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Application of endochronic densification law for designing vibroflotation soil improvement technique

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ABSTRACT

The densification phenomenon in dry or completely drained sands is mainly due to the application of dynamic loading, like an earthquake or other kind of vibrations. This fact causes a reduction of voids volume and in a consequence the compaction of the soil. A finite element model, including the generalized endochronic densification law, formulated in cylindrical coordinates, has been developed for simulating the vibroflotation soil improvement technique the numerical model. Punctual dynamic loadings, like those occurring in vibroflotation treatment, are reproduced in the code. There are several other vibration-compacting soil improvement techniques which could be modelled like an axi-symmetric problem with this new approach, which includes absorbent boundary conditions (silent boundaries).

1. INTRODUCTION

The vibroflotation is a technique for improving the strength and bearing capacity of unsaturated, granular soils. It consists of the application of punctual vibrations at different depths inside a soil layer, produced by an apparatus called "vibrator". These vibrations could have different amplitudes and frequencies, and causes the application of dynamic loadings inside the soil (Fig.1). This technique was first used in Germany in the 1930's, and later, in USA. It is specially indicated for soils with very small fines content, and for deep layers, as deep as the vibrator can be introduced within the soil (until 20m). For a given soil and vibrator (with its particular amplitude and frequency of loading), this procedure requires the definition of several geometrical parameters, i.e. the vertical distance of vibrating points at the same hole, as well as the horizontal distance between them and time of vibration at each point. In practice, these magnitudes are usually defined using empirical relations, making proofs before the complete treatment, and using data from successful past experiences. Although it seems to be necessary the development of a rational design approach, it is possible to affirm that this technique has been applied with success in a number of cases (Mitchell, 1970; Brown, 1977).

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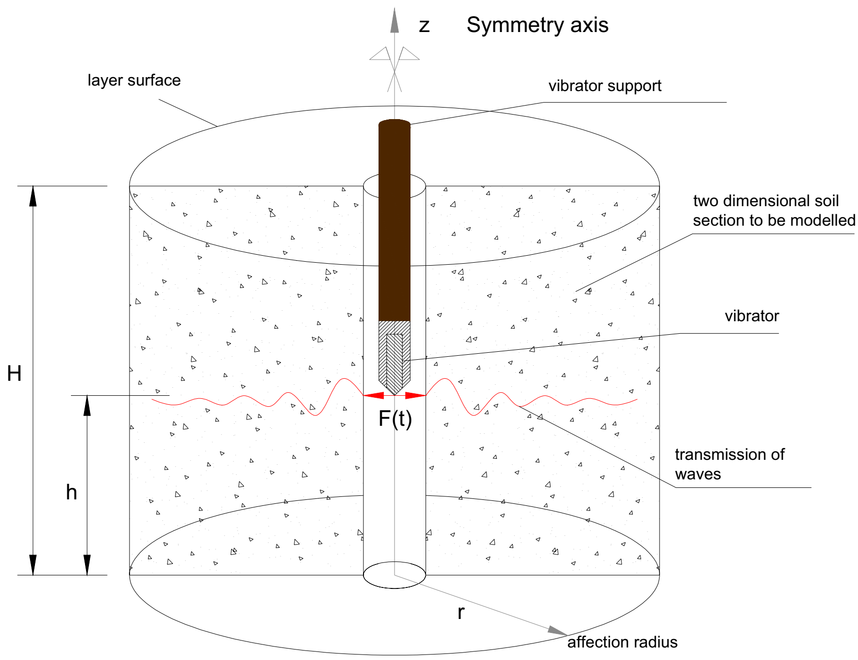
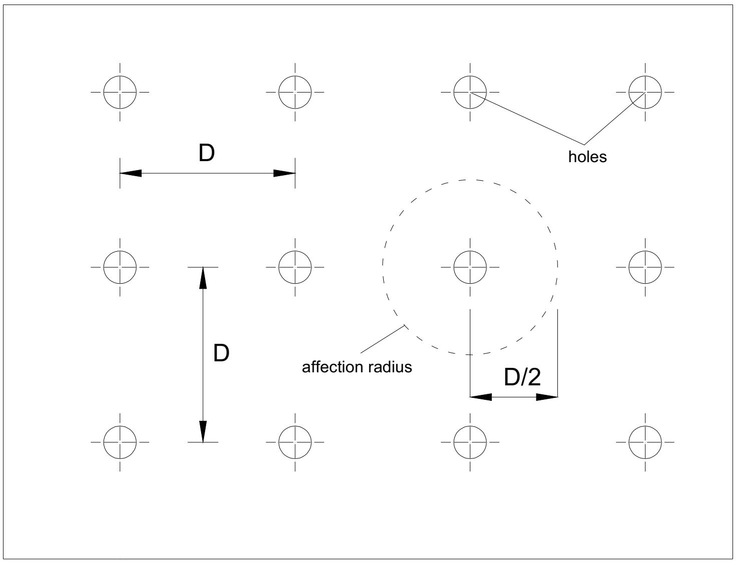


