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FREE SINGULARITY PATH PLANNING OF HYBRID PARALLEL ROBOT

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ABSTRACT

This paper presents a singularity-free path planning approach for a hybrid parallel robot. The hybrid robot is composed of two well-known parallel robots, a hexapod and a tripod, that are serially connected. In this paper a methodology is developed to avoid singularity configurations of the hybrid parallel robot. Nominal polynomial paths are used for motion of end effector, and the strokes of each actuator is calculated by using the developed inverse kinematic. A MATLAB program has been developed to generate the designed paths, and several poses have been tested in a CAD model of the hybrid parallel robot to validate the feasibility of the path planning approach.

Keywords: Parallel Robot, Path planning, Singularity.

1 INTRODUCTION

Parallel robots are close loop mechanisms which are utilized in industry for various applications, such as flight simulation, machining and pick and place operation. Compared with serial manipulators, parallel mechanisms have higher operating accuracy, although they maintain the disadvantage of exhibiting low work volume compared to serial manipulators. The hexapod manipulator is a 6-UPU parallel mechanism, which has six prismatic legs and two circular platforms, the legs are connected to the platforms with universal joints. Each of the six legs can move linearly which is driven by a linear electric motor. Therefore, this parallel mechanism has 6 degrees of freedom (6-DOF). This hexapod parallel robot is designed based on the Stewart Platform manipulator, which is one of the most well-known models in the family of parallel robots. The detailed introduction of the Stewart Platform manipulator has been investigated (Merlet, 2006).The tripod parallel manipulator is constructed on the upper platform of the hexapod robot, and the end-effector is positioned on the upper platform of the tripod manipulator. There have been various investigations of the kinematic analysis of the parallel mechanisms (Chen *et al.* 2009).This hybrid parallel robot is designed to provide a tradeoff of merits and demerits of close loop and open loop mechanisms (Haddab et al. 2013). By serially connecting these two parallel mechanisms together, the hybrid parallel robot gains a larger work space without losing its inherent advantages over serial robots. To plan paths for parallel manipulators, the singularity configurations of the manipulator should be avoided. Kinematic singularities are particular poses of the end-effector where parallel robots lose their inherent infinite rigidity. Two types of serial and parallel singularities have been categorized (Rost et al. 2011). It is widely believed that serial singularities are obtained on the boundary of workspace, while parallel singularity presents when the actuator forces on the legs cannot be supported. The path planning approach of the hybrid parallel robot is developed based on its inverse kinematic analysis. Inverse kinematic analysis gives the relationship between position of joint and end-effector poses based on the physical structure of a robot. For each given pose of the end-effector, the inverse kinematic analysis calculates out the length of actuators on each leg of the robotic mechanism.

A criterion of singularity configurations for the Stewart Platform manipulators is presented (Dasgupta *et al.* 1998). They also developed a singularity-free path planning method based on linear interpolation and conquer algorithm. A singularity-free path planning approach considering the kinetic and potential energy of parallel mechanisms for various types of parallel robots have been introduced (Sen et al. 2003) using Euler-Lagrange equations to solve the singularity-free path, (Anjan et al. 2005) described a singularity-free path planning algorithm by grouping singularity points into various clusters. Currently, a number of researchers have investigated the issue of singularity-free path planning problems for parallel robots, especially for the Stewart platform parallel manipulators (Jazar et al. 2007).

1.1 Inverse kinematic

Positions of joints attached to platforms are calculated by using transformation matrix while initial positions of joints are fully defined. Figs.1 and 2 show the simulated and the physical prototype of the hybrid parallel robot that is considered in this paper.

