PROBABILITY AND UNCERTAINTY IN KEYNES'S GENERAL THEORY

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1. The Post-Keynesians and the Problem

In the last two decades, a great deal of attention has been devoted to the question of probability and uncertainty in Keynes's *General Theory* by a group often referred to as the 'Post-Keynesians'. As I will be making a good deal of use of the researches of this group in the present paper, I will begin by saying a little in general terms about the group and its ideas.

After the second world war, Keynesian economics became dominant in the British academic community, and British governments to a large extent followed the advice of Keynesian economists. Keynesian economics had a similarly important (even if not always quite so dominant) rôle in other advanced capitalist countries in the same period. During the 1970's, however, Keynesian economics came under increasing criticism from the monetarist school, and Keynesian economists began to lose both academic and political influence. In Britain the election of Margaret Thatcher in 1979 signalled the end of the government's use of Keynesian policies, and the adoption instead of free market policies based on monetarist economic theory. Many academic economists went over to the new (or rather revived) free market ideas. However, some remained convinced of the value of Keynesian ideas in economics.

The remaining followers of Keynes were at this point faced with the unhappy situation that the academic and political influence of their ideas was declining, and that these ideas were being increasingly criticized as inadequate. The Post-Keynesians reacted to this crisis in a way which has parallels in other intellectual schools at a time of difficulty. They argued that the Keynesian economics which had prevailed in the period 1945-75, and which was now increasingly being rejected, was not in fact the economics which Keynes himself had proposed in his *General Theory*, but rather a simplified and unsatisfactory version of what Keynes had

said. They suggested that Keynes's approach could be revived by a return to Keynes's original ideas.

The object of the Post-Keynesian attack was the standard text-book account of Keynesian economics based on Hicks's IS-LM diagram. Skidelsky explains the origin of this kind of Keynesianism with characteristic clarity and historical erudition. He writes (1992, 538):

'The IS-LM diagram, first drawn by John Hicks in 1936, *is* the *General Theory* as it has been taught to economics students ever since: 384 pages of argument whittled down to four equations and two curves. Hicks, Harrod, Meade and Hansen in America, the leading constructors of 'IS-LM' Keynesianism, had a clear motive: to reconcile Keynesians and non-Keynesians, so that the ground for policy could be quickly cleared. These early theoretical models incorporated features which were not at all evident in the *magnum opus*, but which conformed more closely to orthodox theory. The constructors of these models also thought they were improving the original building.'

A little later in a section significantly entitled: 'Vision into Algebra', Skidelsky writes (1992, 611):

'The mathematisation of the *General Theory* started immediately it was published but it was left to Hicks to map the mathematics on to a two-curve diagram which became the accepted form of the *General Theory*. His famous paper 'Mr. Keynes and the Classics: A Suggested Reinterpretation' was published in *Econometrica* in April 1937. What Hicks does is to turn Keynes's logical chain of reasoning designed to expose the causes which drive the economy towards a low employment trap into a generalised system of simultaneous equations, devoid of causal significance, with the behavioural characteristics of the propensities to be filled in according to assumption. The 'generalised' system has room for Keynes's 'special theory', but also, for example, for the Treasury view, which Keynes wrote the *General Theory* to refute.'

IS-LM Keynesianism does not include any reference to probability and uncertainty. But the Post-Keynesians argue that probability and uncertainty were central to the real Keynes who wrote a *Treatise on Probability* in 1921, and in his *General Theory* of 1936 made implicit use of probability in his theory of long-term expectation. The Post-Keynesians have accordingly carried out a great deal of valuable historical research on the evolution of Keynes's ideas on probability, and his use of probability in the *General Theory*.

Post-Keynesianism began in the 1980's as a reaction to the decline in academic and political influence of post-war IS-LM Keynesianism. Perhaps the first significant Post-

Keynesian book was the first volume of Skidelsky's masterly life of Keynes which appeared in 1983. This covers Keynes's life up to 1920, and discusses Keynes's early philosophical work on probability and induction - a topic which had been ignored for many years. Other Post-Keynesian books to appear in the 1980's include Carabelli (1988), Fitzgibbons (1988), and O'Donnell (1989). In 1985 a collection of papers edited by Lawson and Pesaran appeared. This contains articles by Victoria Chick, Alexander and Sheila Dow, Tony Lawson, and John Pheby. Somewhat younger Post-Keynesians include Bateman (1987, 1988, and 1996), Davis (1994), and Runde (1994, 1996). In what follows I will make use of this Post-Keynesian work on the reconstruction of Keynes's ideas.[1]

Let us now turn to Keynes *General Theory* of 1936, which I will take in conjunction with his 1937 article: 'The General Theory of Employment', written to summarise and defend his book. In these works Keynes argues that the *amount of investment* is the key factor in determining the performance of the economy as a whole. As we shall see he regards it as the '*causa causans*' of 'the level of output and employment as a whole' (1937, 121). Let us start therefore with Keynes's analysis of investment. We shall consider two of the concepts which Keynes introduces in this connection, namely: *prospective yield* and *demand price of the investment*. Keynes defines these as follows (1936, 135 & 137):

'When a man buys an investment or capital-asset, he purchases the right to the series of prospective returns, which he expects to obtain from selling its output, after deducting the running expenses of obtaining that output, during the life of the asset. This series of annuities Q_1, Q_2, ... Q_n it is convenient to call the *prospective yield* of the investment. ...

