PRIMARY SCHOOL - CHILDREN'S STRATEGIES FOR ADDITION

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ABSTRACT

Children use a range of addition strategies during the primary years and progress from using mainly counting based strategies to retrieval of known number facts.

This thesis looks at the cognitive developmental and social factors which influence children's strategy choices for addition sums during these early years.

Siegler and Jenkins's (1989) model for the distribution of strategies based on the speed and accuracy of a strategy for a particular sum, and Baroody and Ginsburg's (1986) schema based theory of a search for relationships and cognitive economy are challenged. The studies in this thesis reveal a large proportion of children whose conceptualisation of these abstract concepts seems to be at variance with that of adults.

Contrasting theories about the conceptual basis for the transition from counting all to using min are investigated through a comparison of performance on commutativity tasks and strategy choices for sums. The studies trace development over the primary years and show an informal knowledge of commutativity in very young children.

Curriculum interest in number patterns prompted an investigation into possible links between retrieval of number facts for sums and retrieval for number patterns. Performance on the patterns varied, and though a relationship was found more research in this area of curriculum development is needed before any conclusions can be reached.

When questioned, most of the children aspired to using retrieval, though analysis of performance showed that strategy choice was governed by type of sum, age and rated ability.

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CHAPTER 1

INTRODUCTION

Because of its central role in the foundation of mathematical competence simple addition merits past and present study. It forms part of the child's early discovery of the world around him/her through informal play and contact with quantities of objects and relationships among them, e.g., with items of food, activities with toys, etc. This wealth of informal knowledge is brought to the formal task of simple addition in school. Unfortunately, for some children classroom instruction serves to separate symbols from the knowledge they are meant to represent. Hiebert and Lefevre (1986) state:-

> 'For many children the effect of initial instruction on arithmetic symbols is to pry apart conceptual and procedural knowledge and send them in different directions. Up to this point both types of knowledge seem to develop in close synchrony, continually informing each other. But with the introduction of written symbols whose meanings are not well established, the dynamic interaction is broken'. (p.20)

The task of research into simple addition is to seek to discover how the child conceptualises the addition process, and how to connect knowledge of procedures with their conceptual referents.

At the beginning of the century psychological inquiry was based on classroom practice. In 1922 Thorndike proposed the strengthening of arithmetical bonds and associations by putting the child through a series of structured arithmetical exercises which would form his/her response to similar situations thereafter. The task was to formulate lists of arithmetical bonds which were mental habits and connections for performing a particular arithmetical task, e.g., the distinct psychological functions involved in simple addition described in his book 'The Psychology of Arithmetic' (p.52). These bonds would be strengthened through rewarded drill and practice. In this way the 'law of effect', - rewarded practice, would enable learning to take place.

The psychological analysis of distinct mental connections would be used to create a structured practice regime through which the child would strengthen bonds and learn the underlying principle. This idea contrasted with the previous policy of stating a principle which the pupil learned, followed by tasks which he could not do unless he understood the principle. It was left to the pupil to devise ways of understanding the principle, and so solving problems.

Thorndike's psychological aims were to promote accuracy with a view to the world of work:-

> 'If clerks got only six answers out of ten right ... one would need to have at least four clerks make each computation'. (p.105)

The job of the teacher was to provide practice to strengthen the stimulus response reactions so that bonds were integrated into a whole system which developed in complexity, e.g., the co-operation of learned addition and subtraction bonds in solving division problems.

The question of the boredom of drill was addressed by saying that the child would not object to 'bareness' of meaning, so long as the 'bareness' of failure was prevented, and that confidence in accuracy through prolonged practice was reward in itself. He spoke of children having ' a general interest in getting right answers', and of the responsibility of 'time well spent' (p.271) in terms of classroom instruction, placing the responsibility for productive learning firmly in the direction of teachers rather than pupils, as had previously been the case. This approach stimulated psychological inquiry into mechanisms of learning and instruction amongst colleagues of his day.

Thorndike's drill and practice for retrieving from memory was challenged by Brownell (1928) and others (cited in Resnick & Ford 1981 and Carpenter & Moser 1983). They found that children used a variety of strategies for simple arithmetic like finger counting, using known facts, as well as direct retrieval of number facts. He stressed the meaningful approach of the understanding of quantities rather than the automatic retrieval of Thorndike's method.

Both were concerned with the understanding of arithmetical principles, the differences were in the route to be taken.

Brownell proposed instruction based on concepts and relationships, combining and separating concrete quantities, grouping and labelling them, so that the child was able to relate the symbol to the quantity. He was concerned with the transfer of knowledge to novel situations brought about by conceptual understanding linked to procedures. His 'meaningful habituation' rather than 'meaningless repetition' was the basis for instructional schemes. There would be an increase in speed and accuracy with understanding after an initial decline in accuracy during the acquisition phase of new procedures.

Brownell's views were supported by the evidence of further research in the 1930's and 1940's.

It was found that performance on un-taught combinations was more successful in a group taught by the meaningful approach than the drill method, but that the drill groups produced immediate responses to number facts more efficiently.

The argument between the rote learning and practising of number facts and meaningful instruction, stressing ongoing conceptualisation has continued, and remains un-resolved today. As in the past, the aims of instruction are the same, the difficulties lie in integrating methods of instruction which combine the benefits of practice with the insight and creativity of meaningful instruction.

Fleming (1946) emphasised the benefits to individual children of individual textbooks which enabled them to progress at different rates without wasted time on copying from blackboards. It was possible to think in terms of individual step by step mastery, which Fleming considered essential if individual differences like ability, health and attendance were to be adequately coped with.

At the same time as changes in the classroom organisation and materials came studies of success and failure of the case study type. These attempted to see each problem pupil in his/her complete environmental setting, taking into account the pupils physical, mental, social and emotional conditions associated with arithmetical failure. Fleming listed the following examples of pupil error in addition sums as a checklist for teachers concerned with individual needs, and not mass instruction.

1 Ignorance of certain combinations.
2 Addition of the same digit to a second column.
3 Difficulty in bridging the tens.
4 Attempt at wrong operations.
5 Mixture of wrong operations.
6 Ignorance of carrying.
7 Carrying of wrong number.
8 Omission of carrying.
9 Beginning with wrong column.
10 Addition of second column to first.
11 Zero difficulties.
12 Difficulties with unseen numbers.
13 Difficulties with empty spaces in columns.

A significant difference between pre-war and post-war years seems to have been a shift of emphasis from the mass needs of the work place in providing accurate calculators, to the fulfilment of individual needs for competence, contributing to a general well being.

Fleming's summary of research recommendations however was set against a post-war Britain with classes of over forty pupils and shortages of teachers and resources.

In the 1950's research began to focus on the psychological elements of addition and not outcomes. Ilg and Ames (1951) were concerned with developmental stages and the psychological processes of operations. They described development in four stages: in the first, count all was used on all problems, in the second, retrieval of number facts was used on some, and count on from the first addend for the rest. In the third stage, the range of retrieval increased and min (counting on from the larger addend) replaced count on, and in the fourth stage, retrieval was used on most sums with a variety of strategies such as decomposition, (the manipulation of known facts), being used on the rest.

They presented a gradient of the development of the child's concepts and abilities in number and quantities from birth to nine years. Their aim was to plot developmental readiness so that levels of instruction in arithmetic could be matched with the child's actual developmental performance, regardless of age. The focus was on the kinds of errors children made because certain types of error are widespread at certain stages of development, e.g., errors of +1 or -1 are common at five or six years old and so do not have the same significance as at eight or nine, when they warn of basic counting errors needing specific attention. This type of psychological analysis of the conceptual and procedural development of addition strategies has continued.

The 1960's saw changes in the mathematics curriculum of primary schools. The Schools Council Bulletin of 1966, 'Mathematics in the Primary School' endorsed discovery methods, encouraging children to think for themselves and record their findings. Educators foresaw that the development of computers would free the workplace from much of the tedium of calculations, so they emphasised that:-

> 'Mathematics is a discovery of relationships ... and the expression of the relationships in symbolic form'. (p.9)

They summarised their ideas, supported by the Plowden Report (1967) as being that:-

1 Children learn concepts slowly.

2 All pass through stages of development depending on age and experience. (This statement showing the influence of the work of Piaget (1952).

3 Learning can be accelerated by suitable learning experiences.

4 The value of practice is in fixing a concept, supporting Diene's view that practice is the third stage in learning a concept, not the first. (p.9).

The essence of the Bulletin was:-

> 'Perhaps the most important message of 'modern' mathematics at this level (primary) is its ubiquity, the fact that doing sums is only a fraction of the programme envisaged'. (p.27)

The Plowden Report 'Children and their Primary Schools', 1967, welcomed 'progressive' methods with the stress on enhanced pupil choice in work, freedom to move and talk, group work and integrated subjects with less teacher direction and control, in order to foster social and emotional development. There was concern with matching the existing knowledge and ideas of the child to experiences which would develop these skills and concepts. A view which has underpinned educational thinking ever since. Informal teaching methods claimed to recognise 'quickening trends' leading to more progressive approaches in a 'child centred' regime.

In many ways the Plowden Report was a turning point in educational practice in general, and mathematics in particular, with the focus on the individual child's needs setting the pace and content of instruction. This philosophy is summarized in the following quotation:-

> 'There has to be the right mixture of familiar and novel, the right match to the stage of learning the child has reached ... Children can think and form concepts so long as they work at their own level and are not made to feel that they are failures'.

In 1976 the lack of precise description of what was going on in the classroom prompted research into the effects of teaching methods, and the personality characteristics of the pupil, on academic progress. Neville Bennett in his book, 'Teaching Styles and Pupil Progress' (1976), found that pupils taught in a formal class structure were superior in mathematics achievement to their informal and mixed style counterparts. The evidence of mathematics achievement tests showed that:-

> 'Better progress in mathematics understanding is evident with formal teaching styles and is apparent at every level of achievement, except amongst the lowest achieving boys'. (p.93)

What Richards (1982) calls the 'heady idealism of Plowden', gave way to the more 'circumspect, measured aspirations of the 1978 H.M.I. Primary Survey', resulting from the so called educational 'Great Debate' of the 1970's, in which concern for academic standards was expressed.

The survey found that scores achieved by junior school children in the N.F.E.R. mathematics tests were disappointing. Group and class instruction in mathematics rules was recommended to 'quicken the pace of mental responses and encourage accuracy'. The report focused on the 'equality of curricular opportunity'. They identified thirty-six items in the experienced curriculum of 80% of the classes inspected (twelve items concerned with mathematics), and found many of these items lacking in up to 25% of the classes generally. They concluded that:-

> 'the coverage of items varied from class to class and showed no overall consistency'. (para 6.7)

The inspectorate published 'A View of the Curriculum' in 1980 in which they outlined the need for curriculum statements to form a framework of compulsory elements in the range of pupil studies. In 'Mathematics 5-11' (H.M.S.O. 1979) it was recommended that children between the ages of five and eight should begin work on:-

vii The ability to carry out practical activities involving ideas of addition.

viii The ability to perform simple calculations involving mathematical processes indicated by + sign with whole numbers (maintaining rapid recall of the sums, differences and products of pairs of numbers from 0 - 10).

From the ages of eight to eleven:-

i An appreciation of place value and a recognition of simple number patterns.

ii The ability to carry out with confidence, and accuracy, simple examples in the four operations of number, and the addition of numbers up to two decimal places.

The government's response in 'The School Curriculum' 1980, was clearly influenced by the views of the inspectorate, and was the first statement of government guidance since 1944. It made local authorities and schools responsible for policy making and curricula reviews, and laid the foundation for the development of the National Curriculum. The Education Reform Act 1988 in the Education Order of 1989 outlines Mathematics in the National Curriculum. The Document sets out four Key Stages from the ages of five to sixteen, the first two being for ages five to eleven (Primary). In the first Key Stage levels, 1 to 3 of the 14 Attainment Targets are to be taught, and in the second Key Stage levels 2 to 6 are to be taught. These levels are to be taught with reference to the Programme of Study which specifies the subject matter to be covered for each of the 10 levels in the Attainments Targets for the four Key Stages. There is an overlap between the levels in the Key Stages to allow for individual differences in the range of material covered, so that minorities are catered for. For example, by the end of Key Stage two, at eleven years, most pupils should have attained the middle, or close to the middle of the ranges of the levels specified, i.e., levels 1 to 6 to be taught, most pupils should be around levels 3/4 in attainment.

Assessment of the attainments of pupils will take place at the end of each Key Stage with a combination of external standard assessment tasks, (SATS) and the teachers' own assessments.

The range of primary school children's knowledge of addition is specified in the Programme of Study as:-

Level 1 Counting, reading, writing and ordering numbers to at least 10

Understanding the conservation of number using addition with numbers no greater than 10 in the context of real objects

Copying, continuing and devising repeating number patterns.

Level 2 Reading, writing and ordering numbers to at least 100 and using the knowledge that the tens digit indicates the number of tens

Knowing and using addition facts up to 10 solving whole numbers involving addition

Exploring and using patterns in addition facts to 10.

Level 3 Reading, writing and ordering numbers to at least 1000 and using the knowledge that the position of a digit indicates its value; knowing and using addition facts to 20 (including zero)

Finding number patterns and equivalent forms of 2 digit numbers and using these to perform mental calculations

Explaining number patterns and predicting subsequent numbers

Dealing with inputs and outputs from simple function machines.

Level 4 Reading, writing and ordering whole numbers

Adding two 2 digit numbers mentally

Adding mentally single digit numbers

Adding two 3 digit numbers without a calculator

Estimating and approximating to check the validity of addition calculations

Solving addition problems using numbers with no more than two decimal places.

Level 5 Generating sequences

Understand and use simple formulae or equations expressed in symbolic form.

Level 6 Reading, writing and ordering decimals and appreciating place value

Determining possible rules for generating sequences

Using spreadsheets or other computer facilities to explore number patterns.

N.B. Most pupils should have reached levels 3/4 by the second Key Stage at eleven years old.

CURRENT RESEARCH INTO STRATEGIES FOR SIMPLE ADDITION

In parallel with the evolution of teaching methods and classroom management has been research into the psychology of mathematics operation. Following on from the precise psychological descriptions of addition strategies by Ilg and Ames, research inquiry over the past twenty years has been in two main areas, knowledge structures, and operational strategies. Unlike the earlier research described, these studies have not been directly related to classroom practice, but have been more concerned with the psychological mechanisms and developmental aspects of children's addition strategies.

Knowledge Structures

The initial representation of numbers and quantity and the linking of ordinal and cardinal values to written numerals has been the subject of extensive study (Greeno, Riley & Gelman 1985; Fuson, Richards & Briars 1982; Fuson 1983; Sinclair & Sinclair 1986; Gelman and Meck 1986; Hughes 1986; Todd, Barber & Jones 1987). The differing interpretations young children have of number operations in formal arithmetic have been considered by Weaver (1982), who draws attention to the meaning a child attaches to number sentences. For example, adding two discrete sets to form a single set in a binary operation is conceptually different from joining one set to another to form a third in a unary operation. He proposes that these conceptual differences could explain why some children fail to recognise commuted pairs; seeing 3 + 4 as conceptually different from 4 + 3.

It has been proposed that the conceptualisation of part/whole relationships are the basis for the development of min and decomposition. Resnick (1983) describes a possible emergence of min when the child applies a part-whole schema by assigning addends to slots in the whole, whose parts can be added in either order to discover the value of the whole. Other researchers (e.g., Baroody 1987) believe that the invention of min is not so much conceptually based but rather the saving of mental effort. By having students justify or complete correct and incorrect strategies performed by a puppet Putnam, DeBettencourt & Leinhardt (1990) studied the students understanding of part-whole relationships in their use of derived number facts in decomposition.

Information processing psychology has tried to bridge the gap between the skills involved in performance and the conceptual base linked to the performance. Much of the work has developed from attempts to program computers to simulate human behaviour. Theorists seek to understand human thinking in terms of networks of semantic memory where information is organised into related knowledge structures through which new relations amongst existing concepts are found, as well as processing incoming information. For example, the inverse relationship between addition and subtraction where the same quantities are involved, but with different outcomes depending on the operation. This being linked to the procedural knowledge that when setting down the subtraction sum, the larger number is placed first for the smaller one to be subtracted, whereas in addition the numbers can be added in either order (Resnick & Ford 1981).

Operational Strategies

Methods for studying addition strategies have been mainly chronometric analysis, individual interviews, observation or a combination of these techniques.

In 1972, Groen and Parkman found that the reaction time of young children varied as a function of the sum of two addends. They compared the reaction times of adults and children and concluded that a fast access to memorized facts exits which is more efficient in adults than children. This process may be stimulated by visual display, for example, the uniformity of ties, which always had lower latencies for adults and children, and which were not related to addend size. When this process failed the children resorted to a reconstructive process involving counting. The researchers proposed reaction time to be a linear function of the number of steps required to perform a task, and that keeping track of the count influenced all counting models in a uniform way by setting a mental register at nought, and then counting on by incrementing by one each time until the addition sum was reached. If the count began from the first addend then reaction time was a function of the quantity of the second addend; a more efficient procedure being to begin the count from the larger addend, regardless of position, thus requiring fewer counts and reaction time being a function of the minimum addend.

Ashcraft's (1982) chronometric analysis of mental processes suggests developmental trends in the mastery of arithmetical knowledge, with initial reliance in procedural counting followed by a gradual shift to retrieval of number facts, from a network of number facts built up through practice, with reaction time for certain facts, e.g., ties, being quicker.

The reaction time of subitizing is discussed by Resnick and Ford (1981), showing scan time for small arrays of three to four dots being quicker than counting, thus leaving more room in working memory for other necessary operations.

Besides reaction time studies, children have been interviewed and observed to find out what strategies they use.

Some studies seek to discover the operational strategies of children, based on conceptual knowledge, through interviews. Each child is asked to explain or justify his/her responses which are then interpreted, and strategies inferred (Carpenter & Moser 1982; Fuson & Hall 1983; Baroody 1984; Resnick & Ford 1981; Gelman & Meck 1986; Siegler 1987; and many others).

Through informal observation Fuson (1983) discovered that when counting on some children stated the number word for the first addend before counting on the numerals for the second, whist others began with the enumeration of the second addend. When dealing with young children observational techniques are often more appropriate than questioning because of the limited language development of the subject. Case (1982) observed pre-school and older children in order to relate arithmetical performance to the ability of the child's processing capacity to handle the quantitative load that these increasingly complex procedures entail.

A number of studies have combined interview and chronometric analysis. Svenson and Broquist (1975) combined the two methods of enquiry by interviewing their subjects after each timed trial. The evidence of the interviews and inferences drawn from reaction times both suggested that the children were using min. However, Siegler (1987) found that whilst solution times were consistent with the view that children use the min strategy, verbal reports revealed that min was one of five approaches that the children were using. This use of a range of strategies was true for individuals as well as groups.

A possible consequence of different methods is that differing conclusions are reached (Kaye, Post, Hall & Dineen 1986). Though reaction time studies are quantifiably more precise they do not reflect reality in the same way that interviews do, as Siegler found with the min strategy. Interviews do not have the limitations of assuming the type of counting process found in reaction time research, nor do they assume that the time required for various steps is constant for different number combinations. Thus the reaction time best fitting model may be more appropriate for certain number combinations than for others. However, the interpretation of interview data may be flawed because the explanations given by children may not accurately describe what they really did, because of limited language development, (Carpenter and Moser 1983), or that strategy use is not totally under the child's conscious control

(Piaget 1952). Children may not be aware of the distinction drawn by adults between strategies, and find difficulty understanding what responses are required of them. Gelman and Meck (1986) point out that:-

> 'It is well known that young children are sensitive to variations in the social context' (p.47)

and may respond in accordance with their interpretation of situational demands, rather than their understanding of the task, particularly when they are required to identify and correct what they see to be adult errors.

That strategies change is confirmed, research must now discover why they change, with reliable evidence emerging which is supported by different methods of enquiry. As Ashcraft (1982) points out:-

> 'important as chronometric evidence is, our conclusions require support and validation from converging operations ... such mutual validation across substantially different paradigms strengthens both research traditions and will be necessary for an adequate psychology of mathematical cognition'.

Why do strategies change?

Why is there change and development instead of people continuing to use a strategy which has been proved to be perfectly adequate to the task? Opinions vary, though all are agreed on the pattern of change from counting to the retrieval of known number facts.

The transition from using counting all to using min is a source of argument. It is thought to be based on seeking economy of mental effort according to Baroody (1987), and Neches (cited in Resnick &

Ford 1981) suggests that the advantages of min are discovered through trial and error during counting activities. Siegler and Jenkins (1989) believe that new strategies emerge from existing ones, the two key influences being the accuracy and efficiency with which each strategy can be executed on a given problem of class of problems. They found that most of the children in their study discovered min through the shortcut-sum strategy. These views are at variance with the theory that min develops as a result of conceptual understanding. Briars and Larkin (1984) see min as the outcome of understanding the commutativity principle, and Resnick (1983) proposes that the understanding of part/whole relationships underlies its development.

The effects of practice are acknowledged to be crucial to strategy development. Groen and Resnick (1977) taught a group of children to count all and found that after a number of practice sessions half of the group had changed to min through their own choice and without instruction. Siegler and Shrager (1984) emphasised that practice strengthens the association between number combinations and it is generally agreed that the predominant use of retrieval is the outcome of years of practice in number calculations. Yet Carpenter and Moser (1983) acknowledge that:-

> 'little is known about the transitions from informal modelling and counting strategies that children appear to invent for themselves, to the formal algorithms and memorized number facts that children learn as part of the mathematics curriculum'. (p.38)

It is the reasons for these changes in strategy use, based on conceptual development over the primary school years, which are addressed in the following studies.

A BRIEF OVERVIEW OF THE STUDIES TO BE REPORTED IN THIS THESIS

The studies in this thesis use the interview methods adopted by Siegler and Baroody, whose work will be reviewed in Chapter Two.

After an initial survey of the distribution of strategies at primary age, the studies extend to looking at the retrieval of number facts in contexts other than sums, namely number patterns. The children's aspirations towards strategy use at different ages during the primary years compared with their actual practice is a further consideration of the role of the child. Subsequent studies of alternative strategies, the reasons for choosing one strategy as opposed to another for a particular sum, and the child's conception of the speed, accuracy and economy of strategies in relation to each other are investigated. Finally, the linking of the children's conceptualisation of the commutativity principle and its' translation into the procedures of strategy choice for doing sums is examined.

CHAPTER 2

The subject of this chapter is a review of the current work of R. Siegler and A. J. Baroody. During the course of their investigations these two researchers have indicated a number of possible explanations for strategy change, often adopting opposing positions, e.g., on the development of min, and the mechanisms by which number facts are memorized and retrieved.

Siegler's theories for the distribution of strategies in relation to sum type and strategy choice based on accuracy and efficiency are challenged in studies one, two and six to eight, and Baroody's beliefs about the development of min are questioned in studies nine and ten.

Siegler

Siegler and Shrager (1984) investigated multiple strategy use in addition and produced their 'Distribution of Associations Model of Strategy Choice', to account for the variability in children's strategy choices. They proposed three phases: retrieval, elaboration of the representation and counting, the child first makes an effort to retrieve the answer setting two parameters, a confidence criterion and a search length. The confidence criterion defines a value that must be exceeded by the associative strength of a retrieved answer for the child to state the answer. The search length indicates the maximum number of retrieval efforts the child is prepared to make. The probability of any given answer being retrieved on a retrieval effort is proportional to the associative strength of that answer for that problem, e.g., the probability of retrieving 4 for 2 + 2 might be .8 whilst for retrieving 9 for 5 + 4 it might be only .16. If the strength of the retrieved answer exceeds the confidence criteria the answer is stated, if not the child determines whether the number of searches for a retrieved answer is within the pre-set search length. Retrieval continues so long as associated strengths are below the confidence criteria and the number of searches does not exceed the search length. If this is reached then the child proceeds to phase two. Here he/she creates an elaborated representation either externally, e.g., with fingers, or internally with a mental image. Adding the elaborated representation to the already existing association between the problem and various answers prompts further retrieval efforts and if this exceeds the confidence criteria the answer is stated, if not phase three is put into operation. This algorithmic process involves counting the objects in the elaborated representation and stating the number of the last object as the sum.

Subsequently Siegler and Jenkins (1989) proposed modifications to the original model because of it's limitations. The inflexibility of always retrieving first, the identical approach to all problems, and the lack of choices between alternative back-up strategies are problems addressed in the modifications. The original model's procedure of choosing among answers has been generalised to choosing among strategies as well, with consideration for the speed and accuracy of each strategy produced and novelty points for new strategies used in preference to known strategies with a proven track record. For example, Siegler and Jenkins note that the five year olds in their study discovered the min strategy through their existing repertoire of counting strategies. Most children developed min through the 'shortcut-sum' strategy which incorporates features of both the old and new procedures. It is like the sum strategy in counting all the numbers but in one step and not counting out each addend first before summing, as in the sum strategy. It is also like min in that the representation of the second addend and its addition to the running total takes place at the same time. Thus existing strategies form transitional links in the invention of new strategies.

Within the strategy choice phase, strategies are retrieved with the probability proportional to their strength relative to the strength of all of the strategies, based on speed and accuracy in the domain. Once chosen, an attempt is made to use it, if this is not possible, e.g., inability to retrieve, then the process returns to the strategy choice phase; this cycle continuing until a strategy is chosen and executed producing an answer.

In 1988 Siegler examined individual differences in relation to the Siegler and Shrager (1984) model. Children were classified into three groups; good students, not-so-good and perfectionists. Perfectionists were children who had good knowledge of problems and set very high thresholds for stating a retrieved answer, if this threshold was not reached then 'back-up' counting strategies were used to solve the problem. Good students also had good knowledge of problems but set lower thresholds for stating a retrieved answer before using back-up counting strategies. Not-so-good students had less knowledge and low thresholds for stating a retrieved answer. Results showed that perfectionists used retrieval less than the other two groups, but were as accurate and fast as the good students, who used retrieval almost twice as many times, with more errors than perfectionists. The not-so-good students used retrieval almost as many times as the good students but with more errors.

Siegler intuitively related the individual differences of these three groups to Kogan's (cited in Siegler 1988) definition of the 'reflectivity' and 'impulsivity' construct. He saw the construct as similar to the role of the confidence criterion in the decision of whether to state a retrieved answer, or to use a back-up counting strategy which was sure to achieve success.

Geary and Burlingham-Dubree (1989) replicated this work and found that their results supported those of Siegler. They proposed that young children who used back-up counting strategies as well as retrieval were making adaptive choices for solving the addition sums with success, whereas those who did not use back-up strategies very often were frequently guessing.

Baroody

According to the schema based view of Baroody and Ginsburg (1986) the addition strategies of young children are initially estimating. With time their strategies become more sophisticated and estimates more reasonable through the influence of conceptual knowledge, so that different strategies for different types of number calculations are devised. Gradually children apply the conceptual knowledge that addition makes a collection larger, reasoning that the sum must be larger than either of the addends.

The schema based view emphasises the discovering of relationships leading to the mastery of many number combinations by learning the general rule, e.g., adding nought does not change the sum, and adding one is a continuation of the count.

In his analysis of the evolution of counting strategies, Baroody (1987) classifies development into closely related stages. Concrete counting all (cc) is the first stage where fingers or objects representing each addend are counted out separately then totalled for the sum. A labour saving shortcut is when the procedure is the same except for the sum count, when the child sums from the cardinal designation of the first set. Further development occurs when bypassing the sum count by counting out each addend and establishing a sequential finger pattern without counting out the sum from the beginning. A continuation is when one addend is represented simultaneously with a finger pattern then the sum of both addends is counted. This leads to the first and second addends being represented simultaneously by finger patterns and counted. Finally both addends are represented by simultaneous finger patterns and counted from the first addend, progressing to simultaneous finger patterns for both addends being immediately recognised for the sum, either visually or kinesthetically in a similar way to Siegler's 'finger strategy'.

Baroody describes the more sophisticated set of counting strategies that of counting entities (CE), which means creating a set of entities with the cardinality of the second addend (Fuson & Secada 1986). The counting of entities involves the recognition of the particular number pattern devised by the child, thus providing feedback on counting accuracy by recognition of the pattern, and keeping track of the count. Further development leads to pattern recognition which eliminates formal counting, e.g., for 5 + 4 the child may put up the finger pattern for four, realises that if the first pattern is raised it would mean only one finger not used, therefore the sum is one less than ten, which is nine.

As calculations increase the count of the second addend is combined with the counting sum in a single keeping track process (CAF), i.e., objects representing the second addend are used to keep track of how far the sum count must go beyond the cardinal value of the first addend. This stage is followed by CAL where the procedure is the same as CAF but the counting begins at the larger addend. Eventually the cardinal value of the larger addend is stated and the smaller addend is counted on (COL). This strategy is the most economical because it eliminates the need for counting the larger addend by starting from it's cardinal value.

Baroody investigated the relation between the transition from counting from the first addend to counting from the larger addend, which implies a knowledge of commutativity (Resnick & Ford 1981). He found that only four out of seven five to six year olds, who used a strategy which disregarded addend order, were successful on commutativity tasks. However, inconsistency in performance may be the result of 'protocommutativity', an order indifferent adding scheme where numbers can be added in any order producing a correct though not necessarily the same answer.

ANALYSIS OF ERROR PATTERNS: DIFFERENCES BETWEEN ASSOCIATION BASED AND SCHEMA BASED MODELS

Both Siegler and Baroody examined error patterns. Siegler classified the effects of errors into two main types. The first follows the widespread belief that the negative feedback of errors leads the learner to generate alternative strategies, e.g., Van Lehn (1988).

> 'Learning occurs only when an impasse occurs. If there is no impasse, there is no learning'. (p.31)

The second is the generation of new strategies through a search for efficiency, as in the discovery of min from counting all strategies. According to Siegler and Shrager's (1984) model practice results in number traces being built up in long term memory, whether correct or not. Some incorrect answers are more likely to be strengthened through practice than others, e.g., counting string associates like 2 + 4 = 5, where 5 follows 4 in the count, and miscalculations by one, a common error in young children and which could also explain 2 + 4 = 5. However, with time, children learn to add efficiently the correct answers being strengthened with all basic number facts mastered independently.

Baroody (1989) found that children's error patterns were more a result of applying specific strategies. Some estimated, some made 'teens' responses, e.g., 8 + 5 = 18, and some stated a favourite number. A number of children with low developmental readiness on the pretest stated an addend for the answer, whilst children who scored higher on the arithmetic readiness assessment were able to use more genuine estimation strategies and nearly all knew the combinations involving one. He found that new or infrequent responses were associated with shifts in strategy use, some resulting from refinement to estimation strategies. There were few examples of counting string response errors, or a broad range of unaccountable responses. Baroody concludes that a network of numerical associations and practice cannot satisfactorily account for the changes in error patterns which produce correct answers. He quotes Ilg and Ames (1951) 'more an error of method than an error of answer' as a more likely explanation, with retrieval less mechanistic than the Distribution of Associations model suggests.

SUMMARY OF THE MAIN POINTS OF THESE VIEWS WHICH ARE TO BE FOLLOWED UP IN THE STUDIES OF THIS THESIS

Siegler's ideas on the frequency of strategy use in relation to type of sum are investigated in studies one and two with six to nine year old children.

The modified Distribution of Association model (Siegler and Jenkins 1989), where strategy choice is influenced by the speed and accuracy of a particular strategy for a particular problem or class of problems is challenged in studies six to eight with the same age group. In these studies children are asked to give reasons for their original and alternative strategy choices, and are also asked to judge strategies for speed, accuracy and economy relative to each other and different types of sums.

Baroody's belief that the use of min does not necessarily depend on a knowledge of the commutativity principle is explored in the last two studies with children aged five to nine. In these studies the children complete tasks involving concrete materials, numerals and sums, so that comparisons in performance can be made, to discover their knowledge of commutativity, and whether or not this knowledge is reflected in strategy use for the sums.

CHAPTER 3

INTRODUCTION TO STUDY 1 AND STUDY II

As described in Chapters One and Two, there have been several studies where children's strategies for solving addition problems have been inferred from observing what they did or asking them how they did them (e.g., Geary & Burlingham-Dubree 1989; Carpenter & Moser 1984; Goldman, Davis, Mertz and Pellegrino 1989; Siegler 1987 & 1988). The general impression is that strategy use varies between children of the same age, and also within the same child, with different strategies being used on different sums.

Siegler (1987) found that most children reported using at least three of the following; count all, min, retrieval, decomposition and guessing. He also found that the frequency with which particular strategies were reported changed with age; the use of count all declining with a marked increase in the use of retrieval which was the most common strategy for both first and second grade children, overtaking min which showed little increase. Decomposition increased, though it was relatively rare even amongst the oldest children (see Table 3.1A Reproduction of Siegler, 1987, Table 2).

STRATEGY					
GRADE LEVEL	RETRIEVAL	MIN	DECOMP-OSITION	COUNT ALL	GUESS OR NO RESPONSE
Kindergarten	16	30	2	22	30
Grade 1	44	38	9	1	8
Grade 2	45	40	11	0	5
Overall	35	36	7	8	14

Table 3.1A PERCENTAGE OF USE OF EACH STRATEGY BY CHILDREN OF EACH AGE

The oldest children studied by Siegler (1987) were second graders, i.e., 7 or 8 year olds, and they were attending an upper middle class American school in which they received substantial amount of instruction in both single and multiple digit arithmetic problems.

One question arising from Siegler's results is whether British children would report similar proportions of strategy use, or whether min would be replaced by decomposition as the back up strategy used when retrieval failed to yield an answer. British children's instruction in arithmetic may differ in several important respects from American children's: it is probably not so devoted to doing sums, and is possibly more devoted to understanding aspects of number composition, such as part/whole relationships, which should facilitate both retrieval and decomposition.

A further question stems from the large number of young children's responses in the category of 'guess or no response' (see Table 3.1A). For the youngest this was a very common category. Siegler (1987) did not distinguish between trials on which children did not volunteer an answer and those on which they said they guessed, so it is not possible to tell how common the latter were.

The problem of interpreting children's reports was described by Johnson and Wellman (1980) who found that children up to 9 years old used 'know and guess' indiscriminately. Sodian and Wimmer (1987) found that most 4 to 6 year olds used the terms correctly to describe their own state of knowledge, but there was still a sizable proportion ($^{12}/_{48}$) who said 'guess' when they should have said 'know'.

Some of the children who said 'guess' may have known the answer and described their retrieval of the number fact as 'guessed'. The videotaped record would not reveal this because unlike counting strategies, retrieval is not often accompanied by overt behaviour. So the observed increase with age in reported use of retrieval may be partly due to the children's increased ability to communicate their strategy use.

