Investigating the integration of a digital resource in the mathematics classroom: the case of a creative electronic book on Reflection

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This paper explores the potential impact of a full teacher-driven design and implementation cycle of an electronic book (c-book). We analyse data gathered from a school case study and identify the potential of the affordances of the c-book technology that allow the integration of various mathematical widgets and reflective activities. Our conjecture is that encouraging flexibility on playful tasks and reflection on 'bridging' activities early in the structure of the book prepared the students to more complex constructionist tasks around the concept of Reflection. Looking into the full cycle from design to evaluation this study demonstrates a successful integration of a digital resource in the mathematics classroom and highlights some of the successful components of the resource namely: playful activities for students, matched with carefully designed bridging activities, followed by constructionist activities that allow deeper exploration of the subject matter.

Keywords: Mathematical creativity, e-books, transformations, reflection, bridging activities.

Introduction

There is a lot of research and many projects that focus on developing digital resources for the teaching and learning of mathematics. The issue though regarding their successful use and integration in the mathematics classroom still remains (Clark-Wilson, Robutti & Sinclair, 2014; Geraniou & Mavrikis, 2015). One of the issues is whether and how students who may become experts in using a digital tool reflect and consolidate their mathematical knowledge (Geraniou & Mavrikis, 2015). Teachers then may not be convinced of the potential value of using digital tools in their mathematics lessons. In our view, a successful integration of such tools also involves the successful transition from interacting with a digital tool to a metacognitive understanding on behalf of the students that the interaction can support their knowledge 'outside' the tool.

Our work continues to focus on building 'bridges' to the maths involved (and may be 'hidden') in digital resources. We are looking into how we can encourage the consolidation of knowledge within digital tools and the 'transfer' of knowledge aiming at finding strategies to integrate them successfully in the classroom and the learning process. We define bridging activities as short tasks or questions that are used to intervene and encourage students to reflect upon mathematical concepts and problem-solving strategies they use throughout a sequence of activities (or simple interactions) with a digital tool. Such activities could take various arrangements from questions or prompts within the digital tool to paper-based worksheets or verbal teacher's interventions. In this paper, we focus on an electronic book resource and, particularly what the Mathematical Creativity (MC) Squared project (http://mc2-project.eu/) calls 'c-books', which are extended electronic 'creative' books that include widgets i.e. objects, other than text ranging from simple hyperlinks or videos to a broad range of interactive digital environments for mathematics such as GeoGebra and other microworlds (c.f. Kynigos, 2015). The project also includes an authorable intelligent support and data analytics engine that allows designers (e.g. teachers) to author the feedback that the system could provide to a student and the data they would like to see from their interaction (Karkalas & Mavrikis, 2016). The idea behind the MC Squared project is to focus on social creativity in the design of digital media intended to enhance creativity in mathematical thinking (CMT). Researchers collaborating with math educators and teachers join Communities of Interest (COI) that work together to creatively think and design c-book resources reflecting 21st-century pedagogy for CMT.

The focus of the small study presented in this paper has been on designing a c-book including appropriate resources, such as bridging activities (Geraniou & Mavrikis, 2015) with the aim of enabling students to make connections to the mathematical concept the c-book is designed to teach them, in this case Reflection[1]. We conjecture that designing resources that encourage flexibility on playful tasks and reflection early in the structure of the book prepared the students to more complex constructionist tasks. Looking into the full cycle from design to evaluation we demonstrate a successful integration of the c-book in the mathematics classroom and highlight some of its key components namely: playful activities for students, matched with carefully designed bridging activities, followed by constructionist activities that allow deeper exploration of the subject matter.

Theoretical Framework

CMT has been given many definitions by various authors (e.g. El-Demerdash & Kortenkamp, 2009; Mann, 2005). In the MC Squared project, CMT has been drawn on Guilford's (1950) model of fluency (the ability to generate a number of solutions to a problem), flexibility (the ability to create different solutions), originality (the ability to generate new and unique solutions), and elaboration (the ability to redefine a problem). CMT has also been approached as a thinking 'process' that takes place in the context of a mathematical activity in order to produce a 'product' (e.g. a solution to a mathematical problem). As such the product and process are intertwined. For example, the construction of a geometric artefact is seen as a product that was started as a response to a task (problem), continued with the identification of a set of points, lines etc. that are underpinned by some properties that provide an answer to the task (product). Taking the above CMT's aspects as a starting point, we align our views to Papadopoulos et al.'s (2015; 2016) who consider CMT as the (i)'construction' of math ideas or objects, in accordance to constructionism that sees CMT being expressed through exploration, modification and creation of digital artefacts (Daskolia & Kynigos, 2012), (ii) Fluency (as many answers as possible) and Flexibility (different solutions/strategies for the same problem) and (iii) novelty/originality (new/unusual/unexpected ways of applying mathematical knowledge in posing and solving problems). Even though CMT seems to be at the core of mathematical thinking, its development through the use of exploratory and expressive digital media hasn't been thoroughly investigated (e.g. Healy & Kynigos, 2010) and the question about the best possible strategies for developing appropriate resources for integrating such digital media and promoting CMT inside and outside of the classroom remains.