If Q_r is the prospective yield from an asset at time r, and d_r is the present value of £1 deferred r years *at the current rate of interest*, $\Sigma Q_r d_r$ is the demand price of the investment; and investment will be carried to the point where $\Sigma Q_r d_r$ becomes equal to the supply price of the investment as defined above. If, on the other hand, $\Sigma Q_r d_r$ falls short of the supply price, there will be no current investment in the asset in question.'

So any decision to invest depends crucially on the quantity $\Sigma Q_r d_r$ (the demand price of the investment) which is the sum of the prospective annual yields discounted at the current rate of interest. But now the crucial problem arises, because the prospective yield Q_1, Q_2, ... Q_n of an investment is not known, and and consequently $\Sigma Q_r d_r$ cannot be calculated. As Keynes puts it (1936, 149-50):

'The outstanding fact is the extreme precariousness of the basis of knowledge on which our estimates of prospective yield have to be made. Our knowledge of the factors which will govern the yield of an investment some years hence is usually very slight and often negligible. If we speak frankly, we have to admit that our basis of knowledge for estimating the yield ten

years hence of a railway, a copper mine, a textile factory, the goodwill of a patent medicine, an Atlantic liner, a building in the City of London amounts to little and sometimes to nothing; or even five years hence.'

Since the actual future yields are unknown, they must be replaced in calculating $\sum Q_r d_r$ to make an investment decision by expected yields. A decision to invest consequently depends on what Keynes calls *the state of long-term expectation* (the title of the famous chapter 12 of the *General Theory*). Now the notions of expectation and of probability are interdefinable. If we take expectation as the starting point, we can define probabilities in terms of expectations, and vice versa.[2] If then Keynes is using the notion of expectation in its standard sense, he is implicitly operating with a concept of probability, and it is natural to ask what should be the interpretation of the probabilities involved. This then brings us to the fundamental question with which this paper is concerned, namely: 'what is the most appropriate interpretation of probability in Keynes's *General Theory*?' The Post-Keynesians have devoted a great deal of attention to this problem, but, before we can consider their arguments in detail, it will be necessary to give a brief explanation of the various interpretations of probability.[3]

2. The Logical, Subjective, and Intersubjective Interpretations of Probability

Different versions of the logical interpretation of probability have been developed by different authors, but here, naturally, we will be concerned with Keynes's version as expounded in his 1921 *Treatise on Probability*. In the case of deductive logic a conclusion is entailed by the premises, and is certain given those premises. Thus, if our premises are that all ravens are black, and George is a raven, it follows with certainty that George is black. But now let us consider an inductive, rather than deductive, case. Suppose our premises are the evidence (e say) that several thousand ravens have been observed, and that they were all black. Suppose further that we are considering the hypothesis (h say) that all ravens are black, or the prediction (d say) that the next observed raven will be black. Hume argued, and this is in agreement with modern logic, that neither h nor d follow logically from e. Yet even though e does not entail either h or d, could we not say that e *partially entails* h and d, since e surely gives some support for these conclusions? This line of thought suggests that there might be a logical theory of partial entailment which generalises the ordinary theory of full entailment which is found in deductive logic. This is the starting point of Keynes's approach to probability. He writes (1921, 52):

'Inasmuch as it is always assumed that we can sometimes judge directly that a conclusion *follows from* a premiss, it is no great extension of this assumption to suppose that we can sometimes recognise that a conclusion *partially follows from*, or stands in a relation of probability to a premiss.'

So a probability is the degree of a partial entailment. Keynes further makes the assumption that if e partially entails h to degree p, then, given e, it is rational to believe h to degree p. For Keynes probability is degree of *rational* belief *not* simply degree of belief. As he says (1921, 4):

‘ ... in the sense important to logic, probability is not subjective. It is not, that is to say, subject to human caprice. A proposition is not probable because we think it so. When once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively, and is independent of our opinion. The Theory of Probability is logical, therefore, because it is concerned with the degree of belief which it is *rational* to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational.’

Here Keynes speaks of probabilities as being fixed objectively, but he is not using objective to refer to things in the material world. He means objective in the Platonic sense, referring to something in a supposed Platonic world of abstract ideas.

The next question which might be asked regarding Keynes’s approach is the following: ‘how do we obtain knowledge about this logical relation of probability?’ Keynes’s answer is that we get to know at least some probability relations by direct acquaintance or immediate logical intuition. As Keynes says (1921, 13): ‘We pass from a knowledge of the proposition *a* to a knowledge about the proposition *b* by perceiving a logical relation between them. With this logical relation we have direct acquaintance.’

A problem which arises on this account is how we can ever assign numerical values to probabilities. Keynes indeed thinks that this is possible only in some cases, and writes on this point (1921, 41): ‘In order that numerical measurement may be possible, we must be given a number of *equally* probable alternatives.’ So in order to get numerical probabilities we have to be able to judge that a number of cases are equally probable and to enable us to make this judgement we need an a priori principle. This a priori principle is called by Keynes the *Principle of Indifference*, and he gives the following statement of it (1921, 42):

‘The Principle of Indifference asserts that if there is no *known* reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an equal probability.’