Carpenter and Moser (1983) also expressed doubts about children's reports. They found it difficult sometimes to identify strategies from children's comments and even suggested that some children found such difficulty in describing what they had done that they decided to describe another strategy which was easier to explain.

Analogous problems have been encountered in studies of young children's understanding of counting where the aim is to assess children's knowledge of counting principles, and it is recognised that such knowledge may well be in advance of their ability to verbalise these principles. One tactic adopted is to use a puppet to demonstrate conventional, unorthodox and faulty counting (Briars & Siegler 1984; Gelman & Meck 1983).

Using puppet demonstrations of addition strategies makes clear to the child what strategies are considered distinct by the adult and reduce strategy identification to a matter of recognition. It may however distort the process of identification in some way and so the principal aim of this study is to compare the distribution of strategies reported by children when shown strategies to choose from (Video Inquiry), and when they are simply asked how they did the sum (Oral Inquiry).

Subsidiary aims are to explore how strategy use varies with sum type and rated ability.

Expectations of how strategy may vary with sum type can be derived from a priori considerations as well as previous research. Retrieval would be expected to be most common on sums with small addends as these are likely to have been encountered most often. Also Siegler and Shrager (1984) found considerably more use of retrieval by young children than Siegler (1987). In the former studies, all sums had addends less or equal to 5 and even with the range of sums used by Siegler (1987) addend sum was the best predictor of retrieval use.

Decomposition was found by Siegler (1987) to be most common when one of the addends was greater than 10, presumably because this would be decomposed into 10 + n as in 15 + 4 where 5 + 4 = 9 and 10 +9 = 19.

In discussing when min would be used Siegler (1987) considered several possibilities: if the smaller addend is less than 4 it would be easier to execute; if the difference between addends is large, the advantages in speed over count all would be greater. Because his model assumes that children only resort to min if attempts to use retrieval or decomposition fail to deliver an answer he argued that probabilities of min use should be assessed with conditional probabilities rather than unconditional ones. Essentially by using conditional probabilities in the way he did he was actually considering the relative propensity to use min over count all. What he found was that children were indeed more likely to use min than count all on sums with large differences between addends and when the smaller addend was small.

How robust these various findings of variation in strategy use with sum type will be examined in Studies I and II.

Finally, how much strategy use varies from child to child is explored. The approach to be taken is to compare variation in strategy use with variation in the teacher's rating of pupil ability.

In his study of individual differences in strategy use Siegler (1988) found that achievement test performance showed the superior performance of the perfectionists and good students over that of the not-so-good students. However, only the experimental situation discriminated between the perfectionists and good students who were indistinguishable in measures of knowledge, yet showed a considerable difference in their pattern of strategy use, especially in the use of retrieval, which the good students chose more frequently than the perfectionists. The analysis of performance on achievement tests involved one dimension: knowledge, whilst analysis of performance in the experimental situation explored two dimensions: knowledge and confidence criteria for stating a retrieved answer, or cognitive retrieved style.

The decision to use teacher's rating of ability was based on the two dimensional approach. The teacher has considerable day to day experience of the work habits as well as the knowledge levels of the child in addition tasks in the classroom, which are like the ones to be given. It is possible that the teacher's intuitive assessment of the individual differences of the study group will be based on a two dimensional approach of knowledge and cognitive style over a period of time.

To sum up, the particular questions to be answered by these studies are; how method of inquiry (oral or video) will affect strategy use, how strategy will vary with sum type and whether strategy use will vary with rated ability.

In addition there will be the general interest in how frequencies of strategy use by this wider age range of British children will compare with a more socially selected group of American children.

The sums in these studies are set well within the capabilities of the subjects to increase the possibilities of valid responses. For the older group the range of numbers used is one to sixteen, and for the younger children the sums are all composed of single digit numbers. They are presented in writing as they are in ordinary classroom arithmetic in order to reduce the need to maintain a representation in working memory whilst trying to solve it, which may in itself be a cause of error.

STUDY I

METHOD

3.1 Design

The children were placed in four groups with four subjects in each group. Groups differed in the order and combination of conditions and sum sets.

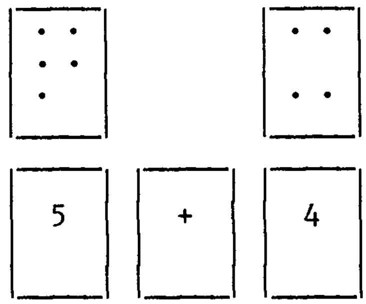
Group I	oral inquiry for Set I	video inquiry for Set II
Group II	oral inquiry for Set II	video inquiry for Set I
Group III	video inquiry for Set I	oral inquiry for Set II
Group IV	video inquiry for Set II	oral inquiry for Set I

3.2 Subjects

There were eight boys and eight girls taken from a first year (8 to 9 yrs) mixed ability class of a middle school. The children were chosen by the class teacher to represent the ability range from below average, average, to above average, on a rating scale 0 to 10 with 5 as average. The mean age was 9 years and 4 months with a standard deviation of 3 months.

3.3 Materials and Apparatus

A video was made with a glove puppet illustrating four strategies on a plain background, with dots for the numbers 4 and 5, and cards with the numbers written on and a plus sign on the fifth card, e.g.,



The puppet calculated the sum 5 + 4 using each of the four strategies, count all, min, retrieval and decomposition. Each child had a sum sheet with twenty-four sums in two sets. Each set of sums consisted of two each of the following types:-

Small and small addends	Numbers 1 to 5
Small with medium addends	Numbers 1 to 9
Medium with small addends	Numbers 9 to 1
Medium with large addends	Numbers 6 to 16
Large with small addends	Numbers 16 to 1
Large with medium addends	Numbers 16 to 6

The experimenter had a similar sheet for noting strategy choice and ongoing comments.

3.4 Procedure

The children came individually in random order depending on the convenience of leaving their classroom activity. In the oral condition the child wrote down the answer to the first sum of the set and was asked "How did you do that sum?" The strategy was noted on the experimenter's sheet with any other relevant comments. This was repeated for each of the twelve sums in the set.

In the video condition the video was shown first then the child wrote down the answer to the first sum in the set. After that the video was re-run and the child identified the strategy used. This was repeated for each of the twelve sums in the set. If the subject said that his/her strategy was not demonstrated, he/she was asked how the sum was done and this reply was noted.

3.5 Results

A Preliminaries

There was only one child with one error in this group, a miscalculation of one (13 + 6 = 18). There was no significant difference between the distribution of strategies of boys and girls, or according to the order in which the sums were worked, or the sum sets.

B Overall Strategy Frequencies

	STRATEGY					
	COUNT ALL	COUNT ON	MIN	RETRIEVAL	DECOMPO-SITION	GUESS
FREQUENCY OF REPORTED USE	5	22	206	104	46	1
NUMBER OF CHILDREN	4	6	16	14	9	1
RANGE OF USE	1-2	1-11	3-24	1-13	1-12	1

TABLE 3.1 FREQUENCY OF REPORTED STRATEGY USE

Min was used for all types of sums by all the children, it was the most frequently used strategy of twelve of the children, and accounted for over half of all strategy choices. During the course of the first interview the first subject used a count on from the first addend strategy. It was decided to categorize this separately so as not to confuse it with the min strategy. The child who reported guessing gave a correct answer which may have been retrieved.

C Variation in Reported Strategy with Method of Inquiry

STRATEGY

	COUNT ALL	COUNT ON	MIN	RETRIEVAL	DECOMPOSITION	GUESS
video	5	9	104	62	12	0
oral	4	13	102	42	34	1

TABLE 3.2 VARIATION IN REPORTED STRATEGY WITH METHOD OF INQUIRY

The only significant difference in reported frequency of strategy use is for decomposition. The results of the Wilcoxon test ($p < .02$ when $N=7$, $T=0$), indicates that there were more identifications in the oral than in the video inquiry. Because decomposition is a manipulative strategy the video demonstration was one of several possible demonstrations, so some children may not have identified their use of decomposition with the portrayal of decomposition on the video because they did not see the connection.

The children used a variety of decomposition strategies such as adding and subtracting and using ties, e.g., for 6 + 5, 5 + 5 = 10 + 1 = 11 or 6 + 6 = 12 - 1 = 11.

The differences in the frequency of decomposition is matched by reported uses of retrieval where there were 33% more identifications in the video condition than the oral, possibly because retrieval is more straightforward to identify with the number fact either known or not.

Min was the most frequently used strategy and was chosen equally in both conditions, and though there was no video demonstrations of count on, the children either said that their strategy was not demonstrated and proceeded to describe it, or they said that they used min but began at the beginning.

D Variation in Strategy Use with Sum Type

It was expected that retrieval would be more frequent with smaller addends; Siegler and Shrager (1984), and Siegler (1987), found retrieval more frequent with addends less than six. Consideration of practice effects also supports the retrieval of addition facts of small numbers, these being memorized from an early age through constant use. Siegler (1987) also found decomposition most common when one addend was greater than ten, this being true for conditional and unconditional probabilities; and the conditional probability of min most common when the problem included small addends of one to three, or there was a difference of more than eight between addends.

SPEARMAN CORRELATION COEFFICIENTS

	TOTAL	ADDGTEN	ADDLFOUR	DIFF	COMB	MINFREQ	RETFREQ
ADDGTEN	.8368**						
ADDLFOUR	-.5458*	-.3427					
DIFF	.4264	.5895*	.0122				
COMB	-.3727	-.0800	.8995**	.2874			
MINFREQ	.4698	.3223	-.3770	.1361	-.4266		
RETFREQ	-.4925*	-.2861	.4930*	.1664	.5865*	-.7735	
DECFREQ	.1683	.1186	-.2685	-.4432	-.3005	.0756	-.5820*

* - SIGNIF.LE.01 ** - SIGNIF.LE.001

TABLE 3.3 VARIATION IN REPORTED STRATEGY USE WITH SUM TYPE

KEY:-

ADDGTEN = addends greater than 10

ADDLFOUR = addends less than 4

MINFREQ = frequency of min

RETFREQ = frequency of retrieval

DECFREQ = frequency of decomposition

DIFF = the size of the difference between addends

COMB = composite of the difference between addends and whether addends are less than four

TOTAL = totals of the sums set

The table shows that retrieval is unlikely with the larger sum totals, rs - .492 ($p < .01$) as Siegler found. Retrieval is also associated with whether one addend is less than four rs.493 ($p < .01$) and with the composite variable of the size difference between addends and addends less than four rs.586 ($p < .01$) which is sums with a small addend and small and large addends. No relationship between decomposition and addends greater than ten was found. There is a suggestion that decomposition is associated with sums where the difference between the addends is small, rs - .443 ($p < .05$), which could be due to the use of ties in decomposition, Min is more associated with sums with large totals, rs.469 ($p < .05$) though used generally with all sum types.

E Variation in Strategy Use and Rated Ability[a]

	CO	MIN	RET	DEC
TR	-.15	-.5*	.8***	.57*
CO		.39	-.17	-.24
MIN			-.69**	-.59 **
RET				.5*

* SIGNIF LEV.05 ** SIGNIF LEV.01 *** SIGNIF LEV.001

TABLE 3.4 VARIATION IN STRATEGY USE WITH RATED ABILITY (Spearman)

[a]. 16 children in the group.

Table 3.4 shows a relationship between teacher's rating of pupil ability and frequency of strategy use reported. The higher the rating the more likely the subject is to use retrieval and decomposition. With count on and min showing negative relationships the reverse is the case, the higher the rating the less likely the subject is to use these two strategies. Subject 8 illustrates this point in that he used min only three times, the lowest score of all, and was rated nine in ability. The negative relationship of min with retrieval and decomposition rs-.69 ($p < .01$) and rs-.59 ($p < .01$) suggests that the pupils using the min strategy to a large extent are unlikely to use retrieval or decomposition often. The positive relationship between retrieval and decomposition .5 ($p < .01$) shows that the children who use retrieval often tend to use decomposition also.

There were only twenty-two uses of count on reported out of a total of 384. This strategy had no significant correlations with other strategies or teacher's rating. There were only five reported uses of count all so these were not included in the table, nor was the one guess, as both strategies represented only 1.5% of the total number of calculations.

3.6 Discussion

There was a significant difference between oral and video inquiry for decomposition with almost three times as many reported uses of decomposition for oral than video inquiry. This is possibly because in the video re-run after each sum the retrieval demonstration came before decomposition and the children may have chosen this strategy because they were using known number facts in decomposition. Because decomposition is a manipulative strategy the children's oral descriptions varied which could have created difficulties for them when identifying from one video demonstration amongst several possible demonstrations, e.g., 5 + 4 could have been demonstrated as 4 + 4 plus one. The oral condition may also have given the children the opportunity to explain their individual strategy variations, or looking at the sums may have suggested a decomposition strategy. The frequency of the reported use of retrieval for the two conditions matches decomposition with a third more video identifications than oral, possibly because the demonstration of retrieval was unambiguous compared with decomposition, in that the number fact in question was either known or not.

Count on, which was not demonstrated on the video was used for nine sums in the video inquiry. The children described their strategy when they found that it was not demonstrated, or some said that it was like the second one (min) but that they began at the beginning.

Results showed that retrieval was rarely used for sums with larger totals, as Siegler found. However, contrary to Siegler, the use of retrieval was also associated with addends less than four and with the composite variable of the difference in size between addends and addends less than four. No relationship was found between decomposition and addends greater than ten, but a moderate relationship was found between decomposition and a small difference in size between addends, possibly due to the use of ties. Min was widely used by all of the children on all of the sums, it was the first preference of twelve of the children, and accounted for over half of the total of strategies reported.

Rated ability was found to be associated with reported strategy use. The higher the rating the more likely the subject was to use retrieval and decomposition, with count on and min showing a negative relationship with ability rating. More able children tend to have more practice in number calculations because they work quicker and cover more examples thus facilitating the use of retrieval. They may also have a more efficient memorisation and retrieval system, or the child's use of retrieval and decomposition may influence the teacher's rating of ability.

SUMMARY OF STUDY I

The addition strategies of eight to nine year olds were investigated. There were eight boys and eight girls from a mixed ability first years middle school class selected to represent a range of ability and rated by their teacher on a 0 to 10 scale with 5 as average. There were 4 groups with 4 subjects in each group in 2 conditions. Twenty four sums with addends up to 16 were calculated by each child who was questioned orally about strategy use or visually, identifying the strategy used from a video of strategies with a puppet demonstrating count all, min, retrieval and decomposition. Results showed a significant difference between oral and video inquiry for decomposition, with more identifications in the oral condition. Reported strategy use varied with type of sum; retrieval was used mainly for sums with small addends and where the difference between addends was large, as well as for sums with small totals. Min was widely used for all types of sums by all of the children. Rated ability was associated with reported strategy use; the higher the rating the more likely the subject was to use retrieval and decomposition strategies.

INTRODUCTION TO STUDY II

Questions to be answered in Study II are the same as those in Study I but with a younger age group of six to seven year olds.

The design and procedure is the same as Study I but the sums are simpler, taking into account the capabilities of younger children. All the sums are single digit, and as a result of pupil responses in Study I, the smaller addend is first in each sum so that the count on from the first addend strategy can be identified.

The puppet video of strategies is colour coded to facilitate the identification of strategies by young children with limited language development, and count on is demonstrated before min, making five strategy demonstrations in all.

STUDY II

Method

3.7 Design and Procedure

The same as for Study I.

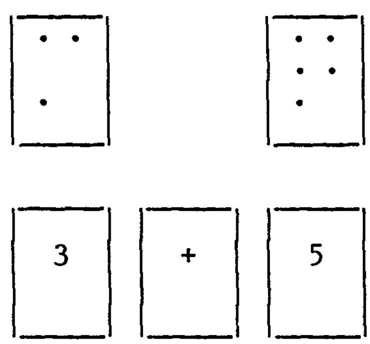
3.8 Subjects

There were nine boys and seven girls from a mixed ability second year infant class, chosen by their teacher to represent the range of ability from below average to above. She rated them on a scale of 0 - 10 with 5 as average. The age range was 6 years to 7 years 1 month, average age was 6 years 8 months with a standard deviation of 3.75 months.

3.9 Materials

A video was made in which a glove puppet demonstrated the five strategies of count all, count on, min, retrieval and decomposition in doing the sum 3 + 5. Each strategy was colour coded. There were five cards, two with the numbers 3 and 5, two with dots representing these numbers and a card with a plus sign on.

e.g.



Each child had a sum sheet with twenty four sums in two sets of twelve. Each set of sums consisted of two each of the following types:

Small with small addends	Numbers 1 to 3
Small with medium addends	Numbers 1 to 6
Small with large addends	Numbers 1 to 9
Medium with medium addends	Numbers 4 to 6
Medium with large addends	Numbers 4 to 9
Large with large addends	Numbers 7 to 9

The experimenter had a similar sheet for noting strategy choice and ongoing comments.

3.10 Results

A. Preliminaries

There were fifty four errors out of a total of 384 sums, representing 14%. The errors were mainly confined to medium and large addend sums, and especially the last two sums 8 + 9 and 7 + 8 with thirteen errors altogether. Most of the miscalculations were plus or minus one.

B Overall Strategy Frequency

	STRATEGY					
	COUNT ALL	COUNT ON	MIN	RETRIEVAL	DECOMP-OSITION	GUESS
FREQUENCY OF REPORTED STRATEGY USE	147	31	132	49	10	15
NUMBER OF CHILDREN	14	9	13	15	5	2
RANGE OF USE	1-21	1-7	1-18	1-11	1-5	1-14

TABLE 3.5 FREQUENCY OF REPORTED STRATEGY USE

Count all and min accounted for over 70% of the total number of strategies chosen. Placing the small addend first separated out the children who used count on from the first addend, though the number represented only 8% of the total. Of the fifteen guesses, fourteen were for one child who wrote 5 and 3 as the answer to each pair of sums.

C <u>Variation in Reported Strategy with Method of Inquiry</u>

	COUNT ALL	COUNT ON	MIN	RETRIEVAL	DECOMP-OSITION	GUESS
VIDEO	77	16	57	30	6	6
ORAL	70	15	75	19	4	9

TABLE 3.6 FREQUENCY OF REPORTED STRATEGY WITH METHOD OF INQUIRY

Reported strategy use did not vary significantly with method of identification, the choices being fairly evenly spread, especially for count all and min which together accounted for the majority of strategy choices. Two children identified their video strategy using the colour code, the others said 'that one', or 'like that', or a reply with similar wording.

D Variation in Strategy Use with Sum Type

SPEARMAN CORRELATION COEFFICIENTS

	TOTAL	ADDLFOUR	DIFF	COMB	COUNT ALL	MIN
COUNT ALL	.210	.09	-.299	-.449		
MIN	.322	.04	.51*	.388	-.384	
RETRIEVAL	-.341	.657**	.054	.483*	-.109	-.384

Key:-

ADDGTEN = Addend greater than ten
ADDLFOUR = Addend less than four
DIFF = The size of the difference between addends
MIN = Frequency of min
RETRIEVAL = Frequency of retrieval
COUNT ALL = Frequency of count all
COMB = Composite of the difference between addends and whether addends are less than four

* - SIGNIF LEV.01 ** - SIGNIF LEV.001

TABLE 3.7 VARIATION IN REPORTED STRATEGY WITH SUM TYPE

Retrieval is likely to be used for sums with a small addend rs .657 ($p < .001$) and with the composite variable of the size difference between addends and whether or not one addend is less than four rs.483 ($p < .01$). However, no significant association was found between retrieval and sum totals as was found with the older children of Study I, and as Siegler found. Min was more likely to be used in preference to count all on sums where the difference between addends was large, but there was no relationship between the use of min and addends less than four as Siegler suggests. There was a moderate negative relationship between count all and the composite variable of the size difference between addends and an addend less than four rs-.449 ($p < .05$) indicating that this strategy was unlikely to be used when differences between addends were large or when an addend was less than four.

E Variation in Strategy Use and Rated Ability[a]

	TEACHERS RATING	COUNT ALL	COUNT ON	MIN	RETRIEVAL	DECOMP-OSITION
TEACHERS RATING	0	-.6**	.356	.493*	.65**	.585**
COUNT ALL			-.27	-.8***	-.89***	-.20
COUNT ON				.266	.362	.335
MIN					.253	.391
RETRIEVAL						.631**

* - SIGNIF LEV.05
** - SIGNIF LEV.01
*** - SIGNIF LEV.001
a - 16 children in the group.

TABLE 3.8 VARIATION IN STRATEGY USE WITH RATED ABILITY (SPEARMAN)

The higher the teacher's rating of pupil ability the more likely the child is to use min, retrieval and decomposition. The negative relationship of count all rs-.6 ($p < .01$) suggest that less able children are using this basic strategy most of the time. There is an association between retrieval and decomposition indicated, though the latter strategy was little used.

Discussion

Errors were mainly plus or minus one and represented 14% of the total number of sums calculated. A quarter of the errors made were for the last two sums involving large addends.

Count all and min accounted for 70% of the total number of strategies reported, and placing the small addend first separated out the users of count on from the first addend, though few children used the strategy.

Reported strategy use did not vary significantly with video and oral inquiry, and only two children identified their strategy on the video by naming the colour, the other children said that they had used "that one", or "like that" or a similar phrase.

Retrieval was more likely to be used with sums where one addend was less than four and where the difference between addends was large with one addend less than four. Min would probably be used in preference to count all when the difference between addends was large as Siegler suggests, but no evidence was found for the use of min for sums with an addend less than four. Count all was found to have a moderate negative association with the composite variable of the size difference between addends and an addend less than four, indicating that it would not be chosen for sums with a large and a small addend, or where an addend was less than four.

Teacher's rating of pupil ability was found to be associated with reported strategy use. The higher the rating, the more likely the pupil was to use min and retrieval strategies, whilst pupils rated as less able continued to rely mainly on counting all.

SUMMARY OF STUDY II

This study was a repeat of Study I except for the age group and composition of the sums. The sixteen children, seven girls and nine boys, were aged between six and seven years and were from a mixed ability infant class. The sums were all single digit with the smaller addend placed first in each sum to distinguish between users of count on and users of min. Results showed that reported strategy use did not vary significantly with method of inquiry. Retrieval was more likely to be used with sums where one addend was less than four and the difference between addends was large, and min was used in preference to count all when the difference between addends was large. Teacher's rating of pupil ability was associated with reported strategy use; the higher the rating, the more likely the pupil was to use min and retrieval strategies, whilst pupils rated as less able continued to use count all.

Sum	Size of Addends	Set	Sum Number	Study	C.A.	C.O.	Min	Ret.	Dec.	Guess
4 + 3	SS	2	2	1	0	0	10	3	3	0
3 + 4	SM	1	4	2	6	2	6	0	1	1
8 + 3	MS	1	5	1	0	1	14	1	0	0
3 + 8	SL	1	8	2	4	0	10	1	0	1
6 + 5	MS	1	6	1	0	1	5	2	8	0
5 + 6	MM	1	9	2	6	1	7	0	1	1
3 + 2	SS	1	1	1	0	0	7	8	1	0
2 + 3	SS	2	2	2	10	2	3	1	0	0
5 + 1	SS	2	1	1	0	1	8	7	0	0
1 + 5	SM	2	4	2	5	1	7	3	0	0

TABLE 3.9 FREQUENCY OF REPORTED STRATEGIES FOR SUMS WITH ADDENDS REVERSED FROM STUDY I FOR STUDY II

Table 3.9 shows 5 sums taken from Study I with addends reversed for Study II in order to separate users of count on from the first addend from users of min. However, choices of count on were small for these sums, and the other sums in Study II.

The differences in the sums in the two studies could have affected the use of retrieval. There were three ties in Study II, 2+2, 8+8 and 5+5, which accounted for almost half of the total choices of retrieval (24 out of 49). There were no ties in Study I, so if there had been ties the gap between the figures for reported uses of retrieval would probably have been wider than it was (49/104).

In both studies the use of retrieval was associated with sums where one addend was less than four, and where the difference between addends was large, and one of the addends was less than four, which was contrary to Siegler et al findings. The only result which did agree with Siegler's findings was that retrieval was more likely to be used for sums with small totals, here there was a significant relationship for the older children, with the same trend though not significant for the younger ones.

Min was widely used in both studies, especially by the older children. However, it was only in the younger group that a relationship between the use of min and sums with large differences between addends was found as in Siegler's studies. No relationship was found between the choice of min and sums with one addend less than four in either study, as Siegler has proposed.

Count all was little used in Study I but was the most frequently chosen in Study II, though not for sums where the differences between addends was large or one addend was less than four, for which retrieval was the preferred choice.

Decomposition was little used by the younger children and where it was chosen by the older group it was not associated with sums with addends greater than ten, but rather where the differences were small which could have been due to the use of ties.

Sum	Size of Addends	Set	Sum Number	Study 1=9/10 2=6/7	C.A.	C.O.	Min	Ret.	Dec.	Guess
1 + 3	SS	1	2	1	0	3	2	10	1	0
1 + 3	SS	1	1	2	12	0	2	1	0	0
1 + 7	SM	1	4	1	0	2	6	7	1	0
1 + 7	SL	2	8	2	5	1	8	2	0	0
4 + 6	SM	2	3	1	1	3	5	2	5	0
4 + 6	MM	2	9	2	5	0	10	0	0	1

TABLE 3.10 FREQUENCY OF STRATEGY CHOICE FOR THE SAME SUMS IN BOTH STUDIES

Frequency of strategy choice in both studies showed a progression over time from count all to more complex min, retrieval and decomposition strategies. This pattern of choices is seen in Table 3.10 which shows the distribution of strategy use for the same sums in each study. The younger children used mainly count all and min for these sums and the other sums in the study, while the older children used min, retrieval and decomposition. These results support the findings of previous research, Siegler (1987) and Fuson (1983) which state that children progress from basic counting strategies to retrieval strategies and continue to use a variety of strategies.

Evidence in both studies showed that age and rated ability also influences strategy choice. The higher the rating the more likely the younger children were to use min, retrieval and decomposition, and less likely to use count all. The higher the rating in the older group the more likely the children were to use retrieval and decomposition. There was a positive relationship shown between min and teacher's rating of ability with the younger children, and a negative relationship for the older children indicating that more able six year olds were using min, whilst less able nine year olds were also mainly using min.

The two methods of eliciting reports of strategy use were equally effective in both studies, except for decomposition in Study I. There was a significantly higher number of reported uses of decomposition in the oral condition than the video. This could be because oral inquiry revealed observational strategies based on previous calculations, as well as manipulation of retrieved number facts, which was the only method demonstrated on the video.

CHAPTER 4

INTRODUCTION TO STUDY III

The aim of the first part of Study III is to discover the aspirations of six to ten year olds toward strategy use. Do children perceive some strategies as better than others, e.g., more approved by their teacher or more adult and mature, and are these social aspects related to strategy choice? The aim of the second part of the study is to see if there is a connection between the retrieval of number facts for sums, and the retrieval of number facts for number patterns going up in a set sequence.

Other studies have inferred strategy change through the study and analysis of strategy use (Siegler and Shrager 1984; Baroody 1987; Fuson 1983; Resnick and Ford 1981; Groen and Parkman 1972; Svenson and Broquist 1975; and many others). This study focuses on the role of the child in the social aspects of strategy change affecting aspirations towards strategy use in the present social context, and looking forward to the future. Little is known about the social constraints involved in strategy choice, yet formal arithmetic is done in a social setting. Observation of other children, the influence of instruction and the awareness of being observed must influence the child's performance and his/her future goals.

In the first part of each interview the child is asked six questions. Three about present strategy use; including observation of older children, perception of instructional demands, and preferred strategy choice. The remaining three are concerned with near and long term future use, and perception of adult performance. The strategies chosen in answer to these questions are identified from the puppet video used in Study I which makes clear the distinction adults draw between strategies.

The second part of each interview involves number patterns and sums. Children learn number patterns from an early age in singing games, stories and songs prior to their introduction to formal arithmetic. These patterns are based on counting and continuous addition. There is a possibility that a connection exists between the retrieval of patterns in informal play and the retrieval of number facts for addition sums.

In cognitive development there are cases where the causes of a particular development are unknown. In various areas such as reasoning, number, reading and memory tasks there is evidence of a transitional phase in which children have relevant skills or knowledge but fail to use them for a task, possibly because the development is incomplete and therefore the child fails to associate one aspect of knowledge with another in the same domain. Number pattern tasks are included in this study because they might reveal children's knowledge of number sequences which could be used to solve sums by retrieval, e.g., using the sequence 5, 10, 15, 20 to solve 15 + 5.

In the second part of each interview, the child is set simple auditory and written number pattern tasks to see if he/she can detect errors, and identify and continue number patterns based on the repeated addition of a constant. The child then completes ten addition sums chosen to elicit retrieval and follow the same number order as the written pattern. Performance on these tasks is compared to see if there is any relationship between the retrieval of number facts in different contexts.

Because strategy use varied with rated ability in the first two studies, it was decided to see if rated ability is associated with the children's aspiration towards strategy use, and their performance on the pattern tasks and the sums.

STUDY III

4.1 Research Questions

1. What are the aspirations of six to ten year olds towards strategy use for addition sums?
2. Is there any relationship between aspirations towards the use of retrieval and actual use for sums?
3. Is there an association between knowledge of number patterns going up in a set sequence and the use of retrieval for sums?
4. Is rated ability connected to aspirations, performance on the pattern tasks and use of retrieval for the sums?

METHOD

4.2 Design

Each of the two groups of children was divided into two equal groups and interviewed individually. The first group had the video and strategy preference questionnaire followed by the auditory pattern task. They continued with the written pattern completion task and ten addition sums, with oral strategy inquiry after each sum. The second group followed the same order as the first up to the written patterns and sums, which were done in the reverse order. The order of the six questions for the strategy preference questionnaire was balanced across children.

4.3 Subjects

There were 36 subjects aged between 6 and 10 years divided into two groups of 18. The younger group of 11 boys and 7 girls was aged between 6 years 8 months and 7 years 6 months; average age was 7 years 2 months with a standard deviation of 3 months. The older group of 10 boys and 8 girls was aged between 8 years 11 months and 9 years 8 months; average age was 9 years 10 months with a standard deviation of 3 months.

All the children were rated for ability by the class teacher on a 0 to 10 scale with 5 as average.

4.4 Materials

The video of strategies for the strategy preference questionnaire was the one used in Study I in which a glove puppet demonstrated the four strategies of count all, min, retrieval and decomposition with the sum 5 + 4.

For the auditory pattern task there was a list of spoken patterns each containing one error which the subject had to identify. These were:-

a)	1	3	5	8	9
b)	2	4	7	8	10
c)	3	6	10	12	15
d)	2	5	8	10	14
e)	4	8	13	16	20

The visual pattern completion task consisted of 5 patterns with the first 3 numbers given, the remaining 3 numbers in the sequence were supplied by the subject. The patterns were:-

a)	2	4	6	_	_	_
b)	1	3	5	_	_	_
c)	5	10	15	_	_	_
d)	1	6	11	_	_	_
e)	10	20	30	_	_	_

Both auditory and visual tasks contained conventional (2, 4, 6, _ _ _) and unconventional (1, 3, 5, _ _ _) examples of pattern sequences.

The ten sums were printed on a separate sheet and were chosen from studies I and II, with some additions, to elicit the retrieval strategy. The order of the sums was the same as the written patterns in ascending order value i.e. 2's, 5's and 10's. The sums were:-

2 + 2 =	6 + 2 =
1 + 2 =	7 + 2 =
5 + 5 =	15 + 5 =
1 + 5 =	16 + 5 =
10 + 10 =	40 + 10 =

4.5 Procedure

The subjects came individually and in random order depending on the convenience of leaving their lesson. The strategy video was shown before each of the following questions was asked:-

1) Which do you think is most grown up?
2) Which do you think your teacher likes you to use?
3) Which do you think you will use when you grow up?
4) Which do you like to use?
5) Which do you think clever children use?
6) Which do you think you will be using next year?

The order of the questions was balanced across subjects.

Following the strategy preference questionnaire the children in each age group did the auditory task then the visual tasks. In the auditory task the child was asked to select the wrong number as follows:- "I am going to say numbers which go together, but I will make one mistake, see if you can find it".

If the response was "no" or there was no response then the experimenter said "Listen carefully, I'm going to say them again, ready?"

If the subject said "yes", then the experimenter said "Which number was wrong?" then, "How do you know?" and "What should it be?"

The child was classified as correct if he/she was able to say which number was wrong and supply the correct one. A nil score was recorded if there was no response, or if the wrong number was not identified and the correct one supplied after the pattern had been repeated once.

The first group of nine subjects then proceeded to the 5 written patterns. Here the child was given a sheet with the first three numbers of each pattern written down. The experimenter explained:- "Here are some more patterns, only this time I want you to write down the three missing numbers which come after these first three numbers in the pattern". These patterns were followed by the 10 sums, after which the child was asked how he/she did the sums and this was noted on the record sheet.

The second group followed the same procedure in the written tasks as the first group but in reverse order, i.e., sums, then patterns.

4.6 Results

a) Strategy Preferences

Questions	9-10 year olds[a]				6-7 year olds[a]			
Future	CA	MIN	RET	DEC	CA	MIN	RET	DEC
Which do you think is most grown up?	0	0	12	6	0	4	10	4
Which do you think you will use when you grow up?	0	3	12	3	0	0	14	4
Which do you think you will be using next year?	1	0	12	5	1	2	12	3
TOTAL	1	3	36	14	1	6	36	11
Present								
Which do you think your teacher likes you to use?	0	1	13	4	8	6	3	1
Which do you like to use?	2	4	7	5	5	4	8	1
Which do you think clever children use?	0	0	16	2	0	2	12	4
TOTAL	2	5	36	11	13	12	23	6

[a] 18 children in each group

TABLE 4.1 REPORTED STRATEGY PREFERENCES

Retrieval and decomposition accounted for 90% of choices in answer to the questions for the older children. In the younger group 70% of choices were for retrieval and decomposition, but the pattern of choices for question two, where the children were asked which strategy they thought their teacher preferred them to use, was significantly different. Cochran's Q tests showed strategy choice varied with questions when question two was included (Q17.86 df=5 p <.01), but not when it was excluded (Q5.33df=4 n.s.). So it would seem that whilst the younger children aspire to use retrieval strategies, they also respond to perceived present instruction, and choose counting strategies.

b) Variation in Strategy Use with Sum Type

		9-10 year olds[a]				6-7 year olds[a]			
	Sums	CA	MIN	RET	DEC	CA	MIN	RET	DEC
Ties	2 + 2	3	-	15	-	-	-	18	-
	5 + 5	1	-	17	-	-	-	18	-
	10 + 10	-	-	18	-	3	-	15	-
Plus 2	6 + 2	1	8	9	-	1	10	7	-
	1 + 2	2	1	15	-	1	-	17	-
	7 + 2	1	10	6	1	2	10	6	-
Plus 5	15 + 5	-	4	13	1	2	9	7	-
	1 + 5	-	7	11	-	-	4	14	-
	16 + 5	-	9	3	6	1	15	1	1
Plus 10	40 + 10	-	4	13	1	2	10	4	2
TOTAL		8	43	120	9	12	58	107	3

[a] 18 children in each group

TABLE 4.2 REPORTED STRATEGY CHOICES FOR THE SUMS

Most of the children in both age groups reported using retrieval for the sums. The only sum with a significant difference in choice of retrieval was 40 + 10 (p < .02), for which several of the younger children used min.