a)



b)

Figure 1. Sketch of the vibroflotation soil improvement technique. a) Detail of a hole, including the vibrator inside. a) A plant vision of the mesh of holes in the soil layer to be improved.

It is well known that, when loose sandy soils are subjected to dynamic loadings, they tend to acquire denser states, reorganizing their grains. This phenomenon is known as densification.

To model this kind of processes, there are several kinds of approaches. On one hand, there are empirical and semi-empirical based models, extrapolating the results of densification found in the laboratory for a particular material. These models usually give good results, but are far from reproducing the physics of the mechanisms governing the real soil behaviour (Seed & Silver, 1972; Martin et al., 1975). On the other hand, it is possible to find in the literature advanced elastic-plastic constitutive laws, very efficient from the computational cost point of view, but which usually require the calibration of a high number of parameters without physical meaning.

In this research, an endochronic based densification law for sandy material is used. The endochronic theory, first developed for metals, was successfully applied to sand under vibrations by Cuéllar (Cuéllar, 1974). This law, based in the definition of two monotonic functions which increase as the number of harmonic shear strain cycles progresses, has been recently updated and generalized for non-harmonic loadings, and high number of cycles (Blázquez & López-Querol, 2006).

The above mentioned endochronic based densification law has been implemented in a Finite Element, axi-symmetric code. One single hole, along with its surrounding soil domain, is modelled. The axis of symmetry is located on the hole, where punctual vibrations take place at different depths. Aiming to avoid the unrealistic reflection of stress waves, those boundaries which do not represent rigid borders, but links to other surrounding material, are established as absorbent (silent boundaries).

The paper begins showing the details of the numerical model, including the employed constitutive law, the axi-symmetric FE code, and the formulation of absorbent boundary conditions. After that, the model is applied to different materials, and the optimal spacing of the vibration points is established.

2. DESCRIPTION OF THE NUMERICAL MODEL

2.1 Generalized endochronic densification law.

In his pioneer work, (Cuéllar, 1974) established that the densification of sand is due to the irreversible rearrangement of grain configurations associated to the application of a deviatoric strain. In mathematical form:

$$d\xi = f(d\varepsilon_{ij}) \tag{1}$$

where ξ is called "rearrangement measure", a function of the deviatoric component of the strain tensor, ε_{ij}. In order to account for the monotonically increasing trend of ξ, the differential expression (2) must be a quadratic and finite function:

$$d\xi = [P_{ijkl} \cdot d\varepsilon_{ij} \cdot d\varepsilon_{kl}]^{1/2} \tag{2}$$

where P_{ijkl} denotes a set of coefficients which depend on the state of the material.

