The mathematical development of children with Apert syndrome

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I, Caroline Hilton confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

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ABSTRACT

Apert syndrome is a rare condition (birth prevalence of 1 in 65000) with associated risks of other physical disabilities. Children with the condition experience major surgery involving the fingers. It has been suggested that these children have greater difficulty with mathematics than with other curriculum subjects.

This study explored the mathematical learning of 10 primary school age children with Apert syndrome over two years. The children in the study had varied sensory disabilities, which included hearing and visual impairments, as well as limited finger mobility. The children were visited five or six times at school, in order to detect change over time.

The children were observed when they were learning mathematics in school. To explore the children's understanding and thinking in mathematics, clinical interviews using items from number skills tests were conducted. Standardized measures of working memory and mathematical achievement were administered. Interviews were carried out with the children's parents and school staff supporting their education.

A central finding of this study is that children with Apert syndrome are heterogeneous. The only factor that the children in the study shared was their initial lack of finger use when engaging with work involving number and arithmetic. In line with contemporary neuroscience, this study suggests that finger knowledge and awareness, or finger gnosis, and finger mobility are important in early number development. Exercises in developing of finger gnosis may enable flexibility in strategy use and development for solving problems in arithmetic. However, children with Apert syndrome will continue to be confronted with many other challenges that impact their learning of mathematics.

Contents

List of tables and figures

1 INTRODUCTION

This study focuses on the mathematical development of children with Apert syndrome.

As the parent of a child with Apert syndrome, I realised quite early on in my daughter's schooling that her development of early number skills seemed to be delayed relative to the rest of her development. On discussing this with other families and with staff at the specialist hospital that she attended on a very regular basis, it became apparent that this was quite common among children with Apert syndrome. Most of the literature on Apert syndrome focuses on medical aspects and not on educational issues. As I had over 20 years' experience as a mathematics teacher and was also a parent of a child with Apert syndrome, I believed that I was well placed to take on an exploration of the mathematical development of a group of children with Apert syndrome. This proved to be a very rewarding experience and it allowed me to get to know a number of very special children and their families, from areas right across the United Kingdom. The work itself revealed insights into how children learn early number skills and how this can be supported. The story that follows began from a very personal place, but the findings will hopefully be used to provide insight, hope and support to children with and without Apert syndrome.

Apert syndrome is a rare syndrome which was first described by Wheaton in 1894, and investigated further by Apert in 1906 (Patton, Goodship, Hayward and Lansdown, 1988). One of the largest studies, carried out between 1983 and 1993 in the USA estimated a birth prevalence of Apert syndrome of approximately 1 in 65000, in North America and Europe (Cohen et al., 1992; Tolorova, Harris, Ordway and Vargervik, 1997). Advances in medical treatment have resulted in better outcomes for children with Apert syndrome and as a result the number of children with Apert syndrome in mainstream schools is likely to increase. In Apert syndrome the cranial (and facial) sutures begin to fuse early (craniosynostosis) during foetal development. This early fusion alters the normal pattern of skull growth and therefore the shape of the skull and face, which can raise pressure within the skull (intracranial pressure) and have consequences for development of the brain. The rate and order in which different skull sutures fuse determines how the skull shape is affected.

Children with Apert syndrome are also born with their fingers and toes fused (syndactyly) and they often have a cleft palate.

As a result of this premature fusion, children with Apert syndrome often spend a significant part of their early years in hospital having surgery on their heads to make room for the brain to grow and separating the fingers. As the fusion of the fingers is quite complicated, children with Apert syndrome do not always have four fingers and a thumb and even when their fingers have been separated, the fingers do not bend, except at the knuckle. As well as these physical characteristics, children with Apert syndrome often have hearing and visual impairments. Later on in their childhood, or as young adults, people with Apert syndrome may have mid-face surgery, usually for cosmetic reasons and to give them a more “normal” face. Children with Apert syndrome are diverse and complex and, therefore, a very heterogeneous population.

1.1 A summary of the cognitive, physical and psychosocial development of children with Apert syndrome

Apert syndrome is caused by a mutation of the fibroblast growth factor receptor 2 (FGFR2) gene (Wilkie et al., 1995). There are two mutations, called S252W and P253R which account for 98% of cases of Apert syndrome (Stark et al., 2014). The two variants of Apert syndrome can be distinguished by the severity of the syndactyly of the fingers and children with the S252W variant are more likely to have less severe syndactyly than patients with the P253R variant (Fearon and Podner, 2013; Wilkie et al., 1995).

Much of the literature on Apert syndrome focuses on the management of the syndrome from a clinical perspective. While there is much in the medical literature about the range of surgical interventions and optimal times for carrying out this surgery, much less is known about the cognitive and social development of children with Apert syndrome.

As in the general population, the cognitive development of children born with Apert syndrome varies. Trying to locate a cause for this variation reveals the complex nature of the profiles of children with Apert syndrome, which may include visual and hearing impairment as well as difficulties with manipulating objects. Consequently, the early research, much of which suggested that the majority of children with Apert syndrome have learning difficulties, has been questioned (Shipster, Hearst, Dockrell, Kilby and Hayward, 2002).

1.2 The cognitive, physical and psychosocial development of children with Apert syndrome: what is typical?

An overview of some of the most significant factors which impact on the development of children with Apert syndrome will be described. This is important because, although children with Apert syndrome show substantial variation in cognitive development and a wide range of IQ scores, it has not been possible to identify a cause for this (Lefebvre, Travis, Arndt and Munro, 1986; Patton et al., 1988; Renier et al., 1996; Yacubian-Fernandez et al., 2005). Issues discussed in previous research, which are thought to have the potential to impact on the cognitive development of children with Apert syndrome, have included:

- Visual impairments
- Hearing impairments
- Poor fine motor skills due to fusion of the fingers
- Speech and language development
- White matter anomalies
- Psychosocial aspects

Some of these issues will be discussed further in order to better understand the challenges faced by children and young people with Apert syndrome.

1.2.1 Visual impairments in children with Apert syndrome

Children with Apert syndrome often experience a range of problems with their eyes and, consequently, with their vision. Children with Apert syndrome always have hypertelorism (i.e. their eyes are always far apart). As a consequence, they are more likely have a squint and other related problems with their vision (Kreiborg and Cohen, 2010). Children with Apert syndrome are also likely to have an astigmatism and/or be long-sighted or short-sighted (Khong et al., 2006). As a result of their complex vision, even when children with Apert syndrome wear glasses to correct for their visual difficulties, many of them do not have visual acuity within the normal range. If the visual acuity obtained with glasses is not within normal limits, there will be other problems with the eye which means that it is not possible to provide "normal" vision. A corrected visual acuity of 6/12 means that, with glasses, the affected person sees at 6 metres what a person with normal vision sees at 12 metres. This could make things like copying from a board and playing football or tennis quite challenging. In addition to these difficulties, children with Apert syndrome often have amblyopia (having eyes that do not work together). In Apert syndrome this usually caused by strabismus (squint) which, when combined with other unusual structures, can cause a lack of stereoscopic vision and, consequently, an inability to judge distances effectively (Read, 2014).

Children with Apert syndrome, then, are likely to be affected by some form of visual impairment. This could affect their ability to judge distances and speed (which could impact on their ability to join in with games such as football and netball). A visual impairment could also impact on a child's ability to scan and track information presented visually, which in turn, for example, could affect their ability to: learn to read and write; keep track of a number of objects they were asked to count or use a number line. A visual impairment might also mean that even if a child can read, they might need to have texts available in larger fonts with a high quality of production, rather than poor quality, enlarged photocopies.

1.2.2 Hearing impairments in children with Apert syndrome

Acquired hearing loss is common in children with Apert syndrome (Rajenderkumar et al., 2005a). This is attributed to poor functioning of the eustachian tubes which often results in glue ear, or the consequences of glue ear (e.g. perforated ear drums) into adulthood. The hearing loss experienced by children with Apert syndrome tends to affect the lower frequencies (Rajenderkumar, Bamiou and Sirimanna, 2005b) so that children are able to hear high pitch noises better than low pitch noises. As human speech is often within the range of lower pitch sounds, this type of hearing loss may affect children's ability to engage in conversation and interact appropriately.

In addition to the acquired hearing loss, children with Apert syndrome often have vestibular impairment which may cause poor balance and delay in reaching certain physical milestones such as standing and walking independently (Mills, Perera, Veness and Raglan, 2014).

As a consequence of the high incidence of persistent hearing loss, children with Apert syndrome are at risk of developmental delays in speech, language and communication (Deafness Research UK, 2009). This can result in social and behavioural problems and gaps in general knowledge, in addition to delays in learning to read and the development of reasoning skills.

Children with hearing impairments are likely to have delayed phonological development (Shipster, et al., 2002). This might not only have an impact on their speech and language development, but could also have an impact on their phonological working memory and make the acquisition of reading and any other learning which requires the processing of aural information particularly challenging.

Children who experience hearing loss are at risk of missing key bits of information during activities in school which could make it very difficult when new learning is built on existing learning. If the child does not hear a key piece of information, then any subsequent work will not make sense. In addition, a hearing impairment might make the learning of new vocabulary difficult because the new words will be missed, or will not be heard correctly. For a young child, for example, this could cause a lot of confusion when learning number words, where it might be hard to distinguish between words such as “thirteen” and “thirty”. Finally, if a child has a hearing impairment, the very act of listening will be much more tiring than for a child without a hearing impairment. The consequence of this could be that a child with a hearing impairment simply “switches off”.

1.2.3 Hand anomalies and outcomes for children with Apert syndrome

Children with Apert syndrome are always born with some fusion of the fingers. These can be categorised into three types:

- Type 1 is syndactyly of the three middle fingers (digits 2 to 4)
- Type 2 is syndactyly of all the fingers (digits 2 to 5)
- Type 3 is syndactyly of all five digits.

(Cohen and Kreiborg, 1995)

In other words, the index, middle and “ring” fingers are always fused in children with Apert syndrome, with boney fusion of the tips of the fingers. The “little” finger is usually, but not always, fused. The last joint on the little finger usually bends. The thumb is usually short and wide and it bends outwards. This makes it very difficult for children to have a “normal” pinch grip, even after surgery.

Surgery is usually performed in stages before a child reaches 5 years of age. Depending on the severity and complexity of the fusion, children usually have either three or four fingers and a thumb following surgery. In an investigation of an adult with Apert syndrome, prior to surgical separation, the fingers were represented in the brain, as one single digit (Mogilner et al., 1993). Within 1 week after surgery, the hand area in the brain had increased and the fingers had more distinct cortical representation locations. This was followed up for 6 weeks after surgery and the changes remained stable. However, “the resulting hand area was smaller than normal and the organization was nonsomatotopic” (Mogilner et al., 1993, p. 3597). This suggests that, in addition to the cosmetic benefits and greater dexterity, separation of digits results in a changed cortical representation of the hand in the brain. Nevertheless the resulting representation of the fingers in the brain will not be the same as that in individuals with a “normal” hand which may be partly explained by the fact that the new fingers will not have “normal” pads (Mogilner et al.).

1.2.4 Speech, language and cognitive development in children with Apert syndrome

Speech development in children with Apert syndrome may be affected by a cleft palate. A cleft palate in a child without Apert syndrome will cause delayed acquisition of speech sounds (Chapman, Hardin-Jones and Halter, 2003). A child with Apert syndrome and a cleft palate will have to deal with the challenges of a cleft palate, in addition to the challenges that result from the other unusual oral structures that are due to the syndrome.

Shipster et al. (2002) reported on a study of 10 children. The aims of the study were:

- To establish the prevalence , range and nature of cognitive, language, speech, attention, resonance/voice and oro-motor difficulties within an age delineated group of children with Apert syndrome
- To explore the relationship between their language and cognitive skills
- To consider cognitive, speech and language skills in relation to associated difficulties

(Shipster et al., 2002, p.328)

The study revealed that eight out of the ten children had language difficulties; six of these children performing better on tests for receptive language than for expressive language. All the children in the study had hearing difficulties and also articulation problems with their speech, due to their unusual oral structures. As a consequence of these factors, it is not surprising that the children exhibited problems with speech and language development.

Difficulties with attention control were identified in nine of the 10 children studied. This finding that is not unusual in children with speech and language difficulties (Shipster et al., 2002). The authors also reported that all the children had phonological delays which, if not identified and addressed, could impact the children's literacy skills in school.

As already mentioned, early studies of the cognitive development of children with Apert syndrome reported a high incidence of lower IQ scores than in the general population (e.g. Patton et al., 1988; Lajeunie et al., 1999, Maliepaard et al., 2014). In a more recent study of 18 people with Apert syndrome (Yacubian-Fernandes et al., 2005) aged between 1 year and 26 years, with an average of 8 years, assessments on developmental or cognitive functions were correlated with the age of the patient, age at which cranial surgery was performed, brain development, and quality of family environment. In this study, 77.8% of the children had cognitive assessment scores within the normal range. The strongest correlation was between cognitive assessment scores and home environment (based on number of family members, level of education, type of accommodation, job and salary). Another study, by Renier et al., (1996), found that the children who had grown up with their families were three times more likely to have a normal IQ when compared with children who had been institutionalized at an early age. Taking these two studies together, it seems likely that the environment in which children with Apert syndrome grow up is a significant factor in their later development. However, it could also be argued that the children who were institutionalized at an early age, had more complex needs from the start.

1.2.5 White matter anomalies in children with Apert syndrome

The brain is divided into grey matter and white matter. Grey matter is mainly on the surface of the brain, while white matter is in the centre of the brain. Grey matter is responsible for processing information and for functions such as muscle control and sensory perception. White matter provides the means by which communication occurs between different areas of grey matter and between grey matter and the rest of the body. White matter also controls functions that we are unaware of, such as, blood pressure and temperature. There have been conflicting findings in relation to the role of white matter anomalies in determining developmental outcomes for children with Apert syndrome.

A number of studies (e.g. Cohen and Kreiborg, 1990; Quintero-Rivera et al., 2006) have concluded that children with Apert syndrome are at greater risk for hypoplasia of white matter and malformations of the corpus callosum, septum pellucidum and the limbic system. It has been suggested that these malformations may put some children with Apert syndrome at greater risk of having problems with memory, cognitive function and behaviour. It has also been suggested that children with anomalies of the septum pellucidum are more likely to have an IQ <70 (Renier et al., 1996).

These studies, however, need to be contrasted with the work of Fearon and Podner (2013), who did not find any correlation between cognitive development and either anomalies of the septum pellucidum or anomalies of the corpus callosum. Moreover, although there may be changes in white matter and other aspects of brain development in children with Apert syndrome (Raybaud and Di Rocco, 2007; Stark et al., 2015), it is hard to identify a causal link between these changes and neuropsychological outcomes (Florisson et al., 2011). This is an area that may still benefit from further study both within the literature on Apert syndrome and in the literature on the role of white matter more generally.

1.2.6 Psychosocial development and social inclusion for children with Apert syndrome

Sarimski (2001) identified the fact that peer relationships for children with Apert syndrome were often unsuccessful and that children often experience rejection and hostility. This may cause children to become withdrawn and anxious in social situations, in order to avoid behaviours such as name calling, staring and pitying. These children are, therefore, more likely to opt out of activities with peers in school and in social situations more generally.

In a study focusing more specifically on behavioural and emotional functioning of children with craniofacial syndromes (including six children with Apert syndrome), Maliepaard et al. (2014) found that children with syndromic craniosynostosis were more likely to have difficulty with social interactions. This could mean that children with Apert syndrome are more at risk for social isolation, poor self-esteem and depression.

Parents of children with Apert syndrome have reported concerns about their children's psychosocial inclusion at school and in the wider community (Shipster et al., 2002). They also reported that their children were far less communicative with people outside the home, especially in group situations and at school. This could result in delayed development of social skills, as well as speech, language and communication skills. This could then be followed by lower expectations of children in school which could impact on children's self-image, goals and expectations.

In the school environment, the issue of social inclusion is important. Children with a visible difference (disfigurement), and unusual speech sounds, are more likely to experience bullying at school than their peers (Turner, Thomas, Dowell and Rumsey, 1997). It is not uncommon for people with a visible difference to have difficulties in social situations, have low self-esteem, be fearful of the negative reactions of others and have negative patterns of behaviour, such as avoiding social interactions (Rumsey and Harcourt, 2004). In the school environment, these issues may create additional stressors for children with Apert syndrome and may, therefore, impact on the development of their social skills, self-image and confidence.

Children with Apert syndrome are clearly likely to benefit from school environments that are well-informed, supportive and accepting of them, in order to support them to build their self-confidence and a positive self-image in relationships outside the family.

1.3 Arithmetic skills

It has only been possible to locate one study, by Sarimski (1997), which provides any specific data on the arithmetic skills of children with Apert syndrome. In this study of 11 children, aged between 2 years and 12 years of age, information on arithmetic skills was available for nine of the children. Of those nine children, seven scored lower in tests testing arithmetic and/or short term memory than they did in tests investigating verbal and perceptual skills.

It seems possible that there are elements of these assessments which could produce confusing results. For example, given the nature of the difficulties children with Apert syndrome experience with fine motor skills due to the children's complex syndactyly at birth, it seems likely that an assessment which involves complicated hand movements may produce unreliable data.

1.4 Personal experiences

A review of this kind would not be complete without some consideration of the reflections and experiences of those experiencing Apert syndrome first hand.

1.4.1 Living with Apert syndrome

In 2012, Christine Clinton, a young woman with Apert syndrome, published a book entitled 'My Life Story with Apert Syndrome in My Own Words'. In her reflections, Clinton discusses her childhood experiences which included: endless teasing and bullying at school; problems with friendships; low expectations from teachers and a general lack of 'acceptance and tolerance'. Clinton highlights the fact that her positive experiences in education seemed to depend on individual teachers and their attitudes towards her - the positive experiences were the exception rather than the norm. This discussion takes place alongside descriptions of frequent hospital visits and surgical procedures. In addition, as Clinton reflects on different stages of her education and the struggles she experienced, references are made to the fact that she always found mathematics much more difficult than other subjects.

1.4.2 Parents' experiences of raising children with Apert syndrome

In a study of mothers' experiences of bringing up a child with a craniofacial anomaly, one of the main concerns of mothers of children with Apert syndrome was to protect their child from the impact of negative attitudes and behaviours (Klein, Pope, Getahun and Thompson, 2006). One mother talking about a trip to a ball game reported that: ‘Usually it’s really fun, and he has a great time...But... you could get someone really mean two rows away from you and some people are really...cruel. I’m not talking little kids who don’t know what they’re saying... I mean adults.’ (Klein et al., 2006, p.593). As a consequence of this lack of consistency in people’s behaviours and reactions, the usual sense of anonymity that other people experience when they are going about their normal lives, is removed, because the child with Apert syndrome stands out from the crowd and is visibly different. This could make every social experience unpredictable and children with Apert syndrome may receive very negative reactions from people just for being who they are. This has been very well explained by a mother who, when talking about the daily challenges her family faces, said: ‘[You never know] how others will react. Going to a place that kids love – an amusement park – that should just be a happy time. But when you go it could be a happy time or a disaster, you never know. Things that people take for granted could always go in opposite ways’ (Klein et al., 2006, p.596).

Mothers of children with Apert syndrome have highlighted the difficulties their children can face when they want to join in with sports activities with their peers (Klein et al., 2006). Because of their physical disabilities, some sports can be difficult. A mother of a boy with Apert syndrome reported that: ‘They’re just faster than him, quicker. So it’s hard...They wanna do their thing and he can’t keep up. So he usually stays with a lot of younger kids’ (Klein et al., 2006, p.592).

Fearon and Podner (2013) highlighted the fact that many parents in their study reported that their children with Apert syndrome had particular difficulty with mathematics when compared to their reading and verbal skills. Parents also reported that many of their children had been diagnosed with attention deficit hyperactivity disorder.

1.5 Final comments

As can be seen, the development of children with Apert syndrome is the result of a complex interaction of their personalities, their physical abilities and disabilities and the world around them. Children with Apert syndrome face many challenges which may impact, not only on the development of their cognitive and academic skills, but also on their social, emotional and behavioural development. Existing studies on children with Apert syndrome are inconsistent, possibly due to the use of unreliable assessment data and the variability and complex needs of children with Apert syndrome. However, what stand out as common to nearly all of these children are the facts they are born with their fingers fused and that there are many accounts, both professional and personal, which suggest that children with Apert syndrome have a particular difficulty with mathematics at school. Is it possible that these are connected?

2 LITERATURE REVIEW

2.1 Learning to work with numbers

The need to count is not instinctive, but is a human creation. The need to compare quantities, however, is a matter of survival (Dehaene, 2011). The discussion which follows will try to explore how children learn to understand how numbers work and how to work with them.

2.1.1 The language of number

We often believe that children from a very young age learn about counting and the language of counting from watching and copying the adults around them, but how does our cultural use of number affect our development of counting and concepts of numerosity? What happens if the adults in the community do not typically count?

In some remote communities in Australia and South America, the local languages often only contain number words for one, two and three and less frequently for four and five (Gelman and Butterworth, 2005; Zhou and Bowern, 2015). The Piraha from Brazil, only have words for “few”, “many” and “much”, but not for exact numerosities; not even for “one” (Frank, Everett, Fedorenko and Gibson, 2008). Frank et al. argue that this provides evidence for the development of the language of number as a form of “cognitive technology” that allows us to manipulate and keep track of our calculations and utterances. Interestingly, the Piraha do not name individual fingers, but do refer to them collectively as “hand sticks” (Everett, 2005, p.624). Could this link to the “sticks” we use in tallies?

A study of the Australian family of languages, Pama-Nyungan, exploring the development of these languages over time, found that number words change over time and with particular patterns (Zhou and Bowern, 2015). Where the languages have number words for one and two, the words for three and four are often made from the words for the smaller numbers (e.g. three may be “two and one” and four may be “two and two”). None of the languages had words for four and not for one, two and three. This suggests that there is some progression in the development of number words. Most of the languages that had number words for five, also had number words up to 10 or 20. The authors argue that five may be a tipping point and that if communities need words for five, they probably need words for bigger numbers as well – hence the rapid development beyond five.

2.1.2 Subitising

The ability to subitise refers to the skill of being able to enumerate small groups of observable objects without counting (Butterworth, 1999). By the age of three years, children can usually subitise for up to three objects (Fuson, 1988). For adults, the maximum number is usually four (Hughes, 1986). There is some discussion, though, as to whether subitising or counting comes first and how this numerical information is processed (Sarama and Clements, 2009).

This form of subitising is sometimes called ‘perceptual’ subtitising, as it is the ability to recognise “a number without consciously using other mental or mathematical processes and then naming it” (Sarama and Clements, 2009, p.44). This needs to be distinguished from an associated form of subitising known as ‘conceptual’ subitising. Conceptual subitising is the ability to recognise a number of objects without counting because of the way the objects are organised, such as in dominoes, dice and fingers (Steffe and Cobb, 1988). Conceptual subitising is linked to perceptual subitising in the ways that the patterns (on dice and dominoes, for example) are presented. For instance, six on a dice is presented as two rows of three and three is a number that can be easily perceptually subitised. In other words, conceptual subitising uses our ability to perceptually subitise in order to present numbers greater than three using arrangements based on our unconscious ability to perceptually subitise groups of objects containing up to about four items.

Clements and Sarama (2009) argue that perceptual subitising is based on a language-independent capacity to judge numersosities and that it is therefore is a precursor to counting.

2.1.3 Learning to count

Learning to count is no trivial task (Baroody, Bajwa and Eiland, 2009; Fuson, 1988; Gelman and Gallistel, 1978; Steffe, von Glasersfeld, Richards and Cobb, 1983) and all the principles of arithmetic that children learn at school are based on a deep understanding of counting. As not many of us have any recollection of learning to count, it is worth exploring models of how this skill is acquired and what it actually means for a child to be considered a competent "counter".

The most commonly used model for identifying the key elements of successful counting is that proposed by Gelman and Gallistel (1978). Gelman and Gallistel proposed five principles that include three "how to count" principles and two "what to count" principles.

The "how to count" principles are:

1. The one-one principle (each object is counted once and once only)
2. The stable-order principle (the number words are said in a stable order, even if a child cannot remember all the words – e.g. one, two, three, five, six)
3. The cardinal principle (the last word in the count indicates the numerosity of the set)

The "what to count" principles are:

4. The abstraction principle (any group of objects can be counted)
5. The order-irrelevance principle (objects can be counted in any order and the answer will always be the same)

Although Gelman and Gallistel's counting principles provide the most commonly cited model, it is worth considering an alternative model proposed by Steffe, von Glaserfeld, Richards and Cobb (1983). This model is based on three key abilities:

1. The ability to be able to say the number word sequence
2. The ability to be able to identify the countables (the objects that will be counted)
3. The ability to be able to match the counting words to the countables

These two models differ in a number of ways. Firstly, they differ in the ways in which they are encapsulated: one is concerned with “principles” while the other is concerned with “abilities”. For Gelman and Gallistel, it is important to observe children applying the principles, while for Steffe et al. it is important to observe children correctly demonstrating their abilities. However, an ability to apply a principle or perform a task does not necessarily imply understanding. For Gelman and Gallistel (1978), once children have grasped the one-one and stable order principles, they are ready to take on board the cardinality principle. The stable order principle is very generous in its definition, in that it allows for children to get the number sequence wrong, as long as they do this consistently (e.g. saying “1,2,3,5,6”, if this is done consistently). The key here is that it is about understanding principles, rather than being judged on observed actions.

In Steffe at al.’s (1983) model, the ability to match words and objects is very similar to Gelman and Gallistel’s one-one principle. The idea of cardinality is implicit in Steffe et al.’s model, while in Gelman and Gallistel’s (1978) model it needs to be understood as a separate principle, once the first two principles have been grasped. While the idea of the abstraction principle comes after the “how to count” principles for Gelman and Gallistel, for Steffe et al., it is implicit in the ability to identify the countables. Similarly, Gelman and Gallistel’s order-irrelevance principle, seems to be implicit in Steffe et al.’s ability to match the counting words to the countables. In other words, where Gelman and Gallistel have tried to distinguish between the “how to count” and the “what to count” principles, Steffe et al. have subsumed these in what are, in effect, observable abilities.

Where Gelman and Gallistel (1978) have provided the abstraction and order-irrelevance principles, Steffe et al. (1983) attempted to defined five counting types:

- Perceptual counters – these counters need to have physical objects to count and calculate, whether they are the original objects in questions or representations, such as fingers
- Figural counters – these counters can count without the need for the actual physical objects. For, example if three counters are covered by a cloth, a child would be able to say how many counters there were altogether if two more were added

- Counters of motor unit items – this describes a method used to perform a calculation where physical actions, such as purposefully moving fingers, become the things that are counted, rather than the perceptual or figural items themselves
- Counters of verbal unit items – these counters no longer need any form of physical action, but can use the number words as countable objects in their own right
- Counters of abstract unit items – these counters can perform calculations without relating their actions to any actual objects or actions

According to Steffe et al. (1983) children move between these different types of counting and can indeed fall into more than one category. One of the issues which Steffe et al. acknowledged with this categorization, is the fact that it is very difficult to observe the different stages with confidence. Thus, they argued later that "at best an observer can make educated guesses, taking into account – as does any experienced diagnostician – several indications collected over an extended period of observation" (Steffe and Cobb, 1988, p.19). Moreover, the process of performing calculations was seen as something that grew out of counting. In order to calculate children need to become familiar with the number words, both forwards and backwards and need to be able to count on from any point in the number sequence (Steffe et al., 1983). This is a very significant observation, but it is a shame that the actual numerical relationships in the number sequence were not explored in relation to how these calculation skills are to be understood.

Counting then, is clearly not trivial and yet the skill of counting underpins all subsequent arithmetic skills. Assessing children's counting skills is no easy task, especially as children's counting strategies vary and change over time. In trying to resolve this problem, Fuson (1988) attempted to identify a developmental path for children from 3½ to 6 years of age. She observed that:

> Both action parts of counting immovable object – pointing and saying number words – undergo progressive internalization with age. Pointing may move from touching to pointing from a distance to using eye fixation. Saying number words moves from saying audible words to making readable lip movements to making

> abbreviated and unreadable lip movements to silent mental production of words.
>
> (Fuson, 1988, p. 85)

Fuson (1988) provides a very neat hierarchy of skill development, but it seems unlikely that the developmental path is so simple and so uniform for all children. Moreover counting is used in many different situations and for different purposes.

It takes time for children to learn that counting is used to enumerate a set of objects and that unequal counts provide information about relative numerosities (Fuson and Hall, 1983). It also take time for children to become confident that counting is more reliable than estimating for providing exact numerosities of groups of objects (Cowan, 1987). However it may also be the case that it takes time for children to make the link between cardinality and counting. Wynn (1990) noticed that some children were able to count sets of objects that were placed in front of them, but when the same children were asked to "give me five cars", for example, would usually just take a handful, rather than count out the number requested.

Learning to count, then, is complex and consequently, reliably assessing whether a child can count and understands the purpose of counting are challenges that have not yet been overcome.

2.1.4 Learning to say and use number words

Our base-10 number system is one which has come about as a result of thousands of years of human development. It is a very sophisticated system, so it is not a surprise that it takes time for children to understand how it works. One of the challenges facing children, is to understand the links between our spoken and written number systems (transcoding) (Nunes and Bryant, 1996). In the English language, this are particularly evident with the teen numbers which do not seem to follow the same rules as for numbers between 21 and 99. For example, when we say the numbers "19" and "91", they both start with "nine" and so it easy to get confused. Children have to learn about these inconsistencies in our language of numbers; they have to learn that the teen numbers do not follow the rules of the tens numbers. Children learn to use numbers verbally before they start to read and write numbers. When children are in the early stages of learning to read and write numbers, problems are often observed due to transcoding errors caused by the confusion between the spoken and the written number words (Nunes and Bryant, 1996).

2.1.5 Addition and subtraction

Children begin to engage with addition and subtraction, involving small numbers of objects, long before they start formal education. Hughes (1986) noticed that when 3 - 5 year old children were presented with a closed box containing a few cubes (for example, two or three), if a change was made (by either adding or taking away one or two cubes), the children could work out how many cubes were left in the box. Hughes also explored what happened when children had to imagine the boxes and he found that the children continued to be successful if the situations were ones that they had already experienced. In these cases, children tended to use their fingers in order to model the problems they were asked to solve. This form of representing one-to-one correspondence is also visible in written number systems, such as our tally system and in many early number systems, such as those from Egypt and Babylon.

Hughes (1986) observed children working with real objects and with their fingers, but the development of more abstract concepts of addition and subtraction is less easy to observe. Firstly, children usually begin to understand how “one more and one less” are related to our number system at around 6 years of age (Gifford, 2014). Being able to do this with confidence requires confidence with the number words and the ability to count forwards and backwards. It also requires knowledge that in our number system, the numbers increase one at a time (e.g. four is three and one more and three is one less than four). This understanding underpins all the later arithmetic that children engage with at school.

Carpenter and Moser (1984), in a longitudinal study of 88 children from Year 2 to Year 4, observed the changes in strategy use when children added and subtracted numbers. For addition, they identified five different strategies:

- Count all (e.g. for 3+4, counting out 3 objects, then counting out 4 objects and finally counting all the objects to get a total of 7)
- Count on from first number (e.g. for 3+4, counting on from 3 to get a total of 7)
- Count on from the larger number (e.g. for 3+4, counting on from 4 to get a total of 7)
- Use known number facts (retrieving a known fact from memory)
- Use derived facts (use a known fact to work out a solution – e.g. using the fact that 3+3=6 to work out that 3+4 must be equivalent to 6+1)

For children to move on from the count all strategy, they need to be able to count on from any number in the number sequence. For children to be able to count on from the larger number, they have to able to quickly identify the larger of two numbers. Therefore, these skills are all underpinned by knowledge and understanding of the counting system. These also underpin successful use of known and, more especially, derived facts. In order to use derived facts, children again need to understand the counting system and the patterns and relationships that exist (for example, if 3+3=6, then 3+4=7 because 4 is 1 more than 3 and 3+4 is equivalent to 3+3+1).

For subtraction, Carpenter and Moser (1984) again observed five different strategies:

- Separating from (e.g. for 5-2, count out 5 objects and take 2 away to leave 3)
- Adding on (e.g. for 5-2, count out 2 objects and then add on objects until 5 objects have been produced. Count out the 3 objects that have been added)
- Matching (e.g. for 5-2, line up the 5 objects and the 2 objects so that they are matched one to one. Count the remaining 3 objects in the unmatched group)
- Counting down from (e.g. for 5-2, count back 2 from 5, keeping track of the counting words, to get the answer 3)

- Counting up from given (e.g. for 5-2, count up from 2 to 5, keeping track of the number of counting words used, to get the answer 3)

It is interesting to note that for subtraction, no use of known facts was observed. This suggests that the children were not aware of the complement principle (so for example, if you know that 2+3=5, you also know that 5-2=3). However, the children did demonstrate a range of interpretations that go beyond the understanding of subtraction as "taking away". All the methods involved some form of counting, whether this was counting of real objects, or keeping track of number words in the counting sequence.

Both within interviews and over time, Carpenter and Moser (1984) found that there was wide variation in the choice of strategies. The main variability could be explained in terms of resources, in the sense that when practical apparatus was made available, the children preferred to use this rather than using more abstract strategies. There were also differences in children's strategies for questions where the mathematical operations required are not clear within the wording of the question (e.g. "Joe has 8 balloons. His sister Connie has 14 balloons. How many more balloons does Connie have than Joe?" and "There are 12 children in the playground. 7 are boys and the rest are girls. How many girls are on the playground?" (Carpenter and Moser, 1984, p.180)).

However, there are more issues to consider when analyzing children's responses to questions. It is not just whether or not the mathematical operation is transparent in the wording of a question, but also whether the meaning of the words are understood within the context of the question. For example, with the question "Joe has 8 balloons. His sister Connie has 14 balloons. How many more balloons does Connie have than Joe?" not only is the mathematical operation not explicit in the wording, the use of the phrase "how many more?" is particularly challenging for children (Nunes and Bryant, 1996). Nunes and Bryant (1986) have argued that one of the reasons for this may be related to the fact that "more" in everyday speech takes on a very different meaning.

Fuson (1982) and Carpenter and Moser (1982) observed that children tended to use a count all strategy when they could see the objects being counted (first addend and second addend), but used a counting on strategy when the objects being used for the first addend were hidden but the objects for the second addend could be seen. It is possible though that although the children were not counting out loud they were counting silently to themselves until they reached the count of the hidden objects.

2.1.6 Is counting always necessary?

Is there a point at which counting is no longer a necessary skill? This is not a straight forward question. This issue was first explored by Baroody and Ginsburg (1986) when they were trying to explore how an integrated understanding of both the principles and procedure of counting could be conceptualised. This was then taken further by Gray and Tall (1991) when they tried to integrate the process of counting with the concept of number. In order to describe this way of conceptualising these different aspects of working in mathematics, Gray and Tall defined the notion of a “process” and a “procept”.

This reconceptualization was the result of the observation that understanding the meaning of a symbol, say “4”, requires more than just knowing how to count to four. For example, we can obtain the answer four by adding one and three or two and two. Thus, “4” can be understood as either the process of counting or the process of addition, but it may also be understood as the concept in its own right. This notion of a symbol that can represent both a process and concept has been called a “procept”. The notion of a procept is particularly useful in arithmetic and algebra, where numbers and symbols become objects which can be manipulated. For example, the count all approach, involves a *process* and a *process*, while the counting on approach, may involve a *procept* and a *process*. Using a known fact, however, involves a *procept* and a *procept,* as long as the known facts are not just rote learned, but are understood in terms of mathematical relationships: “A proceptual known fact should be distinguished from a rote learned fact by virtue of its rich inner structure which may be decomposed and recomposed to produce *derived facts*” (Gray and Tall, 1991, p.74).

2.1.7 How do children "know" their number facts?

Cowan et al. (2011), in their study of children in Years 3 and 4 (aged 7 to 9 years), found that very few children "knew" all their number facts for addition and subtraction up to 20, but that they varied in the proficiency with which they were able to perform calculations. It seemed that children were more likely to derive their answers from a few known facts. Moreover, when working out their answers from their known facts, the children continued to rely on counting procedures, often using their fingers to help. Although, there was a relationship between calculation proficiency and attainment in mathematics, it seems that this relationship is not straightforward.

What is appearing, then, is a very complicated picture. Where attempts have been made to try to conceptualise the mathematical processes that children learn to use and apply, it seems that few provide a complete picture. This confirms Raghubar, Barnes and Hecht's (2010, p.119) suggestion that "what is currently lacking in the field is a sufficiently comprehensive model of mathematical processing, particularly in relation to skill acquisition". There is clearly more to learning to use numbers than can be described with just reference to the mathematical processes alone.

2.2 Looking beyond mathematical processes

2.2.1 The role of working memory

When children engage with numerical calculation, the tasks are often supported by aspects of working memory. Working memory is "the ability we have to hold and manipulate information in the mind over short periods of time" (Gathercole and Alloway, 2008). Poor working memory is often associated with low attainment in mathematics (Geary, 2011). Using the work of Baddeley, Hitch and others, Pickering and Gathercole (2001) suggest that working memory consists of three components. These components include two "slave systems", the Phonological Loop (PL) and the Visuo-spatial Sketchpad (VSSP) and the Central Executive (CE).

The PL holds information that is related to sound and, therefore is associated with spoken and written language and pictures, where these act as triggers for particular patterns of sounds. Information is held in the PL for approximately two seconds, unless it is consciously rehearsed. The VSSP stores non-verbal information which is either visual (such as colour or shape) or spatial (such as movement or position). These slave systems can be thought of as short term memory capacities.

The term “working memory” suggests that there is more going on than just remembering things in short term memory. It is the CE that is responsible for the more complex aspects of working memory. The CE is responsible for activities such as organisation of the flow of information, planning and, when necessary, retrieving information from long term memory. For example, when you want to work out the sale price of an item that is reduced by 10%, you might use the fact that you already know that 10% is equivalent to $^{1}/_{10}$ and that there is a quick way to calculate $^{1}/_{10}$ of an amount of money. This knowledge is stored in long term memory, but the CE can call upon it, so that you can then calculate the new price. However, while you are retrieving the knowledge from your long term memory, you also need to hold on to the original price (in your PL) and once you know how to calculate the new price, you need to be able to manipulate this “in your head” in order to work out the desired new price. Put like this, it does not sound so easy!

In a study of children 6 and 7 year olds in the UK, a correlation was found between children’s working memory and their attainment in mathematics (Gathercole and Alloway, 2008). Most children are expected to score between 85 and 115. Anything below 85 is considered a poor score. In the study, the average score for working memory, for children scoring below average in mathematics, was below 85, but the average score for their short term verbal memory was above 100 and was actually higher than the average for the middle attaining group of children. Unfortunately, this factor was not discussed. It may be that the children had developed this ability in order to compensate for weaknesses in other areas, such as visuo-spatial memory. Another study of children at age 14 years, found again that the children with the lowest attainment had average working memory assessment scores below 85, but, unfortunately for this group, no score for short term memory was shown (Gathercole and Alloway, 2008).

Children with poor working memory, are likely to have particular difficulty with arithmetic:

> There are several reasons for this. First, working memory overload in the individual activities designed to develop numeracy skills will result in frequent errors and task failures, impairing the incremental process of acquiring basic number skills and knowledge, and slowing down the child's rate of learning. Second, mental arithmetic is heavily dependent on working memory….it requires not only the storage of arbitrary numerical information, but also the retrieval and application of number rules that may not yet have been securely learned.
>
> (Gathercole and Alloway, 2008, p.54)

In young children, this kind of difficulty might be observed in counting recall tasks. Cowan (1987) suggested that when children are engaging with counting recall tasks, they try to remember the numerosity of the collections, but once they start to count a second, or subsequent, collection, they forget how many were in the earlier collections. Hitch, Towse and Hutton (2001) investigated speed and performance in counting recall tasks. They argued that children who process information more quickly are less likely to forget. This suggests that a child who processes information quickly will not need to "hold" the information for as long as a child who processes information more slowly. This is not as straight forward as it may at first appear though, as there are individual differences amongst children which make this a non-trivial relationship. Hitch, Towse and Hutton also highlight the relationship between the rate at which children forget individual items or sets and the effect of experiencing this forgetting. It might also be the case, though, that children who are at an early stage of learning about number and counting find that they expend a significant amount of effort in counting the dots and keeping track of each count.

In general, if children have poor working memory, they score poorly in all areas of mathematics (Gathercole and Alloway, 2008), but this is not always the case. For example, children with motor coordination difficulties tend to be more successful with tests of verbal working memory than visuo-spatial working memory, whereas children with language impairments tend to be more successful with tasks of visuo-spatial working memory than tasks involving verbal working memory (Gathercole and Alloway (2008). Cowan, Donlan, Newton and Lloyd (2005), however, when working with a group of 7 to 9 year old children, found that children with specific language impairments (SLI) also had poor visuo-spatial skills as demonstrated by their low scores in the Corsi span task (where children with specific language impairments are defined as those who have been assessed as being significantly delayed in receptive and expressive language while achieving age-appropriate scores on tests of non-verbal skills).

It seems, then, that the relationship between working memory and performance in mathematical tasks is not straightforward (Raghubar, Barnes and Hecht, 2010; Hitch, Towse and Hutton, 2001). The main reason for this is that performance in mathematical tasks depends on a wide range factors in addition to that of working memory. These include:

> ….age, skill level, language of instruction, the way in which mathematical problems are presented, the type of mathematical skill under consideration and whether that skill is in the process of being acquired, consolidated, or mastered.
>
> (Raghubar, Barnes and Hecht, 2010, p.119)

2.2.2 The role of speech and language impairments

It has already been seen that children aged 7-9 years, with SLI are more at risk for poor working memory than their typically developing peers (Cowan et al., 2005). It is worth exploring this in a bit more depth.

In terms of development of mathematics, Cowan et al. (2005) found that children with SLI do less well than their typically developing peers in a range of tasks involving number. They also found that their visuo-spatial skills were below those expected for children of their age. They suggested that working memory is particularly important for learning and that performing less well in the Corsi span task may be linked to the way that children try to hold on to the memory of the paths traced by the tester. It is possible that the rhythm, including the sounds and movements, have an influence on how the paths are remembered.

Cowan et al. (2005) found a relationship between working memory and the children’s ability to solve contextualised word problems. In addition, they found that transcoding tasks were particularly influenced by working memory (especially central executive functioning). In the tasks undertaken by the children, they had be able to switch between spoken and written representations of number, thereby placing a particularly heavy load on short-term memory and executive function.

It has been shown that children with SLI may be delayed in learning the number sequence and calculation skills (such as 17-9) whether presented verbally (Donlan, Cowan, Newton and Lloyd, 2007) or in written form (Cowan et al., 2005). However, when understanding of arithmetic principles were explored (using statements such as given Ɛ+ Ꮑ = Ꮼ is it true that Ɛ+ Ꮼ = Ꮑ), it seems that children with SLI are very successful (Donlan et al.). This suggests that although children with SLI may be poor at doing numerical calculations, they do understand the principles of arithmetic.

What about children with other speech and language difficulties? Stothard, Snowling, Bishop, Chipchase and Kaplan (1998), in a follow-up study of 85 children who had been initially assessed at ages 4 years, 4½ years and 5½ years, found that the story for these children was rather complicated. Some of the children had initially been assessed as having SLI while others who had poor speech and language development in addition to poor non-verbal IQ scores, were described as the “general delay group”. However, the key for all of these children was their level of language development at age 5½ years. At age 15 years, over two-thirds of those children who had “caught up” by 5½ years of age, continued to be successful at school, although with residual phonological processing impairments. The other children, who had appeared to have caught up age 5½ years, but who were not achieving so well at 15 years of age, seemed to be those who struggled more with tasks involving receptive grammar and narrative skills when they were 5½ years old.

2.2.3 Number sense

The term “number sense” was first used by Dantzig in 1954:

> Man even in the lower stages of development, possesses a faculty which, for want of a better name, I shall call *Number Sense*. This faculty permits him to recognize that something has changed in a small collection when, without his direct knowledge, an object has been removed or added to the collection.
>
> (Dantzig, 2007, p.1)

Nowadays, the term “number sense” is used widely and with many differing definitions. In 2000, Julia Anghileri wrote the book “Teaching Number Sense”. The definition Anghileri used was very broad. She wanted to get across the idea that children develop their “number sense” in very different ways, but that number sense is concerned with noticing patterns and relationships, working flexibly and understanding about different representations and meanings (Anghileri, 2006).

Tosto *et al.* (2014b) defined “number sense” as our ability to estimate and work with quantities that are not presented as numerals or symbols. We often use our number sense when deciding which is the best queue to join at a supermarket checkout or which pile of sweets to pick. Using this definition, it could be argued that this is a very useful survival skill and, in terms of human evolution, would precede any need to use numbers for counting exact numerosities.

This demonstrates the range of definitions and lack of clarity that exist for the term “number sense”, and yet it is a term that we use frequently, assuming a common understanding. This issue has been explored by Berch (2005). Berch points out that the definitions we use are not trivial, due to the fact that many of our judgments about children, our testing regimes and our definitions of their progress are often based on our definitions “number sense”. Berch identified two strands of definitions. Firstly, the lower order strands that identify number sense as a skill that is concerned with a representation of quantity, in line with Dehaene’s argument (2011) or secondly, a wider definition that includes a range of skills which are more in line with

Anghileri's definition, which may also be more closely linked to children's educational experiences.

The term "number sense" will be used throughout this review of the literature and definitions provided by the various authors will be identified.

The issue of "number sense" has been the subject of a well-known book (now in its second edition) by Stanislas Dehaene entitled *The Number Sense [How the mind creates mathematics]* (2011). In this book, Dehaene explores the possibility that number is a property to which all small children as well as animals have some access. In this way, he attempts to define "number sense" as a core facility. He describes a number of experiments in which it has been shown that animals, including rats and pigeons, are able to distinguish reliably between the numbers 1, 2 and 3. Rats and chimpanzees appear to be able to add small numbers with accuracy even when the stimuli are mixed—for instance recognising that two flashes and two sounds make 4.

According to Dehaene (2011), research shows that measuring the length of babies gaze or the rate of sucking suggests that they are be able to distinguish the numbers two and three from as early as a few days old in response to auditory or visual stimuli. Babies less than a year old, it is argued, can "calculate" additions and subtractions 1+1 and 2-1 (Wynn, 1992). It has also been suggested that babies are able to match quantities. If faced with two images, one representing three objects and one representing two objects, when a series of drum beats sounds they will give more attention to the image with two objects when hearing two drum beats and to the image of three objects when hearing three (Dehaene, 2011).

Adults also recognise the quantities 1, 2, and 3 reliably almost all of the time with a significant drop off from 4 upwards (this is called "subitising"). Dehaene (2011) and Butterworth (1999), among others, argue that the quantities 1, 2 and 3 are recognised by a neural system specific to these numbers, called the intraparietal sulcus (IPS). When it comes to larger numbers, adult humans take more time to distinguish between groups that are close in number than groups with a large difference. For instance it is easier to distinguish between a group of 2 and a group of 5 than between a group of 4 and a group of 5. It is also easier for us to distinguish between a group of 1 and a group of 2 than between a group of 4 and a group of 5.

This is because the relative difference is greater between 1 and 2 than it is between 4 and 5. Interestingly, this behaviour seems to be present not only when looking at groups of objects, but also when looking at the symbols (numerals) that represent them. Dehaene suggests that human beings somehow use an approximation of the number for comparison rather than relying on an exact knowledge of the relative sizes of quantities represented by each digit.

Our relative difficulty in distinguishing the larger or smaller number in pairs of larger numbers as compared to pairs of smaller numbers leads to a suggestion that human beings (and perhaps animals) represent numbers on a logarithmic rather than a linear scale, which provides a much greater distinction between the numbers 1 and 2 than between the numbers 31 and 32. Such a scale would match with those already established for human perception of continuous measures such as loudness and pitch of sound and the brightness of lights (Dehaene, 2011).

According to Dehaene, the inferior parietal areas on both sides of the brain appear to be involved with understanding the magnitude of numbers. The left hemisphere of the brain is better able to manipulate symbols than the right and it seems to be this ability to process symbols that allows it to perform accurate calculation. The right hemisphere, though, is involved in making approximations.

Dehaene (2011) suggests that once number facts have been learned (for example, multiplication tables) the IPS may not be used at all. Recall of those facts that have been learned (such as 3 x 3 = 9) rely on verbal recall rather than mathematical processing. It has already been seen, however, that children are more likely to derive number facts from a few known facts (Cowan et al., 2011). This brings into question the notion that verbal recall is all that is involved in the apparent recall of number facts.

2.2.4 Visuo-spatial skills and mathematics

It has been suggested that that the intraparietal sulcus supports visuo-spatial working memory (Silk, Bellgrove, Wrafter, Mattingley, and Cunnington, 2010). We know that working memory is important for learning, because it is our working memory that allows us to hold information for a short period of time and manipulate it.

Studies in genetics have also explored the link between visuo-spatial skills and mathematical skills. Tosto et al. (2014a) used standardised assessments to look at a range of mathematics skills (based on the current requirements of the English national curriculum) and visuo-spatial skills (assessed using jigsaw and hidden shape tasks). They found that visuo-spatial skills and mathematical skills were moderately heritable, but more significantly, that, in terms of genetics, there was an almost complete overlap between mathematical skills and visuo-spatial working memory. They also reported that the environmental factors were very significant in determining outcomes in visuo-spatial skills. In addition, they found that "there were no significant genetic effects that were specific to spatial ability once the shared genetic effects with mathematical measures were taken into account" (Tosto et al, 2014a, p.468). Following on from this, it seems reasonable to suggest that activities which develop children's visuo-spatial skills should also benefit their mathematical skills.

2.2.5 Measuring the approximate number system

The approximate number system (ANS) has been a significant area of exploration for Halberda, Mazzocco and Feigenson, and following the publication of their report in 2008, in Nature. Since 6th May 2011, Panamath has been available online as a resource that anyone can use to test their ANS. To test the ANS using Panamath, the participant has to decide which out of two images of dots is the greater. The images flash up for a few seconds only, so there is not enough time to count the dots. Once the test has been completed, a record of the results, including reaction time is provided and the participant can see how their performance relates to others of their age. The image in figure 2-1 may help to clarify how Panamath works:

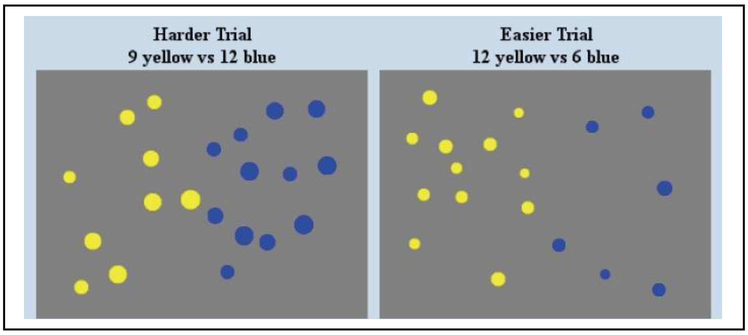


Figure 2-1: Panamath screenshot

According to the website:

> The precision of the ANS drastically improves across an individual's lifespan—being imprecise yet still observable at birth, undergoing a rapid period of improvement from the teens to the 20s, reaching an optimal accuracy around age 30, and progressing through a gradual decline in later adulthood.

(Panamath: what we found. Retrieved from http://www.panamath.org/results.php).

Mazzocco, Feigenson, and Halberda (2011a) found a relationship between children’s achievement in Panamath at 3 years of age and their attainment in mathematics aged 6 and 7 years. They also found that children who struggled with mathematics performed less well in Panamath (Mazzocco, Feigenson, and Halberda, 2011b).

2.2.6 Numbers on a number line

Studies on the ANS could be associated with activities on a number line. For example, Siegler and Opfer, (2003) and Booth and Siegler (2006) used number lines to explore children’s number knowledge, especially in relation to children’s knowledge of the relative position of numbers on a number line. In the studies undertaken, the authors found a shift in children’s positioning of numbers on a 0-100 number line from that of a logarithmic distribution to a linear distribution between ages of 5 years and 8 years. On a 0-1000 number line, they observed the same progression for children between the ages of 8 years and 10 years. This was taken as evidence that numerical estimation is a “common core” attribute and that children without experience tend to rely on “logarithmic rather than linear representations of numerical magnitudes” (Booth and Siegler, 2006, p.199). Booth and Siegler also found that children who were low attainers in mathematics, tended to be those who persisted in using logarithmic representations. Number knowledge, then is knowledge of complex relationships, but the role of our “innate” capacities in supporting this is still a matter of debate.

2.2.7 The relationship between non-symbolic and symbolic representations of number

In considering the role of the ANS, it is useful to understand how it develops in children. This has been explored in the field of genetics by Tosto et al. (2014b). However, in their study, children’s ANS was only modestly associated with genetic factors. Tosto et al. argue that this suggests that a significant factor in developing number sense is associated with environmental factors, such as home life and school experience. If this is the case, then there is much that can be done to support children’s development in this area.

It is important to consider the role of symbolic representations of number. Lyons, Ansari and Beilock (2012) suggest that mental representations of numbers presented as symbols (numbers or words) are only very loosely linked to actual quantities. The brain concentrates more on the relationship between the symbols than on the quantities they represent and it is these, rather than the quantities themselves, that are being manipulated when we make accurate calculations. Sasanguie, Defever, Maertens and Reynvoet (2014) found that the performance of 6 to 8 year old children on mathematical assessments was related to their ability to compare digits rather than their ability to manipulate non-symbolic representations of number (for example, patterns of dots). A similar finding was reported by Fazio, Bailey, Thompson and Siegler (2014). Following their study on children in Year 6, Fazio et al. concluded that while both non-symbolic and symbolic numerical representations uniquely predicted attainment in mathematics, symbolic representations were far more strongly associated than non-symbolic magnitude representations. They used this finding to suggest that as children get older, symbolic magnitude representations are much more important in the mathematics that children do. It seems likely, though, that in order for symbolic notation to become so significant, much work needs to be done to make the link between non-symbolic magnitude representations and symbolic magnitude representations, in the early years of school. If, for some reason, this is disrupted, then it is probable that children will not be able to move on at the same pace as their peers.

In another study on children in Year 2, it was found that activities which focused on non-symbolic approximate number problems, just before working on symbolic, exact number addition calculations, had a positive effect on children's outcomes (Hyde, Khanum and Spelke, 2014). Lyons, Price, Vaessen, Blomort and Ansari (2014), however, in a study of children across Years 2 to 7, found no relationship between the dot comparison task (which out of two arrays of dots has the most dots) and arithmetic skills when they controlled for non-numerical and numerical factors. They argue that it is the fact that they controlled for these factors that makes their results unique. Moreover, Lyons et al. also found that there were developmental changes which altered the significance of particular factors at particular points in time. For children who were just learning about arithmetic, the skills of assessing the relative magnitude of two numbers presented symbolically (identifying which is the larger number) and mapping numbers correctly on a number line were very important. Ordering symbolic numbers (identifying whether a group of three numbers are in ascending order), though, was not a good predictor of skills in arithmetic in Year 2, but increased throughout the school years to Year 7, where it was very closely associated with arithmetic skills. This suggests that, over time, there is a decrease in the importance of cardinal numbers (relative values of symbolically presented numbers) and an increase in the importance of ordinal numbers (symbolic numbers presented in numerical order) in relation to the skills required for success in arithmetic. It seems probable that this observed shift is more likely due to the fact that as children get older, they have to deal with larger numbers, which require better understanding of the place value system. Interestingly, Lyons et al. also noticed that during the period from Year 4 to Year 6, they could not identify any basic numerical skill which dominated. This, they argue, provides evidence for a possible period of consolidation and transition between the cardinal and ordinal stages of numerical development. Finally, once children have started formal education, at “the age of roughly 6-7 years, the importance of the ANS is largely overshadowed by other basic numerical and cognitive abilities” (Lyons et al. 2014, p.724).

There is, then, some disagreement in the literature as to whether there is or is not a link between the ANS and the development of symbolic representations of exact numberosities. Although Halberda et al. (2008) claim that there exists a relationship throughout life, it seems more likely that if a relationship does exist in the early stages of learning to count, that this becomes less significant as children move into the realms of dealing with the more abstract concepts of mathematics, which rely more on patterns and relationships. Nevertheless, the fact that there seems to be an association between visuo-spatial skills and mathematics attainment, taken together with the fact that test for the ANS relies on visuo-spatial skills, does tend to muddy the water. Perhaps the link is not between the ANS and symbolic representation of exact numerosities, but rather between the ANS and some other aspects of mathematical understanding.

2.3 The role of fingers

> “Whenever a counting technique, worthy of the name, exists at all, *finger counting* has been found either to precede it or accompany it”
>
> (Dantzig, 2007, p.9).

If we want to reflect more on the role of visuo-spatial skills, then our fingers seem to be a very good place to start. What it is about our fingers that makes them so special? (where “fingers” are taken to include thumbs). In much of the literature on the learning of number, fingers are recognised as playing an important role in supporting understanding of our number system (Anghileri, 2006; Fuson, 1988; Gelman and Gallistel, 1978; Hughes, 1986; Jordan, Huttenlocher and Levine, 1992). Fingers can help children keep track of items in a count (Fuson, Richards and Briars, 1982) and they can also be used to represent both cardinality and ordinality (Domahs, Krinzinger and Willmes, 2008). As children begin to manipulate numbers, fingers can represent objects in calculations and can thereby help children understand that numbers can also exist as abstract entities in their own right (Anghileri, 2006; Hughes, 1986). In other words, fingers are particularly important, because they “possess simultaneously iconic (i.e., features shared with the referent), symbolic (i.e., conventional meaning shared with other individuals), computational (i.e., used to support calculation procedures), and communicative (i.e., used to communicate numerosities through gestures with other individuals whatever their language) properties” (Di Luca and Pesenti, 2011, p.2). Thus, fingers used as symbolic referents do not require knowledge of any formal written system in order to work as an effective form of communication.