There was no significant relationship found between the choice of retrieval for the sums and choice of retrieval in answer to the strategy preference questionnaire in either group (rs .227 older children, rs .063 younger ones).

c) Auditory Patterns

SPOKEN PATTERN WITH ONE ERROR						NUMBER OF CORRECT RESPONSES	
						n= 18 9-10 year olds[a]	n= 18 6-7 year olds[a]
a	1	3	5	8	9	5	2
b	2	4	7	8	10	11	5
c	3	6	10	12	15	7	2
d	2	5	8	10	14	1	0
e	4	8	13	16	20	7	1
TOTAL						31	10

[a] 18 children in each group

TABLE 4.3 AUDITORY PATTERNS

Both groups found this task difficult, especially the younger ones. There were seven nil scores in the nine to ten group and thirteen in the six to sevens, where there was a total of only ten correct responses. The conventional patterns were more successful, particularly twos; a Cochran's Q test showed that performance varied significantly with pattern type for both groups (Q19.0 df=4 $p < .001$) for the older children, and (Q11.66 df=4 $p < .05$) for the younger ones.

d) Visual Patterns

WRITTEN PATTERN CONTINUATION TASK	NUMBER OF CORRECT RESPONSES	
	9-10 year olds[a]	6-7 year olds[a]
a 2 4 6 - - -	16	16
b 1 3 5 - - -	12	11
c 5 10 15 - - -	13	9
d 1 6 11 - - -	6	1
e 10 20 30 - - -	16	12
TOTAL	63	49

[a] 18 children in each group

TABLE 4.4 WRITTEN PATTERNS

Both groups found the written pattern continuation task easier. As with the auditory patterns, the conventional patterns were more successful, especially the twos, and Cochran's Q tests again showed that performance varied significantly with pattern for both groups: (Q28.7 df=4 $p < .001$) for the older children, and (Q29.3 df=4 $p < .001$) for the six to seven year olds.

Choice of retrieval for the sums was associated with performance on both pattern tasks for the nine to ten years olds: rs.532 written patterns and rs.439 auditory patterns (both $p < .05$), but not for the six to sevens, probably because of their low level of performance, especially on the auditory tasks. A significant relationship was found between performance on both pattern tasks in the older group rs.571 ($p < .01$) but not the younger ones.

e) Rated Ability and Performance

CORRELATION	9-10 years	6-7 years
Rated ability and use of retrieval	.583**	.423*
Rated ability and written pattern performance	.627**	.75***
Rated ability and auditory pattern performance	.761***	.215
Rated ability and frequency of choice of retrieval in the strategy preference questionnaire	.507*	-.21

* - SIGNIF LEV.05
** - SIGNIF LEV.01
*** - SIGNIF LEV.001

TABLE 4.5 SPEARMAN CORRELATION COEFFICIENTS

Rated ability and the use of retrieval was related in this study as in Studies I and II, rs.583 (p <.01) for the older group and rs.423 (p <.05) for the six to seven year olds. Performance on the written pattern task was also associated with rated ability in both groups .627 (p <.01) for the nine to tens, and .75 (p <.001) for the younger children, possibly because both activities are clearly connected with the type of formal classroom arithmetic done from the outset. A strong relationship between ability rating and performance on the auditory patterns was found in the older group .761 (p <.001) as well as a moderate association between ability rating and the frequency of choice of retrieval in answer to the strategy preference questionnaire. However, for the younger children ability rating appeared to have little connection with either auditory pattern performance or choice of retrieval in answer to questions.

An analysis of variance showed that the older children performed significantly better than the younger ones on the pattern tasks ($F=6.47$, df 1,34, $p < .05$) and that the visual task scores for both groups were significantly higher than auditory scores (F58.99, df 1, 34, $p < .001$).

4.7 Discussion

Comparing the results from the two age groups it would seem that all the children aspire to use retrieval, though the influence of instruction can be seen with the six to seven year olds choices of counting strategies in reply to the question about perceived teacher preferences. There was a relationship between rated ability and the choice of retrieval in answer to the questions in the older group but not the younger one. No association between the choice of retrieval in answer to the questions and use of retrieval for the sums was found in either group.

The only evidence of a relationship between the choice of retrieval for the sums and knowledge of number patterns was in the older group, where there was a moderate correlation with both pattern tasks. There proved to be little association for the younger children who found the pattern tasks difficult, especially the auditory one, suggesting that the ability to retrieve number facts in different contexts develops with age and practice. A significant relationship between performance on both pattern tasks was found in the nine to ten year olds, but not the younger children, again probably due to the low level of performance of the six to sevens.

In both studies, performance on the conventional patterns was better than on the unconventional, especially the pattern of twos.

The sums were set to elicit retrieval which was used for most of them in both studies, particularly for ties, the only significant difference in performance being for 40 + 10 where several of the younger children used min.

As in previous studies, there was a correlation between rated ability and the use of retrieval in both studies, as well as a strong association between rated ability and performance on the written pattern tasks. There was a difference in the auditory tasks for each age group with a strong association for the older children but not the younger ones, possibly because many of the younger children did not understand the task, the majority failing to get any right.

The performance of the older children on both of the pattern tasks was significantly better than that of the younger children, and written pattern scores for both groups were significantly higher than auditory scores.

SUMMARY OF STUDY III

Thirty six primary school children aged between six and nine completed four individual tasks; the strategy preference questionnaire, auditory and written number patterns, and ten addition sums.

Whilst strategy change has been investigated through an analysis of strategy use, little is known about the social constraints which influence strategy use and change. To investigate the children's perception of social influences each child answered six questions, three concerned with present observation and use, and three on future goals. The puppet video of strategies from Study I was used for the children to identify a strategy in answer to each question.

The second part of each interview consisted of error detection in five oral number patterns and completion of five written patterns, followed by ten sums set to elicit retrieval and overlap the patterns. Number patterns were included in this study to see if the retrieval of number facts for sums is connected with the retrieval of number facts for patterns based on the repeated addition of a constant.

Results showed that whilst all the children aspired to use retrieval, the younger children responded to the influence of instruction and chose counting strategies in reply to the question on perceived teacher preferences. No relationship was found between the choice of retrieval in answer to the questions and use of retrieval for the sums. The only evidence of a connection between knowledge of number patterns and use of retrieval for the sums was in the older group, suggesting that the retrieval of number facts in different contexts develops with age. As in previous studies, rated ability was associated with the use of retrieval for the sums and was also related to performance on the written patterns, but not the auditory ones which all the children found difficult, especially the younger children, the majority failing to get any right. Conventional patterns in both pattern tasks proved to be most successful especially the pattern of twos.

CHAPTER 5

INTRODUCTION TO STUDY IV AND STUDY V

Studies IV and V are a further investigation into children's performance on number pattern tasks begun in Study III.

Despite curriculum recommendations in 'Mathematics 5-11' (1979) and inclusion in the 'Programme of Study' for the National Curriculum (1989) number patterns in formal arithmetic have received little investigation. They are part of an introduction to formal number work in early schooling in number rhymes, songs and games, and several years of teaching arithmetic have pointed to a possible link between knowledge of number patterns and the use of retrieval for addition sums. Just as retrieval is more likely to be used for sums with small addends, ties and numbers associated with 5's and 10's, so number patterns associated with these numbers would be more successfully retrieved, the two retrieval processes complimenting each other.

Both age groups in Study III found the orally presented error detection tasks more difficult than the graphically presented pattern completion task, and as expected, conventional patterns were more successful. Studies IV and V are designed to determine whether differences in performance are due to the type of task, modality of presentation, or the composition of the patterns. To do this the range of tasks has been extended to cover oral and visual error detection and pattern completion tasks, with six conventional and unconventional patterns for number sequences of 1, 2, 3, 4, 5, and 10.

In short, the studies seek to discover whether differences in performance are due to:-

a) The type of presentation- oral or visual

b) Type of task - error detection or pattern completion

c) The patterns themselves - conventional (2, 4, 6)
or unconventional (1, 3, 5)

d) The type of pattern sequence - 1s, 2s, 3s, 4s, 5s or 10s.

STUDY IV

Method

5.1 Subjects

There were sixteen children, eight boys and eight girls aged between 6 years 3 months and 7 years 9 months, with a mean age of 7 years 3 months and a standard deviation of 5 months. The children were taken from a mixed ability infant class and were rated on ability by their teacher on a 0 - 10 scale, 5 being average.

5.2 Design

A repeated measures design of four tasks each subject; two auditory tasks, one error detection the other pattern completion, and two visual written tasks, also error detection and pattern completion. The order of presentation was balanced across children, and the order of presentation of the twelve number patterns was varied within each of the four tasks.

5.3 Materials

There were four task sheets per child. Two of the sheets, the written pattern continuation and the written error detection were completed by the child in the visual condition. The remaining two sheets, oral pattern continuation and oral error detection, were completed by the experimenter as the child responded in the oral condition.

Each sheet had twelve number patterns, six conventional and six unconventional sequences of 1, 2, 3, 4, 5 and 10. There were four order variations of these patterns within the task sheets for each child.

5.4 Procedure

Before starting all the children were told that we were looking at patterns in numbers, continuing patterns with the next three numbers after the first three were given in two tasks and finding one mistake in each finished pattern in the other two tasks. There were to be two written and two spoken tasks for each child.

The children were split into two equal groups. In the first session one sub-group did the two written pattern tasks and the other sub-group did the two oral pattern tasks. In the second session, two days later, the sub-groups did the other tasks. The written patterns were done in a group and the oral patterns were done individually in a quiet room. The oral patterns were repeated once if the child failed to respond, and there was no time restriction on any of the four tasks.

5.5 Results

MODALITY TASK	VISUAL	ORAL
Pattern Completion	3.56 (3.79)	3.93 (3.13)
Error Detection	4.81 (2.53)	4.125 (2.33)

TABLE 5.1 TABLE OF MEANS (AND STANDARD DEVIATIONS)
MAXIMUM SCORE = 12

An analysis of variance revealed significant main effects for conventionality (F = 37.61, df 1, 15, p <.01) and for sequence type (F = 27.88, df 5, 75 p<.01) suggesting that the composition of the patterns has the greatest influence on performance.

There were interaction effects between conventionality x sequence type (F = 3.37, df 5, 75 p <.01) and between modality x conventionality x sequence type (F 3.67, df 5, 75 p <.01) showing that the composition and modality of the pattern presentation had a significant affect on performance, rather than the type of task, i.e., error detection or pattern completion, which was thought to be a source of difficulty in Study III.

	SEQUENCE						
	1	2	3	4	5	10	Total
Conventional							
Visual Pattern	.75	.19	.25	.19	.19	.31	1.88
Visual Error	.75	.25	.25	.38	.44	.56	2.63
Auditory Pattern	.88	.31	.19	.13	.25	.63	2.39
Auditory Error	.75	.19	.13	.25	.56	.75	2.63
Combined	3.13	.94	.82	.95	1.44	2.25	9.53
Unconventional							
Visual Pattern	.75	.25	.25	.18	.18	.19	1.70
Visual Error	.75	.25	.31	.38	.13	.38	2.20
Auditory Pattern	.81	.25	.13	.19	.06	.13	1.57
Auditory Error	.69	.13	.06	.13	.13	.38	1.52
Combined	3.00	.88	.75	.83	.45	1.08	6.99

TABLE 5.2 TABLE OF MEANS ACCORDING TO SEQUENCE TYPE, CONVENTIONALITY, MODALITY AND TASK

Table 5.2 shows a difference in performance means between conventional and unconventional sequences though overall performance was low. It is interesting to note that the conventional auditory pattern tasks were the most successful, possibly because of the amount of oral as well as written arithmetic at this age.

CONVENTIONAL

Visual

2	3	4	5	10	1
0.4	0.5	0.6	0.6	0.9	1.5

Auditory

3	4	2	5	10	1
0.3	0.4	0.5	0.8	1.4	1.6

UNCONVENTIONAL

Visual

5	4	2	3	10	1
0.3	0.5	0.5	0.6	0.6	1.5

Auditory

5	3	4	2	10	1
0.2	0.2	0.3	0.4	0.5	1.5

NB. Underscoring indicates means that are not significantly different, p <.01.

TABLE 5.3 NEWAN-KEULS ANALYSES OF MEANS FOR SEQUENCE TYPES ACCORDING TO CONVENTIONALITY AND MODALITY

A further analysis was conducted on the means for each level of conventionality and modality.

For the sequences of 1s and 2s, there were no significant differences. For the sequences of 5s and 10s, both unconventional means were less than the conventional visual mean, which in turn was less than the conventional auditory mean. For the sequences of 3s both auditory means were less than either of the visual means, and for the sequences of 4s the auditory unconventional mean was less than either of the visual means, and the auditory conventional mean was less than the corresponding visual mean. All differences reported were at $p < 0.01$.

The counting sequence of one was introduced in this study and was most successful as expected. However, even here there were only 75% correct responses. The children found the pattern tasks difficult with an overall accuracy of only 34%.

	VISUAL PATTERN COMPLETION	VISUAL ERROR DETECTION	ORAL PATTERN COMPLETION	ORAL ERROR DETECTION
Teacher's Rating of Pupil Ability	.206	.497*	.181	.365
Visual Pattern Completion		.6**	.852***	.796***
Visual Error Detection			.53*	.821***
Oral Pattern Completion				.633**

* SIGNIF LEV P<.05
** SIGNIF LEV P<.01
*** SIGNIF LEV P<.001

TABLE 5.4 VARIATION IN PATTERN PERFORMANCE AND TEACHER'S RATING OF PUPIL ABILITY (SPEARMAN)

Rated ability appears to be associated with the error detection tasks, but not the visual ones, though there was an overall relationship between performance and ability rating of .441 ($p < .05$).

5.6 Discussion

The composition of the patterns was found to have the greatest influence on performance. As expected, the conventional patterns were most successful, and particularly the sequences of one and ten. Though overall performance was low, the auditory conventional patterns were generally most successful, probably because informal and formal oral work plays a large part in arithmetic at this age.

It had been thought that the type of task had a significant effect on performance from the results of Study III, where the children found oral error detection much harder than written pattern completion. However, in this study, where the range of tasks was extended, results showed that it was the composition of the patterns, and to some extent modality, that affected performance and not type of task. On reflection, performance in the oral error detection task of Study III could have been adversely affected by the presentation. The task came straight after the questionnaire without any familiarisation or 'warm up' activity, e.g., introducing the task through a number rhyme like 'Two by Two' which would have demonstrated a number pattern already known to the children.

In Study III, sums composed of fives and tens were amongst those sums which accounted for a large percentage of choices of retrieval, which is paralleled in this study by the success of conventional patterns of fives and tens.

SUMMARY OF STUDY IV

Study IV was a further investigation into the ability of six to seven year olds to complete and detect errors in conventional and unconventional number patterns going up in 1s, 2s, 3s, 4s, 5s and 10s, in oral and visual (written) form. The study followed questions raised in Study III in which significant differences were observed in the performance of the children on the oral and written pattern tasks. The range of tasks was extended so that a more precise analysis of performance could be made into the considerable difficulties experienced by some of the children.

There were eight boys and eight girls, and each child completed four tasks, two oral and two written error detection and pattern completion tasks. In each of the four tasks there were twelve patterns, six conventional and six unconventional patterns.

Results showed that the composition of the pattern tasks significantly influenced performance rather than type of task or modality. Conventional patterns were more successful especially the sequences of 1s, 5s, and 10s. The teacher's rating of pupil ability was significantly related to overall performance and especially with the error detection tasks.

INTRODUCTION TO STUDY V

This study was the same as Study IV except for the age group of sixteen boys and girls aged between nine and ten years.

Although the materials and procedure were the same, it was decided to report the two studies separately because of the difference in performance of the two age groups. The percentage of accurate responses in the younger group was 34% whilst for the older children it was 87%, suggesting a developmental gap in knowledge and/or interpretation of the task. A separate analysis of the data would be more likely to show the specific influences on performance outlined in the introduction.

STUDY V

Method

5.7 Subjects

There were 16 children, 8 boys and 8 girls aged between 9 years and 10 years 9 months, with a mean age of 10 years 2 months, and a standard deviation of 3.75 months.

5.8 Materials and Procedure

These were the same as for Study IV (6 to 7 year olds), with all the children completing two visual pattern tasks and two auditory pattern tasks.

5.9 Results

MODALITY / TASK	VISUAL	ORAL
Pattern Completion	11.31 (.876)	10.18 (1.044)
Error Detection	10.81 (1.013)	9.44 (2.27)

TABLE 5.5 TABLE OF MEANS (AND STANDARD DEVIATIONS)
MAXIMUM SCORE = 12

An analysis of variance revealed significant main effects for modality (F=7.80, df 1, 15, p <.05). The children performed better on the visual written task (mean 22.15) than on the oral tasks (mean 19.61). A significant main effect was found for task (F=6.95, df 1, 15, p <.05). Performance on the pattern continuation tasks (mean 21.53) was more accurate than the error detection tasks (mean 20.23).

A significant main effect was also found for conventionality ($F = 36.14$, df 1, 15, $p < .01$) and for sequence type ($F = 5.74$, df 5, 75, $p < .01$), suggesting that the composition of the patterns affected performance. There were more correct responses on the conventional pattern sequences (mean 22.85) than the unconventional (mean 18.91).

The analysis of variance also revealed significant interaction effects between modality x conventionality ($F=12.71$, df 1, 15, $p < .01$) and between conventionality x sequence type, ($F=2.82$, df 5, 75, $p < .05$) showing that the interaction of modality with the composition of the pattern tasks affected performance.

SEQUENCE

	1	2	3	4	5	10	Total
VISUAL PATTERN							
Conventional	1	1	.94	.94	1	1	5.88
Unconventional	.94	1	.81	.81	.94	.94	5.44
Combined	1.94	2	1.75	1.75	1.94	1.94	11.32
VISUAL ERROR							
Conventional	1	.94	.88	.88	1	1	5.70
Unconventional	1	.81	.88	.63	.81	1	5.13
Combined	2	1.75	1.76	1.51	1.81	2	10.83
Combined Visual Pattern/Error	3.94	3.75	3.51	3.26	3.75	3.94	22.15
ORAL PATTERN							
Conventional	1	.88	.94	.94	1	1	5.76
Unconventional	.88	.81	.75	.63	.69	.69	4.45
Combined	1.88	1.69	1.69	1.57	1.69	1.69	10.21
ORAL ERROR							
Conventional	.94	.88	.75	.94	1	1	5.51
Unconventional	.88	.75	.5	.38	.5	.88	3.89
Combined	1.82	1.63	1.25	1.32	1.5	1.88	9.4
Combined Oral Pattern/Error	3.70	3.32	2.94	2.89	3.19	3.57	19.61

TABLE 5.6 TABLE OF MEANS ACCORDING TO SEQUENCE TYPE, CONVENTIONALITY, MODALITY AND TASK

Table 5.6 shows the difference in performance means; visual being more accurate than oral, the pattern completion tasks being easier than the error detection tasks, the conventional patterns having more correct responses than unconventional, and the sequences of ones and tens having the highest total scores for the group.

	CONVENTIONAL	UNCONVENTIONAL
VISUAL	11.56	10.56
ORAL	11.25	8.31

TABLE 5.7 TABLE OF MEANS FOR CONVENTIONALITY AND MODALITY

Follow up tests of means for the interaction between modality and conventionality showed all differences significant at $p < .01$ except for the difference between visual conventional and oral conventional which were significant at $p < .05$ level. Of all the tasks, the children found the oral unconventional patterns the most difficult, possibly because the patterns were read out and did not have the familiarity of conventional patterns which were identified with tables, and also because there was no opportunity to check completed patterns as in the visual written tasks.

	1	2	3	4	5	10
CONVENTIONAL	3.94	3.69	3.5	3.69	4	4
UNCONVENTIONAL	3.69	3.38	2.94	2.44	2.94	3.5

TABLE 5.8 TABLE OF MEANS FOR CONVENTIONALITY AND SEQUENCE TYPE

Follow up tests of the difference between means and interaction between conventionality and sequence type showed that performance on the conventional sequences was significantly higher than corresponding unconventional sequences, ($p < .01$), except for the sequences of ones and twos. The conventional sequences of threes were significantly less than ones ($p < .05$), and fives and tens ($p < .01$). The unconventional sequences of fours were significantly less than fives and threes, ($p < .05$) and twos, tens and ones ($p < .01$).

These results suggest that the children found the conventional patterns of one, five and ten easier than the smaller numbers in between, indicating that fluency with patterns is not based on number value alone. This could be because of the rhyming of the patterns of five and ten; fives being added on to each ten, and the pattern of tens ending in 'ty'.

Successes with the unconventional patterns, whilst being lower overall show a greater variation. The pattern of fours was harder to calculate than tens, possibly because in both conventional and unconventional settings, tens have the rhyming rhythm.

	VISUAL PATTERN COMPLETION	VISUAL ERROR DETECTION	ORAL PATTERN COMPLETION	ORAL ERROR DETECTION
Teacher's Rating of Pupil Ability	.002	.489*	-.041	.162
Visual Pattern Completion		.353	.295	.251
Visual Error Detection			-.094	-.135
Oral Pattern Completion				.627**

* SIGNIF LEV P<.05
** SIGNIF LEV P<.01

TABLE 5.9 VARIATION IN PATTERN PERFORMANCE AND TEACHER'S RATING OF PUPIL ABILITY (SPEARMAN)

Table 5.9 shows the only significant relationship with teacher's rating of ability to be visual error detection, probably because the performance of all abilities showed an adequate knowledge of patterns on all the tasks. The oral patterns were significantly associated, with the same trend for the visual patterns, but there appeared to be little connection between visual and oral tasks.

5.10 Discussion

The results of this study show that children of this age have a knowledge of number patterns not found in younger children. There were 87% correct responses suggesting that all abilities were able to attempt the patterns with a degree of success.

The analysis of the extended patterns showed that the visual patterns were easier, possibly because the children were able to check their finished work and make alterations. Also, the pattern continuation tasks were more successful than the error detection tasks, may be the mental search in error detection was more demanding than retrieving the remaining three numbers in a pattern where the initial three were given.

Besides the anticipated success of the counting sequence of ones, introduced in Studies IV and V, the conventional sequences of fives and tens were the most successful of all the patterns. This could have been because of the connection with tables, rhymes and songs, and also because the two sequences have a rhyming rhythm, five being added to the sequences of tens and tens ending in 'ty'.

Teacher's rating of ability was significantly associated with visual error detection, but surprisingly, not with oral error detection, even though the basis of the tasks was similar.

A strong relationship between the oral tasks was found and the same trend, though not significant, for the visual written tasks. There was no significant relationship found between overall performance and teacher's rating of ability in this study, though there was in Study III.

When comparing these results with Study III, there are other differences. Teacher's ability rating was correlated with performance on both tasks in Study III, but only with visual error detection in this study. The two tasks, oral error detection and visual pattern completion, were significantly associated in Study III, but only oral tasks in this study.

There were similarities in the results of both studies also. The visual pattern continuation tasks were more successful than oral error detection, and the conventional sequences of fives and tens were easier than the other patterns, though the pattern of twos was equally successful in Study III.

The patterns were introduced to see if there is a connection between retrieval and knowledge of number patterns. Both studies showed that conventional patterns of fives and tens are easier, and sums involving these numbers elicited retrieval, so the acquisition of number patterns may give the child an added flexibility with number progressions which could promote the effective use of retrieval.

SUMMARY OF STUDY V

This study was a repeat of Study IV but with the older age group of nine to ten year old boys and girls.

Results showed 87% correct responses suggesting that all abilities had a degree of success and that performance improves with age. Visual patterns were easier and pattern continuation tasks were more successful than error detection tasks. Conventional sequences of 1s, 5s, and 10s were most successful, and teacher's rating of pupil ability was significantly correlated with visual error detection but not oral error detection. Findings indicate a possible connection between retrieval and number patterns, in that pattern sequences of 5s and 10s were most successful and sums involving these numbers elicit retrieval, however, little is known about the acquisition of number patterns at the present time.

COMPARISON OF THE RESULTS OF STUDIES IV AND V

The younger children found the pattern tasks difficult with an overall accuracy rate of only 34% compared with the much better performance of 87% correct responses of the nine to ten year olds.

Whilst the composition of the patterns had the greatest influence on performance in the younger group, main effects of modality, task, conventionality and sequence type affected the performance of the older children. Interaction effects between modality, conventionality and sequence type were also found in both studies.

The conventional patterns were most successful for both age groups and especially the conventional sequences of ones, fives and tens. Although there was a considerable difference in performance between the two groups, the pattern of successful responses was similar. Moreover, this pattern of responses follows the same composition as sums which elicit retrieval, i.e., small addends, fives and tens, so the acquisition of number patterns may run parallel with the development of the choice of retrieval for addition sums. If this is the case, promotion of the learning of number patterns incidentally through games, songs and puzzles could give the child a flexibility with numbers which could assist retrieval.

There were similarities and differences in the correlations of both studies. In Study IV all the tasks were significantly associated whilst only the oral tasks were in Study V, with the same trend, though not significant for the visual tasks. Teacher's rating of ability was significantly correlated with the visual error detection task in both studies, but not with the other tasks, though it approached significance for the oral error detection task in the six to seven year old group. Ability rating was also significantly related with overall performance in the younger group, but not with the nine to ten year olds, suggesting that only the more able younger children could complete the patterns satisfactorily, whilst all abilities experienced a degree of success in the older group.

CHAPTER 6

INTRODUCTION TO STUDIES VI AND VII

Children use a variety of strategies for solving addition sums (e.g., Carpenter & Moser 1983; Resnick and Ford 1981; Siegler and Shrager 1984; Baroody and Ginsburg 1986; and others). There is a consensus that with age and experience children gradually progress from using mainly counting based strategies to using retrieval strategies (e.g., Siegler 1987; Baroody 1985; Groen and Parkman 1972; Ashcraft 1982; Ilg and Ames 1951), yet at any stage they continue to choose from a repertoire of strategies.

The modified Distribution of Association model (Siegler and Jenkins 1989) proposes that strategies are chosen on the basis of speed and accuracy for a particular problem or set of problems. This choice, according to Siegler (1988) is influenced by the child's confidence criteria for stating an answer. For instance, children with equal knowledge 'perfectionist' and 'good' students vary in strategy choice because of their differing thresholds for stating a retrieved answer before using 'back up' counting strategies.

The main purpose of these two studies is to discover the range of possible alternative addition strategies of two age groups, early infant (6/7) and middle junior (8/9) school children, and to investigate the basis on which these strategy choices are made. The individual criteria to be met are:- whether the child can successfully demonstrate his/her chosen alternative strategies for each sum, and to give reasons why the initial choice was made in preference to other possible alternatives.

The schema based theory of Baroody and Ginsburg (1986) proposes that differing cognitive demands and the search for establishing relationships among number combinations leads to variation in strategy choice, rather than the essentially reproductive processes of associative learning models. For example, the invention of min reduces cognitive demands by stating the cardinal value of the largest addend, regardless of position, and is not necessarily dependent on a knowledge of the commutativity principle. The retrieval of plus zero and plus one combinations are mastered early because of the discovery that for plus zero the answer is the value of the other addend, and for plus one, the answer is a continuation of the count. The retrieval of ties could signify rote learning without any real understanding of what the number sentence means, or a child may initially count out a tie, e.g., five fingers and five fingers makes ten for 5 + 5 = 10, and then generalize this to other contexts like dice, where 5 and 5 also make 10, so abstracting a meaningful relationship.

A second aim of these two studies is to examine strategy choice for just such number combinations. Choices in relation to plus zero and plus one are included as well as seven commuted pairs and two ties. The investigation will record whether the children make reference to commuted pairs and calculate only one of them, or whether they make no comment but use the same strategy for both sums. The ability to choose and operate alternative strategies for retrieved ties will throw some light on the possible rote learning of these combinations.

Because teacher's rating of pupil ability has been related to strategy choice in previous studies, these studies will investigate whether there is an association between rated ability and alternative strategy choices.

In conclusion, these exploratory studies seek to investigate the range of possible alternative addition strategy choices of mixed ability six to nine year olds, and the reasons for their selection and rejection of available strategies. The studies will also examine the affects of different types of number combination on strategy choice, and whether the choice and execution of alternative strategies is associated with rated ability.

STUDY VI

METHOD

6.1 Subjects

There were twenty subjects, ten boys and ten girls aged between 8 years 6 months and 9 years 9 months, with a mean age of 9 years 1 month and a standard deviation of 3.5 months. The children were rated for ability by their teacher on a nought to ten scale, five being average.

6.2 Design

A repeated measures design where each subject completed sixteen sums with oral inquiry after each sum. A further inquiry was made with the first two correct examples of count all, min and retrieval to see which other strategies the child claimed she/he could have chosen.

6.3 Materials

There were sixteen sums on a printed sheet; composed of seven commuted pairs with addends from zero to twelve, and two ties. The puppet video of strategies from earlier studies was used to demonstrate count all, min, retrieval and decomposition for the further inquiry into alternative strategy choices.

6.4 Procedure

The children were interviewed individually and came in random order. They were each given a sum sheet and after completing each sum were asked "How did you do that sum?" This response was noted by the experimenter.

When the oral inquiry for the sixteen sums was completed, the first two correctly worked sums on which the child reported count all, min and retrieval were used for the further inquiry into alternative strategy choices.

For the first sum done using count all, the child was shown the sum written in a separate space so that she/he could not see it already worked in the sixteen sums. The experimenter then said:- "You counted all the numbers for this one, see if you could have done it another way". The part of the video showing retrieval was then shown and the child was asked if she/he could have done it that way. If the response was "yes" then the experimenter said "What was the answer to that sum then?" The child then demonstrated the strategy with the sum. If she/he was unable to demonstrate the strategy correctly, the choice was noted but not listed as an alternative strategy choice. If however she/he was able to demonstrate the retrieval strategy by stating the answer spontaneously then she/he was asked:- "Why didn't you use that way of doing it?" The response was noted then the video demonstration of decomposition was seen and the procedure was repeated for this strategy, followed by a demonstration of the min strategy, but the wording at the end of these video demonstrations was "show me" and the child proceeded to demonstrate the strategy with the sum, explaining how it could be done. When the alternative strategies had been worked the child was asked why she/he did the sum the way she/he did it the first time, and this was noted. The procedure was the same for min and retrieval but with a different order of possible alternative strategies demonstrated.

The order of alternative strategy inquiry for the correctly worked initial choices of min was retrieval, decomposition and count all, and for retrieval it was decomposition, min and count all.

So each child completed sixteen sums with oral strategy inquiry. This was followed by the alternative strategy inquiry for each of the first two correctly worked sums on which she/he had reported using count all, then min and then retrieval. The puppet video was used to demonstrate each strategy for the alternative strategy inquiry. Successful and unsuccessful demonstrations of each chosen alternative strategy were noted, and the reason for selecting and rejecting strategies was recorded.

6.5 Results

	NOT CHOSEN	CHOSEN NOT DEMONSTRATED	CHOSEN AND DEMONSTRATED	TOTAL POSSIBLE
FOR RETRIEVAL:				
COUNT ALL MIN DECOMPOSITION	0 1 (1) 18 (11)	2 (1) 1 (1) 20 (13)	37 (19) 37 (20) 1 (1)	39 39 39
FOR MIN:				
COUNT ALL RETRIEVAL DECOMPOSITION	0 10 (7) 12 (8)	3 (2) 5 (4) 19 (13)	37 (19) 25 (15) 9 (8)	40 40 40
	41	50	146	237

TABLE 6.1 ALTERNATIVE STRATEGY CHOICES FOR EACH OF THE INITIAL CHOICES OF RETRIEVAL AND MIN PER CHILD, WITH NUMBER OF CHILDREN IN BRACKETS[a]

[a] 20 children in the group

Table 6.1 summarizes the results of the alternative strategy choices both correctly and incorrectly worked.

In this study, the alternative strategy inquiry was confined to the initial use of min and retrieval because there was only one example of count all by one child when doing the sixteen sums.

All the children except one, subject ten, who was very shy and upset, were able to successfully demonstrate count all and min on at least one of the sums they claimed to have used retrieval on. However, only one girl, subject three, rated eight in ability, was able to demonstrate decomposition successfully out of the thirteen who claimed they could.

On sums where the use of min was reported, all, except for subject ten, showed they could use count all, fifteen showed they could have used retrieval, and eight were able to successfully demonstrate decomposition.

The main type of failure was identifying an alternative strategy, particularly decomposition, but being unable to demonstrate it successfully. A few children failed by either re-working their initial strategy choice again instead of the chosen alternative, or re-working the video example with the chosen alternative, and not the sum in question.

The most surprising result is that fifteen of the children were able to demonstrate retrieval on sums on which they had used min; and four of these children were rated average or below average in ability by their teacher.

As expected, children who used retrieval successfully could also demonstrate count all and min.

The absence of the initial choice of count all suggests that children progress from using counting strategies to retrieval strategies (Siegler 1987), whilst continuing to use a variety of strategies.

The children were asked to give reasons for not using their correctly demonstrated alternative strategies.

Many of the reasons given showed that the children did not base their strategy choices on speed of execution. For example, of the fifteen who could have used retrieval instead of min, six said that they could have done but chose not to, and six did not know why. Only three children, rated above average, gave reasons of accuracy for choosing min, which was that they preferred counting because it was easier to make sure of the answer.

When asked why they did not use the counting strategies instead of retrieval ten of the children gave reasons based on efficiency for their initial choice of retrieval, of these ten, five said that they did not need to count any more because they knew the answer, and five said that they used to count but found retrieval easier. The remaining children gave vague reasons like not knowing why they had chosen retrieval or that they could have used a counting strategy but chose not to.

The number of children who did not know why they had not chosen a particular strategy suggests that strategy choice was not under the conscious control of a substantial number of children (Piaget 1952). The reasons that were given show a concern for economy and accuracy rather than speed, but for several children it seems that their concepts of speed, accuracy and economy in relation to strategy choice are immature.