The previous expression can be decoupled in terms of normal and shear strains:

$$d\xi = [P_1(I_1(d\varepsilon_{ij}))^2 + P_2(J_2(d\varepsilon_{ij}))^2]^{1/2} \tag{3}$$

where $(I_1(...))$ is the first invariant of the tensor in parenthesis, and $(J_2(...))$ is the second invariant of the same tensor.

Taking into account that the volumetric stress has a small influence on the densification, P_1 can be neglected. On the other hand, shear stresses between grains depend on the normal force component between them, and therefore P_2 can be considered independent on $J_2(d\sigma_{ij})$ and $I_3(d\sigma_{ij})$, where I_3 is the third invariant of the incremental stress tensor, $d\sigma_{ij}$. Therefore, P_2 only depends on the first invariant of $d\sigma_{ij}$. Since the invariant is related to the normal stresses, P_2 can be taken as a constant:

$$d\xi = [L \cdot (J_2(d\varepsilon_{ij}))^2]^{1/2} \tag{4}$$

Defining a new variable, named "densification measure", ζ, independent of time, and function of the rearrangement measure and the deviatoric stresses:

$$d\zeta = F_1(J_2(\varepsilon_{ij}))\, d\xi \tag{5}$$

the densification, ε_v, can be expressed as follows:

$$d\varepsilon_v = -F_2(\zeta, Dr_0)\, d\zeta \tag{6}$$

where Dr_0 is the initial relative density of the sand.

The functions F_1 and F_2 are taken by Cuéllar as:

$$F_1 = \frac{n}{4}|\gamma|^{(n-1)} \tag{7}$$

$$F_2 = \frac{1}{1+\alpha\cdot\zeta} \tag{8}$$

where α and n are the two parameters of the densification law, and $\gamma = (8 \cdot J_2(\varepsilon_{ij}))^{1/2}$. Introducing this new variable in (4):

$$d\xi = |d\gamma| \tag{9}$$

and replacing (7) and (8) in (5) and (6), the following densification law (in incremental form) is obtained:

$$d\varepsilon_v = -\frac{d\zeta}{1+\alpha\cdot\zeta} = -\frac{n}{4}\cdot\frac{|100\cdot\gamma|^{(n-1)}}{1+\alpha\cdot\zeta}\cdot|d\gamma| \tag{10}$$

Where

$$\zeta = \int d\zeta = \int \frac{n}{4} \cdot |100 \cdot \gamma|^{(n-1)} \cdot |d\gamma| \tag{11}$$

The negative sign of densification means that the sand behaves in contractive manner.

Although Cuéllar assumed that n and α where independent on the number of cycles, N, some posterior research (Blázquez & López-Querol, 2006) demonstrated their dependence on this magnitude for high values of N, which is of the next form:

$$n = A \cdot \ln(N) + B \tag{12}$$

$$\alpha = C \cdot N^{-D} \tag{13}$$

where A, B, C and D are soil dependent parameters. For quartzitic sands, they can be estimated as follows:

$$A = \frac{1}{2}\left(\frac{e_{max}^2 - e_{min}^2}{1 + e_{max} - Dr_0 \cdot (e_{max} - e_{min})}\right) \tag{14}$$

$$D = \frac{1}{2}(e_{max}^2 - e_{min}^2) \tag{15}$$

$$B = D + 1 \tag{16}$$

$$C = C^* - 50 \cdot Dr_0 \tag{17}$$

In the above equations, e_{min} and e_{max} respectively denote minimum and maximum voids ratio, Dr_0 is the initial relative density at the beginning of the dynamic loading, and C^* is the only remaining unknown parameters, which needs to be calibrated by means of dynamics tests.

The above describe constitutive model has been successfully implemented in FE codes to simulate settlements in granular soils due to earthquakes (Blázquez & López-Querol, 2006).