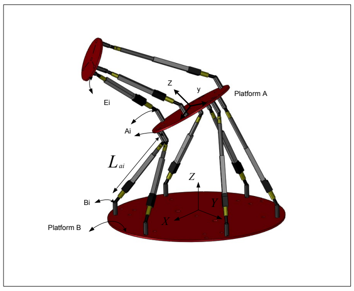

Figure 1: Hybrid parallel robot CAD model

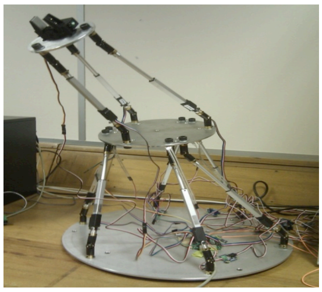

Figure 2: Hybrid parallel robot physical model

Position vectors of the hexapod are developed by equation.1.

$$L_{Ai} = A_i \times T_B^A - B_i \qquad i \in [1 \dots 6] \tag{1}$$

Where, L_{Ai} is position vector of each actuator connecting platforms B and A, T_B^A is transformation matrix for platform A, A_i is initial position of joint on platform A, and B_i is position of joints on platform B.

Moreover, the transformation matrix for hexapod structure was calculated based on three rotations motions and three translation motions as in equation.2 (David F, J. Alan. 1990).

$$T_B^A = T_{l,m,n} \times R_\theta \times R_\varphi \times R_\psi \tag{2}$$

Where, θ, φ, ψ are defined rotational components and l, m and n are linear motions in X, Y, Z directions respectively. The transformation matrix platform E (End Effector) is related to platform B, as given in equation 3.1.

$$T_B^E = T_A^E \times T_B^A \tag{3.1}$$

$$T_A^E = T_n \times R_{\theta_i} \times R_{\varphi_i} \tag{3.2}$$

Where, T_B^E is transformation matrix of platform E related to B, T_A^E is transformation matrix of platform E related to B, and T_B^A is transformation matrix of platform A related to B, as defined in equation 3.2.

Therefore the length of each actuator is calculated by equation 4.

$$L_{Ti} = E_j \times T_B^E - A_j \times T_B^A \qquad j \in [1 \dots 3] \tag{4}$$

Where, L_{Ti} is position vector of each actuator connecting platform A and E, E_j is initial joint position on platform E, and A_j is joint positions on top of platform A.

1.2 Singularity

In general, the singularities happen while some of the vector force of actuators in particular poses and orientations of the end effector are zero. Singularities in workspace can be calculated by identifying the force transfer matrix of the structure. A typical consequence caused by the singularities of parallel robots is the collapse of the parallel structure.

The proposed hybrid structure composes of nine actuators that connect two moving platforms and a fixed platform. Inverses kinematic analysis determines position vector of actuators that are not related to one other. Unit vector of each actuator demonstrates the direction of the force created by the actuators.

In the Cartesian space (global frame), the pose X of the end-effector is determined by the position variables ($x \quad y \quad z$) and orientation variables ($\alpha \quad \beta \quad \gamma$), as can be expressed in equation 5.

$$X = [x \quad y \quad z \quad \alpha \quad \beta \quad \gamma]^T \tag{5}$$

The exerted force on top platform hexapod (platform A) is developed on equation 6.

$$F_{in}(X) = [F_1 \quad F_2 \quad F_3 \quad F_4 \quad F_5 \quad F_6]^T \tag{6}$$

Where, F_{in} is the force matrix of hexapod created by actuators on platform A and F_i actuator force.

In order to calculate the vector force, unit vector of actuators are obtained as shown below:

$$S_i = \frac{L_{Hi}}{\|L_{Hi}\|} \qquad i \in [1 \ldots 6] \tag{7.a}$$

$$F = \sum_{i=1}^{6} s_i F_i \tag{7.b}$$

The created momentum on the platform A due to linear forces of actuators is obtained by equation 8.

$$M = \sum_{i=1}^{6} (A_i \times s_i) F_i \tag{8}$$

The results of actuators force for each motion could be defined in a matrix that is expressed in equation 9.

$$F_{out}(X) = [F \quad M]^T \tag{9}$$

The relations of input and output loads of the hexapod parallel manipulator can be given by

$$F_{out}(X) = H(X) \cdot F_{in}(X) \tag{10}$$

The matrix $H(X)$ is a 6×6 force transformation matrix which indicates how the output load is related to the input forces.