The teacher's rating of pupil ability was significantly correlated with successful demonstrations of alternative strategy choices 0.5 ($p < .05$), and there was a strong negative relationship - 0.66 ($p < .002$) between teacher's rating and unsuccessful demonstrations, suggesting that more able children could identify and successfully operate alternative procedures for their initial choices.

COMMUTED EGS. AND TIES	CA	CO	MIN	RET	DEC	OBSERV. OF COMM. PAIRS	NUMBER OF PUPILS USING SAME STRATEGY FOR BOTH SUMS
3 + 2 and 2 + 3	0	3	17	19	0	1	12
1 + 7 and 7 + 1	0	1	19	17	0	3	13
8 + 3 and 3 + 8	0	1	31	1	2	5	11
0 + 4 and 4 + 0	0	0	17	22	0	1	14
12 + 3 and 3 + 12	0	1	36	1	0	2	16
1 + 3 and 3 + 1	1	2	6	31	0	0	17
2 + 7 and 7 + 2	0	1	28	9	1	1	10
5 + 5	0	0	0	19	1	-	-
2 + 2	0	0	2	18	0	-	-
TOTALS	1	9	156	137	4	13	-

TABLE 6.2 REPORTED STRATEGIES FOR COMMUTED EXAMPLES AND TIES WITH NUMBER OF PUPILS USING THE SAME STRATEGY FOR COMMUTED EXAMPLES

The ties, and commuted pairs with the addition of zero and one accounted for 78% of the total choices of retrieval, indicating the routine recall of these familiar number combinations. For the addition of zero, the children who described adding nothing to four were listed a using min and those who said four because there was nothing to add on were categorized as having used retrieval. Min was the most frequently used strategy, especially for large and small addends (Siegler 1987), e.g., 12 + 3.

Though there were few references to the commuted pairs ($^{13}/_{140}$) over half of the children chose the same strategy for each sum. The children may have refrained from commenting on commuted pairs because they felt it inappropriate to use 'short cuts' in a setting of formal addition sums as Baroody, Ginsburg and Waxman (1983) note, failure to use the commutativity principle does not signify that it is unknown. The children may have been responding to instructions to begin adding from the larger number and using min for some of the commuted pairs without recognising the same addends reversed.

6.6 Discussion

All of the children except one distressed child could demonstrate alternatives of min and count all for their initial choices of retrieval. What was not expected was the number of initial choices of min in preference to retrieval; fifteen children could have used retrieval but chose min instead.

This could be evidence of the 'perfectionist' group whom Siegler (1988) describes as having good knowledge of problems but high thresholds for stating a retrieved answer before using 'back up' counting strategies. This category is in contrast to the 'good' group, who also have good knowledge of problems but who use retrieval more frequently because of having lower thresholds for stating a retrieved answer.

Of these fifteen, only three could give reasons of accuracy for choosing min, the rest could not give a specific reason, and several did not know. These children represented a range of ability, though the three who said that they counted for accuracy were rated above average. Also, in this study the children were asked to justify their selection or rejection of a strategy directly, whereas in Siegler's study indirect methods of observation, video recording and reaction times were used.

A substantial number of children could not give reasons for rejecting or selecting a strategy. Half of the group gave economic reasons for not using a counting strategy instead of retrieval, the remaining ten children were unable to give a precise reason, and most of the children seemed unaware of the speed of retrieval over counting.

The seemingly arbitrary pattern of responses of some of the children and their lack of awareness of the speed, accuracy and economy of strategies in relation to each other and problems suggests that strategy choice may not be totally under their conscious control (Piaget 1952), or they may have difficulty articulating a response (Carpenter and Moser 1983), or concepts of speed, accuracy and economy are not sufficiently developed to be generalised to judgments on addition strategy choices.

In the alternative strategy inquiry, most failures were due to identifying an alternative but being unable to demonstrate it correctly. A few children re-worked their initial strategy again instead of the alternative, or re-worked the video example instead of using the strategy on the sum in question.

Min and retrieval were the main initial strategy choices, with ties and commuted pairs of plus zero and one accounting for over three quarters of choices of retrieval.

There were few references made to commuted pairs, yet over half of the children used the same strategy for each sum in the commuted pairs. They may have refrained from using the commutativity principle thinking it inappropriate in the setting of formal arithmetic. Where min was used this could have been due to the influence of instruction to begin counting from the larger number, or an invention of the child to save cognitive effort (Baroody and Ginsburg 1986), without observing that commuted pairs were the same sum with addends reversed. Knowledge of commutativity is not clear from this study bearing in mind Baroody, Ginsburg and Waxman's (1983) warning that failure to use the principle does not signify that it is unknown.

The teacher's rating of pupil ability was found to be significantly related to successful demonstrations of alternative strategy choices, with a strong negative correlation between teacher's rating and unsuccessful demonstrations, making it likely that the more able pupils in the group could demonstrate a wider range of alternative strategies satisfactorily.

SUMMARY OF STUDY VI

The individual interviews of twenty, eight to nine year old children began with the child completing sixteen addition sums composed of seven commuted pairs and two ties, with oral strategy inquiry after each sum.

The first two correctly worked examples of count all, min and retrieval were used for the alternative strategy inquiry. These pairs of sums were written in a separate space so that the child could not see the original working, and the video of strategies was used to demonstrate alternatives which could have been used. Successful and unsuccessful demonstrations of alternative strategies were recorded.

Results showed that all the children could demonstrate alternatives of min and count all for their initial choices of retrieval as expected. Surprisingly fifteen children who could have used retrieval chose min instead, these two strategies being the only initial choices in this study.

A substantial number of children could not give reasons for their strategy choices, only ten gave economic reasons for not choosing counting in preference to retrieval, and most of the children seemed unaware of speed in relation to strategy use.

There were only thirteen references to commuted pairs yet over half of the group used the same strategy for each of the commuted examples.

Teacher's rating of pupil ability was found to be significantly associated with successful demonstrations of alternative strategies, and there was a strong negative correlation between rated ability and unsuccessful demonstrations of alternative strategies.

INTRODUCTION TO STUDY VII

The design and procedure of this study is the same as Study VI but with six to seven year olds.

There are sixteen sums to be calculated as in Study VI but they are simpler, being composed of single digit numbers to ensure that the problems are well within the children's capabilities. This will help to eliminate distractions from the main purpose of the study which is an inquiry into alternative strategy choices, and not the ability to calculate difficult addition sums correctly.

STUDY VII

METHOD

6.7 Subjects

There were twenty subjects, eleven boys and nine girls aged between 6 years and 6 months and 7 years and 4 months with a mean age of 6 years and 11 months and a standard deviation of 3.25 months. The children were rated for ability by their teacher on a nought to ten scale, five being average.

6.8 Materials

There were 16 sums printed on a sheet for each child with an equivalent sheet for the experimenter with space for comments. The sums were easier for these younger children, taken from Study I and composed of single digits for each of six commuted pairs, with the addition of two ties and a commuted pair involving the addition of zero.

The puppet video of strategies from Study I was used for the alternative strategy inquiry, as in Study VI.

6.9 Results

	NOT CHOSEN	CHOSEN NOT DEMONSTRATED	CHOSEN AND DEMONSTRATED	TOTAL POSSIBLE
FOR RETRIEVAL:				
COUNT ALL MIN DECOMPOSITION	3 (2) 12 (7) 21 (12)	10 (5) 4 (3) 11 (8)	19 (12) 16 (9) 0 (0)	32 32 32
FOR MIN:				
COUNT ALL RETRIEVAL DECOMPOSITION	10 (5) 12 (7) 26 (14)	6 (4) 13 (8) 10 (6)	21 (12) 12 (8) 1 (1)	37 37 37
FOR COUNT ALL:				
MIN RETRIEVAL DECOMPOSITION	9 (5) 15 (9) 16 (10)	7 (4) 2 (1) 9 (6)	9 (6) 8 (6) 0 (0)	25 25 25
TOTAL	124	72	86	282

TABLE 6.3 ALTERNATIVE STRATEGY CHOICES FOR EACH OF THE INITIAL CHOICES OF RETRIEVAL, MIN AND COUNT ALL PER CHILD, WITH NUMBER OF CHILDREN IN BRACKETS[a]

[a] 20 children in the group

Table 6.3 shows successfully and unsuccessfully demonstrated alternative strategy choices.

The alternative strategy inquiry in this study showed that out of the eighteen subjects who reported using retrieval, only seven were able to demonstrate count all and min on at least one of the sums claimed to have been solved with retrieval. Twelve children were able to demonstrate count all, nine min, and four were unable to demonstrate either. There were no successful demonstrations of decomposition.

Four of the nineteen children who reported using min were able to demonstrate retrieval and count all on at least one sum, eight were able to demonstrate retrieval, twelve count all, and one was able to demonstrate decomposition successfully. This was the only successful demonstration of decomposition in this study by a girl rated seven in ability. She could not demonstrate retrieval on the sum 4 + 6 but demonstrated decomposition by saying that 6 + 6 was 12, then take 2 away to make 10. There were two pupils who were not able to demonstrate any alternative strategy for their initial use of min.

Fourteen reported having used count all, of these, three children successfully demonstrated alternative strategies of both min and retrieval. There were six successful demonstrations of min altogether and six of retrieval. None of the children could demonstrate decomposition and five were unable to demonstrate any alternative successfully.

The unexpected pattern of these results shows that these young children do not appear to be progressing from counting strategies to retrieval. They seem to make arbitrary decisions with reference to their personal repertoire of strategies. For example, fourteen children chose counting strategies in preference to retrieval which they subsequently demonstrated successfully, and of the fourteen who reported using count all there were twelve who successfully demonstrated min and retrieval. Conversely, there were nine children who reported using retrieval for 5 + 5, who were unable to successfully demonstrate min as an alternative, suggesting rote learning of this particular tie. Also there were six children who could not demonstrate count all as an alternative to their initial choice of retrieval, and there were seven who failed to demonstrate count all as an alternative to min.

Failures to demonstrate alternative strategies were mainly due to incorrectly worked alternative choices, particularly decomposition which accounted for over forty percent of failures. Some of the children chose alternative strategies, tried to work them out and gave up. Others chose alternatives and said that they could not work them out before making an attempt. A few started to re-work the video example or demonstrated their original strategy again, and not the chosen alternative.

The children were asked to give reasons for not using their correctly demonstrated alternative strategies. Most could not give reasons and replied that they did not know, or simply shrugged their shoulders.

Twelve of the children who successfully demonstrated alternative counting strategies for their initial choice of retrieval did not know why they had not counted, only two said they they knew the answer without counting.

Similarly, seven children said that they did not know why they had not used successfully demonstrated retrieval instead of min, only one child said that she counted to check the answer. Eight did not know why they had not used count all instead of min, the remaining four said that they knew min was better, but did not use it. The girl who successfully demonstrated decomposition did not know why she had not used it.

When asked why they had not used min or retrieval instead of count all, eight of the twelve did not know, three said that they knew count all was better and one said that he knew retrieval was quicker, but decided to use count all instead.

As in Study VI, there was a large number of children in this study who did not know why they had chosen a particular strategy, which again points to strategy choice being partially at the subconscious level. These young children may also have had difficulty expressing themselves, or sophisticated concepts of speed, accuracy and economy in relation to strategy use may not be fully developed at this age.

As in previous studies with this age group, use of retrieval was found to be significantly correlated with the teacher's rating of pupil ability .658 ($p < .01$). The range of correctly worked alternative strategies was also associated with rated ability .684 ($p < .002$).

COMMUTED EGS. AND TIES	CA	CO	MIN	RET	DEC	OBSERV. OF COMM.PAIRS	GUESS	NUMBER OF PUPILS USING SAME STRATEGY FOR BOTH SUMS
1 + 7 and 7 + 1	6	1	17	15	0	0	1	12
2 + 3 and 3 + 2	10	6	14	4	1	3	2	9
1 + 9 and 9 + 1	4	3	17	11	0	4	1	7
4 + 6 and 6 + 4	11	5	17	1	0	0	6	11
1 + 3 and 3 + 1	9	3	14	11	0	1	2	13
4 + 0 and 0 + 4	2	1	14	16	0	4	3	11
3 + 8 and 8 + 3	9	2	19	2	0	3	5	12
5 + 5	0	0	1	17	0	-	2	-
2 + 2	7	0	0	12	0	-	1	-
TOTALS	58	21	113	89	1	15	23	-

TABLE 6.4 REPORTED STRATEGIES FOR COMMUTED EXAMPLES AND TIES, WITH NUMBER OF PUPILS USING THE SAME STRATEGY FOR COMMUTED EXAMPLES

The retrieval of ties, plus zero and one supports evidence for the early learning of these number combinations (Baroody and Ginsburg 1986). Surprisingly, seven children reported using count all for 2 + 2.

There were few references to commuted pairs, yet most of the children used the same strategy for each of the sums. There may have been a genuine ignorance of commutativity, or reluctance to use the principle, or a response to instruction in using min for some of the pairs as suggested in Study VI, where responses of the older group to the commuted pairs was similar.

Nineteen of the twenty-three guesses were for subjects 7 and 8, who had difficulty counting to twenty, were rated below average in ability, and were responsible for twenty out of the total of thirty-six errors for the whole group.

6.10 Discussion

The children reported using a variety of initial strategies which did not appear to be chosen on the basis of speed, accuracy or economy most of the time. For instance, eight reported using min when they could have used retrieval, six could have used retrieval instead of count all, and the same number of children could have used min instead of count all. This could have been evidence of what Siegler called 'perfectionists', who have high thresholds for stating a retrieved answer, relying on counting for accuracy.

Most of the children did not know why they had selected or rejected a strategy and many seemed unaware of efficiency as a basis for choice. They may have had difficulty explaining their reasons, or strategy choice may not have been totally under their conscious control.

Less than half of the children who reported an initial choice of retrieval were able to successfully demonstrate alternatives of min and count all. Almost half of all the chosen alternatives were not demonstrated successfully and many children could not attempt a demonstration, or tried and gave up. A few re-worked the video example or another strategy but not the chosen alternative.

A strong relationship was found between teacher's rating of pupil ability and successfully demonstrated alternative strategies.

As in previous studies, rated ability was associated with the use of retrieval.

Min was widely used for all types of sum, and retrieval was mostly used for ties and the addition of zero and one. Nine children who reported using retrieval for 5 + 5 were unable to demonstrate an alternative successfully, suggesting rote learning of that number combination (Baroody and Ginsburg 1986).

As with the older children, there were few uses of the commutativity principle with commuted pairs, though most children used the same strategy for each of the sums in the pair. There may have been a reluctance to use the principle because of perceived social constraints, or the children may have been responding to instruction in the case of using min, or they may not have conceptualised the principle sufficiently to generalise it to formal addition strategies.

SUMMARY OF STUDY VII

This study was the same as Study VI, but the sums were simpler, being composed of single digits for this younger age group of six to seven year olds.

Results showed that these younger children use a variety of strategies. Some of the children were unable to successfully demonstrate alternatives to their initial choice of retrieval, and several chose basic counting in preference to more sophisticated strategies which they could have chosen.

Most of the children did not know why they had selected or rejected a strategy, and there was little evidence of an awareness of the speed, accuracy or economy of strategies in relation to each other and type of sums.

Almost half of the chosen alternatives were not demonstrated successfully, and teacher's rating of pupil ability was found to be related to successfully demonstrated chosen alternatives.

There were few observations of commuted pairs, but several children used the same strategy for each sum in the commuted pairs.

COMPARISON OF THE RESULTS OF STUDIES VI AND VII

The studies show that the older children were able to successfully demonstrate 'back up' strategies of count all and min for reported initial choices of retrieval whilst less than half of the younger children could. In every case, there was a minority of six to seven year olds who could not demonstrate any alternatives for their initial choices, and nine children who reported using retrieval for a tie were unable to demonstrate an alternative of min successfully.

In both groups, especially the eight to nines, there were a number of children who reported an initial choice of min when they could have used retrieval, possibly showing adaptive strategy choices for solving problems, in a similar way to the 'perfectionist' (Siegler 1988, Geary et al 1989).

There were more unsuccessful demonstrations of decomposition than any other alternative strategy in both groups. In the older group the children attempted to demonstrate all their chosen alternatives, whilst several of the younger children could not begin. Some failures were similar in both groups; re-working the video example or repeating the initial strategy and not the chosen alternative.

When asked to give reasons for selecting or rejecting a strategy many of the children in both groups did not know. In the older group, half of the children gave economical reasons for not using count all or min instead of retrieval compared with only two of the six to seven year olds. Three of the older ones and one of the younger group gave accuracy as a reason for choosing min instead of retrieval. Contrary to Siegler and Jenkins's (1989) proposal, speed did not seem to be a basis for strategy choice in either age group in these studies.

Retrieval was used mainly for ties and the addition of zero and one (Baroody and Ginsburg 1986), and min was widely used for all types of sum in both studies. However, contrary to their suggestion that a search for cognitive economy promotes the use of min over count all, almost half of the six to seven year olds who reported using count all could have used min, and the same number could have used even more economical retrieval.

There were few uses of the commutativity principle in either group, yet most of the children used the same strategy for each of the sums in the commuted pairs.

Teacher's rating of pupil ability was found to be related to successful demonstrations of alternative strategies in both age groups.

It is not clear from these exploratory studies whether children base their strategy choices on the speed, accuracy or economy of one strategy against another for a particular problem or set of problems. It is also unclear whether these abstract concepts are sufficiently developed to be generalised to strategy selection or rejection.

These questions are addressed in the next chapter.

There was little evidence in either group for knowledge of, or use of, the commutativity principle. The final studies are a further investigation into commutativity with an extended age range of five to ten, and covering informal knowledge of the principle applied to concrete quantities, formal understanding in connection with number values and number combinations, and whether knowledge of commutativity is related to strategy choice.

CHAPTER 7

INTRODUCTION TO STUDY VIII

The study arises from questions raised concerning the reasons behind the seemingly unpredictable pattern of choices of some of the children in Studies VI and VII. For example, in both studies, a substantial number of children chose counting strategies in preference to retrieval, and in the six to seven year old group almost half of the children who reported using count all could have used min or retrieval. These results cast doubt on assumptions that strategy choice for most children is based on speed, accuracy (Siegler and Jenkins 1989) and economy (Baroody and Ginsburg 1986) for a particular problem or set of problems.

Siegler and Jenkins's (1989) model predicts that strategies are chosen on the basis of speed and accuracy. In Studies VI and VII there were several examples of more laborious counting strategies chosen in preference to accurately demonstrated retrieval, and in the younger group there were successful alternative demonstrations of min when the initial choice had been the more error prone count all. Many of the children could not justify their selection or rejection of available strategies, and of those who could, there were few who justified their choices on grounds of accuracy or speed of execution.

Siegler's (1988) description of 'perfectionists' with good knowledge of problems who choose 'back up' counting strategies in preference to retrieval for accuracy does not properly describe the children's performance. For example, the 75% of eight to nine year olds who reported using min in preference to retrieval represented a range of rated ability, and only three justified their choices on grounds of accuracy. Of the six to seven year olds, eight children could have used retrieval instead of min and only one of them said that she counted to check; and five of the six children who chose count all in preference to retrieval did not know why they had done so. It should be pointed out however that the children in these studies were asked directly to justify their strategy choices, whereas in Siegler's study, indirect methods of observation, reaction times, tests and analysis were used.

There was little evidence of a search for cognitive economy (Baroody and Ginsburg 1986). If there had been, a majority of eight to nine year olds would not have chosen min when they could have used retrieval successfully, and many of the younger children would not have counted all in preference to min and retrieval, yet several children did.

This study will seek to clarify some aspects of strategy choice connected with concepts of speed, accuracy, economy and superiority. This will be done by having the children judge strategies in relation to each other and different types of sum, from specific video demonstrations of paired comparisons of strategies. Each pair of strategies will be used to calculate a tie, a sum with a small and a large addend, and a third sum composed of two medium addends.

STUDY VIII

METHOD

7.1 Subjects

These were the same children from Studies VI and VII, ten boys and ten girls aged eight to nine and eleven boys and nine girls aged six to seven years.

7.2 Design

A repeated measures design where each subject was questioned on the relative speed, economy, accuracy and superiority of count all, min and retrieval set in a series of paired comparisons.

Each set of paired comparisons was done with three sums, a tie (4 + 4), a small and large addend (1 + 7) and two medium addends (3 + 5). The order of presentation of the three paired comparisons and the three sums was balanced across three groups; 2 of seven subjects and one of 6 for each age group.

7.3 Materials

A video was made where a glove puppet demonstrated the operation of each paired comparison of strategies on each of the three sums, in each of the presentation variations. This was done so that the experimenter could observe the responses of the subjects without having to re-wind the video tape.

Every subject was given a printed sheet with the choices of the strategies 1 or 2 to circle, plus a 'same' and 'don't know' section for each sum in each of the paired comparisons.

7.4 Procedure

Each age group of children came in three separate groups and were seated at tables from which they could see the video and the experimenter, but not each other's work.

The video of the first paired comparison was shown after which the experimenter said "You have seen two ways of doing that sum, do you think one of them is quicker than the other? If you think the first is quicker, put a ring round number one on your sheet. If you think the second way is quicker put a ring round number two on your sheet. If you think they are the same put a ring round the word 'same', and if you do not know whether one is quicker or whether they are the same then put a ring round the words 'don't know'". After the children had put a ring round their choice, the experimenter glanced at their papers to check before showing the next section of the video about economy of effort.

Here the children were asked which was the easiest way of doing the sum and getting it right. The same instructions for circling their choice were given as in the first demonstration, i.e., circle one, two, same or don't know, on the next row on their sheet.

The next section of the video was then shown with the question on accuracy asked, "Which of the two ways is sure to be right?" Instructions on circling were given as before, and the final video demonstration on superiority was then shown with the question, "Which do you think is the better way of doing the sum?"

When the four questions for the paired comparison for that particular sum had been completed, the four questions for the next sum for the same paired comparison were done, and so on until all the paired comparisons of strategies for all of the sums were finished.

7.5 Results

	CA		MIN		RET		SAME		DON'T KNOW	
	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9[a]
4 + 4										
CA and MIN	7	1	13	17	-	-	0	1	0	1
CA and RET	3	2	-	-	14	17	1	1	2	0
MIN and RET	-	-	3	0	12	19	1	1	4	0
1 + 7										
CA and MIN	5	1	14	19	-	-	0	0	1	0
CA and RET	3	1	-	-	14	19	1	0	2	0
MIN and RET	-	-	3	2	12	17	1	1	4	0
3 + 5										
CA and MIN	4	1	14	19	-	-	1	0	1	0
CA and RET	6	3	-	-	13	15	0	1	1	1
MIN and RET	-	-	3	2	14	17	2	0	1	1
TOTALS	28	9	50	59	79	104	7	5	16	3

[a] 20 children in each age group

TABLE 7.1 STRATEGY CHOICES FOR SPEED OF EXECUTION

The older children always judged with a significant amount of consistency showing that by this age most children have the same conceptualisation of speed in relation to strategy choice.

In the younger group, the Binomial test showed varying levels of consistency. For 4 + 4 and 1 + 7 there was a significant difference in favour of retrieval over count all ($p < .01$) and for retrieval against min ($p < .05$). For 3 + 5 retrieval was preferred to min ($p < .01$) and min was preferred to count all ($p < .05$).

A quarter of the younger group thought count all was faster than retrieval for 3 + 5, and count all was faster than min for the other two sums, suggesting that some of the children could not apply concepts of speed to the operation of strategies.

	NUMBER OF CHILDREN					
	8 to 9 years[a]			6 to 7 years[a]		
ORDER	4+4	1+7	3+5	4+4	1+7	3+5
RET < MIN < CA	16*	17*	14*	9*	9*	9*
RET < MIN = CA	1	0	0	0	0	1
RET = MIN < CA	0	1*	0	0	0	0
MIN < RET < CA	0	1	1	0	0	0
CA < MIN < RET	0	1	0	0	1	2
MIN < CA < RET	0	0	2	0	1	1
CA < RET = MIN	0	0	0	0	0	1
MIN < CA = RET	0	0	0	1	0	0
CA < RET < MIN	0	0	0	1	0	0
TOTAL	17(0)	20(0)	17(0)	11(1)	11(2)	14(3)

[a] 20 children in each group

< takes less time than
= equal
* plausible and logically coherent

TABLE 7.2 INDIVIDUAL CHILDREN'S JUDGMENTS
FOR SPEED OF EXECUTION WITH 'DON'T KNOWS' IN BRACKETS

Table 7.2 shows that almost 75% of the older children made logically coherent and plausible judgments, whilst less than half of the younger children did.

There was a small number of children in each group who produced logically coherent but implausible sequences, e.g., two children in each group said that count all and min were faster than retrieval for 3 + 5.

There was a minority in both age groups especially in the younger group, who failed to show consistency, or an understanding of speed in relation to strategy choice and type of sum.

	CA		MIN		RET		SAME		DON'T KNOW	
	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9[a]
4 + 4										
CA and MIN	3	2	13	15	–	–	3	2	1	1
CA and RET	8	3	–	–	11	17	1	0	0	0
MIN and RET	–	–	6	4	12	15	1	0	1	1
1 + 7										
CA and MIN	4	1	12	17	–	–	3	1	1	1
CA and RET	8	3	–	–	9	16	2	1	1	0
MIN and RET	–	–	5	4	13	14	2	1	0	1
3 + 5										
CA and MIN	5	3	12	16	–	–	3	1	0	0
CA and RET	9	5	–	–	11	15	0	0	0	0
MIN and RET	–	–	2	4	12	14	3	1	3	1
TOTALS	37	17	50	60	68	91	18	7	7	5

[a] 20 children in each age group

TABLE 7.3 STRATEGY CHOICES FOR ECONOMY OF EFFORT

Though all choices for the older group were significant, levels varied. Min was preferred to count all for all of the sums ($p < .01$) and for retrieval and count all it was the same except for 3 + 5 ($p < .05$). Retrieval was chosen rather than min for 4 + 4, ($p < .02$) and for 3 + 5 and 1 + 7 ($p < .05$) where the addition of one makes little difference to economy.

There were only two pairs which showed consistency of choice in the younger group; min was chosen rather than count all for 4 + 4 ($p < .05$) and retrieval was preferred to min for 3 + 5 ($p < .01$).

Consistency levels varied for economy, and several of the younger children did not interpret the question in the way that an adult would have judged economy of effort, e.g., three children judged count all and min equal for all of the sums.

NUMBER OF CHILDREN						
	8 to 9 years[a]			6 to 7 years[a]		
ORDER	4+4	1+7	3+5	4+4	1+7	3+5
CA < MIN < RET	13*	9*	11*	8*	7*	5*
CA = MIN < RET	1	0	0	1	1	0
MIN < CA < RET	0	1	0	0	1	0
CA < RET < MIN	1	3	1	1	1	1
RET < MIN < CA	1	0	1	1	1	0
MIN < RET < CA	0	0	1	0	1	3
RET < CA < MIN	0	0	1	3	1	1
RET < CA = MIN	0	0	0	0	1	0
CA < MIN = RET	0	1*	0	0	0	1
TOTAL	16(2)	14(1)	15(1)	14(1)	14(0)	11(2)

[a] 20 children in each group
< less economical than
= equally economical
* plausible and logically coherent

TABLE 7.4 INDIVIDUAL CHILDREN'S JUDGMENTS
FOR ECONOMY OF EFFORT WITH 'DON'T KNOWS' IN BRACKETS

About half of the older group and less than half of the younger children responded to this question in the way expected. Though 25% of the sequences after the first one were logically coherent they did not fit the adult concept of economy, e.g., ten children thought that retrieval was less economical than counting all or min.

A number of children in both groups could not produce logically coherent sequences, nor did they grasp the idea of economy, e.g., three quarters of the younger group were confused when judging economy in relation to strategies for 3 + 5.

	CA		MIN		RET		SAME		DON'T KNOW	
	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9[a]
4 + 4										
CA and MIN	7	9	11	5	–	–	2	4	0	2
CA and RET	6	14	–	–	12	5	0	1	2	0
MIN and RET	–	–	7	11	10	8	2	1	1	0
1 + 7										
CA and MIN	8	8	11	8	–	–	0	4	1	0
CA and RET	8	8	–	–	10	10	1	1	1	1
MIN and RET	–	–	4	12	16	6	0	2	0	0
3 + 5										
CA and MIN	7	6	11	11	–	–	1	3	1	0
CA and RET	6	14	–	–	11	5	3	1	0	0
MIN and RET	–	–	5	11	10	8	3	1	2	0
TOTALS	42	59	49	58	69	42	12	18	8	3

[a] 20 children in each age group

TABLE 7.5 STRATEGY CHOICES FOR ACCURACY

There were more judgments in favour of counting all for accuracy in both groups than there were for speed and economy, probably because it is the predominant strategy for some of the children. However, choices were evenly spread and in only one of the comparisons was the choice of one of the pairs statistically significant; which was the choice of retrieval against min by the younger children for 1 + 7 ($p < .01$) which is surprising considering that adding one is a continuation of counting and so both strategies are likely to be accurate.

NUMBER OF CHILDREN						
	8 to 9 years[a]			6 to 7 years[a]		
ORDER	4+4	1+7	3+5	4+4	1+7	3+5
CA < MIN < RET	2*	5*	2*	6*	9*	10*
RET < MIN < CA	6*	5*	5*	3*	2*	2*
RET < CA = MIN	3*	2*	3*	0	0	0
CA = MIN < RET	0	0	0	2*	0	0
MIN < RET < CA	2*	1*	0	1*	3*	0
CA < RET < MIN	0	1*	1*	3*	0	0
MIN < CA < RET	1*	1*	1*	0	0	0
RET < CA < MIN	1*	1*	1*	1*	0	1*
CA < MIN = RET	0	1*	1*	1*	0	0
MIN < RET = CA	0	0	0	0	1*	0
RET = MIN < CA	0	0	0	0	0	1*
CA = RET < MIN	1*	0	1*	0	0	0
TOTALS	16(2)	17(0)	15(0)	17(1)	15(0)	14(2)

[a] 20 children in each group
< less likely to be accurate
= equally likely to be accurate
* plausible and logically coherent

TABLE 7.6 INDIVIDUAL CHILDREN'S JUDGMENTS
FOR ACCURACY WITH 'DON'T KNOWS' IN BRACKETS

Judging accuracy is difficult because all strategies are potentially accurate when used in the right context, so all the logically coherent sequences produced by the children were plausible. It is interesting that several of the younger children, who are likely to use counting all quite a lot, judged it to be less accurate than min and retrieval, whereas several of the older children opted for counting for accuracy. This was borne out in Langford's research (cited in Lesh and Landau 1983) where he found seventh grade pupils using counting as a 'back up' strategy but with a speed which matched retrieval.

	CA		MIN		RET		SAME		DON'T KNOW	
	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9[a]
4 + 4										
CA and MIN	3	2	14	15	-	-	2	2	1	1
CA and RET	7	2	-	-	9	16	3	2	1	0
MIN and RET	-	-	3	3	12	15	1	1	4	1
1 + 7										
CA and MIN	1	1	15	18	-	-	2	0	2	1
CA and RET	4	3	-	-	12	17	3	0	1	0
MIN and RET	-	-	7	4	12	13	1	1	0	2
3 + 5										
CA and MIN	3	2	15	17	-	-	2	0	0	1
CA and RET	6	5	-	-	12	14	1	1	1	0
MIN and RET	-	-	4	6	11	13	1	1	4	0
TOTALS	24	15	58	63	68	88	16	8	14	6

[a] 20 children in each age group

TABLE 7.7 STRATEGY CHOICES FOR SUPERIORITY

There were significantly more choices in the older group for min than for count all for 4 + 4 ($p < .002$) and 1 + 7 ($p < .001$). All choices for retrieval in preference to count all and min were consistently high ($p < .01$) for 4 + 4 and 1 + 7, except for min and retrieval for 1 + 7 where the significance level was lower ($p < .05$), probably because there is so little difference when adding one. For 3 + 5 there were inconsistent responses where retrieval was concerned, but significantly more choices for min when paired with count all ($p < .002$).

For the younger group none of the choices between count all and retrieval were significant for any of the sums, and only for 4 + 4 was the choice of retrieval as opposed to min significant ($p < .05$). However, the children consistently chose min when paired with count all for all three sums.

NUMBER OF CHILDREN						
	8 to 9 years[a]			6 to 7 years[a]		
ORDER	4+4	1+7	3+5	4+4	1+7	3+5
CA < MIN < RET	12	13	11	6	9	9
CA < RET < MIN	1	2	2	0	2	0
RET < CA < MIN	1	1	2	2	2	2
RET < MIN < CA	1	1	0	0	0	1
MIN < CA < RET	1	0	0	1	0	0
MIN < RET < CA	0	0	0	1	0	0
CA = MIN < RET	2	0	0	0	0	0
CA = RET < MIN	0	0	0	1	1	0
MIN < RET = CA	0	0	1	0	0	0
RET < CA = MIN	0	0	0	0	0	1
CA = MIN = RET	0	0	0	0	1	0
TOTALS	18(1)	17(2)	16(1)	11(3)	15(2)	13(1)

[a] 20 children in each age group

< inferior to
= equal to

TABLE 7.8 INDIVIDUAL CHILDREN'S JUDGMENTS FOR SUPERIORITY WITH 'DON'T KNOWS' IN BRACKETS

Most children thought that retrieval was superior to the counting strategies but there were a few children in each group who preferred min. It is surprising that only six of the younger ones chose retrieval for the tie because ties are amongst the first number combinations to be memorized.

As with the other criteria of judgments, there were a number of children in both groups who were inconsistent and either made what appeared to be random choices, or did not understand what was required of them.

This category of judgments, unlike the others was based on subjective choices so all logically coherent sequences were plausible depending on the personal standpoint of each child.

		SPEED	ECONOMY	ACCURACY	SUPERIORITY	OVERALL
TEACHER'S RATING OF PUPIL ABILITY	8 to 9	.733***	.279	.408*	.366	.363
	6 to 7	.25	.152	.318	.142	.202

SIG LEV * p $<.05$
SIG LEV *** p $<.001$

TABLE 7.9 RANK ORDER CORRELATIONS (SPEARMAN)

The relationship between rated ability and logically coherent sequences for speed and accuracy found in the older group was not seen in the six to seven year olds, possibly because the younger children's judgments were based on limited formal experience in addition.

7.6 Discussion

The results of this exploratory study divide into three sections. The first comprises children who can produce logically coherent and plausible judgments about the speed, economy, accuracy and superiority of strategies for addition sums. The second group are those who, whilst producing logically consistent sequences do not appear to judge strategies in the way adults expect, e.g., several judged retrieval slower and less economical than count all or min. The third group contains the constant minority of children in each category who are inconsistent in their judgments, making what appears to be random decisions not based on any consistent concept of the judgment criteria, e.g., that count all is quicker than min, retrieval is quicker than count all but slower than min for 1 + 7.