2.2 Details on the developed FEM code.

Since it has been previously justified, the vibroflotation is a procedure through which the stress waves move are transmitted inside the soil from the hole where the punctual vibrations are applied. Hence, the problem being axi-symmetric, it seems obvious the convenience of formulating the governing equations using cylindrical coordinates. Figure 2 represents a sketch of the modeled geometry, along with the definition of the employed coordinates (*r*, θ and *z*) and their corresponding displacements (*u*, *v* and *w*) (Oñate, 1992).

The general governing equation of this problem is given by the equilibrium of forces in a differential element of the soil (Zienkiewicz et al., 2000):

$$S^T D^{ep} S\{du\} - \rho \cdot \{d\ddot{u}\} = -\rho \cdot \{db\} + \{df_{ext}\} \qquad (18)$$

where ρ is the soil density, b is the vector of gravitational accelerations, f_{ext} is the vector of external forces, D^{ep} includes the constitutive law of the material, and d represents incremental magnitudes.

To solve the above equation (18), a code programmed in MatLab has been carried out. Triangular, quadratic approximation elements have been used. After applying the Galerkin theory (weighted residuals), the weak formulation of the problem yields:

$$K \cdot \{\Delta u\} + C \cdot \{\Delta \dot{u}\} + M \cdot \{\Delta \ddot{u}\} = \Delta f_{ext} \qquad (19)$$

where {…} represents a column vector, Δ denotes incremental magnitudes between to consecutives instants of time, u is the vector or unknowns (in each node, (u, v, w) - the displacements in cylindrical coordinates), and f_{ext} are the external forces applied to the model. K, C and M are stiffness, damping and mass matrices of the system, after assembled, with dimension 3·N (N is the number of nodes). Due to the nature of the problem, no displacements are expected in θ (v=0), and the problem is two-dimensional (Ottosen & Peterson, 1992).

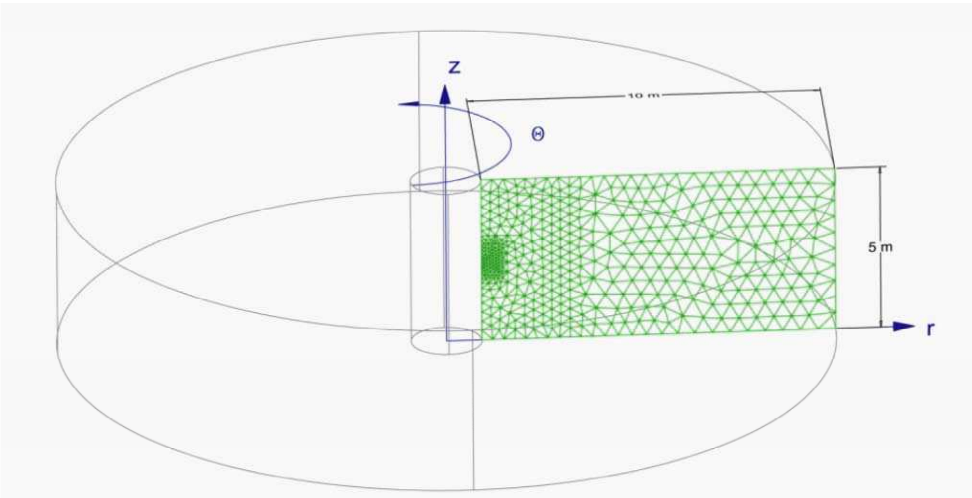


Figure 2. Finite element mesh of a soil layer in cylindrical coordinates system

It is worth to point out that the problem is incrementally solved. In each time step, the soil is considered elastic; after the solution, the increment of γ_{rz} is obtained, and Eqs. (9-11) are applied to calculate the increment of densification, as well as the accumulated densification in each point. In addition, after every step, the soil becomes stiffer, and hence, the value of the shear elastic modulus, G, is updated, by means of the following expressions (Papadimitriou et al., 2001):

$$G = \frac{G_{max}}{T} = \frac{1}{T} \cdot \frac{B_g \cdot P_a}{0.3 + 0.7 \cdot e^2} \cdot \left(\frac{\sigma'_v}{P_a}\right)^{0.5} \tag{20}$$