[1] To distinguish between 'reflection' as a thought process and the mathematical concept 'Reflection', we will use capital letter 'R' for the mathematical concept.

Authoring c-books

As mentioned above, c-books are special electronic books that are designed within the Digital Mathematics Environment (DME)[2] which has been designed to allow teachers to create sequences of activities involving a number of widgets. It allows teachers to change the feedback messages students receive during their interactions with the c-book and stores all user interactions and scores. As part of a teacher training course, and based on our previous work, we encourage teachers to use DME's affordance to design bridging activities that promote students' reflective thinking on their interactions aligned with the various widgets. We expect these activities to 'bridge' the students' transition to the mathematical concepts, that the digital resource is designed to support (Geraniou & Mavrikis, 2015). These are questions presented and directly linked to the widget's tasks and can be viewed as interventions that encourage students' reflections on their interactions throughout a sequence of tasks, but also introduce and encourage the use of mathematical notation, not necessarily presented within the widgets. Authoring bridging activities within the digital medium of a c-book and recognizing the potential value to students' learning progress and outcomes may encourage teachers to use such digital media more often.

The case of a c-book on Reflection

The c-book on Reflection consists of a number of pages involving different tasks mostly in GeoGebra. This c-book (as opposed to others created in COI meetings, during which COI members brainstormed about ideas and activities that could be part of a c-book on a specific mathematical topic), was initially created by the class teacher in this study, who already had a number of prepared resources, which they put together using the affordances of the DME platform to form the c-book. These were resources like book chapters and GeoGebra worksheets. The c-book was also shared with the COI in an effort to gain constructive feedback and improve it.

The learning objective for the c-book was to remind students of the definition of Reflection, which had already been introduced about seven months before, define the Reflection ('mirror') line, consolidate students' prior knowledge and develop their understanding of the concept of Reflection. Even though the c-book technology allows a non-linear browsing of the c-book and students can work on any activity they want, it was designed (and used) as a linear progression for constructing students' knowledge on Reflection by: (i) revising prior knowledge on Reflection through a series of multiple choice questions on certain reflected images where students had to decide which of the four images was the correct reflected image, (ii) revising and practicing on the GeoGebra widget (Figures 1A and 1B), (iii) challenging their understanding of Reflection through a competition task (Figures 1C and 1D) that promoted 'flexibility' in their solving approaches, (iv) challenging further their understanding of Reflection through a problem that challenged further their understanding and took them away from the standard style of questions such as 'Reflect this shape across the given Reflection line' (by not giving them the Reflection line, adding a constraint of the squared frame and giving them a story context to think about) (Figures 1E and 1F), and finally (v) a final

[2] See http://ws.fisme.science.uu.nl/dwo/site/index_en.html and http://mc2-project.eu/

assessment task mostly for those who finish faster aimed at recapping what students should know at the end of this c-book unit. We need to emphasize that all GeoGebra tasks were presented as bridging activities through the use of added text and reflective questions (see Figures 1A, 1C and 1E) on the side. These were designed as such to challenge students' thinking and understanding of Reflection and help them consider carefully their interactions rather than simply undertake the tasks. The feedback provided to students was of different types: (i) as a tick or cross for correct and incorrect responses, (ii) as a score for the GeoGebra competition task, which identified the number of correct Reflections students reached within the 5 minutes timeframe set by their teacher (Figure 1D) and (iii) as a written text to provoke their problem solving.

