MATHEMATICAL KNOWLEDGE FOR TEACHING USING TECHNOLOGY: A CASE STUDY

This paper analyses data from a PhD pilot study to explore the nature of mathematical knowledge for teaching using technology, as represented by the central construct of the TPACK framework. The case study of teacher Alice is used as an illustrative example to suggest that the central TPACK construct may be better understood as a transformation and deepening of existing mathematical knowledge rather than as a new category of knowledge representing the integration of technology, pedagogical and mathematical knowledge.

INTRODUCTION

This paper explores the nature of mathematical knowledge for teaching using technology. In particular, it explores whether teachers' mathematical knowledge for teaching using technology should be conceptualised as a new domain of knowledge integrating knowledge of mathematics, pedagogy and technology or rather as a transformation or re-contextualisation of existing *mathematical knowledge* for teaching using technology. In this paper, technology is used to indicate digital technologies, commonly referred to as Information Communication Technologies (ICT). In recent PME conferences, mathematical knowledge for teaching has been a sustained research interest, see for example the RF1 papers on teacher knowledge (Ball et al., 2009) in the 33rd conference and last year's plenary lecture on designing settings for teachers' disciplinary knowledge (Davis, 2010). Although substantial research effort has been focused on conceptualising teacher knowledge (Rowland & Ruthven, 2011; Sullivan & Wood, 2008), it has rarely considered teachers' mathematical knowledge for teaching in the context of technology use. Correspondingly, research on mathematics teachers' knowledge and use of technology is rarely informed by studies of teacher knowledge in mathematics education or in the wider field of education (see for example Hoyles and Lagrange, 2010), thus such research tends not to build towards a systematic analysis of mathematical knowledge for teaching using technology. These omissions are surprising given widespread recognition of the complexities of technology integration experienced by teachers and the corresponding gap between aspirations for technology use in schools and the classroom reality of technology use (Lagrange & Erdogan, 2008), with teacher knowledge often cited as an explanatory factor. Nevertheless, the nature of teachers' mathematical knowledge for teaching using technology remains unresolved.

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THEORETICAL BACKGROUND: THE TPACK FRAMEWORK

This study adopts Mishra and Koehler's (2006) Technological Pedagogical Content Knowledge (TPACK) framework due to the juxtaposition of technology knowledge alongside pedagogy and content knowledge, enabling an explicit focus on technology and thus an exploration of the nature of teachers' mathematical knowledge for teaching using technology. The TPACK framework represents Shulman's (1986) pedagogic content knowledge diagrammatically as the intersection of two circles representing general pedagogic knowledge and content knowledge. Extending this representation using a Venn diagram with three overlapping circles, they incorporate technology knowledge as a third domain of teacher knowledge, to indicate the skills or knowledge needed to successfully operate technology. The inclusion of technology knowledge introduces two new dyads, technological pedagogical knowledge (TPK) and technological content knowledge (TCK), representing the intersection of technology knowledge with pedagogic knowledge and content knowledge respectively, and a triad representing the intersection of all three types of knowledge: technological pedagogical content knowledge (TPACK, see Figure 1).