Unfortunately the Principle of Indifference leads to a number of paradoxes. Keynes gives a full account of these in chapter IV of his Treatise, and makes an attempt to solve them. Yet is has to be said that his solution is far from satisfactory. This concludes my brief account of

Keynes's version of the logical theory of probability. Let us now turn to the subjective interpretation.

The subjective theory of probability was discovered independently and at about the same time by Frank Ramsey in England, and Bruno de Finetti in Italy. Their two versions of the theory are broadly similar, though there are important differences which are well described in Galavotti (1991). In what follows I will concentrate mainly on Ramsey since his work is directly connected with that of Keynes.

Ramsey was a younger contemporary of Keynes at Cambridge. His fundamental paper introducing the subjective approach to probability was read to the Moral Sciences Club at Cambridge, and Ramsey begins the paper by criticizing Keynes's views on probability. According to Keynes there are logical relations of probability between pairs of propositions, and these can be in some sense perceived. Ramsey criticizes this as follows (1926, 161):

'But let us now return to a more fundamental criticism of Mr. Keynes' views, which is the obvious one that there really do not seem to be any such things as the probability relations he describes. He supposes that, at any rate in certain cases, they can be perceived; but speaking for myself I feel confident that this is not true. I do not perceive them, and if I am to be persuaded that they exist it must be by argument; moreover I shrewdly suspect that others do not perceive them either, because they are able to come to so very little agreement as to which of them relates any two given propositions.'

This is an interesting case of an argument which gains in strength from the nature of the person who proposes it. Had a less distinguished logician than Ramsey objected that he was unable to perceive any logical relations of probability, Keynes might have replied that this was merely a sign of logical incompetence, or logical blindness. Indeed Keynes does say (1921, 18): 'Some men - indeed it is obviously the case - may have a greater power of logical intuition than others.' Ramsey, however, was such a brilliant mathematical logician that Keynes could not have claimed with plausibility that Ramsey was lacking in the capacity for logical intuition or perception - and Keynes did not in fact do so.

In the logical interpretation, the probability of h given e is identified with the rational degree of belief which someone, who had evidence e, would accord to h. This rational degree of belief is considered to be the same for all rational individuals. The subjective interpretation of probability abandons the assumption of rationality leading to consensus. According to the subjective theory different individuals (Ms A, Mr B and Master C say). although all perfectly reasonable and having the same evidence e, may yet have different degrees of belief in h. Probability is thus defined as the degree of belief of a particular individual, so that we should

really not speak of *the* probability, but rather of Ms A's probability, Mr B's probability, or Master C's probability.

Now the mathematical theory of probability takes probabilities to be numbers in the interval [0, 1]. So, if the subjective theory is to be an adequate interpretation of the mathematical calculus, a way must be found of measuring the degree of belief of an individual that some event (E say) will occur. Thus we want to be able to measure, for example, Mr B's degree of belief that it will rain tomorrow in London, that a particular political party will win the next election, and so on. How can this be done? Ramsey argues (1926, 172): 'The old-established way of measuring a person's belief is to propose a bet, and see what are the lowest odds which he will accept. This method I regard as fundamentally sound; ...' Ramsey defends this betting approach as follows (1926, 183):

' ... this section ... is based fundamentally on betting, but this will not seem unreasonable when it is seen that all our lives we are in a sense betting. Whenever we go to the station we are betting that a train will really run, and if we had not a sufficient degree of belief in this we should decline the bet and stay at home.'

The betting approach to probability can be made precise as follows. Let us imagine that Ms A (a psychologist) wants to measure the degree of belief of Mr B in some event E. To do so, she gets Mr B to agree to bet with her on E, under the following conditions. Mr B has to choose a number q (called his *betting quotient* on E), and then Ms A chooses the stake S. Mr B pays Ms A qS in exchange for S if E occurs. S can be positive or negative, but $|S|$ must be small in relation to Mr B's wealth. Under these circumstances q is taken to be a measure of Mr B's degree of belief in E.

If Mr B has to bet on a number of events $E_1, \ldots, E_n$, his betting quotients are said to be *coherent* if and only if Ms A cannot choose stakes $S_1, \ldots, S_n$ such that she wins whatever happens. If Ms A can choose stakes so that she wins whatever happens, she is said to have made a *Dutch Book* against Mr B.

It is taken as obvious that Mr B will want his bets to be coherent, that is to say he will want to avoid the possibility of his losing whatever happens. Surprisingly this condition is both necessary and sufficient for betting quotients to satisfy the axioms of probability. This is the content of the following theorem.

The Ramsey-De Finetti Theorem

A set of betting quotients is coherent if and only if they satisfy the axioms of probability.

This theorem gives a rigorous foundation to the subjective theory of probability. The chain of reasoning is close knit and ingenious. The first general idea is to measure degrees of belief by betting. This is made precise by introducing betting quotients. What is known as the Dutch Book argument then shows that for betting quotients to be coherent, they must satisfy the axioms of probability and so can be regarded as probabilities.