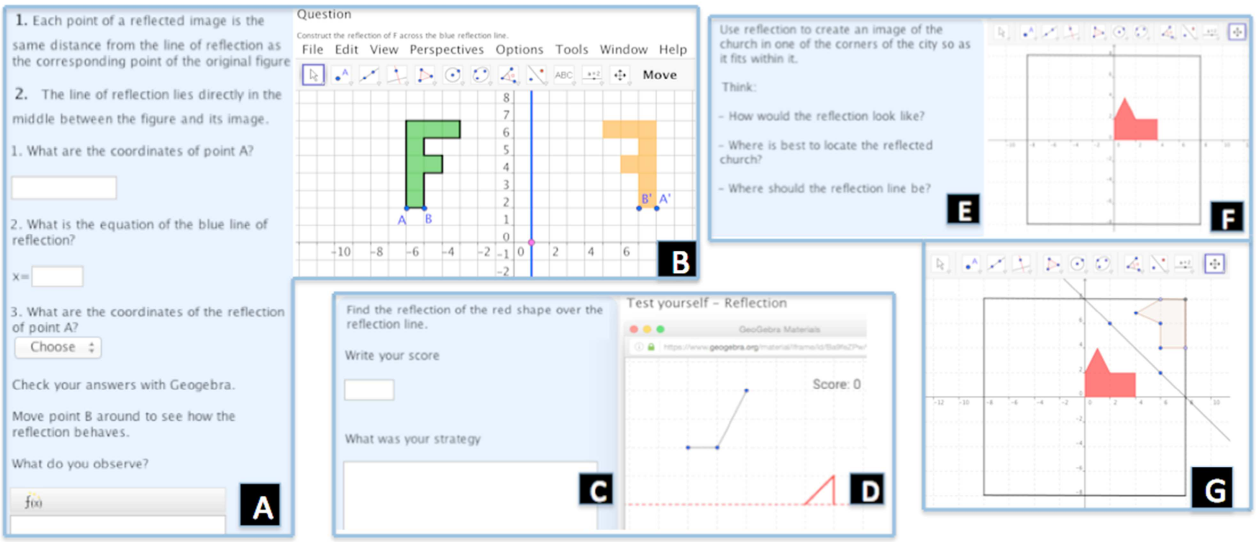

Figure 1: (A – F) Excerpts from the Reflection c-book and (G) a sample solution of (F)

Data Collection

The aim of this case study was to explore the potential of both the Reflection digital book in the light of the affordances of the overall c-book technology i.e. beyond the ability to sequence activities, the potential for automated feedback and reflection that could be used to support bridging activities. The methodological tool used was that of a "design experiment" (Collins et al., 2004), that could act both as a way to 'engineer' and support the didactical situation and to systematically study it (Cobb et al., 2003). In this case, we, as a research team, collaborated with a teacher but left the decisions and responsibility of the classroom to the teacher.

Twenty-one 11-12 year old (Grade-7) students together with their class teacher and two researchers participated in the study, which was completed in two lessons in the school's computer lab. The students had been introduced to the concept of Reflection earlier in the year by working on some simple activities involving reflecting 2D shapes across the Reflection line. According to the teacher, the aim of these two sessions was to revise and consolidate their knowledge, but also to challenge their mathematical thinking against the concept of Reflection. The plan for the first lesson was (i) to remind them of what Reflection is and introduce the mathematical term of 'Reflection line' as opposed to 'mirror line' when they were first introduced to Reflection, (ii) introduce the c-book technology and (iii) allow students to familiarize themselves with GeoGebra through a challenging task, which acted as a bridging activity to recap prior knowledge. It involved working on some bridging activities, which included mathematical questions (such as 'find the coordinates') and reflective questions (such as 'what was your strategy?') within the platform. At the end of the first lesson, most students had reached the 'Church Challenge' task (see Figure 1E and 1F). During the second lesson, students continued to work on the 'Church Challenge' and then answered a questionnaire to evaluate the c-book.

In addition, at the end of the second lesson, they were given a questionnaire to share their feedback on their learning experience with the Reflection c-book. The questionnaire was a Likert multiple-choice questionnaire consisting of questions such as: (1) How satisfied were you after completing the c-book activities?, (2) How easy to use do you think the c-book is?, (3) How free did you feel to experiment with the c-book and try out your ideas?, (4) I feel I understand Reflection now. Another two questions (5 and 6) gave them options to pick on their thoughts on the c-book and their preferred features. The questionnaire finished with three more questions to request suggestions from students (out of the scope of this paper).