The number of children who did not, or could not judge strategies in the same way as adults draws into question assumptions about young children's search for efficiency implied by Siegler and Jenkins (1989), Baroody and Ginsburg (1986) and others. Perhaps researchers have assumed that at an unconscious level distinctions between abstract concepts like speed, economy, accuracy and superiority are made consistently when in fact they are not, and that conceptualisation is at a slower rate than was previously thought. Whilst the older group produced more logically coherent and plausible sequences than the younger group the differences were small, especially for economy and accuracy.

The pattern of responses for accuracy included more choices for counting all by the older children than the younger ones, suggesting that decerning the reliability of 'back up' counting strategies rather than stating uncertain retrieved facts develops with age (Lankford 1972).

The novelty of the task could have affected performance: children are rarely required to make judgments about procedures at this age, but are usually the recipients of advice and instruction in formal schooling. In day to day arithmetic the stress is on accuracy, especially in the early years rather than on speed or economy of effort, so some of the children may have experienced difficulty in judging these concepts against a background of conflicting personal experience.

A consideration of the points raised in this study needs to be taken into account when questioning young children, with limited language and conceptual development, about aspects of formal arithmetic.

SUMMARY OF STUDY VIII

Forty, six to nine year old boys and girls from Studies VI and VII were questioned on the relative speed, economy, accuracy and superiority of count all, min and retrieval, set in a series of paired comparisons with three sums composed of small, medium and large addends, and one tie.

Results divided into three groups. There were children who made logically coherent and plausible judgments according to each criterion. The second group of children produced logically consistent sequences but implausible judgments, e.g., deciding that retrieval was slower and less economical than both counting strategies. The third group made neither logically coherent nor plausible judgments.

Some of the children may have been uncertain because of the novelty of the task or the language used in the context of addition sums. However, evidence from this exploratory study should be taken into account when questioning young children, with limited language and conceptual development, about strategies for addition sums.

CHAPTER 8

INTRODUCTION TO STUDY IX AND X

These studies are an investigation into five to nine year old's knowledge and use of the commutativity principle in formal addition. The age group has been extended to include five year olds so that the performance on commutativity tests of children not exposed to extensive formal arithmetic can be studied.

The studies are a continuation of the exploratory work begun in Studies VI and VII where commuted pairs were included but no reference was made to them, so that the spontaneous response of the children could be observed. In both age groups of those studies, very few children made reference to the commuted pairs, or used the commutativity principle in calculating them.

There have been a number of investigations into commutativity (e.g., Resnick 1983; Weaver 1982; Skemp 1986). In 1981 Langford assessed the development of commutativity longitudinally. He used a game with five to six year olds at intervals over a two year period. The game went as follows:-

> "In this game we put these beans in these boxes. We always put the same number of beans in your green box as in my green box, and always the same number in your yellow box as in my yellow box ... If there were two beans in my green box, how many would there be in yours? Now you take the green boxes and put some beans in them. Make sure you put the same number in each but don't show me how many you put in ... Can you tell me this? I tip all the beans in my yellow box on to my white plate. Then I tip all the beans in my green box on to my white plate. You tip all the beans in your green box on to your white plate. Then tip all the beans in your yellow box on to your white plate. Who will have more beans on their white plate? Can you tell me why?"

Langford used two criteria: one was correct judgments and the other was correct judgements and explanations. He found that most of the children could make correct judgments but could not offer correct explanations until seven or eight years old.

Baroody and Gannon (1984) investigated the relationship between knowledge of the commutativity principle and the development of formal addition strategies which disregard addend order. They used commuted pairs, identical pairs and sums where one of the addends in each pair was the same and the other one different. The children had to say whether the three different types of pairs of sums would add up to the same or a different answer. In the second commutativity task, each child was presented with a problem and asked to calculate it. After that the child was classified according to whether she/he counted all, counted all from either addend or counted on from one addend. When the child had done the sum, the experimenter then presented the same sum with addends reversed and asked if this sum would add to the same answer the child had just given or not, and why. They found that of the five to six year olds who used a strategy which disregarded addend order, 45% were unsuccessful on some or all of the commutativity tasks. They concluded that for some children, the understanding of commutativity may be involved in the invention of strategies like counting all from the largest addend or min, but for others such inventions may occur without such understanding.

They also describe a primitive notion of commutativity as 'protocommutativity' where an order indifferent adding scheme is operated, but which does not imply that commuted pairs are equivalent in sum.

In a further study Baroody (1987) reinforced the conclusions of his previous work through a detailed analysis of strategy development with five to six year olds, which included the role of commutativity. He found that there was a tendency to minimise the cognitively demanding keeping track process by starting adding from the larger addend, which did not appear to be linked to the conceptualisation of commutativity (Briars and Larkin 1984).

In their review of the relationship between addition strategies and a grasp of the commutativity principle, Baroody and Ginsburg (1986) warn that evidence from studies using symbolic problems only may be misleading, and that using concrete materials as well may reveal a knowledge of the principle in younger children.

This study will investigate knowledge of the commutativity principle using concrete objects as in the Langford (1981) study, but extending the range of activities by having commuted and non-commuted items within the abstract task. This will show knowledge of commutativity and the ability to differentiate between commuted and non-commuted arrays. There will be the two criteria: correct judgments, and correct judgments with explanation. Performance on the abstract tasks will be compared with performance on four further commutativity tasks involving symbolic representation similar to the

Baroody and Gannon (1984) study. The outcome of the commutativity tasks will then be compared with the children's strategy choices in four tasks composed of addition problems.

In short, the prime aim of this study is to see if a connection exists between strategy use and a grasp of the commutativity principle both informally and formally. Subsidiary aims will be to identify instances of protocommutativity, and to see if the teacher's rating of pupil ability is associated with performance on the sums and commutativity trials.

STUDY IX

METHOD

8.1 Subjects

There were 48 subjects aged between 6 and 9 years divided into two groups of 24. The younger group of eleven boys and thirteen girls were aged between 6 years, and 7 years 7 months with a mean age of 7 years 2 months and a standard deviation of 4.32 months. The older group of twelve boys and twelve girls were aged between 8 years 9 months and 9 years 8 months with a mean age of 9 years 3 months, and a standard deviation of 3.25 months. All the children were rated for ability by their teacher on a scale of 0 to 10, 5 being average, and most of the children had taken part in Studies VI and VII.

8.2 Design

A repeated measures design where each subject completed twenty sums in four blocks, and twenty-five commutativity trials in five task blocks. The order of the sums and commutativity tests were randomized within blocks and the order of presentation of the sums and the commutativity tests was balanced across subjects. There was oral strategy inquiry after each sum, and commutativity test, with oral inquiry about the principle after each commutativity test also.

8.3 Materials

Yellow and blue counters were used. There were six cardboard boxes; two red, two blue and two black ones. There was a set of yellow and a set of blue cards with two each of the following numbers:- 2, 3, 4, 5, 7 and 8.

There were also two blue cards and one yellow card with each of the following numbers:- 12, 13, 14, 15 and 16.

The sums were:- 3 + 4, 2 + 5, 2 + 7, 3 + 8, 4 + 7 small
2 + 13, 3 + 14, 2 + 15, 4 + 12, 3 + 16 large

These sums were written on separate cards and were written again twice on two other separate cards with the numbers reversed for the commutativity tests. A further set of addition ties were written on eleven cards, the ties were:- 2 + 2, 3 + 3, 4 + 4, 5 + 5, 7 + 7, 8 + 8, 12 + 12, 13 + 13, 14 + 14, 15 + 15, 16 + 16

There were two lots of 15 drawing pins and two lots of 45 drawing pins for the abstract commutativity tests.

8.4 Procedure

Each child completed nine tasks, four sum tasks and five commutativity tasks. The sum tasks and four of the commutativity tasks differed as to whether symbols (SYM) or symbols and objects (CI) were used, and whether the sums were large or small. In the fifth commutativity task unspecified groups of objects (ABS) were used to test knowledge of the principle.

Within each of the nine tasks, there were five trials, the four sum tasks contained the five large and five small sums represented with numerals and counters for two of the tasks CIS and CIL and sums for the other two, SYMS and SYML. There was oral strategy inquiry after each sum. Four of the five commutativity tasks followed the same representation as the sums, while in the fifth, ABS, the drawing pins were used. The five items in each of the 2 symbols and objects

(CIS and CIL) and the 2 symbolic tasks (SYMS and SYML) were set out as follows:

COMMUTED		TIE	TIE	REPETITION
a	b	a	b	c
3 + 4 4 + 3	3 + 8 I've got 11 8 + 3	4 + 7 4 + 4	2 + 5 5 + 5	2 + 7 2 + 7

EXAMPLES OF EACH TRIAL IN THE COMMUTATIVITY
TASKS USING NUMERALS AND SUMS

After each item the child was asked "Have you and I got the same, or has one of us got more?" then "How do you know?" for the numerals and counters, (CI). For the sums, (SYM) the questions were, "Has this sum got the same answer as this sum or a different answer?" then "How do you know?" and "Which answer is more?" where appropriate.

For the abstract (ABS) trials, 2 boxes were called a and b and contained 45 pins each and another 2 boxes were labelled c and d and contained 15 pins each. The wording of each question was "If you have _ and _, I have _ and _ (labelled boxes), would we have the same number of pins or would one of us have more?" "How do you know?" The order of presentation of the boxes was:-

commuted a) a and c and d and b

commuted b) c and b and a and d

non-commuted a) a and b and c and d

non-commuted b) c and d and a and b

non-commuted c) b and d and a and c

8.5 Results

COMMUTATIVITY						
		CONCRETE INVISIBLE SMALL	CONCRETE INVISIBLE LARGE	SPECIFIC NUMBER SYMBOLS SMALL	SPECIFIC NUMBER SYMBOLS LARGE	ABSTRACT
6 to 7 yrs	JUDGMENTS[a]	112	118	116	116	104
	EXPLANATIONS	107	114	111	108	97
8 to 9 yrs	JUDGMENTS[a]	117	119	118	120	111
	EXPLANATIONS	115	119	118	120	106

a 24 children in each group

TABLE 8.1 TOTAL OF CORRECT RESPONSES IN THE COMMUTATIVITY TASKS FOR BOTH GROUPS (24 x 5 = 120 MAXIMUM FOR EACH OF THE 5 TRIALS FOR JUDGMENTS AND FOR EXPLANATIONS)

Table 8.1 shows that these children have an adequate knowledge of the commutativity principle and can explain their judgments in over 80% of cases.

			EXPLANATIONS	
			ALL CORRECT	SOME CORRECT
JUDG-MENTS	6 to 7 year olds	ALL CORRECT	13	1
		SOME CORRECT	0	10
	8 to 9 year olds	ALL CORRECT	16	2
		SOME CORRECT	0	6

TABLE 8.2 TOTAL NUMBER OF CHILDREN IN EACH CATEGORY FOR JUDGMENTS AND EXPLANATIONS IN THE FIVE COMMUTATIVITY TASKS

The older children made few errors, and of the ten younger children who made some incorrect judgments and explanations, four 'did not know' when asked to explain some of their correct judgments, others shrugged their shoulders and made no reply, and one or two gave vague explanations like "I think so" or "I looked at them", and could not elaborate when probed further. Even so, all except one of the children in the younger group judged correctly on over twenty of the trials.

In the abstract tasks, which were similar to Langford's (1981) experiments, results were comparable with his. Most of the children who could not explain their judgments were the youngest in the six to seven year old group.

SUMS

	CONCRETE INVISIBLE SMALL	CONCRETE INVISIBLE LARGE	SPECIFIC NUMBER SYMBOLS SMALL	SPECIFIC NUMBER SYMBOLS LARGE
6 to 7 yrs.[a]	88	73	93	79
8 to 9 yrs.[a]	112	113	117	119

[a] 24 children in each group

TABLE 8.3 TOTAL NUMBER OF CORRECT RESPONSES FOR EACH TASK (Maximum = 120)

As Table 8.3 shows, some of the younger children found the sums difficult, which was reflected in a significant correlation between rated ability and performance on the sum tasks, rs.587 ($p < .002$). Several made counting errors of plus or minus one, and five children would not attempt the sums with large addends. Some of the younger children failed to make the connection between knowledge of commutativity and strategies. Despite high commutativity scores, a quarter of the group laboriously counted all from the first addend, and two other children failed on all four sum tasks.

There was some evidence of protocommutativity in the younger group only; eight children began counting all from the largest addend for some of the sums whilst using min and count all from the first addend for the others.

The older children had little difficulty overall, but were more accurate with the symbolic representation as the results of the

Wilcoxon test shows (T=9, n=11, p <.05). This was probably due to the familiarity of the presentation of the sums. In contrast, there was no difference with the younger children (T=42.5. n=16).

SUMS	CA		CO		MIN		RET		DEC		DON'T KNOW	
	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9	6/7	8/9
2 + 7	17	0	1	3	29	43	0	2	0	0	1	0
2 + 5	16	0	2	4	26	36	4	8	0	0	0	0
3 + 4	16	0	8	8	23	36	1	3	0	1	0	0
3 + 8	15	0	2	3	29	42	0	2	0	1	2	0
4 + 7	17	0	3	4	27	42	0	1	0	1	1	0
2 + 13	5	1	1	1	31	44	0	2	0	0	11	0
2 + 15	3	1	2	1	31	45	0	1	0	0	12	0
3 + 14	5	1	1	1	31	46	0	0	0	0	11	0
3 + 16	5	1	1	1	31	46	0	0	0	0	11	0
4 + 12	5	1	1	1	31	45	0	1	0	0	11	0
TOTAL	104	5	22	27	289	425	5	20	0	3	60	0

TABLE 8.4 REPORTED STRATEGIES FOR EACH SUM
FOR BOTH AGE GROUPS

The sums were set to elicit min so that a comparison with performance on the commutativity tasks could be made, and min was the predominant strategy of both age groups. There was a wider range of strategy choices in the younger group, where difficulties increased with addend size, and where eight of the children used counting all for all of the calculations they attempted. All of the younger children who were in the 'don't know' category for large addend sums used count all for the small addend sums they did.

SUMS	SAME STRATEGY		DIFFERENT STRATEGY	
	6/7	8/9	6/7	8/9
2 + 7	22	21	2	3
2 + 5	20	18	4	6
3 + 4	15	14	9	10
3 + 8	19	20	5	4
4 + 7	19	19	5	5
2 + 13	22	20	2	4
2 + 15	21	21	3	3
3 + 14	22	22	2	2
3 + 16	22	22	2	2
4 + 12	22	21	2	3

TABLE 8.5 NUMBER OF CHILDREN USING THE SAME, OR DIFFERENT STRATEGIES FOR CONCRETE AND SYMBOLIC PRESENTATION OF EACH SUM.

Table 8.5 shows that most of the children used the same strategy for both types of presentation, except for 3 + 4 where several of the younger children used a combination of count all, count on or min, whilst the older children chose between count on, min or retrieval.

No significant relationship was found in either group between knowledge of the commutativity principle and use of min. The correlation between correct judgments in the commutativity tasks and use of min for the older group was rs .319 p <.10, and for the six to seven year olds it was rs .323 p <.10. Performance on the abstract task was then analysed separately to see if there was any connection between an informal knowledge of the principle and use of min, but here again there was no significant association. (8 to 9 year olds rs.327 p <.10, 6 to 7 year olds rs.268 p <.25). All of the older group and most of the younger children used min and were successful on most of the commutativity trials.

The concrete and symbolic tasks were similar to those in the Baroody and Gannon (1984) study, so the performance of the younger group in this study was compared to see if there was evidence of CAL or COL without an appreciation of the equivalence of commuted pairs. Of the 19 who used CAL or COL, 5 (26%) were unsuccessful on some of the commutativity tasks compared with 45% in the Baroody et al (1984) study.

8.6 Discussion

Essentially, most of the children had little difficulty with the commutativity tasks, though success on the abstract tasks was lower than the concrete and symbolic tasks for both groups. The higher success rate of the concrete and symbolic tasks could be evidence of what Hiebert and Lefevre (1986) describe as procedural knowledge based on successful visual symbol recognition, rather than a knowledge of the principle, which was what was required in the abstract task. Contrary to Baroody and Ginsburg's (1986) proposition, the use of unspecified groups of objects in the abstract task proved to be more difficult for the children in identifying commuted pairs.

No significant relationship was found in either group between knowledge of the commutativity principle and the use of min. All of the older children and most of the younger ones used min, and showed a knowledge of the principle on high scores on the commutativity tasks. A comparison of performance in the abstract tasks and use of min in the younger group revealed a minority of children who succeeded on the abstract trials and did not use min, and almost the same proportion who failed on the task yet used min.

When comparing the younger children's performance on the concrete and symbolic commutativity tasks and use of min or count all from the larger addend, as in the Baroody and Gannon (1984) study, the percentage of pupils who used an order indifferent adding strategy and were unsuccessful on some of the commutativity tasks was much smaller than in the Baroody and Gannon study.

There was evidence of protocommutativity in the younger group only, though none of the children in question used the strategy consistently or exclusively, and some used min as well.

Some of the younger children found the sums difficult, and a significant relationship between correct responses and teacher's rating of pupil ability was found in this age group. However, no significant association existed between rated ability and performance on the commutativity tasks in either group.

SUMMARY OF STUDY IX

The aims of the study were to see if there is a connection between knowledge of the commutativity principle and addition strategies.

Each of forty-eight six to nine year old boys and girls, divided into two age groups, completed nine tasks, four with addition sums and five commutativity tasks involving concrete and symbolic representation.

Results showed that the children have an adequate knowledge of commutativity both formally and informally. No significant relationship was found between knowledge of commutativity and the use of min.

There was some evidence of 'protocommutativity' in the younger six to seven year old group, where a relationship between rated ability and performance on the sums was also found. No association between performance on the commutativity tasks and rated ability was found in either age group.

INTRODUCTION TO STUDY X

Five year old boys and girls took part in this study so that knowledge of commutativity in children with little experience of formal addition could be studied.

The number of tasks was reduced to seven. The symbolic sum and commutativity tasks were excluded and two concrete sum tasks were added with counters visible throughout, to see if having available objects to manipulate prompted different behaviour with these young children in their first term of formal schooling.

STUDY X

METHOD

8.7 Subjects

There were twenty four subjects, fourteen boys and ten girls aged between 5 years and 5 years 8 months with a mean age of 5 years and 4 months and a standard deviation of 2.33 months. All the children were rated for ability by their teacher on a scale of nought to ten, five being average.

8.8 Design

A repeated measures design where each subject completed twenty sums in four blocks and fifteen commutativity tests in three task blocks. The order of the sums and commutativity tests were randomised within blocks and the order of presentation of the sums and commutativity tests was balanced across subjects. There was oral strategy inquiry after each sum and commutativity test, with oral inquiry about the principle after each commutativity test also.

8.9 Materials

The materials were the same as those for Study IX except for the sum cards which were not used in this study.

8.10 Procedure

The procedure was the same as Study IX except for:

a) The sums were all presented with counters and numerals, one presentation with the counters visible, and one presentation where they were not visible after counting.

b) There were only three commutativity tasks, two with concealed counters and numerals, and one with unspecified groups of objects (drawing pins) in the abstract task.

8.11 Results

COMMUTATIVITY			
	CONCRETE INVISIBLE SMALL	CONCRETE INVISIBLE LARGE	ABSTRACT
JUDGMENTS EXPLANATIONS	98 63	104 73	89 40

TABLE 8.6 TOTALS OF CORRECT RESPONSES IN THE COMMUTATIVITY TASKS FOR THE WHOLE GROUP (24 x 5 = 120 MAXIMUM FOR EACH OF THE 3 TASKS FOR JUDGMENTS AND EXPLANATIONS)

Table 8.6 shows the children found explaining their correct judgments difficult. Sixteen 'did not know' for some of their judgments, and five did not attempt an answer for their abstract task judgments. In all, 66% of the group had some difficulty explaining their correct judgments.

In contrast, the children showed a knowledge of the principle with a minimum of 75% correct judgments.

		EXPLANATIONS	
		ALL CORRECT	SOME CORRECT
JUDG-MENTS	ALL COORECT	4	2
	SOME CORRECT	0	18

TABLE 8.7 TOTAL NUMBER OF CHILDREN IN EACH CATEGORY FOR JUDGMENTS AND EXPLANATIONS IN THE THREE COMMUTATIVITY TASKS

Fifteen of the eighteen who made some incorrect judgments and explanations succeeded on more than half of the total of fifteen trials, and only one child failed on all of the five abstract trials whilst succeeding on six out of the ten concrete trials for judgments and explanations

	CORRECT JUDGMENTS	CORRECT EXPLANATIONS
CONCRETE INVISIBLE SMALL	12	9
CONCRETE INVISIBLE LARGE	17	12
ABSTRACT	13	5

TABLE 8.8 TOTAL NUMBER OF CHILDREN WHO MADE CORRECT JUDGMENTS AND EXPLANATIONS FOR ALL OF THE FIVE TRIALS IN EACH OF THE THREE COMMUTATIVITY TASKS

The table shows that judging and explaining commuted and non-commuted pairs of large and small numbers was easier than small single digit number combinations. The children found explaining their correct judgments in the abstract task more difficult than the concrete tasks: three children gave no explanation of their correct judgments and simply shrugged their shoulders, and five replied 'don't know' to their maximum total of five correct judgments.

There could have been other reasons unrelated to understanding commutativity which made explanation difficult, for example, inadequate expressive vocabulary, or uncertainty in the novel test situation.

SUMS			
CONCRETE VISIBLE SMALL	CONCRETE VISIBLE LARGE	CONCRETE INVISIBLE SMALL	CONCRETE INVISIBLE LARGE
52	35	54	43

TABLE 8.9 TOTAL NUMBER OF CORRECT RESPONSES FOR EACH TASK (MAXIMUM = 120)

As the table shows, the children found the sums difficult with less than half correct for any of the tasks. Seven children had nil scores, and a further four did not attempt sums with large addends, though they had some success with small addend sums.

SUMS	CA	CO	MIN	DEC & RET	DON'T KNOW	GUESS
2 + 7	28	0	4	1	2	13
2 + 5	29	0	5	1	2	11
3 + 4	27	1	3	1	3	13
3 + 8	25	0	5	1	4	13
4 + 7	27	0	3	0	4	14
2 + 13	16	0	6	0	24	2
2 + 15	14	0	6	2	26	0
3 + 14	17	0	6	0	24	1
3 + 16	16	0	7	0	24	1
4 + 12	17	0	5	0	24	2
TOTAL	216	1	50	6	137	70

TABLE 8.10 REPORTED STRATEGIES FOR EACH SUM
(MAXIMUM = 48)

Ten children used counting all from the largest addend for some of the sums, whilst using other strategies as well, e.g., three of these children used min.

Only six children chose to use the counters in the visible condition, the majority used their fingers for counting. Several made counting errors of plus or minus one, there were four who combined the two addends for the answer, e.g., 2 + 7 = 27, and four children stated an addend as the answer.

SUMS	SAME STRATEGY	DIFFERENT STRATEGY
2 + 7	20	4
2 + 5	20	4
3 + 4	19	5
3 + 8	20	4
4 + 7	20	4
2 + 13	21	3
2 + 15	24	0
3 + 14	21	3
3 + 16	23	1
4 + 12	20	4

TABLE 8.11 NUMBER OF CHILDREN USING THE SAME OR DIFFERENT STRATEGIES FOR BOTH TYPES OF PRESENTATION

Most of the children used the same strategy for the counters visible and the counters invisible conditions.

	CORRECT JUDGMENTS	CORRECT EXPLANATIONS	MIN	SUMS
TEACHER'S RATING	.06	.515**	.521**	.458*
MIN	385*	.558**		

* Signif.lev. p <.05   ** Signif.lev. p <.01

TABLE 8.12 RANK ORDER CORRELATIONS (SPEARMAN)

The relationship between teacher's rating of pupil ability and correct explanations of judgments in the commutativity tasks, correct performance on the sums and use of min suggests that the more able in the class of five year olds could meet the strict criterion of explaining judgments and were competent in addition to the extent of using more economical counting strategies.

The conceptual basis of the development of min is indicated in the correlation between the use of min and correct judgments and explanations of judgments on all commutativity tasks. Knowledge of the principle in judgments in the abstract task and the development of min was analysed separately, and here again there was a strong association, rs.495 ($p < .01$) reinforcing a theory of conceptually based strategy development for most children.

Comparing this study with the results of Baroody and Gannon (1984); in both studies 33% of the group used an order indifferent adding scheme. Of the 8 children in this group who used CAL or COL, 4 were successful on both concrete commutativity tasks, and 4 made no more than two errors on one of the tasks whilst judging the other task correctly. In the Baroody et al study, 11 children used CAL or COL, 6 were successful on both commutativity tasks, 3 had mixed success and 2 failed on both tasks.

8.12 Discussion

A relationship between knowledge of the commutativity principle and strategy use was found in this study. The use of min was associated with correct judgments and explanations on all of the commutativity tasks. Informal knowledge of the principle in the abstract task was also significantly correlated with the use of min.

Comparing the performance of the five to six year olds in this study with the same age group who completed similar tasks in Baroody and Gannon's (1984) study; all the children who used counting all from the larger addend or min in this study succeeded on both or one of the concrete commutativity tasks, compared with a higher failure rate in commutativity tasks in Baroody et al's study.

The children coped with judgments in the commutativity tasks better than explaining their correct judgments as Langford (1981) found. Whilst responses like 'I don't know' could indicate inadequate conceptual development for accurate explanation, the children may have had language difficulties or reacted adversely to the novelty of the test situation.

The majority of children used count all, and there were thirty-one instances of count all from the larger addend by a minority of children who also used other strategies as well, including min.

The teacher's rating of pupil ability was related to performance on the sum tasks, and with judgments and explanations in the commutativity tasks.

SUMMARY OF STUDY X

The aim of this study was to see if five year old children understand the commutativity principle informally and formally, and whether this knowledge is linked to strategy development for formal addition sums.

Twenty-four five year old boys and girls completed three commutativity tasks, and four sum tasks, with concrete materials in two conditions, visible, and invisible after counting.

A relationship was found between knowledge of commutativity and the use of min. Knowledge of commutativity seemed to be in advance of competence in formal addition, with higher scores for the commutativity tasks compared with the sums, where seven children had nil scores.

There was evidence of 'protocommutativity' though the children used other strategies as well, including min.

Teacher's rating of pupil ability was associated with performance on the sum tasks, and with judgments and explanations in the commutativity tasks.

COMPARISON OF THE RESULTS OF STUDIES IX and X

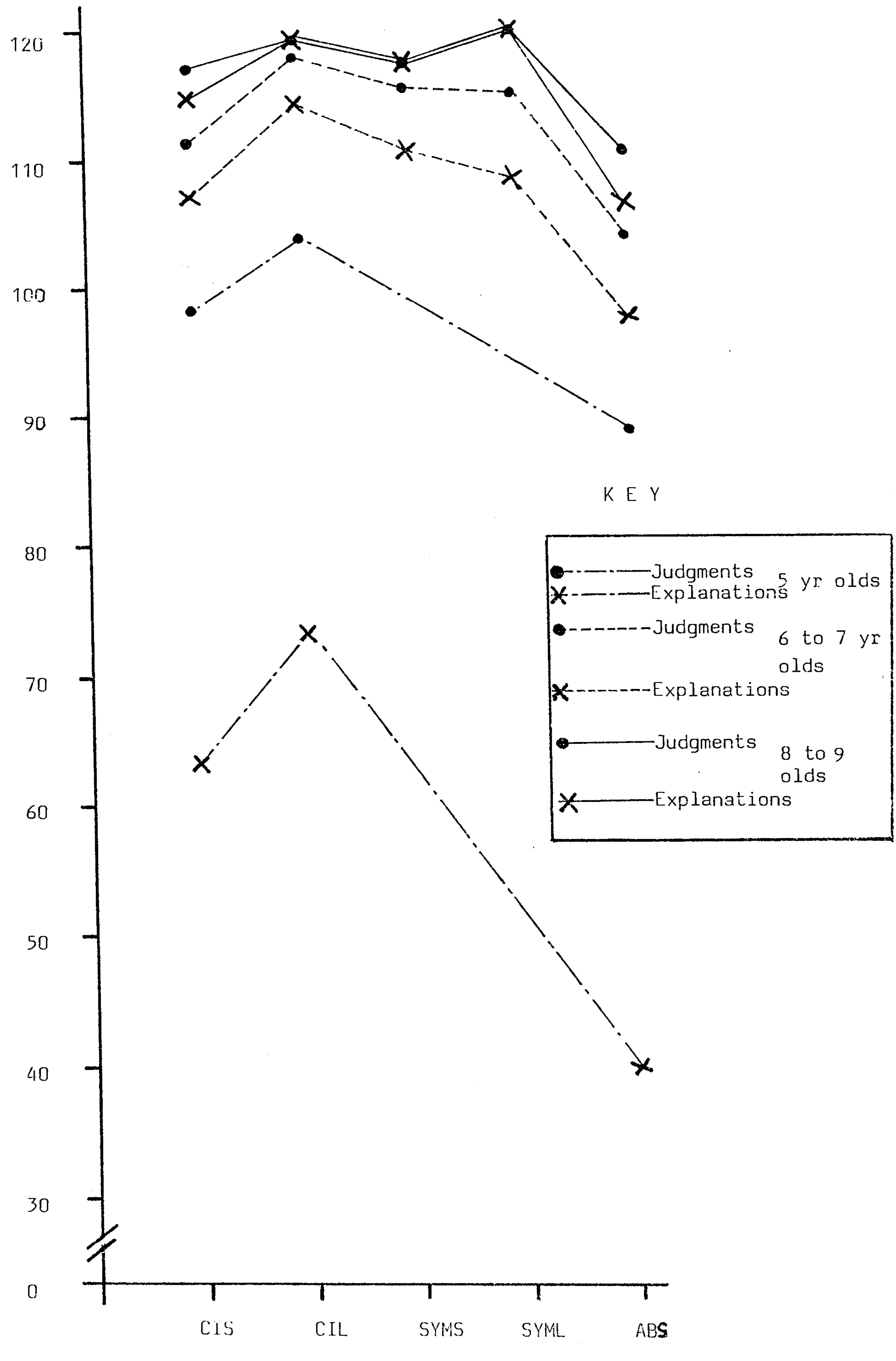
Fig 8.1 (page 181) shows a similar pattern of responses in each age group with concrete and symbolic tasks more successful than the abstract task, and more successful judgments than explanations.

	SUCCEEDED ON ABSTRACT TASK		FAILED ON ABSTRACT TASK	
AGE	SOME USE OF MIN	NO USE OF MIN	SOME USE OF MIN	NO USE OF MIN
5yrs	3	11	1	9
6/7yrs	11	6	5	2
8/9yrs	21	0	3	0
TOTAL	35	17	9	11

TABLE 8.13 PERFORMANCE ON THE ABSTRACT TASK WITH USE OF MIN FOR EACH AGE GROUP

The table shows the gradual development of knowledge of the commutativity principle and strategy use over the primary school years. Informal knowledge of the principle appears to precede strategy development, this is clearly seen in the youngest group where 45% of the children succeeded on all of the abstract trials but did not use min. As Baroody and Ginsburg (1986) proposed, the use of objects in the abstract tasks did reveal a knowledge of the principle in the younger group prior to extensive formal instruction in addition.

FIG. 8.1. TOTAL OF CORRECT RESPONSES FOR JUDGMENTS AND EXPLANATIONS IN THE COMMUTATIVITY TASKS FOR EACH AGE GROUP



There was some evidence of 'protocommutativity' in the five to seven year old group, a minority of children used this strategy along with others including min.

The teacher's rating of pupil ability was related to performance on the sum tasks in the two youngest groups, and with use of min, judgments and explanations in the five year olds. Individual differences in ability seem to affect the rate of development in conceptualisation of the commutativity principle, accuracy in formal arithmetic, and strategy use.

CHAPTER 9

OVERVIEW OF STUDIES I TO X

The main purpose of these studies has been to focus on the role of the child in simple addition. The reasons for strategy change are explored in the wider context, looking at social influences and developmental aspects from the child's perspective rather than the adults interpretation of it. This is seen as crucial to an adequate understanding of the psychology of mathematical cognition.

An intuitive assessment of the day to day work habits and knowledge levels of the children was given by their class teacher. Though crude when compared with the precision of attainment tests, its advantage is in its two dimensional approach: that of assessing knowledge and cognitive style, as opposed to the one dimensional assessment of knowledge in standardized testing. This method of assessment was based on Siegler's (1988) two dimensional approach of assessing knowledge and confidence criteria for stating a retrieved answer. He found that good students and perfectionists were indistinguishable on measures of knowledge, yet their strategy choices were completely different because of differing confidence levels for stating a retrieved answer before using 'back up' counting strategies.

The teacher's assessments of the study group was used in all of the studies and proved useful when related to performance in the experimental condition. For example, higher ability ratings were associated with the reported use of retrieval and with the transition from counting based strategies to retrieval. In the first two studies, a positive relationship was found between rated ability and min in six to seven year olds and a negative relationship in eight to nine year olds, indicating that more able six year olds and less able nine year olds were mainly using min.

Previous research has acknowledged the effects of age on strategy use, e.g., Groen and Parkman (1972) found a significant difference in reaction time between different age groups indicating changes in strategy use from counting to retrieval, Siegler (1987) also reported comparable strategy changes with age and Baroody (1987) detailed the evolution of strategy development over time. The studies in this thesis show that rated ability as well as age is associated with these changes.

As in Siegler's (1987) study, type of sum was found to influence strategy choice. However, contrary to Siegler's findings, retrieval was associated with sums where there was a large difference between addends, and decomposition was used where differences were small.

Because young children's knowledge may be in advance of their ability to verbalise it a puppet video of strategies was made from which the children could identify their strategies. This was done to see if reducing strategy identification to recognition would elicit a different response to verbal questioning. In the event there was no appreciable difference between the two methods in Studies I and II where the children identified the strategy they had just used for a sum, except for decomposition where there were more oral identifications on the few occasions on which this strategy was reported. The puppet video demonstration was found to be useful in subsequent studies for identification of strategies in answer to questions related to conceptual development and social perceptions.

The aims in Study III were to find out more about what children think about strategies, and whether this is related to strategy use. There is a wealth of evidence for changes in strategy use but no clear explanation of these changes. Arithmetic is done in a social setting so it is likely that observing others and the awareness of being observed would affect attitudes to strategy use. Answers to the questionnaire showed that whilst most of the children aspired to using retrieval, there were a number of younger children who responded to perceived teacher preferences and chose counting strategies in answer to that particular question, and no relationship was found between strategy choices in answer to the questions and strategy use for the sums.

