]).

England

In the last ten years, a number of books have been published that focus exclusively on mathematics across the curriculum. These books have concentrated on the benefits of cross-curricular teaching and have also attempted to provide some examples of how this can be managed (e.g. Fox & Surtees, 2010; Pound & Lee, 2011; and Hansen & Vaukins, 2012). There have also been a number of publications that use music and songs to introduce and practise mathematical concepts and principles, such as those by MacGregor (2005) and MacGregor and Chadwick (2011). Although these do not adopt the integrated approach that we are proposing, they do try to use songs to introduce concepts and principles alongside ideas for how to develop these further.

In addition, there are a range of other resources that are available within mathematics and music education to support teachers by providing cross-curricular approaches within the teaching of music and mathematics. For example, NRICH (http://nrich.maths.org), an online mathematics resource run by the University of Cambridge, aims to:

- *Enrich mathematical experiences so that all students have the opportunity to explore, engage with and communicate mathematical ideas.*
- *Offer challenging activities and resources which can develop mathematical thinking and problem solving skills.*

[3] Example of workshop: http://fresno.pntic.mec.es/mrir0002/exposiciomatematiques/web2008/index.htm
Example of exhibition: http://www.mnactec.cat/ofertaeducativa/fitxa/les-matematiques-i-la-vida/

- *Show the use of mathematics in engaging and meaningful contexts.*
- *Contribute to national and international debates relating to maths teaching and learning.*

Similarly, the National Centre for Excellence in the Teaching of Mathematics Education (NCETM) (https://www.ncetm.org.uk/), whose aim is to 'raise levels of achievement in maths, and to increase appreciation of the power and wonder of maths', provides a wide range of resources and references to projects in schools which have focused on making links between music and mathematics.

Sing Up (https://www.singup.org), a not-for-profit organisation promoting good quality music provision in primary schools, is developing a programme of activities exploring the integration of music with other curriculum subjects. A large part of this development is focusing on an integrated approach to the teaching of music and mathematics (e.g., https://www.singup.org/singup-songbank/top-ten-curriculum-songs/mathematics-curriculum-songs/).

The main educational weekly newspaper in England, *The Times Educational Supplement*, provides a forum for teachers to post activities for practitioners to use. Although this is an open forum, there are opportunities for other teachers to provide useful feedback. On this forum, there exist a number of activities which exploit the relationship between music and mathematics (e.g., https://www.tes.com/teaching-resources/search/?q=music+and+mathematics).

Discussion

In addition to the points already made, there exist other factors which are common to both countries. Both the Catalan and English educational systems encourage teachers to pursue on-going professional development, but these exist within certain political and

cultural dimensions which necessarily impact on the choices teachers are able to make (e.g. Lago, 2007; Hardy and Melville, 2013). In this respect, at the time of writing, both countries are beginning collaborative projects between higher education institutions and schools to develop integrated approaches to the teaching of music and mathematics through different projects (mentioned above). This training provides tools, materials and examples of activities for teachers and promotes collective reflection, based on the analysis of experiences in the classroom. We are also developing ways of incorporating integrated approaches in our initial teacher training programmes. For example, at UCL Institute of Education, in London, UK, primary music and mathematics specialists spend time working together in workshops which are designed to facilitate an integrated approach. In many cases this leads to trainee teachers completing master's level projects exploring the integration of music and mathematics. Finally, the existence of the internet has provided the possibility of new ways of sharing information and communicating. Both countries have well respected teacher organisations that support and encourage teachers to share information in a public domain (e.g. the TES, in the UK).

There are, however, some differences between the two countries. For example, in Catalonia, music is taught by music specialists in primary schools and yet experience suggests that music is not valued (Carrillo & Baguley, 2011). From our work with these music specialists, it has become apparent that the music specialists often lack confidence in their own knowledge and understanding of mathematics and are reluctant to teach it (Carrillo and Baguley, 2011). In England, the situation is often quite different. Music in English primary schools is usually taught by generalist teachers who lack confidence in their own knowledge, understanding and expertise in music and who, as a consequence are likely to be reluctant to teach it, especially with older children

(Welch & Henley, 2014). In addition, in Catalonia there are few publications and resources to help teachers develop integrated approaches, but, as has already been seen, this is a developing area in England. In England, there have also been government initiatives, such as the 'Numeracy across the curriculum' (DfES, 2001) which have attempted to encourage integrated approaches, but in Catalonia, these initiatives have been slow to develop over the last 10 years.

In the present climate, in both countries, there remain a number of challenges. For teachers to be confident in their ability to incorporate integrated approaches in the teaching of music and mathematics, teachers may need to reconceptualise what it is that makes an activity 'musical' and 'mathematical'. This may require support from higher education tutors and teachers and is likely to take time to develop. Experience to date indicates that, when experienced teachers and trainee teachers engage with appropriate activities for themselves, they quickly develop the confidence to explore the activities further and even go on create activities of their own. In England, a key challenge often concerns finding time to teach music. Where music is taught by a music specialist, the challenge is ensure that the generalist primary teacher is involved (Welch and Henley, op.cit.).

In changing approaches, we need to support teachers to make time for learning to take place and avoid using music as a quick way of getting children to learn number facts with no meaning! We want to encourage our teachers to allow children in primary and early childhood education to explore and play in both music and mathematics and to experience the synergy of exploring the two subjects as one.

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