Researchers took the role of 'participant observers' focusing on students' interactions with the digital medium and taking field notes. Besides working with the researchers and other COI members to design the Reflections c-book, the teachers' role was to offer assistance in technical issues when required during the two lessons and ensure that all students were on task and answered the bridging activities. Our data consists of the logged answers in DME and voice recordings as students elaborated on their interaction and answers. The data analysis was carried out by retrieving students' interactions with the cbook from the system and interpreting their responses against the CMT criteria presented earlier and by going through their answers on the questionnaire.

Results

The main outcome based on the data from the bridging activities, in particular, was that students were encouraged to reflect on the GeoGebra task from the start of their interactions. The teacher reminded students of the reflective questions (Figure 1A) and encouraged them to record their answers. The designed automated feedback supported all students to identify correctly the missing coordinates for the 'F' shape, its Reflected image and the equation of the Reflection line. In this first bridging activity, students were reminded of what Reflection is and the definition of the 'line of Reflection'. Both these terms were also introduced to the whole class and discussed with the class teacher at the start of the first lesson. But, we envisaged the repetition would give students a sense of familiarity and they would eventually start using mathematical terms in later tasks and would adopt mathematical ways of thinking. Fourteen (14) of the students provided sensible answers to the bridging question in relation to their strategy. Looking at students' responses to the bridging activity questions for the first couple of GeoGebra tasks, students were mostly using informal terminology:

Student: we have to flip the shape.

Student: count how many down from the mirror line.

But, in later bridging activities questions, students started to use mathematical terms, such as "the reflected church" or the "reflection line". For the question on what they notice when they move the 'F' shape, their responses were rather superficial:

Student: if you move the green shape, the orange shape moves with it.

They seemed to have noticed that the two shapes (green and orange 'F') are linked, but only 2 were able to articulate that they maintain the same distance from the Reflection line. Retrospectively, observing the students talking about their strategies, it might have been better to include some explicit scaffolding questions here such as "What is the distance from the 'F' shape to the reflection line?", "What do you notice?" etc. These could be followed up by the teacher to clarify what reflection is and how the reflected images are defined.

The bridging activities questions revealed students' solving strategies and consequently their CMT. For the Competition task, students claimed to use three different strategies: (i) counting boxes across and down, (ii) tilt their head so that the reflection line becomes vertical and (iii) imagine using tracing paper on the screen. In this way, students demonstrated not only that they can come up with some original (for them) solutions but that they can also provide elaborate reflections on their strategies, which is linked to the originality/novelty CMT criteria described earlier. In retrospect, the c-book could have been designed to ask students for different strategies after they come up with one to challenge further their CMT in terms of the fluency and flexibility criteria.

Asking students about their strategy seems to promote reflection on their actions that helped them reach a solution. In particular, the Church Challenge (Figures 1E and 1F) posed a problem that ignited students' thinking 'process' and resulted in a 'product', i.e. the reflected church image. In all the previous activities, students were given the Reflection line and their aim was to reflect a given shape. On the contrary with the Church challenge, students had to find the Reflection line and reflect the church image within the square town (see Figure 1G). By writing down their strategy, they recognized the solution 'steps' they took, questioned their actions and corrected them when needed. This open-ended problem allowed for exploration, construction of mathematical ideas and flexibility, which are all aspects we used to define CMT earlier (e.g. Papadopoulos et al., 2016).

Sixteen students (16/21 or 76%) managed to complete the task, whereas the rest ran out of time in the lesson. 10 of those got a correct answer. To reach the solution or the 'product', students produced creative solving strategies, which they were asked to justify. These strategies involved imagining a tracing paper used on the screen to reflect the church (14% or 3/21), which could be considered original in this context; trial and error technique by reflecting the church in all 4 quadrants and then thinking about reflecting each image within a quadrant to the corner of that quadrant to see which one fits within the square town (33% or 7/21); or another trial and error technique by constructing different Reflection lines and reflecting the church in one or more quadrants (52% or 11/21, see Figure 1G). These two latter strategies demonstrate students' flexibility through the CMT criteria lens.

As far as the questionnaire is concerned, we are mostly interested in this paper on question #4 where most of the students (85% or 18/21) responded with an answer above 4 in the Likert scale. In relation to their thoughts on the c-books about 60% (13/21) answered that it helped them see the idea of reflection in different ways. This is really encouraging as one of our objectives was indeed to help students expand their understanding. About 43% (9/21) said that it included problems that they would not have tried to solve. This is also interesting as we want to encourage students to appreciate their mathematical abilities. In the open-ended questions, most students complimented the affordances of the c-book by commenting on enjoying the free explorations, testing of their ideas, experimenting, working on new questions and being challenged. While some students had comments for aesthetic improvements (fonts, games, colours etc.) three (3) students made comments that showed that they appreciate the advantages of digital technologies:

> Student: the digital book help[s] because you could have actually test[ed] out your ideas and improve if it's wrong or not.