Despite considerable interest in the learning of number patterns on the part of curriculum planners (e.g., Mathematics in the National Curriculum 1989), little is known about children's knowledge of, or use of number patterns in addition sums. Auditory and visual number patterns were included in Study III to see if there was a link between the retrieval of simple number patterns and retrieval of number facts for sums. An association was found in the older group but not the younger ones. Both groups found the patterns difficult, and several children had nil scores.

The next two studies concentrated on knowledge of number patterns going up in a set sequence, and were an extension of those begun in Study III. The gap between the performance of the youngest and the oldest children was considerable, and the composition of the patterns proved to have the greatest influence on performance rather than modality or type of task.

The higher the child's ability rating, the more accurate the performance on the patterns, especially of the six to seven year olds where difficulties were common.

The patterns of 5's and 10's were most successful and sums associated with these numbers are amongst the earliest combinations to be learned and retrieved, (Carpenter and Moser 1983) so the two processes may have a related knowledge base which could be useful in giving an added flexibility with numbers in teaching arithmetic at the primary level. Further research in this area is needed if curriculum development is to be psychologically based.

How children choose amongst alternative strategies was the subject of the next two studies, VI and VII. Only correctly worked examples were used for alternative strategy choices and justification, so that procedural competence did not distract from the main purpose of the investigation, which was why strategies are chosen not skill in executing them.

Siegler and Jenkins's (1989) model states that strategies are chosen at the subconscious level according to their speed and accuracy for a particular problem. This theory was explored in the alternative strategy choices of Studies VI and VII where the children re-worked sums with alternatives to their initial strategy choices. As expected, the older children were able to demonstrate alternative counting strategies to their initial choices of retrieval. What was surprising was the number of children (75%), who could have retrieved an answer but chose min. There were children in the younger group also who chose basic count all in preference to successfully demonstrated min and retrieval.

When asked to justify their choices at the conscious level, many of the children in both groups could not, and replied 'I don't know'.

Baroody and Ginsburg's (1986) schema based theory of a sub-conscious search for cognitive economy was also not proven in these studies, where children in both age groups chose count all in preference to min, and both counting strategies instead of retrieval.

The results of Studies VI and VII can be compared with the questionnaire of Study III where children in both age groups consciously aspired to using retrieval, but at the subconscious level they chose a variety of strategies for calculating their sums.

The results of Studies VI and VII led to the evolution of Study VIII, which was an attempt to ascertain the children's conscious judgments on abstract concepts of speed, economy, accuracy and superiority in relation to strategy choice. The aim was to try and discover whether in fact the adults interpretation of the psychological factors involved in strategy choice based on a knowledge and understanding of these concepts is accurate or not.

A puppet video demonstrating the distinction between the strategies was made from which the children chose according to the judgment criteria.

Here again, results were far from clear cut. A number of children produced logically coherent choices which were not judged on efficiency in the same way that adults would judge, for example, that retrieval was less economical and slower than a counting strategy. There was a further group whose responses seemed confused, and who produced both logically inconsistent and implausible judgments viewed from the adults standpoint. For instance, in answer to the question on speed of execution:- that count all is quicker than min, retrieval is quicker than count all but slower than min.

It is possible in some instances that adults and children think of speed, accuracy, economy and superiority in relation to addition strategies in different ways from each other. This raises questions about the accuracy of the interpretation of children's performance on addition sums and drawing conclusions which may not be a proper reflection of what the children are thinking. More attention needs to be paid to the meaning children attach to addition tasks in order to attempt a fuller understanding of what is going on, and not to draw erroneous conclusions based on the wrong premise.

The final two studies were an investigation of the knowledge of the commutativity principle and its application to strategy use for addition sums. These studies were an indirect consequence of Studies VI and VII where commuted examples were used and to which few children referred or used the commutativity principle as a labour saving 'short cut' in calculating them.

There have been several studies of commutativity, (e.g., Resnick 1983; Baroody 1987; Weaver 1982; Langford 1981; and others). The argument is whether knowledge of the principle precedes strategy development, e.g., use of min, or whether the progression towards economical strategies proceeds without such knowledge.

The age range in Studies IX and X was extended to cover five to nine year olds. Five year olds were included so that the performance on commutativity tests of children with little formal addition experience could be compared with those who have had a number of years of formal schooling in arithmetic.

Following the advice of Baroody and Ginsburg (1986) concrete and symbolic examples were used, and Langford's (1981) criteria of correct judgments and correct judgments with explanations were used. The range of activities was extended to include commuted and non-commuted pairs to show knowledge of the principle, and the ability to differentiate between commuted and non-commuted arrays. Any association between performance and rated ability was also of interest.

Results showed that trends were similar in the three age groups. There were more correct judgments than explanations of correct judgments and symbolic tasks were more successful than the concrete abstract task.

Comparison of the age groups showed a gradual development of knowledge of the commutativity principle and strategy use over the primary years; most of the nine year olds knowing the principle and using min.

Informal knowledge of commutativity in the abstract task appeared to be in advance of strategy use, which was seen in the performance of the five year olds where almost half the group succeeded on the task and did not use min. So the use of concrete examples did reveal knowledge of commutativity in children not exposed to extensive formal instruction.

The effects of ability on the rate of development is suggested in the association found between rated ability and performance in the younger group on the sum tasks, use of min and correct judgments and explanations of commutativity.

Conclusion

Have some of the reasons for the changes in strategy use emerged from these studies as envisaged at the outset?

Whilst findings in the first two studies agreed with Siegler that retrieval is likely to be used for sums with small totals, retrieval was also used in these studies where the difference between addends was large. In the case of min, which Siegler assumes the children resort to if retrieval fails, the children in his group used min for sums where there were large differences between addends, whereas in this study min was widely used for all types of sum. Decomposition was not associated with addends greater than ten, which Siegler found, but was used where differences were small, probably because of the use of ties in the decomposition process.

The sums in these studies were identical to the day to day addition sums done in class by all of the children. Because of this, the two dimensional ability rating of the children by their teacher, based on the child's knowledge and conscientious work habits in class, was used. It proved to be consistently associated with the use of retrieval based strategies, and the rate of progression towards more economical strategy use in younger children. Accuracy in addition and competent performance on the pattern tasks was also associated with rated ability, as well as correct demonstrations of alternative strategy choices, logically coherent sequences in eight to nine year olds and the rate of development in the concept of commutativity. Individual differences in ability assessed in this intuitive way over a period of time seems to go some way towards predicting the rate of change in strategy development linked with age. The ability rating also gives some insight into the possession of components of knowledge which may contribute to this change, e.g., competence in number patterns, and successful demonstrations of alternative strategy choices were associated with higher ability ratings.

Siegler and Jenkins's (1989) modified Distribution of Association model where strategy choice is influenced by the speed and accuracy of a particular strategy for a particular problem or class of problems did not adequately describe the data of Studies VI and VII.

Some of the children did not choose from their repertoire of available strategies on the basis of efficiency in the way adults would expect them to. Nor did some children seem to have conceptualised the efficiency of strategies in the same way as adults. Children may be being credited with the influences of conceptual development on strategy choice, e.g., retrieval chosen because of speed over counting, when in fact their conceptual development is not mature enough to be applied in such a way.

When asked to make judgments in Study VIII on the speed, accuracy, economy and superiority of strategies some of the children made logically coherent but implausible judgments from the adult point of view. These conscious judgments, e.g., that counting is more economical and quicker than retrieval, may partially explain the puzzle of why some nine year olds did not use retrieval when they could have done, and were unable to give adequate explanation when asked to do so in Studies VI and VII.

Adult supposition about what children think about strategy use, and what some of them really do think seems to be at variance.

Some young children believed that they were expected to use counting strategies, and whilst eight to nine year olds showed a good knowledge of commutativity, they did not use it when they had the opportunity in calculating commuted pairs. May be children perceive social constraints in using methods of working which save effort, as Baroody et al (1983) found when children in their study who used the commutativity principle to short-cut computation regarded it as 'naughty'. These aspects of formal learning need further investigation if mastery in problem solving is to be more discovery and less drudgery.

Results in the final studies of commutativity and strategy development showed that children as young as five have an informal knowledge of commutativity before exposure to formal instruction in addition. This contrasts with Baroody's (1987) evidence of children who used an order indifferent adding strategy, yet failed on commutativity tasks.

To sum up, some of the reasons for changes in strategy use which have emerged from these studies are that a possible combination of type of sum, age and rated ability influences strategy choice. Exploratory studies into strategy changes from the child's perspective revealed that children may view the relative speed, accuracy and economy of strategies for addition sums in a different way to that of adults. The progression towards more economical counting strategies appears to be preceded by an informal knowledge of the commutativity principle which may not be apparent because of some children's perceived social constraints on using 'short cuts'.

Whilst previous observations that children use a variety of strategies was borne out in these studies, it is the reasons for these choices and changes which are challenged, and where we need to look again.

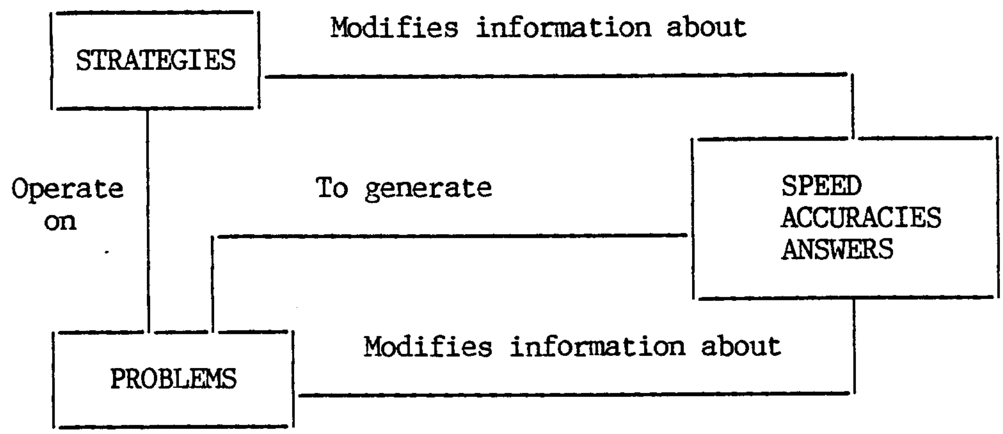
The reasons for the progression from counting to retrieval would seem to be more diverse than Siegler and Jenkins' (1989) model suggests. For instance, social influences in the form of classroom instruction and peer group interaction needs to be added, as well as the ability the child brings to the task by way of prior knowledge, e.g., higher rated six year olds and lower rated nine year olds were mainly using min.

The explanation of these differences may lie in confidence.

Whereas Siegler and Jenkins' (1989) used the notion of a confidence threshold to explain when children might use a back up strategy rather than retrieval, there may also be differences in confidence that explains why some children use count all even when they could use min.

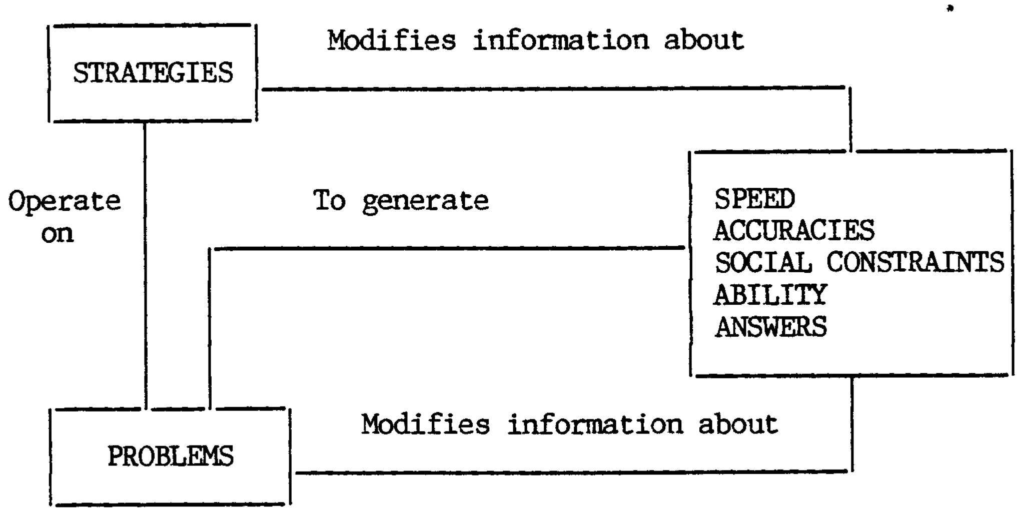
The childrens confidence in the particular use of a strategy rather than its results may well be influenced by the social climate in the classroom in which the teacher's actions deliberately or involuntarily signal to the child beliefs about his/her ability, e.g., the six year olds who reported using count all when they could have used min (Study VII) were likely to have been influenced by what they thought their teacher wanted them to do, as their answers to the questionnaire in Study III suggests. In the same way, the nine year olds who were still using min (Study I) could have believed that their teacher's assessment of their abilities restricted them to counting, or that they had failed on past retrieval attempts and inferred that they were expected to opt for the accuracy of counting.

Siegler and Jenkins' (1989) model could be changed from this:-



(Reproduction of Siegler and Jenkins' 1989) P.42

to this:-



in order to incorporate the wider influences on strategy change.

Contrary to Baroody's (1987) findings, an informal principled knowledge of commutativity preceded strategy change, and a search for cognitive economy was not found in strategy choices, where counting was often preferred to retrieval.

It would seem that the way forward is to explore existing informal knowledge to form the basis of building number relations in formal procedural instruction in order to promote strategy change.

By reflecting on past informal knowledge of addition, progression would be principle driven moving from counting to abstract mental strategies of retrieval and decomposition. Not in the sense of being context disembedded but in the sense of being transcednent, applicable to many problems and contexts. Through reflecting on informal principled knowledge of the addition process formal procedural instruction will be grounded in established schema. Edward and Mercer (1987) describe reviewing past responses and picking out what is relevant to present needs, thus analizing ones own schemata and reconstructing afresh, which is a prominent function of consciousness. Woods (1988) also speaks of initial 'impulsive' responses followed by regulation of the child's own thinking and activity by reformation and simplification of likley solutions, intellectual achievement arising from interaction between novice and expert: child and teacher.

Demands on working memory are a major developmental factor. Again, Woods (1988) draws attention to the memory demands of learning mathematical language in a formal social instructional setting. Case (1982) advises that instruction in addition must centre around diverse opportunities for automaticity of basic operations at every stage so that the child acquires more complex executive schemes at a younger age by reducing memory demands. Drawing the child's attention to new strategies will 'chunk' together items of knowledge in procedural conventions of addition which would otherwise be attended to separately, as in the continued inefficient keeping track of needless counting in counting all.

Educational Implications

The findings of the studies in this thesis point to intervention which encourages children to use their informal knowledge of commutativity to promote min, and relations among number patterns to be linked to the transition from counting to retrieval strategies.

Teaches need information about discrimination between strategies and progression from counting to retrieval, knowledge which teachers who co-operated in these studies did not have before involvement in the research.

The constraints of the social context of formal learning were inferred from the children's perceived teacher preferences for counting, and knowledge of, but reluctance to use labour saving 'short cuts'.

Walkderdine (1988) draws attention to the teacher's control of the learning situation:-

> 'she (the teacher) indicates what kind of response she requires' p.62

and Walkerdine warns against assumptions that schooling serves to facilitate conceptual development when in practice children may be confused by unique classroom practices, e.g., the ambiguity of questioning where the same question is repeated when an answer has been given, or questions asked when the child is aware that the teacher already knows the answer. The child must make sense of activities, e.g., manipulated iconic signifiers (drawings or objects) expressed in symbolic addition.

She concludes:-

> 'Real understanding therefore depends first upon a set of practices in which real understanding is the goal of an explicit framework of activities' p.201

Solomon (1989) echoes the same sentiments in her description:-

> 'School introduces the child to a completely new social context within which arithmetic takes place' P.170.

She describes the confusion of what she calls 'pseudo' questions meant to elicit correct answers, and often causing misunderstanding for children.

Perceived social constraint of the classroom could adversely affect childrens' use of their existing knowledge in promoting strategy change. Edward and Mercer (1987) point out the contrast between learning in formal schooling and informal learning of the child's first language. They describe the gradual handover of control from the teacher to the learner as s/he becomes able to operate without help in informal language learning, which is seldom realised in formal education.

The language of instruction needs to be clear to the child with explanation at each stage clarifying the purposes behind efficient strategy use, goals to be aimed for, and concepts behind operations. When questioned in Study III most of the children's personal aspirations were for retrieval, but they were unclear about instructional goals. This vague unawareness is described by Edward and Mercer (1987) as 'ritualised' responses by children for whom the process of formal instruction remains a mystery. No matter how friendly and informal the manner they are required to learn things without reason.

Retrospective Operational Changes to the Studies; and looking to the Future

Reconsidering the studies with a view to their ecological validity improvements could be made.

The studies reported here have involved children doing sums, completing patterns, detecting errors, judging the equivalence of addition and answering questions about strategies. While the children were tested in a familiar setting by a familiar person, the experimetnter being known to the children as a teacher, it is only the first activity; doing sums that is routinely experienced in the classroom. This overlap suggests that the distinction of reported strategies for doing sums obtained in the studies is likely to be similar to that in the classroom, i.e., the findings are likely to be valid and the children were unlikely to have been unnaturally conservative in their choice of strategies because of intimidation by the experimenter. By the same token, the innovatory nature of the other tasks makes the need for validation by other methods pressing. While the young children's report that count all was what their teacher favoured (Study III) is borne out by observation in Study VII where several children counted all when they could have used min, their views of strategies require confirmation, as at present reliability is unknown. This is seen in the apparant confusion of the minority of children who made logically inconsistent judgments about the speed, accuracy and economy of strategies compared with each other for particular sums (Study VIII). The experimental condition placed the children in the unfamiliar position of making comparison judgments about strategies on the basis of efficiency, whereas in their classroom experience an accurate end point is stressed rather than decision about the efficiency of the means by which that end point is reached.

Looking back on the operation of the studies exploration of the relationship between aspirations towards the use of retrieval and use of retrieval for the sums would have been clearer if inquiry had focused on performance compared with answers to individual questions, e.g., perceived teacher preferences, rather than the whole set of questions. Also, questions about past strategy use incorporated with the questions on present and future aspirations would have presented a more complete picture of what children think about addition strategies.

The auditory pattern task of Study III would probably have been more successful if preceded by a known auditory pattern rhyme familiar to the children, making the requirements of the task clearer.

The video demonstrations of decomposition could have been improved by more than one exposition of the strategy, e.g., 5 + 4 as 4 + 4 and 1, besides 5 + 4 as 1 less than 10.

The teacher's rating of pupil ability could have been given in two 0 to 10 scales, one for knowledge and one for work habits. These being compared separately with performance may have revealed subtleties which were lost by incorporating both measures in the one scale.

Future expansion of the exploratory work in number patterns could be useful as stated earlier, considering the recommendations of curriculum planners. Oral games and songs based on patterns çould be investigated in relation to their written expression and use of retrieval.

A further particular concern would be to investigate auditory discrimination of numbers which does not seem to attract the attention it deserves. Several years of teaching children with learning difficulties has revealed a number of children who are confused with number values because of not discriminating between similar sounding numbers like eighteen and eighty.

Expansion of research on formal addition strategies to include the role of the plus and equals signs would further clarify the children's conceptualisation of symbolic addition. Wood (1988) points out that the plus sign does not bear perceptual resemblance to the operation to which it refers, and Sinclair and Sinclair (1986) remind us that there is nothing 'natural' about the operation of formal addition as taught in schools. Skemp (1982) suggests that research based on teaching experiments in which children experience the application of addition strategies with concrete and symbolic representation, followed by interviews to see if schema are built on and concepts expanded from one stage to another would reveal how strategies change through methodology, and not logical inference. This could be a useful approach considering the evidence of childrens strategy choices in this thesis, which did not follow the clear cut proposals set out in recent research.

Follow on studies of the exploratory work of Study VIII in which the experimental procedure is reversed might further clarify the conceptualisation of the speed, accuracy and economy of different strategies in relation to each other and different types of sum. These studies would state varying types of sum, e.g., tie (4 + 4), medium addends (5 + 4), large and small addends (8 + 1) for which the child would select an appropriate strategy from a video demonstration. They would then show the operation of the chosen strategy and say why it was appropriate for the particular sum. This method of investigation would eliminate the possible ambiguity of direct questioning. Or a puppet video of inappropriate strategy choice, e.g., count all for 12 + 2, could be shown and the child asked to judge the performance and demonstrate a 'better' way if s/he thought it necessary.

The studies of this thesis have suggested that strategies change because of ability as well as age. Social constraints influence development towards, and use of, more economical strategies, and informal principled knowledge precedes strategy development. The progression towards more sophisticated mental strategies based on retrieval is not adequately accounted for by inferred child conceptualisation of the speed, accuracy and economy of strategies, but is also governed by the wider context of the climate of social interaction in formal schooling.

The complexity of this basic and essential element of mathematics education needs further study if recommendations for educational practice are to be effective.

APPENDICES

DATA FOR STUDIES I TO X

Strategy Preferences: 1 – Count All
2 – Min
3 – Retrieval
4 – Decomposition
5 – Count on from 1st Addend
6 – Guess

JULY 1988 STUDY I. 8/9 years.

Teachers Rating 0 – 10	Subject Number	V/O First	Set Order of sums		Set 1 Strategies												Set 2 Strategies												Frequency of Strategy						Strategy Preferences						Age	Sex
					1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	1	2	3	4	5	6		
3	1	0	1	2	M	CO	M	CO	M	M	M	CO	M	M	M	M	M	M	CO	M	R	M	CA	M	M	M	R	R	1	16	3	0	4	0	4th	1st	3rd	0	2nd	0	8/11	F
3	2	0	1	2	M	R	M	M	M	D	R	M	M	M	M	D	R	M	R	M	R	M	M	M	M	M	R	R	0	15	7	2	0	0	0	1st	2nd	3rd	0	0	8/11	M
2	3	0	1	2	M	CO	CO	CO	CO	CO	CO	CO	M	M	M	M	M	M	CO	CO	CA	CA	CO	CO	M	M	M	M	2	11	0	0	11	0	2nd	1st	0	0	1st	0	9/5	M
3	4	0	1	2	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	24	0	0	0	0	0	1st	0	0	0	0	8/11	F
9	5	0	2	1	R	R	CA	R	M	D	R	R	M	R	M	R	R	R	R	M	R	D	D	D	D	D	D	R	1	4	12	7	0	0	4th	3rd	1st	2nd	0	0	9/2	F
9	6	0	2	1	R	R	R	R	M	D	R	M	R	R	R	M	R	M	M	R	R	M	M	D	M	M	R	R	0	9	13	2	0	0	0	2nd	1st	3rd	0	0	8/11	M
8	7	0	2	1	R	R	M	R	M	D	R	M	R	M	R	R	M	D	D	M	M	M	M	D	M	M	M	M	0	13	7	4	0	0	0	1st	2nd	3rd	0	0	9/4	F
9	8	0	2	1	R	R	M	R	M	D	R	D	R	M	R	R	R	D	D	R	R	D	D	D	D	D	D	R	0	3	11	10	0	0	0	3rd	1st	2nd	0	0	9/7	M
3	9	V	1	2	M	R	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	23	1	0	0	0	0	1st	2nd	0	0	0	9/0	M
8	10	V	1	2	D	R	R	M	M	R	R	M	M	M	R	M	M	D	D	M	M	R	D	D	D	M	R	R	0	10	8	6	0	0	0	1st	2nd	3rd	0	0	9/3	F
8	11	V	1	2	M	M	M	M	M	M	R	M	M	M	R	M	R	M	M	M	R	M	CO	CO	M	M	M	R	0	17	5	0	2	0	0	1st	2nd	0	3rd	0	9/5	M
6	12	V	2	1	R	D	M	D	M	D	D	D	D	G	D	M	R	R	D	D	R	D	D	D	R	M	R	R	0	4	7	12	0	1	0	3rd	2nd	1st	0	4th	9/2	M
8	13	V	1	2	R	R	R	R	M	M	R	M	M	M	R	M	R	R	M	M	R	M	M	M	M	M	R	R	0	13	11	0	0	0	0	1st	2nd	0	0	0	9/5	F
8	14	V	2	1	R	R	M	R	M	D	R	M	M	M	R	M	M	M	CO	CO	R	M	M	M	M	M	M	R	0	14	7	1	2	0	0	1st	2nd	4th	3rd	0	9/8	F
7	15	V	2	1	M	CO	M	M	M	D	R	M	M	M	M	M	CO	M	D	M	M	M	M	M	M	M	M	M	0	19	1	2	2	0	0	1st	3rd	2nd	2nd	0	9/6	M
6	16	V	2	1	R	R	R	R	R	R	M	M	M	M	R	R	M	M	CA	M	R	M	M	CO	M	M	R	R	1	11	11	0	1	0	2nd	1st	1st	0	2nd	0	9/2	F
																													5	206	104	46	22	1	0	12	4	1	1	0	– first choice	
																													384	384	384	384	384		2	1	8	3	2	0	– second choice	
																													1%	54%	27%	12%	6%		0	3	2	4	2	0	– third choice	

STUDY I

Name	Age	Number

3 + 2 =	5 + 1 =
1 + 3 =	4 + 3 =
2 + 6 =	4 + 6 =
1 + 7 =	2 + 7 =
8 + 3 =	8 + 2 =
6 + 5 =	7 + 4 =
7 + 10 =	8 + 12 =
9 + 12 =	9 + 11 =
13 + 7 =	13 + 6 =
14 + 8 =	14 + 7 =
15 + 2 =	12 + 3 =
16 + 4 =	15 + 1 =

STUDY I

Name	Age	Number
S 3 + 2 S 1 + 3		
S 2 + 6 M 1 + 7		
M 8 + 3 S 6 + 5		
M 7 + 10 L 9 + 12		
L 13 + 7 M 14 + 8		
L 15 + 2 S 16 + 4		
S 5 + 1 S 4 + 3		
S 4 + 6 M 2 + 7		
M 8 + 2 S 7 + 4		
M 8 + 12 L 9 + 11		
L 13 + 6 M 14 + 7		
L 12 + 3 S 15 + 1		

Strategy Preferences: 1 – Count All
2 – Count on from first Addend
3 – Min
4 – Retrieval
5 – Decomposition
6 – Guess

OCTOBER 1988 STUDY II. 6/7 years.

Teachers Rating 0 – 10	Subject Number	V/O First	Set Order of sums		Set 1 Strategies												Set 2 Strategies												Frequency of Strategy						Strategy Preferences						Age	Sex
					1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	1	2	3	4	5	6		
7	1	O	1	2	CA	CA	M	M	M	M	M	M	M	M	M	M	CA	CA	CA	R	CO	M	CA	CA	M	R	CO	CO	7	3	12	2	0	0	2	3	1	4	0	0	6/11	F
4	2	O	1	2	CA	CA	M	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	M	CA	R	CA	CA	21	0	2	1	0	0	1	0	2	3	0	0	6/11	M
5	3	O	1	2	CA	CO	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	R	CA	CA	R	CA	CA	R	CA	CA	20	1	0	3	0	0	1	3	0	2	0	0	7/1	M
8	4	O	1	2	M	M	M	M	M	M	M	M	CA	M	R	M	R	M	CA	M	M	M	M	R	M	R	M	D	2	0	17	4	1	0	3	0	1	2	4	0	6/8	F
7	5	O	2	1	M	R	M	M	CO	M	R	M	M	M	M	M	M	M	M	M	M	M	D	M	M	R	M	D	0	1	18	3	2	0	0	4	1	2	3	0	6/9	M
5	6	O	2	1	R	R	CO	CO	D	CA	R	M	M	M	M	M	CO	CO	CO	CO	CA	CA	CA	M	CA	R	CA	CA	7	6	6	4	1	0	1	2	2	3	4	0	6/7	M
8	7	O	2	1	CA	CA	CA	CA	M	CO	M	M	M	M	M	CA	M	CA	CO	M	M	R	CO	M	M	R	M	M	6	3	13	2	0	0	2	3	1	4	0	0	6/6	M
6	8	O	2	1	CA	R	CO	CO	D	D	R	M	D	CA	D	CA	CA	CA	CA	CA	CO	CO	CO	CO	CA	R	G	D	8	6	1	3	5	1	1	2	5	4	3	5	6/11	F
5	9	V	1	2	CA	R	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	R	CA	CA	CA	CA	CA	CA	CA	CA	R	CA	CA	21	0	0	3	0	0	1	0	0	2	0	0	6/5	F
7	10	V	1	2	CA	R	R	M	CO	CO	R	R	CO	M	R	CO	CO	CO	M	M	M	M	M	M	M	R	M	CO	1	7	10	6	0	0	4	2	1	3	0	0	6/10	M
5	11	V	1	2	CA	CA	CA	CA	CO	CO	M	M	M	CO	CA	M	M	M	M	M	M	M	M	M	M	R	M	M	5	3	15	1	0	0	2	3	1	4	0	0	6/8	M
4	12	V	1	2	CA	CA	M	M	CA	CA	M	CA	CA	CA	CA	CA	CA	CA	CA	CA	M	CA	M	CA	M	R	CA	CA	17	0	6	1	0	0	1	0	2	3	0	0	6/8	M
4	13	V	2	1	CA	CA	CA	G	G	G	CA	G	G	G	G	G	CA	CA	CA	CA	G	G	G	CA	G	CA	G	G	10	0	0	0	0	14	2	0	0	0	0	1	6/1	M
4	14	V	2	1	CA	R	M	M	CO	M	R	M	M	M	CA	M	CA	CA	CA	M	M	M	M	M	M	R	M	M	5	1	15	3	0	0	2	4	1	3	0	0	6/0	F
4	15	V	2	1	CA	CA	M	CA	CA	CA	R	M	CA	CA	CA	CA	CA	CA	CA	M	CA	CA	CA	M	M	R	CA	CA	17	0	5	2	0	0	1	0	2	3	0	0	6/2	F
9	16	V	2	1	R	R	R	D	M	M	R	M	M	M	R	M	R	R	M	R	M	M	R	R	M	R	M	M	0	0	12	11	1	0	0	0	1	2	3	0	6/11	F

STUDY II

Name Age Number

1 + 3 =	1 + 2 =
2 + 2 =	2 + 3 =
1 + 6 =	2 + 4 =
3 + 4 =	1 + 5 =
5 + 8 =	4 + 9 =
6 + 7 =	5 + 7 =
1 + 9 =	2 + 9 =
3 + 8 =	1 + 7 =
5 + 6 =	4 + 6 =
4 + 5 =	5 + 5 =
8 + 8 =	8 + 9 =
7 + 9 =	7 + 8 =

STUDY II

Name Age Number

S S	1 + 3 2 + 2	
S M	1 + 6 3 + 4	
M L	5 + 8 6 + 7	
S L	1 + 9 3 + 8	
M M	5 + 6 4 + 5	
L L	8 + 8 7 + 9	
S S	1 + 2 2 + 3	
S M	2 + 4 1 + 5	
M L	4 + 9 5 + 7	
S L	2 + 9 1 + 7	
M M	4 + 6 5 + 5	
L L	8 + 9 7 + 8	

STUDY III APRIL '89 6 TO 7 YEAR OLDS

Rating	Subject Number	Spoken Patterns					Written Patterns					Strategies for sums					Strategies for sums					Frequency of strategies				Sex	Age - years/ months
		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	6	7	8	9	10	CA	MIN	RET	DEC		
9	1	✓	✓	✓	x	✓	✓	✓	✓	x	✓	R	R	R	R	R	R	R	M	M	M	0	3	7	0	F	7/5
7	2	✓	✓	x	x	x	✓	x	✓	x	✓	R	CA	R	M	R	M	M	R	M	M	1	5	4	0	F	7/5
8	3	x	x	x	x	x	✓	x	✓	x	✓	R	R	R	R	R	R	M	R	D	R	0	1	8	1	M	6/11
4	4	x	x	x	x	x	x	x	x	x	x	R	R	R	R	R	M	M	M	M	M	0	5	5	0	F	6/8
4	5	x	x	x	x	x	x	✓	x	x	x	R	R	R	R	R	R	CA	M	M	M	1	3	6	0	M	7/2
5	6	x	✓	x	x	x	✓	x	x	x	✓	R	R	R	R	R	R	R	R	R	R	0	0	10	0	M	7/3
5	7	x	x	x	x	x	✓	✓	x	x	✓	R	R	R	R	R	M	M	R	M	M	0	4	6	0	M	7/1
4	8	x	x	x	x	x	✓	✓	x	x	x	R	R	R	R	CA	M	M	CA	CA	CA	4	2	4	0	M	6/11
4	9	x	✓	x	x	x	✓	x	x	x	x	R	R	R	M	CA	M	M	CA	M	CA	3	4	3	0	M	7/5
8	10	x	✓	✓	x	x	✓	x	✓	x	✓	R	R	R	R	R	R	R	R	M	D	0	1	8	1	F	7/2
7	11	x	x	x	x	x	✓	x	✓	x	✓	R	R	R	R	R	M	M	R	M	R	0	3	7	0	M	7/3
8	12	x	x	x	x	x	✓	✓	✓	x	✓	R	R	R	R	R	M	R	M	M	M	0	4	6	0	M	7/6
7	13	x	x	x	x	x	✓	✓	x	x	x	R	R	R	R	R	M	M	R	M	R	0	3	7	0	M	7/5
6	14	x	x	x	x	x	✓	✓	✓	x	✓	R	R	R	R	R	R	R	M	M	D	0	2	7	1	F	7/5
5	15	x	x	x	x	x	✓	✓	x	x	x	R	R	R	M	CA	CA	CA	M	M	M	3	4	3	0	F	7/0
6	16	x	x	x	x	x	✓	✓	✓	x	✓	R	R	R	R	R	M	M	M	M	M	0	5	5	0	M	7/6
6	17	x	x	x	x	x	✓	✓	x	x	✓	R	R	R	R	R	R	R	M	M	M	0	3	7	0	M	6/11
8	18	x	x	x	x	x	✓	✓	✓	✓	✓	R	R	R	R	M	M	M	M	M	M	0	6	4	0	F	6/9
																						12	58	107	3		