The force transformation matrix of the hybrid parallel robot is obtained as:

$$H(X) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ A_1\times s_1 & A_2\times s_2 & A_3\times s_3 & A_4\times s_4 & A_5\times s_5 & A_6\times s_6 \end{bmatrix} \tag{11}$$

The force transformation matrix H represents the relations between the input forces F_{in} and the output forces F_{out}. When $H(X)$ is singular, the transformation equation (2) will be degenerated and some loads (forces and/or moments) on the upper platform cannot be supported by the actuator forces. As a result, the end-effector gains some extra degrees of freedom and become uncontrollable.

Therefore, the criterion of the singularity configuration of the hexapod platform is equivalent to the singularity of the force transformation matrix. Therefore, the criterion of the singularity configuration of the hexapod parallel manipulator is given as given in equation 12.

$$\det(H(X)) = 0. \tag{12}$$

Where, $\det$ is a command defined to calculate the determinant of a matrix in MATLAB.

The boundary singularity configuration is determined by the boundary conditions of the hybrid parallel robot, the boundary conditions are determined by the length limitations of the nine actuated legs. The boundary condition of the hybrid robot can be expressed as given in equation 13.

$$L_{min} \leq L_i \leq L_{max} \quad (i = 1 \text{ to } 9) \tag{13}$$

Where, $L_{min} = 226.11\ mm$ and $L_{max} = 326.11\ mm$.Thus, the criterion of singularity configurations of the parallel robot consists of the criterions of serial and parallel singularities. The singularity-free condition of the hybrid parallel robot can be expressed as:

$$\det(H(X)) \neq 0, \text{ and } L_{min} \leq L_i \leq L_{max} \quad (i = 1 \text{ to } 9) \tag{14}$$

2 SINGULARITY-FREE PATH PLANNING APPROACH

2.1 Initial polynomial paths for end-effector

The path planning approach for the hybrid parallel robot starts with defining an initial nominal path to connect two given poses of the end-effector in workspace. Initially, a straight line is used as the nominal path. However, when the end-effector moves along the considered path, large accelerations are present along the path especially at the start and end pose.

This problem is solved by using a polynomial path as the nominal path. As a polynomial path is smooth the acceleration of the path can be considered using high degree polynomial equations. A detailed description for generating polynomial path is introduced by Reza N. Jazar [9].

In this section, a polynomial path will be formulated to connect a start pose P_0 and an end pose P_1 without considering the singularity and boundary conditions. The path is formulated as a function of a parameter u $(0 \le u \le 1)$ with the understanding that it can be parameterized with respect to time t. The function of the nominal path can be expressed as:

$$P(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \cdots + a_n u^n \; (0 \le u \le 1) \tag{15}$$

Where, $P(0) = X_0$ and $P(1) = P_1$.

Where, the parameter n represents the number of degrees of the polynomial equation which is determined by numbers of constrains the function must fulfill. In order to finish the tasks faultlessly, the end-effector needs to move without velocity and acceleration when approaching the target poses P_0 and P_1. Thus, a rest to rest path with zero acceleration at the start and end poses is considered in this path planning approach, i.e. the first, second and third derivative of the polynomial path at both initial and final poses must equal to zero. Those constrains are given by

$$\begin{gathered} P(0) = P_0 \quad \dot{P}(0) = 0 \quad \ddot{P}(0) = 0 \quad \dddot{P}(0) = 0 \\ P(1) = P_1 \quad \dot{P}(1) = \ddot{P}(1) = 0 \quad \dddot{P}(1) = 0 \end{gathered} \tag{16}$$

To meet these eight conditions, a seven degree polynomial path is used, which can be expressed as

$$P(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5 + \quad a_6 u^6 + a_7 u^7 \quad (0 \le u \le 1) \tag{17}$$

Substituting equation (16) into equation (15), gives the seven coefficients $a_0, a_1, \ldots a_7$ of the polynomial path, and the initial nominal path of the path planning approach given by equation (17). Assuming that $X_0 = 0$, $X_1 = 100$, the sketch of the nominal polynomial path connecting these two poses. The second and third differential of the nominal path represents the accelerations of the nominal path respectively. As can be seen is the graph, the value of accelerations at the start point X_0 and end point X_1 is equal to zero, thus the end-effector will start and stop at the target poses without acceleration when moving along the designed path.