When reflecting on our number system, it can be no coincidence that most cultures operate with a base-10 number system (Hughes, 1986) and yet it seems likely that the use of fingers is a learned, and not necessarily a spontaneous, activity (Crollen, Seron and Noel, 2011). However, as our number system is so much part of our everyday life experience, the relationship between finger-use and the development of number skills needs to be explored.

2.3.1 Fingers and counting

> Finger-counting/montring activities, especially if practiced at an early age, can contribute to a fast and deep understanding of number concepts, which has an impact during the entire cycle of life by providing the sensory-motor roots onto which the number concept grows.
>
> (Di Luca and Pesenti, 2011, p.3)

The term “finger-montring” is the capacity to show particular numerosities with the correct number of fingers, all at once and without counting. How then are finger-montring and finger-counting linked?

It has been shown that touching objects when counting helps pre-school 4 year old children to count correctly (Alibali and DiRusso, 1999). This can help children to understand about one-to-one correspondence and relieves the pressure on working memory because it makes it easier to keep track of the counted items. Fingers can do a similar thing when trying to keep track of items and during calculations. With practice, children learn to map particular patterns onto particular numbers (Morrissey, Liu, Kang, Hallett and Wang, 2016). In other words fingers can provide an “egocentric sensory-motor schema” (Rinaldi, Di Luca, Henik and Girelli, 2016, p.51) developed by repetition and practice. In addition, by providing a tool which aids keeping track of items, fingers can also provide an embodied means of understanding ordinality (Rinaldi, Di Luca, Henik and Girelli, 2016).

For these to be effective requires an awareness of one’s own fingers, or “finger sense”, otherwise known as finger gnosis (Gerstmann, 1940). However, this still leaves open the question of whether it is finger gnosis or the whole sensorimotor experience which is of significance. Citing recent research in the area of neuroscience, Berteletti and Booth (2015a) argue that it is not just finger gnosis, but also the embodied actions of moving fingers which are significant in determining the role of fingers in early arithmetic. Thus fingers are useful to keep track of items in a count (Andres, Seron and Olivier, 2007) or compare numbers presented symbolically (Sato, Cattaneo, Rizzolatti and Gallese, 2007). This evidence supports the findings from observational studies such as those by Hughes (1986) and Jordan, Huttenlocher and Levine (1992).

While there are clearly cultural differences in the ways that children learn and are taught to use their fingers (Di Luca and Pesenti, 2011; Domahs et al., 2010), it has also been suggested that “personal finger-counting habits influence the way numerical information is mentally represented and processed” (Berteletti and Booth, 2015a, p.111) and stored in long-term memory (Di Luca and Pesenti, 2008). It therefore seems likely that if fingers are to be used as a tool to support numerical calculations, the most significant factor is *whether* children learn to use a particular pattern of movements, rather than *what* the particular pattern might be.

If nothing else, then, fingers are a very useful tool, but it is the sensory-motor experiences that make this possible. What happens when this sensory information is disrupted? In order to explore this, the case of Gerstmann syndrome will be discussed.

2.3.2 Gerstmann syndrome

Gerstmann syndrome was first described in 1924 as a syndrome which is characterised by the presence of four conditions: finger agnosia, acalculia, dysgraphia and left-right disorientation. Gerstmann argued that the origin of the difficulties is a consequence of the lack of “finger sense” and he named this condition “finger agnosia”. This term was subsequently described as an “elective disability for recognizing, naming, selecting, differentiating and indicating the individual fingers of either hand, the patient’s own as well as those of other persons” (Gerstmann, 1940, p.398). In other words, not only do people with Gerstmann syndrome have difficulty identifying their own fingers in response to touch and request, they cannot mirror the finger actions of others.

In Gerstmann syndrome it is believed that there is damage to the inferior parietal area of the brain (Dehaene, 2011) or a problem with white matter fibre tracts in the parietal area (Rusconi et al., 2009). As discussed earlier, according to Dehaene, the inferior parietal areas on both sides of the brain appear to be involved with understanding the magnitude of numbers, while the left hand side of the brain seems capable of processes that depend on symbolic manipulation of symbols.

Gerstmann (1940) observed that people with finger agnosia tend to make more finger identification errors with the three middle fingers than with the little finger and the thumb. He also noted that this lack of finger sense seemed to be accompanied by a difficulty in making individual finger movements. In addition to these observations about fingers, Gerstmann made some very interesting observations about the capabilities of the people he studied when asked to make sense of numbers: “The significance of the position of the digit within a given complex figure is not realized; accordingly, the patient is disorientated regarding the sequence and the decimal values of the digits within a figure” (Gerstmann, 1940, p.401). In other words, it is proposed that, not only does finger agnosia seem to affect people’s ability to make sense of numbers up to ten (to match with the number of digits we usually have on both hands), but extends across the whole of our place value system. As further evidence for the link between fingers and numbers, Gerstmann (1940) suggested that it is probably no coincidence that the word “digit” is used to mean both “finger” and to identify any of the Indo-Arabic representations for zero to nine.

2.3.3 Finger gnosis and finger mobility

It has been proposed that finger gnosis is a good predictor of maths attainment in typically developing children and that finger representations become more defined as children get older (Reeve and Humberstone, 2011). In typically developing children, finger gnosis develops quickly up to the age of 6 years and then continues to develop at a slower rate up to the age of 12 years at which point it should be fully developed (Strauss, Sherman and Spreen, 2006). In addition, children with higher IQs have more well-developed finger gnosis (Strauss et al., 2006).

For children and adults with varying degrees of finger agnosia, this relationship may have implications for their mathematical development. Research by Kinsbourne and Warrington (1962) on 12 people with finger agnosia, identified the apparent lack of differentiation of individual fingers, in terms of touch and spatial relationship to each other. The fingers were described as representing an “undifferentiated mass” (Kinsbourne and Warrington, 1962, p.56), or, in other words, as if they were a single digit. This is significant in relation to children with Apert syndrome, who, until their fingers are surgically separated, have a finger representation of a single digit (for the fingers that are fused) (Mogilner et al., 1993). This must have implications for the development of their finger gnosis, but it could also have implications for their understanding of number and arithmetic.

In the classroom, children are often encouraged to use their fingers to help them count and keep track of their counting. Fingers cannot only be seen, felt and moved, but are central to how human beings, as babies begin to explore and interact with the world.

If fingers are important in developing number representations, are there differences in the ways that adults and children perform tasks involving number? Kaufmann et al. (2008) used brain imaging techniques to explore the areas of the brain that are recruited when performing simple tasks involving number in a study involving eight year old children and adults. In tasks involving non-symbolic representations of number, they found that although the children and the adults were able to complete the tasks successfully, children took longer. When this was investigated further, it was discovered that when making numerical comparisons using images of hands showing differing numbers of fingers, the children (but not the adults) recruited additional areas of the brain normally used for fingers. The authors suggest that fingers are an important stepping stone in the development of an abstract understanding of number and that finger-use should be encouraged to help develop fluency and competence in activities involving number.

Finger gnosis and fine motor skills have also been implicated in supporting the development of arithmetic and mathematical skills (Noel, 2005; Gracia-Bafalluy and Noel, 2008). Noel (2005) carried out assessments of finger gnosis with 41 children in Year 2 and compared this with an assessment of their skills in mathematics one year later. A correlation was found between the children's level of finger gnosis in Year 2 and their achievements in tasks involving number identification and simple arithmetic one year later. In fact, the relationship between finger gnosis and their achievement in mathematics was stronger than the relationship between tests of general cognitive ability and achievement in mathematics between Years 2 and 3. This was followed up with an intervention study in which children were provided with a finger-differentiation intervention, twice a week for a period of 8 weeks. The children's finger gnosis and their numerical skills both improved, when compared to a control group (Gracia-Bafalluy and Noel, 2008).

It seems then that finger-use during early number work is not trivial and has the potential to support children's developing understanding. If this is the case, then children with Apert syndrome are likely to be disadvantaged in the early years of school because they will have experienced a number of surgical procedures to create their fingers and they may be only just developing their finger-use and finger awareness.

In terms of finger representation in the brain, Weiss et al. (2000) found that following amputation of the middle and ring fingers of a patient, the somatosensory cortex had undergone significant change after 10 days. Reflecting on other research in the area, Weiss et al. argue that there are three stages of somatosensory reorganisation. The first stage is immediate and involves demarcating distinct representations. The second stage takes longer and goes on for weeks or months. During this stage, new connections are made which create a different mapping within the brain. During the third stage, the reorganisation is focused on how the fingers are now used.

For children with Apert syndrome, then, there are significant changes which need to take place both in terms of finger-use and how this then supports longer-term changes in the brain. If there is a relationship between fingers and the ways in which children come to understand and work with numbers, this is likely to be of significance to children with Apert syndrome.

2.3.4 Numbers, fingers and the developing brain

In the last 20 years, advances in non-invasive techniques, such as fMRI, have made it easier to explore what is happening in the brain without the need for surgery. This has enabled doctors, psychologists and neuroscientists to explore the workings of the brain in typical and atypical development. As a result, there is a growing body of research in the fields of psychology and neuroscience (e.g. Butterworth, 1999; Noel, 2005; Kaufmann, 2008; Penner-Wilger and Anderson, 2008), investigating and hypothesising about the exact relationship between fingers and the development of number concepts.

Although the exact form of this relationship is not yet clear, from a neuroscientific perspective, Butterworth (1999, pp.249-250) has argued that “without the ability to attach number representations to the neural representations of fingers and hands in their normal location, the numbers themselves will never have a normal representation in the brain.” There are a number of models which have been proposed to explain the means by which these representations develop. For the purpose of this discussion, the models proposed by Butterworth (1999) and Penner-Wilger and Anderson (2008) will be reviewed.

Butterworth (1999) has suggested that, over time, children begin to create a link between their fingers and numbers. Through practice, guidance and modelling by others, children learn how to use their fingers to count and to help with calculations.

Eventually, children come to recognise different finger patterns (for example, that four fingers can represent 4) and these become symbolic representations in the same way that letters are symbolic representations of sounds. In other words, the links in the neural networks in the brain, between fingers and numerosities arise as a consequence of activity. This relationship is described as functional, because the networks created are a consequence of links being made between finger representations and numerosities, rather than one being integral to the development of the other.

Penner-Wilger and Anderson (2008) have proposed an alternative view, and suggest that the link between finger gnosis and number is due to the redeployment of finger representations. They suggest that the neural networks in the brain have evolved in such a way that the structures used for finger gnosis are concurrently used to support the development of concepts of numerosities. This has been termed the "Massive Redeployment Hypothesis" (MRH) (Penner-Wilger and Anderson, 2008). MRH provides an evolutionary explanation of this hypothesis based on the knowledge of the brain's plasticity - its ability to adapt and change in response to significant environmental changes. The MRH is based on the suggestion that the brain has evolved to employ the neural circuits used for finger gnosis with the neural circuits for the more abstract concepts of number, as greater use of these concepts have become integral to our way of life. In other words, the same neural circuits in the brain have evolved over time to provide representations of both fingers and numbers.

Penner-Wilger and Anderson (2008) have sought to provide evidence from several perspectives to support their MRH. One of these, which is of particular significance here, relates to their suggestion that "individuals *without* finger agnosia who could not or did not use their fingers to represent quantities during development, should nevertheless show activation in the finger circuit (left angular gyrus) during tasks requiring the representation of number such as magnitude comparison" (Penner-Wilger and Anderson, 2008, p.1649). While there is no evidence from brain imaging to support this suggestion, Penner-Wilger and Anderson discuss findings from research which has been undertaken with children with spina bifida and children with developmental coordination disorder (DCD) to support this proposal.

Studies of children with spina bifida are thought to be useful because they identify a group of atypically developing children who have difficulty with moving their fingers, in addition to having varying degrees of finger agnosia (Penner-Wilger and Anderson, 2008; Reeve and Humberstone, 2011). In a study reported by Cermak and Larkin (2001, as cited in Penner-Wilger and Anderson, 2008), children with spina bifida were found to have no significant difficulties with mathematics. This, however needs to be contrasted with Barnes et al. (2006), who, in a study of 98 children with spina bifida myelomeningocele (the most common form of spina bifida), reported that all the children in their study exhibited problems with arithmetic, irrespective of whether they had any other learning difficulties. It is has also been reported that children with spina bifida myelomeningocele tend to have stronger language skills and weaker motor and visuo-spatial skills, in addition to their weaker mathematical skills. However, children with spina bifida myelomeningocele also have disruption to their brain development which results in quite specific brain anomalies (Barnes et al., 2006) and consequently the identification of any specific causes which may explain the children's difficulties with mathematics becomes hard to locate.

Alloway (2007) also reporting on a study of 55 children with DCD, found that the children's numeracy and literacy skills were poor. Alloway (2007) also reported that the children performed poorly in tasks requiring visuo-spatial memory and working memory, compared with their performance in tasks requiring verbal short-term memory. In addition, Alloway and Warner (2006) found that even after implementing an intervention focusing on everyday tasks, such as balancing and throwing, with a group of children with DCD, improvements in coordination and visuo-spatial skills were not followed by improvements in numeracy and literacy (as cited in Alloway, 2007). In an attempt to address these findings, Penner-Wilger and Anderson (2008) have suggested that this poor performance in tasks involving arithmetic could be due to the children's lack of control of finger movements, rather than to a poor representation of number. Again, this group of atypically developing children may provide useful observations concerning finger agnosia and the development of understanding of numerosities, but the existence of other common traits makes it hard to identify any causal relationships. What about children with other disabilities involving fine motor skills?

Children with cerebral palsy (CP) are predominantly affected by disorders affecting their motor performance, although “it is increasingly acknowledged that children with CP also display disturbances in sensation, behaviour, communication and cognition.” (Rosenbaum et al., 2007, as cited in van Rooijen, Verhoeven and Steenbergen, 2015, p.1). In particular, children with CP often have difficulties with work on number and arithmetic (Jenks, van Lieshout and de Moor, 2008; van Rooijen, Verhoeven and Steenbergen, 2011) in addition to their other disabilities.

Jenks, van Lieshout and de Moor (2009), found that in children with CP, left-hand impairment was related to the speed at which children were able to answer questions involving addition and subtraction. It has also been shown that in primary aged children with CP there appears to be a relationship between fine motor skills and skills in addition and subtraction (Van Rooijen et al., 2012). In a study of 6 year old children with CP, a positive relationship was found between number skills and fine motor skills and also between working memory and number skills (van Rooijen, Verhoeven and Steenbergen, 2015a). In the latter study, the authors found that “counting mediated the relation between working memory and early numeracy” (van Rooijen, Verhoeven and Steenbergen, 2015a, p.9). However, in a longitudinal study in which children were assessed once a year between the ages of 6 and 8 years, the effect of counting on numeracy and arithmetic skills disappeared (Van Rooijen, Verhoeven and Steenbergen, 2015b). As the children got older, the effects of working memory and fine motor skills were much more closely related to performance in arithmetic. In fact, at 8 years of age, arithmetic achievement could be predicted by fine motor skills and working memory at 7 years of age.

It has been shown that interventions which are designed to improve the function of the upper limbs in children with CP are effective. Consequently, van Rooijen, Verhoeven and Steenbergen (2015b) suggest that it would be useful to explore whether this improved functionality has an effect on the development of skills in arithmetic.

In a study of 7 to 9 year old children with hemiplegia (children who are wholly or partially paralysed on one side of their bodies), Thevenot et al. (2014) noticed that the hemiplegic children did not tend to use their fingers for calculations and had poorer finger gnosis on their paretic (paralysed or partially paralysed) hand than on their non-paretic hand. Thevenot et al. also found that in a test involving counting syllables, children with hemiplegia did less well than their typically developing peers. In addition, in a test involving counting dots as quickly as possible (looking for evidence of subitising), the hemiplegic children were much slower than their typically developing peers, thus suggesting that the hemiplegic children were counting the dots more often than they were subitising.

It seems then that poor finger gnosis and poor finger mobility are often associated with difficulties in work on number and arithmetic. This association appears to be a common finding for children with a very wide range of complex disabilities, suggesting that this is more than just a coincidence. However, the association of arithmetic skills with both finger mobility and finger gnosis provides evidence against the massive redeployment hypothesis, but would support a view that suggests that fingers can play an important role in the development of early number concepts and relationships.

2.3.5 Using fingers and counting in arithmetic calculations

Jordan, Huttenlocher and Levine (1992), in a study of kindergarten children who had received no formal education, identified finger counting as the strategy that distinguished the higher achieving middle income children from the lower achieving lower-income children. When they adjusted for income differences, they suggested that “income-group differences on conventional calculation problems are associated with language and approaches to problem solving rather than with basic mathematical abilities” (Jordan, Hanich and Uberti, 2003, p.366). However, they still acknowledged that the low-income children may have had greater difficulty, not because they failed to understand the verbal input, but rather because they had no strategy for identifying object referents, such as fingers (Jordan, Huttenlocher and Levine, 1992). One year later, when the same children were followed up, the low-income children had caught up in relation to work on number combinations and were using their fingers effectively to help with this. The same children were still, however, underperforming in story problems, which it was argued was due to the children’s continued limited language development (Jordan, Levine, and Huttenlocher, 1994).

Finger counting was associated with higher performance levels on verbal calculation tasks; children in the middle-income group used their fingers to calculate on the verbal problems, whereas children in the low-income group did not. While on further statistical analysis it was suggested that language had a greater impact (Jordan, Huttenlocher and Levine, 1992; Jordan, Levine, and Huttenlocher, 1994), it is clear that both factors clearly influenced the children's ability to achieve in early mathematical problem-solving.

In a later study, Jordan, Kaplan, Ramineni and Locuniak (2008) found that in kindergarten, children who used their fingers in calculations provided more accurate answers to questions. However, by the end of Year 3, those children who tended to be more accurate, used their fingers less frequently than those who made more calculation errors. As in the earlier study, Jordan et al. found that children from low-income families started kindergarten with less confident finger-use than their middle-income peers. Consequently, as the children from middle-income families were beginning to use their fingers less, children from low-income fingers continued to depend on their fingers for performing calculations. This suggests that it takes a considerable amount of time (in the region of 2 to 3 years) for children to move on from relying on fingers to help with arithmetic calculations to confidently using known facts and other strategies to support work with numbers. On the basis of their findings, Jordan et al. suggest that finger-use should be actively encouraged and supported in early years' education.

This use of fingers as a concrete referent, Hughes (1986) suggests, is something that children learn initially from home, and is therefore a cultural artefact. Fingers are first of all used to represent real objects and later to represent numbers, when the more abstract language of arithmetic is introduced. Hughes (1986, p.51) argues that "Fingers can thus play a crucial role in linking the abstract and the concrete, because they can be both representations of objects and objects in their own right." Hughes argues that as children begin to use their fingers more efficiently, the use of fingers supports the idea of one-to one correspondence.

Fuson et al. (1982), observed that once children are able to count-on with calculations involving numbers less than 10, they use several different strategies. When adding two one-digit numbers, children can usually count-on one or two (e.g.

4+1, or 6+2) using their knowledge of the counting sequence. However, once the second addend is greater than two, children usually use some sort of strategy for keeping track. Fuson (1982) noticed that these strategies usually involve fingers, where for example, three fingers may put up in order to calculate a problem such as 4+3 (Fuson, 1982). Other less common strategies often involve a form of double counting, where the second addend is counted (e.g. "five is one, six is two and seven is three"). This is interesting because it suggests that when working memory becomes overloaded (i.e. when the second addend is greater than two), children need some way of off-loading. It seems that if children 'know' their fingers, they do not need to count them when using them to help with keeping track.

Thompson (1995), in a study of children aged between 6 and 8 years of age, exploring the role of counting in children's mental calculation strategies, found that counting strategies often persisted, alongside the use of known facts and derived strategy use. Many of the children observed, used their fingers to aid them with their counting. Children used their fingers to help with 'counting-on' for addition, 'counting-out' for subtraction (counting the minuend and then taking away the subtrahend), 'counting-down-from' for subtraction (the most common strategy for subtraction) and 'counting-up-from' for subtraction (the least common strategy). When children did not have enough fingers, they often used other objects around them such as newspapers or their own legs, to make numbers greater than 10. For counting-down-from, children had a number of strategies that involved keeping track of the count. For example:

Richard found 7 – 3 by saying:

"*7……….6..5..4*."

Whilst putting up three fingers, and Rebecca correctly worked 23-9 by counting backwards starting from 22 and tallying the count on nine of her fingers.

(Thompson, 1995, p.13)

Thompson (1995) observed that the children in his study continued to use counting alongside the mathematical knowledge and understanding that they were acquiring at school. He suggests that counting "comprises a variety of sub-skills…..in that it is often combined with existing skills and knowledge to generate other new skills and knowledge" (Thompson, 1995, p.15). As a result, Thompson argues that the development of these more efficient counting strategies, may support children to understand a range of number concepts.

Although Thompson observed children using their fingers, he did not suggest that their use per se, was of significance. However, their role in counting and aiding keeping track was clearly significant. This role of fingers was explored in more depth by Fuson and Secada (1986). They explored the use of two instructional interventions for counting-on with mixed attaining children in Year 2 and low attaining children in Year 3. One intervention with 107 children, explored counting-on using dots. The other one with 106 children involved the use of finger patterns for counting-on.

For finger counting, children were taught a one-handed method of counting up to nine which was based on "the way children spontaneously use all 10 of their fingers to keep track" (Fuson and Secada, 1986). The method seemed to require the children to touch each successive finger on the table, as they counted on. Fuson and Secada found that the finger counting strategy was effective for all children and that "most children spontaneously related counting-on with finger patterns to their schemas of addition and thus counted-on with finger patterns to solve addition word problems" (Fuson and Secada, 1986, p.256). They found that the children could use the fingers as abstract representations to support them to count on using the known number sequence. They also found that the children learned how to use their fingers to count-on for subtraction. Following the dots intervention, children were not more likely to draw diagrams to help with sequence counting-on for problems such as 5+4. However, the children who had learned finger counting, could easily use their fingers to count on from five to nine using the learned finger patterns to help. It therefore seems that fingers provide more than just a tool, since when an alternative tool was demonstrated, it did not have the same impact, in terms of linking mathematical concepts and strategies, as the use of fingers.

The use of multi-touch devices has been explored in relation to children's number knowledge and counting skills. Working within the context of embodied cognition, Baccaglini-Frank and Maracci (2015), explored the use of two applications (apps) with children aged between 4 and 5 years. These apps were designed for use with iPads and both required finger touch. In an activity called “Ladybug Count (Finger Mode)”, children were required to place fingers on the iPad screen to represent the number of spots on the image of the ladybug. If this is done successfully, the number of spots is announced. For 1-5 spots, the results suggest that children could easily recognise two or three spots and could place the same number of fingers on the iPad screen. For numbers greater than these, children tended to place two or three fingers first and then added the other fingers one at a time. Children usually did not appear to count the dots, but rather used a matching strategy. Where counting did take place, this was either accompanied by children placing their fingers on the screen one at a time as they said the numbers, or tapping the screen the same number of times as the count. Children were quick to recognise repeating patterns. For numbers of spots greater than five, especially in the range 7-10, children first remarked on the fact that there were a lot of spots. Children were able to be successful with these larger numbers of spots by first placing all 10 fingers on the screen and removing fingers one at a time until they received a response from the iPad that indicated that they had placed the correct number of fingers on the screen.

Reporting on the other activity “Fingu”, which required fingers to be placed on the iPad screen to match a floating number of fruits, Baccaglini-Frank and Maracci (2015) reported that the children found it hard to select the right number of fingers within the given timeframe. In this case the children did not have time to recognise particular patterns and match these with particular arrangements of fingers.

During a project in a day care setting exploring the use of the app ”TouchCounts”, Sinclair and Pimm (2015) worked with four 3 and 4 year old children. In this activity an adult would say a number to ‘make’ and the children would try to mathc that number. TouchCounts can be used to produce numbers through serial finger taps or through simultaneous or all-at-once representations. Most of the children discussed seemed to find it easier to make simultaneous gestures using two or three fingers than they did with four fingers. The children often found it more reliable to tap the screen serially to make the number that was requested by the adult. In these cases, the children just had to remember which number had been asked for, in order to know when to stop tapping the screen, as the app provides a running verbal count. After five minutes of playing together with TouchCounts, all four children in the group were able to place four fingers on the screen all-at-once. Sinclair and Pimm (2015, p.107) argue that this is evidence of "a developing finger gnosis about fourness". It seems more likely that the children had learned what the finger-number representations looked like through seeing it done correctly and, through practice, were better able to coordinate their own fingers to create the same arrangements (this is more akin to pattern recognition). The authors suggest that:

> This form of finger gnosis differs from the more ordinally-based differentiation of fingers that is discussed in the psychology literature, but seems mathematically significant as a form of 'knowing about and through one's fingers'.
>
> Sinclair and Pimm (2015, p.108)

However, this seems to ignore the evidence from neuroscience which focuses on the fine-grained detail of individual finger representations in the somatosensory cortex, as opposed to the relational connections described above, that may be thought of in terms of "ordinally-based differentiation". In fact, the activities designed by Gracia-Bafalluy and Noël (2008) to develop finger gnosis are very varied and are not based on any form of ordinal differentiation. While the definition used by Sinclair and Pimm (2015) is certainly significant in relation to embodied cognition, the links with finger gnosis are a bit more tenuous. In terms of the children's developing understanding, perhaps this could be described as a growing awareness of the "fourness of four".

When exploring subtraction and multiplication with older children, Berteletti and Booth (2015b) found that finger gnosis but not finger motor activation was related to performance in problems involving subtraction, in children aged between 8 and 13 years. There was no corresponding relationship with multiplication. This suggests that the children were similar in the way the motor areas of their brains were activated when engaging in subtraction problems, but where they differed was in the degree of activation of their somatosensory areas associated with finger gnosis. Bertelletti and Booth (2015b) also found that as finger gnosis improves, the somatosensory activation becomes more fine-grained and specific to each finger and seems to be associated with better performance in questions involving subtraction.

Even when children do use their fingers for solving problems in mathematics, some problems are more prone to error than others. For example, when working with 6 to

8 year old children, Domahs, Krinzinger and Willmes (2008) found that children made “split-five” errors when doing numerical calculations that required keeping track of whole hands during a calculation (e.g. 7+8). The issue here is, it has been suggested, is with working memory and remembering how many “full” hands are needed for the calculation. These errors are more typical for children and adults who use typical Western finger strategies, rather than the hand formations used in places such as China. These formations are similar to most Western for the numbers 1-5 but are more symbolic (i.e. use hand gestures) for the numbers 6-10 (Morrissey, Liu, Kang, Hallett and Wang, 2016).

Finally, it has been argued that there is:

> …..an age-independent correlation between numerosity comparison abilities and finger gnosis, and between finger gnosis and visuo-spatial abilities…..Thus, children who can easily mentally represent their own fingers will also perform well in approximate numerosity comparison, and also have a high short-term visuo-spatial memory span.
>
> (Chinello et al., 2013, p.389)

However, closer inspection of the graphical data reported in Chinello et al.’s study, suggests that there is a closer correlation between finger gnosis and visuo-spatial skills (such as Block Recall), than between finger gnosis and approximate numerosity comparison, in children (aged 3-6 years). This view is supported by Costa et al. (2011) who found that in children aged 8 to 11 years, there was no relationship between finger gnosis and the ANS. They argue that finger gnosis makes a unique contribution to the acquisition of number skills and understanding where children can use “fingers to transiently represent magnitudes, tagging to be counted objects, and thereby reducing cognitive load necessary to solve arithmetic problems” (Costa et al., 2011, p.10).

It seems then, that whether we take a neuroscientific perspective or a psychological perspective, finger-use in early number activities can be a very important feature to support understanding of number and arithmetic relationships.

2.4 Mathematical learning difficulties

The area of mathematical learning difficulties and disabilities has received more interest in recent years following a number of reports which have highlighted the impact on life outcomes for adults with poor numeracy skills (e.g. Parsons and Bynner, 2005). When these difficulties have been explored, it has been argued that difficulties with mathematics are quite common and show significant individual variation (Dowker, 2004). Dowker has suggested that the difficulties may be evidenced through difficulty with the language of mathematics, difficulty remembering number facts or knowledge of number facts but difficulty knowing how to apply them.

In an attempt to be more specific, Geary and Hoard (2005, p.258) have identified several characteristics of children with mathematical disabilities (MD).

> In comparison to their normal peers, children with MD often: (a) rely on developmentally immature strategies, such as finger counting, (b) frequently commit counting errors, (c) use immature counting procedures [they often use counting-all rather than counting-on], and (d) have difficulties retrieving basic facts from long-term memory.

According to Fuson (2009), children with mathematical difficulties often have difficulties with single digit addition and subtraction, although she admits that it is hard to know to what extent this is due to a genuine difficulty with mathematics and what extent it is due to poor teaching. As can be seen, then, within the literature on mathematical difficulties there is much debate concerning what characterises a child as having difficulty in mathematics and whether there is a root cause to this difficulty.

Geary (2011), Gersten, Jordan and Flojo (2005) and Szűcs (2016) have attempted to review much of the literature on mathematical learning disabilities. Geary distinguishes between those children who can be characterised as having a mathematical learning disability (MLD) from those who have persistent low achievement (LA) in mathematics. “Children who score at or below the 10th percentile on standardized mathematics achievement tests for at least 2 consecutive academic years are typically categorized as MLD in research studies, and children scoring between the 11th and 25th percentiles, inclusive, across at least 2 consecutive years are categorized as LA" (Geary, 2011, p. 251). Geary is keen to point out that, to date, the mathematical difficulties experienced by children and adults with different levels of intelligence have not been found to be qualitatively different. In an unpublished piece of research (Geary et al., 2010, cited in Geary, 2011) it was found that the gap between the attainment of children with MLD and typically developing (TA) children increases as children get older. This is contrary to the gap in reading, which tends to get smaller. On number line activities, in the same piece of research, while the LA children had caught up with their peers by Year 5, the MLD group had still not caught up with their TA peers by Year 6.

According to Geary (2011) children with MLD, and some children who are LA, have a delay or a deficit in number representation and processing. What seems to have been a common finding in a number of studies, is the fact that children with MLD and LA use the same approaches to problem-solving as their TA peers. What distinguishes all the children with MLD and some of the LA children, however, is their persistent difficulty remembering, and retrieving, number facts and procedures, rather than their understanding of the principles of mathematics. For example, a child may know that to solve a problem they have to add two numbers together, but the process of doing this may be challenging.

Yet it seems that children with LA and MLD seem to be part of the normal distribution of attainment in mathematics (Geary, Hoard, Nugent and Bailey, 2012). Following a long-term study of children from Year 2 to Year 6, Geary et al. (2012, p.23) suggest that a core deficit in children with MLD is "some combination of poor access to representations of quantities associated with small sets and Arabic numerals, deficits in the mechanisms that enable children to map numerals to quantities, or poor ability to add these representations". Consequently, these early number issues should be addressed in pre-school settings, before children start their formal education.

Aspects of this issue has been addressed by Dowker and Sigley (2010). Focusing on the counting principles specifically, they argue that Gelman and Gallistel's (1978) order irrelevance principle is usually the last principle to be acquired and that it is not unusual for 6 year old children to continue to struggle with this. Linked to this, is the notion that adding or subtracting items to be counted will affect the cardinal value. Therefore, understanding of the order irrelevance principle is associated with understanding of repeated additions and subtractions of one – i.e. children need to know which changes will alter the cardinal value of a set and which ones will not. This view focuses much more on the development of counting skills as a pre-requisite to the arithmetic that seems to characterise much of the research on mathematical difficulties. Dowker and Sigley (2010), adopting the view that arithmetic skills are not developed hierarchically, argue that a "child may perform well at an apparently difficult task (e.g. word problem solving) while performing poorly at an apparently easier component (e.g. remembering the counting word sequence)" (Dowker and Sigley, 2010, p.67). Consequently, any intervention needs to be targeted and specific. In fact, Dowker and Sigley report that when interventions are specifically targeted, they require less time and are more likely to be successful than interventions that assume all children's difficulties in mathematics are the same.

It has also been observed that children with mathematical difficulties continue to use counting strategies and known arithmetic methods for longer than their peers who develop typically (Ostad, 2008). Moreover, according to Ostad children with difficulties in mathematics perform better in tests which assess arithmetic skills only than they do in tests which assess problem-solving skills. This contradicts the findings of Geary (2011) and raises the issue of knowing what tests are actually testing and whether they are appropriate and meet the needs of the children being tested.

According to Geary (2011), children in Year 2 who have been identified as having MLD or being LA, tend to use their fingers more to keep track of counting in calculations than do their typically developing peers. The delay in development of mathematical skills is approximately one year for the LA children and two to three years for the children with MLD. However, Ostad argues that it is not adequate to describe the children's strategies using descriptions such as "immature", as this does not explain what the issues are and how they can be overcome.

What about the role of number facts? Baroody, Bajwa and Eiland (2009) argue that if children start Year 2 with a poor "number sense" (defined as "meaningful and well-interconnected knowledge about numbers" (Baroody et al., 2009)), they are already severely disadvantaged. However, if children do have a well-developed sense of number, they have a basis on which to build "mastery" and "fluency", as a result of being able to make patterns and connections and build on their existing knowledge. This starts with understanding about number. Thus, if children cannot fully understand the counting principles, they are more likely to have difficulty learning number combinations such as 3+2 (Jordan, Kaplan, Oláh, and Locuniak, 2006). Baroody et al. paraphrasing the mathematician Poincaré, state that: “Mastery with fluency is built up of facts as a house is of stones, but a collection of facts is no more [such fluency] than a pile of stones is a house” (Poincaré, as cited in Baroody et al., p.70).

Geary (1993) suggested that MLDs are characterised by two subtypes of difficulty – procedural deficits (due to “developmental delay”) and retrieval deficits (due to “developmental difference”). Chong and Siegel (2008) explored this further and found that fact fluency (retrieval) difficulties persisted more than procedural difficulties. In addition, they found that children with procedural difficulties also had phonological processing difficulties. Children with MLD were additionally found to have slower processing speeds and working memory difficulties (although these were not specified).

Thus it seems that there are many factors with impact on children’s learning of mathematics. However, if it is the case that children who are labelled as LA or MLD do actually understand the principles of mathematics, as suggested by Geary (2011), do they also understand the principles on which our number system is based? It is worth remembering that in Donlan et al.’s (2007) study children with SLI who were delayed in learning the number sequence and calculation skills, met age-related expectations in their knowledge of the principles of arithmetic. Therefore, it is important to take care and try to be aware of what is being assessed when considering children’s knowledge and skills in mathematics and to remember that difficulties with calculation may be due to a poor understanding of the meaning of numbers (Jordan, Glutting and Ramineni, 2008).

2.4.1 Reading skills, working memory and language development in children with difficulties in mathematics

Gersten, Jordan and Flojo (2005) found that in many studies, children with poor skills in mathematics also have poor skills in reading. In a more recent study of existing literature on MLD, Szűcs (2016) developed this further and argued that the two types of MLD are associated with different aspects of memory – one is associated with visual memory problems, while the other is associated with verbal memory and reading difficulties. Szűcs also suggests that in both subtypes there is likely to be some form of executive functioning difficulties. This, Szűcs argues, can explain why some children seem to have difficulties with both reading and mathematics, while others have specific difficulties with mathematics only.

This has recently been added to by Sowinski et al. (2015) in a study of children, in Years 3 and 4 (aged between 7 and 9 years), who had been studied over 3 or 4 years (depending on their age at the beginning of the study). Sowinski et al. were surprised with the mixed results they obtained. They found that only backward counting (counting back from a named number) and arithmetic fluency (addition and subtraction fluency) were associated with the three areas of: linguistic skills (receptive vocabulary and phonological awareness); quantitative skills (subitising and counting, and symbolic magnitude comparison); and working memory. However, calculation (paper and pencil calculations) and number system knowledge (number line estimation and number knowledge) were only associated with quantitative skills and linguistic skills. This again is evidence of the complexity involved in mathematics and the difficulty in identifying what is and what is not significant, in general and for individuals.

This has been supported by Cowan and Powell (2014). In their study of 7 to 10 year old children, they concluded that both numerical skills and domain-general skills (including all aspects of working memory) contribute to number knowledge and understanding, and arithmetic skills. Cowan and Powell found that, although single-digit processing difficulties were more common in the MLD group, they were always present alongside other difficulties and so could not explain the issues observed. In addition, they argue that estimation (based on number line estimation tasks on a 0-1000 number line) and multidigit skills (e.g. “What number comes five numbers after 49?”), while both related, are also very important in their own right, with regard to their impact on arithmetic skills.

Geary (2011) cites studies which have identified difficulties in working memory capacities linked to central executive capacity, phonological loop and visuo-spatial sketch pad, in children with MLD. These problems could be compounded by difficulties with the inhibition of irrelevant information in working memory in tasks involving information retrieval. There is also evidence to suggest that children with MLD and LA take longer to solve mathematical problems than their TA peers. However, to what extent this is a consequence of a problem with processing speed is not known.

An issue that is often neglected in discussions about difficulties with mathematics, is that of the language of numbers and its association with the base-10 number system (Fuson, 2009). In spoken English, it is not clear that ‘eleven’ is made from ‘one-ten-and-one’ or that ‘twelve’ is made from ‘one-ten-and-two’, even though the written versions of these are based on this notion. Consequently, for children with delays with receptive and/or expressive language development, there is a greater likelihood that the very language used in the classroom will be enough to present a very early barrier to their learning of the base-10 number system and its associated inconsistent language. This suggestion is supported by Cowan, Donlan, Newton and Lloyd (2005) who found that children aged 7 to 9 years with speech and language impairments, were at greater risk for difficulties with number than their age-matched peers and younger children who were matched for language skills.

Fuson (2009) emphasises the importance of visual representations to support understanding of the base-10 number system and arithmetic. These visual representations can take the form of images or fingers which can model a calculation and the strategy used to solve it. The central aim in supporting children to develop strategies that use some form of representation, is to provide them with a means of calculating involving keeping track.

2.4.2 Visual impairments and mathematical learning difficulties

Visual impairments are very varied and can impact in very different ways (Harley, Lawrence, Sanford, Burnett, 2000). Blind children tend to do less well in mathematics than in other curriculum subjects at school (Beal and Shaw, 2008). This, Beal and Shaw (2008) argue is due to the demands placed on working memory and is especially the case when children are asked to solve word problems.

In a study by Crollen, Mahe, Collingnon and Seron (2011), blind and sighted children between the ages of 7 and 13 years, were asked to engage in several activities involving counting and keeping track of items. The blind children tended to use their fingers less often for counting than for finger montring (showing particular numerosities with the correct number of fingers) and when they did use them they tended to use their own idiosyncratic finger representations. In several of the tests involving counting, the blind children did less well than their sighted peers. Both the blind children and their peers had similar finger gnosis. What was missing from this study was any indication of whether the children had been taught to use their fingers in counting activities. Blind children, then, provide insight into some of the issues when working memory is overloaded.

In a later study by Crollen et al. (2014) with early blind, late blind and sighted adults, all the late blind and sighted adults used their fingers for counting, but only a few of the early blind adults did. This time the early blind adults were asked about finger counting and all of them reported that they had never been taught to use their fingers to count. As a result of their findings, the authors suggest that vision is important for a finger-number link to develop. However, as finger counting is a learned process for sighted children, it seems likely that the same could be true for blind children. For blind children though, it is perhaps more likely that they will rely on their finger gnosis, as the visual cue will not be available. Thus without some sort of representation, blind children struggle with mathematics when their working memory is overloaded.

2.4.3 Hearing impairments and mathematical learning difficulties

In a review of 23 studies undertaken between 1965 and 2011, comparing the mathematics achievement of hearing children with hearing impaired and deaf children, it was found that deaf children are generally delayed in their achievement in mathematics (Gottardis, Nunes and Lunt, 2011). Gottardis, Nunes and Lunt (2011) also found that the amount of delay increases with age up to about 16 years and that children with mild hearing loss are less likely to be delayed than those with more significant hearing loss. Even before starting school deaf children are generally delayed in their mathematical development, when looking at mathematics overall. Pagliaro and Kritzer (2012), however, found that pre-school children who were deaf or hard of hearing had age-appropriate knowledge and understanding in geometry, but were delayed by 2 years in their counting skills.

In one report, Gottardis, Nunes and Lunt (2011) found that children who had hearing loss in addition to other special educational needs, were even more likely to be delayed in their mathematics achievement. Interestingly, when Gottardis, Nunes and Lunt looked at studies which explored deaf children's non-symbolic number representations, they found that all the studies found that the deaf children were at least as good, if not better than their hearing peers. This, they argued, might be because they miss out on so much coincidental talk. However, it could also provide additional evidence to question the view that there is a relationship between skills in non-symbolic number representation and attainment in mathematics.

2.5 Assessing knowledge and understanding in mathematics with a focus on number and arithmetic

This brief introduction to assessment in number and arithmetic provides some reflections on the issues which abound when children's knowledge and understanding of number and arithmetic are assessed.

2.5.1 What is mathematical understanding?

It seems important to try to think about this question before reflecting on how knowledge and understanding in number and arithmetic might be assessed.

Hiebert and Wearne (1996, p.253) noted that:

> Efforts to improve our understandings of how conceptual understanding influences skilled performance have been plagued by

> several conceptual and methodological problems. A first problem is that clear definitions of understanding and skill are difficult to formulate and even more difficult to operationalize. The notion of conceptual understanding has been especially difficult to define and measure. Without clear definitions that can be operationalized through assessment tasks, it is difficult to interpret empirical findings.

Following on from this, Hiebert and Wearne (1996, p.253) considered "understanding from both a cognitive and a mathematical point of view. We borrowed from the common cognitive view of understanding as the internal construction of connections or relations between representations of mathematical ideas."

This view of understanding has been developed further by Barmby, Bilsborough, Harries and Higgins (2009) who proposed a model for mathematical understanding where "understanding is built up from connections between mental representations, the connections being made by the reasoning processes that we carry out" (Barmby, Bilsborough, Harries and Higgins, 2009, p.3). This definition is useful as it includes reasoning processes, but it is also a challenge because we do not know how this reasoning should be recognised or expressed. Hiebert and Wearne (1996) found that the children that they termed "understanders" were not only better able to be flexible in their strategy use, but were also able to explain their thinking verbally.

While this seems quite clear, it needs to put within the context of the relationship between conceptual understanding and procedural knowledge. If we use the definition of "conceptual understanding" as the one described above, we now need to define "procedural knowledge". Hiebert and Wearne (1996) describe procedural knowledge as knowledge of procedures which can be applied in order to solve problems in mathematics. They found that children who learned procedures only may at first appear to make more progress. However, these children may also forget the procedures quickly and be less able to apply them in unexpected circumstances which differ from those which have been practised. Rittle-Johnson and Siegler (1998) investigated four possible relationships between what they called "procedural knowledge" and "conceptual knowledge":

1. Procedural knowledge develops before conceptual knowledge.
2. Procedural knowledge develops after conceptual knowledge.

3. Procedural and conceptual knowledge develop concurrently.
4. Procedural knowledge and conceptual knowledge develop iteratively, with small increases in one leading to small increase in the other, which trigger new increase in the first.

(Rittle-Johnson and Siegler, 1998, p.77)

In evaluating a range of studies, Rittle-Johnson and Siegler (1998) concluded that most of the research they explored showed that either procedural knowledge develops before conceptual knowledge or procedural knowledge develops after conceptual knowledge. It was hard to find studies that explored the development of the two processes together or iteratively. It could be argued, though, that it is very hard to know where the boundaries are between understanding and the ability to apply a learned procedure (Hiebert and Wearne, 1996).

It is important to point out that in studies exploring children's understanding, "understanding" has usually been assessed by the children's performance in particular tasks. For example, in the study carried out by Hiebert and Wearne (1996, p.253) defined understanding "for purposes of investigating multidigit addition and subtraction, as the construction of connections between the key ideas of the base-10 number system". Skill was defined as "performing addition and subtraction procedures" (Hiebert and Wearne, 1996, p.254). In recognition of the fact that this is not very satisfactory, Hiebert and Wearne (1996) interviewed the children as they were working on the problems, in order to gain more insight into what they were thinking. The fact that Hiebert and Wearne chose to interview the children in addition to assessing their performance as right or wrong, challenges the view that a conventional experimental approach alone provides rich enough data from which conclusions can be drawn.

2.5.2 What does assessment in mathematics tell us?

Williams and Ryan (2000) found that while standardised tests for 7 year old children were useful in providing some information about areas of difficulty in mathematics, they did little to provide information on the nature of the difficulty. They also found that the children left out many questions and often failed to provide any written evidence of their calculations. Consequently, even if children got the answers correct, it was hard to identify whether or not they actually understood the mathematics. Williams and Ryan also noted that it was hard to distinguish between poor test design and poor mathematical understanding.

Denvir and Brown (1987) carried out a study with children in Years 3 to 6 (aged 7 to 11 years) which explored the impact of environment on outcomes when children were assessed. In the first assessment, a whole class was required to take a formal assessment. Later a group of six children, who had performed at a similar level in the test were asked to answer similar questions, but this time the assessment was more like an interview and included discussion with the assessor. When Denvir and Brown compared their results, they noticed some discrepancies. In order to explain these, they suggested that perhaps when there was discussion, learning might actually take place. They also proposed that testing in a formal environment might yield different behaviours than testing in a less formal set-up. For example,

> Seb wrote in the class assessment: 52 – 36 = 24, but in the interview he said:
>
> S: Sixteen. I took 30 from 50 that gives 20, then took two from the six and took away four from the 20.
>
> (Denvir and Brown, 1987, p.105)

Denvir and Brown explored the factors that they thought could account for the differences in outcomes between the two assessment conditions. They came up with a number of different causes which included: the expectation of feedback in an interview situation; poor short-term memory (important in whole class assessments); greater anxiety in the whole class assessment situation; and inability to see or hear or attend properly in the whole class assessment.

Their conclusions, as can be seen, are very similar to those of Williams and Ryan (2000). They too believed that the more formal assessment provided useful information which could form the basis of a more detailed diagnostic assessment.

The idea of attempting to develop a schema which outlined specific developmental stages was explored by Denvir and Brown (1986a). Denvir and Brown worked with children aged 7 to 9 years. They identified 47 skills and then attempted to explore the links between these (for example, which skills appeared to be pre-requisites for other skills). The skills were assessed through an interview and seven of the children were then assessed regularly over a 2 year period. It was observed that all the children did make progress, but that a less sensitive test may not have identified this, as the progress was very slow. Denvir and Brown (1986b) also argued that this detailed assessment made it possible for interventions to be appropriately targeted.

Standardised tests enable us to observe how children perform in relation to their typically developing peers in a particular test. It is also possible to compare children according to identified trajectories, such as those proposed by Sarama and Clements (2009, p.17):

> Learning trajectories have three parts: a goal (that is, an aspect of a mathematical domain children should learn), a developmental progression, or learning path through which children move through levels of thinking, and instruction that helps them move along that path.

Knowledge of these developmental paths, according to Sarama and Clements (2009) should support teachers to better understand children's thinking and assess their understanding. This approach is also underpinned by a view of mathematical learning as being hierarchical and predictable. However, even if these typical trajectories exist, given that children with genetic conditions such as Down syndrome and Williams syndrome, appear to have atypical trajectories when compared with a typically-developing population (Ansari and Kamiloff-Smith, 2002), it is possible that children with Apert syndrome will also have their own trajectory. More significantly though, the suggestion that children do follow particular trajectories conflicts with Denvir and Brown's (1987) study of mathematical skills in 7 – 9 year old children. Denvir and Brown attempted to provide a framework which could represent children's acquisition of understanding in number and early arithmetic. While they were able to identify a potential hierarchy of skills, the relationships between these skills were quite complex and very interrelated. With the framework created by Denvir and Brown, no clear trajectories can be identified. Finally, in an 18 month longitudinal study of 29 7 – 9 year old children, Jordan, Mulhern and Wylie (2009) found that the children's trajectories were very different in terms of "initial status, final status, growth trajectories, and growth rates" (Jordan, Mulhern and Wylie, 2009, p.466). Taken all together, these findings suggest that learning in number and arithmetic is complex and varied, thus making assessment challenging and dependent on individual children's particular strengths and weaknesses as identified by Denvir and Brown (1986b) and Dowker (2009).

2.6 Summary

The intention of the first part of the literature review was to provide an overview of Apert syndrome and demonstrate its complex and heterogeneous nature. By identifying the range of potential disabilities experienced by children with Apert syndrome, it has been shown that the school environment may present particular challenges for both the children and the adults working with them.

When reviewing the literature on the learning of mathematics, the main focus has been on how children learn about number and early arithmetic. A number of different models have been reviewed, but those cited here serve only to provide a background and not an exhaustive review of the issues. By looking at typically developing children and atypically developing children, it has been shown that early experiences and in particular use of fingers are believed to play a crucial role. It has also been demonstrated that particular disabilities can have significant impacts on the ways in which children can access mathematical learning in the school environment.

Although there is still much that is not known about how children learn mathematics, it seems that children with difficulties with mathematics and who are identified in school as being delayed in their mathematical development need individual assessment, as the specific mathematical difficulties that they may exhibit and the reasons for those difficulties, are not all the same and so their learning trajectories are likely to differ.

3 RESEARCH DESIGN AND METHODS

The review of the literature on Apert syndrome and on the acquisition of early number skills, led to the development of the following research questions:

a) What strategies do children with Apert syndrome, from Reception to year 7, use to help them solve numerical problems in mathematics?
b) Do the children's hand anomalies, with specific reference to fingers, impact the range of strategies available to them?
c) Are there other factors which have an impact on the mathematical learning opportunities of children with Apert syndrome?

In order to explore these questions, I had to reflect on the research framework and methods that would best suit the study.

3.1 Choosing a research framework

Choosing a research framework which best suited what I wanted to explore was not straightforward. As Apert syndrome is a rare condition, I knew that finding participants could be a challenge. I also knew that I would be working with children with very diverse profiles. Given this situation, I needed an approach that would enable me to explore the children's understanding and strategy-use in mathematics, which was also informed by their school and home lives. As I wanted to see how the children changed and developed over time, a longitudinal approach was necessary.

3.1.1 Case Study

I decided on a case study approach, as this seemed to suit the exploratory type of research I wanted to undertake with a very heterogeneous population and the likelihood of only a few participants. The case study approach is particularly useful for issues that will be traced over a period of time, where the focus is on what is happening in the here and now, and may involve interviews and observations of people. In addition, a case study can involve both qualitative and quantitative data, so it allows for a broad range of data collection (Yin, 2009).

I wanted to get an in-depth understanding of the children I was working with, over time and from a number of different perspectives. I wanted children of different ages and with a range of educational experiences. I also wanted to gain additional insights into the children from their parents and from the staff that worked with them in school. In the early discussions about my project, it was suggested that it would be useful to conduct a more experimental-type study, with a control group. However, for my purposes this would have been particularly challenging as it would have been very difficult to decide what to control for with a group of children who presented with varied and complex disabilities. Case studies have the potential to explore situations in ways that are not always possible with quantitative data analysis (Cohen, Manion and Morrison, 2007). In addition, "case studies opt for analytic rather than statistical generalization, that is they develop a theory which can help researchers to understand other similar cases, phenomena or situations" (Robson, cited in Cohen, Manion and Morrison, 2007, p.253). I felt, therefore, that my research lent itself very much to the case study method.

Case study research is particularly useful for asking "how" or "why" questions (Yin, 2009)

Yin suggests that:

> A case study enquiry
>
> - Copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result
> - Relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result
> - Benefits from the prior development of theoretical propositions to guide data collection and analysis.
>
> (Yin, 2009, p.18)

In my study I wanted to find out “how” the children were developing in their mathematical thinking and “why” this might be happening. In exploring this, I needed to gain insight into their early experiences in infancy, their home lives and their educational experiences. I wanted to explore the fine-grained detail that is a particular strength of the case study method (Cohen, Manion and Morrison, 2007).

Yin (2009) highlights the importance of developing a solid theoretical understanding of the issues, prior to starting any case study. This is important, as it is likely to inform the methods and procedures used. In this study, it was important to gain an understanding of the issues from a range of disciplines. As a teacher with more than 25 years’ experience in the classroom, I was reasonably comfortable with exploring the educational literature, but the literature in the fields of psychology, neuroscience and medicine were far more challenging. However, by reading widely and across disciplines, I was able to reflect on the issues I wanted to explore from a range of perspectives. This informed, not only the issues I explored with the children, but also the on-going analysis throughout the study.

The case study approach may not provide evidence from which to make generalisations based on probabilities, but it does provide evidence from which theoretical propositions can be based and to provide analytical generalizations which are based on findings (Yin, 2009). Case studies can also provide insights which may not be obtained through more experimental approaches, as they are incidental rather than planned. The challenge, however, is to represent the evidence fairly and to recognise that this is a potential threat to validity.

Yin (2009) is keen to move away from the notion that case studies are observations that happen in the field, without prior planning and organisation. In doing this, Yin states that it is essential that any case study is an organised and systematic piece of work. In my study, it was not always possible to conduct the work as I had intended, due to other factors that were happening within the schools on the days I visited. For example, it was not always possible to have access to a quiet room for my one-to-one interviews, while on other occasions, rehearsals for school performances meant that lesson observations were not possible. Nevertheless, I always planned in advance and based each new interview on work that had been done previously and tasks that remained to be completed.

Yin (2009) suggests that in order to be skilled in the methods used in case study research, it is necessary for the researcher to be trained. It is hard to know what this training should look like. In my case, I drew on years of working in a number of capacities within the field of education. For example, for over 10 years, I was involved in working with children with special educational needs and consequently, have a lot of experience of doing focused observations of children and of working with parents/carers and school staff at all levels. In order to observe changes in children's learning and other behaviours, it is necessary to engage with the process of analysing and evaluating, as you go along. This same process of analysing and revisiting (Rapley, 2011) was a necessary part of the study and so became incorporated into my methodology. The process of moving from description to "exploring and explaining the underlying essences, patterns, processes and structures" (Rapley, 2011, p.288) was revisited throughout and helped to support the different decisions that were made on each school visit.

The strengths of interviews are that they focus directly on the case being studied and provide insights into aspects such as personal views and attitudes. However, interviews are also at risk of bias from both the interviewer and the interviewee and there may also be inaccuracies when the interviewee is reporting on incidents that have happened in the past. There is also the possibility that the interviewee will say what they believe the interviewer wants to hear.

Direct observations and participant-observation can be useful because of their immediacy and because they happen within a particular context which may also be under investigation. Participant-observation has the additional benefit of providing insight into interpersonal behaviour and relationships. However, both direct observations and participant-observations are very time-consuming and can only cover a selective number of cases. Moreover, it is possible that a person's behaviour will be atypical if they know they are being observed and that there may be bias due to the observer's manipulation of events.

I would have to agree that all of these factors, both positive and negative, impacted on my study at some points and to varying degrees. However, I think that the fact that I carried out all of the observations and interviews myself, meant that I had some control over my role as a researcher. In terms of time, this project was extremely time-consuming, especially as all the children I was working with, lived in different parts of the U.K. The weaknesses in terms of the interviews with the children, sometimes related to “poorly articulated” questions, but more often, when transcribing, I felt that I had missed opportunities to follow up on particular issues that the children had raised. When interviewing the teachers and support staff, I tried to be non-judgemental and to bear in mind that I was a guest in their schools and that they had given up time to be with me and support me with my research. They were often very interested in what I was doing and tried to ensure that I was given opportunities to observe the children and work with them on an individual basis outside the classroom.

3.1.2 Theoretical framework for the study

In deciding the theoretical framework for my research, I considered a range of perspectives. Because I wanted a framework that allowed me to incorporate unexpected and unanticipated findings, I concluded that constructivist grounded theory would provide a suitable starting point.

Grounded theory was first proposed by Glaser and Strauss in the 1960s. This method of qualitative enquiry is based on on-going data collection and analysis in order to elucidate common themes and strands. My data were collected over a 2 year period, but the detail of the data collected was led by my on-going reflections in relation to my interactions and experiences with each child. This reflexive and iterative practice fits well within the grounded theory method described by Charmaz and Bryant (2011).

In my research, though, this was taken further, as there was an acknowledgement that for each child their distinct learning environments and their personal experiences had an impact on their learning and their social and educational inclusion. In addition, I too came to the research with my own knowledge, experiences and views of the world which would necessarily impact on my interpretation of the findings. Consequently, the theoretical framework of the more recently defined “constructivist grounded theory” (Charmaz and Bryant, 2011), provided a more appropriate reference point for this study. This approach acknowledges that the process of data collection and the way data are analysed are both determined by social constructions and existing structural conditions (Charmaz, 2014). In my study, the schools each had their own cultures and their own ways of working - no two schools were the same. For my part, I brought my own baggage to this research. Thus, neither the researcher nor those being researched can be viewed as occupying a neutral position. Any observations and conclusions must be viewed as responses to personal interpretations of the situations observed (Charmaz, 2014).

Constructivist grounded theory requires that data collection methods should be those that are most appropriate for the research question and ensure that sufficient data are collected in order to meet the research goals. Using a case study approach which included a range of different assessments, I was able to revisit my data and respond to each child individually over the 2 year period of the study. This iterative approach to data collection was an important strategy in my methodology.

According to Charmaz and Bryant (2011, p.302):

> Starting with ‘What’ and ‘How’ questions brings an analytical edge to the data collection, even in the very early stages of research, and maintaining the grounded theory emphasis on process helps the researcher to link events that otherwise might seem disparate.

This was very much the position I took, as I wanted to explore what mathematics the children could do and how they engaged with it. I was interested in how the children had come to the methods and strategies they used and to consider what might have influenced these developing approaches and to observe changes over time.