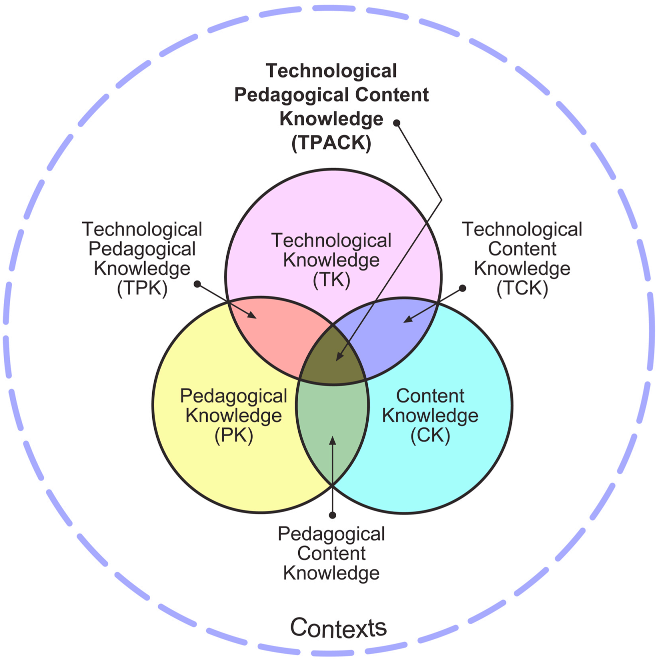

Figure 1. The TPACK framework, source http://tpack.org/

The TPACK framework was developed in the field of educational technology, hence its components require contextualising in the field of mathematics education. Mishra and Koehler (2006) define TCK as knowledge about the manner in which technology and content influence and constrain one another. TCK can be conceptualised as knowledge of how software models mathematical concepts and relations and of how the software design may therefore affect both the substantive and syntactic structures of mathematics. TPK comprises knowledge of the existence, components and capabilities of various technologies for use in teaching and learning settings and pedagogical considerations for their selection (Mishra & Koehler, 2006). For example, teachers need to be able to reinterpret the function of generic software and hardware, such as word-processing, spreadsheet or presentational software or interactive whiteboard hardware, to suit their own pedagogical purposes. This might include how to manage changes in the *working environment* and *activity format* (Ruthven, 2009), requiring the adaptation of strategies for classroom management and organisation. Finally, Mishra and Koehler (2006) suggest that TPACK is a special form of knowledge, different from that of the technology expert, subject matter specialist or the general pedagogic knowledge shared by teachers across disciplines. In teaching mathematics, TPACK could be exemplified by the knowledge underlying a teacher's selection of spreadsheet software for the capability to manipulate variables and formulae dynamically for the pedagogic purpose of supporting an investigative approach to learning algebra, whilst understanding the limitations of the mathematical representation, such as the discrepancies between spreadsheet and standard algebraic notation, and recognising and developing strategies to deal with the pedagogical implications of these limitations.

The nature of the central TPACK construct remains weakly conceptualised (Graham, 2011). For example, Bowers and Stephens (2011) conclude that the central TPACK construct may represent the empty set in terms of particular knowledge or skills. Instead, they suggest TPACK should be regarded as an orientation or disposition towards viewing technology as a critical tool for identifying mathematical relationships. In contrast, Niess et al (2009) propose TPACK as integrated knowledge, representing the intersection and interconnection of content, pedagogy and technology knowledge. As a result, the nature of teachers' mathematical knowledge for teaching using technology, represented by the central TPACK construct, remains unresolved.

DATA COLLECTION AND CONTEXT

As part of a pilot study for the author's PhD project, three case study teachers were observed teaching a lesson involving technology and subsequently asked to reflect on the lesson in a post-observation interview. Initially presented in XXXX (2009), the data has been re-analysed using the TPACK framework for the purposes of this paper. Here the case study of Alice is used as an illustrative example to explore the nature of teachers' mathematical knowledge for teaching using technology, as represented by the central TPACK construct. Alice was an experienced mathematics teacher, working at a private girls' school in the UK. She was teaching a group of 14 girls aged 14-15 years, who had just sat their end of school-year exams. Alice noticed that the majority of the group had incorrectly answered a standard question on the *n*th term of linear sequences and this lesson was intended as a revision lesson of the topic. In Alice's selective school, this group were regarded as low-attaining in mathematics, although according to their predicted grades for the national school-leaving exam (GCSE[1]) they would generally be considered as having average or above-average attainment. The lesson took place in a computer room specially booked for the occasion. Alice used a PowerPoint presentation on the interactive whiteboard to introduce the topic, demonstrating the differencing method to find the *n*th term of a linear sequence, followed by a pencil-and-paper worksheet. After going through the answers to the worksheet on the interactive whiteboard, the students worked on a spreadsheet exercise where they had to provide the *n*th term for a series of sequences.

ANALYSIS

Demonstrating TPK: generating questions randomly as classroom management

In the interview, Alice demonstrated technological pedagogical knowledge (TPK), articulating how she uses her knowledge of the existence and capabilities of the PowerPoint and spreadsheet software to enhance her pedagogy. She explained how her use of technology enhanced her classroom management, helping her to maintain students' engagement in the tasks she set them and contributing to her smooth handling of the lesson. In particular, the downloaded spreadsheet had an important pedagogic advantage over non-ICT resources such as a textbook or paper worksheet: it incorporated a button that when clicked would re-generate all the questions to be different. For Alice, this was the "cleverness of the spreadsheet…, the thing that I couldn't have written personally" which meant that, during the lesson, she could prevent one student from copying another without a disruptive intervention such as moving her to another seat. Alice used her knowledge of this capability of the spreadsheet to allow her to maintain a less intrusive style of classroom management.