2.2 Serial optimization method

After a nominal polynomial path connecting the initial pose P_0 and final pose P_1 has been obtained, the path shall be optimized to avoiding the singularity configurations of the end-effector. In this section, an optimization method is developed for the hybrid parallel robot.

According to the kinematic analysis of the hybrid parallel robot, the poses of the end-effector along the nominal path are determined by the equation shown below:

$$P(u) = P_H(u) + P_T(u) \tag{18}$$

where $P_H(u)$ and $P_T(u)$ are the poses of the centre points of upper platforms of hexapod and tripod respectively. And the values of position and orientation variables of $P_H(u)$ and $P_T(u)$ are based on the global coordinate system.

By defining a proportional factor Δ for the equation (19), the poses of the hexapod and tripod manipulator can be obtained with respect to the poses of the end-effector, which is expressed as:

$$\begin{gathered} P_H(u) = \Delta P(u), \\ P_T(u) = (1 - \Delta) P(u) \qquad \text{where} \quad \Delta = \Delta_0 \end{gathered} \tag{19}$$

Thus, initially, the path of the upper platforms of the hexapod and tripod manipulator can be expressed as:

$$P_H(u) = \Delta_0 P(u), \qquad P_T(u) = (1 - \Delta_0) P(u). \tag{20}$$

While the end-effector moves along the nominal polynomial path, the poses of the end-effector will be checked with the singularity-free condition of the parallel robot, the condition has been obtained. When a specific pose is determined to be singular, an optimization method is developed to adjust the poses of both hexapod and tripod parallel manipulator. This method is referred to as "Serial optimization method". This method is designed to solve the problem of singularity by adjusting the proportional factor Δ of the hybrid parallel manipulator. Therefore, the pose of the hexapod $P_H(u)$

and the pose of the tripod $P(u)$ will be adjusted to support the singularity of the hybrid robot. Meanwhile, the end-effector will always stay at the same pose $P(u)$, and the nominal path for the end-effector will not be changed in this method. Thus, the smoothness and continuity of the seven degrees polynomial path are maintained in the final singularity-free path. This is also a practical advantage of the hybrid parallel robot described in this paper over the general type of parallel manipulators.

2.3 Detour optimization method

When the “Serial optimization method” fails to find a singularity-free path, then the pose of the end-effector has to be changed. In this category of situations, another optimization method, referred to as “Detour optimization method”, will be applied to generate a singularity-free path.. The pose X_{via} will replace the initial singular pose on the nominal polynomial path. Therefore, this optimization method is designed to obtain a singularity-free path by optimizing the nominal path locally, and the final path will detour the singular poses presented on the nominal polynomial path. While the end-effector moves along the nominal polynomial path, the poses of the end-effector will be checked with the singularity-free condition of the parallel robot when a singular pose $X_{singular}$ is presented on the nominal path, a via point X_{via} will be obtained by applying a loop algorithm. During the procedure of the loop, intermediate poses will be determined to replace the singular pose on the nominal path, and the intermediate poses are given by optimizing the position variables (x, y and z) of the singular pose. The singular pose found on the nominal polynomial path is expressed as:

$$P_{singular} \quad P(u) \quad [x_u \quad y_u \quad z_u \quad \alpha_u \quad \beta_u \quad \gamma_u]^T \tag{21}$$

Intermediate poses are found by changing the coordinate value of the singular pose starting from x, y and z respectively. Finally, a non-singular via pose is found, which is expressed as:

$$P_{via} = [x_{via} \quad y_{via} \quad z_{via} \quad \alpha_u \quad \beta_u \quad \gamma_u]^T \tag{22}$$

The output of this detour optimization method is:

$$P(u) = P_{via} \tag{23}$$

Thus, after the procedure is completed, the singularity pose on the nominal path is replaced by a non-singular via pose. The nominal polynomial path has been locally optimized and becomes a singularity-free path for the end-effector. In this method a parameter ∂ is utilized to ensure that the via poses do not locates too far away from the singular pose on the nominal polynomial path. This parameter is designed to guarantee the smoothness of the final non-singularity path.