In terms of data analysis, however, I veered away from constructivist grounded theory and took more of a thematic approach, rather than coding my interviews and observations, as is recommended in constructivist grounded theory. For my work this seemed to be more appropriate as a means of exploring trends in dialogic episodes and observations.

Thus, constructivist grounded theory provided me with a useful theoretical framework for my data collection which allowed the possibility of unexpected and unanticipated findings and accounted for my position as a researcher and for the situated positions of the children, families and schools with whom I worked. Thematic analysis, though, was more appropriate for the process of data analysis.

3.1.3 The role of the researcher when working with children

In the last 25 years there has been a shift in thinking on carrying out research involving children. The traditional view of children as lacking competence has moved to one where children should be viewed more as participants with expertise and in-depth understandings (Fraser, Flewitt and Hammersley, 2014). In addition, changing views on children's rights, which were spearheaded by the 1989 UN Convention on the Rights of the Child, have, by necessity changed our views on the responsibilities of those carrying out research involving children (Brooker, 2001). The articles which have had the greatest influence on practice are: Article 12 (Respect for the views of the child) and Article 3 (Best interests of the child) (UN Assembly, 1989). If we take these views on board, it seems to makes more sense to think about research involving children as an example of an opportunity for "data construction" rather than "data collection", thereby acknowledging the fact that the data are co-constructed by the researcher and the participants. After all, the data collection is an evolving process, rather than being something that already exists and is waiting to be discovered.

My role as a researcher within the project required me to reflect on a number of issues. Firstly, as a participant-observer in class, I took on a number of roles, depending on the lesson, the teacher and the needs of the children. I was often in lessons other than my focus mathematics lessons and then I very much took on the role of a teaching assistant. In the mathematics lessons, I wanted to be mostly observing, but this was not always possible and consequently made the task of observing more challenging. I also needed to be aware of the "baggage" I was bringing from both a personal capacity, not least as a parent of a child with Apert syndrome, and a professional capacity, as a teacher with over 25 years of experience working in schools, colleges and higher education.

The role of the participant-observer is recognised as a problem within constructivist grounded theory. In exploring the literature on the role of the researcher, I was drawn to the notion of the researcher as a “bricoleur” with the product of the research being a “bricolage”, as discussed by Denzin (1994). Discussing the work of earlier qualitative researchers such as Levi-Strauss and Nelson, Triechler and Grossberg, Denzin (1994) proposes that the researcher (the bricoleur) has to use a range of skills in order to collect data using a multi-method approach, in order to construct the end product (the bricolage). The methods of the researcher as bricoleur, may change as the research progresses and as the researcher begins to try to interpret their findings. “The researcher-as-*bricoleur* -theorist works between and within competing and overlapping perspectives and paradigms” (Denzin, 1994, p.17). Moreover, “The *bricoleur* understands that research is an iterative process shaped by his or her personal history, biography, gender, social class, race and ethnicity, and that of the people in the setting” (Denzin, 1994, pp.17-18).

The role of the researcher working with children brings with it some particular challenges and ethical issues that need to be addressed. For instance:

> ….how far children can really consent and choose to be part of research, the extent to which they can separate your role as a researcher from that of a friend, and what responsibilities a researcher has to children in the short and longer term, as well as perhaps, more importantly, what benefit is in this research for them.
>
> (Montgomery, 2014)

In my project, all the children discussed the research I was doing with their parents in the first instance and then with me. The children also all gave their consent before I started working with them in school and they were all told that they could opt out at any time and for any reason. During each of my one-to-one sessions, I tried to be sensitive to the children’s feelings about the work we were doing and tried to adapt the sessions to ensure that they did not feel uncomfortable. Nevertheless, I have to recognise that there was inevitably a power imbalance and that the children’s behaviour may have been influenced by factors such as a desire to please the adult (Brooker, 2001). It has also been argued that the inevitable power imbalance leads to adults being effectively “other” in relation to children (Bucknall, 2014). The challenge then, is for the researcher to “consider how they can best meet, or position themselves in relation to, their participants in order to build a rapport and thus help to address ‘otherness’” (Bucknall, 2014, p.81).

It is also possible that the children may have felt pressurised to participate by their parents and this may have influenced their decision to consent. I tried hard to maintain an appropriate professional position, while also realising the importance of building a positive relationship with the children, the parents and the schools. When planning my visits, I checked the dates with the parents first, before contacting the schools. In addition, the parents always told their children when I was visiting, so they were always expecting me when I arrived.

The children were told that I wanted to find out more about how they were doing in maths. I informed all the parents and all the schools about my interest in the use of fingers to support development in number work. As a result, some of the parents and schools worked with the children to improve their finger-use and/or finger gnosis (discussed in more detail later). While this will definitely have made a difference to my findings, I felt it would have been unethical to withhold this information, as it had been so influential in providing a theoretical underpinning to my study, as a whole. Some parents asked for the results of my assessments and when parents gave consent, I also shared this information with the schools.

The last interview was very important for me. With each of the children, I gave time during my last visit to talk about the fact that I would not be coming again. It was important that we had time to address this issue and to say “good bye”, as I had been a regular visitor for about 2 years.

3.1.4 The importance of triangulation

The use of a range of different forms of data and data collection, allowed me to explore the children from many different perspectives and for common lines of enquiry to be identified, explored and analysed. “By developing convergent evidence, data triangulation helps to strengthen the *construct validity*” (Yin, 2014, p.121) of a case study. Nevertheless, in trying to construct credible theories from the data collected, a question still exists concerning the number of participants that is required (Charmaz and Bryant, 2011). For my study, I worked with all 10 children whose parents responded positively to my invitation to participate. All the children stayed in the study until the end of the project. I had the issue, not only of numbers (did I have enough children to make my findings representative?), but also, would the fact that my children varied in age, have an impact on the validity of my findings?

3.1.5 Interviews

> An interview is a moral enterprise. Moral issues concern the means as well as the ends of an interview enquiry. The human interaction in the interview affects the interviewees, and the knowledge produced by an interview inquiry affects our understanding of the human condition.
>
> (Kvale and Brinkmann, 2009, p.62)

Kvale and Brinkmann (2009) propose that an interview is an opportunity for the interviewer and the interviewee to share views, thereby making the interview an "inter view" (Kvale and Brinkmann, 2009, p.2) – a very human activity in the search for greater knowledge and understanding. However, it is important to remember that it is important to create a trusting relationship between the interviewer and the interviewee. This places particular responsibilities on the interviewer when potentially sensitive issues will be discussed. For me this meant thinking carefully about how I asked parents about their children's development and trying not to be judgemental when interviewing school staff. I was also always happy to answer questions that I was asked and I think this was important with both parents and schools.

There are different ways that the role of the interviewer can be conceived. Kvale and Brinkmann (2009) proposed the notions of the interviewer as either "miner" or "traveller". The miner tries to collect knowledge that already exists by digging deep, while the traveller constructs knowledge as he or she travels through, and among, different lands and different peoples. "These two metaphors for the interviewer – as a miner and as a traveler– represent the contrasting ideal types of interview knowledge as respectively given or constructed" (Kvale and Brinkmann, 2009, p.49).

During my interviews, it is probably fair to say that I was attempting both, with a far greater emphasis on the interviewer as a traveller. For example, I wanted information on when the children had had particular operations, but most of the knowledge developed over the time I was working with the children with interpretations and descriptions ever-changing. Moreover, because I had time to reflect on my interviews between visits, I was able to interpret as I went along and thereby ensure that the interviews were part of an iterative and evolving process (Kvale and Brinkmann, 2009).

It is important to remember that there are ethical issues at every stage of a research project involving interviews. Kvale and Brinkmann (2009) identified potential issues at seven stages of the research process:

- When identifying a theme
- When designing the research project, involving considerations such as confidentiality and consent
- The actual interview itself and the impact on the individuals involved
- The process of transcription, especially around issues of confidentiality
- During the analysis of the interview data, concerning issues such as interpretation of statements made
- Verification of the reported interview and whether the questions were appropriate
- Reporting, concerning issues of confidentiality and any consequences for the interviewees

Moreover, if the results are to be valid, then each of these seven aspects must be considered again, with a focus on rigour and fair and honest reporting.

It is important that the interviewees are clear about what sort of questions they are going to be asked before the interview commences (Cohen, Manion and Morrison, 2007). I made sure that my participants were all informed about the sort of information I would be seeking before any visits took place. During the interviews, I had to take on board the fact that it was my responsibility to make the interviewees feel at ease and to develop a good rapport. I also tried to be sensitive to the possible emotional impact of what I was asking or doing (for example, in the case of the finger gnosis assessments) and I always made sure that I gave time for any questions (Cohen, Manion and Morrison, 2007).

3.1.6 Clinical interviews

Although I have already outlined my rationale for the use of interviews, it seems important to consider the role of the clinical interview, as this was an important method in my research, in relation to exploring children's strategies and understanding of the mathematics I did with them. Although I usually used existing assessments as a basis for my interviews, I often extended and adapted them and even asked additional questions, in order to elicit more information about what the children were thinking.

The notion of the "clinical method" (or "clinical interview") was introduced by Piaget (1929) as a method for assessing children's understanding. This method was proposed in response to his observation that standardised tests do not provide insight into a child's thinking. Moreover, Piaget observed that often the questions asked in standardised tests, are not the sort of questions that children would naturally ask themselves (Piaget, 1929). The clinical method is based on the methods used in psychology, where questions are used in order to explore personal situations and, thereby, bring about deeper understanding. Piaget proposed that this method of focused and individualised questioning can enable children to explain and articulate their thinking in ways that are not possible in a standardised test which is the same for everyone.

In adopting Piaget's ideas, and attempting to take them further, Ginsburg (1981) argues that the clinical interview is the most effective method for assessing children's mathematical thinking. Studies of mathematical thinking he suggests can be broken down into three key areas: discovering the cognitive processes children actually use; identifying and describing these processes; and assessing competence. While standardised assessments and naturalistic observation have their uses, Ginsburg (1981) argues that the clinical interview can do more than any of these methods can ever hope to achieve alone.

3.2 Observing learning in mathematics

In order to explore mathematical learning, it is important to investigate mathematical thinking? Here the views of Ginsburg (1981) have been quite helpful:

> At the beginning stages of research it makes no sense to define “mathematical thinking” in some arbitrary fashion; instead a process of discovery must be employed to determine the main developmental features of children’s mathematical thought. As these are discovered – and many findings will surprise us – our conceptualization and definitions of mathematical thinking must necessarily evolve.
>
> (Ginsburg, 1981, p.5)

For me this was a very helpful position to take, as it allowed me construct the stories for each of the children I worked with and be open to the responses they gave.

As I was observing children being taught mathematics in class, it would be useful to briefly consider what mathematical learning might look like. Learning mathematics:

> ……involves learning powerful mathematical ideas rather than a collection of disconnected procedures for carrying out calculations. But it also entails learning how to generate those ideas, how to express them using words and symbols, and how to justify to oneself and to others that those ideas are true.
>
> (Carpenter, Franke and Levi, 2003, p.1)

As I was only able to get a snapshot, at particular points in time, it was going to be a challenge for me to be sure that learning of the sort described above was actually taking place. However, over time, I did see changes in the children’s understanding of mathematical ideas and I did see how they were able to use their changing understanding to develop new approaches to problem-solving.

3.3 Planning the case study

The first decision I had to make concerned the age of the children I would be working with. I decided that children of primary school age should be the main focus of the study because I wanted to explore the children's early experiences of learning mathematics, as these would provide the foundations for their later development.

I knew that I would need to get consent from parents, children and schools and that this may be very time consuming.

3.3.1 Finding participants

As Apert syndrome is a rare syndrome, finding children to work with was going to be a challenge. In order to help with this, I contacted Headlines, a charity which supports anyone affected by craniosynostosis. I explained my research interests and asked if they would be able to send out letters for me to families with children with Apert syndrome. They suggested that, in the first instance I should come to one of their family weekends to talk to some of the parents and carers of children with Apert syndrome. If there was an interest, then they would happily send letters to all the families on their list.

I attended the family weekend in September 2012 and spoke to a large group of parents of children with Apert syndrome. There was a lot of interest in my study and Headlines agreed to send out letters to parents, once the letter had been approved by their trustees (see Appendix A for letter and expression of interest form). As a result of the mailing, approximately 100 parents/carers of children with Apert syndrome on the database of Headlines were sent information about the project. By the end of October 2012, 13 parents/carers of children under 11 years of age had returned the expression of interest form. The distribution of ages of the children was:

2 years old – 1 child (1 girl)

3 years old – 2 children (1 girl and 1 boy)

4 years old – 1 child (1 boy)

5 years old – 4 children (2 girls and 2 boys)

6 years old – 1 child (1 girl)

8 years old – 1 child (1 girl)

9 years old – 3 children (2 girls and 1 boy)

In selecting the children to include in this study, consideration was given to their age, as I had decided that I wanted to focus on children who were attending primary school in the U.K. consequently, I decided to work with the 10 children who were aged between 4 and 9 years of age in October 2012.

I decided that I would try to continue my study of the children for approximately 2 years, in order to observe their development over at least two school years. I had assumed that I would lose some of my participants during the 2 year period that I would be working with them, but in practice, all the families were very supportive from the beginning to the end of the project.

Once these decisions had been made, I was in a much stronger position to identify the methods I would use and the assessments that would support this. I decided that each case study would include:

- Parents' views of how their children were doing at school and how their children were progressing more generally
- Classroom observations of the children doing mathematics
- One-to-one interviews with the children using a clinical interview approach
- Teacher assessments of the children's mathematics, as well as other areas of the curriculum and their general engagement with school life

3.4 Research methods

The research questions were explored using the following methods and assessment tools:

1) What strategies do children with Apert syndrome, from Reception to year 7, use to help them solve numerical problems in mathematics?

Methods included clinical interviews with tasks assessing number and arithmetic skills and classroom observations.

2) Do the children's hand anomalies, with specific reference to fingers, impact the range of strategies available to them?

In addition to the clinical interviews, classroom observations and information from parents and teachers, supplementary assessments of finger gnosis were conducted.

3) Are there other factors which have an impact on the mathematical learning opportunities of children with Apert syndrome?

The investigation of these other factors included child assessments of ANS functioning and working memory.

In order to obtain the information I needed, I made six or seven visits over the 2 year period that I worked with the children. Although I always planned what I wanted to do during each visit, this was not always possible, due to circumstances beyond my control (such as no room being available for the one-to-one interviews; no mathematics lessons due to rehearsals for school plays; or the well-being of the children).

3.4.1 The visits

The first visit was to the family homes of the children I was hoping to work with. The purpose was to introduce myself; talk about the project; begin to get to know the parents and their children and confirm whether they would like to be part of the study. All the parents were interested and agreed to participate. Often, the reason given for wanting to participate was because there was so little known about the educational experiences of children with Apert syndrome and they thought it would be helpful to explore this in more depth. On this visit voluntary informed consent was obtained from all the parents and their participating children (see Appendix B for parent and child consent forms). All the parents and their children were informed that they could choose to opt out at any time. During the interview with parents, I asked for consent to contact the children's schools, in order to arrange to observe the children in school and to meet with school staff.

All the subsequent visits were to the children's schools. Prior to the first school visit, I wrote to the schools explaining what I was doing and to request their consent to work in the school (see Appendix C for a copy of the consent form). Before each visit, I contacted the parents to arrange dates and only then contacted the schools. At each visit, the children knew I was coming and they always seemed to be happy to see me. Where possible, the school visits included: observation of the children in their mathematics lessons; discussions with staff at school with regard to progress and any particular strategies used; individual interviews and assessments of the children. During the lesson observations, the plan was to take verbatim notes, recording conversations and events as they took place. These notes focused on the children's strategies and their relationships and interactions with their peers and with other adults in the classroom. Interviews with school staff were semi-structured and varied in length, depending on the time available to school staff. The interviews with the children were all audio recorded. At first, the children were very interested in hearing the recordings, but this usually became less interesting as the visits became more familiar. I had considered the use of video, but decided that this might be too much of a distraction and could be difficult to set up on each visit. In practice, I was often working with the children in open spaces in their schools, so it is possible that the use of a video recorder would have been very problematic. On each visit, I sought permission from the children to record our sessions, before I turned on the audio recorder.

Although I planned activities for each of the one-to-one interviews, I was always prepared to make a professional judgment to change these if I believed it was in the best interests of the child.

3.4.2 Classroom observations

In deciding to observe the children in class, I wanted to attempt to cause as little disruption as possible to their normal classroom routines. I decided to use verbatim recording of observations of the target children in mathematics lessons. I always tried to sit close to the children and just observe, but this was not always possible. Sometimes, the teachers asked me to work with the children, in order to find out how they were coping with the mathematics they were doing. Very often, the children themselves, and their peers, asked for my assistance during the lessons. In these instances, it was very hard for me to refuse and so I adopted the role that best suited each situation. In my notes, I tried to record occasions where I had any input into the work with the children.

When I visited the children, I usually spent the whole day with them in school. This gave me a chance to spend time with them at break times and lunchtimes and in other lessons. Consequently, I was able to get a better sense of what school was like for them and how they were coping socially as well as academically. I was also able to observe the children's behaviour and approaches to classroom learning; the amount of time on-task; their engagement with pair and/or group work (when this formed part of the lesson plan); any additional interventions.

3.4.3 Interviews with parents

I wanted the interviews with parents to allow for differences in responses. For this reason I used a semi-structured interview style. I did not record the interviews. I had initially wanted to audio-record these interviews, but after the first two parents said that they would prefer it if the interviews were not recorded, I decided to just take notes during all of my visits.

The proforma I used as a support for these interviews can be found in Appendix D

3.4.4 *Interviews with teachers, support staff and special educational needs coordinators (SENCos)*

When I visited the children in school, I tried to see the other adults who regularly worked with them. The interviews I conducted with schools staff varied in terms of the questions asked and the information they could provide. This was because different staff had very different roles and responsibilities in relation to the children I was observing and there were often time constraints which impacted on what could be achieved. I was interested in finding out how the children were managing in school and what, if any, support they had. If the children had the support of a teaching assistant or learning support assistant, I was interested in finding out how that support was used. I also asked about any interventions and input from outside agencies, such as speech, language and communication therapy.

I was interested in obtaining information on school attainment across subjects, using the Early Years and Foundation Stage profile for children from 4 to 5 years and national curriculum level descriptors for children from 6 to 12 years.

3.4.5 *One-to-one interviews with the children*

As a result of the literature review and my experience as a teacher of mathematics, I focused my interviews with the children on aspects that would provide information on their developing understanding of number and early arithmetic. I adopted a clinical interview approach (Ginsburg, 1981), in order to try to gain some understanding of the children’s thinking when they were solving problems in mathematics. The interviews were audio recorded and later transcribed.

In addition to the mathematics-based questions, I asked the children how they felt they were doing in mathematics and about their feelings about mathematics. Some of the children found this hard to answer and, sometimes, I think they gave me the answer they thought I wanted to hear.

For the purpose of reliability, I decided to base the mathematics-focused questions on existing assessments that had been reported in some of the literature. I needed a range of assessments, as I was going to be working with children of varying ages and at varying stages of development. I needed assessments that would focus on number system knowledge, skills in arithmetic and strategies used for solving problems. I also wanted to assess children's ANS, in order to see if there was a relationship between children's skills in this area and their knowledge and understanding in work on number and arithmetic.

I wanted to assess the children's working memory, as this has been implicated as a potential reason for children's low attainment in mathematics. The children's finger gnosis was also assessed, as this was likely to be delayed in children with Apert syndrome and has been associated with knowledge and skills in number and arithmetic.

The assessments used were:

- Counting skills and early arithmetic - using activities adapted from Gelman and Gallistel's (1978) counting principles and Hughes' (1986) box task

- "Number sense" - using activities adapted from Jordan, Glutting and Ramineni's (2008) number sense screening tool

- "Number knowledge" - using activities adapted from the "Number Knowledge Test" (Griffin and Case, 1997) (see Appendix E)

- Number line estimation - using number line assessments adapted from Siegler and Opfer, (2003) and Booth and Siegler (2006) (for examples see Appendix F)

- Measures of attainment for numerical operations and mathematical reasoning using the **Numerical Operations** and **Mathematical Reasoning** components of the "WIAT- II" (Wechsler, 2005)

- Approximate number system (ANS) - using “Panamath” (Halberda et al., 2008)

- Working memory - using the “Working Memory Test Battery for Children (WMTB-C)” (Pickering and Gathercole, 2001)

- Finger gnosis - using an assessment based on that used by Gracia-Bafalluy and Noël (2008)

Only a few of these have been included in the appendices, due to copyright restrictions.

3.4.6 Counting skills and early arithmetic

Using the clinical interview process for these assessments meant that the assessments were attempted rather informally. In all cases, I asked the children to count several groups of objects to look for evidence of Gelman and Gallistel’s counting principles. I chose these as they are the most commonly referred to principles that underpin early counting.

In order to explore the children’s early arithmetic skills I used an adaptation of Hughes (1986) box task. I particularly wanted to explore this aspect of the children’s understanding, as Hughes had identified this aspect of development as being an important precursor to children’s later understanding of the more abstract representations of number and arithmetic that children would encounter later. Instead of a box, I used a little bag. I put a few counters into my bag (usually two or three to start with) and then added one more counter so the child could see. I asked the child how many counters were now in my bag. We would then check to see if the child’s calculation was correct. I repeated this adding two, or sometimes three counters. I then tried taking counters out, again starting with one and moving on to two or three.

3.4.7 Number sense

As has been seen in chapter 2, this is a very contentious term and one that has many different interpretations. However, I will use the term here alongside the definition provided by Jordan et al. (2008, p. 46) “Number sense” in 3 to 6 year old children:

> ….involves interrelated abilities involving numbers and operations, such as subitizing……quantities of 3 or less quickly, without counting; counting items in a set of at least five with knowledge that the final count word indicates how many are in the set; discriminating between small quantities (e.g., 4 is greater than 3 or 2 is less than 5); comparing numerical magnitudes (e.g., 5 is 2 more than 3) and transforming sets with totals of 5 or less by adding or taking away items.

“The streamlined number sense screening tool” (Jordan et al., 2008) was included because it explores early number and arithmetic skills and is suitable for children in the early stages of acquiring knowledge and understanding in number and arithmetic. This test is also underpinned by much of the literature outlined in chapter 2, in relation to knowledge and understanding in number and arithmetic, with a particular focus on trying to provide a meaningful assessment for children who may be delayed in their acquisition of early mathematics. The assessment has undergone considerable scrutiny, in terms of validity and reliability.

Jordan et al.’s (2008) screening tool includes items on counting and number recognition which were assessed by asking the children to: count a group of five objects; recite the number sequence; identify correct counts and miscounts; read the numbers 13, 37, 82 and 124. Knowledge about number comparisons was assessed by asking the children to say which of two numbers were bigger/smaller or closer to another number. Number operations were assessed by asking the children to: use a combination of hidden and seen objects to make calculations such as 2 + 1; solve contextualised word problems (e.g. Jose has three biscuits. Sarah gives him two more. How many does Jose have now?); answer questions presented in formal mathematical language (e.g. how much is three and one?).

When using this screening tool as a basis for my interviews, I would decide where to start based on my observations and existing knowledge of the children. If I made an incorrect judgement, I would make the appropriate changes.

3.4.8 Number knowledge

For this assessment the Number Knowledge Test (Griffin and Case, 1997) was used as a basis for the interviews (see Appendix E). This assessment is designed to be used with children aged 3 – 10 years. The assessment was chosen as suitable to follow on from the Jordan et al.'s assessment described above. The style of questions is very similar and it is referred to in much of the literature on knowledge and understanding in number and arithmetic. The questions particularly focus on knowledge of numbers and the number system, as well as arithmetic. It has also been constructed with key features which demonstrate particular aspects of number knowledge. For example, the question "Which number is closer to 21: 25 or 18?", requires number skills that can cross the tens boundary and demonstrates secure knowledge of relative number size.

The preliminary activity requires children to count to 10. The test items are then split into four levels. At Level 0, children are asked questions involving quantifying and counting small groups of objects. Level 1 questions are designed to assess: knowledge of the number sequence; simple additions with both real objects and numbers represented symbolically; subtraction using mathematical language; magnitude comparisons using numbers represented symbolically. At Level 2, there are more questions exploring number knowledge, but this time with 2-digit numbers. There are symbolically represented addition and subtraction questions with 2-digit numbers and questions concerning which numbers are closer to other numbers (e.g. "Which number is closer to 21: 25 or 18?"). Finally, the Level 3 questions introduce

3-digit and 4-digit numbers and bridging through 99 and 999. There are also questions on differences using 2-digit numbers and calculations that require decomposition and regrouping.

3.4.9 Number line estimation

In chapter 2, it was suggested that numerical estimation is a “common core” (Booth and Siegler, 2006). It has been argued that as children learn about numbers, they move from having logarithmic representations to more linear representations. In other words, when asked to place numbers on a 0-10 number line, a young child may put bigger gaps between numbers at the lower end of the number line than further up the number line. For example, a child will tend to have a bigger gap between 1 and 2, than between 8 and 9, on a 0-10 number line. As I wanted to explore children’s estimation skills to get a broader view of their understanding of number and their number knowledge in relation to the relative positions of numbers on a number line, I used number line assessments adapted from Siegler and Opfer, (2003) and Booth and Siegler (2006). I used number lines within the ranges: 0-10, 0-100, 0-1000 (see Appendix F).

3.4.10 Assessing numerical operations and mathematical reasoning

As much of the literature cited in chapter 2 used some form of standardised test in mathematics, I decided to include this in my assessments. I decided to use the UK version of the Numerical Operations and Mathematical Reasoning components of the “WIAT- II” (Wechsler, 2005), as these are commonly used for assessing children in the UK.

The Numerical Operations component aims to assess “the ability to identify and write numbers, count using 1:1 correspondence, and solve written calculation problems and simple equations involving the basic operations of addition, subtraction, multiplication and division” (Pearson, n.d.). The tests start with questions involving the ability to identify numbers and then to write numbers. Later questions explore rote counting and counting with 1:1 correspondence. Finally, questions are presented in formal written ways that would normally be introduced at school. These are quite limited in terms of their probing of children's understanding. Within the Numerical Operations component, the questions quickly move on to those involving understanding of symbolic notation and the application of formal operations. In this regard, success depends very much on what aspects of the curriculum have been covered. Nevertheless, they provided a standardised score for the children's achievement in the particular components, relative to what may be expected of their peers at a particular point in time.

In the Mathematical Reasoning component:

> The examinee counts, identifies geometric shapes and solves single- and multi-step word problems, including items related to time, money, and measurement in response to both verbal and visual prompts. The examinee solves problems with whole numbers, fractions or decimals, interprets graphs, identifies mathematical patterns, and solves problems related to statistics and probability.
>
> (Pearson, n.d.)

Although these questions are also quite limited, the fact that many of them are supported by diagrams, means that there is more of an opportunity for children to talk about what they can see. This can help with trying to understand how the children are trying to make sense of the problems they are asked to solve.

3.4.11 Approximate number system (ANS)

As already discussed in chapter 2, the ANS has been widely reported in the literature as being associated with children's knowledge and understanding of number and arithmetic. As Panamath (Halberda, et al., 2008) has been widely used in these studies, I decided to use it in my own research. I downloaded Panamath onto my laptop and this enabled me to take it to all of my visits. There is no recommended screen size on the website and no indication of any adaptations that have been made for children or adults with visual impairments. Consequently, although I used Panamath, I am not sure whether the screen size and the children's own visual impairments impacted on any of the results obtained.

3.4.12 Working memory

As working memory has been widely cited in the literature reviewed in chapter 2 as being a common area of underperformance in children who are low attainers in mathematics, I wanted to include an assessment of this in my study. The Working Memory Test Battery for Children (WMTB-C) (Pickering and Gathercole, 2001) is commonly used in the UK by educational psychologists and teachers to assess children's working memory capacities. The WMTB-C includes assessments of phonological loop (verbal short-term memory), visuo-spatial sketchpad (visual short-term memory) and central executive. As it is very broad, in terms of the aspects of working memory and central executive that are studied, it seemed to be a good choice for this study.

For the phonological loop assessments, I initially tried all the tests, but I found that it was hard to know exactly what the children had heard (due to their hearing impairments). In addition, even if the children had heard the words and nonwords correctly, it was hard to know if they could correctly articulate all the sounds (due to their articulation difficulties). Consequently, the tests involving words and nonwords were not used. This left just digit recall for the assessment of the phonological loop. As number words are more distinctly different from each other, the children seemed more confident with the tests involving number words. For visuo-spatial working memory I used the Corsi block recall task and for the central executive assessment, I used counting recall and backward digit recall.

3.4.13 Finger gnosis

As has been seen in chapter 2, it has been suggested that there is a relationship between finger gnosis and number skills. Children with Apert syndrome have limited finger mobility due to the fact that they are born with their fingers fused. In addition, the fact that these children also have several operations on their hands during the first four or five years of life, the development of the representations of their fingers in their brains is likely to be delayed and unlike those of typically developing children. At a very basic level, the fact that children with Apert syndrome do not have typical finger pads undoubtedly affects the way that they experience the world through touch. If a relationship does exist between finger gnosis and the development of number skills, it makes sense to explore this in children with Apert syndrome.

In order to assess the children's finger gnosis, I used an assessment based on that proposed by Gracia-Bafalluy and Noël (2008). When assessing finger gnosis on each hand, I started by covering the hand being assessed with a piece of paper, so that the child could not see it. I touched one finger and then removed the paper so that the child could show me which finger I had touched. I tried each finger in each hand at least twice. This was then followed by a similar process, but this time I touched two fingers at one time.

3.5 Managing the data

All the interview and test data were safely stored. The paper-based tests and field notes were placed in separate files for each child and stored in a locked office. The audio interviews and transcripts were stored on a password protected workplace computer. In the analysis, pseudonyms have been used for all the people concerned.

3.6 Data collection

Case study "often involves relatively open-ended, exploratory approach, in which the aim is to document the distinctiveness of particular cases as well as their similarities with one another" (Hammersley, 2014). Mine was very much a study of individuals and so the collection of a range of data was necessary in order to triangulate and attempt to see things from a range of perspectives and in a range of voices.

3.6.1 Transcription

I transcribed all the interviews. I tried to do this as soon as possible after the interviews had taken place. I found that by transcribing the interviews myself, rather than paying someone else to do it, I was able to reflect on each interview and see some of the subtleties I had missed during the actual interview process. I was able to really focus in on the children's voice and reflect on their actions. I could also see, for example, when there had been long pauses and when I had missed opportunities to delve further into a child's understanding.

I audio recorded the WMTB-C assessments. This was useful and provided additional information when I reviewed the children's results.

3.6.2 Assessments

All the verbal assessments were audio recorded to allow for more detailed analysis. Results from the Panamath assessments were saved. In cases where the children had attempted Panamath more than once, I took the best results for that day.

3.6.3 School-based assessments

Whenever possible I collected data from schools on how the children were progressing. This was not consistent, as schools had different systems in place for collecting and recording data. More importantly, though, it was not always possible to get this information from the schools, because the staff often did not have time to locate the information on all of the days I visited.

3.6.4 Observations

During my observations of the children, I took verbatim field notes of what I saw and heard. I tried to take note of interactions that the children had with their peers and with other adults. I was also interested in how the children manipulated any physical resources and in anything they wrote down. I often observed the children during their break times, but at these times I did not take notes at the time, but rather when we returned to the classroom. When analysing this data, I attempted to identify key moments and events which I felt provided insight into the children's learning and behaviours.

3.6.5 Effect of the researcher

During the time I worked with my focus children, I built positive relationships with the children, their parents and their schools. The quality of my interactions will inevitably have been affected by the strengthening relationship, but I think in all cases this served to create a more relaxed atmosphere, where the children felt able to answer my questions as honestly and openly as possible.

During my observations, the children knew I was there and they knew I was there to observe them. Therefore it is likely that the "Hawthorne effect" will have resulted in behaviours that were perhaps not typical.

The fact that I was a teacher I think helped the teachers feel that they were talking to someone who understood some of the tensions that exist in trying to meet the needs of all the children in a class. They also knew that I understood the rhythm of the school day and the rhythm of the school year.

Finally, I was very mindful of the fact that my position as a parent of a child with Apert syndrome would impact on the way parents, children and schools would view me. This, though, brought with it additional ethical issues, as I still needed to maintain an appropriate level of professionalism and distance in all my relationships.

3.7 Data analysis

> There is no particular moment when data analysis begins. Analysis is a matter of giving meaning to first impressions as well as to final compilations.
>
> (Stake, 1995, p.71)

This very much expresses the process I experienced with the analysis of my data. Each time I met with any of the participants in my research, I was mindful of my aims and the open structure of my dialogues allowed me to follow-up on questions both as they arose and in subsequent meetings. Thus the on-going, informal data analysis served to inform aspects of each meeting. The emphasis on this iterative process during research is also fundamental to constructivist grounded theory.

According to Stake (1995), case studies can be explored in two main ways. They can be explored as individual cases through “direct interpretation” which then allows for an “aggregation” of properties within each case and across different cases. With the children in my study, I wanted to explore each child in depth and identify any patterns or relationships in their development over time. I also wanted to then compare all my case studies to try to identify any common themes or strands that they shared.

After each visit, I transcribed the interview, reviewed my field notes and looked over any assessments I had done. As I transcribed each interview, I began to identify gaps and questions that I wanted to follow up in later visits. I also reviewed my notes from my meetings with staff, in order to identify areas of questioning which may have been missed. As I did this, I made notes to highlight points that seemed particularly significant. Thus some element of data analysis was on-going and iterative. This on-going analysis was also informed by my initial review of the literature, but as time went on and I read more, the detail of my analysis became more refined and fine-tuned, enabling me to explore more of the detail in the children's responses and to find areas that had not been explored in the literature in such depth.

The process of analysis was challenging and, as stated by Cohen, Manion and Morrison (2007, p.368):

> The great tension in data analysis is between maintaining a sense of the holism of the interview and the tendency for analysis to atomize and fragment the data – to separate them into constituent elements, thereby losing the synergy of the whole, and in interviews often the whole is greater than the sum of the parts.

As I analysed the data, it became clear that the children's responses could not easily be reduced to just a few sentences. Rather, in order to understand the depth of the conversations, it was necessary to include complete episodes. In this way, it was possible to present aspects of the children's personalities and observe their development over time. Over the 2 years that I worked with the children, I built relationships and watched them grow and change. It was important for me that the personalities of the children came through, as this was part of who they were and, therefore, an important part of the "context" for my analysis.

3.7.1 Interviews with the children

My first attempt to analyse the data as a whole took place during the spring, summer and autumn of 2014. During this time, I reviewed all my raw data (including transcripts) and my notes. This helped me to decide which children I was going to focus on. I chose the five children for whom I had the richest data, in relation to my three research questions.

My methods of data analysis were drawn from discursive analysis and thematic analysis, in order to identify themes to exemplify features, patterns and relationships that were relevant to my three research questions. The discursive approach enabled me to pay “more respectful attention to, and faithful representation of, participants’ meanings and understandings” (Wilkinson, 2000, p.454), rather than imposing my own. In thinking about themes, I was cognisant of the need to choose extracts carefully, in order to ensure that they illustrated the points being made (Braun and Clarke, 2006). With the transcripts for each child in chronological order, I read and reread them, in order to become more familiar with the data. This iterative process, enabled me to get a sense of each child’s journey, and to begin to identify extracts of dialogue that seemed to be mathematically significant, in terms of what I could infer about the child’s understanding and approach to problem-solving within the context of the mathematics we had explored. This was broadly a process of identifying themes according to “theory-driven” criteria (Braun and Clarke, 2006). In other words, although I was exploring each child’s mathematical journey and did not know what I would find in terms of the children’s mathematical development, I was working with the background knowledge of certain theories about mathematical learning that have been discussed in chapter 2. I therefore had some models to help identify particular characteristics of mathematical development in order to begin to explain the emerging trends in my data. The aspect that was not so developed in the literature, related to the development of the link between finger gnosis and finger mobility and understanding and skills in number and arithmetic.

A key focus of my analysis was an exploration of the children’s methods used during the interviews. This meant that transcription also had to include notes on the methods used by children to help them work out their solutions to the problems posed.

Analysis of the transcripts also enabled me to identify changes in other areas, such as language development, which were not my focus, but which made a difference to how the children were able to approach the problems posed and articulate their thoughts. As these themes emerged during the process of analysis for each individual child, they were noted and then checked for all the children.

Once this process had been completed, I was able to reflect on what was missing from my data. This reflects the theoretical sampling technique within constructivist grounded theory (Charmaz and Bryant, 2011). Following this I arranged to visit the children again, in order to complete my data collection and then finalise the analysis of the interviews, repeating the process outlined above. This process helped me to choose the five focus children for whom I had the richest data, in relation to the research questions.

My five focus children were spread across the age range and included four girls and one boy. At the beginning of the study, the girls were aged 5 years (Emily), 8 years (Isabelle), 9 years (Hannah) and 9 years (Tania) and the boy was aged 5 years (Luke). All the names are pseudonyms.

3.7.2 Interviews with parents and staff

The interviews with parents were reviewed to try to identify themes within each interview and also across the interviews. The interviews with staff have been used to provide some background information and have not been analysed in great depth. This was done in order to maintain the ethical position that the teaching and the provision was not being scrutinised. My role in the school was to observe the children and any information provided by the school was provided voluntarily and in confidence.

3.7.3 In-class observations

In-class observations were analysed in a similar way to the interviews, where the search was for opportunities to identify moments of significance, in relation to the mathematics that the children were engaging with. Again themes were identified for later analysis and for comparison with the one-to-one interviews that I undertook. Although aspects of interactions with the staff have been identified, these are rarely discussed in a critical manner, in order to maintain the ethical position of the researcher.

3.7.4 Assessments

The standardised assessment scores were analysed in relation to what they told me about each child's learning at a particular point in time. When exploring the data, I also reflected on the strategies and processes the children used.

The tests were carried out according to the instructions in the relevant manuals. For the WMTB-C all the practice trials were used when carrying out the tests. I moved on after four correct trials and observed the discontinuation rule if three or more errors were made in one block. For the WIAT-II Mathematical Reasoning test, I observed the discontinuation rule if three consecutive errors were made. For the WIAT-II Numerical Operations test, I told the children that they should complete the parts that made sense to them. Consequently, the children said when they could not complete any more of the calculations (usually because the calculations were presented in a way that was unfamiliar to them). If I felt that they could do more, I asked them to move on to questions which I knew they were more familiar with.

For Panamath, I was particularly interested in the children's final scores and their reaction times. I also looked for any changes over time.

The number line estimation tasks were analysed individually, but also over time, so that changes in understanding could be more easily identified.

3.7.5 Putting all the results together

Once I had analysed all my data, I put all the information together for each child in chronological order. This helped place my interview notes and observations within the context of the views of the staff that were working with the children at the time. I was then able to look across the results for all the children to attempt to identify any patterns or similarities that were common for some or all of the children in the study. In other words, were any of my findings generalizable for the children? Had I found anything that could be described as a common feature of mathematical development in children with Apert syndrome?

3.8 Ethical considerations

In conducting this research, BERA's (2011) guidelines were adhered to. Many of the issues covered have already been discussed, especially issues relating to consent and confidentiality.

A key aspect of my work with the children was my attempt to always act in the best interests of the children. Following BERA's guidance (point 16, p.6)

> … in all actions concerning children, the best interests of the child must be the primary consideration. Article 12 [of the United Nations Convention on the Rights of the Child] requires that children who are capable of forming their own views should be granted the right to express their views freely in all matters affecting them, commensurate with their age and maturity. Children should therefore be facilitated to give fully informed consent.

By trying to act in the best interests of the children and treat their informed consent with the respect it deserved, I often had to stop assessments early (or not even start them). The children very rarely said that they did not want to continue, but if I felt that they were getting tired or needed a change of activity, I made the decision to stop, even if this meant that my data would perhaps not be complete.

4 THE CHILDREN, THEIR SCHOOLS AND LEARNING MATHEMATICS

This chapter aims to bring together the views of parents and schools within the context of the children's learning of mathematics.

Throughout the discussions, extracts from my transcripts will be used. In all of these, the initials of my first name (C) and the initials of the children's first names have been used. Where other people have participated, their full names have been included. In the transcripts, pauses greater than 4 seconds have been specified. Where there was a very short pause between words, three dots (…) have been used (e.g. Umm…four). Where there was a pause of up to 4 seconds, "[pause]" has been inserted. Finally, all transcript extracts represent conversations which took place without a break.

4.1 Analysis

The analysis is presented in two stages. The first stage of the analysis presented in this chapter provides a detailed analysis of each child in chronological order. In chapter 5, these analyses are brought together, to identify any common themes and patterns across all the children.

Each child will be considered individually in order of age, beginning with the youngest child. At the end of each discussion of individual children, some reflection on their developing understanding of mathematics and the strategies they used for numerical calculation will be presented. The children are Luke, Joe, Emily, Isabelle and Hannah. All the names used are pseudonyms.

The discussion of each child will begin with a summary of the interviews with parents. These are then followed by extracts from each school visit and include in-class observations, interviews with school staff and interviews with the children.

Extracts of the interviews with the children are presented in order to identify significant aspects of dialogue and actions that provide data relevant to the research questions. By approaching this from the point of view of discursive analysis, these extracts provide more than an analysis of the children's utterances, they also situate them clearly within the context of a dialogue with the researcher. This additionally provides an opportunity to observe more subtle changes over time, especially in relation to the development of language and communication skills of individual children.

In-class observations are reported in relation to the research questions and to provide a sense of how the children experienced their mathematics learning environments. The extracts and descriptions from the in-class observations are taken from the notes that were made during lessons. As far as possible these are accurate records of what was observed, but observer bias cannot be ruled out.

Interviews with school staff are summarised, in order to highlight the key points raised. Results from the standardised assessments and Panamath (Halberda et al., 2008) are presented within the context of the relevant school visit.

Finally, for each of the focus children a summary of the findings in relation to the three research questions is presented at the end. The other factors that were considered include:

- mathematical development
- the role of fingers in supporting work on number and calculations
- working memory
- visual impairments and hearing impairments
- language development
- home environment
- school provision

However, each child was very different and so there is some variation in terms of the factors that were explored for each child.

4.2 Luke

My first visit to see Luke was at home, in November 2012. He was 5 years old and attending his local special school, in Year 1. Luke's school catered for children from the ages of 2 to 19 with complex learning difficulties. Before going to the special school, Luke attended a local nursery school.

During the visit I met with Luke, Luke's mum (Cath) and his older sister (Tess), who was then 9 years old. Luke had been late reaching many of the early milestones. For example, he was crawling at about I year of age, pulling himself up at 18 months and walking at 2 years and 4 months. Luke had been very late to start talking, but could understand most things that were said to him. He had been using Makaton since he was between 9 and 12 months of age. Luke had started to speak much more during the last 6 months, but was quite hard to understand, due to structural difficulties caused by his cleft palate and airways. Luke generally spoke in very short utterances, but full sentences nevertheless.

At the time, Luke had mild to moderate conductive hearing loss in both ears. He had been wearing a bone conduction hearing aid, but, according to Cath, it did not help much. He was now trying the more conventional "behind the ear" hearing aids, but was only wearing these at school. Luke also wore glasses. His right eye was much worse than his left eye and, at the time, he was wearing a patch on his left eye for two hours a day to try to improve his eyesight in his right eye.

Following over three years of surgery, Luke had five fingers on each hand. While I was at Luke's house, he played with his sister, Tess. They played with cars and later watched television together. Cath said that Tess was very protective of Luke and played with him a lot at home. Luke used both hands while playing and eating. Luke was also wearing glasses.

Luke had no corpus callosum and Cath had been told by doctors that this was affecting his balance, especially in the dark. Cath encouraged Luke to play on their trampoline, in order to help with this. Luke was happy to play on his own, but more recently he had started playing football and joining in with other activities with other children, especially at school.

4.2.1 First school visit (January 2013)

Luke was in a mixed Year 1/2 class.

I arrived just after the morning break. The children came in from the playground and had a snack around a table. During this time Luke used Makaton signing, in addition to spoken words. There were 10 children in the class and they all had very different special needs. There were 4 adults in the class, one teacher (Jill) and three support staff.

<u>Observation</u>

The maths lesson began with a starter activity where the teacher asked the children to show her:

Five fingers
Four fingers
Three fingers
Two fingers
One finger

These were done in order and for each of them the teacher modelled and the children copied.

Luke appeared to try hard to show Jill the correct number of fingers. He looked very hard at his fingers for each of the different arrangements. For arrangements of three fingers and two fingers, he had to hold down the irrelevant fingers with his other hand.

Jill then asked the children to make ten claps, ten stamps and ten whispers. Luke joined in with all of the activities with confidence and enthusiasm. This was followed by whole class singing of number songs with actions. Luke knew the words and the actions.

Jill introduced the main activity which she said was to do with patterns. Jill laid out five red and yellow bricks on the table (Figure 4-1)

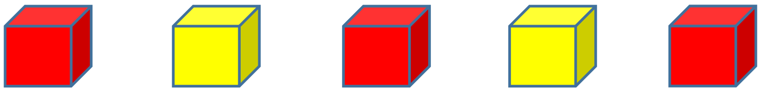


Figure 4-1: Five red and yellow bricks

Jill asked Luke what brick would come next. Luke picked up a red brick and placed it down next to the yellow one and then picked a yellow brick and placed it next to the red one he had just placed earlier. The teacher then asked him what brick would come next. Luke did not respond, so the teacher described the pattern, “Red brick, yellow brick, red brick, yellow brick, red brick, yellow brick….”

Luke then responded with “Red brick.”

Jill repeated the sequencing activity with a range of different objects, such as animals and cars, as well as the bricks. Luke appeared to be focused and joined in when asked a question. However, when the sequencing involved coloured cars, he found the task more difficult. For example, Jill laid out four cars on the table. Luke then independently asked what the next car would be. Jill got Luke to work it out by asking him questions relating to the colour. This was more of a challenge for Luke than the earlier tasks with the blocks. Perhaps this was because the cars were not all the same. Luke had a particular interest in cars and so may have focused on a range of features, not just the colour. With the cubes, however, each cube was exactly the same apart from the colour, so it was easier to decide which was the significant feature. This issue highlights a difficulty with attempting to present problems in “realistic” contexts. Often these problems are not as realistic as we think they are. In addition, the expected solutions do not allow for variation, due to the range of interpretations and problem-solving strategies (Inoue, 2005; Verschaffel, De Corte and Lasure, 1994).

For the next 20 minutes, Luke worked with a teaching assistant. The teaching assistant asked Luke about what would come next with coloured hoops and with blocks. Luke was correct each time. She then tried him again with blue and yellow cars, laid out in alternating colours. This time when he was asked what should come after the blue car, he said, “A blue car.” He was asked to think again and still he wanted to put another blue car next to the one that was already there.

At times, Luke needed to be refocused, but in the main, he stayed on task.

At the end of the lesson, the children came together for another counting activity. This time they had to count their fingers. Luke appeared to concentrate very hard on counting his fingers one at a time, touching each finger once and once only with his index finger, first on his left and then his right hand. When he had counted them all, he announced, “I’ve got ten fingers!” He was praised and the lesson ended. Luke and I then left the class and went to a smaller room where I carried out my first interview with him.

Later, during lunchtime play, I observed Luke playing with other children in the playground and riding a tricycle.

Staff views

Jill described Luke as a delightful child with a great sense of humour; always happy to learn and a joy to teach. He tended to develop interests in particular objects and would then want to play with them all the time. She felt that his levels were similar across all the subjects and that he was currently working towards level P5 (P-scales). Luke’s greatest difficulty was with attention and concentration. Luke loved music, singing and swimming. His best friend was a girl in the Reception class and Jill said they played very well together and looked out for each other, but she said that he did not have any particular friends in his own class.

Jill was very interested in finger gnosis and said she would try to develop activities to develop finger gnosis for all the children in her class.

Interview

Much of the first interview involved counting skills. At the beginning of the interview, I put five counters onto the table:

C: Can you count the counters?

L: One...two...three...four...five [raises voice] [touches the counters as he counts]

C: Fantastic. Now if I pick them up

L: Yes

C: If I pick them all up and I put them in my hand [I pick up the counters and put them in my hand]...How many have I got in my hand? [Pause] Do you remember?

L: [says nothing]

C: Do you need to see?

L: Yes, please

[I opened my hand]

L: Yeah

C: How many are there?

L: Five [raises his voice and shows 5 fingers without counting]

I then put three counters onto the table:

C: Luke, Can you count…How many is that? [as I point to the counters]

L: One, two, three [counting slowly and touching the counters as he counts]

I: Fantastic. Now if I put these three [as I collect them in my hand]

L: Yes

C: How many have I got here, Luke? Luke, how many have I got? [with the counters in my hand]

L: One, two, three [counts slowly, touching the counters as he counts]

C: Fantastic. Now if I put them into a bag [I put the counters into a little bag] how many did I put into the bag?

L: One...Can I see?

C: Do you need to see again?

L: Yes, please.

C: OK. Let's see once more

L: Yeah

[I tip the counters out onto the table]

C: How many are there?

L: One, two, three [counts slowly, touching the counters as he counts]

In this extract, Luke demonstrated that he understood the significance of the last number in the count, as representing the cardinal value. However, he appeared to understand the question “How many?” as an instruction to count. It is possible that Luke has not yet integrated counting with the idea of cardinality (Fuson, 1988).

In an attempt to explore this further, I put the two counters into my little bag and tried adding one more:

C: How many am I putting into the bag?

L: One, two

C: OK, so I’ve put two into the bag. If I put one more in [I put one more counter into the bag]

L: Yes

C: How many have I got in my bag?

L: One, two, three

C: You think I’ve got three?

L: Yes

Having done this with such confidence, the next episode demonstrates that Luke was not always able to make the calculation required:

C: Right, if I put those into my bag

L: Yes

C: How many am I putting in? [I put the counters into the bag]

L: One, two, three

C: If I put one more into my bag [I put one more counter into the bag]

L: Yes

C: How many do I have in my bag?

L: I need to have a look?

C: How many do you think I've got?

L: One, two, three

C: You think there are three?

L: Yes

C: Let's have a look [I empty the counters onto the table] How many are there?

L: One, two, three

C: Check

L: Three

C: Check, count them again

L: One, two, three…oh no, no...one, two, three, four [raises voice] [touches the counters as he counts]

And yet this was followed by:

C: Now if I put the four...how many are here? [I point to the four counters]

L: One, two, three, four [raises voice] [touches the counters as he counts]

C: Now if I put the four counters into the bag

L: Yes

C: How many have I just put into the bag?

L: One, two, three, four [raises voice]

C: Good boy. If I put one more in [I put one counter into the bag]

L: Yes

C: How many have I got in my bag now?

L: One, two, three, four, five [raises voice]

These episodes demonstrate an interesting development. At first it appears that Luke may have had a lack of trust in the counting process (Cowan, 1987), and was not confident to answer without actually seeing the counters. However, as we continued to work together, his confidence grew and he was able to work out how many counters were in the bag without seeing the actual counters. Luke's persistence with counting from one, may represent an example of Fuson's (1988) "unbreakable list". In order for Luke to know what came after four, he needed to recite all the counting words from one.

After these successes, I thought I would try taking one counter out:

C: Fantastic. So if I take these five

L: Yes

C: How many are there here again? [I point to the counters]

L: One, two, three, four, five [raises voice] [touches the counters as he counts]

I: If I put them in my bag [spoken as I put them into the bag]

L: Yes

C: This time I'm going to take one out

L: OK

C: [I take out a counter and show Luke]. This one is going out [as I place it on the table]...How many do you think I've got left in the bag?

L: One, two

C: Do you think I've got two left?

L: Yes please

C: Do you want to have a look?

L: Yes please

[I tip the counters out]

C: How many are there?

L: One, two, three, four, five [raises voice] Ahh [touching the counters and double counting the first counter]

C: Count them again

L: One, two, three, four, five [spoken very quickly and not touching the counters]

C: Where's five?

L: Ahh

C: Count again

L: One, two, three, four [raises voice and giggles, touching the counters as he counts]

This apparent teasing by Luke has been discussed by Gifford (2005) as a way of enabling children to take control. It also suggests that Luke did actually remember that I had originally put five counters into the bag.

I felt that Luke was getting a bit tired. He had been focused for the 15 minutes that we had been working. We took a break and then tried some activities involving reading numbers. Luke could read all the numbers from 1 to 10, but when I asked him to read some two digit numbers, it became hard to understand what he was saying, so I decided to stop.

During this interview, Luke demonstrated that he could confidently add one onto small numbers when they were presented in a situation that he could see and could make sense of. This was observed by Hughes (1986) when he worked with 3-5 year old children in a nursery setting. In this interview I only explored adding cubes, but Luke behaved in exactly the way that Hughes' work suggested. However, unlike many of the children Hughes observed, Luke did not use his fingers to model the problem, he did it without any concrete representations.

The picture with subtraction was not so clear. When I took a counter out of the bag (I put five counters into the bag and took one out), Luke clearly had an idea that the number would be less than the number we started with (he said "two"), but he seemed to be "estimating" rather than calculating. This demonstrates an awareness of the process of subtraction as "taking away". This may have been due to Luke's lack of familiarity with subtraction and how this relates to the number sequence. If Luke had estimated his answer, rather than working it out, it might help to explain why when he counted, he did not appear to be surprised that he obtained a different answer. Could this be in some part due to a lack of confidence in the reliability of the counting process (Cowan, 1987) or due to an incomplete understanding of one more/one less (Cross et al., 2009, as cited in Gifford, 2014)?

4.2.2 Second school visit (May 2013)

On my second visit, I arrived in the morning, after break.

<u>Observation</u>

The children were sitting around a table with their teacher, Jill and three teaching assistants. The maths lesson began again with a “show me” activity using the numbers one to five. For “Show me five digits,” Luke showed his full left hand. For, “Show me four,” Luke put his thumb in. For, “Show me three,” Luke seemed to struggle, as he was not able to move his index finger or little finger down, like Jill. Luke had similar difficulties with the number two. For “Show me one,” however, Luke was able to individuate his index finger and lower the other fingers.

This activity was followed by ten stamps and ten whispers, all of which Luke participated in fully. There were then several number songs based on the number five, which involved counting up or down. Luke seemed to enjoy the songs and could remember the words. However, during all of the songs, Jill modelled the numbers with her fingers and clearly wanted the class to copy her. While most of the children did copy Jill, Luke did not use his fingers for any of the songs.

The main activity on this occasion was focused on “full and empty”. The teacher used a range of containers and asked the children whether the containers were full or empty. Whenever Jill asked Luke whether containers were full or empty, he was always able to answer correctly. This session ended and Luke and I went to a smaller room where I carried out the interview with Luke.

In the afternoon, I observed Luke in a design and technology lesson. In this lesson, Luke was making a bus out of a cardboard box. Part of the process of making the bus involved sticking on four wheels. Luke collected his four wheels (counting out four correctly) and stuck one on. Once he had done this I asked him how many more he had to stick on. In reply, Luke said 'three…three more'. After he had stuck on the next one, I asked him again how many more he needed to stick on. This time he replied with "Two more." Finally, I asked after he had stuck on the third wheel and he replied with "One."

I did not get a chance to talk to Jill on this visit, but I did look in Luke's maths workbook. The work on numbers and counting to five was all done successfully. However, Luke's work on repeating patterns revealed difficulties when three items were being repeated.

<u>Interview</u>

This time we started with some counting activities. It soon became apparent that Luke was developing his understanding of cardinality, although he was not yet consistent:

C: We're going to do some more activities with counters

L: OK

C: [I put out 2 red counters] How many red counters are there?

L: One, two

C: Good boy. [I put one more counter onto the table] How many are there now?

L: Three [very quick and with no evidence of counting]

C: That was very quick! [I put one more counter down] How many are there now?

L: One, two, three, four [touches the counters as he counts]

Next, I wanted to explore Luke's understanding of "more", based on the Number Knowledge Test (Griffin, 1997):

C: Now if I put down these yellow ones [I put 2 yellow counters onto the table next to the four red counters that were already on the table]. How many yellow ones are there?

L: One, two [touches the counters as he counts]

C: And are there more red or more yellow?

L: [pause] Yeah

C: More red…

L: Yeah

C: Or more yellow?

L: [pause] Red [spoken with confidence]

C: Good boy [as I put down 4 more yellow counters]. Now are there more red or more yellow?

L: One...one, two [up to] ten [counts the yellow counters, touching the counters as he counts each one]

C: Very good. And are there more red or more yellow? [pause]. Which one is more? Is this more or is this more? [as I point to each of the piles]

L: This more [pointing to the yellow pile]

C: Yellow?

L: Yellow

Both of these were interesting, because even when Luke was challenged, he was confident in his responses. Luke was able to repeat this task successfully with four red compared with nine yellow counters; four red compared with five yellow counters; seven red compared with five yellow counters. I then placed seven red and fourteen yellow counters on the table:

C: This time I don't want you to count, so see if you can do it without counting this time. This time have we got more red or more yellow [I placed 7 red and 14 yellow counters onto the table]

L: More yellow [quick and confident response]

C: That was very quick well done

L: One [Luke starts to count the yellow counters]

C: Can you count them?

L: One, two [up to] ten [Luke touches each counter as he counts, but stops at 10 and raises his voice as if he has counted all of them]

C: Are there ten? There might be more than ten. Can you count further than ten, Luke?

[7 second pause]

I: What comes after ten?

L: [starts counting the counters again from the beginning] One, two [up to] eleven [and again he stops]

[short pause]

C: What comes after eleven?

L: Twelve

C: And what comes after twelve?

L: Yeah

C: And what comes after twelve? What comes next?

L: Thirteen

C: Good boy...and then?