Alice also identified the provision of immediate feedback as a significant feature of the spreadsheet exercise in enhancing her teaching as compared with traditional tools. When the students entered a potential *n*th term for a sequence, the spreadsheet provided immediate feedback: 'well done' for a correct answer and 'try again' for an incorrect one. She explained that the spreadsheet improved pupils' confidence, thereby having a positive impact on their engagement and productivity.

> Once they've done three or four, they know they can get the next few right. It tells them immediately that they have got them right, and then they feel that here's something I can do.

Linked to increasing the students' engagement and productivity, the immediate feedback from the spreadsheet enhanced Alice's capacity for effective classroom management. It freed her from constant requests from students asking for validation, allowing her to target her own skills more efficiently to ensure the smooth running of the lesson.

> …they must all have done more than 18 questions. Now with that group, that's quite a lot of questions for them to have done in a 10 minute time, because of this thing that they tend to stop after one question and wait for reassurance before they carry on to the next.

Significantly, Alice did not indicate how the mathematical knowledge she makes available to her pupils in the lesson is altered by the transformation of pedagogical techniques she is able to enact through her knowledge and use of technology. Indeed, during the interview, she appears to suggest that the mathematical content of the lesson remains unaltered, identical to a lesson conducted without digital technology, using a traditional whiteboard and textbook exercise on sequences.

> I used presentation software with a little bit of interaction in it, you know, a few claps when they got something right, and I could just as well have done that on the board though it might not have [appeared] so neatly and it wouldn't have looked so neat, but the spreadsheet that they used, that was essentially just like doing a series of questions from a book except that they got immediate feedback.

Alice's focus on enhancing her pedagogy through her use of technology indicates a lack of depth in her consideration of the changes to the mathematical content made available to the students through her teaching using technology. Her apparent demonstration of TPK in fact serves to highlight the shallowness of her TPACK, since her belief that the mathematical content of the lesson remains unaltered suggests a weakness in the transformation of her *mathematical knowledge* for teaching using technology. Thus it is not that Alice has a thorough grasp of TPK but has yet to integrate her knowledge of mathematics with her knowledge of technology and pedagogy to achieve TPACK. Rather it is that the *depth* of her mathematical knowledge is insufficient to appreciate and critique the changes in her teaching of mathematics brought about by her use of technology. For example, Alice's use of the capability of the spreadsheet to randomly generate a set of questions to enhance her classroom management suggests an explicit disregard for the pedagogic advantages and disadvantages of choosing specific examples over others. Indeed, she explained during the interview that what this class needed was "lots and lots of questions that are all identical, so it builds confidence". Rowland et al (2009) suggest that random generation of examples might be reasonable as a means of demonstrating the efficacy and general application of an established method. However in this lesson, Alice's aim was to counter a particular misconception she had noticed in the pupils' recent exam, namely that if a linear sequence has a common difference of a between one term and the next, then it has an nth term of $n + a$. Random generation of examples may be inappropriate here since it may give rise to examples like $3n + 3$ which obscure the role of variables and may unintentionally act to reinforce such a misconception. In addition, Alice did not explain the benefits of the immediate feedback provided by the spreadsheet in terms of the mathematical insight her pupils might gain. Instead, she struggled to find a rationale based on mathematics pedagogy, hoping that the pupils' increased productivity might improve retention, whilst acknowledging that it might not. Thus for Alice, the significance of the spreadsheet's provision of immediate feedback lay solely in enhancing her capacity for classroom management and not in the possibility of altering the mathematical knowledge made available to her pupils through her teaching.

Demonstrating TCK: identifying discrepancies in spreadsheet notation

Alice also demonstrated TCK in her lesson and interview, recognising discrepancies between standard algebraic notation and the algebraic input accepted by the spreadsheet as a valid nth term. Articulating these discrepancies demonstrates Alice's understanding of how the capabilities of the software may alter the presentation of mathematical content, hence her TCK. During the lesson, she raised the pupils' attention to the issue that the spreadsheet would, for example, only accept $3n + 0$ as a valid nth term for the three times table, rejecting the standard $3n$ as invalid. In this instance, she suggested they ignore the spreadsheet, remembering that for the exam they would need to write $3n$. In another departure from standard notation, the spreadsheet accepted both $1n + 5$ and $n + 5$ as equally valid answers. Alice did not raise this issue with pupils in the lesson. During the interview, she explained why she had raised one issue but chose to ignore the other.