3 RESULTS AND DISCUSSION

A MATLAB program has been developed to implement the singularity-free path planning approach based on developed inverse kinematic analysis of the hybrid parallel robot. The step interval τ in the program is considered to be 0.01. A motion profile is applied to the software as an example to determine possible singularities. The developed program is applied for both hexapod and hybrid structures. Besides, after testing MATLAB program with various trajectories, the best value of the default proportional factor is determined to be 0.5.

$$P_H(u) = 0.5P(u), \quad P_T(u) = 0.5P(u) \tag{24}$$

The MATLAB program has been tested to generate singularity-free path with various poses in workspace, where one is shown below.

The planed path for the end-effector of the hybrid parallel robot starts from a start pose P_0 to a final pose P_1,where, $P_0 = [10 \quad 10 \quad 10 \quad 0 \quad 0 \quad 0]^T$, $P_1 = [100 \quad 80 \quad 110 \quad 0.15 \quad 0.25 \quad 0]^T$.The determinant of the transformation matrix $H(X)$ and a path connecting these two poses has been generated by the path planning MATLAB program. After the singularity-free path is obtained from the MATLAB program, the CAD model of the hybrid parallel robot is utilized to validate the planned path. The path of the platform A by applied optimization method for hybrid parallel robot are compared with simulation data in Fig.3, 4 and 5 while obtained path of platform E in theoretical and simulation in X, Y and Z are demonstrated in Fig.6, 7 and 8.

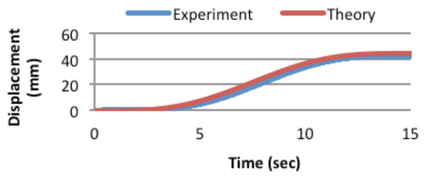

Figure 3: motion of Platform A in X-axis

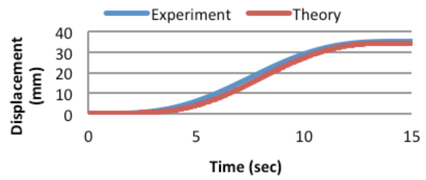

Figure 4: Motion of Platform A in Y-Axis

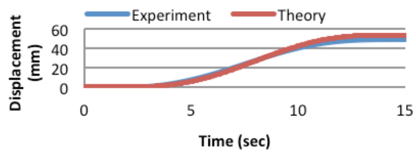

Figure 5: Motion of platform A in Z-axis

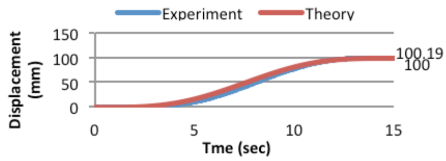

Figure 6: Motion of platform E in X-axis

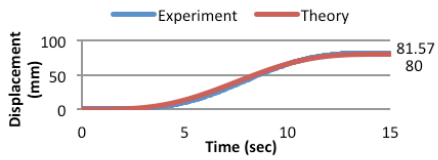

Figure 7: Motion of platform E in Y-axis

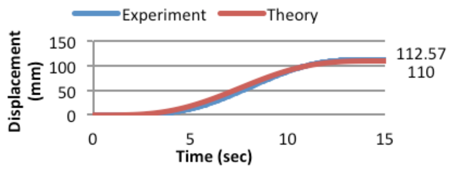

Figure 8: Motion of platform E in Z-axis

4 CONCLUSIONS

This paper addresses the developed path motion for new configuration of hybrid parallel robot. The developed polynomial path applied to inverse kinematic of system in order to calculate stroke sizes of actuators that were imported to CAD model. The theory developed for non singular path was successfully validated by obtained results from the simulation.

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