L: [says something I could not understand]

C: Is that it? [I was concerned that he seemed uncomfortable, so I thought I would give him the chance to stop]

L: Yeah

We had a short break and then I wanted to explore Luke's knowledge of written numbers:

C: Now, I want to ask you [as I got out my number cards] can you read this number? What number is that? [as I point to 13 on the number card]

L: Fifty-three

C: Fifty-three...What number is that [58]?

L: Fifty-three...Fifty-eight

C: Fifty eight. Can you have another look at this one? [I went back to 13] have another look at this one...What’s that one?

L: Thirty-three

C: Thirty-three? What number’s this [17]?

L: Seventeen

C: Seventeen. Good boy…What number is this [12]?

L: Uhhhh. Twelve

C: Twelve. Good boy...What number is this [13]?

L: Fifty-three

C: Fifty-three...What number is this [29]?

L: Twenty-nine

C: Twenty-nine. Good boy...Now this is one you might not know. What number is this one [134]?

L: Forty-three

C: What is it?

L: Thirty-four

This extract demonstrates that Luke was in the process of making sense of the written number system. The difficulties he experienced are typical of children who are beginning to make links between our spoken and written number systems (Nunes and Bryant, 1996). The teen numbers are more confusing as there is no verbal cue in the words themselves. Luke was less secure with the teen numbers, so perhaps it is not surprising that he did not know what “13” represented.

We took a break, after which I wanted to ask Luke to show me different numbers on his hands, using his fingers:

C: Can you show me five fingers?

[Luke puts up one hand his left hand]

C: Can you show me ten fingers?

[Luke puts up both hands]

C: Can you show me two fingers?

[Luke struggles to hold up his index finger and his middle finger on his left hand]

C: Can you show me…

L: One...

C: One finger?

[Luke isolates his index finger on his left hand]

C: Can you show me three fingers?

L: Three

[Luke tries to isolate 3 fingers on his left hand]

C: Can you show me four fingers? Four...Show me four...Five [as Luke puts up his whole left hand]...Where's four?

[Luke puts down his thumb to show 4 fingers]

L: There it is

Luke was able to show these numbers on his fingers without having to count, he seemed to "just know". This is was also found by Hughes (1986) in his work with children in the early years.

<u>Finger gnosis</u>

I used a method based on Gracia-Bafalluy and Noël (2008). Luke's right hand was covered and I touched his index finger. When asked to show which finger I had touched, Luke showed his whole hand. I modelled the activity with my hand, but this did not seem to help. I tried again four times, touching different fingers on his right hand, but each time I asked Luke which one of his fingers I had touched, he showed me his whole hand. It is hard to know whether Luke did not understand what was being asked, or whether it felt to him as if I had touched his whole hand.

4.2.3 Third school visit (November 2013)

Luke was now in Year 2 and with the same teacher (Jill). The class was still a mixed Year 1/2 class, with 10 pupils. I arrived in the morning, as the maths lesson was about to start.

Observation

The starter was “10 wiggly fingers”. Jill had devised these exercises, in response to our discussions about finger gnosis. An aspect of these exercises involved individuating each finger to a song. Luke did not participate in these activities and appeared disinterested and off-task.

The exercises were followed by number activities such as clapping three times; standing up six times and tapping your head two times. Luke joined in with all of these activities.

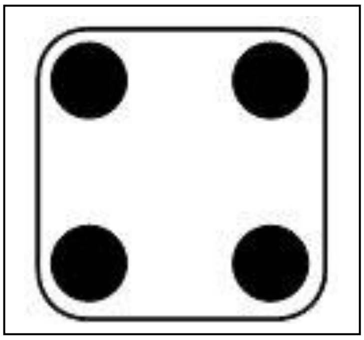
The next activity was on number recognition, counting and ordering of numbers. Firstly, number cards with digits from 1 to 10 were given out. The children had to place themselves in order. Luke had the number 8 and had no difficulty standing up at the right moment and joining the line. The children then had to pick up a specified number of bricks off the carpet. When Luke was asked to pick up two bricks he did this quickly and without counting out loud. For the next activity, Jill built a tower with the bricks in twos. The children had to count up in twos with her. Luke was correct up to ten, but then said “Eleven” instead of twelve.

The main part of the lesson involved counting fish and saying how many there would be if there was one more. Luke worked out one more than five without counting. The next activity required the children to match a digit with the correct number of items, up to ten. Luke completed these activities with ease.

Jill then did an activity with Luke which involved writing number sentences, such as 3+1=4. This activity used dice to create numbers for the number sentences. Luke was expected to write the number sentences and work out the answers.

To get started, Luke was asked to roll one die (figure 4-2).

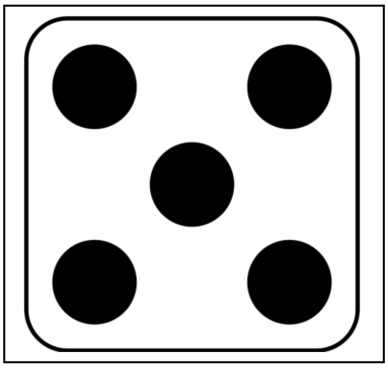
Figure 4-2: Luke's first die



Jill asked Luke, “How much is that?” Luke replied, “I think it’s three.” Jill asked him to check and he then counted correctly to four.

Luke rolled another die (figure 4-3).

Figure 4-3: Luke's second die



When asked how much this was, Luke said, “Four.” Jill asked him to check and again he counted correctly and got five.

Jill then wrote:

5+4=

on a mini-whiteboard. Jill asked Luke how many dots there were altogether. Luke did not seem to know what to do, so Jill modelled for him how to count all the dots to get a total of nine. Jill tried two more of these activities with Luke, but he had to be taken through each one by Jill.

During the dice activity Luke seemed to be unfamiliar with dice and dice patterns, so the process of working out how many dots were on each of the dice was quite time-consuming, especially as he did not seem to consider that counting would be the most reliable way of finding out how many dots there were; in fact he appeared to prefer to guess. This could have been a distraction from the activity of learning about mathematical symbols, especially if he was not yet confident with translating or linking the formal written language of mathematics with the activity he was engaged in (Hughes, 1986).

Staff views

According to Jill, the most significant improvement had been with Luke's language. He was now more confident when speaking and no longer used Makaton, but it was sometimes hard to understand what he was saying. She said she had been surprised with his attempts at the dice activity, because he had recently done activities requiring the recognition of dice patterns up to six. However, the use of mathematical symbols to represent calculations was still quite new to him.

Interview

The main focus for this session was to try the WIAT-II Mathematical Reasoning test (Wechsler, 2005). Before we started, however, I just wanted to see if Luke would still count whenever he was asked "How many?" I used a bag and some counters, as before. I started by putting four counters on the table:

C: How many counters are there? [as I point to the counters on the table]

L: One, two, three, four [raises his voice]

C: Now if I put them in my bag [I put them into the bag] and now I take one out [I took a counter out of the bag and put it on the table] how many counters are there in the bag?

L: Three [spoken very quickly]

As before, Luke then checked that this was correct. I put the four counters back into the bag and put in one more:

C: How many are in the bag now?

L: Four, five

C: How many?

L: Five

As in the last visit, Luke did not use his fingers to model the calculation.

Luke then attempted the WIAT-II Mathematical Reasoning test (Wechsler, 2005). During the assessment, Luke was able to subitise for two and three and then state that the group with three objects had more than the group with two objects. He understood about taller and could recognise a triangle from a group of shapes that included a circle, a triangle, a square and a rectangle. He was very quick to say what number came after eleven. Luke provided an interesting response for the question: “Neil had five marbles. Then his mother gave him three more marbles. How many marbles did he have then?” This question was supported by a picture showing the marbles. At first Luke was not sure what he should do. However, when I explained that Neil now has all the marbles in the picture, Luke knew to count all the marbles and he completed the task successfully. This supports the experience in the earlier session when Jill asked Luke how many dots there were “altogether” and he did not seem to know how to proceed. Is this a problem with his understanding of mathematics or with the language of mathematics?

Luke’s standard score for the WIAT-II Mathematical Reasoning test was 78.

<u>Finger gnosis</u>

I attempted the finger gnosis assessment again, Luke allowed me to test his index finger on his right hand and he correctly identified the finger I had touched. After this Luke did not want to do any more, so we put everything away and went back to class.

4.2.4 Fourth school visit (March 2014)

I arrived in the morning in time for maths.

<u>Observation</u>

At the start of the lesson, Jill asked Luke, “What’s four add four?” Luke replied “Six”. Jill then asked Luke to check, but Luke did not visibly do anything to check and answered, “Five.” Jill then asked Luke to put up four fingers on each hand and count them all. Luke did this and got the correct answer.

The main part of the lesson was on patterns. Luke could easily provide an example of a pattern with two colours (red, white, red, white, red, white...) and then with three colours. The session today did not involve individual work, because of other activities that had been scheduled.

Staff views

Jill felt that Luke had not made much progress this term. She felt that he was losing concentration more easily in class and appeared to “zone out”. Jill was concerned that Luke was often fiddling with things and found it hard to sit still. During my visit, I observed Jill reminding Luke to stay focused and concentrate on the activities. Jill did reflect that perhaps Luke was finding the work too easy and that maybe she should try to challenge him more.

Luke’s levels were P7 or P8. Luke was now able to read some words and was making progress with his phonics.

Interview

The interview took place after the lesson, in a small room. Luke did the WMTB-C (Pickering and Gathercole, 2001). Luke did not seem to understand what was required for Backward Digit Recall. When I asked him to, “Say five, six backwards”, he said “Five, six backwards.” I repeated the process and modelled the response I wanted, but when I asked him again, he just repeated what I said, as before. As a result, I did not to record the score. Luke’s final scores are shown in table 4-4.

Test component	**Standard scores**
Digit Recall	127
Block Recall	103
Counting Recall	71

Table 4-4: Luke's WMTB-C scores

There is a significant difference between all of these. For the Counting Recall task, Luke counted each image correctly, but this took time. Consequently, it may have been hard for him to remember previous counts, as the process of counting subsequent images required concentration and effort (Cowan, 1987) or perhaps there was too much information for him to process in the time available (Hitch, Towse and Hutton, 2001).

Finally, Luke tried Panamath (Halberda et al., 2008). I asked him to tell me whether there were more blue or more yellow dots. I then typed the appropriate keys on the keyboard for him, so that he only had to concentrate on solving the problem and not on locating the correct keys. Luke had a weber score of 0.69 (with the 10th percentile at 0.92 and the 90th percentile at 0.24). His response time was 4546 ms (with the 10th percentile at 2272 ms), making him very slow relative to his peers.

Finger gnosis

On this visit Luke allowed me to test his fingers on both hands. He was able to successfully identify single fingers on both hands, but he was not able to identify pairs. When I touched two fingers at a time, Luke often identified fingers that I had not touched and fingers that were next to each other.

4.2.5 Fifth school visit (June 2014)

I arrived in the morning, in time for the maths lesson.

Observation

For the starter, a 1-20 number line was introduced. Jill, asked Luke to count backwards from 10. Luke did this with ease. Jill then asked Luke, “What’s one less than one?” Luke replied, “Two.” This was followed by some counting activities:

J: Luke, can you take my frog to number five?

L: OK [as he moves the frog to number 5, counting as he goes]

J: He wants to go one more

L: Six

J: He wants to go two more

[Luke starts counting from one again]

J: My frog wants to go two more from six

L: Seven [spoken quickly and confidently]

J: Count on two more [emphasising “two more”]

L: Seven, eight

This observation represented real progress for Luke, as he demonstrated his ability to count on one more from five and two more from six (even though some prompting was required).

For the main part of the lesson, Luke did individual work with a teaching assistant who was supporting three children. The main activity involved counting groups of objects and picking up a piece of paper with the digit that matched the number of objects. Luke knew what to do, but had difficulty picking up the pieces of paper and sticking them down, due his limited fine motor skills. At times, he seemed to get frustrated, but he persisted nevertheless and completed the task.

I noticed how much Luke’s language skills had improved since my last visit. Luke was much easier to understand and used longer sentences with more detail. For example, in response to a question about what he had been doing at home, Luke responded, “Because I like to play.” However, Luke sometimes made grammatical errors. For example, he used the present tense when talking about what he had done at the weekend.

<u>Staff views</u>

Jill said that Luke had made “…good progress in all areas, except focus and attention and willingness to write.” She was concerned that Luke seemed to be easily distracted and often needed adult support to stay on-task.

Luke was able to count objects and add groups of objects, but he did not yet understand number sentences such as “2+3=”. Jill felt that the targets which were set by the school were not challenging enough for Luke, as he had met his annual targets by the end of the autumn term. In mathematics, Jill thought that Luke was now working at level P6 in most areas of the curriculum. This may have been a mistake, as she had previously said that Luke was working at P7 or P8.

Interview

For this interview, I decided to try some of the items from the Number Knowledge Test (Griffin, 1997). As Luke had previously successfully achieved the assessments for 4 year olds, I decided to move on to the next block which are designed for 6 year olds. Luke found the questions hard to understand, but he attempted them and gave some interesting responses:

C: You can use these counters to help you if you want to [I put out some yellow counters]….right if you had four chocolates

L: Yeah

C: Should I give you counters for your four chocolates?

L: Yeah

C: If I give you three more [I put down another three counters, close to the other four counters]…how many chocolates do you have altogether?

L: One, two, three, four [raises his voice] [counts the first four counters, touching the counters as he counts]

[4 second pause]

C: How many do you have altogether?

L: One, two, three, four, five [2 second pause], six

C: Do you want to check?

L: Yeah…one, two, three, four, five, six, seven [touches the counters as he counts]

C: How many?

L: Seven [spoken loudly and with confidence]

Luke was now much more confident at providing single word answers to the question "How many?" rather than reciting the whole number sequence. In addition, Luke was able to answer an "altogether" question independently. Interestingly, Luke wanted to first estimate his answers, rather than counting. It is possible that if Luke had been asked to work out “four plus three” or “4+3”, he would not have known what to do. However, by couching the problem in an imagined situation that made sense, Luke was able to work out the answer (Hughes, 1986).

I then asked Luke to complete the WIAT-II Numerical Operations test (Wechsler, 2005). He achieved a standard score of 78. He was able to answer all the number questions, but did not attempt any of the calculation questions (for example, 3+3= or 8+5=) – contrast this with the chocolate problem above. For the questions that required written answers, Luke asked me to write the answers for him, as he did not want to try to write them himself.

4.2.6 Sixth school visit (January 2015)

I arrived in the morning and went straight to Luke’s new class. He had a new teacher, Megan. There were 11 children in Luke’s class and two teaching assistants.

<u>Observation</u>

The mathematics lesson started with a recap of yesterday’s lesson on bar graphs. Megan asked the children to sort out some Starburst sweets into different colours, so that the information could be represented on a bar chart. The scales on the bar chart were already prepared and Megan wanted to create the bars in a systematic way, working with the colours from left to right. Luke was asked to sort out the orange ones. He did this quickly and with apparent ease.

Megan then asked the class how many orange sweets there were in the pile. The class counted the sweets together and Luke joined in. Megan then drew an orange bar to represent the sweets they had counted. Megan asked Luke, “What colour is next?” Luke correctly replied, “Green” and then proceeded to sort out the green sweets and count them. Luke moved the green sweets from the main pile to his green pile one at a time, counting the sweets as he moved them (figure 4-5). This time, Megan drew a green bar to represent the sweets.

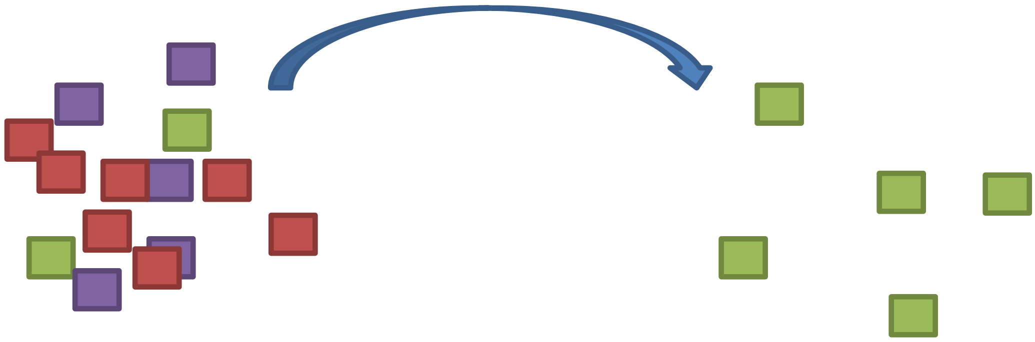


Figure 4-5: Luke sorting out the green sweets

Once Megan was satisfied that the children knew what to do, she explained the next activity. Luke was provided with a template for his bar graphs and was asked to repeat the whole class activity with a small box of Smarties. I was asked to work with Luke.

Luke quickly sorted out the Smarties into groups of different colours. He decided to count the five orange Smarties first; he counted them correctly, touching them as he counted. The next stage was more problematic for Luke. He was not sure how he could show this on the bar chart. He started to colour in some orange in the appropriate place but he did not know how much to colour in. I discussed this with him and he coloured in the bar correctly, but I was not sure if he would be able to repeat the process for the following bars.

At this point, Luke became more interested in just counting the different coloured Smarties. This led to him taking on the role of the teacher – perhaps this was because he was bored or finding the work challenging. Taking on the role of the teacher enabled him to take control of the activity (Whitehead, 1995).

He asked me to count the blue Smarties. There were four blue Smarties. I said there were “seven” and Luke responded with, “Excellent!” I asked him if he was sure that I was right and he said, “Yes.” I said I just wanted to check and counted them correctly, touching them as I counted to four. Luke did not seem surprised and just accepted the new result. This links to observations made on earlier visits and continues to relate to Cowan’s (1987) suggestion that it takes time for children to be confident that counting will provide a more reliable result than estimating.

During this visit, I noticed that Luke had made more progress with his language. He was now very chatty and used word like “awesome” and “excellent”. The development in Luke’s language is particularly evident in the transcripts discussed later.

Staff views

Megan was very pleased with the way Luke had settled into his new class; he was doing very well and would have a go at most things. She said he had a great sense of humour, was very inquisitive, enjoyed learning new things and was lovely to have in the class.

Megan thought that mathematics was Luke’s favourite subject. She said he was better than most of the other children in his class at one-one correspondence and number recognition. Luke was still on single figures for addition and subtraction, but he could use a number line and a calculator to help with working things out. Luke was beginning to use his fingers to help with his mathematics by tapping on the table. According to Megan, Luke’s mum had been working with him on this at home. Luke’s writing had also improved and he was developing good pencil control.

Luke’s levels were:

Number – P8

Using and Applying – P8

Shape, Space and Measure – P8

Reading – P6

Writing – P5

Speaking – P7

Listening – P8

Interview

The interview was based on the Number Knowledge Test (Griffin, 1997). I started by asking Luke how far he could count:

C: How far can you count Luke? Can you count up to twenty?

L: Yeah

C: Can you show me?

L: One, two, three [continues confidently and fluently to twenty]

C: Fantastic. What comes after twenty? What comes next?

L: Hmmm

C: Twentyyyyy

L: One! Twenty-two, twenty-three, twenty-four…I think that's it actually

C: Is that it?

L: Yeah

C: Is there nothing after twenty-four?

L: Umm. I don't know. We'll find out later

This extract is particularly interesting, especially in relation to the Luke's expectations of what will be taking place during our session and what the social conventions are. His statement that, "We'll find out later," reinforces my experiences during the class session, in relation to Luke's keen awareness of social convention within a school setting.

Later on in the interview, I again asked Luke to count as far as he could. This time he counted up to 39 and then said he could not go any further. When I presented Luke with different numbers to read, he could read all the numbers I presented to him up to 100, but he could not read any of the numbers I showed him beyond 100. It is likely that he was not familiar with numbers beyond 100 and so did not know how to read them.

Luke's confidence with number was demonstrated further as the interview progressed:

C: OK. What number comes after seven?

L: Eight [quickly and confidently]

C: Good boy. What number comes two numbers after seven?

L: Uhh…eight and nine

C: OK. Which is bigger, five or four?

L: Umm...five [using his fingers to make four and five]

C: Which is bigger seven or nine?

L: Nine

C: And how do you know that?

L: I don’t know

It was very interesting to see Luke use his fingers to help him to determine which number was bigger (four or five?). However, when I then asked about which numbers were smaller, Luke was not able to answer correctly. This suggests that he may not yet fully understand about “bigger” and “smaller” when working with numbers as abstract entities.

Later I thought I would try to explore Luke’s understanding of the conventional language of maths in relation to addition:

C: OK. Do you know how to do two add one?

L: Two add one

C: Two add one. You can use the counters or you can use your fingers if you like?

L: I can use my fingers?

C: You can use your fingers. How do you think you could do two add one?

L: Ummm two add one [5 second pause]

As this was not going well, I thought I would try using counters:

C: OK, let’s try something else. Can you count out three counters from this pile?

L: OK. One, two, three, four, five, six [takes six counters and places them in front of him as he counts]

C: Luke, can you put three counters in front of you…just three?

L: OK. [Takes some counters]. Oh that was close.

C: How many counters did I ask you to put in front of you? Do you remember?

L: Umm five

C: I asked you to put three. Can you put three counters in front of you?

[Luke takes two counters]

C: How many's that?

L: One, two [doesn't touch the counters]

C: Can you make it three?

[Luke takes one more]

C: How many have you got now?

L: One, two, three [touches the counters as he counts and puts them in a line]

C: Now, can you get another two counters?

[Luke takes two more counters and places them in the line with the others]

C: You've put them all in a line

L: Yeah

C: How many have you got altogether?

L: One, two, three…oh no…one, two, three, oh no I'll do it again. One, two, three, four, five [touches the counters as he counts]

It seems from this episode, that when Luke was asked to take a particular number of counters, at some point during the process of counting out the counters, he forgot how many he was supposed to be taking. The consequence of this was that he appeared to demonstrate a lack of understanding, but perhaps the task was too demanding, or took too long, and it was this that prevented him from being successful (Hitch, Towse and Hutton, 2001). Alternatively, it could be that Luke did not understand the cardinality principle and his ability to provide two counters when asked was just evidence of subitising (Wynn, 1990). However, I would argue that given his success with the other counting tasks, this explanation seems unlikely.

I decided to move on to the questions about chocolates. When I tried these before, Luke had great difficulty imagining his chocolates. This time, however, he approached these questions with confidence:

C: If you had two chocolates

L: Yeah

C: And I gave you one more

L: Yeah

C: How many chocolates would you have?

L: Ummm

C: Do you remember what I said?

L: Umm three

C: That was very good. That's fantastic. Do you know how you worked that out?

L: Umm yeah

C: What did you do?

L: Ummm ummm I was just thinking

C: Can I ask you another one? [Luke nods] So if you had four chocolates and I gave you one more, how many would you have?

L: [after 4 seconds] Five

C: Very good. What if you had four chocolates

L: Yes

C: And I gave you two more

L: [after 3 seconds] Six

It is interesting to compare these responses with the ones discussed earlier when Luke was working with counters. In the earlier extract, Luke used a count all strategy when one group of counters was added to another. In the extract above, where there were no counters, it seems likely that Luke used a counting-on strategy, although he did not say any counting on words out loud. This is similar to observations made by Fuson (1982) and Carpenter and Moser (1982).

I then tried to explore the idea of subtraction with chocolates:

C: If you started off with three chocolates…imagine three chocolates in your head

L: Yeah

C: So imagine those three chocolates. Now I'm going to take one of those chocolates and I'm going to eat it. How many chocolates will you have left?

L: Uhhh don't know

C: What if you had one chocolate and I ate your chocolate. How many chocolates would you have left?

L: Umm zero [very hard to understand]

C: What was that?

L: Zero! [very emphatic]

C: Very good, excellent. What if you had two chocolates and I ate one of them, how many would you have left?

L: Let's think. I don't know

C: How many do you think you'd have left, if you had two chocolates? So imagine your two chocolates in your head. Can you see two chocolates? [Luke nods] Just imagine two chocolates and then see me taking one of those chocolates…how many would there be left?

L: Hmmm five

C: Five, OK let's pretend these are your two chocolates [I take two counters]

L: OK

C: If I eat one of them no if you eat one of your chocolates

L: Yeah

C: How many will there be left?

L: Umm one

C: One, exactly. Right what if you had three chocolates and you ate one of them, how many would be left?

L: [says something inaudible followed by the answer] Two

C: Very good. What if you had four chocolates and I ate one of them, how many would be left?

L: [takes four counters, moves one away, then counts the left over counters, touching them as he counts] One, two, three [pause] one, two, three [very quick and confident]

Luke found it hard to work with an idea of taking chocolates away when I asked him to imagine the chocolates. It is hard to speculate on the reasons for this, as I cannot be sure that Luke understood what I was asking him to “imagine”. Looking back at my questions, it is possible that by repeating certain aspects of the question several times, rather than helping Luke, I may actually have confused him. When I modelled the problem with counters, Luke was quick to learn the strategy and to then apply it to new situations.

I then tried using the counters with the bag, to explore Luke’s understanding of subtraction further:

C: Can you count out four counters for me?

L: Yes one, two, three, four [moving the counters as he counts]

Luke seemed more at ease when asked to, “Count four counters for me,” as opposed to being asked to, “Put three counters in front of you.”

I was interested in exploring some other aspects of mathematics with Luke, so I asked Luke if he could work out:

$$2+3=$$

Luke said that he did not know what to do. I raised this with Megan later and she was surprised, as they had been doing calculations like this very recently. She wondered if he did not know what to do because he was seeing it in an unusual context.

I followed this by putting up my two index fingers and asking Luke how many fingers there were. He said, “Eleven”. I then put up four fingers on each hand and asked him again how many fingers I had. This time Luke said, “Forty-four.” This seems to be an attempt to link my finger representations to his knowledge of place value.

4.2.7 Conclusions

Luke had developed and changed in many ways over the 2 years that I worked with him. His confidence with number grew and his understanding of the meaning of the number sequence and how to work with it also improved. By the end of the 2 years, Luke was confident with adding and subtracting one and two from numerosities less than 10, although it seemed that for subtraction, he was more confident when he had real objects to actually “take away”.

Teacher assessments, did show progress over the 2 year period, as Luke moved from “working towards P5” in all subjects to P8 in mathematics and a mixture of P-scales in other areas of the curriculum. However, Luke did not make smooth progress and actually seemed to go backwards during Year 2 (from P8 to P6 in all subjects). This regression was contrary to my own findings, but I was only focusing on number work and perhaps, during this time, Luke did not make progress in the other areas of mathematics covered at school. My own evaluation, as a result of the work I did with Luke, would have put him at beyond P8 in Number and Using and Applying.

Luke had limited opportunities for social interactions with his peers and even in the playground was more often playing “alongside”, rather than “with” them.

The overall conclusions will be considered with reference to the research questions.

1. What strategies did Luke use to help him solve numerical problems in mathematics?

Luke did not tend to use any concrete or pictorial aids to help him with his calculations unless prompted, thereby putting a greater load on his working memory (Beller and Bender, 2011). However, by the end of the 2 year period, Luke had moved from having to recite the whole number sequence (in an “unbreakable list” (Fuson, 1988)) every time he was asked a question, to being able to count on mentally, without any concrete aids (compare the interviews in 4.2.1 with those in 4.2.3). At the end of the 2 years, Luke was not yet confident with the more formal language of mathematics, whether spoken or written. However, it is hard to know to what extent this was as a consequence of lack of experience rather than any specific difficulties with arithmetic.

2. Did Luke’s hand anomalies impact the range of strategies available to him?

Did the fact that Luke did not use his fingers impact his learning of number? From quite early on in my visits, it seemed that Luke could use his fingers to represent numerosities up to five, but he did not use them for calculations. On the early visits (see 4.2.1 and 4.2.2) when I had observed Luke in class, he had seemed to find it hard to physically create particular finger patterns, but this seemed to become easier with time, as his finger gnosis improved (see 4.2.4 and 4.2.5). Perhaps with encouragement, Luke may have been ready to begin to use his fingers to solve problems in mathematics.

3. Are there other factors which had an impact on Luke’s mathematical learning opportunities?

Luke was in a special school for children with complex needs. One thing that became apparent from very early on was the fact that the language used in the classroom was very simple and very repetitive. This cultural environment must have had an impact on Luke’s development, but it is not within the scope of this thesis to discuss this in any depth. It has been mentioned, in order to provide some additional contextual information and to highlight the fact that Luke was not in a language-rich learning environment.

Luke’s language development was additionally hindered by his complex oral structures and cleft palate and his hearing impairment, but over the time I worked with him, his language and communication skills developed in many areas. For example, he began to talk in much longer sentences and his social confidence grew. This was particularly evident in his later attempts to tease and play with language. He was also able to talk through his actions which made it easier for him to self-correct.

Luke also had hearing and visual impairments for which he wore hearing aids and glasses. As he was in a small class and in a special school with experienced staff, there was a cultural awareness of the impact of such disabilities in relation to accessing learning.

Assessments of Luke's working memory suggest that his phonological loop and visuo-spatial skills are within the normal range, while his central executive skills are well below the normal range (with a score of 58 for Counting Recall). However, this result needs to be treated with caution, given the amount of effort and time that was required to count each set of dots. Luke's scores in the WIAT-II (Wechsler, 2005) assessments, show a disparity between his mathematical reasoning skills (with a score of 78) and his knowledge of numerical operations (with a score of 66). This low score for numerical operations, was consistent with my own observations and experiences of working with Luke.

My experience of working with Luke highlighted his ability to reason mathematically. However, Luke was restricted in his problem solving strategies by the lack of use of any aids to support with these.

Luke's first teacher Jill did comment on the fact that she believed that perhaps she was not challenging Luke enough during my visit in March 2014. At the time, Luke was displaying a range of off-task behaviours and appeared to be easily distracted. The possible lack of challenge could have resulted in the behaviours described and may have impacted on the progress Luke made in mathematics.

4.3 Joe

I first met Joe in December 2013, when he was 5 years old. Joe had a non-identical twin sister and a 4 year old younger brother. Joe attended his local primary school and was in Year 1. On this visit, I met Joe's mum Sarah and Joe.

According to Sarah, Joe was very considerate and affectionate and had a good sense of humour. Joe was very active and enjoyed making things and taking them apart; imaginative play; reading and being read to; and playing with toys that were noisy. Joe also loved singing and was very good at remembering the lyrics to songs. Joe could ride a bike (with stabilizers) and was learning to swim. Sarah said Joe was not very good at team games. Although Joe would happily play with anyone, he had a "…tendency to be self-sufficient" and was very comfortable with adults.

In terms of milestones, Sarah said that Joe moved from crawling to walking between the ages of 2 to 3 years and that he started speaking between 2½ and 3½ years, but soon caught up with his peers.

Joe had a mild hearing loss and had had grommets at 3 years of age. These were not very successful, so at 3½ years, he was given in-ear hearing aids which seemed to help. At his last assessment, Joe had age appropriate receptive and expressive language skills. His eye sight though was more complicated and he had optic nerve damage that was worse in his left eye. Joe received support from a speech, language and communication therapist once a week at school, on a half-termly cycle. The therapist often provided activities to do at home, which focused on Joe's pronunciation. Joe also had regular school visits from an occupational therapist. Joe had a statement of SEN and full-time support from a teaching assistant.

Joe had had surgery on his hands and had four fingers on his left hand and five fingers on his right hand. There was no more surgery planned for Joe at this time.

In terms of work on number, Sarah said that Joe was always adding things up. She said he enjoyed number games like Snakes and Ladders and playing with dominoes. She said that they tried to use numbers at home whenever possible, but that Joe relied on his memory to work things out and never used his fingers (or any other concrete aids). Joe's sister, on the other hand apparently used her fingers a lot to help with doing calculations involving number. Sarah said that Joe had "an excellent memory" and this was very helpful.

Joe and his twin sister were in different classes at school and this was working well.

4.3.1 First school visit (February 2013)

On my first visit I arrived in the morning. It was a special themed day, so no specific mathematics lessons were planned.

Staff views

I spent some time during the afternoon talking with Joe's teaching assistant, Bea.

She said that Joe was in the "…middle level in all subjects," but that, "The biggest battle is to get him to try, because he knows he finds things difficult." She also said Joe, "Doesn't go looking for children to play with…maybe because he always has an adult with him, but he will often play alongside other children." In Reception, Joe had not been very stable on his feet, but this was improving. Bea said she had noticed his eyesight was poor, so he usually sat near the board. She also thought that background noise made it harder for Joe to hear what people said. Joe did not usually volunteer answers to questions when the children were on the carpet, but he would answer if asked.

Bea felt Joe needed one-to-one support all the time, especially to keep him safe. Nevertheless, Joe, "…copes really well and takes everything in his stride."

Later on I spoke with Joe's teacher, Sylvia, who said many of the same things as Bea. However, she did add that Joe found open questions particularly difficult and had "…good avoidance strategies." She thought that he was probably "…slightly below average in attainment."

While I was there, a girl asked Joe a question. When she did not understand the reply, she asked Bea what he had said and Bea told her. Speaking for Joe in this way was meant to be helpful, but it meant that a conversation was then had between the girl and Bea and not the girl and Joe.

Interview

The first interview took place in the morning, in a small side room, next to Joe's class. Bea was present. She did not interfere, but tried to encourage Joe, when he was asked questions, she thought he should be able to do. To start, I wanted to know how confident Joe was with counting:

C: Can you count as far as you can…starting from one? How far can you count?

J: One, two, three [continued correctly to twenty and stopped]

C: Can you go further?

J: Twenty-one, twenty-two [continued correctly to fifty and stopped]

C: And can you get up to a hundred?

J: Fifty-one, fifty-two...

C: You don't have to count, you can just tell me if you can count up to a hundred. Do you want to count up to a hundred? [J nods]. OK keep going. Sorry

J: [counts correctly from fifty to one hundred]

C: And then? Can you go up beyond a hundred?

[pause]

C: Do you know what comes after a hundred?

J: [pause] Yeah

As Joe did not offer any numbers beyond a hundred, I decided to move on.

C: Well I think you've done very well. I'm very impressed. Can you count backwards from ten?

J: Ten, nine, eight, seven, six, five, four, three, two, one

C: Brilliant. Can you count backwards from twenty four?

J: [counts back correctly and quickly down to one]

C: Well done. That's fantastic. How do you remember it all Joe? Do you know?

J: No

I then tried some questions based on Hughes' (1986) box task:

C: If I put these three into this envelope [puts in the counters]

J: Yeah

C: And I put one more in [puts one more counter into the envelope]. How many will there be in the envelope now?

J: [pause] Four

C: How did you know that?

J: I counted in my head

Joe could also do this successfully when I added two counters. With subtraction, it was harder to know what Joe was thinking:

C: Well done…If I put these six back in [as I put the counters back into the envelope] and this time I'm going to take one out [as I take out one counter] so I've taken one out [showing Joe the counter]. How many are left in here?

J: Five [no hesitation]

C: Very quick…should we check? [as I tip the counters out] How many are there?

J: Five [without counting]

C: Excellent. OK if I put the five back in [as I put the counters back into the envelope] and this time I'm going to take two out [as I take two counters out] how many will be left in?

J: Four

C: Should we check how many are there [as I tip the counters out]?

J: One, two, three, four [touching the counters and double touching one of them]

C: Where's the fourth one? [everyone giggles] Should we try one like that again? How many should we put into the envelope this time?

J: Er two

C: Just two alright then…if I put two in [putting in two counters]…if I then take two out [as I take out two counters]…how many are left in there?

J: None

I then asked Joe some questions based on those used by Jordan, Glutting and Ramineni (2008). Joe could answer questions on which numbers were "bigger" or “smaller”. We then moved onto questions using the more abstract language of mathematics:

C: OK are you happy doing sums like two plus one?

J: Yeah

C: So what’s two plus one?

J: Three

C: And four plus three?

[4 second pause]

C: Do you want to use some of these to help? [pointing to the counters] Do you want me to write it down as well? Would that help?

J: Yeah

[I write 4+3 on a piece of paper for Joe]

J: [looks at what I’ve written and doesn’t use the counters] Five

C: Five. Why do you think it’s five? [pause] What makes you think it’s five?

J: Dunno

C: Were you just guessing?

J: Yeah [giggles]

It seemed that Joe did not know how to use the counters and relied on his ability to calculate mentally. I thought I would try to get him to see the number pattern, when we keep adding one:

C: What would four plus one be?

[2 second pause]

J: Five

C: And how do you know that?

J: Cos I said it is

C: And what about four plus two?

[3 second pause]

J: Six

C: So what do you think four plus three might be?

J: Seven

C: Well done and what do you think four plus four might be?

J: Eight

The next set of questions were contextualised and required addition and subtraction. Joe was able to calculate the answers as long as no more than three was being added. With subtraction, he could subtract up to two items but when this was more, he struggled:

C: Kisha has six pennies and Peter takes four of her pennies away. How many does she have left?

[4 second pause]

J: None

C: You think she has none…do you want me to read it again just to be sure?

J: Yeah

C: Kisha has six pennies and Peter takes four of her pennies away…how many does she have left?

J: None [very quick response]

I then tried some more addition questions using number only and working within 10. Joe could add on up to three, but could still not do four.

Joe said he liked school and enjoyed doing work on numbers, but did not enjoy writing.

Overall, Joe was very confident when working within his limits and he was not afraid to say when he could not do something. However, when Joe could not work out an answer in his head, he did not have any other strategy to fall back on. He could work comfortably with numbers with both contextualised word problems and with the more abstract language of mathematics. It is possible that the games and activities that Joe engaged with at home, such as playing Snakes and Ladders, had helped him to develop his confidence with numbers (Siegler and Ramani, 2008, 2011).

4.3.2 Second school visit (May 2013)

There was no mathematics lesson on this visit, as Sylvia was doing one-to-one assessments with all the children. During Joe's assessment, it was interesting to observe the difficulty Joe had with subtraction. When asked addition sums, such as:

Sylvia: Can you write six add two?

[Joe writes 6+2]

Sylvia: What comes next?

[Joe writes = in the correct place]

Sylvia: And the answer?

[Joe writes 8 after the equals sign and shows no evidence of how he worked out the answer]

However, with subtraction:

Sylvia: Can you write eight take away three?

J: [writes 8–3] Six

Sylvia: Would you like some cubes? [as she passes them to Joe]

Joe: [counts out three cubes] Six

Sylvia: What do you need to count out first?

Joe: Eight [Joe slowly counts out eight cubes and takes away two cubes to leave six cubes]

Sylvia: Look at the question again [pointing to the 8–3 that Joe has already written]

Joe: [looks at the two cubes he has already taken away and takes away one more] Five

Joe seemed to be very confident with adding numbers and did not need any concrete resources to help. With subtraction he was less confident. He did not use any of his known addition facts, did not count back and only used concrete resources when prompted. As in my first visit, Joe continued to rely on his working memory to help with calculations.

Interview

The interview took place in a quiet room, early in the afternoon and Bea joined us. During the interview, Joe did the WMTB-C (Pickering and Gathercole, 2001) (Table 4-6).

Test component	**Standard score**
Digit Recall	137
Block Recall	133
Listening Recall	Did not seem to understand what to do
Counting Recall	108
Backward Digit Recall	84

Table 4-6: Joe's WMTB-C scores (first attempt)

We attempted the tests in order. During the assessment Joe did not use his fingers. For Listening Recall, Joe did not seem to understand the instructions, so we moved on. Towards the end of the assessment, Joe was getting tired, so the results for the last two tests, especially the Backward Digit Recall, may not be particularly reliable. For Counting Recall, Joe used a pen to point at the dots. Consequently, the process of counting the dots took quite a long time, and may explain why Joe scored less well in this part of the assessment (Cowan, 1987).

The fact that Joe's Digit Recall score was so high may go some way to explain how he was so good at working out calculations that required the recital of the number sequence. For Digit Recall, Joe's maximum number of items recalled was six. For Backward Digit Recall, Joe could say two numbers in reverse order reliably, but began to make errors with three numbers.

4.3.3 Third school visit (January 2014)

Joe was now in Year 2, with a new teacher, Nina, but the same teaching assistant, Bea. Joe's levels were:

Mathematics: 2c

Reading: 2c+

Writing: 1a (but this was particularly challenging for Joe and speed was a real problem)

The focus for the school was on building Joe's independence. Two days a week there was a buddy system in place to make sure that Joe had someone to go to in the playground.

<u>Observation</u>

The focus of the lesson was on making different numbers with digit cards. Joe needed lots of prompts from Bea to stay on task. Joe was fine with 2-digit numbers, but seemed lost with 3-digit numbers. It seemed that he was not sure what 3-digit numbers represented and, therefore, did not know what he was supposed to do.

<u>Interview</u>

This time the interview took place in a side room. Bea joined us.

Joe did the WIAT II Numerical Operations test and the WIAT II Mathematical Reasoning test (Wechsler, 2005). For the Numerical Operations test, Joe achieved a standard score of 113. However, this is possibly not a very reliable score, as he used a hundred square (on Bea's advice) for many of the questions. At first when using the hundred square he counted forwards and backwards (for addition and subtraction) in ones, but after Bea reminded him how to add 10 on a hundred square, he was able to do this with ease. As I was an observer, it is unclear how much “relational” understanding (Skemp, 1977) Joe was employing when using the hundred square.

When Joe tried to work out 8x5, he demonstrated that he could count up in fives, but he did not know where to stop. I suggested that he could use his fingers to help him keep track. When he tried this however, it became clear that he did not know when he had reached his eighth finger.

For the Mathematical Reasoning test Joe achieved a standard score of 103. He approached these questions with confidence and often articulated what he was doing as he went along. Joe did not use any aids to support him with this part of the assessment.

Joe also tried the Panamath assessment (Halberda, et al., 2008). He got a Weber fraction of 0.41 (with the 90^{th} percentile at 0.24 and the 10^{th} percentile at 0.92) and a response time of 2011 ms (with the 90^{th} percentile at 1271 ms and the 10^{th} percentile at 2272 ms).

After the school visit I went to see Joe’s mum, Sarah. She said that Joe was finding it hard to work things out in mathematics, because if he did not know the answer to a question, he had no strategy to fall back on. She said he did not use his fingers and did not seem to have much idea of how a number line worked. As a result, when Joe was finding his homework hard, Sarah was not sure how best to help him. I talked to her about Gracia-Bafalluy and Noël’s (2008) research on finger gnosis.

4.3.4 Fourth school visit (July 2014)

I arrived in the morning when the children were doing English. Joe was working with a partner but his partner complained that Joe was not doing any of the work. Bea explained to the child that Joe finds it harder to write and so he should be patient. At break time, Joe spent all his time with Bea and other adults. Bea said that Joe preferred to spend time with adults.

Staff views

Joe's levels were now:

Mathematics: 2b

Reading: 2a

Writing: 2b

Bea said that Joe was doing well in all areas of the curriculum. She said he did not use his fingers to help with mathematics because he found it hard to move his fingers individually and the children were now being encouraged to use other strategies.

Observation

The main lesson was on measuring and drawing lines with a ruler. Joe found it hard to do this accurately. He also struggled with writing measurements that had partial amounts, such as 4.3cm. Throughout the lesson, Joe needed prompting from Bea to move on to the next question.

I noticed that Joe's writing was much better and that he was doing joined-up writing with confidence.

Interview

Joe tried Panamath (Halberda et al., 2008) on this visit, but Joe did not have his glasses at school and he seemed to find it quite uncomfortable to look at the screen. I decided that I would not try any more assessments.

4.3.5 Fifth school visit (January 2015)

Joe was now in Year 3 with a new teacher, Jennie and Bea as his teaching assistant.

Staff views

Jennie said she thought that Joe was doing well, but that he could be lazy. He was learning to touch-type and to use software such as Clicker 6 to help him. Jennie said Joe was doing well in mathematics, but that he was sometimes a bit slow because he did not like getting things wrong. Jennie also said that Joe had difficulty in English with comprehension and inference. Joe’s levels were:

Mathematics: 2b

Reading: 2a

Writing: 2b

These levels were exactly the same as on my last visit.

Bea said that Joe was finding mental mathematics hard and was now in an intervention group for 10-15 minutes twice a week, to help with this. Joe did not seem to have strategies to work things out in his head and he needed reminding to use his fingers (compare this with the last visit and finger-use was being discouraged).

According to Bea, Joe still tended to prefer to play on his own. At break time, he would rush out and go over to the big Connect 4 apparatus, so he could get there before anyone else.

Observation

The focus of the lesson was on drawing bar charts. This was particularly hard for Joe, because he found it hard to locate positions on the page and to draw the bars in the right place with accuracy. At each stage he needed prompting from Bea to move on to the next part. Even after he had drawn a bar, he would not do the next one until Bea had told him to.

Interview

We had some time at the end of the morning to work together and Bea joined us. Joe used his fingers a lot during the session today. Each of his fingers had varying degrees of movement, but Joe was able to use them with confidence and accuracy.

Joe seemed to favour his right hand (with five fingers), but he would use his left hand as well, if needed.

We started with some questions from the Number Knowledge Test (Griffin, 1997). The first example demonstrates Joe's ability to keep track of calculations mentally:

C: Right, which number is closer to seven, is it four or nine? [using visual arrray]

[9 second pause and then Joe points to the 9]

C: Nine is closer. Why?

J: Because...ummm…nine minus two is seven

C: Yep and what about the four?

J: Four…plus three

C: So is that why nine is closer?

[Joe nods]

Having seen this confidence the next example was a surprise:

C: OK, how much is two plus four? [Joe is still for 5 secs] you can use your fingers, or I can give you some counters. How much is two plus four? [spoken slowly]

[Joe is motionless]

C: Do you know what it would look like? Should I write it down for you?

J: Yeah

[I write 2+4 on a piece of paper]

C: Do you know how to do it?

J: No

C: Joe, have you done things like this before?

J: I have done two plus two before

C: So what's two plus two?

J: Four [no hesitation]

C: Lovely. Can you write that underneath? [pointing to a space underneath 2+4]

[Joe writes 2+2=4] [takes about 15 secs]

C: If you know this [pointing to 2+2=4], how do you think you could work out this? [pointing to 2+4] You can use your fingers to help

J: I don't really know

C: You don't really know. OK, what if you had two counters Joe counts out 2 counters] and another four counters [Joe counts out another 4 counters, one by one]. Now can you work out what it is altogether?

[Joe moves the counters, counting on from the two counters, but speaking inaudibly]

J: Six [appears to count without touching the counters]

C: Six. How did you work that out?

J: Cos I counted

It seems that while Joe was confident with adding and subtracting up to three, he did not have a strategy for adding four. This suggests that he either did not know the answer, or did not have the working memory capacity to work it out. In addition, Joe did not use the commutative property of addition, or the knowledge that he could count on from the largest addend (Carpenter and Moser, 1984). Joe did not attempt to use his fingers and only used the counters when prompted. This lack of representation seemed to leave Joe in a very insecure position.

Later, Joe demonstrated his ability to self-correct:

C: Now, I've got another one of these pictures [show visual array] which number is closer to twenty-one, is it twenty five or eighteen [pointing to the numbers]

[pause]

J: Eighteen

C: Why do you say that?

[4 second pause]

J: Because…eighteen in one le…two less than twenty-one

C: Is it two less than twenty-one? Are you sure it's two less?

J: Yeah [spoken quickly and confidently]

C: Can you prove it?

J: Actually three

C: How did you just work that out?

J: Cos umm. I counted in my head

This shows Joe's ability to articulate the fact that he can keep track of a count for up to three items.

The final example illustrates Joe's ability to use his knowledge of number relationships to solve problems:

C: Right can you do forty-seven take away twenty-one?

[Joe writes 47 37 27 26]

C: How did you do that?

J: Because I started from forty-seven

C: Yep

J: Then I counted back ten…and ten…and then I counted back one [pointing to the numbers as he explains]

Joe successfully used a sequencing approach for subtraction. This seemed to demonstrate a more secure understanding of numerical relationships than I had observed during my third visit.

Joe redid two parts of the WMTB-C (Table 4-7) (Pickering and Gathercole, 2001). For Digit Recall, with six or more digits, Joe used his fingers to keep track of the numbers.

Test component	Percentile rank
Digit Recall	144
Backward Digit Recall	134

Table 4-7: Joe's WMTB-C scores (second attempt)

4.3.6 Conclusions

Joe started off with great confidence with his numbers. Indeed, it seemed that he had a good understanding of number and number relationships, and was beginning to use his knowledge of the base-10 number system to help him solve problems. Joe could very confidently use numbers as objects, as described by Gray and Tall (1991). He seemed to understand about addition and subtraction and when to apply these calculations to “real life” contexts. However, as Joe was moving up through the school, his “mental maths” was becoming a problem and his understanding of the number system and strategy deployment was failing to develop.

Joe’s teacher assessments, placed Joe below the average for his class, but always within nationally expected levels. Nevertheless, Joe’s levels for mathematics tended to lag behind his levels for reading.

The overall conclusions will be considered with reference to the research questions.

1. What strategies did Joe use to help him solve numerical problems in mathematics?

When working on numerical problems, Joe seemed to rely very much on his long term memory and his working memory, as seen in the interviews in 4.3.2, 4.3.3 and 4.3.5. When these failed him, however, he had nothing to fall back on. When he was prompted to use objects, Joe tended to use a count all strategy when he could see the objects he was counting, but could count on when there were no objects visible. This was also observed by Fuson (1982) and Carpenter and Moser (1982).

2. Did Joe's hand anomalies impact the range of strategies available to him?

Joe could use his fingers very well to help him keep track of items in the WMTB-C and he did this without prompting. However, Joe did not use his fingers, or any other concrete resources to help with calculations, unless prompted. Although I did not test Joe's finger gnosis during any of my interviews, it seems likely that he was developing quite good finger awareness, due to the fact that he did use his fingers to keep track of items. Unfortunately, this "intuitive" strategy was not being exploited more to help Joe develop the "mental maths" skills with which he seemed to be struggling.

3. Are there other factors which had an impact on Joe's mathematical learning opportunities?

Joe had visual and hearing impairments. Although he wore hearing aids and glasses, it is very likely that these impacted his ability to manage the classroom learning environment and to manage a busy playground. During my observations of Joe in the classroom, he seemed to struggle to focus on the main teaching activities and relied very much on his teaching assistant to tell him what he had to do. In fact, Joe was very dependent on his teaching assistant and he found it hard to work independently. Although the school was aware that this may be a problem, his teaching assistant found it hard to leave him alone and watch him do nothing, even for short periods of time. Where possible Bea tried to provide "low support" (Radford, Bosanquet, Webster, Blatchford and Rubie-Davies, 2014) and just prompt Joe. However, when there were activities that were particularly challenging for Joe, because of his poor eyesight and poor manual dexterity, more help was provided. This highlights some of the tensions for those in a support role, when trying to enable independence, while also ensuring that tasks are completed.

Joe's working memory scores were all within, or above, the expected range, apart from the score for Counting Recall (see tables 4-6 and 4-7). Joe found it very laborious and time-consuming to count all the dots and this may account for his low score in this assessment of his central executive. In the Backward Digit Recall assessment, Joe scored 134, so whatever the reason for his low score in the Counting Recall assessment, he was clearly able to use his central executive skills successfully in some tasks. Joe's scores in the WIAT-II assessments (Wechsler,

2005) placed him within the expected range for his age for both Numerical Operations and Mathematical Reasoning.

From my assessments, it seemed that Joe relied very much on his long term memory and working memory to help with calculations and rarely used any aids unless prompted. Joe would sometimes write down parts of calculations to help him, such as when he wrote down “47 37 27 26” to work out forty-seven take away twenty-one during my last visit. It may be that as he develops this strategy it will help him to support his learning in mathematics.

Joe’s opportunities for social interactions were limited and he tended to avoid playing with his peers, preferring to be with adults or else doing activities “alongside” his peers. It is unclear why this situation had come about and the school seemed unsure as to how to address it. The school was aware that Joe was becoming very dependent on his teaching assistant and they were trying to support him to become more independent. It is likely that this lack of opportunity to work with peers would have had an impact on Joe’s learning opportunities.

4.4 Emily

On my first visit to meet Emily’s parents, Helen and John, Emily was at school, so I did not meet her. Emily was 5 years old, in Year 1 and attending a mainstream primary school. Emily had a statement of SEN, but did not have one-to-one support from a teaching assistant.

Emily was described by her parents as “…socially confident, determined, happy, funny, very independent and will have a go at anything.” She enjoyed many activities, including reading, playing with dolls, computers, drama, dancing and painting. Emily was described as “not clingy”. She enjoyed playing on her own and with her sister (who was 2 years older) and had friends at school who were mostly boys.

Emily had her first major surgery at 10 months of age when she had her cleft palate repaired. At the same time, she also underwent her first hand surgery to release one of her thumbs (the other thumb was already free). At 15 months of age Emily had a vault expansion and between the ages of 1½ and 4 years, Emily had operations on her hands to separate her fingers. She now had five fingers on her left hand and four fingers on her right hand.

In terms of milestones, Emily "bum-shuffled" from about 8 months of age and walked at about 2 years of age. Emily started talking after she was 2 years old and, until very recently, received regular support from a speech and language therapist. Emily's hearing fluctuated a lot, especially in the winter when she tended to have more colds. To help with this, Emily had grommets. When she was 3 years old, Emily tried wearing in-ear hearing aids, but the grommets seemed to be better. Emily wore glasses. Emily had problems with her balance and this was a particular concern at school.

Emily's parents engaged in a lot of activities involving number at home. These included games such as Snakes and Ladders, bingo and dominoes and activities such as counting out cutlery at meal times. Emily knew her numbers up to 100, but could not add up. She could count up in twos and tens, but could not, for example, say that two tens are twenty. Emily's parents felt that her mathematics was "…still a work in progress." Emily was doing well with reading and could answer questions about stories, but she needed to practise retelling stories. Emily was "good at phonics" and was starting to write.

4.4.1 First school visit (December 2012)

On my first visit I was in class with Emily for the whole day, so did not do any one-to-one work with her. Emily was in a Year 1 class. There was a teaching assistant attached to the class for most of the day and Emily sometimes worked with her. Emily had also recently started wearing a bone conduction hearing aid which she was using every day.

Observation

The mathematics lesson started after the morning break. The focus of the first activity was on sorting. All the children were on the carpet with the teacher, Katia, leading the session. Katia used soft toys and encouraged the children to think of different ways that they could sort the toys. When Emily was asked, she said that she could sort them into different positions. Katia prompted Emily to try to group some of the animals according to shared properties. Emily then collected a group of animals together, but did not explain her reasoning. Katia asked her to collect a different group of animals and this time Emily collected the three elephants. When Katia asked her why she had sorted them in that way, Emily said "Because they're both the same." Katia asked other children questions about how they could sort the animals. When Emily was not involved in the discussion, she started playing with the toys. Katia noticed this and asked her a question in order to refocus her attention.

After the carpet session, the children went to their tables to work on some sorting activities. Katia had suggested that they could work in pairs, but Emily worked on her own. Katia had put a range of different coloured animals on each of the tables and she wanted the children to sort them according to different properties. Emily played with the toys, but did not appear to be sorting them. Katia came over to Emily's table and tried to encourage her to sort some of the toys. Emily sorted out a set of blue teddies and a set of red teddies. Katia asked Emily if she could sort the animals in any other ways. Emily collected 3 orange teddies, 5 purple teddies and 7 blue teddies and proceeded to count the number of teddies in each group. Emily did this correctly for each group, by touching the teddies, but not moving them. Even with prompts from Katia, Emily did not seem to understand that the animals could be sorted according to different properties; for example, sorting a group of animals according to colour only. While children naturally organise and sort objects on their environment, it is seems that in this case Emily did not fully comprehend what it was she was being asked to do, perhaps because of the number of possibilities available to her (Montague-Smith and Price, 2012).

The lesson ended and the children went out to lunch. Emily left with her friend.

Staff views

Katia said that Emily was definitely one of the class and was totally accepted by her peers. Even so, Emily tended to be very quiet and had one special friend, who she always chose to be with and who was very protective of her. Katia reported that, "Emily is very nosey and is always interested in what other people are doing." Emily had a very "can do" attitude to school. She was also very observant and always noticed if, for example, one of the children was missing or had a new hairstyle.

According to Katia, Emily had difficulty staying focused and on task in class and needed constant refocusing. Although there was no specific provision for in-class support for Emily, she did get one-to-one support for about 30 minutes a day to develop her physical coordination. This was delivered by a teaching assistant, but as a result, Emily regularly missed important parts of lessons.

Emily's levels at the time were P8 (working towards level 1) for mathematics and level 1 for reading. Writing was very hard for Emily, so she was not given a level for this. Emily was good at reading and recognising words. She could also understand things that had just been read to her, but found it harder to recall longer passages. Emily was "….not so good on 'why' and 'what' type questions that require thinking rather than recall."

Katia was concerned about Emily's understanding of number and described this as "poor". She said that Emily was good at counting; could order numbers to 20 or 30 and could spot missing numbers in a number sequence. Emily could also say which numbers in a pair was more or less, or bigger or smaller. Emily was able to count in twos and tens and could say "one more" than a number, but found it harder to say "one less". However, given her age, this is perhaps not such a surprise (Gifford, 2014). Emily could add by using a count all strategy with apparatus and could subtract by taking away groups of objects and counting what was left. Katia was concerned that Emily found it very hard to explain her reasoning and would not discuss her work.

Katia felt Emily was "doing fine socially," but she was concerned that Emily was very quiet in class and tended to be "very passive when she worked with other children." Emily's speech had improved a lot since she started school, but Emily was becoming more aware of her differences. Emily was not very confident in the playground and tended to stay close to the adults, where she felt safer.

4.4.2 Second school visit (July 2013)

I arrived at school first thing in the morning.