B_g is a model parameter which depends on the type of sand, e is the void ratio, σ'_v is the effective confining pressure, and p_a is the atmospheric pressure (σ'_v, p_a and G_{max} are given in the same units). The degradation parameter, T, can be expressed as:

$$T = 1 + 2 \cdot C_t \cdot |\eta - \eta_0| \text{ (first loading)}$$
$$T = 1 + C_t \cdot |\eta - \eta_{SR}| \text{ (unloading and reloading)} \tag{21}$$

where C_t is a material constant that depends on the degree of non linearity of the soil response, and η, η_0 and η_{sr} are, respectively, the current, initial and at last reversal values of τ/σ'_v.

Since the modelled problem changes in time, and the governing equation (18) jointly involves displacements along with their first and second derivatives in time, a time integration scheme is required to completely define the solution. The step-by-step Newmark's method has been implemented in the code, formulated with $\beta_1 = \beta_2 = 0.5$ (Zienkiewicz et al., 2000).

Aiming to reproduce the physics of the problem, and to avoid unrealistic reflection of waves at those boundaries which do not represent rigid borders (on the left and right of the modelled geometry, where more surrounding soil is confining the domain – see Fig. 3), absorbent boundary conditions have been implemented, according to (Toshinawa & Ohmachi, 1992).

3. OPTIMIZING THE SPACING OF VIBRATION POINTS

As it was previously mentioned, one of the key features on designing vibroflotation treatments is optimizing the horizontal spacing of holes where vibration takes place. Using the above described numerical model, and three different sands for which the densification constitutive law has been previously calibrated (Crystal Silica with Dr_0=45% - CS 0.45 - , Crystal Silica with Dr_0=80% - CS 0.80, Ottawa Sand with Dr_0=77% - OT 0.77), the optimal horizontal spacing has been obtained in each of them. The values of the parameters used for these sands are given in Table 1.

Table 1. Constitutive law parameters for the analysed sands.

Sand	A	B	C	D	B_g	C_t
CS 0.45	0.1497	12.720	2.00	0.2720	200	0.5
CS 0.80	0.1602	12.720	20.00	0.2720	200	0.5
OT 0.77	0.1072	11.656	1.66	0.1656	200	0.5

The model has been applied to a 5 m deep layer. The modelled geometry is a 5m high rectangle, with different widths to reproduce several spacing. For instance, if D (horizontal spacing, according to Fig. 1b) is 4 m, the model is D/2, hence, 2m (like in Fig. 3). Distances of D=1, 2, 3 and 4m have been tried. Just for comparison purposes, a unique point of vibration has been applied (Fig. 3). For this single vibrating point, the amplitude of horizontal displacement is 5 mm, the frequency is 60 Hz, and the total time of vibration is 10 seconds.

For optimizing the spacing between holes, a comparison between the measurements of the mean densification in each one of the geometries, along with the total time of treatment, will be carried out:

- For a given spacing (and its correspondent geometry), after the total time of vibration, we define the "mean densification", ϕ_y, as:

$$\phi_y = \frac{\sum_{i=1}^{Ne} A_i \cdot \varepsilon_{v\,i}}{\sum_{i=1}^{Ne} A_i} \tag{21}$$

where N_e denotes the total number of elements.