> Coming back to the [GCSE] exam, I think they would get the mark for 1n+5 and one other mark for n+5, so the fact that the spreadsheet would take either didn't seem to me to be a problem. I thought it was more of a problem […] it wouldn't take 3n, it would only take 3n + 0. That is a problem because obviously, you know, because 3n+0 is not nearly as good an answer as 3n.

Thus she intentionally overlooked this discrepancy between standard algebraic notation and the spreadsheet notation, whilst drawing attention to the issue of the spreadsheet accepting $3n + 0$ but rejecting $3n$. This suggests an explicit ignorance on Alice's part of the pedagogic advantages or disadvantages of her *choice of examples* (Rowland et al., 2009). In addition, by asking students to ignore the spreadsheet, Alice reinforces her position of authority as the source of mathematical knowledge, undermining her argument that the immediate feedback provided by the spreadsheet can act as an alternative source of mathematical knowledge for the students to rely on. There is no point in the students following the spreadsheet's instruction to 'try again' when they appear to get a question incorrect, since it may be the spreadsheet in error. Instead, from the students' point of view, they are better off turning once again to Alice for ultimate validation. Further, by asking students to ignore the spreadsheet and rely instead on her judgement of what is expected in the exam, she misses an opportunity to examine *why* $3n$ may be conceived as an equally valid, if not better notation for the nth term of the three times table. She therefore misses the opportunity to build her students' ability to rely on themselves as a source of mathematical knowledge. Thus it seems the depth of Alice's *mathematical knowledge* is insufficient for her to recognise the pedagogic value in discussing explicitly the discrepancies spreadsheet and standard algebraic notation. In particular, Alice's demonstration of TCK serves to highlight the shallowness of her TPACK, again indicating a lack of depth in her consideration of the potential changes to the mathematical content made available to the students through her teaching using technology. Although she recognises using technology may lead to alterations in the presentation of mathematical content, she fails to consider the implications for mathematics pedagogy of such alterations. That she does not see the changes to mathematical content through technology use as impacting on her teaching of mathematics suggests a weakness in the transformation of her *mathematical knowledge* for teaching using ICT. Significantly it is not that Alice has a thorough grasp of TCK but has yet to integrate her pedagogical knowledge with her knowledge of technology and mathematics to achieve TPACK. Instead, it is that the *depth* of her mathematical knowledge is insufficient to appreciate and develop the changes in the presentation of mathematical content through technology use for pedagogic purposes.

DISCUSSION

A major advantage of the TPACK framework is that by emphasising technology as a knowledge domain alongside pedagogy and content knowledge, the existence of teachers' mathematical knowledge for teaching using technology is highlighted through the central TPACK construct. To an extent Alice exhibited some level of TPACK. She demonstrated sufficient mathematical knowledge to select appropriate technological resources to teach the given mathematical topic with some degree of competence to her students. However, her demonstrations of TPK and TCK both serve to highlight the shallowness of her TPACK, by indicating a lack of depth in her consideration of the potential changes to the mathematical content made available to the students through her teaching using technology. Importantly, in each case it was not that she had a thorough grasp of the dyadic components, TPK and TCK, but had yet to integrate her knowledge of content and pedagogy respectively. Instead, it is that the *depth* of her mathematical knowledge was insufficient to appreciate and develop the changes in the presentation of mathematical content through technology use for pedagogic purposes. Explicit recognition of how changes in the presentation of mathematical content could be transformed for pedagogic purposes would entail a deepening of Alice's existing mathematical knowledge for teaching using technology. The analysis presented above suggests that the central TPACK construct may be better understood, not as a new category of knowledge representing the integration of technology, pedagogy and mathematical knowledge, nor as an orientation towards using technology, but rather as a transformation and deepening of existing *mathematical knowledge* for teaching using technology. A further hypothesis is that the dyadic constructs TPK, TCK and also PCK may not exist as distinct categories of knowledge in the actuality of classroom practice. However, these constructs do provide useful analytical tools for identifying weaknesses in teachers' *mathematical knowledge* for teaching in the context of a particular technological tool.

Notes

1. GCSE stands for General Certificate of Secondary Education.

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