Let us now turn to the intersubjective interpretation of probability.[4] The subjective theory is concerned with degrees of belief of particular individuals. However this abstracts from the fact that many, if not most, of our beliefs are social in character. They are held in common by nearly all members of a social group, and a particular individual usually acquires them through social interactions with this group. If we accept Kuhn's analysis (1962) then this applies to many of the beliefs of scientists. According to Kuhn, the scientific experts working in a particular area, nearly all accept a paradigm, which contains a set of theories and factual propositions. These theories and propositions are thus believed by nearly all the members of this group of scientific experts. A new recruit to the group is trained to know and accept the paradigm as a condition for entry to the group. Much the same considerations apply to other social groups such as religious sects, political parties and so on. These groups have common beliefs which an individual usually acquires through joining the group. It is actually quite difficult for individuals to resist accepting the dominant beliefs of a group of which they form part, though of course dissidents and heretics do occur. One striking instance of this is that individuals kidnapped by a terrorist organisation do sometimes, like Patty Hearst, adopt the terrorists' beliefs. All this seems to indicate that as well as the specific beliefs of a particular individual, there are the consensus beliefs of social groups. Indeed the latter may be more fundamental than the former. What will be shown next is that these consensus beliefs can be treated as probabilities through an extension of the Dutch Book argument.

Earlier we imagined that Ms A (a psychologist) wanted to measure the degree of belief of Mr B in some event E. To do so, she gets Mr B to agree to bet with her on E, under the following conditions. Mr B has to choose a number q (called his betting quotient on E), and then Ms A chooses the stake S. Mr B pays Ms A qS in exchange for S if E occurs. S can be positive or negative, but |S| must be small in relation to Mr B's wealth. Under these circumstances q is taken to be a measure of Mr B's degree of belief in E.

In order to extend this to social groups, we can retain our psychologist Ms A, but we should replace Mr B by a set $\mathbf{B} = (B_1, B_2, \ldots, B_n)$ of individuals. We then have the following theorem.

Theorem. Suppose Ms A is betting against **B** = (B_1, B_2, ... , B_n) on event E. Suppose B_i chooses betting quotient q_i. Ms A will be able to choose stakes so that she gains money from **B** whatever happens *unless* $q_1 = q_2 = ... = q_n$.

Informally what this theorem shows is the following. Let **B** be some social group. Then it is in the interest of **B** as a whole if its members agree, perhaps as a result of rational discussion, on a common betting quotient rather than each member of the group choosing his or her own betting quotient. If a group does in fact agree on a common betting quotient, this will be called the *intersubjective* or *consensus* probability of the social group. This type of probability can then be contrasted with the *subjective* or *personal* probability of a particular individual.

The Dutch book argument used to introduce intersubjective probability shows that if the group agrees on a common betting quotient, this protects them against a cunning opponent betting against them. This then is a particular mathematical case of an old piece of folk wisdom, the claim, namely, that solidarity within a group protects it against an outside enemy. This point of view is expressed in many traditional maxims and stories. A recent example occurs in Kurosawa's film *Seven Samurai*. In one particular scene Kambei the leader of the samurai is urging the villagers to act together to repel the coming attack by bandits. 'This is a rule of war.' he says 'Collective defence protects the individual. Individual defence destroys the individual.'

One helpful way of regarding the intersubjective interpretation of probability is to see it as intermediate between the logical interpretation of the early Keynes, and the subjective interpretation of his critic Ramsey. According to the early Keynes, there exists a single rational degree of belief in some conclusion c given evidence e. If this were really so, we would expect nearly all human beings to have this single rational degree of belief in c given e, since, after all, most human beings are rational. Yet in very many cases different individuals come to quite different conclusions even though they have the same background knowledge and expertise in the relevant area, and even though they are all quite rational. A single rational degree of belief on which all rational human beings should agree seems to be a myth.

So much for the logical interpretation of probability, but the subjective view of probability does not seem to be entirely satisfactory either. Degree of belief is not an entirely personal or individual matter. We very often find an individual human being belonging to a group which shares a common outlook, has some degree of common interest, and is able to reach a consensus as regards its beliefs. Obvious examples of such groups would be religious sects, political parties, or schools of thought regarding various scientific questions. For such groups the concept of intersubjective probability seems to be the appropriate one. These groups may be small or large, but usually they fall short of embracing the whole of humanity.

The intersubjective probability of such a group is thus intermediate between a degree of rational belief (the early Keynes) and a degree of subjective belief (Ramsey).

The three views we have considered so far have in common that they regard probability as a measure of human belief, whether it is degree of rational belief, degree of individual belief, or the degree of a consensus belief of a group. Such theories are called *epistemological* theories of probability, and they can be contrasted with *objective* theories of probability. Here objective does not, as in Keynes, mean objective in the Platonic sense, but rather in the sense of belonging to the objective material or physical world. The probability of a radioactive atom disintegrating in a year is an example of an objective probability in this sense. It is an objective feature of the physical world, and does not depend on human beliefs. Such objective probabilities are to be found in the natural sciences in situations where we have a set of repeatable conditions.