<u>Observation</u>

When the mathematics lesson started, the children were all at their tables. Katia put the following questions on the board as a starter:

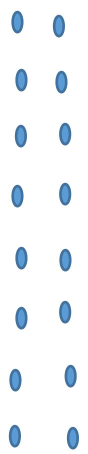
2 x 6

4 x 2

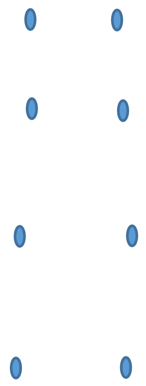
3 x 4

5 x 2

The children were asked to work on their own to work these out. Emily straight away wrote the first one with the answer (2x6=12). For the next question, she wrote 4x2=12. Katia noticed this and asked Emily to explain what she had done. Emily laid out pairs of counters up to 16:



Katia left this arrangement and tried to convince Emily that it would make more sense to “put out four lots of two.” Emily did not appear to understand what she was being asked to do, so Katia made four groups of two for Emily to see:



Katia asked Emily to count all the counters. Emily did this quickly and confidently. Katia then asked Emily to do the same thing for the next question (i.e. “Make three lots of four.”). Katia tried to prompt Emily by reminding her of how she had represented 4x2, but again she did not appear to know what she was being asked to do. Katia asked Emily to think of another way that she could work it out. Katia then left Emily alone to work on the problem, but Emily just played with the counters until the lesson ended.

This deserves some exploration. Emily seemed to know what the answer was to the mathematical problem 2x6, but could not extend this to other numbers. Was this because she did not fully understand the way that mathematical symbolism works? Hughes (1986, p.133) suggests that this is a problem of translation and that the “ability to translate appropriately and correctly between concrete and written representations appears to be fundamental to understanding arithmetic”. It seems that Emily could not make the link between the concrete representations and the number sentences they were supposed to represent. Additionally, the model used assumed that Emily understood multiplication as repeated addition, which may not have been the case (Nunes and Park, 2001).

Staff views

Katia continued to be concerned that Emily was very “passive” when working with others in the class and seemed to prefer to work on her own. Emily found it hard to stay on task and she was not able to give Emily the time she needed. However, Emily was coping much better in the playground and was becoming much more confident. Emily's English skills continued to be better than her mathematics skills.

<u>Interview</u>

Emily was very keen to come out of class with me, so I had two short sessions with her, one in the morning and one in the afternoon. Emily was very chatty and very interested in what we were doing. She was able to focus for short periods of time but needed breaks after about 5 or 10 minutes. This could have been due to poor concentration, but could equally well have been due to boredom with the activities.

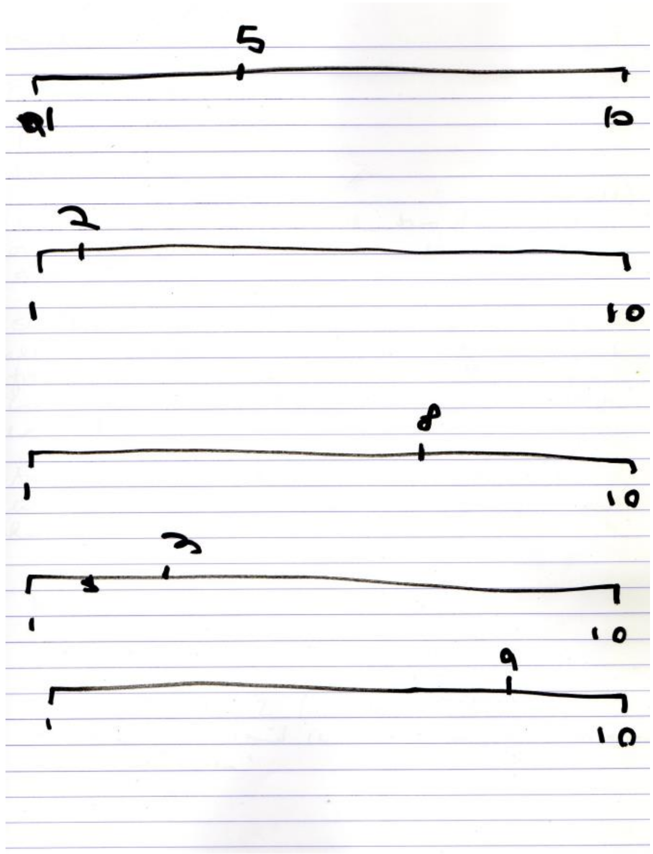


Figure 4-8: Emily's 1-10 number lines

I started the interview with the adapted number line estimation task (Siegler and Opfer, 2003). Emily decided that she would prefer it if the 0-10 lines started from 1, so I made some new lines for her to work on (see figures 4-8 and 4-9 for samples of Emily's attempts).

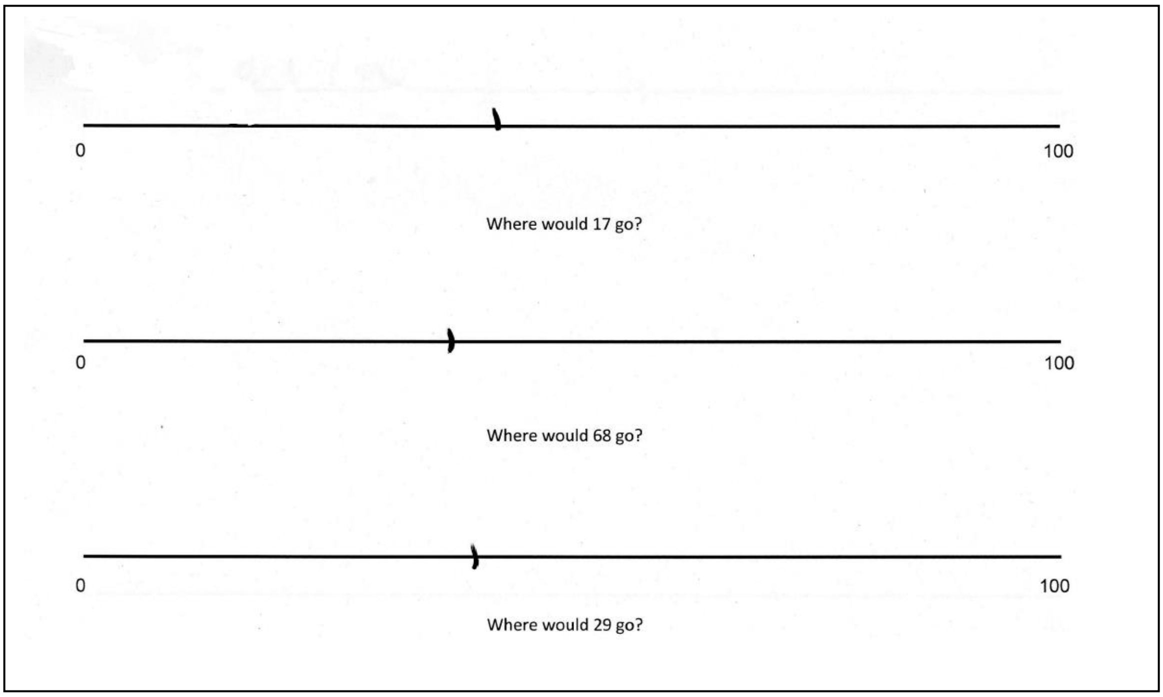


Figure 4-9: Emily's 0-100 number lines

Emily demonstrated a sound knowledge of the relational positions of numbers up to 10, but did not do the same for numbers up to 100 (e.g. placing all three numbers in very similar positions on the 1-100 number line). However, she was not able to demonstrate knowledge that half of ten is five. In addition, Emily's number lines do not demonstrate the expected logarithmic relationships found by Siegler and Opfer (2003), but they do show greater confidence with lower numbers, which is expected.

This was followed by an adapted version of the box task (Hughes, 1986):

C: [I put out 3 yellow counters] Emily how many yellow counters are there?

E: Three [with no evidence of counting]

C: Did you count them or did you just know?

E: I just know

C: Right if I put these three counters into the bag [as I put the counters into the bag]

E: Yeah

C: How many are in the bag [as I shake the bag]

E: Three

C: Good girl. If I put one more in [as I put one more counter into the bag]…

E: Four

C: That was very quick. Should we check?

E: Yeah

[I tip the counters out]

E: Four [very quick response and no evidence of counting]

Followed by:

C: If I put them in here [as I put them into the bag] how many's in here?

E: Four

C: Right if I take two more [as I take two counters and put them into the bag]

E: Six [very quick and before I had asked the question]

C: How did you work that out so quickly?

E: [pause] I don't know

Emily seemed to be able to subitise for up to three or four items. She was also very quickly able to work out how many counters would be in the bag when I added one or two counters. Interestingly, though, she was not able to explain how she had worked these out. Perhaps, it would have helped if I had given Emily the chance to represent her thinking in a drawing, as suggested by Carruthers and Worthington (2006). This might have supported her to explain her thinking in ways that made more sense to her, rather than in the more abstract way that I had asked her to reflect on her methods.

Following these achievements, I explored what would happen when I removed counters from the bag:

C: Right if I put the six counters back in here [as I put the counters back into the bag]. How many's in here?

E: Six

C: OK if I take one out [as I take a counters out of the bag] how many are in here now?

E: Five [very confident]

C: Should we check?

E: Yeah

C: [I tip the counters out] Were you right?

E: One, two, three, four, five [touching the counters as she counts] yeah

C: If I put the five back in here [as I put the counters into the bag]

E: Yeah

C: How many are in here? [as I shake the bag] [pause] How many did I put in?

E: Five

C: If I take out two [as I take out 2 counters] how many will be left?

E: Three [very confident]

C: Well done. You did those really quickly. Do you do a lot of things like this at home?

E: Yeah

C: With counters?

E: No with buttons

I already knew that Emily did a lot of work at home with her parents, so it was interesting to discover that she was familiar with this sort of activity and may go some way to explain her competence and confidence. At no point during the activity did Emily use her fingers to model her problem-solving.

I based the next section of the interview on the Number Knowledge Test (Griffin, 1997). Emily was able to add on three to a starting number less than five, without using her fingers or any other concrete aids. She also demonstrated knowledge of the positional relationship of numbers, in terms of which ones were bigger or smaller. However, she was more confident counting up from a target number than counting down when she had to answer questions on which numbers were closer. This was possibly because Emily was more familiar with counting up than with counting down.

Emily tried Panamath (Halberda et al., 2008). She enjoyed it and saw it as a game. Emily had several attempts. Her scores were all high and she typically achieved a Weber fraction of 0.16 (with the 90^{th} percentile at 0.24 and the 10^{th} percentile at 0.92) with an average response time of 3105 milliseconds (with the 90^{th} percentile at 1271 milliseconds and the 10^{th} percentile at 2272 milliseconds).

In the afternoon, I attempted some more activities exploring “more” using counters. Following my experience in the morning, I decided to explore this area in greater depth. I wanted to see if using concrete resources would help Emily to demonstrate her strategies.

C: How many counters have you got? [as I pass five counters to Emily]

E: Five [spoken very quickly, with no evidence of counting]

C: And if I have these [I point to my three counters] who’s got the most?

E: Me

C: How many more do you have?

E: Two more

C: Good girl. How did you work that out Emily?

E: Because I just know

C: You just know. Can I ask you another question?

E: Yeah

C: Right how many counters have you got this time? [as I give seven counters to Emily]

E: One, two, three, four, five, six, seven [touches the counters as she counts]

C: OK and if I’ve got these three [pointing to my three counters] who’s got the most this time?

E: Me

C: And how many more do you have?

E: Two more

I then tried with fewer counters:

C: OK what if [as I give Emily 4 counters] how many have you got there?

E: One, two, three, four

C: OK if I've got these [pointing to my seven counters]. How many have I got?

E: One, two..one, two, three, four, five, six, seven [touching the counters as she counts]

C: Who's got the most?

E: You

C: And how many more do I have?

E: Three

C: Why is it three?

E: Because I got three and you got three

Emily seemed to have no difficulty identifying who had more, but whether she understood what I was asking when I asked about "how many more?" is less clear. At points there was a sense that Emily was getting there, but it is hard to be certain of this. The counters may have provided an emergent model from which Emily was beginning to make links with the real world problem and the mathematical operation she was being asked to work out (Gravemeijer, 1999).

Emily did not move any of the counters and only touched them to count how many there were. Emily's difficulties with answering the "why" and "how" resonated very well with the comments made by Katia. It is likely that this issue was closely linked to her delayed speech and language development (Shipster et al., 2002).

<u>Finger gnosis</u>

Before we went back to class, I tried the finger gnosis assessment based on Gracia-Bafalluy and Noël (2008).

Emily had four fingers on her right hand and five fingers on her left hand (Figure 4-10).

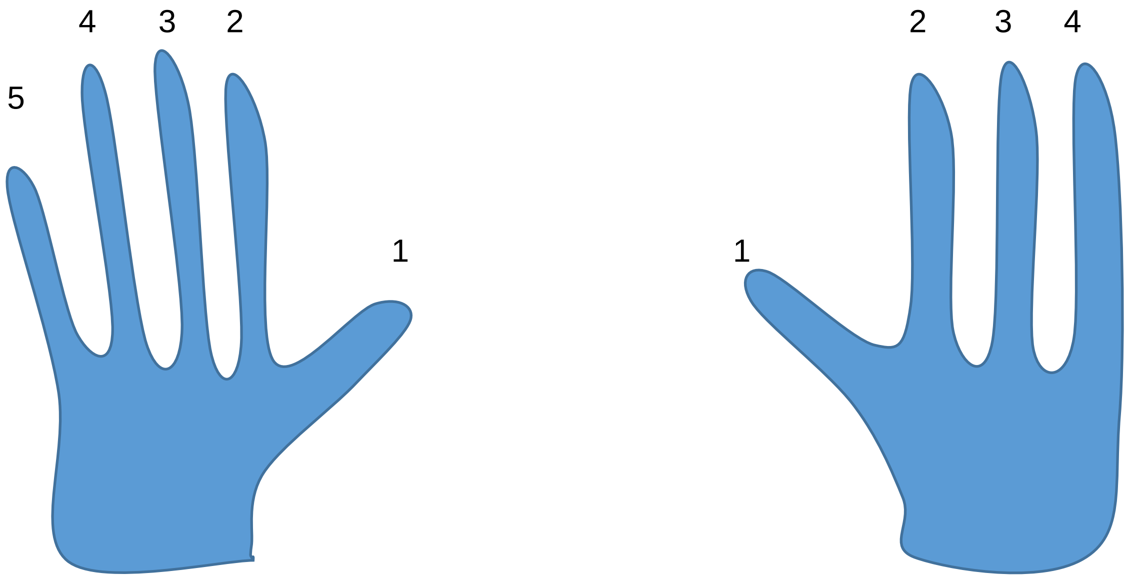


Figure 4-10: Diagrammatic view of Emily's hands

On the left hand, with one finger identification, Emily was able to correctly identify fingers 1, 2 and 5, but consistently mixed up fingers 3 and 4. When two fingers were touched, Emily was able to identify the correct fingers for the combination of 1 and 5 on her left hand, but seemed to pick any fingers when other combinations were touched. On the right hand, Emily could correctly identify fingers when one finger was touched. With two fingers, Emily could correctly identify her thumb when it was touched, but was not consistent with any other fingers.

4.4.3 Third School visit (December 2013)

I arrived in the morning, in time to meet with Emily's new teacher, Wilma. Emily was now in Year 2. She was also attending intervention classes for mathematics three times a week.

<u>Observation</u>

Emily sat at a table with four other children for her mathematics lesson. The focus of the lesson was on halving. I worked quite closely with Emily, so it was hard to take notes during this lesson.

During the first part of the lesson, after the introduction, the children were doing questions on “halving”. Emily had cubes to help her. The first task was to halve four. Emily counted out four cubes. She then wrote “half of 4 is 1”. Next she tried to work out half of six. This time, Emily counted out six cubes and then wrote “half of 6 is 2”. I discussed the meaning of “half” with Emily and explained that it was not just about making sure that there were two parts, but that the two parts had to be the same. She could then halve four successfully and halve six successfully when reminded that the two parts had to be the same.

The lesson ended with some multiplication questions:

4x2p

10x5p

10x2p

8x2p

Emily attempted all these questions. This time, and for the first time during my visits, Emily used her fingers to keep track when she was counting in fives and twos. After the lesson, the children left the class for break.

Staff views

Wilma felt that Emily was working within level 1 for mathematics, reading and writing. Emily was, however, at the lower end for mathematics and writing.

Wilma felt that Emily was doing very well to manage all her difficulties and would always try everything herself. She was, however, concerned that Emily seemed to find it hard to stay focused and needed frequent reminders to stay on task.

Wilma was particularly worried about Emily’s skills in mathematics. She said that Emily “…knows a lot of number facts, but finds it harder to apply her knowledge.” These comments were very similar to those of Emily’s previous teacher. Emily found two-step problems very difficult in mathematics and she was having difficulty with division and found mental mathematics hard.

Wilma was very interested in finger gnosis and wanted to talk to Emily's parents about using some of the activities we discussed, based on the work of Gracia-Bafalluy and Noel (2008).

Interview

This time we had to work in the corridor, so there were many distractions for Emily. During this interview, Emily attempted the WIAT II Mathematical Reasoning test (Wechsler, 2005) and achieved a standard score of 99. Of particular interest was Emily's response to the task requiring her to say how many wagons there were in a picture ("How many wagons do you see? You can touch them as you count, if you wish."). In response to the question, Emily said "Three, no four." When I checked, Emily confirmed that it was four and, when asked how she knew, she said that she had counted. If she did count, she did it very quickly. It is possible, though, that she subitised for two or three and then counted on. For questions involving addition and subtraction, Emily used counting strategies to solve the problems.

For the rest of the interview, I repeated some of the questions from the Number Knowledge Test (Griffin, 1997), in order to see if there was any change in the strategies Emily used. On this occasion Emily drew images to support one of the questions:

C: If you had four chocolates

E: Yeah

C: You can write things down if you want to [as I pass Emily some paper]….if you had four chocolates and I gave you three more

[Emily draws four rectangles and then three more (Figure 4-11)]

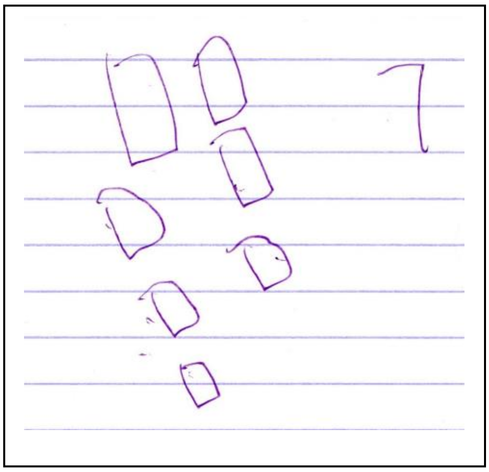


Figure 4-11: Emily's drawing of "Four chocolates and three more"

C: How many would you have altogether?

E: One, two, three, four, five, six, seven [pointing to the images as she counted]

C: Do you want to write that down?

[Emily writes 7 next to the rectangles]

Emily seemed at ease constructing her own image to help with the calculation and once she had completed the drawing, she used a count all, rather than a count on approach. Next I asked her about numbers in the number sequence:

C: What number comes right after seven?

E: Eight

C: What number comes two after seven? What number comes two after seven?

E: Three, four

C: Sorry

E: No, eh

C: Two numbers after seven

E: Nine

C: How did you do that?

E: I counted it

When I asked her about other more conventional calculations, the response was much less confident:

C: How much is two plus four?

E: Six

C: How did you work that out Emily?

[pause]

C: Do you know how you worked that out?

E: Yeah

C: What did you do?

E: [says something inaudible and then says nothing, so I move on]

That was interesting because it was not clear if she had worked it out, or if it was a number fact that she already knew. With the next questions, Emily did use a counting strategy:

C: What number comes five numbers after forty-nine? [pause] Do you know what comes five numbers after forty-nine? Five numbers after [emphasising "after"]

E: Forty-six, forty-seven, forty-eight, forty-nine, fifty

C: OK…What number comes four numbers before sixty? Four numbers before sixty

E: Fifty-five…no, fifty-nine, fifty-eight, fifty-seven, fifty-six

Emily did not use her fingers to help with this and yet she was able to keep track. It seems likely that for the first question, Emily heard "forty-five" instead of "forty-nine". She clearly knew her number sequences very well and was confident to count forwards and backwards.

4.4.4 Fourth school visit (March 2014)

I arrived in the morning as usual. Emily now attended a mathematics intervention group three times a week.

Observation

Emily was at a table with four other children, so I joined them. The starter activity involved halving. This time Emily was successfully able to halve 4, 8, 10 and 6. It looked like she was working these out from memory. At one point, when she was working out 6÷2, she touched her forehead and then got the answer.

The main activity was on contextualised word problems. After a brief introduction, the children had to work from worksheets on questions involving money. The questions required the use of a range of mathematical operations. Emily used her fingers for each question to keep track of the counts (forwards and backwards). When Emily did not answer questions correctly, I encouraged her to read the questions out loud. She did this with ease, but when I asked her to explain what each question was asking, she struggled. This then made it harder for her to make links between the questions and the mathematical operations she was expected to use.

Wilma noticed that Emily was not writing out the mathematical sentences, so she asked Emily to do this. Emily wrote all of them as subtractions, even though some of the questions required addition and multiplication. This was interesting, given Emily's earlier performance in the WIAT-II Mathematical Reasoning test (Wechsler, 2005), where she could answer word problems of a similar nature. The difference with the test was that each question was accompanied by an image, but the questions Emily was given in class were just presented as contextualised word problems. It is possible that if Emily had had some way of representing the problems with concrete objects or images, she may have found it easier to make sense of what she had to do.

Fyfe, McNeil, Son and Goldstone (2014) and Fyfe, McNeil and Borjas (2015) have discussed this relationship between objects and images in mathematical problem-solving. Taking the ideas of Bruner, they argue that for children to be able to make sense of contextualised word problems, it is helpful to introduce problems with a concrete element first which then "fades" to an image. This image should be used alongside the more abstract symbols of mathematics, in order to support the transition to solving problems with only abstract notation. As Emily had previously demonstrated the ability to use diagrams to make sense of questions in my last visit (see 4.4.3), it seems likely that if this had been demonstrated, she may have been able to make sense of the contextualised word problems and been able to relate them to the abstract symbols of mathematics and the mathematical models that represented the problems.

Staff views

Wilma felt that Emily was making progress and was now working at level 2 in mathematics, reading and writing, but, as before, was at the lower end in mathematics and writing. Emily had started doing the finger gnosis exercises and Wilma felt that it was making a difference to Emily's mathematics. Emily was using her fingers more in calculations and Wilma felt that she had gained in confidence as a result. On Emily's right hand, which had four fingers, she used both sides of finger 3, in order to make the whole hand up to five. Although Wilma continued to be concerned about Emily's progress in mathematics, she felt that the finger gnosis exercises were definitely helping.

Emily's comprehension and reasoning skills continued to be a problem in all subject areas. Emily could still be easily distracted and found it hard to stay focused without adult support.

Interview

On this visit, we had access to a quiet room, so I did the WMTB-C (Pickering and Gathercole, 2001) with Emily (Table 4-12).

Test component	**Standard score**
Digit Recall	142
Block Recall	108
Listening Recall	145
Counting Recall	81
Backward Digit Recall	72

Table 4-12: Emily's WMTB-C scores

Emily's high scores in some of the tests may explain why she was able to perform simple calculations in mathematics using the knowledge that she had about numbers, without relying on fingers or manipulatives. The lower score for Counting Recall, may have been due to the fact that it took Emily a long time to count the dots on each image and then had to remember these for each subsequent count. Thus the time taken meant that she was at risk of forgetting (Hitch, Towse and Hutton (2001). Emily found it very hard to know what to do for the Backward Digit Recall test, and could not say three digits in reverse order.

Emily also attempted the WIAT-II Numerical Operations test (Wechsler, 2005) and achieved a standard score of 103. I would argue that the results here are not necessarily indicative of mathematical understanding, as the requirement in the test at this level is for children to be able to complete calculations such as 3+2 and 3-2, with which Emily was probably very familiar.

Finally, Emily had another attempt at Panamath (Halberda et al., 2008) and achieved a Weber fraction of 0.32 (with the 90^{th} percentile at 0.22 and the 10^{th} percentile at 0.70). Emily's average response time was 3807 milliseconds (with the 90^{th} percentile at 1174 milliseconds and the 10^{th} percentile at 2042 milliseconds). Scores in both areas were lower than last time, but this was the end of a long session, so perhaps it is not surprising that Emily did not focus as well as before.

Finger gnosis

The finger gnosis assessments were very interesting. As part of the finger gnosis training, Emily had numbered her fingers. Consequently, when I touched Emily's fingers, she was able to tell me which fingers she thought I had touched by naming them.

When one finger was touched, Emily was correct with every trial on her right hand, but mixed up fingers 3 and 4 on her left hand. When two fingers were touched, Emily made errors with every pair and even made some errors when her thumb was touched. When fingers 1 and 5 on her left hand or 1and 4 on her right hand were touched, Emily could identify one of the fingers correctly. Emily's finger gnosis seemed very similar to the last assessment which had taken place in July.

4.4.5 Fifth school visit (June 2014)

I arrived in the morning. Emily was no longer wearing a bone conduction hearing aid, as it was thought that it was not helping very much.

<u>Observation</u>

This time I observed Emily in her intervention class. The class had 12 children and Emily was at a table with three other children. The focus of the lesson was on finding fractions of money. The teacher Wilma started by asking the class what a fraction was:

E: It's to do with dividing….because it's like sharing

Wilma: Does anyone remember what a third is?

E: I don't know what it's called but is it a quarter?

This seems to suggest that Emily had some confused ideas about the language of fractions. The children were then asked to make 20p using more than two coins (i.e. not two 10p coins).

Wilma: I want you to get twenty pence but I want you to use more than two coins so what don't I want?

E: You don't want a ten p and a ten p

The children had a selection of plastic coins which represented 1p, 2p and 5p coins. In Emily's first attempt, she had four 5p coins and three 2p coins. Wilma noticed the mistake and came over to help Emily. Wilma got Emily to count out her coins. Emily did this correctly and realised that she had made more than 20p. On her next attempt, Emily picked up four 5p coins. Wilma then asked the children to check each other's coin collections. Emily checked her partners and correctly added up his coins, starting with the 5p and double counting the 2p's.

Wilma: Here's a problem for you…what's half of twenty p? Put half of twenty p in your hand

[Emily puts two 5p coins in her hand]

Wilma: What's half?

E: Half

Wilma: What's half of twenty?

E: [looks at her hand] five…ten p

Wilma: In your hand put a quarter [writes ¼ on the board] of twenty p. Put a quarter of twenty p in your hand

E: [Emily puts all four 5p coins in her hand] I've got it!

Wilma: So Emily what's a quarter of twenty?

E: Ten [Emily shows two 5p coins]

Wilma: So can we make four groups? So what's a quarter of twenty?

E: One…five pence

Wilma followed this up with some questions on the board

1. ½ of 20p=
2. ½ of 10p=
3. ½ of 18p=
4. ¼ of 16p=

Emily did the first three questions very quickly. When I asked how she worked them out, she said, "I just thought them in my head." However, when Emily got to question 4, she said, "I'm stuck." Wilma came over and asked Emily what she could do to help. Emily started making different combinations of coins and finally got 16p by using three 5p coins and a 1p coin. This was then offered to the class as a problem. "How can Emily split her 16p into four groups that are the same?" One child suggested that it might be better to make 16p using only 1p coins. Emily followed this advice and then split her coins into groups of 4p, 3p, 4p and 5p.

Wilma: Are they all the same?

E: No

As this was towards the end of the lesson, Wilma quickly explained what needed to be done, but it is hard to know if Emily was able to take this on-board. However, she was quickly able to say that the groups were not all the same size, even if she was not clear how this linked to what she was being asked to do. This whole episode suggests that Emily was in the process of learning that when we split something up into fractional parts, each of the parts needs to be the same size. This early misconception has been discussed by Hansen (2014).

Staff views

Wilma was pleased that Emily had achieved level 2 in her end of year assessments. She did comment, however that Emily continued to have difficulty with problem solving and deduction skills in mathematics and with inference and deduction in reading activities.

Wilma felt that Emily was a bit scared in the playground, especially when the children were running around. Emily tended to stay at the side, where she felt safe and although she had three friends, all boys, they did not tend to play with her in the playground, but were very protective towards her in class.

Wilma reported that "The finger gnosis exercises have really helped with Emily's number work, because she can now visualise the numbers much better and seems to have a better sense of numbers."

Interview

On this visit, we did not have a room to work in, so our interview took place in the corridor. Consequently, I kept the interview short. I wanted to try the Backward Digit

Recall test of the WMTB-C (Pickering and Gathercole, 2001) as Emily seemed to have such difficulty with it on the last visit. This time Emily achieved a standard score of 125. When I asked Emily how she was remembering the numbers, she said that she tried to "see" the numbers to help her remember them.

Emily tried the number line estimation task (Siegler and Opfer, 2003) again on this visit. Emily showed more accuracy with numbers from 0 to 10 and greater awareness for numbers from 0 to 100. Emily's "5" was now much more central and she continued not to demonstrate a logarithmic relationship for numbers up to 10 (Figure 4-13).

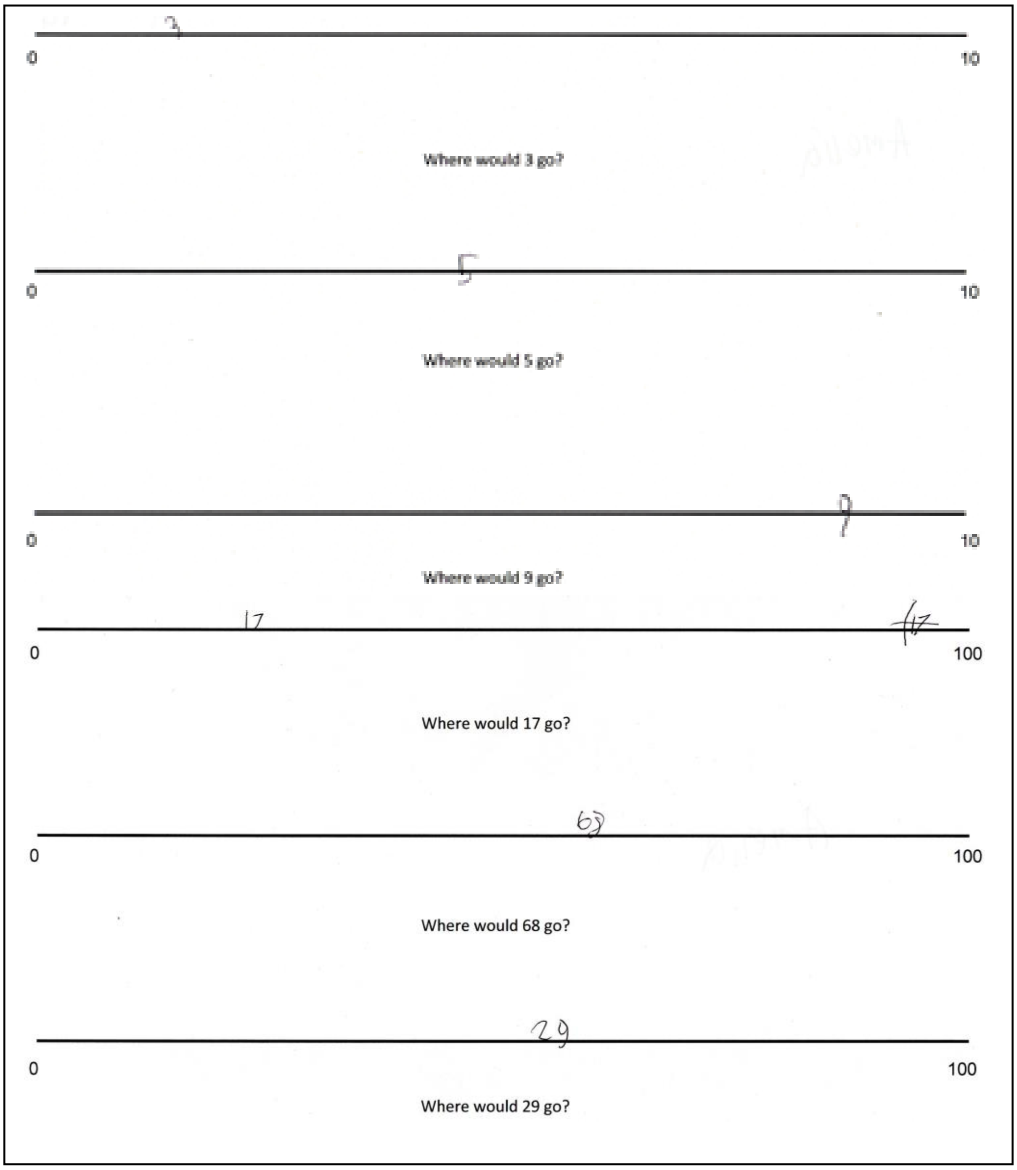


Figure 4-13: Emily's 0-10 and 0-100 number lines

Finger gnosis

This time Emily was very successful with both hands and with one and two finger touches. On her left hand she made a few mistakes when fingers 3 and 4 were touched, but on other trials, she identified them correctly. This was a significant change from the last visit in March.

4.4.6 Sixth school visit (January 2015)

Emily was now in Year 3. The class was shared by two teachers, but on the day I visited, her teacher was Carys.

Observation

For mathematics, the two classes in Year 3 were in sets which were grouped by “ability” (meaning attainment). Emily was in the lower of the two sets. The lesson began with a starter activity with the children all sat on the floor, at the front of the class opposite the teacher. The children all had mini whiteboards on which they were asked to write their three times table. Emily wrote:

1x3=3

2x3=6

3x3=8

4x3=12

5x3=15

6x3=18

She then paused for a breath and for the rest of the questions, up to 12x3, she copied off her neighbour.

Carys began the main lesson which was on tallying and drawing “pictograms” (which were in fact block graphs) to illustrate the information. Emily was very quiet during the introduction and often appeared be unfocused (e.g. drawing on her mini whiteboard). She did not volunteer any answers to the questions that were addressed to the whole class. Carys read out the months of the children’s birthdays and each group had to record their results in the form of a tally. Emily was able to tally her information. She hesitated once she reached the fifth person, but then correctly put a line through her existing four tallies to indicate five.

The children were asked to construct a pictogram (block graph). Emily began to draw her axes, but found it very hard to locate the correct position on the page and then line up with the lines on her page. Nevertheless, Emily persisted and did not ask for any assistance.

Staff views

Carys felt that although Emily was very quiet in class, she was doing well socially. Carys felt Emily needed to become more independent, as she was too reliant on adult support. Carys also reported that, “Focus, concentration, amount of work done are issues for Emily.” This was not helped by Emily’s poor organisational skills. Other children in the class were very helpful to Emily - Emily was definitely accepted as part of the class and was regularly invited to children’s birthday parties.

Emily was working at level 2 in all subjects, including science.

In the first term, Emily had been in an intervention group for English, three days a week, but this had now stopped, as Emily was doing better than the other children in the intervention group. Instead, Emily was being monitored to make sure that she made progress. Although Emily was a confident, fluent reader, she continued to struggle more with comprehension skills.

Carys felt that Emily was “pretty good at maths and good at mental maths, place value and times tables.” Although Emily was in the bottom set for mathematics, she was in the middle of the group, in terms of “ability”.

Interview

The interview took place in the library - an open space which we shared with other children from another class. Emily and I sat at our own table and Emily was able to focus well. I wanted to review the Number Knowledge Test (Griffin, 1997) to see if Emily could get further, but also to review Emily’s strategies for working out her answers. Throughout the interview, I noticed that Emily used her fingers much more than on previous visits and she seemed to rely much less on known number facts. It is possible that this gave her more space in her working memory to focus on the problems and how she could solve them, as she could represent the numbers on her fingers rather than having to hold so many numbers in her head. For example:

C: If you had four chocolates

E: Yeah

C: And someone gave you three more

E: Yeah

C: How many would you have altogether?

E: Four, five, six, seven [counting on her fingers]

However, when adding two, she could do it without her fingers and could explain her working:

C: OK ummm how much is two plus four?

E: Eight…six

C: Which one, eight or six

E: Six

C: Good girl. Why is it six?

E: Because six is before…six is after four

C: Yes. How many numbers after four is six?

E: Two [said with no hesitation]

For subtraction, Emily used her fingers to help her count back:

C: Lovely. How much is eight take away six?

E: Seven, six, five, four, three, two [using her fingers]

In the following questions, Emily's finger-use was interesting because when these questions had been asked before (in December 2013), Emily was able to answer the questions without the use of her fingers. On this occasion, though, she chose to use her fingers:

C: What numbers comes five numbers after forty-nine...five numbers after forty-nine?

E: Forty-nine?

C: Yes, forty-nine

E: Fifty, fifty-one, fifty-two, fifty-three, fifty-four [using fingers]

C: OK lovely…what number comes four numbers before sixty? Before sixty

[counting back using fingers inaudibly]

E: Forty-six

C: Forty-six. Do you want to try that one again? What's the number before sixty?

E: Ummm fifty-nine

C: OK, so try that one again. Four numbers before sixty?

E: Fifty-nine, fifty-eight, fifty-seven, fifty-six [using fingers]

C: So do you think it's forty-six or fifty-six?

E: Fifty-six

This extract suggests that Emily was able to self-correct when prompted and then make a choice about which answer was most likely to be correct.

Emily was quick to calculate 12+54 and 47-21 and used her fingers to help. When she was asked to work out a calculation that involved bridging through 10, she was able to use her knowledge in an interesting way to solve what at first sight appeared unsolvable:

C: Ummm can you work out thirteen add 39? You can write it down if it helps thirteen add thirty nine [spoken slowly as Emily writes 13+39]

[pause]

C: Do you know what it will be?

E: No I don't know what the answer is because…the trouble is the twelve….and I've got to add another ten on

C: Yeah so what do you think this might be? [as I point to the calculation that Emily has written down] [pause] What's the strategy you could use to work it out?

E: Umm...nine and three...nine, ten, eleven, twelve [using fingers]. Now…fifty add two is fifty two [writes = 52]

C: How did you get the fifty Emily? Where did you get the fifty from?

E: The tens

C: The tens. Can you explain to me how you got it from the tens?

E: Ermm like..like…thirty and ten is forty. Then I added another ten on from the twelve

Emily used her understanding of place value with confidence and was able to clearly articulate her reasoning. Could it be that the fact that she was using her fingers to model her calculations, meant that she was able to see what she was doing and, therefore, better understand how she was working out the problems?

Emily's understanding of place value was used again for the following questions:

C: What number comes ten numbers after ninety nine? Ten numbers after ninety nine

E: It's a hundred and nine

C: That's very good…that's very quick. How did you do that?

E: Because it's…it turns into a hundred and so it's a hundred and nine

C: Can you write that down for me?

E: What's the sum?

C: It's ninety nine

E: Ninety-nine [as she writes 99]

C: Plus or add ten

[Emily writes +10, to get 99+10=109]

C: Lovely. What number comes ten numbers after nine hundred and ninety nine?

E: Ummm A thousand and nine [picks up the pen and begins to write]. Nine hundred and ninety nine add nine [writes 999+9=10009, as she says "One thousand and nine." She looks at what she has written but says nothing and makes no changes]

This is interesting as it demonstrates a clear mismatch between Emily's application of her number knowledge when she is speaking and when she is writing. Conversely, it could be that her own voice is interfering with the way that she is interpreting her answer. In this case her answer is "one thousand and nine" which it could be argued is exactly what she has written. This is an example of the concatenation that Nunes and Bryant (1996) found and which they argued is due to the use of “and” in the spoken utterance of the number “1009”. At this stage, Emily is probably more familiar with working on numbers in the hundreds and could, therefore, write these numbers more reliably than numbers in the thousands. It also seems likely that Emily counted up to get her answer, rather than using knowledge of number patterns. It seems likely that Emily is in the process of making sense of the place value system and this is evidence that she still has some way to go.

The next example demonstrates an interesting use of mixed methods, including the use of known number facts combined with finger strategies:

C: Right, can you do thirty-six? You might want to write this down...thirty-six take away eighteen [Emily writes down 36-18 as it is read out to her]

E: Thirty six take away ten is twenty six

C: Yep

E: And twenty-six, twenty-five, [counts correctly down to eighteen using her fingers]

[Emily writes = 18 on the end of the calculation]

What was striking on this visit was Emily’s confidence and ability to articulate what she was doing. She also demonstrated flexible and interchangeable use of number relationships, number facts and more informal methods. In terms of visible strategies, the most distinct one was her increased use of fingers in problem-solving which possibly provided her with the ‘mental space’ to focus on the methods she was using.

Emily had another go at Panamath (Halberda et al., 2008). This time she scored a Weber fraction of 0.43 (with the 90^{th} percentile at 0.22 and the 10^{th} percentile at 0.70) with an average response time of 1865 milliseconds (with the 90^{th} percentile at 1174 milliseconds and the 10^{th} percentile at 2042 milliseconds). Both of these scores are of interest. Emily’s accuracy had decreased, but her average speed of response had increased. Perhaps she had realised that the aim of the task was to answer as quickly as possible, or perhaps she was guessing on more occasions, as she may have lost interest in Panamath as a “game”.

Finger gnosis

On this visit, Emily was correct on every trial on both hands with one finger and with two.

4.4.7 Conclusions

Emily was always very lively and enthusiastic when I visited. She was initially described by her teachers as knowing how to perform certain arithmetic tasks, but without understanding. In addition, Emily's teacher assessments suggested that her mathematics levels were consistently below her levels for reading. This description of her mathematical performance persisted through Year 1 and into Year 2. Observations and interviews with Emily during this period presented a mixed picture.

Emily's scores in the WIAT-II assessments (Wechsler, 2005) placed her within the expected range for her age for both Numerical Operations and Mathematical Reasoning (see 4.4.4).

Emily had friends in the class who were very protective of her, but she rarely played with her peers in the playground, preferring to be with the adults instead. Over the time that I visited, her teachers reported a growing confidence, but it seemed that this was an area the school saw as an area for further development.

The overall conclusions will be considered with reference to the research questions.

1. What strategies did Emily use to help her solve numerical problems in mathematics?

In 4.4.2, Emily demonstrated that she could answer questions involving addition and subtraction with numbers from 1 to 10 and addends or subtrahends between 1 and 3, but that she found it hard to make the link between abstract number sentences and concrete representations. Contextualised problems sometimes presented a problem for Emily (as in 4.4.3 and 4.4.4), but this depended on the demands of the question, in terms of mathematical language and representation and in particular whether she was expected to use symbolic notation or more informal approaches. Emily relied very much on her working memory, but by the end of my visits, as was seen in 4.4.6, Emily was using her fingers as an additional resource, thereby reducing the load on her working memory and supporting her to demonstrate how she was engaging with the mathematics.

2. Did Emily's hand anomalies impact the range of strategies available to her?

When I first visited Emily she did not use her fingers to help with problems involving number. In Year 2 Emily's teacher began the finger gnosis training, as described by Gracia-Bafalluy and Noël (2008). Her teacher at the time believed that it was helping and Emily was very happy to use her fingers (see 4.4.4 and 4.4.5). By the time Emily was in Year 3 (see 4.4.6), she was using her fingers with great confidence, and had found a way to "create" five fingers on each hand. Of particular interest at this time, was the flexibility with which she used her fingers, in amongst the other knowledge and strategies adopted. In addition, on the last visit, Emily was able to clearly articulate how she had worked out the problems and how she had overcome aspects that at first seemed more challenging. Her fingers seemed to provide a model which enabled her to focus more on the mathematics rather than focus on the mechanics of solving problems by relying on her working memory capacities.

3. Are there other factors which had an impact on Emily's mathematical learning opportunities?

As seen in table 4-12, Emily's working memory scores were all within, or above, the expected range, apart from the score for Counting Recall. Emily demonstrated that she had particularly strong working memory capacities for the tests for Digit Recall and Listening Recall. Emily found it very time-consuming to count all the dots and this may account for her low score in this assessment of her central executive - below average counting speed for age makes this task more challenging as the information must be retained for longer. In the Backward Digit Recall assessment, Emily scored 125, so she was clearly able to use her central executive skills successfully in some tasks. This may explain how she was able to work out answers to number problems, involving counting forwards and backwards, with such apparent ease. Emily practised number activities at home with her parents, and this seemed to give her a lot of confidence with number activities involving addition and subtraction of small numbers of objects.

During Year 1 and Year 2, Emily found it hard to explain her working. Emily was always very chatty, but her ability to explain what she was doing in her mathematics developed significantly when she was in Year 3. This made it much easier for me to understand her thinking as she became more able to explain the processes she was going through when solving problems.

Emily had visual and hearing impairments. She always wore glasses and had tried a range of hearing aids. These difficulties would inevitably have impacted on her ability to access the learning environment. Emily had friends with whom she sometimes worked and was well-liked within the class. Emily had very limited in-class support, so her teachers had had to support her throughout to develop strategies to be as independent as possible.

Isabelle

I first saw Isabelle in November 2012. I met initially with Isabelle's parents (Ian and Alice) and later met Isabelle when she returned from school. At this time, Isabelle was 8 years old, in Year 4 and attending a local two form entry primary school. Isabelle had a statement of SEN and was in receipt of full-time support from a teaching assistant (including break times). Isabelle's parents chose the school because they thought it would provide a very caring environment for Isabelle.

Isabelle's parents said that Isabelle was very active and was always busy. Amongst the activities Isabelle enjoyed were: swimming; walking; going to the park; playing with her Wii and riding her bicycle. Isabelle also enjoyed reading and was faster at reading than anyone else in her class.

Isabelle's parents said she was "…very resourceful and has great survival skills, but knows her limitations." It was always important for Isabelle to know why she had to do things. She also had a "…very strong character and could be very intuitive." Isabelle could be, "…quite selfish with things she likes, although she does share with other children." Isabelle's parents said that she could get quite upset if other children did not want to play with her. Isabelle, "feels what's going on around her and can empathise with others and pick up the emotions of others."

Isabelle's parents pointed out that Isabelle had a very good memory for things she has seen or heard and, as an example, pointed out that she was able to find all the keys on a piano. Ian said that Isabelle enjoyed music and could "feel" music and move in time with it. Ian said that Isabelle was "…really good to talk to" and that talking with her was more like talking to an adult than a child.

In terms of milestones, Isabelle's parents remembered that Isabelle walked at about 2 years of age – prior to this she shuffled. Isabelle also began to talk, using just single words, at about the same time. Isabelle had a vault expansion at 6 months of age and had operations on her hands to separate her fingers between the ages of about 1½ and 4 years to give her five fingers on each hand. Isabelle had recently received a diagnosis of autistic spectrum disorder. Isabelle wore glasses and in-ear hearing aids. Isabelle's hearing was described as "up and down". She had reduced peripheral vision and was long-sighted, but could read without her glasses. Isabelle was left-handed.

Isabelle's parents were very concerned about her development in mathematics. Although Isabelle knew her times tables and many number facts, they did not feel that she understood the mathematics that underpinned this knowledge and they were concerned that she did not seem able to apply this knowledge to day-to-day, situations. As a result, Isabelle's parents had recently arranged for a maths tutor to come once a week, after school.

Isabelle's parents did remember that Isabelle sometimes used her fingers for number problems when she was younger, but they felt that she now relied on her memory and did not did not use her fingers to help with solving problems in maths. When they heard about my interest in children's use of fingers in mathematics, they said they would talk to her maths tutor and see if he would be interested in encouraging Isabelle to use her fingers more.

I asked Isabelle's parents if she engaged with activities involving number, such as telling the time, cooking and board games, when she was at home. Both parents said that Isabelle did not and Ian said "Not if she can avoid it!" He did, however, point out that Isabelle liked playing Snakes and Ladders (if she won) and enjoyed playing picture card matching games.

According to Isabelle’s parents, Isabelle also struggled with writing. In addition, there had recently been concerns at school about her behaviour and social skills. The school had reported that Isabelle sometimes made inappropriate comments to the other children. However, they also mentioned that Isabelle was particularly good at science, enjoyed PE and was improving with her art work.

When Isabelle came home from school, she was very chatty and interested in why I was there. We talked about the work I was hoping to do with Isabelle at school and she was keen to participate.

4.5 Isabelle

4.5.1 First school visit (January 2013)

I arrived at school in the morning, just as the children were going out to play. The mathematics lesson was due to start after break.

<u>Staff views</u>

On this visit I spoke with the SENCo, Rachel, as well as Isabelle’s teacher, Anna and Isabelle’s teaching assistant, Frances. Rachel felt that Isabelle’s understanding in mathematics was very poor. She was concerned that Isabelle relied on her memory to recall number facts, but without understanding the concepts. At this point, Isabelle’s levels were:

Reading 3c

Writing 2b

Mathematics 2a

Anna did not think that Isabelle’s understanding in mathematics matched the level she had been awarded and thought that she was actually working at a lower level. Isabelle was in Year 4 at this time, so was working well below expected levels for her age. Isabelle found it hard to write, due to her poor manual dexterity and it was believed that this was the reason for her lower level for writing.

Anna described Isabelle as very set in her ways and good at retaining number facts (for example times tables), but she was not sure if she understood the mathematical concepts. Anna also said, “Isabelle can be very lazy and her mind does wander.” She said that she was trying to get Isabelle to be more independent and not rely so much on Frances. Anna was concerned that she did not always know how much of the work Isabelle had done on her own.

In terms of relationships with peers, Anna said that, “For paired work, Isabelle always chooses another child with SEN” and that, “…during golden time, she’ll [Isabelle] join in but doesn’t really have friends.” Anna was also worried about the fact that when she asked questions to the whole class, Isabelle rarely put her hand up to answer.

Frances had been working with Isabelle since she was in Reception and had built up a positive relationship with her. She also described Isabelle as being, “…very set in her ways” and remarked that, “…if she’s left on her own, she does nothing.”

Anna and Frances both pointed out that Isabelle did not really have any friends in her class. Anna also said that some of the children could be “quite unpleasant” to Isabelle, but that Isabelle could also be “rude” in class.

Observation

Isabelle came in from break and quickly found her seat. The class was grouped by “ability” for maths and Isabelle was in one of the lowest groups. Frances sat next to Isabelle and I sat behind them.

The first activity was a paired activity. The children were shown a pentagon on the interactive whiteboard and they had to say how many sides and vertices the shape had and then try to name the shape. Isabelle correctly counted the number of sides and vertices, while her partner tried to guess the name of the shape.

Anna went through the names of different triangles and their properties, using the interactive whiteboard to illustrate them. The children were then required to name a range of triangles on a worksheet. For this part of the lesson, Isabelle worked alone with the support of Frances. With support and prompts, Isabelle was able to name all the triangles correctly. The next activity involved measuring the lengths of lines. For this activity, Frances checked that Isabelle knew what to do and then left her to work alone. Isabelle was able to use the ruler correctly for the first three lines, but then started to measure the lines from **1** on the ruler, rather than **0**. When she did not self-correct, I asked her if she needed some help, to which Isabelle said, “No thank you.” As this was my first observation and I did not want to embarrass Isabelle, I did not interfere, but raised it later with Isabelle’s class teacher. The lesson ended and Isabelle followed the instructions to pack up.

Throughout the lesson, Isabelle sat quietly and when Anna was teaching, she seemed to be focused on what was being taught. When she was working independently, Isabelle responded to Frances’s prompts, but did not manage so well when Frances was not there.

Interview

The interview took place in a side room and Frances joined us. For the first part, I first of all wanted to find out about Isabelle’s counting skills:

C: Can you first of all count as far as you can?

I: Ummm...

C: Start from one

I: One, two, three, four [Isabelle went up to forty, counting very quickly and correctly]

C: OK. Brilliant. Thank you. Can you count on from eight- four?

I: Eighty-four, eighty-five, eighty-six [Isabelle counted on quickly and correctly to one hundred and stopped]

C: Keep going....

I: One hundred and one, one hundred and two, one hundred and three, one hundred and four…

C: Fantastic...Can you count backwards from ummmm thirty-eight?

I: Thirty-eight, thirty-seven, thirty-six [counts down correctly to sixteen when I stop her]

C: Brilliant...That’s fantastic. When you’re doing it…because you know the numbers really well don’t you? Do you just know them or do you see a number line or have some other way?

I: I just know them

C: You just know them. Do you know what the biggest number is you can count up to?

I: Uh huh

C: What is it?

I: It is one thousand [spoken with great confidence]

C: What comes after a thousand?

I: A billion

C: What's one more than a thousand? [emphasising "one"]

[Isabelle says nothing]

C: Do you know?

[Isabelle sighs and shrugs, so I move on]

Isabelle seemed happy to be participating and willing to continue. The next few questions focused on reading numbers up to three digits. Isabelle read all of these quickly and correctly. She was also able to very quickly tell me that one hundred and thirty came after one hundred and twenty nine.

Next I tried to explore her addition and subtraction knowledge and understanding using smaller numbers:

C: And what comes two numbers after seven?

I: [short pause] Nine [more confident]

C: What number comes two numbers before eighteen?

I: Sixteen?

C: Uh huh. And two numbers after eighteen?

I: Twenty?

C: Yeah. How are you working those out?

I: Ummm I'm just counting in twos

C: You're counting in twos. OK, fantastic. Do you use your fingers at all to help? [I had noticed that Isabelle seemed to be using her fingers, but I wanted to see what she would say if I asked]

I: Yeah

As Isabelle's parents had said that she did not use her fingers much for mathematical calculations, I wondered whether they had started to encourage her to use her fingers following my first meeting with them.

I then started to explore Isabelle's knowledge and understanding more formally, using questions based on those used by Jordan, Glutting and Ramineni (2008). The more formal language of mathematics, provided some insights into Isabelle's thinking and additionally, her relationship with Frances:

C: OK what's two plus one?

[I pauses]

C: Two plus one? Two add one?

I: Two plus one is three

C: Four plus three?

I: Four plus three...Four plus three...Eight?

[in both of these examples, the process of repeating the question out loud was clearly helpful for Isabelle, but the strategy she used was not obvious]

C: How would you check that?

I: You just have to do ones

Frances: What do you normally do?

I: Count in your head

C: Do you remember what I asked you?

I: Yeah

C: I've got these if you want to use them [I put some counters out on the table]......Four plus three

I: One, two, three, four [as she counts out four counters] plus three

C: Yes

I: Five, six, seven [adds three more counters] It’s seven

This was interesting in a number of ways. Firstly, Isabelle knew quickly how to add one more and two more. She was also able to switch between different forms of mathematical vocabulary. Surprisingly, she quickly jumped to an answer for “four plus three” (“Eight”) which was incorrect and yet she did not express any doubts about her response. When prompted to check her answer, however, Isabelle knew when she had counted three more counters and was able to do this by counting on without having to count out the three counters first. Was she able to subitise for three?

It was interesting to observe the relationship between Isabelle and Frances. Rather than allowing Isabelle to try to solve the problem for herself, Frances immediately stepped in to support. This may have been done with the very best of intentions, but it may also have created a situation where Isabelle was not being given the opportunity to develop the problem-solving skills that she needed. Frances continued to “assist” throughout the interview.

This episode was followed by a few more questions involving simple addition and subtraction. In these questions, Isabelle was able to calculate her answers and successfully explain her methods:

C: And three take away one?

I: That’s easy...

C: What is it then?

I: It’s two

C: Fantastic...Right, I’ve now got some questions that are problems, so you just need to listen and you can use those [pointing to the counters] if you want them to help you. So, Jill has two pennies. If Jim gives her one more, how many does Jill have altogether?

I: Three

C: Very quick...Sally has four crayons. If Stan gives her three more, how many does Sally have now?

I: Seven

C: Good girl. How did you work that out?

I: Ummmm I just counted in my head

C: Good. And how did you do the counting in your head?

I: I just counted in my head and used my fingers

C: And did you start from one, or did you start from one of the other numbers?

I: I started from...did you say four crayons?

C: I did

I: I started from four

Later, I asked Isabelle some questions involving subtraction:

C: Kisha has six pennies. Peter takes away four of her pennies. How many pennies does Kisha have left?

[Isabelle takes time]

I: Two

C: Good girl. How did you work that one out?

I: I just used my fingers and counted back

C: OK...from?

I: Did you say six?

C: I did say six...that’s very good

I: Yeah, I counted back from six

The next series of questions involved operations with two digit numbers. With these questions, Isabelle seemed less confident with her methods, but was successful when prompted by Frances:

C: OK what number comes five numbers after forty nine? If you want to write anything down Isabelle, you know you can?

I: I can use the counters if I want?

[Isabelle seems lost]

Frances: How do you normally count?

I: In my head

Frances: Not in your head...You put the...

I: Number in my head

Frances: Yeah what number do you put in your head?

M: Forty-nine

Frances: Right and then you...?

I: Umm count on

C: Do you want to write anything down?

I: No thank you

C: OK. So what number comes five after forty nine?

I: Forty nine [counts on inaudibly]. Fifty four?

C: And how did you work that out.

I: I counted on my fingers

C: You counted on your fingers…Brilliant…What number comes four numbers before sixty?

I: Sixty

C: Four numbers before sixty

I: I don’t know [counts inaudibly] fifty six [using her fingers]

This was followed by questions using pairs of two digit numbers. The questions required Isabelle to say which of the two numbers were bigger or smaller. Isabelle answered these questions quickly and confidently.

When Isabelle remembered to use her fingers, she could calculate answers to problems with confidence and she had a means of checking her solutions. It seemed that Isabelle was not yet in the habit of using her fingers and in some cases continued to rely on her memory. This worked well for numbers plus or minus one or two, but when the addend or minuend was bigger, she was not so confident or accurate. Isabelle could work within certain limits, but seemed to be quite bounded by the number facts she knew.

When Isabelle was asked to do some calculations using two digit numbers, she seemed less sure about which strategies to apply:

C: Umm what is...Isabelle [to get attention]...twelve plus fifty four?