ORDER OF QUESTIONS FOR THE AUDITORY
PREFERENCE QUESTIONNAIRE STUDY III WITH ANSWERS
6 TO 7 YEAR OLDS

Questions						Subjects
1	2	3	4	5	6	
M	M	R	M	R	R	1
R	CA	R	R	D	R	7
R	CA	R	R	R	R	13
2	3	4	5	6	1	
D	R	D	R	D	D	2
R	R	CA	R	R	R	8
M	R	R	R	R	R	14
3	4	5	6	1	2	
R	CA	R	D	R	M	3
R	R	R	R	M	CA	9
R	R	D	M	R	CA	15
4	5	6	1	2	3	
CA	R	R	R	R	R	4
R	D	R	D	M	D	10
M	M	M	M	CA	R	16
5	6	1	2	3	4	
M	R	M	M	D	M	5
R	R	R	CA	R	R	11
R	R	D	CA	D	R	17
6	1	2	3	4	5	
R	R	R	D	CA	D	6
CA	D	CA	R	CA	R	12
D	R	M	R	M	R	18

STUDY III MARCH 1989 9 TO 10 YEAR OLDS

Subject Number	Rating	Age	Sex	Spoken Patterns					Written Patterns					Strategies										Frequency of strategy choice			
				1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	6	7	8	9	10	CA	MIN	RET	DEC
1	8	9	F	✓	✓	✓	x	✓	✓	✓	✓	✓	✓	R	R	R	R	R	M	M	R	M	R	0	3	7	0
2	3	9	M	x	x	x	x	x	x	x	x	x	x	CA	CA	R	M	R	M	M	M	M	M	2	6	2	0
3	4	10	M	x	x	x	x	x	✓	✓	✓	✓	✓	R	R	R	R	R	R	M	R	D	R	0	1	8	1
4	9	9	F	x	✓	✓	✓	✓	✓	✓	✓	✓	✓	R	R	R	R	R	R	R	R	D	R	0	0	9	1
5	2	10	M	x	x	x	x	x	✓	x	x	x	✓	CA	CA	R	M	R	M	M	M	M	M	2	6	2	0
6	4	10	M	x	✓	✓	x	x	✓	✓	✓	x	✓	CA	M	CA	M	R	M	M	D	D	D	2	4	1	3
7	8	10	F	✓	✓	✓	x	✓	✓	✓	✓	x	✓	R	R	R	R	R	R	D	R	R	R	0	0	9	1
8	8	10	F	x	x	x	x	x	✓	✓	✓	x	✓	R	R	R	M	R	R	M	R	M	R	0	3	7	0
9	3	9	F	x	x	x	x	x	x	x	x	x	x	R	R	R	R	R	M	M	M	M	M	0	5	5	0
10	3	9	F	x	x	x	x	x	✓	✓	✓	x	✓	R	R	R	M	R	M	M	R	M	M	0	5	5	0
11	3	9	M	x	x	x	x	x	✓	✓	✓	x	✓	R	R	R	R	R	R	R	R	M	R	0	1	9	0
12	6	9	M	✓	✓	✓	x	x	✓	✓	✓	✓	✓	R	R	R	R	R	R	M	R	D	R	0	1	8	1
13	7	10	M	x	✓	x	x	✓	✓	x	✓	x	✓	R	R	R	M	R	M	M	M	D	R	0	4	5	1
14	6	9	M	x	✓	✓	x	x	✓	x	x	x	✓	R	R	R	M	R	CA	CA	R	M	R	2	2	6	0
15	8	10	F	✓	✓	✓	x	✓	✓	✓	✓	✓	✓	R	R	R	R	R	M	R	R	M	R	0	2	8	0
16	6	9	F	✓	✓	x	x	x	✓	x	✓	x	✓	R	R	R	R	R	R	R	R	R	R	0	0	10	0
17	9	10	M	x	✓	x	x	✓	✓	✓	✓	✓	✓	R	R	R	R	R	R	R	R	R	R	0	0	10	0
18	7	10	M	x	✓	x	x	✓	✓	✓	✓	x	✓	R	R	R	R	R	R	R	R	D	R	0	0	9	1
																								8	43	120	9

= 180

ORDER OF QUESTIONS FOR THE AUDITORY
PREFERENCE QUESTIONNAIRE STUDY III WITH ANSWERS
9 TO 10 YEAR OLDS

Questions						Subjects
1	2	3	4	5	6	
R	R	R	D	R	R	1
R	R	R	R	R	R	7
R	R	M	R	R	R	13
2	3	4	5	6	1	
R	M	R	R	CA	R	2
R	R	R	R	D	R	8
D	M	M	R	R	D	14
3	4	5	6	1	2	
R	CA	R	R	R	R	3
R	M	R	D	R	R	9
R	M	R	D	R	R	15
4	5	6	1	2	3	
R	R	R	R	R	R	4
D	R	R	D	R	D	10
R	D	R	D	D	R	16
5	6	1	2	3	4	
R	R	D	R	R	CA	5
R	D	R	R	R	M	11
R	R	R	D	R	R	17
6	1	2	3	4	5	
R	R	D	D	D	R	6
D	D	R	R	D	D	12
R	D	M	D	D	R	18

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1
2	1	1	0	1	1	1	5	1	0	0	0	0	1	2	2	1	0	1	1	2	7
3	0	1	1	0	1	0	3	1	0	0	0	0	0	1	1	1	1	0	1	0	4
4	1	0	0	0	1	1	3	1	0	1	1	0	0	3	2	0	1	1	1	1	6
5	1	0	0	1	1	1	4	1	1	0	0	1	1	4	2	1	0	1	2	2	8
6	1	0	0	0	0	1	2	1	0	0	0	0	0	1	2	0	0	0	0	1	3
7	1	1	0	0	1	1	4	1	0	0	0	0	1	2	2	1	0	0	1	2	6
8	1	0	1	1	1	1	5	0	1	0	1	1	0	3	1	1	1	2	2	1	8
9	1	0	0	1	0	1	3	1	0	0	0	0	1	2	2	0	0	1	0	2	5
10	0	0	0	0	1	1	2	1	0	0	0	0	0	1	1	0	0	0	1	1	3
11	1	0	0	0	0	1	2	1	0	0	0	0	0	1	2	0	0	0	0	1	3
12	0	0	0	0	1	1	2	0	0	0	0	0	1	1	0	0	0	0	1	2	3
13	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1
14	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1
15	1	0	0	0	0	1	2	1	0	0	0	0	1	2	2	0	0	0	0	2	4
16	1	0	0	0	1	0	2	1	0	0	0	0	0	1	2	0	0	0	1	0	3
TOT	12	3	2	4	9	12	42	11	2	1	2	2	6	24	23	5	3	6	11	18	66

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
2	1	1	1	0	1	1	5	1	1	0	1	0	0	3	2	2	1	1	1	1	8
3	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
4	1	0	0	0	1	1	3	1	0	0	0	0	0	1	2	0	0	0	1	1	4
5	1	1	1	1	1	1	6	1	1	1	1	0	1	5	2	2	2	2	1	2	11
6	0	1	0	0	0	0	1	0	1	0	0	0	0	1	0	2	0	0	0	0	2
7	0	1	1	0	0	1	3	0	0	0	0	0	0	0	0	1	1	0	0	1	3
8	1	1	0	1	1	1	5	1	1	1	1	1	1	6	2	2	1	2	2	2	11
9	1	0	0	0	0	1	2	0	0	0	0	0	0	0	1	0	0	0	0	1	2
10	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
11	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
12	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
13	1	0	0	0	0	1	2	1	0	0	0	0	0	1	2	0	0	0	0	1	3
14	1	0	0	0	0	1	2	1	0	0	0	0	0	1	2	0	0	0	0	1	3
15	1	0	0	0	0	1	2	1	0	0	0	0	0	1	2	0	0	0	0	1	3
16	1	0	0	0	0	1	2	1	0	0	0	0	0	1	2	0	0	0	0	1	3
TOT	14	5	3	2	4	10	38	13	4	2	3	1	2	25	27	9	5	5	5	12	63

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	0	0	0	1	1	0	2	0	0	0	1	0	1	2	0	0	0	2	1	1	4
2	1	1	0	0	1	1	4	1	1	1	1	0	0	4	2	2	1	1	1	1	8
3	0	0	0	0	0	0	0	1	1	0	0	0	1	3	1	1	0	0	0	1	3
4	1	0	1	0	1	1	4	1	1	0	1	0	1	4	2	1	1	1	1	2	8
5	1	1	1	1	1	1	6	1	0	1	0	0	1	3	2	1	2	1	1	2	9
6	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
7	1	1	0	0	0	1	3	0	0	1	0	0	1	2	1	1	1	0	0	2	5
8	1	1	0	1	1	1	5	1	1	1	0	1	0	4	2	2	1	1	2	1	9
9	1	0	0	0	1	1	3	1	0	0	1	0	1	3	2	0	0	1	1	2	6
10	1	0	0	0	1	1	3	1	0	0	0	0	0	1	2	0	0	0	1	1	4
11	0	0	1	1	0	1	3	1	0	1	0	0	0	2	1	0	2	1	0	1	5
12	1	0	0	1	0	1	3	0	0	0	1	0	0	1	1	0	0	2	0	1	4
13	1	0	0	0	0	0	1	1	0	0	0	1	0	2	2	0	0	0	1	0	3
14	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1
15	1	0	0	1	0	0	2	1	0	0	1	0	0	2	2	0	0	2	0	0	4
16	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
TOT	12	4	4	6	7	9	42	12	4	5	6	2	6	35	24	8	9	12	9	15	77

VISUAL WRITTEN PATTERN 6/7 YEARS STUDY IV

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	1	1	1	5	1	1	1	1	0	1	5	2	1	2	2	1	2	10
3	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
4	1	0	0	0	0	1	2	1	0	0	0	0	0	1	2	0	0	0	0	1	3
5	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
6	0	1	0	0	0	0	1	0	1	0	0	0	0	1	0	2	0	0	0	0	2
7	0	0	1	0	0	0	1	0	0	1	0	0	0	1	0	0	2	0	0	0	2
8	1	1	1	1	1	1	6	1	1	1	0	1	1	5	2	2	2	1	2	2	11
9	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
10	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
13	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
14	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
15	1	0	0	0	0	1	2	1	0	0	0	0	0	1	2	0	0	0	0	1	3
16	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
TOT	12	3	4	3	3	5	30	12	4	4	2	2	3	27	24	7	8	5	5	8	57

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	1	0	0	1	1	0	3	0	0	0	1	0	1	2	1	0	0	2	1	1	5
2	2	2	0	1	2	2	9	2	1	1	1	0	1	6	4	3	1	2	2	3	15
3	0	1	1	0	1	0	3	2	1	0	0	0	1	4	2	2	1	0	1	1	7
4	2	0	1	0	2	2	7	2	1	1	2	0	1	7	4	1	2	2	2	3	14
5	2	1	1	2	2	2	10	2	1	1	0	1	2	7	4	2	2	2	3	4	17
6	2	0	0	0	0	1	3	2	0	0	0	0	0	2	4	0	0	0	0	1	5
7	2	2	0	0	1	2	7	1	0	1	0	0	2	4	3	2	1	0	1	4	11
8	2	1	1	2	2	2	10	1	2	1	1	2	0	7	3	3	2	3	4	2	17
9	2	0	0	1	1	2	6	2	0	0	1	0	2	5	4	0	0	2	1	4	11
10	1	0	0	0	2	2	5	2	0	0	0	0	0	2	3	0	0	0	2	2	7
11	1	0	1	1	0	2	5	2	0	1	0	0	0	3	3	0	2	1	0	2	8
12	1	0	0	1	1	2	5	0	0	0	1	0	1	2	1	0	0	2	1	3	7
13	2	0	0	0	0	0	2	1	0	0	0	1	0	2	3	0	0	0	1	0	4
14	0	0	1	0	0	1	2	0	0	0	0	0	0	0	0	0	1	0	0	1	2
15	2	0	0	1	0	1	4	2	0	0	1	0	1	4	4	0	0	2	0	2	8
16	2	0	0	0	1	0	3	2	0	0	0	0	0	2	4	0	0	0	1	0	5
TOT	24	7	6	10	16	21	84	23	6	6	8	4	12	59	47	13	12	18	20	33	143

PATTERN (ORAL AND VISUAL) 6/7 YEARS STUDY IV

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
2	2	1	2	1	2	2	10	2	2	1	2	0	1	8	4	3	3	3	2	3	18
3	2	0	0	0	0	0	2	2	0	0	0	0	0	2	4	0	0	0	0	0	4
4	2	0	0	0	1	2	5	2	0	0	0	0	0	2	4	0	0	0	1	2	7
5	2	2	2	2	2	2	12	2	2	2	2	1	2	11	4	4	4	4	3	4	23
6	0	2	0	0	0	0	2	0	2	0	0	0	0	2	0	4	0	0	0	0	4
7	0	1	2	0	0	1	4	0	0	1	0	0	0	1	0	1	3	0	0	1	5
8	2	2	1	2	2	2	11	2	2	2	1	2	2	11	4	4	3	3	4	4	22
9	2	0	0	0	0	1	3	1	0	0	0	0	0	1	3	0	0	0	0	1	4
10	2	0	0	0	0	0	2	2	0	0	0	0	0	2	4	0	0	0	0	0	4
11	1	0	0	0	0	0	1	1	0	0	0	0	0	1	2	0	0	0	0	0	2
12	2	0	0	0	0	0	2	2	0	0	0	0	0	2	4	0	0	0	0	0	4
13	2	0	0	0	0	1	3	2	0	0	0	0	0	2	4	0	0	0	0	1	5
14	2	0	0	0	0	1	3	2	0	0	0	0	0	2	4	0	0	0	0	1	5
15	2	0	0	0	0	2	4	2	0	0	0	0	0	2	4	0	0	0	0	2	6
16	2	0	0	0	0	1	3	2	0	0	0	0	0	2	4	0	0	0	0	1	5
TOT	26	8	7	5	7	15	68	25	8	6	5	3	5	52	51	16	13	10	10	20	120

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	2	0	0	0	0	0	2	1	0	0	0	0	0	1	3	0	0	0	0	0	3
2	2	2	1	1	2	2	10	2	1	0	1	0	1	5	4	3	1	2	2	3	15
3	1	1	1	0	1	0	4	2	0	0	0	0	0	2	3	1	1	0	1	0	6
4	2	0	0	0	2	2	6	2	0	1	1	0	0	4	4	0	1	1	2	2	10
5	2	1	1	2	2	2	10	2	2	1	1	1	2	9	4	3	2	3	3	4	19
6	1	1	0	0	0	1	3	1	1	0	0	0	0	2	2	2	0	0	0	1	5
7	1	2	1	0	1	2	7	1	0	0	0	0	1	2	2	2	1	0	1	3	9
8	2	1	1	2	2	2	10	1	2	1	2	2	1	9	3	3	2	4	4	3	19
9	2	0	0	1	0	2	5	1	0	0	0	0	1	2	3	0	0	1	0	3	7
10	1	0	0	0	1	1	3	2	0	0	0	0	0	2	3	0	0	0	1	1	5
11	2	0	0	0	0	1	3	2	0	0	0	0	0	2	4	0	0	0	0	1	5
12	1	0	0	0	1	1	3	1	0	0	0	0	1	2	2	0	0	0	1	2	5
13	2	0	0	0	0	1	3	1	0	0	0	0	0	1	3	0	0	0	0	1	4
14	1	0	0	0	0	2	3	1	0	0	0	0	0	1	2	0	0	0	0	2	4
15	2	0	0	0	0	2	4	2	0	0	0	0	1	3	4	0	0	0	0	3	7
16	2	0	0	0	1	1	4	2	0	0	0	0	0	2	4	0	0	0	1	1	6
TOT	26	8	5	6	13	22	80	24	6	3	5	3	8	49	50	14	8	11	16	30	129

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	0	0	0	1	1	0	2	0	0	0	1	0	1	2	0	0	0	2	1	1	4
2	2	1	1	1	2	2	9	2	2	2	2	0	1	9	4	3	3	3	2	3	18
3	1	0	0	0	0	0	1	2	1	0	0	0	1	4	3	1	0	0	0	1	5
4	2	0	1	0	1	2	6	2	1	0	1	0	1	5	4	1	1	1	1	3	11
5	2	2	2	2	2	2	12	2	1	2	1	1	2	9	4	3	4	3	3	4	21
6	1	1	0	0	0	0	2	1	1	0	0	0	0	2	2	2	0	0	0	0	4
7	1	1	1	0	0	1	4	0	0	2	0	0	1	3	1	1	3	0	0	2	7
8	2	2	1	2	2	2	11	2	2	2	0	2	1	9	4	4	3	2	4	3	20
9	2	0	0	0	1	1	4	2	0	0	1	0	1	4	4	0	0	1	1	2	8
10	2	0	0	0	1	1	4	2	0	0	0	0	0	2	4	0	0	0	1	1	6
11	0	0	1	1	0	1	3	1	0	1	0	0	0	2	1	0	2	1	0	1	5
12	2	0	0	1	0	1	4	1	0	1	0	0	0	2	3	0	1	1	0	1	6
13	2	0	0	0	0	0	2	2	0	0	0	1	0	3	4	0	0	1	1	0	5
14	1	0	1	0	0	0	2	1	0	0	0	0	0	1	2	0	1	0	0	0	3
15	2	0	0	1	0	1	4	2	0	0	1	0	0	3	4	0	0	2	0	1	7
16	2	0	0	0	0	0	2	2	0	0	0	0	0	2	4	0	0	0	0	0	4
TOT	24	7	8	9	10	14	72	24	8	10	7	4	9	62	48	15	18	16	14	23	134

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	2	0	0	1	1	0	4	1	0	0	1	0	1	3	3	0	0	2	1	1	7
2	4	3	2	2	4	4	19	4	3	2	3	0	2	14	8	6	4	5	4	6	33
3	2	1	1	0	1	0	5	4	1	0	0	0	1	6	6	2	1	0	1	1	11
4	4	0	1	0	3	4	12	4	1	1	2	0	1	9	8	1	2	2	3	5	21
5	4	3	3	4	4	4	22	4	3	3	2	2	4	18	8	6	6	6	6	8	40
6	2	2	0	0	0	1	5	2	2	0	0	0	0	4	4	4	0	0	0	1	9
7	2	3	2	0	1	3	11	1	0	2	0	0	2	5	3	3	4	0	1	5	16
8	4	3	2	4	4	4	21	3	4	3	2	4	2	18	7	7	5	6	8	6	39
9	4	0	0	1	1	3	9	3	0	0	1	0	2	6	7	0	0	2	1	5	15
10	3	0	0	0	2	2	7	4	0	0	0	0	0	4	7	0	0	0	2	2	11
11	2	0	1	1	0	2	6	3	0	1	0	0	0	4	5	0	2	1	0	2	10
12	3	0	0	1	1	2	7	2	0	0	1	0	1	4	5	0	0	2	1	3	11
13	4	0	0	0	0	1	5	3	0	0	0	1	0	4	7	0	0	0	1	1	9
14	2	0	1	0	0	2	5	2	0	0	0	0	0	2	4	0	1	0	0	2	7
15	4	0	0	1	0	3	8	4	0	0	1	0	1	6	8	0	0	2	0	4	14
16	4	0	0	0	1	1	6	4	0	0	0	0	0	4	8	0	0	0	1	1	10
TOT	50	15	13	15	23	36	152	48	14	12	13	7	17	111	98	29	25	28	30	53	263

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	1	0	0	1	1	1	4	1	0	0	1	0	1	3	2	0	0	2	1	2	7
2	1	1	1	1	1	1	6	1	0	1	0	1	1	4	2	1	2	1	2	2	10
3	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
4	1	1	0	1	1	1	5	1	1	1	0	1	1	5	2	2	1	1	2	2	10
5	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
6	1	1	1	1	1	1	6	1	1	1	0	0	1	4	2	2	2	1	1	2	10
7	1	1	1	1	1	1	6	1	1	0	0	0	1	3	2	2	1	1	1	2	9
8	1	1	0	1	1	1	5	1	0	0	0	0	1	2	2	1	0	1	1	2	7
9	0	1	1	1	1	1	5	0	1	0	0	1	1	3	0	2	1	1	2	2	8
10	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
11	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
12	1	0	0	1	1	1	4	0	0	0	0	0	0	0	1	0	0	1	1	1	4
13	1	1	1	0	1	1	5	1	1	0	0	0	0	2	2	2	1	0	1	1	7
14	1	1	1	1	1	1	6	1	1	1	0	0	1	4	2	2	2	1	1	2	10
15	1	1	1	1	1	1	6	1	1	0	0	1	1	4	2	2	1	1	2	2	10
16	1	1	1	1	1	1	6	1	1	0	1	0	1	4	2	2	1	2	1	2	10
TOT	15	14	12	15	16	16	88	14	12	8	6	8	14	62	29	26	20	21	24	30	150

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	1	0	1	0	1	1	4	1	1	0	1	1	0	4	2	1	1	1	2	1	8
2	1	1	1	1	1	1	6	0	1	1	1	0	1	4	1	2	2	2	1	2	10
3	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
4	1	1	1	1	1	1	6	1	0	1	0	1	1	4	2	1	2	1	2	2	10
5	1	1	1	1	1	1	6	0	1	1	1	1	1	5	1	2	2	2	2	2	11
6	1	1	1	1	1	1	6	1	1	1	1	0	1	5	2	2	2	2	1	2	11
7	1	1	0	1	1	1	5	1	1	1	0	1	0	4	2	2	1	1	2	1	9
8	1	1	1	1	1	1	6	1	1	1	1	1	0	5	2	2	2	2	2	1	11
9	1	1	1	1	1	1	6	1	0	1	0	1	1	4	2	1	2	1	2	2	10
10	1	1	1	1	1	1	6	1	0	0	1	1	1	4	2	1	1	2	2	2	10
11	1	1	1	1	1	1	6	1	1	1	0	1	1	5	2	2	2	1	2	2	11
12	1	1	1	1	1	1	6	1	1	0	0	1	0	3	2	2	1	1	2	1	9
13	1	1	1	1	1	1	6	1	1	1	0	0	0	3	2	2	2	1	1	1	9
14	1	1	1	1	1	1	6	1	1	0	1	0	1	4	2	2	1	2	1	2	10
15	1	0	1	1	1	1	5	1	1	1	1	1	1	6	2	1	2	2	2	2	11
16	1	1	1	1	1	1	6	1	1	1	1	0	1	5	2	2	2	2	1	2	11
TOT	16	14	15	15	16	16	92	14	13	12	10	11	11	71	30	27	27	25	27	27	163

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	1	0	1	1	1	1	5	1	1	1	1	1	1	6	2	1	2	2	2	2	11
2	1	1	0	0	1	1	4	1	1	1	0	1	1	5	2	2	1	0	2	2	9
3	1	1	0	1	1	1	5	1	0	1	1	1	1	5	2	1	1	2	2	2	10
4	1	1	1	0	1	1	5	1	1	1	0	1	1	5	2	2	2	0	2	2	10
5	1	1	1	1	1	1	6	1	0	0	1	1	1	4	2	1	1	2	2	2	10
6	1	1	1	1	1	1	6	1	0	1	0	0	1	3	2	1	2	1	1	2	9
7	1	1	1	1	1	1	6	1	1	1	0	1	1	5	2	2	2	1	2	2	11
8	1	1	1	1	1	1	6	1	1	1	1	0	1	5	2	2	2	2	1	2	11
9	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
10	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
11	1	1	1	1	1	1	6	1	1	1	0	1	1	5	2	2	2	1	2	2	11
12	1	1	1	1	1	1	6	1	1	1	0	1	1	5	2	2	2	1	2	2	11
13	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
14	1	1	1	1	1	1	6	1	1	0	1	0	1	4	2	2	1	2	1	2	10
15	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
16	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
TOT	16	15	14	14	16	16	91	16	13	14	10	13	16	82	32	28	28	24	29	32	173

S	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
2	1	1	1	1	1	1	6	0	1	1	1	1	1	5	1	2	2	2	2	2	11
3	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
4	1	1	1	1	1	1	6	1	1	0	1	1	1	5	2	2	1	2	2	2	11
5	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
6	1	1	0	1	1	1	5	1	1	1	0	1	1	5	2	2	1	1	2	2	10
7	1	1	1	0	1	1	5	1	1	1	1	1	1	6	2	2	2	1	2	2	11
8	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
9	1	1	1	1	1	1	6	1	1	1	0	1	1	5	2	2	2	1	2	2	11
10	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
11	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
12	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
13	1	1	1	1	1	1	6	1	1	0	1	1	1	5	2	2	1	2	2	2	11
14	1	1	1	1	1	1	6	1	1	0	0	0	1	3	2	2	1	1	1	2	9
15	1	1	1	1	1	1	6	1	1	1	1	1	0	5	2	2	2	2	2	1	11
16	1	1	1	1	1	1	6	1	1	1	1	1	1	6	2	2	2	2	2	2	12
TOT	16	16	15	15	16	16	94	15	16	13	13	15	15	87	31	32	28	28	31	31	181

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	2	0	1	2	2	2	9	2	1	1	2	1	2	9	4	1	2	4	3	4	18
2	2	2	1	1	2	2	10	2	1	2	0	2	2	9	4	3	3	1	4	4	19
3	2	2	1	2	2	2	11	2	1	2	2	2	2	11	4	3	3	4	4	4	22
4	2	2	1	1	2	2	10	2	2	2	0	2	2	10	4	4	3	1	4	4	20
5	2	2	2	2	2	2	12	2	1	1	2	2	2	10	4	3	3	4	4	4	22
6	2	2	2	2	2	2	12	2	1	2	0	0	2	7	4	3	4	2	2	4	19
7	2	2	2	2	2	2	12	2	2	1	0	1	2	8	4	4	3	2	3	4	20
8	2	2	1	2	2	2	11	2	1	1	1	0	2	7	4	3	2	3	2	4	18
9	1	2	2	2	2	2	11	1	2	1	1	2	2	9	2	4	3	3	4	4	20
10	2	2	2	2	2	2	12	2	2	2	2	2	2	12	4	4	4	4	4	4	24
11	2	2	2	2	2	2	12	2	2	2	1	2	2	11	4	4	4	3	4	4	23
12	2	1	1	2	2	2	10	1	1	1	0	1	1	5	3	2	2	2	3	3	15
13	2	2	2	1	2	2	11	2	2	1	1	1	1	8	4	4	3	2	3	3	19
14	2	2	2	2	2	2	12	2	2	1	1	0	2	8	4	4	3	3	2	4	20
15	2	2	2	2	2	2	12	2	2	1	1	2	2	10	4	4	3	3	4	4	22
16	2	2	2	2	2	2	12	2	2	1	2	1	2	10	4	4	3	4	3	4	22
TOT	31	29	26	29	32	32	179	30	25	22	16	21	30	144	61	54	48	45	53	62	323

PATTERN (ORAL AND VISUAL) 9/10 YEARS STUDY V

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	2	1	2	1	2	2	10	2	2	1	2	2	1	10	4	3	3	3	4	3	20
2	2	2	2	2	2	2	12	0	2	2	2	1	2	9	2	4	4	4	3	4	21
3	2	2	2	2	2	2	12	2	2	2	2	2	2	12	4	4	4	4	4	4	24
4	2	2	2	2	2	2	12	2	1	1	1	2	2	9	4	3	3	3	4	4	21
5	2	2	2	2	2	2	12	1	2	2	2	2	2	11	3	4	4	4	4	4	23
6	2	2	1	2	2	2	11	2	2	2	1	1	2	10	4	4	3	3	3	4	21
7	2	2	1	1	2	2	10	2	2	2	1	2	1	10	4	4	3	2	4	3	20
8	2	2	2	2	2	2	12	2	2	2	2	2	1	11	4	4	4	4	4	3	23
9	2	2	2	2	2	2	12	2	1	2	0	2	2	9	4	3	4	2	4	4	21
10	2	2	2	2	2	2	12	2	1	1	2	2	2	10	4	3	3	4	4	4	22
11	2	2	2	2	2	2	12	2	2	2	1	2	2	11	4	4	4	3	4	4	23
12	2	2	2	2	2	2	12	2	2	1	1	2	1	9	4	4	3	3	4	3	21
13	2	2	2	2	2	2	12	2	2	1	1	1	1	8	4	4	3	3	3	3	20
14	2	2	2	2	2	2	12	2	2	0	1	0	2	7	4	4	2	3	2	4	19
15	2	1	2	2	2	2	11	2	2	2	2	2	1	11	4	3	4	4	4	3	22
16	2	2	2	2	2	2	12	2	2	2	2	1	2	11	4	4	4	4	3	4	23
TOT	32	30	30	30	32	32	186	29	29	25	23	26	26	158	61	59	55	53	58	58	344

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	2	0	1	1	2	2	8	2	1	0	2	1	1	7	4	1	1	3	3	3	15
2	2	2	2	2	2	2	12	1	1	2	1	1	2	8	3	3	4	3	3	4	20
3	2	2	2	2	2	2	12	2	2	2	2	2	2	12	4	4	4	4	4	4	24
4	2	2	1	2	2	2	11	2	1	2	0	2	2	9	4	3	3	2	4	4	20
5	2	2	2	2	2	2	12	1	2	2	2	2	2	11	3	4	4	4	4	4	23
6	2	2	2	2	2	2	12	2	2	2	1	0	2	9	4	4	4	3	2	4	21
7	2	2	1	2	2	2	11	2	2	1	0	1	1	7	4	4	2	2	3	3	18
8	2	2	1	2	2	2	11	2	1	1	1	1	1	7	4	3	2	3	3	3	18
9	1	2	2	2	2	2	11	1	1	1	0	2	2	7	2	3	3	2	4	4	18
10	2	2	2	2	2	2	12	2	1	1	2	2	2	10	4	3	3	4	4	4	22
11	2	2	2	2	2	2	12	2	2	2	1	2	2	11	4	4	4	3	4	4	23
12	2	1	1	2	2	2	10	1	1	0	0	1	0	3	3	2	1	2	3	2	13
13	2	2	2	1	2	2	11	2	2	1	0	0	0	5	4	4	3	1	2	2	16
14	2	2	2	2	2	2	12	2	2	1	1	0	2	8	4	4	3	3	2	4	20
15	2	1	2	2	2	2	11	2	2	1	1	2	2	10	4	3	3	3	4	4	21
16	2	2	2	2	2	2	12	2	2	1	2	0	2	9	4	4	3	4	2	4	21
TOT	31	28	27	30	32	32	180	28	25	20	16	19	25	133	59	53	47	46	51	57	313

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	2	1	2	2	2	2	11	2	2	2	2	2	2	12	4	3	4	4	4	4	23
2	2	2	1	1	2	2	10	1	2	2	1	2	2	10	3	4	3	2	4	4	20
3	2	2	1	2	2	2	11	2	1	2	2	2	2	11	4	3	3	4	4	4	22
4	2	2	2	1	2	2	11	2	2	1	1	2	2	10	4	4	3	2	4	4	21
5	2	2	2	2	2	2	12	2	1	1	2	2	2	10	4	3	3	4	4	4	22
6	2	2	1	2	2	2	11	2	1	2	0	1	2	8	4	3	3	2	3	4	19
7	2	2	2	1	2	2	11	2	2	2	1	2	2	11	4	4	4	2	4	4	22
8	2	2	2	2	2	2	12	2	2	2	2	1	2	11	4	4	4	4	3	4	23
9	2	2	2	2	2	2	12	2	2	2	1	2	2	11	4	4	4	3	4	4	23
10	2	2	2	2	2	2	12	2	2	2	2	2	2	12	4	4	4	4	4	4	24
11	2	2	2	2	2	2	12	2	2	2	1	2	2	11	4	4	4	3	4	4	23
12	2	2	2	2	2	2	12	2	2	2	1	2	2	11	4	4	4	3	4	4	23
13	2	2	2	2	2	2	12	2	2	1	2	2	2	11	4	4	3	4	4	4	23
14	2	2	2	2	2	2	12	2	2	0	1	0	2	7	4	4	2	3	2	4	19
15	2	2	2	2	2	2	12	2	2	2	2	2	1	11	4	4	4	4	4	3	23
16	2	2	2	2	2	2	12	2	2	2	2	2	2	12	4	4	4	4	4	4	24
TOT	32	31	29	29	32	32	185	31	29	27	23	28	31	169	63	60	56	52	60	63	354

COMBINED (VISUAL AND ORAL) (PATTERN AND ERROR) 9/10 YEARS STUDY V

	CONVENTIONAL (C)							UNCONVENTIONAL (U)							COMBINED (C/U)						
S	1	2	3	4	5	10	T	1	2	3	4	5	10	T	1	2	3	4	5	10	T
1	4	1	3	3	4	4	19	4	3	2	4	3	3	19	8	4	5	7	7	7	38
2	4	4	3	3	4	4	22	2	3	4	2	3	4	18	6	7	7	5	7	8	40
3	4	4	3	4	4	4	23	4	3	4	4	4	4	23	8	7	7	8	8	8	46
4	4	4	3	3	4	4	22	4	3	3	1	4	4	19	8	7	6	4	8	8	41
5	4	4	4	4	4	4	24	3	3	3	4	4	4	21	7	7	7	8	8	8	45
6	4	4	3	4	4	4	23	4	3	4	1	1	4	17	8	7	7	5	5	8	40
7	4	4	3	3	4	4	22	4	4	3	1	3	3	18	8	8	6	4	7	7	40
8	4	4	3	4	4	4	23	4	3	3	3	2	3	18	8	7	6	7	6	7	41
9	3	4	4	4	4	4	23	3	3	3	1	4	4	18	6	7	7	5	8	8	41
10	4	4	4	4	4	4	24	4	3	3	4	4	4	22	8	7	7	8	8	8	46
11	4	4	4	4	4	4	24	4	4	4	2	4	4	22	8	8	8	6	8	8	46
12	4	3	3	4	4	4	22	3	3	2	1	3	2	14	7	6	5	5	7	6	36
13	4	4	4	3	4	4	23	4	4	2	2	2	2	16	8	8	6	5	6	6	39
14	4	4	4	4	4	4	24	4	4	1	2	0	4	15	8	8	5	6	4	8	39
15	4	3	4	4	4	4	23	4	4	3	3	4	3	21	8	7	7	7	8	7	44
16	4	4	4	4	4	4	24	4	4	3	4	2	4	21	8	8	7	8	6	8	45
TOT	63	59	56	59	64	64	365	59	54	47	39	47	56	302	122	113	103	98	111	120	667