This concludes my brief survey of some main interpretations of probability. Let us now see how these views might be applied to Keynes's economics.

3. Probability in Keynes's Theory of Long-Term Expectation

In his *Treatise* of 1921 Keynes advocated the logical interpretation of probability as degree of rational belief. Should we therefore adopt the natural supposition that he is implicitly using this logical interpretation of probability in the *General Theory*? Or are there reasons for thinking that Keynes changed his views on probability between 1921 and 1936? These questions have been the subject of a fascinating debate among the Post-Keynesians. One point of view is the *continuity thesis* that Keynes held much the same view of probability throughout his life. This thesis is advocated by (among others) Lawson (1985), Carabelli (1988), and O'Donnell (1989). Opposed to this is the *discontinuity thesis* that Keynes changed his views on the interpretation of probability significantly between 1921 and 1936. This thesis is advocated by Bateman (1987 & 1996), and Davis (1994). I am in favour of the discontinuity thesis, and will next present the main arguments in its favour.

As far as the interpretation of probability is concerned, a most important intellectual event took place between 1921 and 1936. As we have seen in the previous section, Ramsey in his 1926 paper 'Truth and Probability' subjected Keynes's logical interpretation of probability to an extensive criticism. There is strong evidence that Keynes, who had the greatest respect for Ramsey, took this criticism very seriously, and altered his views on probability in the light of Ramsey's objections.

Ramsey died in 1930 at the age of only 26, and Keynes paid a tribute in Chapter 29 of his 1933 *Essays in Biography* to this remarkable Cambridge philosopher, mathematician, and economist. This is what Keynes says about Ramsey's treatment of probability (1933, 338-9):

'Ramsey argues, as against the view which I had put forward, that probability is concerned not with objective relations between propositions but (in some sense) with degrees of belief, and he succeeds in showing that the calculus of probabilities simply amounts to a set of rules for ensuring that the system of degrees of belief which we hold shall be a *consistent* system. Thus the calculus of probabilities belongs to formal logic. But the basis of our degrees of belief - or the *a priori* probabilities, as they used to be called - is part of our human outfit, perhaps given us merely by natural selection, analogous to our perceptions and our memories rather than to formal logic. So far I yield to Ramsey - I think he is right. But in attempting to distinguish 'rational' degrees of belief from belief in general he was not yet, I think, quite successful.'

We see that Keynes was prepared to yield to Ramsey on a number of points, but yet did not agree with Ramsey about everything. Bateman in his interesting 1987 article on 'Keynes's Changing Conception of Probability' argues that Keynes did adopt the subjective interpretation of probability. After quoting the above passage from Keynes, he writes (1987, 107): 'While he (i.e. Keynes - D.G.) had originally advocated an *objective epistemic* theory of probability in *A Treatise on Probability* he was now willing to accept a *subjective epistemic* theory.'

I agree with Bateman that Keynes abandoned the logical interpretation of probability, but I will argue that Keynes moved towards an intersubjective epistemic theory rather than a subjective epistemic theory of the kind advocated by Ramsey. Intersubjective probability is in fact closer to Keynes's original position, for, as I argued in the previous section, the intersubjective probability of a group is intermediate between a degree of rational belief (the early Keynes) and a degree of subjective belief (Ramsey).

Before discussing intersubjective probability in this context, however, I will present a further piece of evidence that Keynes did abandon the logical interpretation of probability in his *General Theory*. As we saw earlier, Keynes's version of the logical interpretation of probability makes use of what he called the *Principle of Indifference*. Admittedly Keynes does give a full discussion of the paradoxes to which this Principle leads, and he is not very successful in resolving these paradoxes. Yet in his 1921 *Treatise on Probability*, he still regards the Principle of Indifference as essential for probability theory, as the following remarks about it show (Keynes, 1921, 87):

'On the grounds both of its own intuitive plausibility and of that of some of the conclusions for which it is necessary, we are inevitably led towards this principle as a necessary basis for

judgments of probability. In *some* sense, judgments of probability do seem to be based on equally balanced degrees of ignorance.'

By contrast, in the *General Theory*, Keynes wrote (1936, 152):

'Nor can we rationalise our behaviour by arguing that to a man in a state of ignorance errors in either direction are equally probable, so that there remains a mean actuarial expectation based on equi-probabilities. For it can easily be shown that the assumption of arithmetically equal probabilities based on a state of ignorance leads to absurdities.'

This amounts to a complete repudiation of the Principle of Indifference, and it is interesting to note that Keynes may here be echoing Ramsey who wrote (1926, 189):

'To be able to turn the Principle of Indifference out of formal logic is a great advantage; for it is fairly clearly impossible to lay down purely logical conditions for its validity, as is attempted by Mr Keynes.'

All this establishes that Keynes did abandon his logical interpretation of probability in the light of Ramsey's criticisms. But what interpretation of probability is then appropriate for Keynes's use of expectation in the *General Theory*? I think we can obtain an answer to this question through an analysis of Keynes's views on long-term expectation, as set out in his 1936 and 1937.