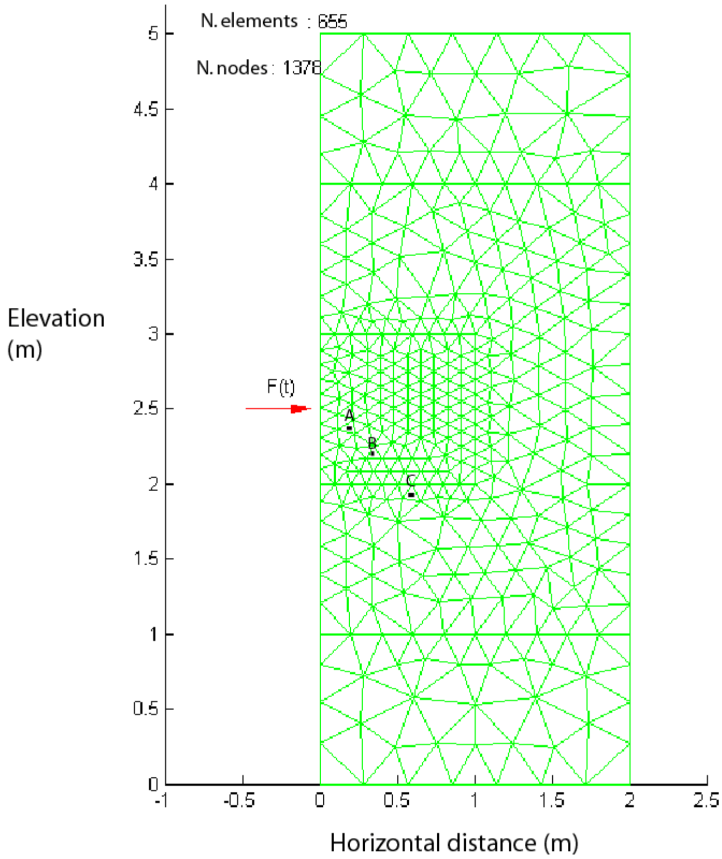


Figure 3. Finite element mesh for a geometry with an affection radius of 2 m.

- The time of treatment is directly related to the number of holes in a particular zone. For instance, if we need to densify a layer which, in plant, is 100m x 100m square, the horizontal spacing between the holes will determine the number of them, and hence, the total time spent on completing the ground improvement. If in each hole we vibrate only once (at the middle of the layer depth), and in each point the vibrator works 10 seconds, the total time of treatment for the analysed spacing is calculated in Table 2. By the inspection of this table, it is self evident the huge difference in time (in other words, in cost) of this technique as a function of the selected separation between holes.

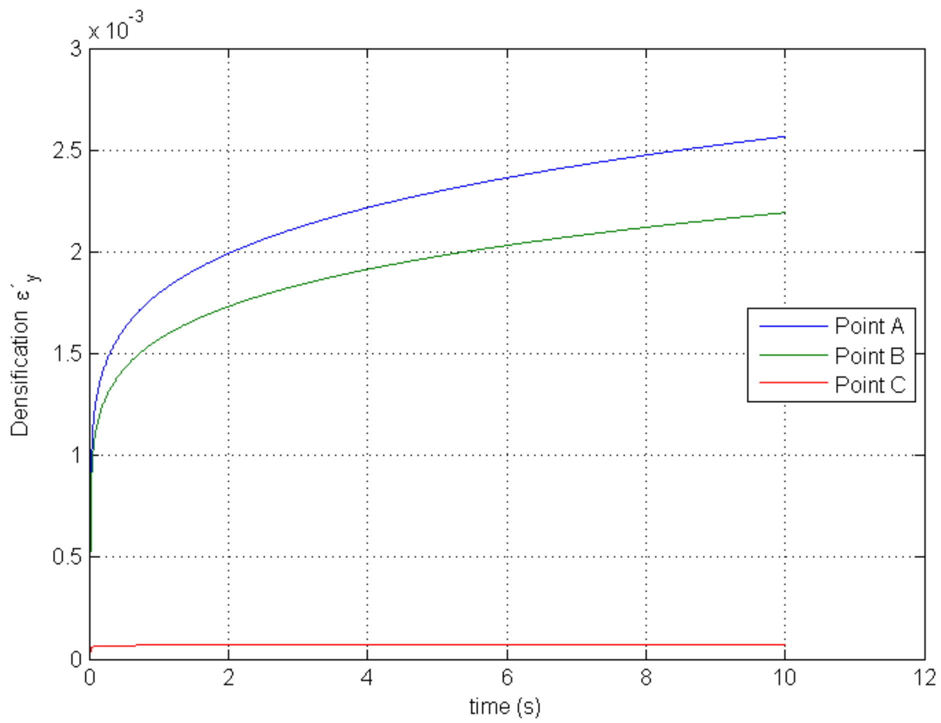


Figure 4. Evolution of the densification in time of vibration at points A, B and C (see figure 3) for CS-0.45 sand.