[Isabelle speaks inaudibly while trying to work out the answer in her head]

I: Fifty six

C: Fifty six and how did you work that out?

I: I just added the tens and then I added the units

Frances: Hold on

[6 second pause]

C: Do you want to check that one again, or do you want to…

I: No

Frances: Look [writes 12+54] We do the units [draws a line joining the 2 and the 4]...four and two?

I: Ummm

Frances: Four and two

I: Four, five, six [spoken very quietly] seven...No six

Frances: And your tens [draws a line joining the 1 and the 5]

I: Fifty [very quietly]. So it's sixty...five

Frances: No you're not looking [5 second pause] you're not actually even looking...here [points to the calculation] [5 second pause] do you remember how we do it sometimes in the classroom? You add the units...two add four

I: Six

Frances: And then the tens...ten add fifty

I: [speaks inaudibly] Fifty-four…fifty-six

Frances: Ten add fifty?

I: Ten add...twenty, thirty, forty [then inaudible]...Sixty...sixty-six [writes 66 next to the calculation with no equals sign]

Frances: Are you sure...Are you sure now?

I: Yeah

This is an interesting episode which suggests that there may have been a mismatch between Isabelle's understanding of number and the methods which she had been taught and was expected to use in school. Isabelle clearly had strategies that worked most of the time. These included the use of some known facts, together with counting strategies, using both fingers and counters. The problem seems to be that Isabelle had not yet pieced together her own understanding together with the "school" methods. It does not seem to be the case that Isabelle only wanted to stick rigidly to the methods she knew worked (Ostad, 2008), but rather that she was in the process of coming to understand the abstract number system and the arithmetical relationships, she was trying to apply.

As the interview was drawing to a close, I wanted to see if Isabelle could go beyond one thousand:

C: What comes nine after nine hundred and ninety nine?

[long pause]

I: That will be uhhhh

Frances: Do you want to use these [points to the counters]

I: Yeah

Frances: So right, what number is in your head?

I: Nine hundred and...what did you say?

C: Nine hundred and ninety nine [writes down 999] and Isabelle we're going to go nine more [writes 9]

I: OK

Frances: Go on then

I: [counts out nine counters and then moves them one at a time, counting inaudibly] A thousand and eight.

C: Fantastic. When I asked you before what came after a thousand, you said you didn't know...but you do

This was followed by more questions on addition which again highlighted Isabelle's lack of familiarity with the methods that had been taught at school:

C: Umm. What is thirteen plus thirty-nine? [writes 13+39]

I: This is a really hard question...why don't we use some counters?

Frances: I don't think we've got enough

C: I've got more, if you don't mind mixing red and blue?

I: Yeah

[I put out more counters]

Frances: How many's there? [pointing at the calculation]. You can put that number [pointing at the 39] in your…

I: Head

Frances: And then you can count on

I: Thirteen [short pause]. I need thirty-nine counters

Frances: Mmmm

I: That will be all of them. [begins to count out the counters inaudibly] six, seven, [continues up to thirteen, moving the counters one at a time].

Frances: Mmmm…right…come on then

I: Thirteen...umm...thirty-nine?

Frances: Mmmm

[pause for 20 seconds]

Frances: Do you need to start again?

[pause for 8 seconds, I moved the counters towards Isabelle on the table]

I: I need some more help with this.

Frances: You don't

[pause for 13 seconds]

Frances: Look [pointing at the 39]. Put this number in your head and then count on...No, you move the counters

I: Forty, forty-one [Isabelle counts up to fifty-two moving the counters as she counts]

These extracts go some way to highlight the confusion experienced by Isabelle when having to select methods to use for calculations – should she "put a number in her head", use counters, use her fingers or use a more formal method? These were not easy choices for Isabelle and she was not able to articulate what the actual problem was (or perhaps she was not given the chance to do this). Frances tried to use prompts to direct Isabelle to use the calculation methods that she had used in class. Frances wanted Isabelle to get the answers correct, and she knew that Isabelle had been able to answer questions of this type before. This sort of "low support" (Radford et al., 2014), has the focus on prompting rather than on providing more of a scaffold. Isabelle, though, was being encouraged to use particular skills that had been taught, rather than focusing on her understanding of the problems posed. Isabelle also appeared quite stressed and anxious. Perhaps if she had had a break from the task, or if she had had a chance to try to articulate why she thought this was a "really hard question", she may have been able to solve the problem without prompts. The impact of this environment on Isabelle's ability to complete the tasks cannot be ignored (Jordan, Huttenlocher and Levine, 1992).

Isabelle tried some of the number line estimation tasks (Siegler and Opfer, 2003). She started by trying to count on with numbers less than 10, but then moved to a more estimation-type approach for numbers greater than 10. However, she did not seem to use known facts in order to place marks for numbers such as 5 on a 1-10 number line and 45 on a 1-100 number line (Figure 4-14 and 4-15).

Although not consistent with the earlier attempt for 45, the examples in figure 4-15, suggest that Isabelle could appreciate the relative sizes of numbers, but there is no evidence of the expected logarithmic relationship (Siegler and Opfer, 2003).

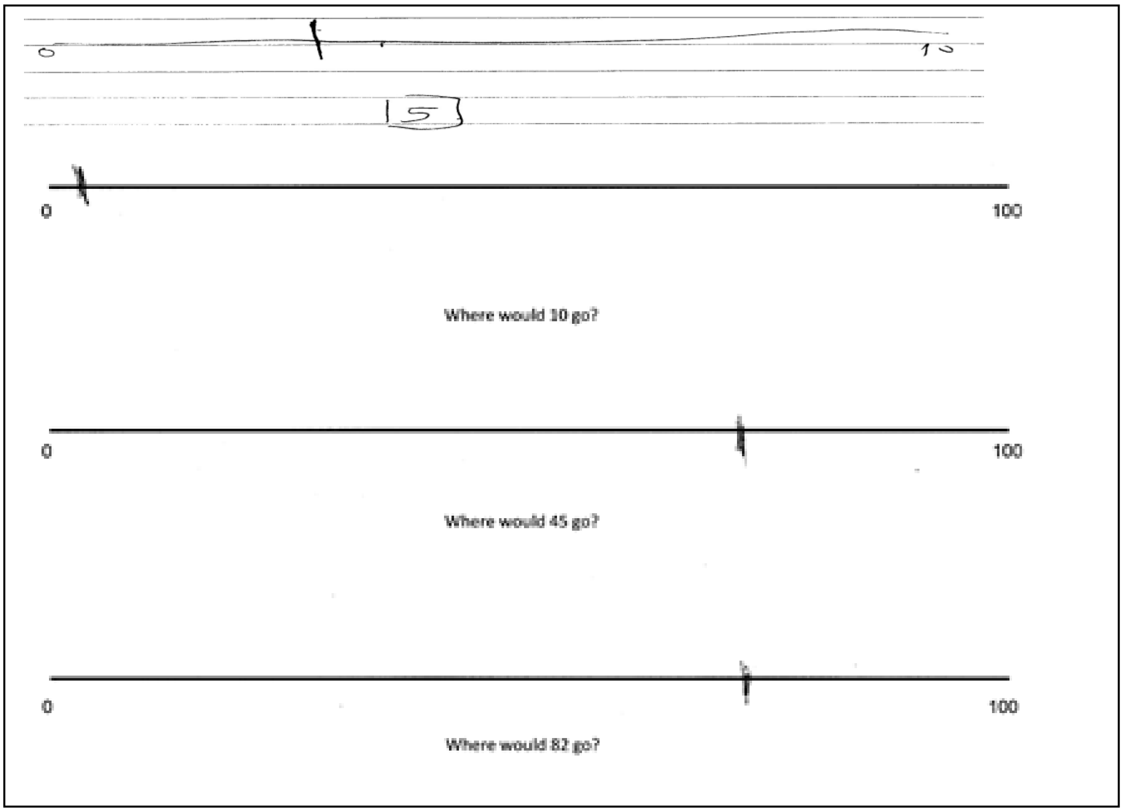


Figure 4-14: Examples of Isabelle's 0-10 and 0-100 number lines

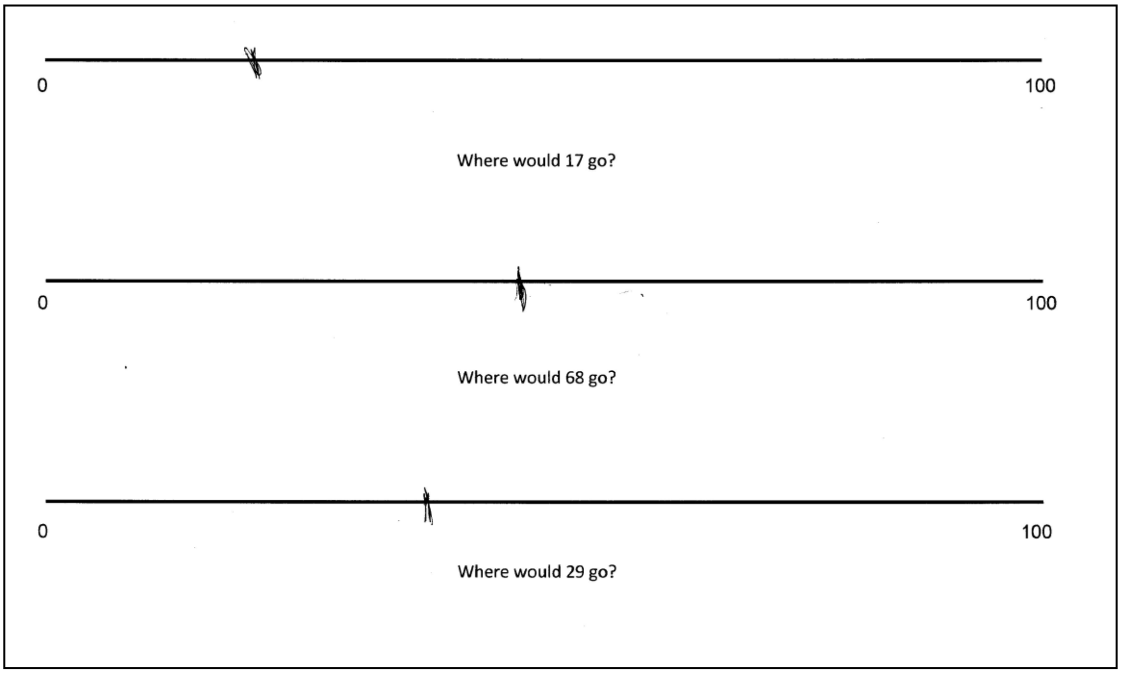


Figure 4-15: More of Isabelle's 0-100 number lines

The session finished with a brief chat. During this time, Isabelle said that she liked maths but could not say why. She said that her favourite thing at school was PE and that she did not like writing stories.

4.5.2 Second school visit (June 2013)

When I arrived, I discovered that Isabelle now had a new teacher, Aniqa

<u>Observation</u>

The children came in from break time and sat at their tables. Worksheets were given out for “times table” practice. The sheets had a range of questions on them which Isabelle was able to answer. Isabelle seemed to “know” most of the questions that were asked. For the ones she did not know, she put the number in her head (by physically modelling) and then counted on, touching her fingers with a pencil, as she counted. Isabelle was sitting at the table with the lowest attainers in the class and I was told that most of the children had special educational needs.

After about 10 minutes, the children were asked to come to the carpet. Isabelle did as asked. The teacher was demonstrating equivalent fractions using diagrams. Isabelle did not seem focused and Frances reminded her to look at the board.

After the introduction, the children went back to their tables and worked independently. Frances sat with Isabelle the whole time. Isabelle did not appear to have difficulty colouring in the images, to illustrate the equivalent fractions (e.g. example $^{3}/_{4}$ and $^{6}/_{8}$ as in figure 4-16).

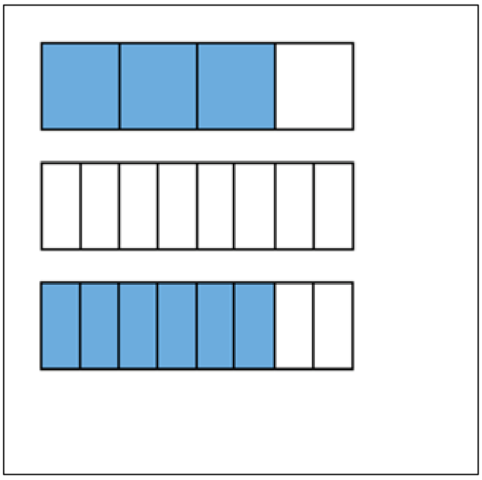


Figure 4-16: Isabelle's drawings for 3/4 and 6/8

Throughout the lesson, Isabelle worked quietly and in isolation. When the children packed up to leave the classroom at the end of the maths lesson, Isabelle collected her things and left on her own, while the other children went off in groups.

Aniqa said she did not yet know Isabelle very well, so could not give me any information about her progress.

<u>Interview</u>

This time Isabelle did the WMTB-C (Pickering and Gathercole, 2001). Frances stayed with us and watched. Isabelle seemed to enjoy the activities and completed them with a high degree of confidence. Isabelle was focused for the whole time and managed to complete all the tests without a break (Table 4-17).

It is clear from these scores that Isabelle’s working memory is at least within the expected range for her age, with her Listening Recall score being above the average range. These scores go some way to explain how Isabelle was able to successfully manipulate numbers mentally.

Test component	Standard score
Digit Recall	109
Block Recall	93
Listening Recall	130
Counting Recall	102
Backward Digit Recall	107

Table 4-17: Isabelle's first WMTB-C scores

Finger gnosis

For this I used a finger gnosis assessment model based on Gracia-Bafalluy and Noël (2008). On both hands, Isabelle was very able to identify fingers 1, 2 and 5 whether they were touched on their own, or in conjunction with another finger (Figure 4-18). However, on both hands, when fingers 3 and 4 were touched, Isabelle was not able to identify the correct finger (i.e. finger 3 for finger 4 or finger 4 for finger 3).

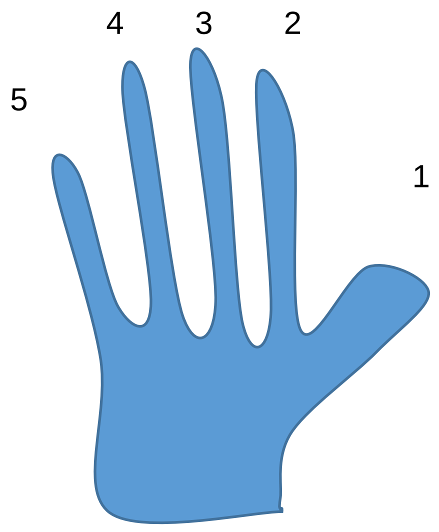


Figure 4-18: Diagrammatic view of Isabelle's left hand

4.5.3 Third school visit (December 2013)

I arrived at school in time for the morning mathematics lesson. Isabelle was now in Year 5 with a new teacher, Gemma and her teaching assistant Frances.

Observation

The topic for the lesson was measuring. Gemma asked the class what centimetres and millimetres measure. Isabelle put her hand up and was asked to answer. Isabelle said that they measure time. The teacher said “Not time,” and asked another child in the class.

As the lesson progressed, it became clear that Isabelle was not confident with the relationship between centimetres and millimetres. However, with support from Frances, Isabelle worked out that there are 10mm in 1 cm and she was then able to answer the questions correctly.

For the last activity on drawing lines accurately, Isabelle found it hard to hold the ruler, so her lines were not very straight and were not the right lengths. Isabelle seemed to find this part of the activity frustrating and kept asking if she had done enough. This was probably largely a consequence of the difficulty she was having with managing the process of actually drawing the lines. Her difficulties were possibly due to a combination of her poor fine motor skills and her poor eyesight.

When the lesson ended for lunch, Isabelle was the last child to leave the room and during play time sought adult attention from Frances, rather than playing with other children.

Staff views

On this visit I spoke with Gemma and Frances. At this point, Isabelle’s levels were:

Reading 3a

Writing 3b

Mathematics 3a

Gemma felt that Isabelle was “….good at computational tasks, but can’t explain how she [Isabelle] has calculated her answers.” Isabelle found contextualised word problems more challenging and had difficulty with the language of mathematics (e.g. understanding and using words such as altogether, sum, total and product). In order to address some of these issues, Gemma said that she was trying to ask Isabelle more questions in class. Isabelle often appeared unfocused, but when she was asked a question, it was clear that she had been listening.

Frances felt that, although Isabelle's writing was improving, Isabelle continued to find writing challenging and time-consuming. The school had been given advice by the educational psychologist to try to make things more visual for Isabelle, due to her autism. Frances had been trying to work on this, but she was not sure if it was helping.

Both Frances and Gemma expressed their concern for Isabelle's transition to secondary school and were working with Isabelle's parents to find the best solution.

<u>Interview</u>

The interview took place in the afternoon. Isabelle was again pleased to come out of class with me to do some work. This time Frances did not join us.

On this visit, I wanted to try the WIAT-II (Wechsler, 2005) Mathematical Reasoning test with Isabelle. I started with the questions for children aged 8 years, to give Isabelle an easy start. The fifth question on the test was supported by a picture of ducks in a pond:

C: Five ducks were swimming in a pond. Three flew away. Then two more came to swim. How many ducks were in the pond?

[5 seconds passed]

I: Five

C: Do you want me to read it again?

I: Yes please

C: OK. Five ducks were swimming in a pond. Three flew away. Then two more came to swim…..

I: Four…no six

C: Which?

I: That one, that one [pointing to the ducks in the picture], that one by the reeds, that one in the middle of the pond and that one close to that one there [pointing again to each duck in the picture]

C: So they [emphasised] flew away?

I: Pardon

C: How many are left in the pond?

I: Four [spoken very quickly]

C: How did you work that out?

I: Because I knew that …no hang on three, three

C: OK. How did you work that out?

I: Five take away three is three…no it isn't it's two...two

C: Should I read the whole question out again?

I: OK

C: Right. Five ducks were swimming in a pond.

I: OK

C: Three flew away and then two more came to swim.

I: Four

C: How many were left? How did you work that out?

I: Five take away three is two add two is four [spoken very confidently]. Easy

C: Lovely

This shows an interesting transition in strategies, in order to solve the problem. On the first reading of the question, Isabelle appeared to be confused. After the second reading, she began to make sense of the problem by using the picture and labelling the ducks that could have flown away. From this point, Isabelle quickly abstracted the problem and used a mathematical model to work out how many ducks would be in the pond.

Isabelle seemed to have moved through several stages. At first she used the images of the ducks to make sense of the problem, in much the same way as Hughes (1986) found when working with 3 – 5 year old children. However, once Isabelle had understood the problem, she was able to abstract the real-world problem to create a mathematical model. From there, she was able to use the model to find the solution. The transition through these stages is very interesting and provides insight into the thinking that was required to make sense of the problem, before a mathematical model could be created.

With some of the later questions, there seemed to be some confusion with the language and perhaps it is this that caused Isabelle to make mistakes. As a result, Isabelle made mistakes with what appear to be more simple problems. It is useful here to compare Isabelle's attempts to answer two of the questions. Firstly:

C: What's worth more: seven pence, six five p's, or ten p? So which of these piles is worth more [pointing to the pictures]

I: That one….the ten p coin [pointing to the 10p coin]

Compared with:

C: How much have you got altogether here? [pointing to the picture] This is supposed to be a ten p [pointing to a 10p]. This is supposed to be a five p [pointing to a 5p] and this is supposed to be a one p [pointing to a 1p].

I: Ten p…five...five...ten...ten [speaking inaudibly]...fifty...are these a pound? [pointing to the 1ps]

C: No these are pennies [pointing to the 1p coins]

I: Fifty-one, fifty-two, fifty-three, fifty-four, fifty-five, fifty-six. Fifty-six

It seems that in the first question, Isabelle was simply focusing on the values of the individual coins and had missed the point about being asked to find the values of each of the piles of coins. "More" was clearly understood as a comparative term (Nunes and Bryant, 1996), but the question as a whole was misunderstood, perhaps because the required calculation was disguised. In the second question, Isabelle was only asked to work out how much there was altogether and she did this with relative ease.

A similar issue arose later, where again, Isabelle seemed to struggle with a rather wordy question. Isabelle could see the question herself, so she did not need to remember it word for word.

Firstly:

C: Eric had four pounds on Monday. On Tuesday he earned two pounds mowing the lawn. On Thursday he spent three pounds at the cinema. How much money did he have left?

[3 second pause]

C: Do you want me to read that again?

I: No, hang on, I think I know…four and two is six and threeee…Nine pounds [very loud]

Compared with:

C: Mrs Ryan's classroom has four rows of desks. Each row has the same number of desks. There are a total of twenty four desks….

I: Six [interrupting]

C: I didn't even get to the end of the question. How did you work that out?

I: Six times four is twenty four. Twenty four divided by four is six

In the second example, Isabelle seems to have anticipated what she would have to do, before I had even finished reading out the question. The question was far less language based and Isabelle's confidence is noticeably different from her attempts with the more wordy and more linguistically complex questions. This sort of difficulty with more complex language-based questions, is not unusual (Dowker, 2008; Jordan, Hanich and Uberti, 2003).

For the completed test, Isabelle achieved a standard score of 93. Throughout the test, Isabelle used her fingers to help her, rather than counters. Although her strategies could be called "immature" (Ostad, 2008), her knowledge of mathematical relationships does not seem to suggest that she has a poor understanding of arithmetic. This conflicts with Ostad's suggestion that the use of immature and rigid strategies, is indicative of poor understanding of arithmetic, if by "arithmetic" we mean, the “principles of arithmetic”. This, after all, may develop separately the ability to learn number words, understand number notation and learn calculation skills (Cowan, Donlan, Newton and Lloyd, 2005)

Perhaps, Dowker’s suggestion that “there is no such thing as arithmetic ability: only arithmetic abilities” (Dowker, 2012, p.26) would be a more appropriate perspective to take when reviewing Isabelle’s strategies. This will help to focus on her strategies and their effectiveness, rather than categorising them using terms such as “appropriate”, “immature” or “rigid”.

Isabelle also did the Panamath assessment (Halberda et al., 2008) on this visit. She appeared to concentrate well and achieved a Weber fraction of 0.3 (with the 10th percentile being at 0.48 and the 90th percentile at 0.18). Isabelle’s average response time, however, was 3348 ms, which put her below the value for the 10th percentile, at 1630 ms.

<u>Finger gnosis</u>

On this occasion, Isabelle was almost perfect on every trial with one finger and then two fingers, on both hands. Isabelle made one mistake when fingers 1 and 3 were touched on her right hand, but was able to correct herself when the trail was repeated.

4.5.4 Fourth school visit (May 2014)

I arrived as usual in the morning, just before morning break.

<u>Staff views</u>

I spoke to Gemma, Isabelle’s teacher, before the start of the lesson. Gemma told me that Isabelle had had an operation on both her feet in January and that she had not settled back very well. Isabelle now had specially made shoes, but she did not always seem comfortable and often said that her feet hurt, so that she could go home. Gemma also felt that Isabelle was finding it hard to focus on her work and was very easily distracted. The school was also trying to prepare Isabelle for her transition to secondary school. One of the things they were doing was to try to make her less dependent on Frances by giving her another teaching assistant during some of her lessons. Frances supported her in the lesson I observed.

After the lesson, Frances told me that she thought that Isabelle had made no progress since January. She also said that Isabelle had no friends in her class, but did have someone she called a friend in the year above. However, because this girl would be leaving in a few months, the school thought it was best for Isabelle to be encouraged to stop playing with her, because she needed to get used to being on her own when she was in Year 6. The school was, however, thinking about providing some social skills training for Isabelle when she was in Year 6.

Observation

This time I noticed that Isabelle did not appear very focused, or very interested, from the start of the lesson. Isabelle was supported by Frances. The lesson began with Gemma revising the link between decimals and fractions. As Frances was talking, Isabelle was more interested in what was going on around her and asked Frances a number of questions about other children in the class. Frances repeatedly tried to refocus Isabelle and asked her to listen to Gemma. The children were asked to write down what the decimal would be for 5/10. Isabelle quickly said, “It would be zero point 5”. Gemma then put up more questions for the class to attempt. For each of the questions Isabelle put up her hand, but she was never asked to respond, so I do not know if she was able to answer correctly or not.

Next, Gemma put the following numbers on the interactive white board:

$1^{3}/_{5}$ $1^{3}/_{4}$ $1^{65}/_{100}$

The children were asked “What would we need to do to know which is the biggest number?”

The class was asked to discuss this with a partner. Isabelle did not discuss this with anyone and seemed distracted. After a few minutes, Isabelle asked Frances if she could go to the toilet. On her return, Frances told Isabelle that she needed to convert $1^{3}/_{5}$ into an improper fraction, with a denominator of 100. Isabelle had a mini whiteboard on which she wrote:

$^{5}/_{5}$ $^{3}/_{5}$

Frances prompted Isabelle to write $^8/_5$. Isabelle then stopped. Frances tried to get Isabelle to work out how she could make the denominator 100. "You need to make the five a hundred" she said. Isabelle then wrote $^8/_{500}$.

Frances took the mini whiteboard and wrote:

$$1^3/_5 \qquad \begin{array}{r} 1\text{x}5=5 \\ +3 \\ \hline 8 \end{array} \longrightarrow \quad ^8/_5$$

While she was writing, she explained to Isabelle what she was doing. Frances then gave the mini whiteboard to Isabelle. Isabelle added to this, writing:

$$^8/_5 \text{ x } 20 = 1000$$

She then crosses out a zero to give:

$$^8/_5 \text{ x } 20 = 100\cancel{0}$$

Frances was clearly finding it difficult to know how to progress and continued to tell Isabelle what she should do. Isabelle rubbed out her work and started to doodle on her mini whiteboard. She then wrote:

$$^{100}/_{100} = 100$$

Frances asked Isabelle to clean her board and try the next question. Frances then got up and went over to help another child. Isabelle wiped her board clean and then sat still until Gemma asked the class to stop working while she recapped the key points of the lesson. Again, Isabelle did not appear focused on Gemma and was looking around the room. The lesson ended and the class went to lunch.

According to Fosnot and Dolk (2002), when children learn mathematics they are often not allowed to explore the problem and construct meaning for themselves. This example highlights that issue very clearly. There was no opportunity to explore the problem and for Isabelle this made finding a solution a challenge that she was not able to resolve.

Interview

Again, Isabelle was very cooperative and seemed to concentrate well throughout. I thought it would be interesting to try the WMTB-C (Pickering and Gathercole, 2001) again, as Isabelle had done so well on the previous attempt (Table 4-19).

Test component	**Standard score**
Digit Recall	145
Block Recall	118
Listening Recall	133
Counting Recall	Not attempted
Backward Digit Recall	107

Table 4-19: Isabelle's second WMTB-C scores

Isabelle seemed tired after attempting most of the tests and did not want to attempt the Counting Recall test. However, her scores were much higher than the last attempt, almost one year earlier. The most significant changes were in Digit Recall (109 to 145) and Block Recall (93 to 118). This amount of progress is not generally expected in this type of assessment (Gathercole and Alloway, 2008).

Isabelle also attempted the WIAT-II (Wechsler, 2005) Numerical Operations test. In this test, she achieved a standard score of 85. This compares with a standard score of 93 in the Mathematical Reasoning test attempted during the last visit. This mismatch occurred because of Isabelle's lack of familiarity with methods for calculations involving combinations of 2-digit numbers.

Finally, Isabelle attempted the number line estimation task (Siegler and Opfer, 2003). Isabelle demonstrated progress since the last time that she had done these tasks in January 2013. On this occasion, Isabelle started marking out places for identifying the position of the 5 on a 1-10 number line, but then stopped and placed a mark right in the middle of the 1-10 line. Isabelle's other estimates were also much more accurate (Figures 4-20 and 4-21).

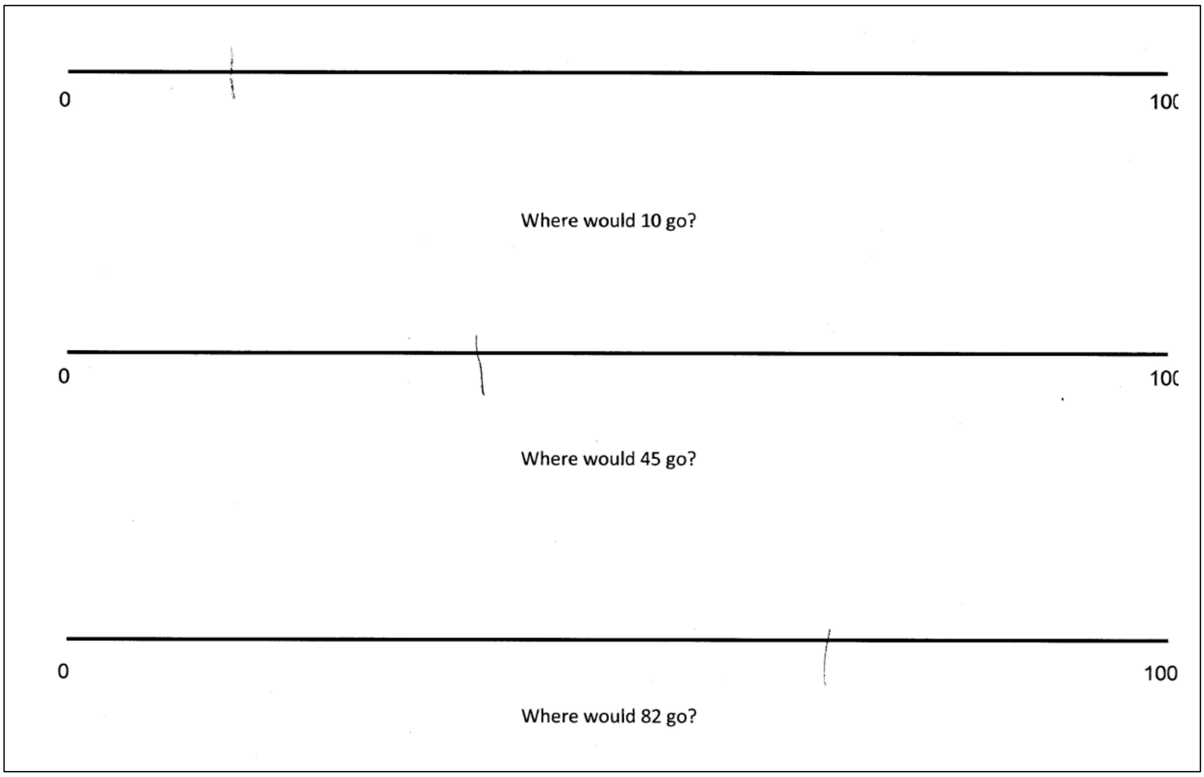


Figure 4-20: Isabelle's 0-10 and 0-100 number lines

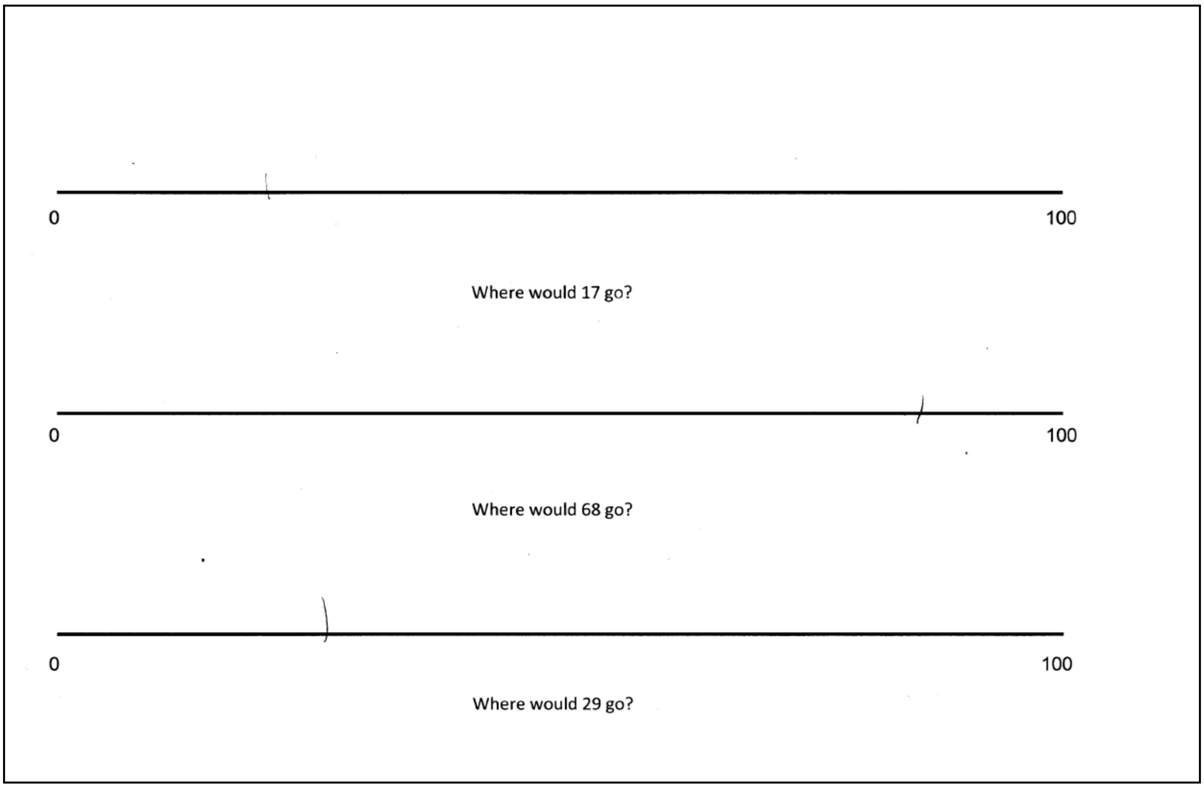


Figure 4-21: Isabelle's 0-10 and 0-100 number lines

On arrival, I learned that Isabelle was now in a mixed Year 5/6 class. I arrived just before the class was due to start their English lesson. I stayed and observed Isabelle in class. The main activity was supposed to be carried out with a partner. However, Isabelle's partner took very little notice of Isabelle and both of them worked independently throughout the lesson. In this lesson, Isabelle was being supported by a new teaching assistant, Jess. Isabelle did not participate when the teacher, Vicky addressed questions to the whole class. Isabelle was very quiet throughout the whole lesson. I did, however, notice that Isabelle's writing had significantly improved since my last visit.

The mathematics lesson was after morning break. The focus of this lesson was addition and subtraction of fractions. Isabelle was sat at a table with other children with additional needs.

At the beginning of the lesson, Isabelle's neighbour noticed that Isabelle was having difficulty taking the lid off her pen and offered to help. Isabelle allowed her neighbour to help and thanked her.

The first question was:

$1^1/_3 + {}^1/_4$

Followed by:

$1^1/_3 - {}^3/_4$

And then:

$2^1/_4 - {}^2/_3$

Vicky asked the class "What do we need to do to make this all into quarters?" Isabelle put hand her up and Vicky asked her to answer.

I: Use the four times table

Vicky: How many fours have we got?

I: Three

Vicky: How many whole ones have we got?

I: Two

Vicky: So how many quarters have we got?

I: Two times four

Vicky: Yes

The solution had not yet been found. Vicky then addressed further questions to the rest of the class.

Following this worked example, the children were given a range of questions to do independently. Jess was working with Isabelle. The first question they had to answer was:

$2^{1}/_{2} - {}^{4}/_{5}$.

An interesting conversation followed:

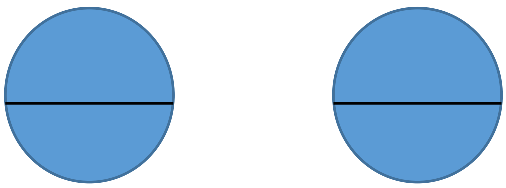
Jess: Do you know what to do?

I: I'm thinking

Jess: How many halves in two?

I: One

Jess drew two circles with lines through the middle:



Jess: How many halves in two?

I: Two

Jess: Look at the picture

I: [Isabelle does as asked] Four

Jess: How many halves in two and a half?

I: Two

At this point, Jess modelled how to convert $2^1/_2$ to $^5/_2$. She then wrote:

$^5/_2 - ^4/_5$

Jess: What's next?

I: Times by four

Again Jess tried to model the method, explaining what she was doing as she went along. Isabelle appeared to be getting quite agitated and did not understand what Jess was trying to show her. Finally, Jess wrote:

$^{25}/_{10} - ^8/_{10}$

She then guided Isabelle to work out 25 – 8. Isabelle did this using the column method. When calculating 15 – 8, she "put the 15 in her head" and counted back eight, using her fingers. She did this three times to check that she had calculated it correctly.

The next question was:

$1^3/_4 + ^5/_6$

Jess: Isabelle how are you going to do this one?

I: I do not know [very emphatic]

Jess: How are you going to turn this [pointing to the $1^3/_4$] into an improper fraction?

I: Times it by ten

[this was followed by some discussion between Jess and Isabelle and then]

I: I don't know what to do

At this point Isabelle seemed quite distressed. Jess told Isabelle what she had to do and worked through the problem for her. Isabelle watched and said nothing.

As in the previous visit, it seems that the problem Isabelle had was that she did not understand and she made that very clear. The problem was then exacerbated as the teaching assistant attempted to support her. According to Askew (2012, p.100):

> Coming to ‘see’ fractions within diagrams is not merely a perceptual activity, not a case of recognizing, but a cognitive activity of re-cognizing…..The ‘knowledge’ of fractions is not something ‘out there’ that has to be ‘delivered’ and which children have to ‘internalize’.

Staff views

Vicky felt that Isabelle was working at level 3a across all her subjects. Looking at Isabelle’s levels alone, it looked as though she had made almost no progress since December 2013. This was an important time, as all the children were working towards their SATs (Standard Attainment Tests). Vicky explained that because of her disabilities, Isabelle would be getting double time and would also receive examination papers with an enlarged font. The school was hopeful that Isabelle would achieve a level 4 (the expected level for children to achieve in their SATs).

Vicky felt that concentration was a continuing problem for Isabelle and that she still needed the full-time support of a teaching assistant. Isabelle now had two teaching assistants that worked with her, in order to support her to manage the transition better and to learn to work with a wider range of adults.

Jess was at a bit of a loss after the lesson. She explained that she felt that she did not have the skills to explain the work properly to Isabelle and that it not been a very satisfactory lesson for either of them. She would have liked to have understood Isabelle’s difficulties better, so that she could be of more help to her.

Jess also said that Isabelle did not tend to play with other children in the playground. Sometimes children would try to play with Isabelle, but when this happened, Isabelle may play for a bit, but would soon want to move on. Jess felt that Isabelle would only play with other children if she could get her own way. If she did not get her own way, Isabelle would tell the adults that she was being bullied. Jess was working with Isabelle on this and was trying to understand what was going on. Jess reported that some of the children found Isabelle annoying. This was all particularly interesting, given that during the last visit the school had said that they were going to be implementing some social skills support for Isabelle. However, I did not ask about this and so cannot comment further.

Interview

I wanted to review some of the assessments I had done in the last interview to see if there had been any changes or developments. The first part of the interview was at the end of the morning and we continued in the afternoon. Isabelle was happy to come out with me both times.

This time, Isabelle was a bit more confident with modelling problems than she had been before. For example (Wechsler, 2005, Mathematical Reasoning, Item 30):

> Robert has 6 stones.
>
> Together Robert and Max have 15 stones.
>
> How many stones does Max have?

Isabelle had no problem with this question and immediately answered “nine”. When I asked how she had worked it out she said, “I realised that six and ten was sixteen, so six and nine would be fifteen.”

This response seems to show progress from the pure counting-based methods used previously. In this case, Isabelle appears to be using a derived strategy (Thompson, 2000) based on her knowledge of number combinations and probably place value (“I realised that six and ten was sixteen”) and her understanding of n-1 principles (“so six and nine would be fifteen”) (Dowker, 2012). Although this may be based on counting, it requires an integrated understanding of both the principles of counting and the procedure of counting (Baroody and Ginsburg, 1986) together with an integration of the process of counting and the concept of number (or numerosity) (Gray and Tall, 1991). It seems likely that this was the stage that Isabelle had reached. She no longer relied completely on counting and she was able to use her known facts to derive new ones.

For a later question:

> Jeff gave some biscuits to five of his friends. He gave three biscuits to each friend and had two left over for himself. How many biscuits did Jeff have in the beginning?
>
> (Wechsler, 2005, Mathematical Reasoning, Item 38)

Isabelle drew each of the five friends (with faces) and then drew out all the biscuits (Figure 4-22). Once she had done this, she worked through the solution, adding on in threes to get the correct solution.

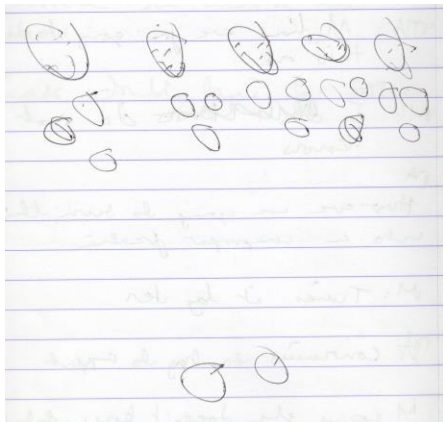


Figure 4-22: Isabelle's drawing for the biscuit problem

The model was an essential part of the problem-solving process for Isabelle. She then easily counted on in threes and was confident with her solution. It is hard to know how Isabelle would have coped, had she not been able to construct a meaningful model of the situation. Perhaps the images helped Isabelle by providing objects to refer to, thus enabling her to think about the problem and to support her to make links with her knowledge of number and the number system (Drews, 2007).

At the end of the interview, we talked a little bit about transition to secondary school. Isabelle said that she was not sure which school she would be going to but she was concerned that one of them was a very big school. She was also worried about leaving all her friends. Isabelle talked about the fact that she has difficulty hearing and very bad eyesight. She said that one eye was stronger than the other and that as a result she tended to use that one most of the time. She clearly had concerns and did not talk about any of the things she was looking forward to.

The difficulties Isabelle continued to have with fractions are not uncommon and it seemed that for Isabelle, the connection between her understanding of fractions as numbers and of fractional quantities needed further development (Nunes, Bryant and Watson, 2009).

Once we had completed this part of the interview, Isabelle tried the Counting Recall test of the WMTB-C (Pickering and Gathercole, 2001). Isabelle was able to get up to four images correct, but then did not want to try any more. This gave her a score of 44. However, because of her age, Isabelle was right on the cusp for scoring – had she done the test two weeks earlier, she would have scored 55 and had she successfully completed one trial with five dots, she would have scored 53. It therefore seems that, broadly speaking, there was little change from the last time that this test was attempted, when Isabelle scored 55. While doing the test, I noticed that Isabelle took much longer to work out how many dots there were when there were more than six dots. For six dots and below, she was very quick and had strategies for grouping dots into groups of two or three. When there were more than six dots, Isabelle counted each one individually which was much more time consuming. For four dots, Isabelle said that she did not need to count and just "knew" how many dots there were. This suggests that Isabelle was able to subitise to four.

I then asked Isabelle to look at her previous attempt with the Numerical Operations Test (Wechsler, 2005) and see if she could try any more than last time. She was unable to do more. This gave her a standard score of 78. It is worth considering whether this truly represents a decline in Isabelle's performance, or whether it indicates a problem with the test. On the Mathematical Reasoning Test (Wechsler, 2005), Isabelle achieved a standard score of 96 (An increase of 3 points on her last score). This was quite consistent with her last score.

Finger gnosis

I tested Isabelle's finger gnosis on her left hand only. Isabelle has recently had an operation on one of the web spaces on her right hand and it was still sore. Isabelle was very quick and very accurate with one and two fingers.

4.5.6 Conclusions

Over the 2 year period that I worked with Isabelle, she made progress in several areas. Isabelle's understanding of number seemed to improve alongside her development of finger gnosis. This is reflected in her teacher assessments. When I first visited Isabelle, it was suggested that the level she had been awarded for mathematics (2a), was actually not reliable. At this time her level for reading was 3a. By the end of my visits, Isabelle was still being awarded 3a for her reading, but her level in mathematics was also 3a. Her confidence and understanding in mathematics and her ability to explain her thinking made significant qualitative changes.

The overall conclusions will be considered with reference to the research questions.

1. What strategies did Isabelle use to help her solve numerical problems in mathematics?

Isabelle used a range of strategies right from the start. She sometimes used her fingers, sometimes used counters, sometimes used known facts and sometimes she worked things out mentally. However, she was also expected to use strategies that she had been taught in class. This was not always very easy for her and as was seen in 4.5.1, caused her to appear confused and frustrated.

Isabelle scored either within or above the expected range for all aspects of working memory (see table 4-19, table 4-17 and 4.5.5). As seen in 4.5.3 and 4.5.4, she also scored within expected range for the WIAT-II assessments (Wechsler, 2005), but had a lower score for Numerical Operations. This might reflect the struggle that she often had applying the formal, school-taught techniques when trying to solve problems. Isabelle always seemed happier to use the more informal methods that she was comfortable with for solving numerical problems. Isabelle's secure working memory skills seemed to help her to work out her calculations mentally. Initially, as in 4.5.1, Isabelle was reluctant to check her answers, even when prompted. However, as time went on, and as seen in 4.5.3, for example, she became more confident and would self-correct and seek clarification when engaged in problem-solving.

2. Did Isabelle's hand anomalies impact the range of strategies available to her?

From observations, Isabelle was becoming more confident with her mathematics as her finger gnosis was developing and as she was using her fingers more. By the time of my last visit, though, Isabelle was using her fingers less, but had acquired a useful set of known number facts from which she was able to derive others. In other words, she was beginning to see the patterns and relationships in mathematics that had not been obvious to her before.

While any assessments, formal or informal, provide limited information and take place at just one point in time, I did observe that, over time, there were qualitative differences in the way Isabelle attempted the mathematics we did together. What was it that made this happen? Isabelle had hearing and visual impairments and she also started with poor finger gnosis. Throughout the whole time I worked with Isabelle, she received in-class support from learning support staff and was socially isolated from her peers. The thing that changed for Isabelle was her finger gnosis. Did this have a role in supporting the development of Isabelle's deeper mathematical understanding in mathematics?

3. Are there other factors which had an impact on Isabelle's mathematical learning opportunities?

Isabelle had hearing and visual impairments and wore hearing aids and glasses to help with these. Isabelle was always very isolated in the class, so it is hard to know to what extent this was due to her hearing and visual impairments and to what extent this had just become her "normal" experience of school.

Isabelle tended to work on her own and did not seem to have any friends in her class. She was used to having teaching assistants to help her, but the school was trying to support her to become more independent, by providing her with a few support staff, in order for her to become used to working with a range of different people.

An area that was not much of a focus in discussions about Isabelle's learning relates to her diagnosis of ASD. There have been a number of studies that have explored the mathematical development of children with ASD (e.g. Wei, Christiano, Jennifer, Wagner and Spiker, 2014; Bae, Chiang and Hickson, 2015; Troyb et al., 2013). These studies have used a range of data sets and forms of assessment to explore children's academic outcomes. In the studies, children with so called "high functioning" autism seemed to score within the average range in mathematics. According to Ehlers, Gillberg and Wing, it is generally believed that "Asperger syndrome is differentiated from autistic disorder by specifying that there is no clinically significant general delay in spoken or receptive language or cognitive development up to 3 years of age" (1999, p.130). This definition, they argue is not unproblematic, but, for the purpose of this discussion, it provides a means of making a distinction.

In the studies on children's academic outcomes that compared problem-solving skills and numerical calculation skills, the so-called "higher functioning" children scored better in questions involving calculations only, than they did in mathematical word problems, where they had to apply mathematical skills in unknown situations (Wei, Christiano, Jennifer, Wagner and Spiker, 2014; Bae, Chiang and Hickson, 2015). In Wei et al.'s study, the children were categorised into one of four groups depending on their overall attainment profile. In each of these groups, apart from the group classified as "hyperlexia", the scores for calculation were higher than those for applied problems. In the group designated "hyperlexia", the scores were the same in both areas.

As my time with Isabelle was coming to an end, she was achieving average levels of attainment for her age across all curriculum subject areas. Although it was never made explicit, it seems likely that Isabelle would be classified as having high functioning autism rather than Asperger syndrome, due to her delayed acquisition of language. However, it is hard to know whether this was due to the autism or Apert syndrome. Isabelle's profile in mathematics, though, does not seem to be typical of a child with high functioning autism. Does this mean that Isabelle did not have high functioning autism, or was there something else that was impacting on her mathematical development? If the latter were the case, was the difference related to internal (something about Isabelle), external (environmental) factors or some combination of the two?

This now serves to complicate the picture. Isabelle had a statement of special educational needs and as a result had an atypical experience of school. Jordan, Hanich and Uberti (2003) point out that there are many factors which impact on children's learning both inside and outside the classroom. It is important to be aware of the existence of these influences on children's learning when making judgements about their strengths and weaknesses and how any concerns can be addressed.

4.6 Hannah

My first visit to see Hannah was at home, in November 2012. At this time, she was 9 years old and attending her local primary school, in Year 5. Hannah was described by her parents (Sam and Matthew) as an active, independent, caring and very friendly child. However, her parents were concerned that she did not have many friends at school. Although there were a number of girls who liked to “help” Hannah, they were not “friends”. Sam said that Hannah liked playing games with other people when she was in a good mood, but when she was “cross” she preferred to play on her own. Sam also felt that Hannah spent a lot of time on her own at school, so she was pleased that she sometimes played with her five year old sister, Louise.

Hannah had a diagnosis of ADHD on the basis of behaviours that were only apparent at home. School staff described her as very quiet and hard-working, but at home, she was always on the go and had difficulty sleeping. This discrepancy was puzzling and the home behaviours were stressful for the whole family. Hannah used a bone anchored hearing aid (BAHA). Prior to having the BAHA, Hannah had many problems with glue ear and had had 3 sets of grommets. Hannah also wore glasses, but, according to her parents, her eyesight was not too bad. Hannah has four fingers on each hand (three fingers and thumb), but had very limited mobility. Hannah had glasses, but she would only agree to wear them at school.

Hannah’s parents had specific concerns about her progress in mathematics: they had been told she was working at Level 1c in mathematics. Hannah’s reading and writing skills were apparently much better, but Hannah’s parents felt that Hannah did not always pick up the detail in stories and had a lack of imagination when writing.

Hannah’s parents felt that she would benefit from a statement of SEN, but the school disagreed. During this visit, Sam asked Hannah to “work out ten plus 6”. Hannah wrote:

10 + 6

At first she did not seem to know how to proceed. We suggested a drawing could help. She drew 10 circles, counting out loud as she drew:

Hannah then completed the calculation with:

10 + 6 = 4

Hannah seemed to know that "plus" should be represented with the symbol "+", but not what it meant.

I found Hannah to be friendly and chatty. She told me she enjoyed playing games, both inside and outside. She particularly enjoyed being with animals and was very fond of the family's chickens. Hannah also said that she liked reading, swimming, music and going to Brownies.

4.6.1 First school visit (January 2013)

On my first visit to see Hannah, the class was working on line symmetry. The class was not set for maths, but the children sat at tables in nominal 'ability' groups, according to national curriculum levels. Hannah's group was towards the back of the class.

<u>Staff views</u>

The school's (SENCo) said Hannah had always struggled with maths. She identified as particular areas of difficulty: place value, patterns and "basic number knowledge. She said that Hannah, "has made progress in terms of independence. When she started she needed to wear a helmet in school due to a hole in her skull". She also believed that Hannah was making good progress and did not need a statement of special educational needs. Hannah's hand writing was very good and "very neat" but slow.

Hannah's class teacher, Basia, reported that she had, "…made progress in maths, but very small steps." Hannah was able to do work at Level 2c, with support. At the time, Hannah was receiving one hour a day of one-to-one support for her mathematics (not during mathematics lessons). Hannah had also been given a special maths pack to take home, with a range of activities and resources to help her practice skills at home.

Basia said, “Hannah can’t use her own hands, so uses cubes, dots and other resources or other people’s fingers” and needed lots of repetition to embed skills. Basia thought Hannah did not perceive herself as being the worst in the class at maths. She felt the one-to-one support was making a big difference and it was often helpful to “pre-teach” aspects of maths that were to be covered in later lessons.

Basia felt that Hannah’s writing composition was age appropriate and she usually had lots of good ideas. She also thought that Hannah was not seen by her peers as “different” and that she had lots of friends. This contrasts with Hannah’s parents’ views. During my visit, I did not see Hannah have conversations with other children, except when directed to, and she did not play with anyone during break time.

Observation

When the teacher was introducing the lesson, Hannah did not appear to be focused on the teacher; her gaze was wandering all over the room and she was not looking at the board. As part of the introduction, the children were asked to talk to their partner. Hannah did not. On a number of occasions, the class teacher asked for responses from the class (using hands up). Hannah did not volunteer any. The children were then asked to work in pairs on an activity. Hannah did find a partner; however, throughout this activity, she was very quiet and followed the directions of her partner. When Hannah noticed that her partner had only found one line of symmetry on a pentagon, she told her partner that she thought there were more lines of symmetry. In response, Hannah’s partner took hold of a mirror and placed it on the pentagon incorrectly to try to show that there were no more lines of symmetry. Hannah gave in.

When the children were asked to write down what they had learned, Hannah’s partner told her what to write. The children were then asked to “traffic light” their learning. Again, Hannah’s partner told her what to do and Hannah obeyed.

Interview

My first interview with Hannah yielded several insights. It was based on items from the Number Knowledge Test (Griffin, 1997) and number line estimation tests (Siegler and Opfer, 2003).

Throughout the session, which lasted for about an hour, Hannah was focused and cooperative. I started by asking about her attitude to maths:

C: Can you first of all tell me...what you think about maths?

H: I like maths...because...ummmmmm

C: Which bits do you like best?

H: Umm...I like counting numbers and all that

C: And are there any things that you don't like?

H: Nnnno

The next section of the interview was very revealing:

C: First of all, can you count as far as you can count?

H: Yes. One, two, three [counts very fluently to twenty and stops]

C: Can you count any further?

H: No

C: OK. Do you know what comes after twenty?

H: [pause] Twenty-one [spoken very slowly]

C: And then?

H: Twenty two, twenty-three [Hannah continued up to twenty-nine and stopped]

C: And...

H: Thirty

C: Fantastic. So you can go further!

H: Yeah

C: Can you count up to a hundred?

H: No [spoken very quietly]

I decided not to pursue this.

The nature of Hannah's confidence (and sudden lack of confidence) is curious, especially as she could do more than she initially thought. Hannah had learned the pattern when counting in the twenties, but she paused when she got to twenty-nine and needed thinking time to remember what comes next. It seems likely she was simply not well-practised with the number system beyond thirty. She seemed to understand how the system works.

As Hannah seemed confident with numbers up to 20, I asked her if she could count backwards:

C: Can you count backwards from twenty?

H: Nineteen, eighteen, seventeen, sixteen, fifteen, fourteen, thirteen, twelve, eleven [pause]. Ten…nine, eight, seven, six, five, four, three, two, one [9-1 done very quickly]

C: When you were doing that, what was helping you? Because it looked like you were really concentrating

H: Ummmmm

C: Was there anything you were doing that was helping you, or did you just know the words in that order?

H: I just know the words

The following activities were based on the Number Knowledge Test (Griffin, 1997). The first questions required Hannah to count red and blue counters when they were arranged in different orders or to say whether there were more red or more blue counters. None of these problems presented any difficulties for Hannah.

The activities, however, were not so straightforward and raise a number of questions about Hannah's strategies for working with discrete quantities:

C: OK, if you had four chocolates.

H: Yeah

C: And someone gave you three more

H: Yes

C: How many would you have altogether?

H: Umm...so I had chocolates

C: Yes if you had four chocolates and someone gave you three more.... [Hannah takes 4 blue counters and then 3 more]

H: Ten [spoken loudly with confidence].....ten [spoken quietly and with less confidence and with no evidence of counting]

C: Ten. How did you work that out?

H: Because I counted the blue ones and then I added them up

Hannah had modelled the problem correctly with the counters and then gave an incorrect answer. As I had not actually observed any evidence of counting, I decided to try a similar question, but with a smaller total.

C: OK so if you had two chocolates and I gave you four more

H: Yeah

C: How many would you have then?

H: [takes 2 counters] Two chocolates

C: And I give you four more

H: [takes four more counters, one at a time, then counts on]....three, four, five, six [touching the counters as she counts]

This time Hannah modelled the problem appropriately and then used the additional counters to help her to count on. As Hannah had been successful, I thought I would try a problem with subtraction.

C: OK, now if you had seven chocolates

H: Yeah

C: And you ate three of them

H: [Hannah giggles] Yeah

C: How many would you have left?

[H takes 2 more counters and adds them to the 6 she has already, but does not show any evidence of counting. She then removes 3 counters, one at a time]

H: It'll be five [without counting]

H: Because I took three chocolates away.

C: If you had ten chocolates

H: Yeah

C: And ate four of them. How many would you have left?

H: OK

[regroups the 8 counters from before and removes 4 counters, one at a time]

H: Four [without counting]

C: How did you work that out?

H: Because I imagined I got ten there [pointing to the counters]

C: Yeah

H: And then I took four away.

The above extract demonstrates that Hannah could count very reliably and could use representations (here counters to stand for chocolates). Where Hannah appeared to be less reliable was with the idea that a given number has to represent a specific numerosity. Maybe Hannah had not yet fully understood the purpose of counting (Fuson, 1988). Perhaps her use of the phrase "I imagined I got ten there" when she had only started with eight counters begins to provide a rationale for her approach. Hannah was very reliable when she actually counted the counters and seemed to apply the five counting principles identified by Gelman and Gallistel (1978). However, she did not always respond to a "how many?" question with a count procedure. Could this be because she was not yet convinced that counting yields the numerosity of the number of objects in a set (Nunes and Bryant, 1986) or that counting is a very reliable procedure (Cowan, 1987)?