In his 1937, Keynes argues that our knowledge of the future yields of investments is 'uncertain' in a sense which he distinguishes from 'probable'. This is what he says (1937, 113-14):

'By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know. Nevertheless, the necessity for action and for decision compels us as practical men to do our best to overlook this awkward fact and to behave exactly as we should if we had behind us a good Benthamite calculation of a series of prospective advantages and disadvantages, each multiplied by its appropriate probability, waiting to be summed.'

Keynes here uses 'uncertain' in the same sense as Knight, who in 1921 had distinguished between risk and uncertainty. Knight put the point as follows (1921, 233):

'The practical difference between the two categories, risk and uncertainty, is that in the former the distribution of the outcome in a group of instances is known (either through calculation *a priori* or from statistics of past experience), while in the case of uncertainty that is not true, the reason being in general that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique.'

Keynes next asks, regarding situations of uncertainty in the above sense, (1937, 114): 'How do we manage in such circumstances to behave in a manner which saves our faces as rational, economic men?' He answers this question by saying that we resort to 'a variety of techniques' of which the most important is the following (1937, 114):

'Knowing that our own individual judgment is worthless, we endeavour to fall back on the judgement of the rest of the world which is perhaps better informed. That is, we endeavour to conform with the behaviour of the majority or the average. The psychology of a society of individuals each of whom is endeavouring to copy the others leads to what we may strictly term a *conventional* judgment.'

Keynes's point is that because of lack of information and because of the general uncertainty of the future, entrepreneurs cannot form a rational expectation, which then determines their investment decisions. As a result, their expectation is largely conventional, and because of this, it is subject to waves of optimism or pessimism, the general state, that is of the famous animal spirits, which Keynes describes as follows (1936, 161-2):

' ... there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits - of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities. ... Thus if the animal spirits are dimmed and the spontaneous optimism falters, leaving us to depend on nothing but a mathematical expectation, enterprise will fade and die; - though fears of loss may have a basis no more reasonable than hopes of profit had before.'

Keynes does not postulate, as a strict follower of Ramsey might have done, that each entrepreneur forms his or her own individual expectation which differs from that of every

other entrepreneur. On the contrary, the entrepreneurs imitate each other so that the group comes to have more or less the same expectation. However this expectation is not based on a rational assessment, but depends on factors like the state of the animal spirits. What we are dealing with is the intersubjective degree of belief of a group of entrepreneurs, which, through a process of social interaction, reaches a consensus. Keynes's long-term expectation is the intersubjective expectation of a group of entrepreneurs, and implicitly involves the notion of intersubjective probability.

This view is reinforced by the way Keynes sees the role of expert professionals who deal in stock market investments (1936, 154):

' ... most of these persons are, in fact, largely concerned, not with making superior long-term forecasts of the probable yield of an investment over its whole life, but with foreseeing changes in the conventional basis of valuation a short time ahead of the general public. They are concerned, not with what an investment is really worth to a man who buys it 'for keeps', but with what the market will value it at, under the influence of mass psychology, three months or a year hence.'

Although intersubjective probability is largely an explication of what Keynes says, I think that it does improve on Keynes's position at one point. Both Keynes and Knight seem to assume that uncertainty is a qualitative concept which cannot be quantified, but, if we use the method of betting quotients and the Dutch book argument, we can quantify uncertainty and treat it using the standard mathematical theory of probability. To see this, let us consider two of Keynes's examples of uncertainty, namely (1937, 113): 'the price of copper and the rate of interest twenty years hence'. Although it is obviously very uncertain what the rate of interest will be in twenty years' time, there is nothing to prevent us getting a particular individual, or a social group, to propose a betting quotient on this price lying in a specified interval in twenty years' time. Thus we can by the standard Dutch book procedure introduce probability distributions for the rate of interest in twenty years' time. These probabilities will, however, be subjective (or intersubjective), and not objective. Thus we can say that uncertainty in the sense of Keynes and Knight can be handled using subjective (or intersubjective) probabilities based on betting; while Knight's risk corresponds to an objective probability.

This analysis in fact accords quite well with what Keynes and Knight themselves say. Keynes says about examples such as the rate of interest in twenty years' time (1937, 113): 'About these matters *there is no scientific basis* on which to form any calculable probability whatever.' (my italics - D. G.) Certainly there is no scientific basis to form a calculable probability, and so we cannot have an objective probability, but there is nothing to prevent individuals (or groups) betting, and so forming a subjective (or intersubjective) probability. Knight associates risk with situations in which (1921, 233) 'the distribution of the outcome in

a group of instances is known', and claims that uncertainty occurs when (1921, 233) 'it is impossible to form a group of instances, because the situation dealt with is in a high degree unique.' This concurs exactly with the position that objective probabilities (corresponding to Knight's risks) should be associated with sets of repeatable conditions, while single events, not uniquely characterised by a set of repeatable conditions, can only be assigned probabilities in the sense of degrees of belief. Indeed Knight does actually say (1921, 233):

'We can also employ the terms "objective" and "subjective" probability to designate the risk and uncertainty respectively, as these expressions are already in general use with a signification akin to that proposed.'