As an example of the obtained numerical results, Fig. 4 shows the evolution in time of vibration in three points of the geometry, close to the vibrating point (see Fig. 3), for the material CS-0.45. From this figure, it can be concluded that the closer the point to the source of vibration, the higher the final densification result. On the contrary, the closer that point, the farer from achieving a final asymptotic value of densification. However, 10 seconds seem to be enough to get a final result very close to the maximum possible in all the points, even in A.

Figure 5 shows the values of the mean densification for the different times of treatment for the three analysed sands. We can conclude that, the denser the mesh of holes, the more the efficient the densification achieved. However, the shapes of the calculated curves show that, for the lower times of treatment, it is worth to increase the number of holes, since the slope is steeper.

As a practical hypothesis of this research, we will assume that the minimum possible separation between holes which permits this treatment to be used is 1m. Hence, let's assume that the maximum mean densification is that produced for an affection radius of 0.5m. Let's take the 75% of the maximum mean densification as our final objective of treatment (called $\phi_{y,75}$). Making the inverse computation, to obtain the distance between holes which would give us this objective densification, we get the final optimal results of spacing which are presented in Table 3.

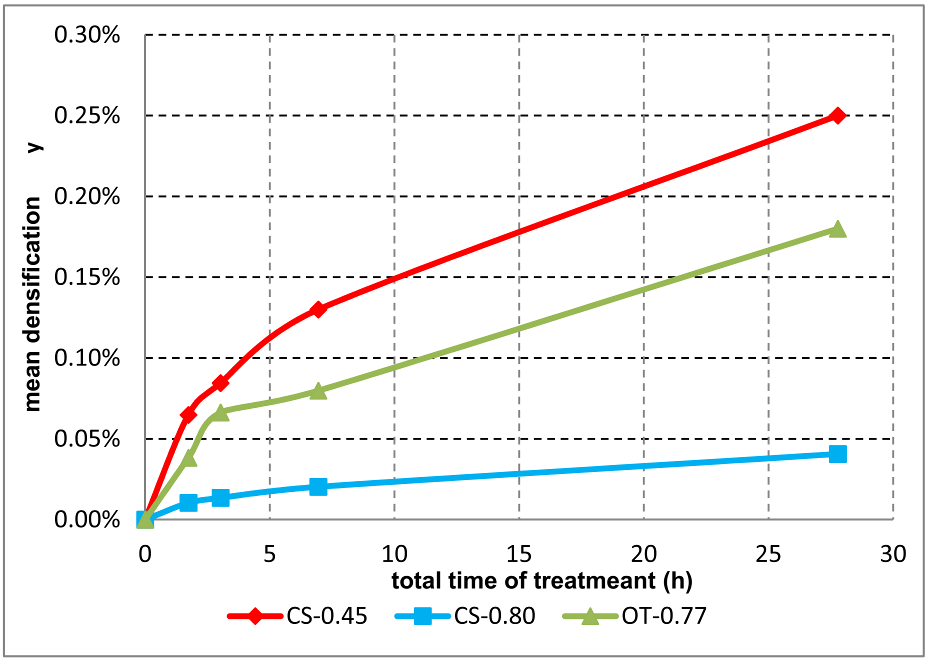


Figure 5. Evolution of the mean densification with the total time of treatment

Table 2. Total time of treatments for different spacing of holes in a 100mx100m square