Hannah later demonstrated an apparent lack of familiarity, or confidence, with number knowledge and number patterns:

C: What number comes five numbers after forty nine?

[H pauses and does nothing]

C: You can write on here if that’s helpful [as I give Hannah some paper]

H: Oh yes. Umm...

C: Five numbers after forty-nine...

H: So [writes down 49]. Forty-nine

[H then continues to write 50 51 52 53 54 55 (Figure 4-23)]

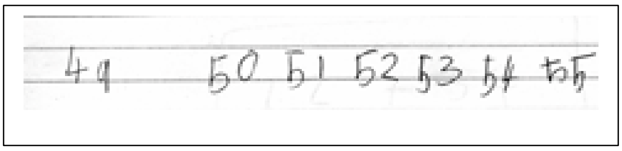


Figure 4-23: Hannah's writing for “What comes five numbers after forty-nine?”

C: Do you want me to tell you what the question was again?

H: Yeah

C: What number comes five numbers after forty-nine? Five numbers after?

H: Fifty-five

C: OK…… How did you work that out?

H: Cos I counted fifty, fifty-one, fifty-two, fifty-three, fifty-four, fifty-five

Is it possible that in this extract, the “five” was more significant than the request for “five more”?

C: OK...What number comes four numbers before sixty?

H: [Writes 60 on the left hand edge of the page, pauses for a few seconds] Before sixty [pauses again] Before?

C: Before. So if you had your number line

H: Yeah

C: And here’s 60 [I points to where Hannah has written 60]

H: Yeah

C: Which way would before be?...The numbers that are before sixty?...Would it be this side or that one? [pointing right and then left]

H: [Hannah points to the left of the 60 she has written] This side

C: So maybe if you put the sixty over here [pointing to the other side of the page]

H: Yeah [writes 60 on the right hand side of the page]

C: And then you’re going to go four numbers

H: Before

C: That side aren’t you [pointing to the left hand side of the 60]

H: [whispers and writes the numbers as she says them going from right to left] sixty one, sixty two, sixty three, sixty four. Sixty four [out loud]. That’s it, it’s sixty four. [H writes each two-digit number from right to left (Figure 4-24)]

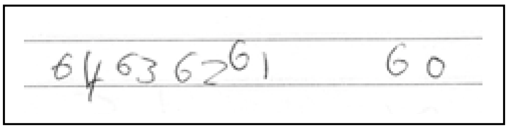


Figure 4-24: Hannah's writing for “What number comes four numbers before sixty?”

It seems that yet again the “four” seemed to be more significant than “four less”.

In this last episode, Hannah demonstrated a lack of confidence when working with numbers as abstract entities. She attempted the activities, but needed prompts and seemed to struggle to understand what was actually being asked. It is unclear whether the model of a number line was very useful for Hannah and whether she had learned how this artefact relates to our number system.

Overall, Hannah presented with little confidence and with little knowledge of number facts. She could easily subitise for numbers up to three or four, but made errors when there were more than four objects. Reliance on enumerating sets by looking is known to be less reliable than touching or pointing (Threlfall, 2008). However, Hannah did not check her results and consequently did not seem to know when she had made a mistake. Hannah seemed to have difficulty with understanding the number line as directional, but perhaps this was not a meaningful representation for her, or perhaps she was not confident with working with numbers beyond twenty, as she had suggested herself early on in the interview. Perhaps, the first time I had asked her to go beyond twenty, I was already pushing her beyond her comfort zone. What seemed to be missing from Hannah's approaches to problem-solving, whether concrete or abstract, was any recognition of pattern in number, beyond the pattern of the number sequence from twenty-one to ninety-nine. Hannah did not use her fingers at all during the assessment. When I asked her if ever used her fingers, she said she preferred to use counters. Finally, Hannah did not use knowledge of any number bonds to help her with her calculations.

For the final activity, Hannah was asked to complete the number line estimation task (Siegler and Opfer, 2003). Hannah was asked to mark the listed numbers on each of the number lines. Two examples will be discussed here. The first is the 1-10 number line (Figure 4-25).

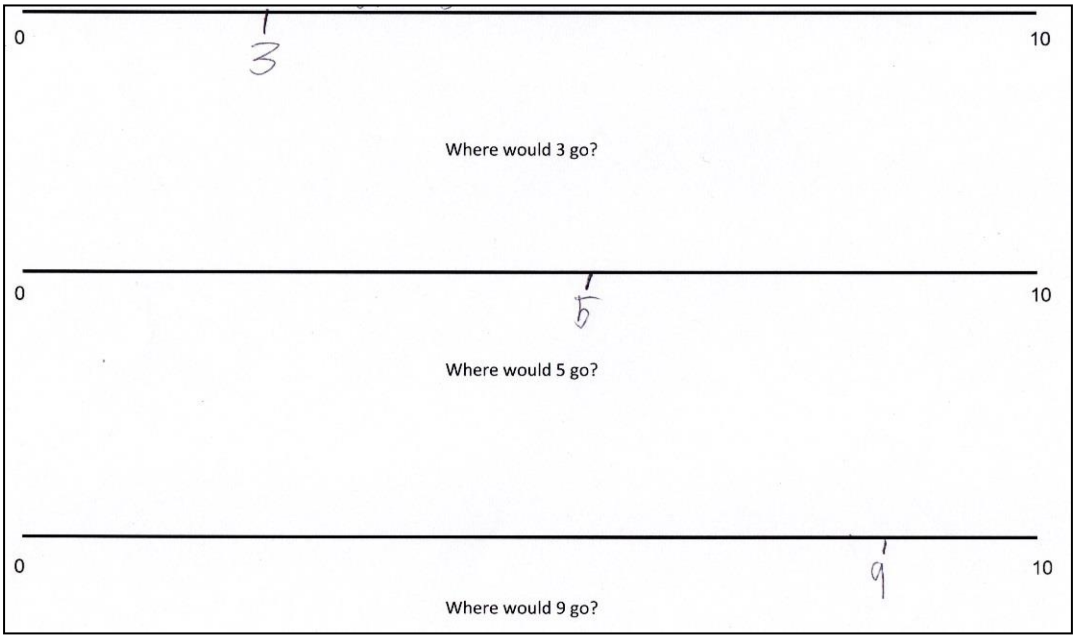


Figure 4-25: Hannah's 0-10 number lines

These lines suggest that Hannah had quite a good sense of the relative positions of the numbers from 1 to 10 on a number line, although, it is not clear whether she used the fact that 5 is half of 10 to position the "5".

The next group of numbers were placed on a 1-100 number line (Figure 4-26).

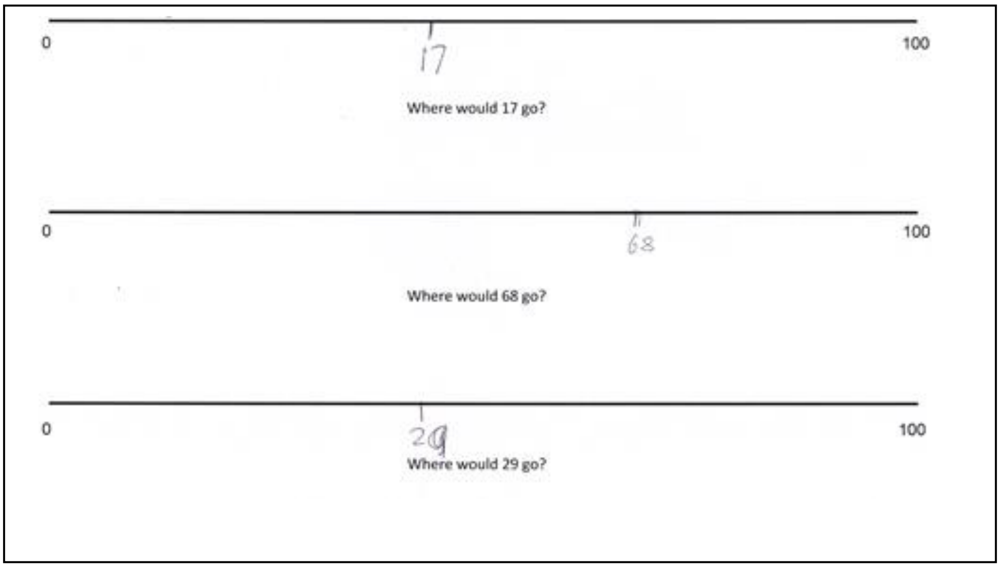


Figure 4-26: Hannah's 0-100 number lines

This time Hannah's knowledge of the relative positions of the numbers is less clear, although it does not seem that the numbers have been randomly placed. Reflecting back on her difficulty with counting back from 60, these examples support the proposal that Hannah was not yet very confident with numbers greater than 20 or maybe even 10.

4.6.2 Second school visit (June 2013)

The next visit to Hannah's school was about 4½ months later.

Observation

The children were working on a quiz designed to help them with a test they would be doing after morning break. Hannah was working with a partner and the teacher spent time with the pair throughout the morning. Hannah needed support with every question in the quiz, apart from one, which was to do with naming 3D shapes. At a number of points, the teacher stopped the class to ask for solutions to the problems. Hannah never put her hand up, and as the lesson progressed, appeared to focus less and less when the teacher was addressing the whole class. The questions in the quiz were mostly concerned with number and number operations. Many of the questions were presented as contextualised word problems. Hannah seemed confused when having to choose which operations to apply to particular word problems. This issue with contextualised word problems has been discussed by Dowker (2008). Hannah also had difficulty with placing decimal numbers on a number line, but perhaps this was not surprising, given that she did not seem very confident placing whole numbers on a 0-100 number line during my previous visit.

The session ended at break time and Hannah went outside to play. For the whole time, Hannah played alongside a group of children who were chasing each other, but she was not an active participant.

After break, the children started their test. Hannah was taken out of the classroom to work with a teaching assistant (Tina). Tina explained that she had been doing one-to-one work with Hannah for one hour a day to help her with her maths. I felt very uncomfortable observing Hannah attempting the test as she was not able to complete a single question successfully. Tina occasionally tried to prompt Hannah, but this was not enough to help her to get any of the answers correct. Sometimes, Tina asked Hannah if she would like to skip a question. Hannah always took up this offer. Tina got Hannah to read out many of the questions. Hannah did this fluently every time. With some of the questions, Hannah would exclaim, “I’m confused” or “I’m not sure.” For a question involving an oblique line of symmetry, Hannah seemed to know that she had not drawn the image correctly, but did not make any changes and continued on to the next question. On occasion, Hannah did use her fingers to count on in ones.

In the afternoon, I attempted the WMTB-C (Pickering and Gathercole, 2001), but Hannah seemed to find it quite hard to stay focused, so I decided to repeat the test on another visit. For that reason, the results have not been reported here.

4.6.3 Third school visit (February 2014)

The next time I visited Hannah, she was in Year 6 and in a new school. The move was part of the transition process in the area Hannah lived, as she had moved from first school to middle school.

Staff views

Although the SENCo (who was also her mathematics teacher) reported that Hannah had settled in well, there were concerns about Hannah's progress, especially in mathematics. At the time, Hannah's level in mathematics was 2c currently, but her teacher felt that this was not a true reflection: "She has achieved this in the most recent test but doesn't always operate at this level consistently."

Although Hannah seemed to be accepted by her peers, her teacher did not feel that Hannah had any particular friends at school.

Hannah had been placed in the bottom set for maths and Hannah's teacher reported that she found maths hard. She commented that Hannah struggled with number bonds and that she was beginning to realise that she was not working at the same level as her peers.

Observation

Hannah was observed in a lesson on coordinates. Hannah was sitting at a table with a teaching assistant and four other children with special educational needs. Hannah had no problems with the lesson, which was on plotting points and identifying and labelling points, and was able to complete all the activities successfully and quickly.

Looking through Hannah's exercise book, it was clear that she had been doing a lot of work on number bonds to 10 and 20. Much of the work had been done on number lines, but these had generally been completed by counting on in ones (this was apparent because of the images Hannah had drawn to support her calculations), rather than using any known number facts. Hannah's mother had written in her homework book that she helped her with her homework because she did not want her to feel that she was falling behind.

Interview

Again Hannah was very pleased to come and work with me and we did this straight after the lesson I had observed.

I decided to try the WIAT-II Mathematical Reasoning test (Wechsler, 2005) first. Hannah achieved a standard score of 57. Hannah was able to quickly and successfully answer parts of the test involving images to solve number patterns or additive reasoning problems. When asked, Hannah reported that she used a counting strategy. However, she was often very quick and it is possible that she used some form of pattern recognition (perhaps involving subitising), rather than counting each item. Hannah did sometimes use her fingers to count on and to keep track of things, such as days of the week.

The test items which Hannah found easiest were those that were accompanied by images, so she could "see" what the problem was asking. For problems without images, she seemed unsure of to how to proceed. It seems likely that Hannah had not yet fully grasped the idea that numbers can exist as abstract entities, without being attached to particular objects. This difficulty was observed by Hughes (1986) when working with much younger children. It also became clear that Hannah did not know how many pence there were in a pound and, therefore, questions involving money were problematic.

Hannah used her fingers occasionally when trying to count on and to keep track of where she was, for example when counting the days of the week.

Some of these issues will be illustrated using a few examples of Hannah's responses. The first example is of a question which is supported by an image of ducks in a pond.

C: Five ducks were swimming in a pond. [Hannah laughs]….. Three flew away and then two more came to swim. How many ducks were in the pond?

H: Four [very quick and confident response]

C: How did you work that out so quickly?

H: Because I took those three [pointed at three of the ducks] and then added the other two that flew away

Although Hannah said that she “added the other two that flew away”, I think that it was just a mistake, as she clearly seemed to know what she was doing.

The next example is of a question with no images. The question asks for the missing number in the sequence 1, 3, 5, __, 9, 11:

C: What number should go in the empty circle?

[silence]

C: If you read them out

H: One, three, five…six…No…I don’t know

C: What do you notice happening?

H: One…

C: There’s no two is there?

H: No

C: There’s three

H: Three, four, five, six [spoken very quietly]. Seven [spoken with confidence]

Hannah was very quick to spot the pattern when she had been given a strategy. However, in earlier questions where there were number patterns supported by images (e.g. figure 4-27), Hannah was very quick to respond and was correct.

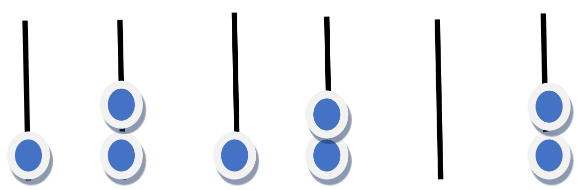


Figure 4-27: Example of image in WIAT Mathematical Reasoning test

The third example, helps to illustrate Hannah's difficulty when questions were presented without images and where the mathematical operations were not obvious from the wording of the question:

C: Robert has six stones and together, Robert and Max have fifteen stones. How many stones does Max have?

[silence]

H: Fifteen? [spoken tentatively]

I then suggested that Hannah could write something down to help with this. Hannah agreed and proceeded to write "6 stones" and underneath "15 stones". This was followed by a period of silence. I then asked Hannah if perhaps she could draw a picture to help her. She paused and then said she was, "…not sure". In an attempt to try to help Hannah, I drew a picture which was similar to figure 4-28.

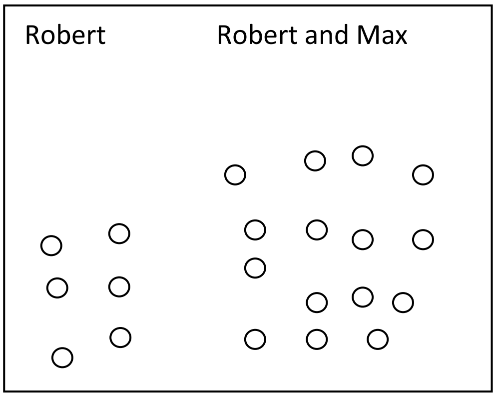


Figure 4-28: Image I drew to support marbles question

This did not help Hannah, so we moved on to the next question.

It seems that Hannah had no strategy for representing the problem and my image did not help her. Somehow the words and the image did not match up for her. Perhaps concrete objects would have been more effective, but as I did not offer Hannah this option, I do not know whether they would have helped. While it is not unusual for contextualised word problems to be more of a challenge than purely numerical problems (Dowker, 2008), it is perhaps the nature of the problem where the mathematical operation is not explicit in the wording of the question that makes this more of a challenge for Hannah (Nunes and Bryant, 1996).

Finger gnosis

Hannah's finger gnosis was assessed using the adapted version of the Gracia-Bafalluy and Noël (2008) finger gnosis assessment. Hannah had four fingers on both hands, as in figure 4-29.

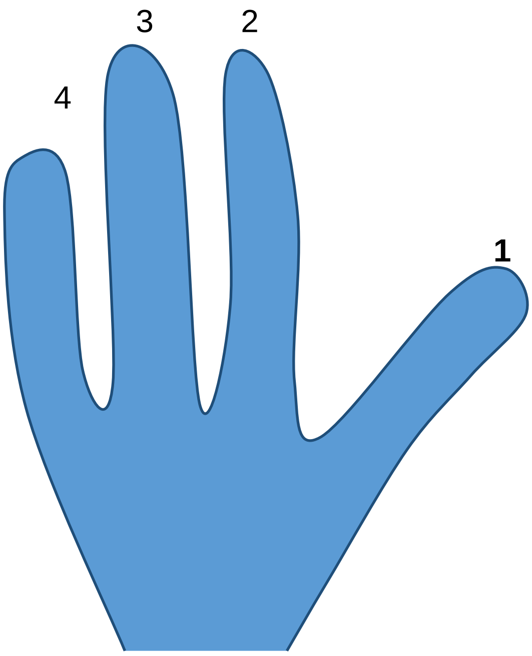


Figure 4-29: Diagrammatic view of Hannah's left hand

Hannah was always able to identify all her fingers when just one finger was touched. With two finger touches, Hannah made no mistakes on her right hand, but sometimes made mistakes on her left hand, repeatedly pointing to fingers 2 and 3 when fingers 3 and 4 had been touched.

When interviewed later, Hannah reported that she liked work on shapes and enjoyed the work on coordinates. She did, however, say that she did not enjoy "work with numbers". This represented a significant change from the first visit.

Panamath

During this session Hannah also did the Panamath assessment (Halberda et al., 2008). Hannah's results were quite remarkable. She achieved a Weber Fraction of 0.01 (with the 10th percentile, 0.44 and 90th percentile, 0.16). Her response time, however, was consistently slow, with an average of 3718 ms (with the 10^{th} percentile, 1444 ms and 90^{th} percentile, 929 ms). She did the assessment twice and got a similar score for each trial. As can be seen, Hannah was very accurate, but her response times were slow.

Number line activity

On this visit, Hannah had another attempt at the number line activity. This time there was a definite change in her approach to working on the number line. Looking at some of her attempts, a number of interesting features are revealed. Firstly for 1-10 in figure 4-30.

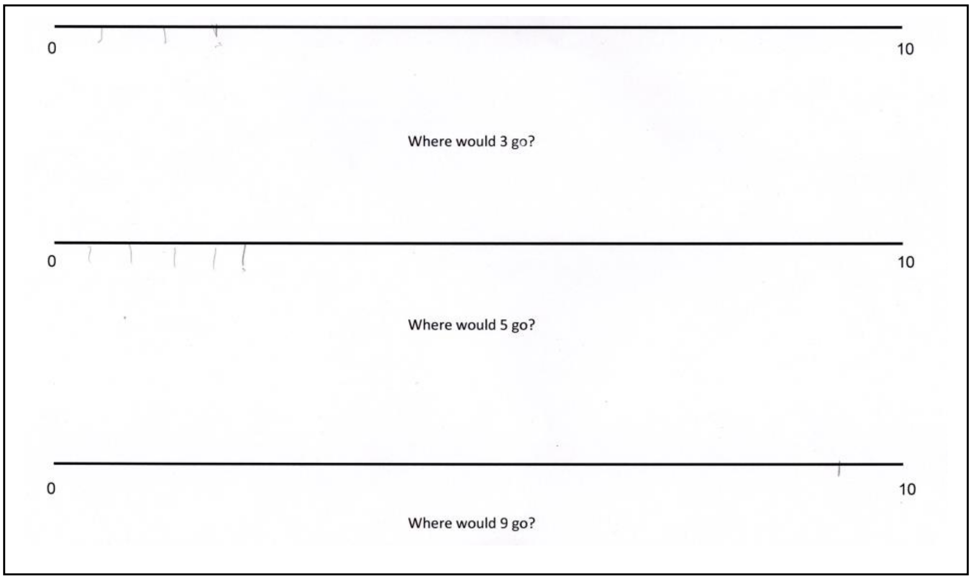


Figure 4-30: Hannah's 0-10 number lines

This time Hannah started by marking the numbers on the line. She did not seem to be concerned that the 3 and the 5 were in very similar positions. As before, when Hannah had attempted the task, she did not use the fact that 5 is half of 10 to help her to decide where to put the “5”. After she had done the first two, I suggested that Hannah should try to think about where she thought the “9” should go, without drawing in each point. The position she chose is 0.92 of the length of the line (not a bad estimate).

An example of the 1-100 number line, again shows some interesting features (Figure 4-31).

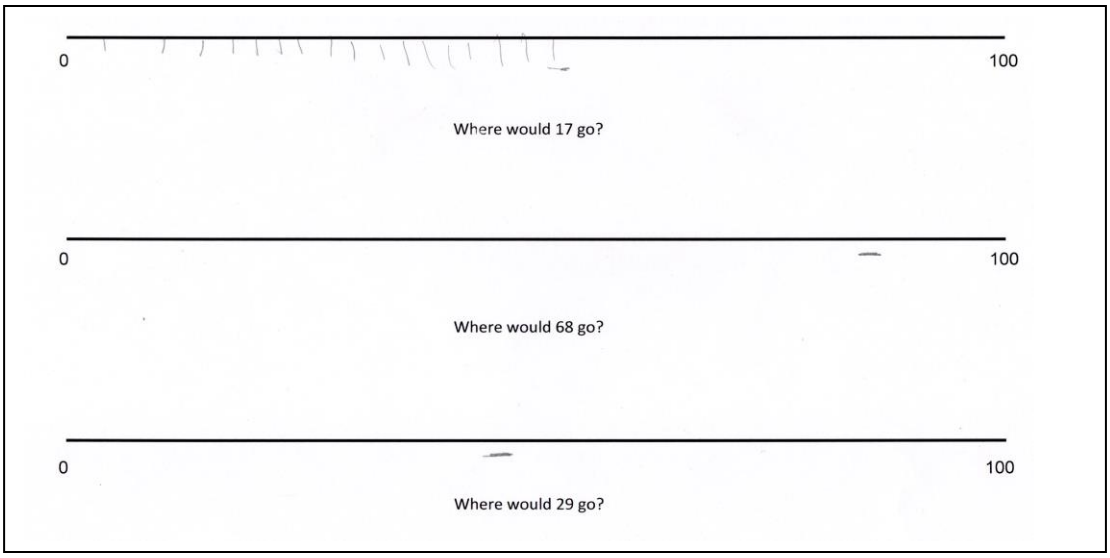


Figure 4-31: Hannah's 0-100 number lines

Hannah went straight back to counting in ones to try to work out where the “17” should go. I suggested that she should try to place the numbers without marking each of the points. For “29” and “68”, Hannah quickly marked the positions. With these numbers, there is definitely evidence that she is aware that they are significantly different and that 68 is much closer to 100 than 29.

4.6.4 Fourth school visit (June 2014)

This visit took place 3½ months after my last visit. On arriving at the school, I was told that Hannah was now in an intervention group for mathematics with four other children which was being taken by Helen, a higher level teaching assistant (HLTA).

<u>Staff views</u>

Helen talked to me briefly before and after the lesson. She thought that Hannah was finding maths very difficult, but that she tried hard. However, she said that was still in the process of getting to know the children and that no-one had yet carried out any assessments of the children’s knowledge and understanding of mathematics. All the children, apart from Hannah, had identified special educational needs. Hannah had not been entered for any of the SATs, because the school felt that she would not reach a level 3 in any of the subjects. Mathematics was definitely worse than English.

Later, I spoke to the head teacher who said that the school thought that Hannah would benefit from a statement of special educational needs and they were in the process of applying to the local authority. Hannah's parents had been requesting an assessment for years and yet, it was only now that Hannah was in secondary school, that this request had been made to the local authority.

<u>Observation</u>

This was the first lesson of the day. The first activity involved adding and taking away 10 from numbers on a 200-rectangle. Helen asked the children questions one at a time. Hannah was able to answer all the questions that were directed at her, with the aid of the 200-rectangle.

The next activity involved rounding numbers to the nearest 10. This time Helen used a number line to model how to round numbers less than 100 to the nearest 10. She only used numbers without a "5" in the "ones" column. The use of the number line meant that the nearest tens number could always be found by inspection, rather than having to use knowledge of numbers and place value. Hannah attempted to answer some of the questions without the number line and always made a mistake. When she was asked to check with the number line, she was always able to self-correct.

The final activity was a game called "Place Value Bingo". For this game, each of the children were given a bingo board with six numbers that were in the thousands. A spinner was used to select the item to look for in each turn. The first spin asked for a number with a "4" in the hundreds column. Hannah pointed to a number with a "4" in the thousands column. The next spin asked for a number with a "4" in the tens column. Hannah pointed to "5304" (perhaps this was a misreading - "04" instead of "40"). The next spin asked for a number with a "3" in the ones column. Hannah pointed to "3214" (again a possible misreading). This time Helen asked Hannah to check and Hannah then pointed to the number "5213" instead. The final spin asked for a number with a "2" in the thousands column. This time Hannah correctly identified the number "2516".

Interview

This time I started with the WMTB-C (Pickering and Gathercole, 2001). Hannah's scores can be seen in table 4-32.

Test component	Standard score
Digit Recall	111
Block Recall	109
Listening Recall	127
Counting Recall	98
Backward Digit Recall	85

Table 4-32: Hannah's WMTB-C scores

Hannah demonstrated areas of strength, with Backward Digit Recall being the only area of weakness. It was interesting to observe her strategies on the Counting Recall component. Within each set of items Hannah attempted, she demonstrated that she could easily subitise to at least 4. Hannah used this effectively to quickly work out how many dots there were in images with larger numbers. For example, when working out how many dots there were on a picture with six dots, Hannah said, "I know that's four [pointing to the four dots], so two more is six." On another occasion, Hannah self-corrected when she miscounted six dots: "…it's six…no seven." Hannah could consistently remember the sequences of numbers for four items, but could not manage five.

With Backward Digit Recall, Hannah found it very hard to recall the numbers in reverse order and had to be reminded to remember the numbers in the correct order, before trying to reverse the order. The Listening Recall, Counting Recall and Backward Digit Recall tests, are all designed to assess central executive processes. Hannah was particularly successful with the Listening Recall test which requires verbal information to be held while other unrelated questions are asked. When I asked her later how she remembered the words for this test, she said she did not know. Despite there being some variation in Hannah's scores, there was nothing that would suggest she had a significant problem. Hannah's difficulties in mathematics, then, could not be explained by her performance in the working memory assessment.

After the WMTB-C, Hannah attempted the WIAT II Numerical Operations test (Wechsler, 2005). She achieved a standard score of 50. Hannah only attempted the questions with single digit addition and subtraction. She did not check any of her answers and she used her fingers to help with most of the calculations, but she never went back to check her answers – perhaps she had not been encouraged to this. As a result she made a calculation error with "2 + 3 + 1 + 4" and got the answer "11". However, for the question "4 – 2 =", Hannah wrote the answer "1". Hannah appeared to pull this answer out from memory, rather than using any strategy for working it out. When asked if she wanted to check it she said that she had worked it out and was happy with her answer, but she did not offer an explanation of her strategy.

Later, for the question: **10**

- 6

Hannah used her fingers to count back from 10 to get to the answer.

It is worth remembering that when doing the Counting Recall test in the WMTB-C, Hannah was very confident with her application of numbers and seemed to make quick calculations based on the images. This suggests that when she was using numbers with images, she was able to make the connections between the numbers, such as four and two making six, described earlier, in ways that she was not doing when she was presented with calculations using abstract notation, such as "3 + 3 =".

Hannah did not attempt any of the questions involving two digits (e.g. "41+14="). Perhaps she did not have a strategy to do these and was not confident enough to use her existing knowledge to explore ways of making sense of the problems. It seemed that Hannah would attempt questions if she was confident that she could work them out, but would not attempt questions where she did not already have a strategy in place.

Overall, Hannah seemed to be developing confidence with using her fingers. She also knew how to apply the methods represented by the abstract symbols for addition and subtraction with single digits.

During this session Hannah also did the Panamath assessment (Halberda et al., 2008). This time Hannah achieved a Weber Fraction of 0.09 (with the 10^{th} percentile, 0.40 and 90^{th} percentile, 0.17). Her response time, however, was consistently slow, with an average of 2087 ms (with the 10^{th} percentile, 1532 ms).

There were areas of number work that presented real difficulties for Hannah, but there were also some areas of strength. Hannah was most comfortable when she was attempting questions that were supported by images and in these cases she was often able to make quick mental calculations. This confidence was not exhibited with calculations that were presented in abstract notation only. It seems that in many ways Hannah was still making sense of the link between concrete and abstract described by Hughes (1986). This presented as a lack of understanding of number, which definitely does not seem to be the case.

Following this visit, I had arranged to See Hannah's mum, as she was very concerned about Hannah's progress in mathematics. When we met, Sam (Hannah's mum) said that she felt Hannah was not making progress with her mathematics and that numbers were particularly difficult for her. We discussed some of the issues Hannah was having. I talked to Sam about the possible benefits of encouraging Hannah to use her fingers when working with number and I directed her to the activities in the Gracia-Bafalluy and Noël (2008) paper.

4.6.5 Fifth school visit (February 2015)

Hannah was now in Year 7 and was still in an intervention group for mathematics with Helen. She had just received her statement of SEN and the school was reviewing their provision for her.

Staff views

Helen said she was still concerned about Hannah's progress in mathematics. She said that, "Hannah finds it hard to retain things she has learned. One day she has it and then it's gone." She also said that she, "…..would like to say she's working at 2a in maths, but I'm not sure. Recently, I have been working on time and money with Hannah and her coin recognition is now much better." I was shown a copy of Hannah's progress review. Her levels were:

Reading 3c

Writing 3c

Mathematics 2

She continued to do much less well in mathematics than in English and all her other subjects.

Observation

Hannah was still in her intervention class with four other children. At the beginning of the lesson, Hannah gave Helen her homework and said that her mum helped her a bit. The lesson was on probability, using lines to represent the range, initially from 0 - 1, and later from impossible-certain. When Helen had drawn the 0 - 1 probability line, she asked the class what went in the middle. Everyone, apart from Hannah put their hand up. Helen then moved on to talk about the impossible-certain probability line. Hannah was able to participate fully when asked about the likelihood of events. Next, Helen introduced a six-sided die and asked the class what the probability was that the die would land on an even number. Hannah wrote down "C" for "certain". Helen spotted this and then told Hannah that even numbers had to be in the two times table. Hannah then told Helen that the numbers must be, "…six, two, four."

Worksheets were then given out and the children were asked to write the WALT (We Are Learning To) in their books. The WALT for this lesson was "To understand probability". Hannah wrote this down very carefully and checked the spelling for "understand" and "probability". For the main activity, the children had to toss a coin 20 times and record their answers using tally marks. Hannah did as asked, but did not put a line through four tally marks to represent five. As a result, she found it hard to keep track of the number of tosses of the coin and had to keep counting the individual tally marks. At one point, Helen asked Hannah, “How many heads would you expect to get?”

Hannah replied with “Ten.”

Helen then asked Hannah, “How many tails have you got?”

Hannah counted each of the tallies and said, “There are nine; I need one more for the tails.”

This interaction suggest that Hannah thought she had to get 10 heads and 10 tails. This sort of confusion with understanding how probability works is not uncommon, but it does serve to highlight the fact that the teaching of probability requires significant mathematical knowledge and understanding (Batanero and Díaz, 2012). Helen counted up all of Hannah’s tallies and told her that she needed to do three more throws. Hannah followed this instruction and ended up with nine heads and 11 tails.

The lesson ended and Hannah and I went to a quiet room to conduct our interview.

<u>Interview</u>

Hannah said she was enjoying maths more and felt she had made progress in all areas and that there were no parts of maths that she did not like.

I started with questions based on the Number Knowledge Test (Griffin, 1997) that I had used before during my first visit to see if there were any changes.

C: If you had four chocolates and someone gave you three more. How many chocolates would you have altogether?

[counting on silently using her fingers]

H: Seven?

C: Good girl and how did you work that out?

H: I used my fingers

Hannah did this quickly and confidently.

This was followed by questions about the number sequence:

C: That’s good…now some of these early questions will be easy for you so…what number comes right after seven?

H: Eight [very quick response]

C: And what number comes two numbers after seven?

H: Nine

Hannah answered these questions with ease and did not need to count. This was followed by questions exploring number relationships in more depth:

C: Right with this one [pointing to the visual array] which is closer to seven is it four or nine?

H: Nine

C: Because

H: Because nine comes two things after seven

C: And what about the four?

H: Four is close to to seven…no [counting on her fingers] three

C: Uh huh so it’s three here and how much is it between seven and nine?

H: Two [using fingers]

C: So which one’s closer?

H: Nine

Hannah was now much better able to reflect on the processes she was using and justify her answers. Her ease with numbers and the confident and repeated use of her fingers was very encouraging to see.

I then moved on to questions involving more use of formal mathematical language:

C: That’s good…now how much is two plus four?

H: Not sure

C: Ok so what if I write it down [as I get out some paper]. Now what if I write this two plus four [as I write 2+4 and pass it over to Hannah]

H: Ohhh

C: Have you any idea how you might work that out

H: Six

[Hannah writes =6 to complete the number sentence]

C: How much is eight take away six?

H: [Hannah counts back, whispering so the words are barely audible] Two

Hannah seemed to still not be making the link between the spoken language of mathematics and the written form (i.e. she did not immediately see that “two plus four” can be written as “2+4”).

Later, we looked at questions involving 2-digit numbers:

C: Great and can you tell me which number comes five numbers after forty-nine…five numbers after forty-nine?

[5 seconds while Hannah counts on her fingers]

H: Sixty-four

C: How did you work that out?

H: I used my fingers

C: So what comes straight after forty-nine?

H: [whispers forty-nine] six…fifty

C: So do you want to have another go?

H: Yeah

C: So five numbers after forty-nine

H: Ummmm, no

C: You can do it Hannah

H: Forty-nine

C: Should I write it down?

H: Yeah

C: So it’s basically forty-nine [as I write 49] plus five [as I write + 5]

H: Forty-nine, fifty, fifty-one, fifty-two, fifty-three, fifty-four

C: Great do you want to write it down?

H: Yep [as she takes the pen and completes the number sentence with = 54]

C: Lovely and do you know what comes four numbers before sixty?

[9 second pause]

H: [whispers in audibly while counting on her fingers] Fifty-six

It seemed that Hannah was able to use her knowledge well. She was now taking what I would describe as “thinking time” before answering questions. However, her confidence, as can be seen by the episode above, was still quite fragile and she could easily return to the position of “not knowing”.

I asked Hannah if she could write down the “maths calculation” for what she had just worked out. This was very hard for her, but she was able to work it out with support. There seemed to be a tension between her confidence with number and her confidence with the symbolic form of written mathematics. At a number of points in some of the later questions, I suggested to Hannah that she could write things down to help, but this did not seem to be something that she was in the habit of doing and, therefore, she often struggled to keep everything in her head, placing a great load on her working memory.

Later questions suggested that Hannah had been using column methods for addition and subtraction, but with little understanding of how these methods linked to the base-10 number system. This issue of rote learning methods and the consequent implications for understanding and further development in mathematics were discussed by Skemp (1977). His terms “relational” and “instrumental” understanding can be well applied to the knowledge that Hannah attempts to use:

C: Now can you work out any way you want to twelve add fifty-four? [using the visual provided] you can use the paper if you like

H: Is it add?

C: Yes

[Hannah writes:

12

+54

____

And then draws dots to represent the “2” and the “4” (Figure 4-33).

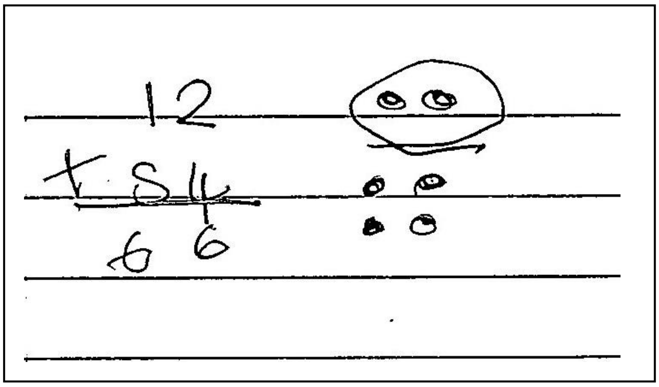


Figure 4-33: Hannah's calculation for 12+54

Hannah counts the dots using a count all approach and writes “6” below the “2” and the “4”.

Hannah begins to draw dots for the “1” and the “5”, but I interrupted with:

C: Can I just stop you for one minute?

H: Yeah

C: Right this is a very good strategy [pointing to Hannah’s drawing], but what is five plus one?

H: Six

C: Yeah

H: Ohhh

C: And what is forty-seven take away twenty-one? [using the visual provided]

H: OK

Hannah writes:

47

-29

She then proceeded to draw out twenty-nine dots, cross out seven of the dots and then write her answer (Figure 4-34).

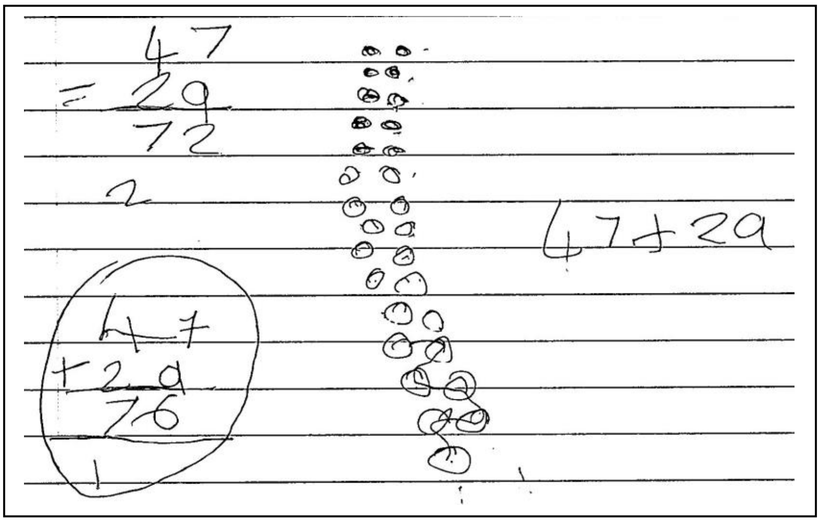


Figure 4-34: Hannah's calculation for 47-29 and 47+29

C: Did you take those away or add them?

H: Take away

C: How did you do the first bit? Can you talk me through what you did?

H: I drew…I started from twenty-seven…I started from the dots

C: So how many dots have you got here [pointing to the Hannah's diagram]

H: Twentyyyy-nine

C: So you've got twenty-nine dots…

H: Yeah take away seven [then says something inaudible]

C: So where does this two come from? [pointing to the "2" in the ones column]

H: I'm not sure

On checking the diagram later (see figure 4-32), I noticed that there were 22 dots left, once Hannah had taken away the seven dots. This probably accounts for the "2" in the ones column and the "2" that has been carried. This suggests that Hannah had been exposed to the formal column methods for addition and subtraction, but she did not seem to understand how they worked. This left her with an insecure method for doing calculations which was prone to error.

I asked Hannah about other methods for working out questions of this type. It seemed that Hannah was aware of other methods, but was unsure of what they were and how they could be applied.

As Hannah was now using her fingers with greater confidence, I wanted to explore how she was using them to make ten.

C: How when you're counting ten and you're using your fingers

H: Yeah

C: How do you know when you've counted ten?

[2 second pause]

C: Because you've got eight fingers altogether

H: Yeah yeah

C: So how do you know when you've counted on ten more?

H: Cos if I get to…say I had eight

C: Yeah

H: And there's two more [banging two fingers on the table to indicate the "two more"]

C: OK

H: Eight, nine, ten [banging fingers 3 and 4 of her left hand on the table – finger number 8 and then fingers 7 and 8 again, to make 9 and 10]

C: Great, OK so you know that

H: Yeah

C: So now try again try to count ten numbers after ninety-nine

H: A hundred, one hundred and one, one hundred and two [continues up to one hundred and nine, banging her fingers on the table as she counts, double counting the last two fingers, as described above]

However, when it came to adding 9 onto 999, Hannah did not know what came after 999, so we moved on.

During the interview, Hannah said that her mother had been encouraging her to use her fingers more and that she felt it was helping her to understand mathematics better because she could work things out on her fingers. It is of course possible that she thought this is what I wanted to hear, but I think that had she not found finger-use helpful, she would not have used them with such ease.

Hannah attempted the WIAT Numerical Operations (Wechsler, 2005) test again and achieved a standard score of 60.

During this session Hannah also did the Panamath assessment (Halberda et al., 2008). She did this after working with me for about 45 minutes without a break, so she did very well to maintain her level of concentration. This time Hannah achieved a Weber Fraction of 0.17 (with the 10^{th} percentile, 0.40 and 90^{th} percentile, 0.17). Her response time, however, was consistently slow, with an average of 2441 ms (with the 10^{th} percentile at 1532 ms).

<u>Finger gnosis</u>

The last assessment we tried was the adapted finger gnosis assessment (Gracia-Bafalluy and Noël, 2008). This time Hannah was correct on all one finger and two finger touches on her left hand. On her right hand, Hannah said that I had touched fingers 2 and 3, when I had actually touched fingers 2 and 4. This was done twice and the same mistake was made. Hannah made no other mistakes with one or two finger touches.

4.6.6 Conclusions

Over the time that I visited Hannah she definitely did seem to make progress with her mathematics, particularly in terms of understanding numbers and how they work. However, as far as her national curriculum levels were concerned, she made almost no progress. This seems to be an unfair representation of her achievements.

Throughout the time that I visited Hannah, her teacher assessments suggested that her skills in mathematics were below those for other areas of the curriculum. In fact, from January 2013 to January 2015, in terms of the levels she was awarded in mathematics, Hannah appeared to make almost no progress at all. This, however, was not reflected in the work I did with her. This is particularly evident in 4.6.5, where it was seen that Hannah had definitely made progress in her understanding of mathematics and in her application of a wider range of problem-solving strategies and skills.

The overall conclusions will be considered with reference to the research questions.

1. What strategies did Hannah use to help her solve numerical problems in mathematics?

When I first met Hannah, she seemed to have very limited understanding of the number system and of number relationships. Hannah also seemed unclear about how to relate the abstract language associated with number to real objects. For example, in 4.6.1, when asked about the number of chocolates, it did not seem to matter whether she guessed or counted them. In other words, Hannah seemed unsure of the purpose of counting (Fuson, 1988). Perhaps what stood out most was the fact that numbers seemed to have very little meaning for Hannah. There are many possible reasons for this. One of these might be that Hannah did not play number games at home and, therefore, missed out on opportunities to learn about number through playing games and engaging with number activities with adults and peers (Munn, 2008; Gifford, 2008). However, she might also have actively avoided playing games involving number, if numbers had little meaning for her.

During the 2 years that I visited Hannah, she began to develop a range of problem-solving strategies, such as using her fingers (as in 4.6.3, 4.6.4 and 4.6.5) and then drawing images (as in 4.6.5) to help with numerical calculations. These helped her to make sense of the problems as well as finding solutions.

2. Did Hannah's hand anomalies impact the range of strategies available to her?

When I first met Hannah, she said that she did not like to use her fingers to help with her mathematics. Her teacher also said that it was too hard for Hannah to use her fingers, so she was encouraged to use counters. My observations though, suggest that Hannah's confidence and skills grew as she was increasingly and more reliably using her fingers to help with calculations, initially as a means of keeping track (for example in 4.6.3) and then for counting on and counting back, as seen in 4.6.4 and 4.6.5. Not only did Hannah become better at understanding number relationships, but she could also apply this knowledge appropriately to contextualised word problems. Hannah's fingers provided her with a way of representing problems that supported her to see number relationships (for example that eight plus two equal ten), in ways that her drawn dots did not.

3. Are there other factors which had an impact on Hannah's mathematical learning opportunities?

Hannah had working memory skills at least within the expected range for all the assessments tried (see 4.6.4). Although the central executive tasks involving number words, were her least successful tasks, she still scored 102 and 107 for Counting Recall and Backward Digit Recall respectively. It is possible, though, that these lower scores may be due to Hannah's anxiety around tasks involving number.

In 4.6.3, 4.6.4 and 4.6.5, it was seen that Hannah was particularly successful at Panamath (Halberda et al., 2008). This suggests that her ANS was particularly strong and if Halberda et al. are right, this should have resulted in at least average attainment in arithmetic. For Hannah, this was not the case. This brings into question the association between ANS and success in arithmetic skills.

Hannah had a visual impairment for which she wore glasses and she also wore a bone anchored hearing aid (BAHA) to help with her hearing impairment. It is hard to know what impact these had on her development, but the fact that she had a BAHA suggests that her hearing was particularly compromised.

In the two schools Hannah attended, she was socially isolated, both in class and in the playground. In her primary school she played “alongside” rather than “with” her peers. Even when Hannah worked in intervention groups, she worked on her own and tended to only engage in conversation with the member of staff. This would have impacted on her opportunities to learn with her peers.

Hannah had a diagnosis of ADHD, but this was never apparent at school. When we worked together, Hannah was always very attentive and focused. Hannah had just received a statement of SEN as our time together was coming to an end. Throughout her education, Hannah had received very little individual support.

5 ANALYSIS, RESULTS AND EVALUATION

This study aimed to explore the following questions:

a) What strategies do children with Apert syndrome, from Reception to year 7, use to help them solve numerical problems in mathematics?

b) Do the children's hand anomalies, with specific reference to fingers, impact the range of strategies available to them?

c) Are there other factors which have an impact on the mathematical learning opportunities of children with Apert syndrome?

This study on the mathematical development of children with Apert syndrome, is, as far as I am aware, the first to explore any area of academic development in children with this syndrome. Given the complex nature of the children in my study, I decided on a qualitative approach. This approach has also been the one of choice in the clinical management of children with complex craniosynostoses:

> …given the small number, phenotypic variability, and complexity of so many complicating "third variables," ……the craniofacial community will continue to look for clinical guidance based on the linking together of often small observational studies in the hope that a more securely evidence-based management algorithm will eventually emerge.
>
> (Hayward, Britto, Dunaway and Jeelani, 2016, p.6)

The so-called "third variables" which are likely to have an impact on children's cognitive development were taken to include aspects such as chronic airway obstruction, hearing impairments, visual impairments, low expectations and teasing.

Although I originally worked with 10 children, I did not have the space to discuss them all here. Consequently, I have discussed, in detail, the five for whom I had the richest data, based on the research questions. Working with the children over the 2 year period enabled me to see them grow, develop and change. I was able to observe changes in aspects, such as their social skills and their language skills and I was able to build positive relationships with all of them. For some of the children, particularly Hannah, the fact that I had a daughter with Apert syndrome was very significant and she would often come up in our conversations.

5.1 Reflections on the use of a longitudinal approach

In terms of the data collected, the longitudinal nature of the study enabled me to collect very rich and varied data. For example, as I got to know the children better, I was able to adapt my approaches and my questions to each individual child, thus enabling me to delve deeper into their thinking and provide opportunities for them to demonstrate a range of strategic approaches (for example, with Emily's changing finger-use). This was sometimes made more difficult when a child's teaching assistant was present and wanted to demonstrate what the child could do; as was often the case with Isabelle.

The fact that the study was longitudinal also enabled me to identify small changes in the children's mathematical development that might not be picked up through more formal methods of assessment and which also might be harder for a teacher in a busy classroom.

The longitudinal nature of the study enabled me to observe the varying approaches and attitudes of different staff and the impact these had on the children. These were often made apparent by the discourses used in relation to the children, their work and their relationships with their peers.

Due to the fact that I made five or six visits to the children's schools, I was able to follow up on aspects of the interviews and could revisit some of the assessments that were more demanding for the children. This was especially the case with the WMTB-C assessments (Pickering and Gathercole, 2001), which required a great degree of concentration to complete at one sitting.

5.2 Evaluation of methods and data collection

This study took me all over the U.K. This enabled me to see a wide variation of provision which seemed to be predicated more on where children lived and the schools they attended, than on their individual needs. I usually visited schools for a whole day, so that I could maximise the time I spent with the children.

The nature of the study enabled me to triangulate my information gathering by including: interviews with parents, school staff and the children; clinical interviews with children; in-class observations; and a range of assessments.

5.2.1 Interviews with parents and staff

It was very useful getting the views of both parents and school staff. The parents were very open and willing to discuss their children and their experiences of parenting children with complex needs. I believe the fact that I had a daughter with Apert syndrome made parents feel more comfortable about being interviewed and allowing their children to be involved in the project.

The interviews with school staff were very varied. This variation often depended on the amount of time staff had available, but also on their interest in my study. Nevertheless, school staff made an effort to make time to talk to me on my visits and seemed to be very open and helpful in their responses to my questions.

Having the views of both parents and schools included in my study, gave me a much richer view of the children, although probably the most telling insights came from spending time with the children themselves.

5.2.2 The clinical interview

The clinical interview proved to be a very useful method. It allowed me to adapt questions and to explore responses in ways that provided greater insight into the children’s thinking and understanding. I was able to ask further questions to explore particular areas of interest and I could be guided by the children’s’ responses. Often this meant that I explored areas that I had not originally planned. The clinical interview allowed me great flexibility in terms of time and organisation. As described by Ginsburg (1981), the clinical interview did help to identify and explore some the children’s problem-solving strategies. However, to what extent I was actually observing the full extent of the children’s “cognitive” processes is harder to know.

The fact that I was very comfortable with this approach may well be due to the fact that I have had many years of experience working with children, and especially with children with special educational needs. This approach was, therefore, not one that was new to me and so using it in this study seemed like a very natural method to adopt. The difference with my role in this study was that I did not follow-up my findings with teaching approaches aimed at developing the children’s understanding. This meant that when I re-explored questions that I had asked in previous interviews, I had no knowledge of the teaching that had taken place, since my last visit. Many of the questions I took as starting points in the clinical interviews were based on those from a range of tests exploring number knowledge, as discussed in chapter 3.

Nevertheless, there were some drawbacks. Some of the children I worked with had poorly developed language skills, especially in the early visits, so asking them to articulate their thinking was a challenge for them and at times made them feel uncomfortable. Also, the interview is inevitably affected by children’s moods and by what else is going for them. These things cannot be factored in and often cannot be seen, but their existence cannot be ignored.

5.2.3 The case study

The use of case studies proved to be appropriate for this study. As each child had very different circumstances and experiences, the use of case studies demonstrated the individuality of each child and the heterogeneity of children with Apert syndrome. The use of case studies also allowed for the flexibility that was needed to continue the study over a 2 year period, given the nature of schools and the frequent changes of plan that are necessarily a part of school life. The use of case studies also allowed me to identify very small changes in the children's approaches to mathematics which might not have been identified through large scale testing without detailed interactions and observations.

Looking across my case studies has allowed me to identify any commonalities, both in terms of approaches to mathematics and experiences of school, but it also highlighted the enormous variation in terms of school provision and children's responses to these. The case study approach also highlighted the differences between all the children and, therefore, the limitations with respect to making generalisations in terms of best practice for children with Apert syndrome. In fact, in terms of generalisations, this study highlights the individual variability in children with Apert syndrome.

5.2.4 In-class observations

The observations of the children in their classes allowed me to see how the children related to their peers and to the school staff. Where children had full-time one-to-one support, it was interesting to see what this "support" looked like. The observations also allowed me to see how the children managed their learning in the classroom environment. However, it was not always possible for me to just sit and observe; sometimes the children or the staff would ask me to help. When this happened, I made notes whenever I could, but it did mean that my notes were not always truly notes of observed lessons. I have always made it clear when I have been engaged with the children during the planned observation periods.

5.2.5 WIAT-II, WMTB-C and Panamath

The standardised tests used provided a range of opportunities to explore the children's knowledge and understanding of mathematics, as well as providing information on aspects of their working memory. Of the two tests I used from the WIAT-II (Wechsler, 2005), the Numerical Operations test mainly revealed what arithmetic procedures the children could carry out and whether they recognised the formal mathematical symbols. In addition, the questions in this test move on very quickly, in terms of difficulty and depend entirely on the children's experience and exposure to the world of formal arithmetic operations. The Mathematical Reasoning test was more informative with regard to children's approaches to solving problems involving number and arithmetic skills. Some of the later questions, though, were less helpful as they could only be answered if the children had already covered the topics in class (e.g. aspects of fractions, probability and perimeter).

The use of the standardised tests highlighted the limitations of formalised testing, especially for children with SEND. For example, the lack of finger-use and the difficulty that many of the children had with the Counting Recall task may have resulted in unreliable test scores and, consequently, incorrect assumptions about the children's capacities could have been drawn. This may call into question the validity of these results in relation to the usefulness of the standardised scores, but they are still useful in terms of identifying changes over time for individual children.

I was surprised at how cooperative the children were when I asked them to do the Panamath test (Halberda et al., 2008). Some of them initially thought it was a game, but they soon realised that it was not very game-like. Nevertheless, they appeared to mostly try hard and stay focused whenever they attempted it and their results support this view. They were, however, not told that they needed to respond as quickly as possible (as this is not a requirement for Panamath), and this may account for the fact that they were slow in their response times. Alternatively, it could be the case that the children's slow response times on Panamath are an indication of slow processing at a visual and/or cognitive level. Again, this may call into question the validity of the results, in terms of the weber fractions and relative reaction times, but as above, the results still provide useful data in terms of individual children's performances over time.

5.2.6 Finger gnosis assessment

This assessment was straight forward to carry out. I tried to approach the assessment with sensitivity, as I was aware that some of the children may be self-conscious about their hands. I always asked for the children to consent before I started the assessment. Following my assessment of the children's finger gnosis, the children sometimes wanted to test my finger gnosis as well. I did not score the children, as described by Gracia-Bafalluy and Noël (2008), as it would have been hard to compare results, given the fact that the children did not all have the same numbers of fingers. I did, though, make sure that I tested each finger twice, with both one finger and two finger touches, as described by Gracia-Bafalluy and Noël.

5.3 Evaluation of findings

I will first of all discuss my findings, drawing from the more detailed reports in chapter 4. The discussion will consider:

- mathematical development
- the role of fingers in supporting work on number and calculations
- working memory
- visuo-spatial skills
- visual impairments and hearing impairments
- language development
- what we can learn from children with complex physical disabilities
- home environment
- school provision

5.3.1 Mathematical development

Anecdotal reports suggested that children with Apert syndrome have difficulties with mathematics. For this reason, I wanted to explore the mathematical development of children with Apert syndrome.

When I started visiting the children, I received information from both parents and schools. These highlighted a range of perceptions of the children's abilities, their needs and how these were being met. For Isabelle and Emily both parents and school staff felt that although they could do much of the work, they did not have a good understanding of the principles. Joe was described as being just below average in all subjects, including mathematics, but needing constant one-to-one support. Hannah's parents were very concerned about how she was doing in mathematics, but the school felt that she was making progress with the support they had in place for her.

During the time that I worked with the children, they all made progress in relation to the national curriculum for mathematics, which their schools were following. My own assessments also showed progress. Prior to my visits, only one of the 10 children in my study used their fingers to support with calculations in mathematics. However, most of the children could "show" a given number using their fingers all at once and without counting.

Assessments of mathematics

As identified in chapter 2, there is some question as to whether or not there is a link between children's arithmetic skills and their problem-solving skills. The children in my study achieved a wide range of scores in the WIAT-II (Wechsler, 2005) tests that I carried out. These can be seen in table 5-1 (all the scores are standard scores). Scores are typically expected to be between 85 and 115. Scores below 85 are considered low.

Table 5-1: WIAT-II standard scores for all 10 children, youngest to oldest (children in italics are those who were not selected for case study reports)

Name	**WIAT-II Mathematical Reasoning (MR)**	**WIAT-II Numerical Operations (NO)**
Wesley	*87*	*63*
Yasmin	*83*	*78*
Luke	78	66
Joe	103	113
Emily	99	103
Uma	*82*	*82*
Isabelle	93	85
Tania	*53*	*58*
Hannah	57	60
Nathan	*82*	*88*

I have included the results for all 10 children in the original study - the five that were not discussed in chapter 4 are in italics. Uma, Wesley and Yasmin were among the youngest children that I worked with. Uma was 6 years of age at the beginning of the study, Wesley was 4 years of age, Yasmin was 5 years of age and Nathan and Tania were both 9 years of age. Of the five additional children, their schools expressed concern about the progress of all of them in mathematics, apart from Uma. However, by the time I finished the study, Nathan (who was then 11 years of age) had reached age-expected levels of attainment.

The results highlight the fact that the children were all very different, even though they all had Apert syndrome. Emily and Joe show scores in the typical range on both tests. Four out of the 10 children did better in the Mathematical Reasoning test (focused more on problem-solving) than in the Numerical Operations test (focused more on arithmetic) and five of the children did better in the Numerical Operations test than in the Mathematical Reasoning test (although one of these was Joe, who used a 100-square to help). Of the children who appear to show difficulties in both areas (Luke, Hannah, Tania, Uma and Yasmin), only Yasmin and Hannah (both low attainers) follow the trend highlighted by Ostad (2008) of better performance in tests which assess arithmetic skills only. It is also worth remembering that Hannah did initially score 50 in the Numerical Operations test, so at that time, the picture was reversed. This study then, does not support the proposal that children with difficulties in mathematics have better arithmetic than reasoning skills. It seems from my observations that the problem for some of the lower scoring children was actually with the use and application of the abstract written language of mathematics, as discussed by Hughes (1986).