Knight was writing in 1921 before Ramsey and De Finetti had developed the method of betting quotients for making subjective probabilities measurable, and the Dutch book argument for handling these subjective probabilities using the standard mathematical theory of probability. It was thus natural for Knight to think of subjective probability in his sense, i.e. uncertainty, as (1921, 46): 'indeterminate, unmeasurable'. This is no longer necessary today.

Thus we can take subjective (or intersubjective) probability to correspond to the uncertainty of Keynes and Knight, and objective probability to correspond to Knight's risk. There are advantages in so doing, since it avoids the need to use any concepts which cannot be handled by the ordinary mathematical calculus of probability. One qualification is needed, however.[5] Knight's risk does not correspond to a situation in which an objective probability exists, but to one in which the value of this objective probability is known. There might be a case in which there is an objective probability, whose value is not known, perhaps because of a lack of statistical data. Such a situation would be one of uncertainty in the sense of Knight and Keynes, that is to say, in our analysis, a situation in which use would have to be made of a subjective or intersubjective, but not objective, probability.

4. Some Concluding Remarks in favour of the Post-Keynesians

I will conclude this section by observing that Keynes's 1937 paper from which I have quoted quite extensively provides very strong evidence in favour of the Post-Keynesian interpretation of Keynes's economics. Keynes states that the aim of the paper is to summarise the main ideas of his book, and to explain the principal points in which his theory differs from the standard economics of his time. Keynes indeed characterises what he calls 'orthodox theory' or 'classical economic theory' as a view held in common by recent authors such Edgeworth and Pigou, and their predecessors such as Ricardo and Marshall. He then explains the first point in which he diverges from this tradition as follows (1937, 112):

‘But these more recent writers like their predecessors were still dealing with a system in which the amount of the factors employed was given and the other relevant facts were known more or less for certain. This does not mean that they were dealing with a system in which change was ruled out, or even one in which the disappointment of expectation was ruled out. But at any given time facts and expectations were assumed to be given in a definite and calculable form; and risks, of which, though admitted, not much notice was taken, were supposed to be capable of an exact actuarial computation. The calculus of probability, though mention of it was kept in the background, was supposed to be capable of reducing uncertainty to the same calculable status as that of certainty itself; ...’

Keynes then goes on to observe that (1937, 113) ‘we have, as a rule, only the vaguest idea of any but the most direct consequences of our acts.’ This may not matter for most of our actions, but is important for the accumulation of wealth, which is concerned with a comparatively distant, or even *indefinitely* distant future. Keynes concludes (1937, 113): ‘Thus the fact that our knowledge of the future is fluctuating, vague and uncertain, renders wealth a peculiarly unsuitable subject for the methods of the classical economic theory.’

All this gives strong support to the Post-Keynesian interpretation. When Keynes sets out to explain how his theory differs from that of the orthodox theorists, the very first point which he emphasizes is that he takes account of uncertainty which they fail to do. The Post-Keynesians are thus correct to emphasize the crucial importance of uncertainty in Keynes’s economics, and to criticize IS-LM Keynesianism for failing to mention, let alone discuss, uncertainty.

Keynes devotes section II of his 1937 paper to the question of uncertainty, and it is only in section III that he mentions effective demand, which he describes as (1937, 119) ‘my next difference from the traditional theory.’ Moreover in his treatment of effective demand, the issues connected with uncertainty, far from being forgotten, are strongly emphasized. Keynes divides effective demand into investment expenditure and consumption expenditure, but he then argues that it is investment expenditure which is the crucial factor in determining the performance of the system as a whole. This is because consumption expenditure is a fairly simple function of aggregate income, whereas investment expenditure is liable to violent fluctuations owing to uncertainty about the future. It is thus the considerations regarding uncertainty which lead Keynes to regarding the level of investment as playing the most important rôle in determining how well or badly the economy as a whole functions. This is how he summarizes the argument (1937, 121):

‘The theory can be summed up by saying that, given the psychology of the public, the level of output and employment as a whole depends on the amount of investment. I put it in this way, not because this is the only factor on which aggregate output depends, but because it

is usual in a complex system to regard as the *causa causans* that factor which is most prone to sudden and wide fluctuation. More comprehensively, aggregate output depends on the propensity to hoard, on the policy of the monetary authority as it affects the quantity of money, on the state of confidence concerning the prospective yield of capital assets, on the propensity to spend and on the social factors which influence the level of the money wage. But of these several factors it is those which determine the rate of investment which are most unreliable, since it is they which are influenced by our views of the future about which we know so little.

This that I offer is, therefore, a theory of why output and employment are so liable to fluctuation.'

So Keynes was not a Keynesian, though he may have been a Post-Keynesian!

Notes

1. The term 'Post-Keynesianism' is rather vague, and not everyone would use it in the way adopted here. Indeed several of those whom I have included in the group might deny that they are Post-Keynesians. I am certainly using the term 'Post-Keynesian' in a broad sense to cover a number of authors with very different views on economics and politics. The right wing of the Post-Keynesians is represented by Skidelsky who holds that Keynes's ideas, though very interesting and important historically, are no longer applicable in the changed conditions of today. The left wing of the group, on the other hand, favour an integration of Keynes with Marx, and very left-wing policies.