STUDY VI MARCH 1990 8/9 yrs

Teacher's Rating	Subject Number	Age Y	Age M	Sex	3+2	1+7	8+3	0+4	12+3	5+5	1+3	2+7	3+8	7+1	3+12	2+2	3+1	7+2	4+0	2+3	CA	CO	M	R	D	ob	
7	1	9	2	M	M	R	M	R	M	R	R	M	ob	ob	M	R	R	R	R	ob	0	0	5	8	0	3	
5	2	9	3	M	R	R	M	R	M	R	R	M	D	R	M	R	R	D	ob	R	0	0	4	9	2	1	
8	3	8	8	F	M	M	M	R	M	D	R	M	M	M	M	R	R	M	R	R	0	0	9	6	1	0	
6	4	9	2	F	M	M	M	R	M	R	R	M	M	R	M	R	R	M	R	R	0	0	8	8	0	0	
2	5	9	5	F	M	CO	M	M	M	R	CO	CO	CO	M	CO	M	M	M	M	CO	0	6	9	1	0	0	1 error
7	6	9	1	M	M	M	M	M	M	R	CO	M	M	M	M	R	M	M	M	CO	0	2	12	2	0	0	
6	7	8	6	F	M	M	M	M	M	R	R	M	ob	ob	M	M	CA	M	M	M	1	0	11	2	0	2	
6	8	9	3	F	M	M	M	M	M	R	M	M	ob	M	ob	R	M	ob	M	M	0	0	11	2	0	3	
3	9	9	5	M	M	R	M	R	M	R	R	M	M	R	M	R	R	M	M	R	0	0	8	8	0	0	
3	10	9	0	F	M	M	M	M	M	R	R	M	M	M	M	R	R	R	R	R	0	0	9	7	0	0	
7	11	9	4	F	R	M	M	M	M	R	R	M	M	R	M	R	R	M	R	CO	0	1	8	7	0	0	
7	12	9	1	M	R	R	M	M	M	R	R	M	M	M	M	R	R	R	M	R	0	0	8	8	0	0	
6	13	9	2	M	R	R	D	M	R	R	R	R	M	R	M	R	R	M	R	R	0	0	4	11	1	0	
5	14	8	10	M	M	M	M	M	M	R	R	M	M	M	M	R	R	M	R	M	0	0	11	5	0	0	
5	15	9	2	M	R	M	M	R	M	R	R	R	M	M	M	R	R	M	R	R	0	0	7	9	0	0	
8	16	9	2	F	R	R	M	R	M	R	R	M	ob	R	ob	R	R	R	R	R	0	0	3	11	0	2	
7	17	8	6	M	M	R	M	R	M	R	R	M	M	R	M	R	R	R	R	M	0	0	7	9	0	0	
8	18	9	3	F	R	R	M	R	M	R	R	R	R	R	M	R	R	R	R	R	0	0	3	13	0	0	
4	19	9	4	M	M	M	M	R	M	R	M	M	M	M	M	R	M	M	R	M	0	0	12	4	0	0	1 error
9	20	8	9	F	R	R	M	M	M	R	R	M	ob	ob	M	R	R	M	M	R	0	0	7	7	0	2	

STUDY VI
8/9 YEARS

\# chosen but incorrect
o chosen and correct, with reason for not using it
dk don't know

SUBJECT	SUM	STRATEGY USED	REASON	ALTERNATIVE STRATEGIES WITH REASON FOR USING THEM							
				CA		MIN		RET		DEC	
1	1 + 7	R	I know it	o	I can	o	I can				
	0 + 4	R	I know it	o	If I was young I would					#	
	3 + 2	M	Easier	o	I could			o	I didn't		
	8 + 3	M	Easier	o	I can			o	I didn't		
2	3 + 2	R	I knew it	o	I didn't want to	o	I didn't want to			#	
	1 + 7	R	I knew it	o	I didn't want to	o	I didn't want to			#	
	8 + 3	M	I can	o	Its babyish					#	
	12 + 3	M	It's easier	o	Its babyish			#		#	
3	0 + 4	R	I knew it	o	There was nothing	o	There was nothing			#	
	2 + 2	R	Its easy	o	I could have	o	I could have			o	2 + 2 = 5 - 1
	3 + 2	M	Quicker	o	dk			o	dk	o	3 + 3 = 6 - 1
	1 + 7	M	Just one more	o	dk			o	dk	#	
4	0 + 4	R	Quicker	o	dk	o	No number			#	
	5 + 5	R	Quicker	o	dk	o	dk			#	
	3 + 2	M	It was the first way I thought of	o	I can			o	I could	#	
	1 + 7	M	dk	o	I can			o	I could	#	
5	5 + 5	R	dk	o	Five little dots then 5 little dots	o	dk			#	
	3 + 2	M	dk	o	dk			#		#	
	8 + 3	M	dk	o	dk			#		#	

chosen but incorrect
o chosen and correct, with reason for not using it
dk don't know

STUDY VI
8/9 YEARS (cont)

SUBJECT	SUM	STRATEGY USED	REASON	ALTERNATIVE STRATEGIES WITH REASON FOR USING THEM							
				CA		MIN		RET		DEC	
6	5 + 5	R	I knew it	o	dk	o	dk			#	
	2 + 2	R	I knew it	o	dk	o	The other is easier			#	
	3 + 2	M	Shrugged	o	dk			o	dk	o	4 + 1 is the same
	1 + 7	M	Shrugged	o	dk			o	dk	#	
7	5 + 5	R	I knew it	o	Began to cry	o	Began to cry			#	
	1 + 3	R	Began to cry	o	Began to cry	o	Began to cry			#	
	3 + 2	M	dk	o	Began to cry			o	I know I could	#	
	1 + 7	M	dk	o	Began to cry			o	I know I could	#	
	3 + 1	CA	dk			o	dk	o	dk	o	5 - 1
8	5 + 5	R	I knew it	o	Because I knew it	o	Because I knew it			#	
	2 + 2	R	I knew it	o	Because I knew it	o	Because I knew it				
	3 + 2	M	Easier	o	dk			o	I didn't want to	o	2 + 2 +1
	1 + 7	M	Easier	#				o	I didn't want to		
9	1 + 7	R	We've done it a lot	o	I can	o	dk			#	
	5 + 5	R	I knew it	o	I knew it	#					
	3 + 2	M	Easier	o	I can			o	No response	#	
	8 + 3	M	Easier	o	I can			o	I will	#	
10	5 + 5	R	dk	#		o	dk				
	1 + 3	R	dk	#		o	dk			#	
	3 + 2	M	dk	#	Began to cry					#	
	1 + 7	M	dk	#	Began to cry					#	

chosen but incorrect
o chosen and correct, with reason for not using it
dk don't know

STUDY VI
8/9 YEARS (cont)

SUBJECT	SUM	STRATEGY USED	REASON	ALTERNATIVE STRATEGIES WITH REASON FOR USING THEM							
				CA		MIN		RET		DEC	
11	3 + 2	R	dk	o	dk	o	dk				
	5 + 5	R	dk	o	dk	o	dk				
	1 + 7	M	dk	o	dk			o	I did!		
	8 + 3	M	dk	o	dk					#	
12	3 + 2	R	Easier	o	dk	o	I can if I like				
	1 + 7	R	Easier	o	dk	o	No response				
	8 + 3	M	I like doing it that way	o	dk			o	dk	#	
	0 + 4	M	dk	o	I can't			o	I knew it		
13	3 + 2	R	I knew it	o	dk	o	dk			#	
	1 + 7	R	I knew it	o	dk	o	dk			#	
	0 + 4	M	No other number	o	dk			o	There's only a 4		
	3 + 8	M	Best way	o	Could do					o	8 + 2 + 1
14	5 + 5	R	I knew it	o	I didn't want to	o	I used to				
	1 + 3	R	I knew it	o	I didn't want to	o	I knew it		the other way		
	3 + 2	M		o	I didn't want to			o	I wanted to do it		
	1 + 7	M		o	I didn't want to			o	I like counting		
15	3 + 2	R	dk	o	dk	o	It is another way				
	5 + 5	R	dk	o	dk	o	I could have				
	1 + 7	M	dk	o	dk			o	I didn't	#	
	8 + 3	M	dk	o	dk			#		o	8 + 2 + 1

\# chosen but incorrect
o chosen and correct, with reason for not using it
dk don't know

STUDY VI
8/9 YEARS (cont)

SUBJECT	SUM	STRATEGY USED	REASON	ALTERNATIVE STRATEGIES WITH REASON FOR USING THEM							
				CA		MIN		RET		DEC	
16	3 + 2	R	Quickest way	o	Takes too long	o	I knew it				
	1 + 7	R	Quickest way	o	Takes too long	o	I knew it				
	8 + 3	M	I didn't know them	o	Takes too long					o	8 + 2 + 1
	12 + 3	M	I didn't know them	o	Takes too long					o	10 + 5
17	1 + 7	R	dk	o	dk	o	dk			#	
	5 + 5	R	dk	o	dk	o	dk			#	
	3 + 2	M	dk	o	dk			o	Counting is easier		
	8 + 3	M	dk	o	dk			o	Counting is easier		
18	3 + 2	R	I knew it	o	Easier to know it	o	Easier to know it				
	1 + 7	R	I knew it	o	Easier to know it	o	Easier to know it				
	8 + 3	M	Making sure	o	Easier			o	Making sure by	o	8 + 2 + 1
	12 + 3	M	I needed to count	o	Easier				counting	#	
19	5 + 5	R	dk	o	dk	o	dk			#	
	2 + 2	R	dk	o	dk	o	dk			#	
	3 + 2	M	dk	o	dk			#			
	1 + 7	M	dk	o	dk			o	dk		
20	3 + 2	R	Other ways take longer	o	You don't have to	o	dk				
	1 + 7	R	Other ways take longer	o	You don't have to	o	dk				
	8 + 3	M	Other ways take longer	o	Too long					#	times do it
	12 + 3	M	Other ways take longer	o	Too long					o	10 + 2 + 3 - I some

STUDY VI

Name Age Number

3 + 2 =

1 + 7 =

8 + 3 =

0 + 4 =

12 + 3 =

5 + 5 =

1 + 3 =

2 + 7 =

3 + 8 =

7 + 1 =

3 + 12 =

2 + 2 =

3 + 1 =

7 + 2 =

4 + 0 =

2 + 3 =

STUDY VII MARCH 1990 6/7 yrs

Teacher's Rating	Subject Number	Age Y	Age M	Sex	5+5	1+7	2+3	1+9	4+6	1+3	4+0	2+2	3+8	9+1	3+2	0+4	8+3	6+4	3+1	7+1	CA	CO	M	R	D	ob	G	
3	1	6	6	M	R	M	CA	M	CA	CA	M	R	M	M	CA	M	M	M	M	M	4	0	10	2	0	0	0	4 errors
4	2	7	4	M	R	M	CO	CO	CA	M	M	CA	CA	M	ob	ob	M	CA	M	M	4	2	7	1	0	2	0	
7	3	7	2	F	R	M	D	M	M	CA	M	CA	M	ob	M	M	ob	M	ob	M	2	0	9	1	1	3	0	
6	4	6	9	F	R	R	M	M	M	CA	M	CA	M	ob	CA	ob	M	M	CA	R	4	0	7	3	0	2	0	
5	5	6	8	M	R	R	M	R	M	R	M	R	M	M	M	R	M	M	M	M	0	0	10	6	0	0	0	
5	6	6	10	M	R	R	CA	R	CA	CA	M	R	M	R	M	ob	M	M	R	R	3	0	5	7	0	1	0	
4	7	6	6	M	G	M	G	M	G	CO	M	R	G	ob	G	R	G	G	R	M	0	1	4	3	0	1	7	8 errors
2	8	7	4	M	G	M	M	M	G	G	G	G	G	G	M	G	G	G	G	G	0	0	4	0	0	0	12	12 errors
7	9	6	11	F	R	R	CO	CO	CA	R	R	R	CA	R	ob	ob	ob	M	R	R	2	2	1	8	0	3	0	
5	10	6	11	F	R	CA	CA	CA	CA	CA	M	CA	CA	M	M	M	M	M	CA	CA	9	0	6	1	0	0	0	
4	11	7	0	F	R	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	15	0	0	1	0	0	0	4 errors
8	12	7	4	M	R	R	M	R	M	R	R	R	M	R	R	R	M	M	R	R	0	0	5	11	0	0	0	
8	13	7	0	F	R	CA	CO	R	CO	R	R	R	M	R	R	R	M	M	R	R	1	2	3	10	0	0	0	
8	14	7	3	F	R	R	R	R	CO	R	R	R	R	M	R	R	R	R	R	R	0	1	1	14	0	0	0	
7	15	6	10	M	R	R	M	M	M	M	R	CA	CO	M	M	R	M	M	M	M	1	1	10	4	0	0	0	1 error
4	16	6	9	M	R	M	CA	M	CA	M	M	R	CA	R	CA	R	CA	CA	M	M	6	0	6	4	0	0	0	
2	17	6	6	M	R	R	CO	M	G	M	M	R	M	M	CA	CO	M	G	M	M	1	2	8	3	0	0	2	4 errors
5	18	6	9	F	M	CA	M	CA	CO	CO	M	CA	CA	M	M	G	G	M	M	M	4	2	8	0	0	0	2	2 errors
6	19	6	8	M	R	R	CO	R	CO	M	R	R	CA	ob	ob	R	ob	CA	M	M	2	2	3	6	0	3	0	1 error
6	20	6	8	F	R	CO	CO	CO	CO	CO	R	R	CO	M	M	R	M	M	M	M	0	6	6	4	0	0	0	

STUDY VII
6/7 YEARS

\# chosen but incorrect
o chosen and correct, with reason for not using it
dk don't know

SUBJECT	SUM	STRATEGY USED	REASON	ALTERNATIVE STRATEGIES WITH REASON FOR USING THEM							
				CA		MIN		RET		DEC	
1	5 + 5	R	dk	o	dk						
	2 + 2	R	I've forgotten	o	dk, didn't want to						
	1 + 7	M	dk								
	1 + 9	M	dk								
	2 + 3	CA	dk								
	4 + 6	CA	dk								
2	5 + 5	R	Quicker	o	dk	o	dk			#	Can't remember
	1 + 7	M	Quicker	o	dk			o	dk		
	1 + 3	M	Quicker	o	dk			o	dk		
	4 + 6	CA	I don't know another way			o	dk				
	2 + 2	CA	dk			o	dk	o	It could be quicker	#	dk
3	5 + 5	R	I knew it	o	dk	#					
	4 + 6	M	dk	o	dk			#		o	6 + 6 − 2
	3 + 8	M	dk	o	dk					#	8 + 8 − ?
	1 + 3	CA	dk			#		o	dk		
	2 + 2	CA	dk					o	dk		
4	5 + 5	R	My mum told me	o	I knew it the other way	o	I know it			#	
	1 + 7	R	dk	o	I did it the other way	o	I knew it			#	
	2 + 3	M	dk					#		#	
	1 + 9	M	dk					#			
	1 + 3	CA	dk			#		o	I knew it!	#	
	2 + 2	CA	dk			#		o	I knew it!	#	

chosen but incorrect
o chosen and correct, with reason for not using it
dk don't know

STUDY VII
6/7 YEARS (cont)

SUBJECT	SUM	STRATEGY USED	REASON	ALTERNATIVE STRATEGIES WITH REASON FOR USING THEM							
				CA		MIN		RET		DEC	
5	5 + 5	R	dk	#		#				#	
	1 + 7	R	dk	#		o	dk			#	
	2 + 3	M	dk	o	dk			#			
	4 + 6	M	dk	#				o	I didn't know		
6	5 + 5	R	I felt like it	o	Could have						
	1 + 7	R	I felt like it	o	dk					#	
	3 + 8	M	dk	o	dk						
	3 + 2	M	dk	o	dk						
	2 + 3	CA	I felt like it			o	I could have				
	4 + 6	CA	I felt like it			o	dk				
7	2 + 2	R	dk	#							
	0 + 4	R	dk	#							
	1 + 7	M	dk					o	dk	#	
	1 + 3	CO	dk					o	dk	#	
8	1 + 7	M	dk					#			
	1 + 9	M	dk					#			

chosen but incorrect
o chosen and correct, with reason for not using it
dk don't know

STUDY VII
6/7 YEARS (cont)

SUBJECT	SUM	STRATEGY USED	REASON	ALTERNATIVE STRATEGIES WITH REASON FOR USING THEM							
				CA		MIN		RET		DEC	
9	5 + 5	R	dk			o	dk			#	
	1 + 7	R	dk	o	dk	o	dk				
	6 + 4	M	dk	o	I knew another way			o	Counted to check		
	4 + 6	CA	I wanted to			o	I knew another way			#	
	3 + 8	CA	I wanted to			o	I knew another way			#	
10	5 + 5	R	dk	o	dk					#	
	9 + 1	M	dk	o	dk						
	3 + 2	M	dk	o	dk						
	1 + 7	CA	dk			#					
	2 + 3	CA	dk			#					
11	5 + 5	R	dk	o	dk					#	
	1 + 7	CA	dk					#		#	
	1 + 9	CA	dk					#		#	
12	5 + 5	R	I knew it	o	dk	o	I knew it already				
	1 + 7	R	I knew it	o	dk	o	I knew it already				
	2 + 3	M	Quicker	o	Not quick			o	dk		
	4 + 6	M	Quicker	o	Not quick			#			

chosen but incorrect
o chosen and correct, with reason for not using it
dk don't know

STUDY VII
6/7 YEARS (cont)

SUBJECT	SUM	STRATEGY USED	REASON	ALTERNATIVE STRATEGIES WITH REASON FOR USING THEM							
				CA		MIN		RET		DEC	
13	5 + 5	R	dk	o	dk	o	dk				
	1 + 9	R	dk	o	dk	o	dk				
	3 + 8	M	dk	o	dk			#		#	
	8 + 3	M	dk	o	dk			#		#	
	1 + 7	CA	dk			o	dk	o	dk		
14	5 + 5	R	dk	o	Knew it already	o	dk				
	1 + 7	R	dk	o	Knew it	o	dk				
	4 + 6	CO	dk	o	Knew it			#		#	
	3 + 8	M	dk	o	Knew it			#		#	
15	5 + 5	R	dk	#		o	dk				
	1 + 7	R	dk	#		o	dk				
	2 + 3	M	dk	o	dk			o	dk	#	
	1 + 9	M	dk	o	dk			o	dk	#	
	2 + 2	CA	dk			o	dk	o	dk	#	
16	5 + 5	R	dk	#							
	2 + 2	R	dk	#							
	1 + 7	M	dk	#							
	1 + 9	M	dk	#							
	2 + 3	CA	dk								
	4 + 6	CA	dk								

chosen but incorrect
o chosen and correct, with reason for not using it
dk don't know

STUDY VII
6/7 YEARS (cont)

SUBJECT	SUM	STRATEGY USED	REASON	ALTERNATIVE STRATEGIES WITH REASON FOR USING THEM							
				CA		MIN		RET		DEC	
17	5 + 5	R	dk			#					
	1 + 7	R	dk			#					
	1 + 9	M	dk	#				#			
	1 + 3	M	dk	o	Too hard			#			
	3 + 2	CA	dk			o	dk				
18	5 + 5	M	dk					o	I didn't know at the start		
	2 + 3	M	dk								
	1 + 7	CA	dk								
	1 + 9	CA	dk								
19	5 + 5	R	dk	#							
	1 + 7	R	dk	#							
	1 + 3	M	dk	#				o	dk		
	7 + 1	M	dk	#				o	dk		
	3 + 8	CA	dk			#					
	6 + 4	CA	dk			#				#	
20	5 + 5	R	dk	o	dk	o	dk			#	
	4 + 0	R	dk	o	dk	o	dk			#	
	9 + 1	M	dk	o	dk						
	3 + 2	M	Mine was a better way	o	dk						

STUDY VII

Name Age Number

$5 + 5 =$

$1 + 7 =$

$2 + 3 =$

$1 + 9 =$

$4 + 6 =$

$1 + 3 =$

$4 + 0 =$

$2 + 2 =$

$3 + 8 =$

$9 + 1 =$

$3 + 2 =$

$0 + 4 =$

$8 + 3 =$

$6 + 4 =$

$3 + 1 =$

$7 + 1 =$

STUDY IX 8/9 YR OLDS
JULY 1990 COMMUTATIVITY TESTS

SUBJECT	CIS					CIL					SYMS					SYML					ABS				
	•a	•b	a	b	c	•a	•b	a	b	c	•a	•b	a	b	c	•a	•b	a	b	c	•a	•b	a	b	c
1	y	y	y	y	y	y	y	y	y	y	y	y	n	y	y	y	y	y	y	y	y	y	y	y	y
2	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
3	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
4	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
5	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
6	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
7	n	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
8	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
9	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
10	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
11	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
12	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
13	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
14	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
15	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
16	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
17	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	y	y	y
18	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
19	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
20	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
21	y	y	y	y	y	y	y	y	y	y	y	y	y	n	y	y	y	y	y	y	n	n	n	n	n
22	n	n	y	y	y	y	n	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
23	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	y	y	n
24	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y

KEY
. = commuted
y = correct
n = incorrect

KEY:-

$\overline{M}$ = incorrect calculation

8/9 STUDY IX JULY 1990
SUMS

Subject Number	Teacher's Rating	Age Y	Age M	Sex	CIS					CIL					SYMS					SYML					CA	CO	M	R	D	CORRECT
1	7	9	5	M	M	M	M	M	M	M	M	M	M	M	M	R	R	M	M	M	R	M	M	M	0	0	17	3	0	20
2	5	9	6	M	D	CO	M	M	M	M	M	$\overline{M}$	M	M	M	M	M	M	M	M	CO	M	M	M	0	2	17	0	1	19
3	8	8	11	F	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	20
4	6	9	5	F	M	R	M	M	M	M	$\overline{M}$	M	M	M	R	M	M	M	M	M	M	M	M	M	0	0	18	2	0	19
5	2	9	8	F	$\overline{M}$	M	M	M	M	$\overline{M}$	M	M	M	M	M	M	M	M	M	M	M	$\overline{M}$	M	M	0	0	20	0	0	17
6	7	9	4	M	M	M	M	M	M	M	$\overline{M}$	M	M	M	CO	CO	CO	CO	CO	CO	CO	CO	M	CO	0	9	11	0	0	19
7	6	8	9	F	M	$\overline{M}$	M	M	M	M	$\overline{M}$	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	18
8	6	9	6	F	M	M	M	M	M	M	M	M	M	M	R	M	M	M	M	M	M	M	M	R	0	0	18	2	0	20
9	3	9	8	M	R	$\overline{R}$	R	$\overline{R}$	CO	R	R	M	M	M	CO	CO	R	R	R	M	M	M	M	M	0	3	8	9	0	18
10	3	9	3	F	M	M	M	M	M	M	M	M	M	M	CO	M	M	M	M	M	M	M	M	M	0	1	19	0	0	20
11	7	9	7	F	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	20
12	7	9	4	M	CO	M	M	M	M	$\overline{M}$	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	1	19	0	0	19
13	6	9	5	M	M	M	R	M	CO	M	M	M	M	M	R	M	M	M	M	M	M	M	M	M	0	1	17	2	0	20
14	5	9	1	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	20
15	5	9	5	M	M	M	M	M	$\overline{M}$	M	M	M	M	M	CO	CO	M	M	$\overline{M}$	M	M	M	M	M	0	2	18	0	0	18
16	8	9	5	F	M	M	R	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	19	1	0	20
17	7	8	9	M	M	M	M	M	M	M	M	M	M	M	R	M	M	M	M	M	M	M	M	M	0	0	19	1	0	20
18	7	9	6	F	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	20
19	4	9	7	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	20
20	9	9	0	F	M	M	M	M	M	M	M	M	M	M	M	M	CO	M	M	M	M	M	M	M	0	1	19	0	0	20
21	2	9	3	F	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	20
22	2	8	11	F	CO	$\overline{CO}$	$\overline{CO}$	CO	CO	CA	$\overline{CA}$	CA	CA	CA	M	M	M	M	M	M	M	M	M	M	5	5	10	0	0	17
23	3	9	1	M	D	D	M	CA	$\overline{M}$	M	M	M	M	M	$\overline{M}$	M	$\overline{M}$	M	M	M	M	M	M	M	1	0	17	0	2	17
24	6	8	11	M	M	M	M	M	M	M	M	M	M	M	CO	M	M	M	M	M	M	M	M	M	0	1	19	0	0	20
	M	9/3																												

STUDY IX 6/7 YR OLDS
COMMUTATIVITY TESTS

SUBJECT	CIS					CIL					SYMS					SYML					ABS				
	ȧ	ḃ	a	b	c	ȧ	ḃ	a	b	c	ȧ	ḃ	a	b	c	ȧ	ḃ	a	b	c	ȧ	ḃ	a	b	c
1	y	y	n	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
2	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
3	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
4	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
5	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	y	n
6	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
7	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
8	y	y	y	y	n	y	n	y	y	y	n	n	y	n	y	n	n	y	n	y	n	n	y	y	n
9	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
10	y	y	n	y	y	y	n	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	y	y	y
11	y	y	n	n	y	y	y	y	y	y	y	y	n	y	y	y	y	n	y	y	y	y	y	y	y
12	n	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	n	n	y
13	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
14	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	y
15	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	y
16	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
17	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
18	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
19	y	n	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
20	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
21	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	y	n	y	y
22	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
23	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
24	y	y	n	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y

KEY
. = commuted
y = correct
n = incorrect

KEY:-

$\overline{M}$ = incorrect calculation

6/7 STUDY IX
SUMS

Subject Number	Teacher's Rating	Age Y	Age M	Sex	CIS					CIL					SYMS					SYML					CA	CO	M	R	D	CORRECT	
																														DK	
1	3	6	9	M	M	M	M	M	M	M	M	M	M	M	$\overline{CO}$	CO	CO	M	M	M	M	M	M	M	0	3	17	0	0	0	19
2	4	7	7	M	M	M	M	$\overline{M}$	M	M	M	$\overline{M}$	$\overline{M}$	M	M	M	M	M	M	M	M	M	$\overline{M}$	M	0	0	20	0	0	0	16
3	7	7	5	F	M	M	M	M	M	M	M	$\overline{M}$	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	0	19
4	6	7	0	F	CA	CA	CA	CA	CA	–	–	–	–	–	CA	CA	CA	CA	CA	–	–	–	–	–	10	0	0	0	0	10	10
5	5	6	11	M	M	M	M	M	M	M	M	M	M	$\overline{M}$	CA	CA	M	M	M	M	M	M	M	M	2	0	18	0	0	0	19
6	5	7	1	M	R	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	19	1	0	0	20
7	4	6	9	M	$\overline{CA}$	$\overline{CA}$	CO	$\overline{CA}$	CO	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	–	$\overline{CA}$	CA	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	–	–	–	–	13	2	0	0	0	5	3
8	2	7	7	M	–	–	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	CA	$\overline{CA}$	$\overline{CA}$	CA	$\overline{CA}$	$\overline{CA}$	CA	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	18	0	0	0	0	2	2
9	7	7	2	F	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	0	20
10	5	7	2	F	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	CA	CA	$\overline{CA}$	CA	–	CA	$\overline{CA}$	–	CA	$\overline{CA}$	CA	CA	CA	CA	CA	CA	$\overline{CA}$	18	0	0	0	0	2	11
11	4	7	3	F	$\overline{CA}$	$\overline{CA}$	CA	$\overline{CA}$	CA	–	–	–	–	–	CA	CA	CA	CA	CA	–	–	–	–	–	10	0	0	0	0	10	7
12	8	7	7	M	$\overline{M}$	M	M	M	M	M	M	M	M	M	M	M	CO	M	M	M	M	M	M	M	0	1	19	0	0	0	19
13	8	7	3	F	M	$\overline{M}$	M	M	M	M	M	M	$\overline{M}$	M	M	M	M	M	$\overline{M}$	M	M	M	M	M	0	0	20	0	0	0	17
14	8	7	7	F	R	M	R	M	M	M	M	M	M	M	CO	CO	CO	R	CO	CO	CO	CO	CO	CO	0	9	8	3	0	0	20
15	7	7	1	M	M	M	M	M	M	M	$\overline{M}$	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	0	19
16	4	7	0	M	$\overline{M}$	M	CO	$\overline{CO}$	M	M	M	$\overline{M}$	M	$\overline{M}$	M	M	M	CO	M	M	M	M	M	M	0	3	17	0	0	0	16
17	2	6	9	M	M	M	M	M	M	M	$\overline{M}$	M	M	M	M	M	M	$\overline{M}$	M	M	M	M	M	M	0	0	20	0	0	0	18
18	5	7	0	F	$\overline{CA}$	CA	CA	$\overline{CA}$	$\overline{CA}$	–	–	–	–	–	CA	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	–	–	–	–	–	10	0	0	0	0	10	3
19	6	6	11	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	0	20	0	0	0	20
20	6	6	11	F	M	CO	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	0	1	19	0	0	0	20
21	6	6	10	F	M	M	M	$\overline{M}$	M	M	M	$\overline{M}$	M	$\overline{M}$	M	CO	M	M	M	M	M	$\overline{M}$	$\overline{M}$	M	0	1	19	0	0	0	15
22	8	6	10	F	CA	CA	CA	CA	CA	M	M	M	M	M	CA	M	CO	M	R	M	M	M	M	$\overline{M}$	6	1	12	1	0	0	19
23	4	6	11	F	CO	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	–	–	–	–	–	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	–	–	–	–	–	10	0	0	0	0	10	1
24	3	6	0	F	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	–	–	–	–	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	$\overline{CA}$	–	–	–	–	–	11	0	0	0	0	9	0

STUDY X 5 YR OLDS
SEPT 1990 COMMUTATIVITY

SUBJECT	CIS					CIL					ABS					CORRECT
	•a	•b	a	b	c	•a	•b	a	b	c	•a	•b	a	b	c	
1	y	y	y	y	y	n	n	y	y	n	y	y	y	y	y	12
2	y	y	n	y	y	y	y	n	n	y	n	n	y	y	n	9
3	y	y	n	n	y	y	y	n	y	y	n	n	n	n	n	7
4	y	y	y	y	y	y	y	y	y	y	n	n	y	y	n	12
5	y	y	y	y	y	y	y	y	n	y	n	y	y	y	n	12
6	n	y	n	y	y	y	y	y	y	y	y	y	y	y	y	13
7	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	15
8	y	y	y	y	y	y	y	y	y	y	n	n	y	y	n	12
9	n	y	y	y	n	n	n	y	n	n	n	n	y	y	y	7
10	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	15
11	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	15
12	n	y	y	n	y	y	y	y	y	y	y	y	y	y	y	13
13	n	n	y	y	y	y	y	y	y	y	n	n	y	y	n	10
14	y	n	n	n	y	y	y	y	y	y	n	n	y	y	n	9
15	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	15
16	y	y	n	y	y	y	y	n	y	y	y	y	y	y	y	13
17	y	y	y	y	y	y	y	y	y	y	n	n	y	y	y	13
18	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	15
19	y	y	n	y	y	y	y	y	y	y	y	y	y	y	y	14
20	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	15
21	n	n	y	n	y	n	n	y	n	n	n	n	y	y	n	5
22	y	y	n	n	y	y	y	y	y	y	y	y	y	y	y	13
23	y	y	y	y	y	y	y	y	y	y	y	y	n	n	y	13
24	y	y	y	n	y	y	y	y	y	y	y	y	y	y	y	14

KEY
• = commuted
y = correct
n = incorrect

KEY:-

$\overline{M}$ = incorrect calculation

STUDY X SEPT 1990
5 YEAR OLDS - SUMS

Subject Number	Teachers Rating	Age Y	Age M	Sex	CVS					CVL					CIS					CIL					CA	CO	M	R	D	G	DK	CORRECT
1	6	5	6	M	CA	CA	CA	M	CA	M	$\overline{M}$	M	M	$\overline{CA}$	M	M	$\overline{M}$	M	M	M	M	M	M	M	5	0	15	0	0	0	0	17
2	6	5	1	M	G	G	G	G	G	-	-	-	-	-	G	G	G	G	G	-	-	-	-	-	0	0	0	0	0	10	10	0
3	5	5	2	M	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	0	0	0	20	0
4	7	5	4	M	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	20	0	0	0	0	0	0	15
5	6	5	2	M	CA	CA	CA	CA	CA	CA	CA	M	CA	M	M	M	M	M	M	M	M	M	M	$\overline{M}$	8	0	12	0	0	0	0	12
6	5	5	2	M	G	G	G	CA	CA	G	G		-	-	G	G	CA	G	G	CA	CA	-	-	-	5	0	0	0	0	9	6	2
7	6	5	6	M	CA	CA	CA	CA	CA	CA	CA	CA	D	D	CA	CA	CA	R	R	D	M	M	M	M	11	0	4	2	3	0	0	20
8	6	5	6	M	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	20	0	0	0	0	0	0	15
9	5	5	4	M	G	G	G	G	G	-	-	-	-	-	-	-	G	G	G	-	-	-	-	-	0	0	0	0	0	8	12	0
10	5	5	7	M	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	20	0	0	0	0	0	0	17
11	6	5	4	M	CA	CA	CA	CA	CA	G	-	-	-	-	-	G	CA	CA	CA	G	G	G	G	-	8	0	0	0	0	6	6	4
12	7	5	4	M	R	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	19	0	0	1	0	0	0	15
13	6	5	3	M	CA	CA	CA	CA	CA	-	-	-	-	-	CA	CA	CA	CA	CA	-	-	-	-	-	10	0	0	0	0	0	10	4
14	6	5	0	M	CA	CA	CA	CA	CA	-	-	-	-	-	CA	CA	CA	CA	CA	-	-	-	-	-	10	0	0	0	0	0	10	2
15	6	5	7	F	M	CO	M	M	M	M	$\overline{M}$	M	M	M	M	M	M	M	M	M	M	M	M	M	0	1	19	0	0	0	0	19
16	5	5	7	F	G	G	G	G	G	-	-	-	-	-	G	G	G	G	-	-	-	-	-	-	0	0	0	0	0	9	11	0
17	4	5	7	F	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	20	0	0	0	0	0	0	15
18	4	5	5	F	G	CA	G	G	G	-	-	-	-	-	G	G	G	G	G	-	-	-	-	-	1	0	0	0	0	9	10	0
19	6	5	8	F	CA	CA	CA	CA	CA	-	-	-	-	-	CA	CA	CA	CA	CA	-	-	-	-	-	10	0	0	0	0	0	10	3
20	6	5	7	F	G	G	G	G	G	-	-	-	-	-	CA	G	G	G	G	-	-	-	-	-	1	0	0	0	0	9	10	1
21	6	5	3	F	G	G	G	G	G	-	-	-	-	-	G	G	G	G	G	-	-	-	-	-	0	0	0	0	0	10	10	0
22	5	5	0	F	-	-	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	18	0	0	0	0	0	2	11
23	6	5	5	F	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA	20	0	0	0	0	0	0	13
24	5	5	3	F	CA	CA	CA	CA	CA	-	-	-	-	-	CA	CA	CA	CA	CA	-	-	-	-	-	10	0	0	0	0	0	10	0

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