They recognized the dynamicity of such resources and how seeing the immediate feedback on their actions helps them validate their solution. At the end of the two lessons, the teacher also shared his reflections with the researchers and later with the COI. The teacher was impressed with how students were so engaged with the c-book, compared to past lessons without any digital resource and commented on the value of bridging activities and shared ideas on how to improve them.

Conclusion

This paper provides a good indication of the value of having a digital medium that combines free exploration, but encourages students to reflect upon their actions and make a link between their interaction in a digital environment and their mathematics through bridging activities. Such activities focus on mathematical terms, the definition of concepts, but also the justification for their solutions, throughout their work and 'bridge' the actions to solving a problem in the digital tool to the underlying mathematics (which could otherwise be 'lost').

Authoring activities using various widgets, designing Bridging Activities and in general, participating in the creation of the Reflection c-book re-enforced the teacher's keenness to continue to use digital technologies in their classroom. As a result of this study the teacher and the COI revisited the c-book that led in further improvements in the book. The most notable of those was breaking down the bridging questions to smaller questions with guidance, and using the feedback affordances to encourage flexibility in terms of the strategies, as an aspect of CMT.

To conclude, this case study demonstrates how the c-book technology can be integrated in the mathematics classroom and promote a positive learning experience through the use of playful activities for students, matched with carefully designed bridging activities, followed by constructionist activities that allow deeper exploration of the subject matter.

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References

Clark-Wilson, A., Robutti, O. & Sinclair, N. (2014). (Eds.) *The Mathematics Teacher in the Digital Era*. Dordrecht: Springer.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher, 32*(1), 9–13.

Collins, A., Joseph, D., & Bielaczyz, K. (2004). Design Research: Theoretical And Methodological Issues. *The Journal of Learning Sciences, 13*(1), 15–42.

Daskolia, M., & Kynigos, C. (2012). Applying a Constructionist Frame to Learning about Sustainability. *Creative Education, 3,* 818–823.

El-Demerdash, M. & Kortenkamp, U. (2009). The effectiveness of an enrichment program using dynamic geometry software in developing mathematically gifted students' geometric creativity. Paper presented at the *9th International Conference on Technology in Mathematics Teaching - ICTMT9*.

Geraniou, E. & Mavrikis, M. (2015). Crossing the Bridge: From a Constructionist Learning Environment to Formal Algebra. In K. Krainer & N. Vondrová (Eds.), Proceedings of the Ninth Conference of the European Society for Research in Mathematics Education (4-8 February 2015) (pp. 2494–2500). Prague, Czech Republic.

Guilford, J.P. (1950). Creativity. *American Psychologist, 5*, 444–454.

Healy, L., & Kynigos, C. (2010). Charting the microworld territory over time: design and construction in mathematics education. *ZDM - The International Journal on Mathematics Education, 42*(1), 63–76.

Karkalas S. & Mavrikis M. (2016). Feedback Authoring for Exploratory Learning Objects: AuthELO. In the *Proceedings of the 8th International Conference on Computer Supported Education*, 144–153. DOI: 10.5220/0005810701440153

Kynigos, C. (2015). Designing Constructionist E-Books: New Mediations for Creative Mathematical Thinking? *Constructivist Foundations, 10*(3), 305–313.

Mann, E. L. (2005). *Mathematical creativity and school mathematics: Indicators of mathematical creativity in middle school students*. Unpublished doctoral thesis, Connecticut University, USA.

Papadopoulos, I., Barquero, B., Richter, A., Daskolia, M., Barajas, M., & Kynigos, C. (2015). Representations of Creative Mathematical Thinking in Collaborative Designs of C-book units. In K. Krainer & N. Vondrová (Eds.), Proceedings of the Ninth Conference of the European Society for Research in Mathematics Education (4-8 February 2015) (pp. 2381–2387). Prague, Czech Republic.

Papadopoulos, I., Diamantidis, D. & Kynigos, C. (2016). Meanings around angle with digital media designed to support creative mathematical thinking. *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 4, pp. 35–42). Szeged, Hungary.