Horizontal spacing (D, in m)	Affection radius (D/2, in m)	Number of holes	Time of treatment (sec.)
1.0	0.5	10000	100000
2.0	1.0	2500	25000
3.0	1.5	1089	10890
4.0	2.0	625	6250

Table 3. Optimal distance between holes of vibration for the analysed sands

Sand	$\phi_{y,75}$ (%)	Optimal time of treatment (for $\phi_{y,75}$) (in hours)	Optimal distance (m)
CS 0.45	0.19	16.9	1.3
CS 0.80	0.03	17.4	1.5
OT 0.77	0.14	18.4	1.4

CONCLUSION

This paper presents a new numerical model as a tool aiding on obtain an optimized design of the vibroflotation soil improvement technique for several sandy soils. The finite element method model, formulated in cylindrical coordinates to represent the axi-symmetry of the problem, includes the generalized densification constitutive law, previously used with success to reproduce settlement of sandy soils after earthquakes. For those borders limiting to the surrounding soil, the boundaries have been formulated as absorbent.

The conclusions derived from this research can be summarized as follows:

- The used constitutive law is not only valid for earthquakes, but also for higher frequency loadings, like for instance, the vibrations produced in this technique. The obtain results are in the range of the results observed both in the laboratory and in the field.
- Horizontal Distances between holes of 3-4 m produce very small total densification, which improves very much as the spacing becomes denser.
- A practical horizontal distance in the range of 1.3-1.5 m has been obtained as optimal to get the 75% of the mean densification in a given sand. For the same material, it can be concluded that, the denser it is, the higher could be the distance.

More analyses, modelling deeper layers, more points of vibration in the same hole, and other materials, as well as comparisons with real field cases, would be desirable.

REFERENCES

Blázquez, R., and López-Querol, S. (2006). “Generalized densification law for dry sand subjected to dynamic loading”, *Soil Dyn. Earthq. Eng.* Vol. **26**, 888-898.

Brown, R.E. (1977). “Vibroflotation compaction of cohesionless soils”, *J. Geot. Eng. Div. ASCE.* Vol. **103**(GT12), 1437-1451.

Cuéllar, V. (1974). “Rearrangement measure theory applied to dynamic behavior of sand”, Ph.D. dissertation, *Civil Engineering Department, Northwestern University*, Evanston, Illinois.

Martin, G.R., Finn, W.D.L. and Seed, H.B. (1975). “Fundamentals of liquefaction under cyclic loading”, *J. Geot. Eng. Div. ASCE.* Vol. **101**(GT5), 423-438.
Mitchell, J.K.(1970), “In-place treatment of foundation soils. *J. Soil Mech. Found. Div. ASCE.* Vol. **96**(SM1):73-110
Oñate, E. (1992). “Cálculo de estructuras por el método de elementos finitos”, *CIMNE, Universidad Politecnica de Catalunya* (ISBN: 84-87867-00-6). (In Spanish).
Ottosen, N. and Petersson, H. (1992). “Introduction to the finite element method”, *Prentice Hall Internatonal* (UK) Ltd.
Papadimitriou, A.G., Bouckovalas, G.D. and Dafalias, Y.F. (2001). “Plasticity model for sand under small and large cyclic strains”, *J. Geot. Eng. Div. ASCE.* Vol. **127**(1), 973-983.
Toshinawa, T. and OPhmachi, T. (1992). “Ground motion simulation by using simplified three-dimensional finite element method”. *Proc. 10th World Conf. Earthq. Eng. Balkema*, Netherland, 851-856.
Seed, H.B., Silver, M.L. (1972), “Settlement of dry sands during earthquakes”, *J. Soil Mech. Found. Div. ASCE.* Vol. **98**, 381-396.
Zienkiewicz, O.C., Chan, A.H.C., Pastor, M., Scherefler, B.A., Shiomi, T. (2000). “Computational geomechanics with special reference to earthquake engineering”, *John Wiley & Sons Ltd, England*