The test scores provide limited insight into the children’s skills in mathematics and areas of strength and weakness. Of the two assessments, the Mathematical Reasoning assessment provided more opportunities to provide insights into the children’s mathematical thinking. The Numerical Operations assessment is very much based on children’s ability to perform written calculations using formal methods. As Hughes (1986) suggests, if children are not familiar with the abstract language and representations of mathematics, then assessments of this kind tell you very little. Greater insight, however, was provided by the observation and interrogation of the strategies they used during the clinical interviews. As Ginsburg (1981) argues, standardised assessments assess competence only.

Teacher assessments

Apart from Luke, all the children were initially assessed by their teachers as achieving less well in mathematics than in other curriculum areas, especially reading (The children’s writing scores often lagged behind their reading scores because of the difficulty they had with manual dexterity, especially in Key Stage 1). At the end of the study, Isabelle was assessed as doing equally well in mathematics as in English. For Luke, mathematics was described as his favourite subject by his teacher. For the other three children, their teacher assessments in mathematics continued to lag behind those for other curriculum subjects.

Assessment of the Approximate Number System (ANS) using Panamath

The Panamath (Halberda, Mazzocco and Feigenson, 2008) scores for all the children varied considerably. Yasmin, Uma, Nathan, Isabelle, Joe and Emily's scores were well within the expected range for their ages. Hannah's scores were always very high. Luke did not do so well, but initially found it hard to understand the instructions, and when he finally took the test, there were serious concerns about his eyesight. Tania achieved a poor score and Wesley tried but did not seem to understand the instructions and gave up very quickly. In terms of response times, all the children were very slow, suggesting that there may be a trade-off between speed and accuracy. The only time Emily was fast enough to be in the normal range was when she was least accurate. This suggests that Panamath results can be affected by the impulsiveness of the participants.

Taking the results overall, the link between the children's Panamath scores and their attainment in mathematics, as claimed by Halberda, Mazzocco and Feigenson (2008), was not evident in my study.

Number line activities

The number line activities were included in response to Siegler and Opfer's (2003) work on children's understanding of number and the relative position of numbers on a number line. The children often tried to space the numbers out on the number lines, rather than estimate the position of the numbers. The number lines were helpful from the point of view of giving me a sense of what numbers the children were comfortable working with and whether they had a sense of where the numbers should go. They were not useful, though, for providing insight into whether the children had a linear or a logarithmic sense of the numbers' relative positions as described by Siegler and Booth (2006). This is at least partly due to the fact that the lines were adapted versions of those Sielgler and Opfer (2003) and that a 1-10 number line was added. In addition, the results were not analysed using the procedures outlined in Siegler and Booth (2006). However, most importantly, it was not appropriate to administer the full set of items and so comparison of linear and logarithmic fits to estimates could not be conducted.

5.3.2 The role of fingers in supporting work on number and calculations

All the children had Apert syndrome and had experienced many operations in order to provide them with fingers, I was particularly interested in exploring finger-use during mathematics activities. If the children did use their fingers, I was interested in how they did this, especially for those that had fewer than 10 fingers. The children I worked with did not exhibit the “spontaneous” finger-use as described by Di Luca and Pesenti (2011) in chapter 2. The children I observed that had fewer than 10 fingers all found ways of compensating for this, often using some form of double counting, so that they could keep track of the tens. In other words, their understanding that ten was a significant number was built into their finger-counting strategy. However, the children I worked with all came to finger-counting late in their development, at a point when most of them already knew that “10” was significant. Consequently, when they were developing their personal base-10 counting strategies, they had existing knowledge about the number system which they were able to build on with their finger counting.

As seen in chapter 2, there is much evidence suggesting that finger gnosis is related to visuo-spatial skills. These findings are interesting to reflect on in relation to my study. For Isabelle, it was certainly the case that she achieved a higher score in the Block Recall test (the percentile rank went up from 33 to 89) of the WMTB-C (Pickering and Gathercole, 2001) when her finger gnosis had improved (the Block Recall test is supposed to assess visuo-spatial memory). The case was similar for Hannah (the percentile rank of the Block Recall test went up from 51 to 72). This supports the view that finger gnosis may be related to visuo-spatial skills for some children, but not all.

What about the relationship between finger-use and working memory? This study suggests that finger-use can reduce the load on working memory. A key finding in my study related to the role of fingers in enhancing strategy use when doing numerical calculations. Fingers provided more than a representation of a given numerosity, they became a tool to support both representation *and* calculation.

With Emily and Isabelle, for example, as their finger gnosis and finger-use increased, so too did their range of strategy use. They could both use their fingers to support their calculations, but they could also do calculations without their fingers. It seemed that their models or representations had become more sophisticated and so the strategies that they had at their disposal were varied and adaptable. Even Hannah, who continued to find number work challenging, was developing a wider range of methods and representations that also included her fingers. This change took place alongside her developing finger gnosis.

Joe presents an example of how a lack of finger-use seemed to be holding him back. In my early visits Joe seemed to be keeping up with his peers, but after 2 years, he seemed to be struggling to keep up, especially with "mental maths". Joe was only just beginning to use his fingers when I stopped working with him and it seems likely that this was being encouraged by his parents and not by the school. Joe could apply his understanding of mathematical relationships without difficulty:

C: Right…can you do…forty-seven take away twenty-one?

[Joe writes 47 37 27 26]

C: How did you do that?

J: Because I started from forty-seven…

C: Yep…

J: Then I counted back ten…and ten…and then I counted back one [pointing to the numbers as he explains]

And yet Joe could not add on four. This suggests that his understanding of the principles was not matched by his strategies for representing and working with small numbers, such as adding or subtracting four from a given starting number. It seemed that the load on his working memory was too much when he was asked to add four and he seemed to have had no strategy to fall back on. Luke also was not in the habit of using his fingers and did not intuitively use any other models or concrete objects to help. Again, the observed difficulty seemed to relate to his working memory capacity. Consequently, in my last visit, he could subtract one or two from a given starting number "in his head", but when he was asked to subtract three, he was stuck.

This study provides evidence to support the view discussed in chapter 2, that finger gnosis becomes more closely related to children's arithmetic skills as children get older and the mathematical problems become more demanding and require more flexible application of skills. The scaffold provided by fingers enables children to do calculations which go beyond their working memory capacities, thus enabling them to rely less on learned number facts and allowing to them to continue to explore and exploit the patterns and relationships in mathematics, rather than relying on rule-based approaches to problem-solving. Given that in typically-developing children, finger gnosis develops quickly up to the age of 6 years and then continues to develop more slowly up to the age of 12 years (Strauss, Sherman and Spreen, 2006), it seems more likely that finger gnosis would be related to scores in arithmetic and word problems in children over 6 years as proposed by Costa et al. (2011) and Newman (2016).

For most of the children I worked with, school staff often said that the children did not use their fingers because it was too hard for them to move them independently. Yet when the children were actively encouraged to do so, they quickly increased their finger awareness together with greater mobility. It seems, then, that the two things go hand-in-hand and can be developed together. This supports the findings of Andres, Di Luca and Pesenti (2008), Berteletti and Booth (2015b) and Costa et al. (2011) outlined in chapter 2.

Looking more broadly at the experience of children with Apert syndrome, it is worth reflecting briefly on the potential impact of hand surgery on the children's development of finger gnosis and mobility. All of the children in the study had had several operations on their fingers before the age of 4 years, with the first operation taking place at around 1 year of age. It is likely that the children will experience pain when they begin to use their new fingers and may, therefore, learn to use their fingers in ways that cause as little pain as possible. If these actions become habits, then the children may explore only a limited range of movements. In addition to the surgical management of the hand, it is interesting to consider how the brain responds to the changes experienced.

It has already been noted that following finger separation in an adult with Apert syndrome, there were changes in the representation of the fingers in the brain within one week (Mogilner et al., 1993).

If children who “gain” fingers (like children with Apert syndrome) need to progress through the stages of development suggested by Weiss et al. (2000), it seems likely that greater use of individual fingers will result in better representations in the brain. Although there appear to be some very immediate changes, this study suggests that the process of improving the somatosensory representations of individual fingers is likely to take several weeks, if not months, as suggested by Weiss et al. (2000) and will require individual use. This study also suggests that if children are not encouraged to use their fingers individually, the finer tuned sensorimotor representations in the brain will be slower to develop. In this study, it seemed to take months of finger gnosis training for finger gnosis to improve to a point where the children could identify two finger touches reliably for all their fingers. This suggests that children with Apert syndrome should benefit from engaging with activities which focus specifically on developing individual finger awareness and finger mobility.

I found that the children often had greater difficulty identifying the middle and ring fingers and in fact, often got these mixed up in the finger gnosis assessment. This is perhaps not surprising, given that even adults typically take longer to identify their middle and ring fingers, compared to their index and little fingers (Andres, Di Luca, Pesenti, 2008).

Strengthening the link: reflections on Nathan and Tania

At this point, it is worth briefly discussing two of the children (Nathan and Tania) who were part of the original study, but who have not been included here as case studies. Nathan and Tania were 9 years old when I started working with them and they were both described as being at least 2 years below age-related expectations in mathematics. Nathan (who had five fingers on his right hand and four on his left) had just started to use his fingers when I began the study and he was the only one to have done this independently. Nathan had a similar double finger counting strategy to Emily on his hand with four fingers thus maintaining the five-five split on his hands. My first tests of his finger gnosis showed that he mixed up his last two fingers when two fingers were touched, on both hands. However, by the end of the 2 year period, he made no mistakes on the finger gnosis assessment and achieved expected levels of attainment in mathematics.

Tania, who had five fingers on both hands did not use her fingers at all when I first met her. When I first tested her finger gnosis Tania made mistakes with one finger touches on one hand and with two finger touches on both hands and she could not show me five fingers without counting her fingers one at a time and touching them with the index finger of her other hand. Following my visit in March 2014, Tania's support teacher took a particular interest in finger gnosis. By my next visit in July 2014, Tania's calculation skills had improved; she was using her fingers more often and she no longer needed to count them before doing a calculation. Tania was also able to individuate her fingers much better and her finger gnosis had improved - she could identify all one finger touches and could do two finger touches when fingers 1 and 5 (little finger and thumb) were touched on both hands.

Tania then moved to secondary school, where I visited her in February 2015. Surprisingly, Tania's finger gnosis seemed to have deteriorated. Some of the tests with one finger had to be repeated and on the tests with two finger touches, she was only correct with fingers 1 and 5 on her right hand (her dominant hand). During my interview with her, Tania used her fingers for questions involving addition and for keeping track of counts. However, when she was asked to work out eight take away six the following conversation took place:

C: How much is eight take away six?

[12 second pause during which Tania writes 8-6]

C: You can use your fingers, or I've got some counters if that would help?

T: Counters

Tania then completed the task successfully, but when asked if she could have done it with her fingers, she shook her head.

However, in a later exchange, Tania did choose to use her fingers:

C: OK…now what number comes four numbers before sixty?...That's going backwards…do you want to write the sixty down…would that help?

[T writes 60 and then pauses for 5 seconds]

T: [using her fingers] fifty-nine, fifty-eight, fifty-seven, fifty-six [writes 56 next to the 60]

Tania could clearly use her fingers successfully to keep track of four numbers when counting down. Perhaps the problem with the first question was that she was not confident about how to represent “six”, which would cross the five boundary and require two hands.

Nathan provides further evidence in support of the view that finger-use, finger gnosis and finger mobility can all be developed alongside developing number awareness. Tania also provides evidence to support this, but her case is not so straight forward. Tania seems to have been at a key point in her understanding of the relationships between numbers in addition and subtraction when finger-use ceased to be a strategy that was encouraged. It also shows clearly that confidence with addition is not necessarily followed by confidence and understanding of subtraction.

<u>Fingers and subitising</u>

The relationship between fingers and subitising needs closer inspection in relation to this study. When I first visited Luke, for example, he counted his fingers one at a time when asked how many fingers he had. For me as an observer, it appeared as if he had to concentrate very hard to identify each individual finger. This concentration on what he could <u>see</u> was very important, but he must have also been making links with what he could <u>feel</u>, both in terms of the sensory and the motor experience. The fact that he had to touch each of his fingers as he counted was a very important part of the counting process. By the next visit, after his teacher had been working on finger awareness, Luke was able to put up a given number of fingers, even though it was physically quite a challenge. It seemed that he was beginning to recognise patterns in his fingers that related to particular numbers and yet when subitising for other groups of objects, Luke was not always so reliable. For example, he could tell me that I had put down two or three objects without counting, but when I asked him to give me three counters he seemed to struggle more.

It seems that Luke’s fingers provided a more easily accessible representation to illustrate a given number than he was able to provide with concrete objects – an association had already been built. When I asked Luke to give me three counters, on the other hand, he had to remember how many counters I had asked for while also counting out the counters. This was bound to require more effort than showing a finger pattern for three which had become very familiar and did not require counting.

It is possible that Luke's understanding of the cardinal value of a number was not secure, but in terms of visual subtising, the embodied representation provided a much more accessible model than that provided by external objects, including counters and dice. This suggests that when children are learning to subitise beyond three, they should be actively encouraged to use their fingers to support the relationship between subtising and cardinality.

Working in base-10

Given that we work with a base-10 number system and that most people have 10 fingers, there are likely to be implications for children with less than 10 fingers, as was the case for some of the children I worked with. As has already been discussed in chapter 2, children often make "split-five" errors, but the five-five split (i.e. five on each hand with counting extending from one hand to the other as counting increases from six to ten) is typically a method that is taught in schools in the U.K. There are clearly implications here for children with Apert syndrome who have less than five fingers on each hand. For Emily, the strategy on her hand with four fingers was to count both sides of her third finger to make five. For Hannah, though, the strategy of double counting the last two fingers in a count that went beyond eight was perhaps more likely to result in calculation errors. In terms of thinking of future implications for children with Apert syndrome, Emily's strategy of counting both sides of one finger would seem to be the most effective, in terms of relating best to the base-10 system made from two lots of five fingers and it would provide the sensory and motor inputs required to enable an appropriate association to develop.

This suggests that if children have two hands with less than 10 fingers, they would benefit from being supported to develop a finger counting strategy that provides some form of five-five split between both hands. This, though would only work for counting activities and not for subitising using finger montring (Di Luca and Pesenti, 2008) (i.e. showing particular numerosities with the correct number of fingers).

Some potential implications

My findings suggest that fingers can provide a representation to demonstrate the link between subitising, cardinality and one-to-one correspondence. Firstly, fingers can be used as immediate representations of one, two, three and so on, but as children begin to use their fingers for keeping track of items, they can see that each object (finger) is counted once and once only, in order to obtain the exact numerosity of the set.

The experience of working with, and observing Luke suggests that fingers have the potential to play a role not yet discussed. It has been observed that children take time to become confident that counting is more reliable than estimating (Cowan, 1987) and that counting provides information about numerosity and unequal counts provide information about relative numerosities (Fuson and Hall, 1983). Perhaps this process can be supported by the use of fingers. After all, fingers can provide children with more than a referent, they can provide children with a representation of numerosities up to 10, which are consistent, reliable and embodied. It could be these features of fingers that can help children to move on from subitising to counting, and, therefore, to understand the links between numerosity, number words and number relationships. This could be what is meant by “deep understanding of number concepts” identified by Luca and Pesenti (2011). Luke was at the beginning of this journey; he could successfully use his fingers for finger-montring, but was not yet able to use them to help with calculations and counting. However, it was nevertheless helping him to make sense of the relationship between number words and their associated relationships.

If it is the case that subitising comes before counting and is a skill we share with other animals (Dehaene, 2011) that is initially associated with visually presented objects, we need a way of transitioning from seeing numbers in their subitised “groups”, to understanding cardinality in relation to counting. The question is how to link the capacity to subitise with the capacity to use fingers to keep track. These are clearly very different skills. After all, we can subitise and use fingers to keep track of items without any knowledge of number (at a conscious level) or number words.

If subitising with fingers comes before counting, it would seem that the visual cue is initially stronger than the embodied representation. With practice, it is likely that visual and then physical patterns and relationships emerge. In other words, as Butterworth (1999) suggests, with practice, children become familiar with the finger patterns that represent particular numerosities or number words.

“Finger montring” (Di Luca and Pesenti, 2008) characterises the way people demonstrate particular numerosities using finger patterns. These patterns are often not the same as the ways in which people use their fingers for finger-counting. For example, when showing “three” someone might raise their index finger, middle finger and ring finger. However, if the same person was to count from one to three, they might start with their thumb and count on in order to three. Therefore, finger montring and finger counting must have different developmental routes. In this study, it has been seen that if children do not know the finger patterns that match particular numbers, fingers are not a very helpful resource. This study has also demonstrated that developing finger gnosis enables children to use their fingers for keeping track and then for counting.

Where does finger gnosis fit in? Finger gnosis in children typically develops very quickly up to the age of 6 years and then develops more slowly until it has reached adult levels, at about 12 years of age (Strauss, Sherman and Spreen, 2006). In addition, children with higher IQs have more well-developed finger gnosis (Strauss et al., 2006). The question one could ask is: which comes first, the higher IQ or the finger gnosis?

In this study, as children developed their finger gnosis, they became more able to use their fingers constructively to help with arithmetic problems. Once they could use their fingers for keeping track of counts, rather than for showing particular numerosities, they were better able to use a wider range of problem solving strategies involving counting, together with known facts and other numerical relationships. This study seems to make clear that finger montring and finger counting are very different skills which are only brought together by our cultural use of numbers. This supports Crollen, Mahe, Collignon and Seron (2011), who found that blind children were more likely to use their fingers for finger montring than they were for finger counting. The blind children also seemed to rely much more on their auditory working memory than the sighted children. This study takes this work further by highlighting the gains that can be made by developing finger gnosis and encouraging finger-use.

I would argue, then, that subitising for collections of two or three objects and being able to name those numerosities comes before counting because of our ability to “see” those objects all at once and to name them. This means that children can quickly learn the patterns associated with different numbers of fingers without being able to count and without understanding the numerical relationships. Counting is a different act and, in terms of finger-use, is more likely linked to the use of fingers for keeping track. However, to use fingers successfully for keeping track of number counts, it is important to "know" one's fingers (i.e. to have good finger gnosis). At the route of this, then, is the fact that finger montring and finger counting are not the same, but they can support each other by developing finger gnosis and developing finger-use, initially for keeping track, and later for one-to-one correspondence.

5.3.3 Working memory

During my early visits, I noticed that the children tended to rely very much on their working memory for solving numerical calculations and that when this failed, they often had no strategies to fall back on. The children's standard scores for the Working Memory Test Battery for Children (WMTB-C) (Pickering and Gathercole, 2001) can be seen in table 5-2 (where tests were repeated, I have taken the highest scores in order to illustrate the children's optimal performance).

These tests were not always straightforward for some of the younger children. Luke, Joe, Uma, Wesley and Yasmin did not understand what they had to do for the Listening Recall test, so no scores were obtained. Luke, Uma and Yasmin also did not understand what to do for the Backward Digit Recall test, so again no scores could be recorded. Hannah's poor score in the Backward Digit Recall test may have been due to anxiety linked to a test involving the manipulation of numbers. For the other missing results, there was just not enough time. Again, these tests are designed for typically-developing populations, so it may not be a surprise that there is such a spread of results for individual children. The lack of consistency is particularly pertinent for the central executive scores (CE) where scores are typically expected to be very similar. None of the children in this study had similar scores across the central executive tests.

Name	Digit Recall	Block Recall	Listening Recall	Counting Recall	Backward Digit Recall	WIAT-II MR	WIAT-II NO
	PL	VSSP	CE				
Wesley	*80*	*98*	--	*70*	*84*	*87*	*63*
Yasmin	*122*	--	--	*63*	--	*83*	*78*
Luke	127	98	--	58	--	78	66
Joe	144	86	--	58	134	103	113
Emily	142	108	145	81	125	99	103
Uma	*77*	*110*	--	--	--	*82*	*82*
Isabelle	145	118	133	102	107	93	85
Tania	*81*	*86*	*68*	*71*	*80*	*53*	*58*
Hannah	111	109	127	98	85	57	60
Nathan	*138*	*118*	*90*	--	*107*	*82*	*88*

Table 5-2: Combined WMTB-C and WIAT-II standard scores for all 10 children, youngest to oldest (children in italics are those who were not selected for case study reports)

Although there is a spread, some patterns do emerge. The Counting Recall test, for instance, stands out as being particularly challenging, with only Hannah and Isabelle scoring within the average range. Cowan (1987) found that children aged 3 to 5 years often had difficulty with dot counting activities. He argued that it could be due to the fact that the strategies the children used required a significant amount of effort and were particularly demanding on their working memory. For the children in my study, it was the younger ones who tended to find the Counting Recall task particularly difficult. Consequently, the issue with the Counting Recall test could be that the children were very slow at counting and so forgot earlier counts as they made successive counts, or it could be that each count took a lot of effort and used a significant amount of working memory. As suggested by Cowan. Both of these seem more likely than a problem with processing speed which Hitch, Towse and Hutton (2001) found in their study of 9 to 11 year olds.

The Block Recall test stands out as a test in which all the children were well within the level expected for their age and for some this was an area of great strength. The memory skills required demand accurate recall of a path traced by the tester. Unlike the Counting Recall test, although this uses visuo-spatial skills, it does not combine these with use of the central executive. Thus, even if the children need time to recall the path, they are only required to do one path at a time. A possible explanation for this apparent area of strength could be the development of a compensatory mechanism to recall visual patterns and sequences, when auditory information is less reliable.

When comparing the WMTB-C results with the WIAT results, an obvious pattern did not emerge. Emily and Joe both scored more highly in the Numerical Operations than in the Mathematical Reasoning tests and they both have strong working memory skills in most of the areas. Therefore, it seems that those children who are good at arithmetic type tasks (such as the Numerical Operations test), benefit from strong working memory skills. However, Hannah and Isabelle also had strong working memory skills and yet they both scored more highly in the Mathematical Reasoning test than in the Numerical Operations test.

Unlike Gathercole and Alloway (2008), the children in my study with the poorest scores in the WIAT-II assessments did not have poor scores across all aspects of the WMTB-C (Pickering and Gathercole, 2001). Luke, Yasmin and Uma had mixed scores, while Hannah had scores well within the expected range across all aspects. These findings, then, present a more mixed picture concerning the relationship between working memory and attainment in mathematics than would have been expected. For each child, the results present interesting patterns of development, highlighting the lack of homogeneity (or great diversity) among this group of children with Apert syndrome.

5.3.4 Visuo-spatial skills

Although the children achieved more highly in the Block Recall test than in the Counting Recall test, it is also worth remembering that they all had very slow reaction times when they did the Panamath test (Halberda, Mazzocco and Feigenson, 2008). Even when the children obtained high scores for accuracy, their reaction times were always exceptionally slow for children of their age. This suggests that there could have been an issue with processing speed. This might also provide a reason for the poorer scores in the Counting Recall than in the Listening Recall. However, it was also the case that when counting the dots in the Counting Recall test, the children took a long time and usually used a counting strategy to work out how many dots there were in each image, rather than a strategy involving some form of subitising followed by a calculation (for example, a child might see five dots as a pattern with two dots and three dots which together make a total of five dots). Therefore, the large amount of time taken to identify the number of dots could have been due to problems with subitising or could have been associated with the children's visual impairments and, therefore, the speed at which they can process visual information. This would not be a surprise given that the majority of children with craniosynostosis have some form of visual pathway disruption (Thompson et al., 2006).

5.3.5 Visual impairments and hearing impairments

All the children in the study had some form of visual impairment and/or hearing impairment which was often improved, *but not corrected*, by the wearing of hearing aids and/or glasses.

<u>Visual impairments</u>

As discussed in chapter 2 and in 5.8 above, visual impairments are very varied and can impact in very different ways (Harley, Lawrence, Sanford, Burnett, 2000). However, it seems from the reports of blind adults who have never been taught how to use their fingers to support with arithmetic calculations that a far greater load is placed on working memory.

The parents of the children I worked with often described their children as having problems with their vision and only some of them were able to tell me the exact diagnosis. During the time I was working with the children, Luke and Joe were both being reviewed by the hospital because of specific concerns about their vision and concerns that it was deteriorating. Exactly what form this deterioration took was not made clear. I was also told that Isabelle had reduced peripheral vision. None of the children received any specific advice from an advisory teacher for the visually impaired. It is, nevertheless, likely that their visual impairments did have an impact on their ability to manage written texts whether at a distance on a whiteboard or interactive whiteboard or on paper, close up.

<u>Hearing impairments</u>

All of the children in the study had hearing impairments and they all used some form of hearing aid. Their hearing impairments were probably in the mild to moderate range. While this does not appear to place them in the “at risk” group (as described in chapter 2) on the basis of their hearing impairment alone, when their other additional needs are taken into account, they are at risk for being delayed in their mathematical development, as well as in their development in other areas of the curriculum.

5.3.6 What can we learn from children with complex disabilities?

In the literature review in chapter 2, some of the difficulties experienced by children with cerebral palsy (CP), developmental coordination disorder (DCD) and spina bifida were discussed, because of claims made that their specific disabilities may shine some light on the relationship between finger gnosis and the representation of number in the brain. However, it was argued that it is hard to identify any causal relationships, due to the complex nature of the children’s disabilities.

Just like the children in my study, children with any named disability or syndrome, such as CP, are a very heterogeneous group (Van Rooijen, Verhoeven and Steenbergen, 2015b) and yet there seems to be an association between motor skills, finger gnosis and attainment in arithmetic. I explored finger gnosis and I also observed changes in the manual dexterity of the children I worked with, although I did not do any formal assessments of finger mobility. Taking these examples together presents a strong case for the importance of the whole sensorimotor experience to support skills in early number development. It could also be the case that poor finger gnosis can lead to a conflict between visual and sensory input which could, in turn, make finger counting unreliable and therefore not a method of choice. Alternatively, poor finger gnosis and/or poor motor control could lead to an overdependence on finger counting, if the finger patterns are not well represented in the brain.

The results from my study suggest that focusing specifically on the development of fine motor skills and finger gnosis can support the development and understanding of numerical calculation skills, and that they can be developed alongside each other.

Children with complex disabilities, then, have helped to throw some light on the potential relationship between fingers and numerical calculation skills. While the literature in the area of neuroscience, discussed in chapter 2 seems to favour the view that finger gnosis is more important than finger mobility, it could be argued from this study that both are important.

This could have significant implications for primary and early years' teachers. This study has highlighted the importance of encouraging finger-use in early number work and especially developing finger gnosis as a specific focus. It seems possible that if finger gnosis is not developed alongside finger mobility, there may be a mismatch between what children can see and what they can feel. This might cause confusion and, therefore, an over reliance on the visual representation that fingers can provide, as this may be the more reliable source of information.

For children with Apert syndrome, it seems likely that they should have very early interventions to develop their finger gnosis. As far as I am aware, it is not common practice to provide children with physiotherapy following hand surgery. Perhaps this needs to be reconsidered in the light of the findings here.

5.3.7 Language development

It is worth reflecting briefly on the children's language development. For Luke, his language developed globally over the time that I was visiting him, so his developing use of a wider range of language in our interviews was not such a surprise. For the others, though, increased confidence in mathematics seemed to be accompanied by a growing ability to articulate their mathematical thinking. Even when the children could get answers correct, early on in my research, they often struggled to articulate what they had done and why. Consequently, it was much easier to make sense of the children's thinking when they had the language to express their thoughts. However, it would be wrong to assume that if a child could not articulate their thinking verbally, that they do not understand. This is perhaps a potential limitation of the clinical interview method.

Isabelle had been identified as having an ASD. While this has been mentioned and discussed, it is hard to know to what extent this affected her ability to engage with mathematics. I was provided with very little information about this by the school and it was not possible for me to draw any conclusions about the impact of this on Hannah's development from my own observations.

5.3.8 Home environment

The parents I interviewed spoke of their children's achievements and of their concerns for their futures. The parents all thought very carefully about which schools would be best for their children, given their needs and disabilities. Some parents did extra work with their children at home and/or made a point of playing games with them. This was the case for the three most highly attaining children in my study. This is in line with research described in chapter 2, so it is likely that these activities had a positive effect on the children's skills and confidence in mathematics when they first started school.

Some parents also took a particular interest in the role of fingers as a strategy to support work on number and arithmetic. Although I know that some parents actively encouraged their children to use their fingers and some also did work on finger gnosis, I do not have any specific data on how this was done in the home.

5.3.9 School provision

One thing that struck me when I first visited the children, was the variation in school provision. All the children but one (Hannah) had statements of special educational needs (now education health and care plans), but they did not all receive in-class support. For the children in the study, specific in-class support was either full-time or non-existent. For Hannah and Emily, who did not receive in-class support, there were some intervention groups. In Hannah's primary school, there was a feeling that she would not receive a statement of special educational needs from the local authority, but when she went to middle school, they were successful in obtaining one. Luke was in a special school, so his experience was different from that of the other children.

In terms of the children's experiences, these differed very much depending on the teacher and the support staff. There was enormous variation within schools as well as across schools. The language used to describe the children was also indicative of the attitudes of staff and whether they perceived any difficulties to be due to within-child factors or the classroom environment.

An aspect of the study that should be noted, was the fact that a number of the teachers expressed concerns about being observed by a "mathematics specialist". I tried to reassure teachers that I was observing the children and not their teaching, but it is unclear to what extent this may have influenced their practice and consequently, my observations of the children.

Some of the teachers were very interested to know about my research and took a great interest in the work I was doing. Some of the schools, such as Emily's school even tried to take on board the activities from the paper by Gracia-Bafalluy and Noël

(2008). Luke's first teacher also tried to implement some activities to encourage finger-use.

Support staff

Many of the children I worked with had in-class support or interventions provided by non-teaching staff. The number of hours of in-class support was often stipulated in the children's statements of special educational needs, while other support (usually in the form of some sort of short-term intervention) was organised on more of a needs basis and again varied enormously from school to school.

The role of the teaching assistants (or learning support assistants) who usually provide the support is unclear and they often find themselves making important decisions about how best to support the children they work with (Webster and Blatchford, 2015).

Both Joe and Isabelle had full-time support from a teaching assistant. In both schools, there was concern that that the children were becoming too dependent on their support staff to help them stay on-task and focused. Moreover, the fact that they had an adult sitting with them all the time meant that they did not interact socially in the same way as their peers. In Joe's case, there were a few occasions when a child did not understand what Joe was saying. Rather than ask for clarification from Joe, the children asked his teaching assistant to tell them what Joe had said. The teaching assistants I observed and talked to all wanted to do their best for the children they supported. When they were present in my sessions, they sometimes tried to help the children to do the work and often guided them to use the methods the children had been taught in class. Although I did not ask about training, one of Isabelle's teaching assistants did say that she did not feel she had the knowledge to support Isabelle adequately when Isabelle was finding her work on fractions very difficult. The children who had in-class support also had support at break times and during lunch times. The children tended to spend most of their time with the adults who were supervising, or else they played on their own.

Due to the variation in the nature and level of support provided, and the focus in this study on the learning and understanding of mathematics rather than teaching and support, it is not possible to make specific associations between particular types of support and specific outcomes for children in the area of mathematics.

Social interactions

In all the schools, apart from Isabelle's school, the staff reported that the children were accepted as one of the class. Some of the issues with social interactions were made more difficult when support staff were always present in class, but there were other factors that also impinged on the children's ability to join in with all the activities in the playground. Firstly, the children's visual impairments would most likely have made running around the playground quite challenging. Their hearing impairments would also have made conversations more challenging, especially in a noisy classroom, or in the playground. In the case of Hannah, Joe and Emily, early concerns over their well-being meant that the children were actively discouraged from engaging in the usual playground activities because of fears for their safety. A possible consequence of this was that they did not learn early on how to interact socially with their peers. In addition, Emily and Joe seemed to be quite fearful in the playground and preferred to be with the adults, or in a quieter space. When I observed Hannah in the playground, she appeared to be playing alongside a group of girls, but did not actually interact with them for the whole period of time that they were in the playground.

The speech and language impairments experienced by the children would also have made things difficult for the children to interact, especially when they were younger and were not yet able to articulate all the speech sounds. All the children were late to start talking and some of them continued to be hard to understand when they started school. Without careful handing in school, early experiences of being unable to communicate with their peers could have implications for social inclusion.

The attitudes of staff also seemed to make a difference to the way that the children were perceived. For example, Isabelle was described by the staff as "rude" and "unpleasant" to others. I never saw evidence of this, but more importantly, this seemed to be a way of making it Isabelle's fault that she had no friends in her class. Out of the children who attended mainstream schools, only Yasmin, Uma, Tania and Emily had friends that they played with at school. For Hannah, Isabelle, Joe, Nathan and Wesley, interactions with peers took place very infrequently and often only when the children had been directed to work in groups or pairs. Friendships and informal social interactions at school for these children were very limited.

The issues raised in relation to social inclusion could have implications not only for the children’s’ psychosocial well-being, but also for their education. More specifically, if children with Apert syndrome have fewer opportunities for learning together with their peers, this is likely to impact on their opportunity for dialogue and shared exploration during all classroom activities, including mathematics.

5.4 Conclusions and final reflections

5.4.1 Weaknesses of the study

There are a number of areas that have been addressed which could have been explored in more depth, using different methods or viewed from alternative perspectives.

The relationship between finger gnosis and mathematics was a key part of the study. Given this fact, it may have been useful to have explored the relationship between finger gnosis and mathematics attainment in a range of typical classrooms and across the primary age range, before undertaking the study with children with Apert syndrome.

When I visited the children, it was not always possible to do the same activities. This meant that sometimes the assessments were spread over a period of time and often took place at different points in the study for each child. In addition, I did not always observe the children doing work on number, if the focus of the curriculum was on another area of mathematics.

As explained in chapter 1, there are two types of Apert syndrome. Towards the end of the study, it was suggested that it may be worth investigating any differences between the two groups in my study. When I asked the parents for this information, none of them knew what type of Apert syndrome their child had and most of them were not aware that there were different types. A number of the parents said they would ask the staff at the specialist hospitals that they attended, but only one parent came back to me with a named variant.

The study focused on the mathematical development of children with Apert syndrome. While I did obtain achievement levels in other curriculum subjects, especially reading and writing, I did not carry out any assessments in these areas. Some more data on these aspects would have added to the richness of the data.

Although I spoke to parent about my ideas about finger gnosis and I know that some parents did actively encourage aspects of finger gnosis development and finger use, I did not collect any data on how this was done. It would be useful to explore the role of parents in supporting the development of finger gnosis and finger mobility in children with Apert syndrome in any future work in this area.

Finally, as the study progressed it became clear that language development was a key feature that impacted the children's abilities to communicate their thinking. It may have been useful to have assessed the children's receptive and expressive language skills as part of the study, in order to gain greater insight into their language abilities at particular points in time.

5.4.2 What next?

There are a number of areas that could be explored in response to this study.

It would be very interesting to make contact with one of the national centres that specialise in care for children with Apert syndrome and carry out a study of the effects of physiotherapy, following hand surgery, on the children's finger mobility and on their finger gnosis. This could then be followed up to see if this made it easier for children to participate in the early number activities in their early years settings and at school.

As mentioned earlier, it may be useful to explore whether there are any key differences in the mathematical development of children with the two forms of Apert syndrome.

A study of children who fall into the group of "continuing to rely on finger counting" would be of interest, in order to establish whether or not their physical fine motor skills are accompanied by equally well-developed finger gnosis.

It has been suggested that using fingers for subitising and using fingers to keep track of counts are different, although related, processes. This should be explored further, in order to better understand this relationship.

Finally, a longitudinal study which explored the relationship between finger gnosis, fine motor skills and attainment in number and calculation may benefit and extend our understanding of how these capacities interact and support each other. This could be followed up by an assessment of appropriate interventions for those children with identified areas of weakness.

5.4.3 Final reflections

My initial intention was to explore the development of early number skills in children with Apert syndrome. I had some information suggesting that children with Apert syndrome do less well in mathematics than in other curriculum subjects at school. I knew that children with Apert syndrome were a very heterogeneous group, but that they would all have been born with their fingers fused and would have had several operations before they started school to separate their fingers. It was with this knowledge that I wondered whether there would be any differences in the way children with Apert syndrome engaged with work on early number at school.

The longitudinal nature of the study meant that I was able to build positive relationships with the children. Taken together, these enabled me to adjust my approaches to each individual child, and thereby observe very subtle changes in their strategy use, as their conceptual understanding developed.

Out of the 10 children that I followed over a 2 year period, only one had begun to use his fingers spontaneously, at the age of 9 years. Of the other nine children, none of them used their fingers before I began my study. My observations suggest that those children who were higher attaining in mathematics tended to have strong working memory skills, but it was not the case that all the lower attainers had poor working memory skills. I also observed that most of the children relied on their working memory for numerical calculations. This was, though, not without its problems, as once the children reached a point where their working memory was overloaded, they had no alternative representations, strategies or resources, to fall back on. Consequently, it was possible to identify a “tipping point” beyond which numerical calculation became impossible.

Most of the children in the study did begin to use their fingers at some point and this resulted in a number of interesting observations relating to finger gnosis and finger mobility, as well as to work on number and arithmetic. It appears that finger gnosis is delayed in children with Apert syndrome, as well as finger mobility. However, both finger gnosis and finger mobility improve with practice, though it takes months, rather than weeks. I noticed that as finger gnosis and finger-use improved, so too did the range of skills that children had at their disposal for numerical calculation. As the children became more skilled, their fingers became one of a number of strategies that they would use and they could move between these with great flexibility. Children who had fewer than five fingers on each hand found ways to “make” 10, with the most effective being to count both sides of one finger, to create a model for “five” on each hand.

This study suggests that the relationship between fingers, subitising, one-to-one correspondence and counting could be explored in more depth. In addition, this study suggests that fingers first of all provide a visual representation of numerosities. As the patterns of particular numbers (or numerosities) become embodied, it becomes easier to make links between the patterns of different finger combinations and their associated numerical relationships. This embodiment seems to require both finger gnosis and finger mobility, in order for finger individuation and the associated one-to-one correspondence to develop. Some of the children also provide evidence to suggest that using fingers for keeping track of items develops at a more “intuitive” level than using fingers in a more formalised way to help with calculations.

The findings presented, especially from the clinical interviews, suggest that children can have better developed understanding of the principles of mathematics than assessments of their arithmetic skills would suggest. The study has also shown that our expectations of what children with Apert syndrome can and cannot do with their fingers need to be questioned.

Importantly, these findings highlight the enormous variation in the development of mathematical skills in children with Apert syndrome. Therefore, I need to conclude that there is no such thing as a typical developmental path in mathematics for a child with Apert syndrome. In terms of adding to the literature on Apert syndrome, I am drawn back to Hayward et al. (2016) who highlight the importance of the so-called “third variables” such as chronic airway obstruction, hearing impairments, visual impairments, low expectations and teasing which are far more likely to impact on children’s learning journeys, than any predisposition they may be born with.

Finally, it was my intention to present the findings in a way that would be respectful to the children and would present them with dignity. With “special” populations, such as children with Apert syndrome, it is easy to lose sight of the fact that behind every piece of research data are real people, with real experiences and a range of strategies for coping with the challenges that life throws at them.

The interviews with parents provided detail about the children from birth. They also told very personal stories and often raised issues that were seen as on-going challenges. Many of the parents raised the fact that one reason for wanting to participate in the research was the fact that they had been told very little about potential issues in relation to the education of their children and they were aware that there was not much literature on this issue. It seemed that having the opportunity to discuss their concerns was an important part of the whole process.

The interviews with staff at school provided information that was focused more on how to ensure that the children were accessing the curriculum and how to best support them. The schools identified different challenges. However, what became increasingly apparent as I began my study was the fact that provision across the country was very inconsistent and depended very much on how resources were made available within each school.

One of the challenges when writing up my findings was deciding how to summarise my research with the children. I hope that my descriptions and analyses have done them justice.

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Appendix A: Expression of interest information and letter to parents/carers

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The mathematical development of children with Apert syndrome: A research project

Information for parents/carers and your children
Please will you help with my research?

My name is Caroline Hilton. I have been a teacher since 1985 and have taught mathematics, and other subjects, in a wide range of schools including primary schools, secondary schools and special schools. Since 2009, I have been a tutor on the Primary PGCE course at the Institute of Education, in London. I have a 19 year old daughter with Apert syndrome. Throughout my daughter's schooling, there were many hurdles, but the one that we all found particularly challenging was finding a way to help her with maths, both at school and at home. Talking to other parents of children with Apert syndrome, and staff at Great Ormond Street Hospital, we found out that many children with Apert syndrome seem to have difficulty with maths.

As confidence with numbers is so important in everyday life, it is important to find out more about how children with Apert syndrome learn about using numbers and if there are some strategies or interventions that are particularly helpful. As it is during the primary school years that the basics of arithmetic and mathematics are usually taught in school, it will be particularly useful to focus on children in this phase of their education.

In addition, over the last few years, there has been a growing interest in the role of fingers in learning maths. If the use of fingers plays an important role in developing understanding about maths, then children with Apert syndrome are more likely to have difficulty. This makes it especially important to try to find out whether there are any strategies or interventions that are particularly helpful for children with Apert syndrome, to make sure that they get the best start possible in learning maths.

This research project will attempt to find out how children with Apert syndrome, who are currently in primary education (mainstream school, special school or unit), are getting on with learning maths. This will involve collecting information from the children, their parents/carers and the children's schools. I would, however, also be very interested in interviewing older children and young adults and their parents/carers to find out how they got on when they were younger and how they are coping with the mathematical demands of life and/or school, college or work.

What will happen during the research?

I will be collecting information about the children over a period of two years. I would like to interview you and your child/children (if appropriate) about your child/children's experiences of school and, in particular about their feelings about learning and doing maths. I would like to do this at the beginning of each year of the study and once at the end. I would also like to do some assessments with your child/children, in a place to be agreed (this could for example be in your home, or at your child/children's school). These assessments will be to investigate:

- Working memory (which has often been thought to be important for the learning of maths)
- Mathematical knowledge, understanding and skills, with a focus the methods and strategies children use for doing things like counting objects and doing sums.
- General cognitive development using a standardized test
- Each child's strengths and difficulties across a range of areas, both academic and non-academic, at home and at school.

These assessments should take between 20 minutes and 40 minutes each and will each be done once, at the beginning of the study. I can be very flexible with the organisation of the assessments and the well-being and comfort of the children will always take precedence in any decision about whether or not to continue with an assessment. I would also like to observe the children in school during a maths lesson, once a term for two years. I would like to speak to their class teachers and obtain National Curriculum levels of attainment in maths and other National Curriculum subjects.

What will happen if you agree to take part?
If you agree to take part, I will first of all contact you to arrange to meet, at a time and place that are convenient for you and your child/children, so you can have an opportunity to find out more about me and what participation in this project will involve and agree which aspects of the project you are happy to participate in. I have listed below, the things I would like to do with your agreement.

If you agree, I will interview you and your child/children and carry out some assessments (as outlined above).

If you agree, I will audio record the interviews that form part of the study and will type them up later. The transcripts will be sent to you and your child/children to be checked before they are included in any report.

If you agree, I will contact your child/children's school and request attainment data.

If you agree, I will ask you, your child (if they are between 11 years and 17 years of age) and your child's teacher to complete a questionnaire on your child's strengths and difficulties.

If you agree, I will contact your child/children's school and ask for permission to observe your child/children in maths lessons. I would like to observe each child once a term over a period of two years. If permission is granted, I will also interview the staff working with your child/children and request updates on each visit to school.

The information obtained from you, your child/children and the school will remain confidential and in any report or publication anonymity is guaranteed. Any data will be stored securely and any data held on a computer will be password protected.

Participation in the project is completely voluntary and you and your child/children will be free to withdraw at any time.

The project has received ethical approval from the Institute of Education, where I am based. As my job requires me to go in and out of primary schools all over London, I have an up to date enhanced Criminal Records Bureau check, which is a requirement for anyone working with children and young people.

If at any time you have any concerns, please contact me by email at c.hilton@ioe.ac.uk, by phone on 0207 612 6237, or by post at the Institute of Education, Dept of Early Years and Primary Education, 20 Bedford Way, London WC1H 0AL.

You can use the same contact details if you or your child/children decide to withdraw at any point.

Final points
Each child will be investigated individually. This may help to see if any similarities emerge. Any written report will not name the child, the parent/carer or the school. The study is being done as part of my PhD. The results will, therefore, be published and will be in the public domain.

The support of Headlines will be acknowledged in my final report, which will also be made available to Headlines and to all the participating members.

If you would like to take part, please complete the attached form and return it in the self-addressed envelope. If you would like to find out more, please feel free to contact me.

I hope to hear from you soon.

Caroline Hilton

Leading education and social research
Institute of Education
University of London

Caroline Hilton
Department of Early Years and Primary Education

20 Bedford Way
London WC1H 0AL
Tel: 020 7612 6000

The mathematical development of children with Apert syndrome: A research project

Parent/carers expression of interest form

I have read the information sheet about the research and I would like to find out more.

☐ (Please tick)

Your name(s) and your relationship(s) to your child:

____

____

Your child's name:
____

Your child's age: _____ Your child's year group in school:_____

Your child's school:____

You can contact me at (please leave your preferred contact details):

Your signature(s):

____

Date:_______________________________

Appendix B: Parent/carer and child consent form

Caroline Hilton
Department of Early Years and Primary Education

Leading education and social research
Institute of Education
University of London
20 Bedford Way
London WC1H 0AL
Tel: 020 7612 6000

The mathematical development of children with Apert syndrome: a research project

Parent/carer consent form (page 1 of 2)

I have read the information sheet about the research. Yes/No (delete as appropriate)

I have discussed the project with my child. Yes/No (delete as appropriate)

I am happy for my child to be asked whether he/she would like to take part in the project. Yes/No (delete as appropriate)

I am happy for my child to take part in the project and I understand that she/he can withdraw at any time. Yes/No (delete as appropriate)

I am happy for you to interview me and audio record the interview. Yes/No (delete as appropriate)

I am happy for you to interview my child and audio record the interview. Yes/No (delete as appropriate)

I am happy for you to undertake assessments of my child and audio record these sessions. Yes/No (delete as appropriate)

I am happy to complete a questionnaire on my child's strengths and difficulties. Yes/No (delete as appropriate)

I am happy for you to contact my child's school to ask for information on her/his progress and I am happy for you to observe my child during maths lessons at school. Yes/No (delete as appropriate)

I understand that I will get a copy of this consent form. Yes/No (delete as appropriate)

I understand that I can contact Caroline Hilton (c.hilton@ioe.ac.uk, 0207 612 6237) to discuss this study at any time. Yes/No (delete as appropriate)

The mathematical development of children with Apert syndrome: a research project

Consent form (page 2 of 2)

Name of child: ______________________________ ____

(First name) **(Family name)**

Male ☐ **Female** ☐

Date of birth: ______________________________ **Year group:** ______

School (please include address):

____

____

____

Name of Head Teacher: ____

Name(s) of parent/carer(s):

____

____

Contact email: ____

Contact phone number: ____

Contact address:

____

____

____

Signature(s):

____

Today's date: ______________________________

Caroline Hilton
Department of Early Years and Primary Education

Leading education and social research
Institute of Education
University of London

20 Bedford Way
London WC1H 0AL
Tel: 020 7612 6000

The mathematical development of children with Apert syndrome: a research project

Participating child's consent form

My name is:____

I am ____ years old.

I have discussed the information about the project with______________________

I am happy to take part in the project. ☐ *(Please tick)*

I am happy to do some work with Caroline Hilton and I know that I can stop at any time. ☐ *(Please tick)*

I am happy for Caroline Hilton to observe me in school. ☐ *(Please tick)*

Signature:__________________________________

Date:____________________

Appendix C: School information and consent form

Leading education and social research
Institute of Education
University of London

20 Bedford Way

London WC1H 0AL

Tel: 020 7612 6000

The mathematical development of children with Apert syndrome: a research project

Information for schools

My name is Caroline Hilton. I have been a teacher since 1985 and have taught mathematics, and other subjects, in a wide range of schools including primary schools, secondary schools and special schools. Since 2009, I have been a tutor on the Primary PGCE course at the Institute of Education, in London.

I am currently undertaking a study of the mathematical development of children with Apert syndrome. Because Apert syndrome is not very common, not much is known about how children with Apert syndrome learn maths.

XXX the parent/carer(s) of **BBB**, who is currently attending your school in year____, has agreed to participate in my research and has agreed for me to contact you. As part of my research, I would very much like to be able to observe **BBB** in class, during maths. I would like to conduct observations once a term for a total of two years. My focus will be on observing **BBB** engaging in maths activities and not on the teaching. If possible, I would also like to talk to **BBB**'s class teacher and any support staff that work with **him/her** to get more information about how **BBB** is getting on in all subject areas and how **BBB** is engaging in school life more generally. I would also like to ask the class teacher to complete a questionnaire on **BBB**'s strengths and difficulties.

The project has received ethical approval from the Institute of Education, where I am based. As my job requires me to go in and out of primary schools all over London, I have an up to date enhanced Criminal Records Bureau check.

The information obtained from the school will remain confidential and in any report or publication, neither the school nor the location will be named. Any data will be stored securely and any data held on a computer will be password protected.

If you would like to discuss this further, please feel free to contact me (c.hilton@ioe.ac.uk, or 0207 612 6237).

If you and **BBB**'s class teacher are happy for me to come and observe **BBB** in maths lessons and share information about how she/he is getting on, please complete the consent form overleaf and return it in the stamped addressed envelope provided. You can of course withdraw your consent at any time.

Thank you.

Caroline Hilton

Leading education and social research
Institute of Education University of London
20 Bedford Way
London WC1H 0AL
Tel: 020 7612 6000

Caroline Hilton
Department of Early Years and Primary Education

The mathematical development of children with Apert syndrome: a research project

School consent form

I have read the information sheet about the research.

Yes/No (delete as appropriate)

I am happy for Caroline Hilton to observe **BBB** in maths lessons.

Yes/No (delete as appropriate)

I understand that I can withdraw my consent at any time.

Yes/No (delete as appropriate)

I am happy to complete a questionnaire on **BBB**'s strengths and difficulties.

Yes/No (delete as appropriate)

I understand that I can contact Caroline Hilton (c.hilton@ioe.ac.uk, 0207 612 6237) to discuss this study at any time.

Yes/No (delete as appropriate)

I understand that I will get a copy of this consent form.

Yes/No (delete as appropriate)

Name: ______________________________

Signature: ______________________________

Head Teacher

School address: ______________________________

Date: ______________________________

Name: ______________________________

Signature: ______________________________

BBB's class teacher

Date: ______________________________

Appendix D: Parent/carer questions used to support the interviews

Parent/carer questions (to be used to support interview)

These questions should help to gather some background information about your child.

Your name(s):____

Relationship to your child:__________________________________ Date:____________

1. Child's name:______________________________________ Sex:______

   Date of birth:______________________________________ Age:_____________

   Country of birth:____

   Languages(s) spoken at home:___________________________________

   School:____

   School year group:________

   Teacher:____

   Head Teacher:____

Left-handed/Right-handed/No hand preference

2. Who lives at home with you?

Name	Age	Relationship to your child

3. Developmental milestones
   What particular things stand out as 'firsts' for your child (e.g. first word spoken, first time she/he walked alone)? Why have you chosen these?

4. Your child now – including recent tests, results and overall functioning.

What sort of things is your child interested in?	
Does your child like to read (with you/alone/both)?	
Does your child get involved with activities involving number at home? (eg board games, card games, telling the time, weighing things for cooking, etc)	
Does your child prefer to play on his/her own or with others? (children/adults/family/friends/etc)	
What do you think are your child's strengths?	
Hearing **-** recent tests, results and overall functioning.	
Does your child wear hearing aids (if yes, what sort, and what is your child's hearing like with them?)	
Vision recent tests, results and overall functioning.	
Does your child wear glasses? (if yes, what is your child's vision like with the glasses?)	
Speech, language and communication development -recent tests, results and overall functioning.	
Does your child regularly see a speech, language and communication therapist? (If yes, how often and where?)	

School achievements – general comments	
How is your child doing in English?	
How is your child doing in Maths?	
How is your child doing in Science?	
How is your child doing in other areas of the curriculum?	

Is there anything else you would like to share?

Appendix E: Number Knowledge Test: Test Items

Preliminary

Let's see if you can count from 1 to 10. Go ahead.

Level 0 (4-year old level): Go to Level 1 if 3 or more correct

1. Can you count these chips and tell me how many there are? (Place 3 counting chips in front of child in a row)

2a. (Show stacks of chips, 5 vs. 2, same color). Which pile has more?

2b. (Show stacks of chips, 3 vs. 7, same color). Which pile has more?

3a. This time I'm going to ask you which pile has less.

(Show stacks of chips, 2 vs. 6, same color). Which pile has less?

3b. (Show stacks of chips, 8 vs. 3, same color). Which pile has less?

4. I'm going to show you some counting chips (Show a line of 3 red and 4 yellow chips in a row, as follows: R Y R Y R Y Y). Count just the yellow chips and tell me how many there are.

5. Pick up all chips from the previous question. Then say: Here are some more counting chips (Show mixed array [not in a row] of 7 yellow and 8 red chips). Count just the red chips and tell me how many there are.

Level 1 (6-year-old level): Go to Level 2 if 5 or more correct

1. If you had 4 chocolates and someone gave you 3 more, how many chocolates would you have altogether?

2. What number comes right after 7?

3. What number comes two numbers after 7?

4a. Which is bigger: 5 or 4?

4b. Which is bigger: 7 or 9?

5a. This time, I'm going to ask you about smaller numbers.

Which is smaller: 8 or 6?

5b. Which is smaller: 5 or 7? 2

6a. Which number is closer to 5: 6 or 2? (Show visual array after asking the question)

6b. Which number is closer to 7: 4 or 9? (Show visual array after asking the question)

7. How much is 2+4? (OK to use fingers for counting)

8. How much is 8 take away 6? (OK to use fingers for counting)

9a. (Show visual array - 8 5 2 6 - and ask child to point to and name each numeral). When you are counting, which of these numbers do you say first?

9b. When you are counting, which of these numbers do you say last?

Level 2 (8-year-old level): Go to Level 3 if 5 or more correct

1. What number comes 5 numbers after 49?

2. What number comes 4 numbers before 60?

3a. Which is bigger: 69 or 71?

3b. Which is bigger: 32 or 28?

4a. This time I'm going to ask you about smaller numbers.

Which is smaller: 27 or 32?

4b. Which is smaller: 51 or 39?

5a. Which number is closer to 21: 25 or 18? (Show visual array after asking the question)

5b. Which number is closer to 28: 31 or 24? (Show visual array after asking the question)

6. How many numbers are there in between 2 and 6? (Accept either 3 or 4)

7. How many numbers are there in between 7 and 9? (Accept either 1 or 2)

8. (Show card 12 54) How much is 12+54?

9. (Show card 47 21) How much is 47 take away 21? 3

Level 3 (10-year-old level):

1. What number comes 10 numbers after 99?

2. What number comes 9 numbers after 999?

3a. Which difference is bigger: the difference between 9 and 6 or the difference between 8 and 3?

3b. Which difference is bigger: the difference between 6 and 2 or the difference between 8 and 5?

4a. Which difference is smaller: the difference between 99 and 92 or the difference between 25 and 11?

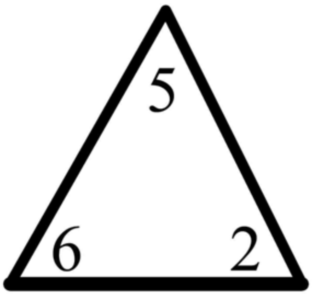
4b. Which difference is smaller: the difference between 48 and 36 or the difference between 84 and 73?

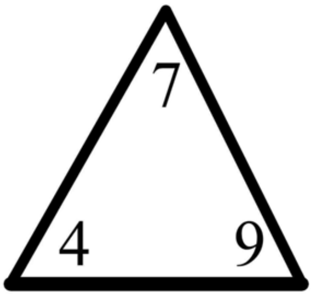
5. (Show card, “13, 39") How much is 13 + 39?

6. (Show card, “36, 18") How much is 36 - 18?

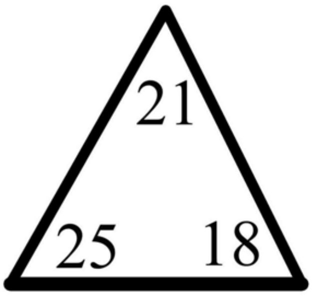
7. How much is 301 take away 7?

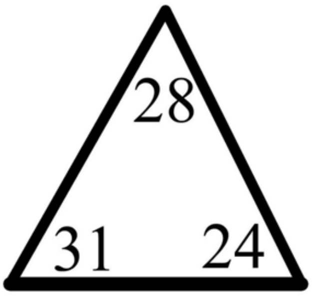
Visual arrays to support questions





8 5 2 6





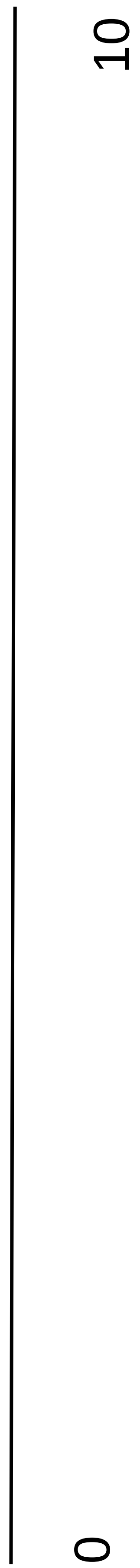
12 54

47 21

13 39

36 18

Appendix F: Number lines



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