2. In the *General Theory*, Keynes does use the terms 'uncertain' and 'uncertainty' quite often. He does also sometimes, though not often, use the word 'probability'. Characteristically, however, he speaks of 'expectation' rather than 'probability'. Now in standard probability theory, expectation can be defined in terms of probability, and vice versa. Suppose, for example, that a random variable X can take on the values a_1, a_2, ... , a_n, with probabilities p_1, p_2, ... , p_n. Then the expectation of X, $E(X) = a_1p_1 + a_2p_2 + ... + a_np_n$. Similar definitions can be given for random variables with more complicated distributions. Conversely let A be an event. We can define the indicator of A by

$$Y(\omega) = 1 \text{ if } \omega \varepsilon A$$
$$Y(\omega) = 0 \text{ if } \neg(\omega \varepsilon A)$$

Then Y is a random variable, and the probability of A, $P(A) = E(Y)$. Indeed one can develop probability by introducing expectation as the primitive concept which appears in the axioms, and defining probability in terms of expectation. For these reasons, I will assume that when Keynes speaks of expectation, he is making an implicit reference to probability. However, it was suggested to me by Tomohide Suzuki that Keynes may be using expectation in a non-standard sense which is not definable in terms of probability. This suggestion leads to a different interpretation of Keynes's writings which seems to me worth exploring, but which I will not consider further in the present paper.

3. For more detailed accounts of the various interpretations, see Gillies (2000).

4. What follows is an informal sketch of the intersubjective interpretation of probability. A more detailed account with full proofs of the relevant theorems is contained in Gillies and Ietto-Gillies (1991). This is a joint paper written with my wife who is Professor of Applied Economics at the University of South Bank, London. The theory of intersubjective probability as applied to economics was worked out by the two of us together. An account of the theory is also to be found in Gillies (2000, 169-180).

5. This point was made to me by Jon Williamson in an informal discussion.

References

Bateman, B.W. (1987). Keynes's Changing Conception of Probability, *Economics and Philosophy*, **3**, 97-120.

Bateman, B.W. (1988). G.E.Moore and J.M.Keynes: A Missing Chapter in the History of the Expected Utility Model, *American Economic Review*, **78**,1098-1106.

Bateman, B.W. (1996). *Keynes's Uncertain Revolution*. University of Michigan Press.

Carabelli, A. (1988). *On Keynes's Method*. Macmillan.

Davis, J.B. (1994). *Keynes's Philosophical Development*. Cambridge University Press.

Fitzgibbons, A. (1988). *Keynes's Vision: A New Political Economy*. Clarendon Press.

Galavotti, M.C. (1991). The Notion of Subjective Probability in the Work of Ramsey and de Finetti, *Theoria*, **LXII**(3), 239-59.

Gillies, D.A. (2000). *Philosophical Theories of Probability*. Routledge.

Gillies, D.A. and Ietto-Gillies, G. (1991). Intersubjective Probability and Economics, *Review of Political Economy*, **3**(4), 393-417.

Keynes, J.M. (1921). *A Treatise on Probability*. Macmillan. 1963.

Keynes, J.M. (1933). *Essays in Biography*. Macmillan, Cambridge University Press for the Royal Economic Society. Paperback edition. 1985.

Keynes, J.M. (1936). *The General Theory of Employment, Interest and Money*. Macmillan, Cambridge University Press for the Royal Economic Society. Paperback edition. 1993.

Keynes, J.M. (1937). The General Theory of Employment. In *The Collected Writings of John Maynard Keynes. Vol. 14*, Macmillan, Cambridge University Press for the Royal Economic Society, 1973, 109-23.

Knight, F.H. (1921). *Risk, Uncertainty and Profit*. Reprint by the University of Chicago Press. 1971.

Kuhn, T.S. (1962). *The Structure of Scientific Revolutions*. The University of Chicago Press.

Lawson, T. (1985). Uncertainty and Economic Analysis, *The Economic Journal*, **95**, 909-27.

Lawson, T. and Pesaran, H.(Eds.) (1985). *Keynes' Economics. Methodological Issues*. Croom Helm.

O'Donnell, R.M. (1989). *Keynes: Economics, Philosophy and Politics*. Macmillan.

Ramsey, F.P. (1926). Truth and Probability. In Ramsey, 1931, 156-98. Reprinted in H.E. Kyburg and H.E.Smokler (eds.), *Studies in Subjective Probability*, Wiley, 1964, 61-92.

Runde, J. (1994). Keynes After Ramsey: In Defence of *A Treatise on Probability*, *Studies in History and Philosophy of Science*, **25** (1), 97-121.

Runde, J. (1996). On Popper, Probabilities and Propensities, *Review of Social Economy*, **LIV**(4), 465-85.

Skidelsky, R. (1983). *John Maynard Keynes. Vol. 1. Hopes Betrayed 1883-1920*. Macmillan.

Skidelsky, R. (1992). *John Maynard Keynes. Vol 2. The Economist as Saviour 1920-1937*. Macmillan.

Skidelsky, R. (2000). *John Maynard Keynes. Vol.3. Fighting for Britain 1937-1946*. Macmillan.