Income and Outcomes: A Structural Model of Intrahousehold Allocation

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There is evidence from several sources that one cannot treat many-person households as a single decision maker. If this is the case, then factors such as the relative incomes of the household members may affect the final allocation decisions made by the household. We develop a method of identifying how "incomes affect outcomes" given conventional family expenditure data. The basic assumption we make is that household decision processes lead to efficient outcomes. We apply our method to a sample of Canadian couples with no children. We find that the final allocations of expenditures on each partner depend significantly on their relative incomes and ages and on the level of lifetime wealth.

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I. Introduction

In microeconomics textbooks, the chapter on consumer theory shows how the preferences of the consumer can be represented by a utility function, which is then maximized subject to a budget constraint. This framework provides an essential support for empirical analyses of behavior, as well as for normative recommendations. Unfortunately, the relevance of these conceptual tools is somewhat hindered by the absence of adequate data. What one generally observes is household consumption or labor supply. But consumer theory does not say much on household behavior if there is more than one person in the household.

The way one invariably deals with this problem is rather simple: one simply ignores it. In most empirical implementation, it is assumed that the tools of consumer theory apply at the household level, without any particular justification. It is thus implicitly assumed that the household systematically behaves "as if" it is a single agent. Casual observation, though, suggests that this may not be a very good assumption. From a more fundamental viewpoint, taking the "unitary" representation of the household as a benchmark is certainly disputable. After all, individualism is supposed to lie at the foundation of micro theory, and individualism obviously requires one to allow that different individuals may have different preferences. Thus the methodologically correct attitude should be to consider first the general, "multiutility" framework. Whether any particular simplification—such as the existence of a household utility function—is acceptable then becomes an open question. The answer depends on both the strength of its theoretical foundations and the adequacy of its predictions for observed behavior.

From a theoretical viewpoint, classical results in aggregation theory strongly suggest that a group does not behave as a single individual except under very strong, specific assumptions. Among the few theoretical attempts to reconcile the single-utility framework with the existence of several individuals within the household, one must cite Samuelson's (1956) household welfare index and Becker's (1981) rotten kid theorem. Both, however, can be shown to rely on restrictive hypotheses. For instance, Samuelson's approach crucially depends on the (very) ad hoc assumption that the members' respective weights in the household index are independent of the environment (wages, prices, and incomes). On the other hand, though the rotten kid theorem has stronger justifications, recent work has stressed that it holds only for transferable utilities (see Bergstrom 1989).

What, then, is the empirical support for the unitary model? Four distinct consequences have been tested in empirical demand systems:

"income pooling," symmetry of the Slutsky matrix, negativity of price responses, and the exclusion of income variables from demand equations that condition on total expenditure.

A prediction of the single-criterion model is that only joint (or household) income should matter for allocation decisions and not who receives the income. This is usually referred to as the income pooling hypothesis. The results of Schultz (1990), Thomas (1990), Bourguignon et al. (1993), and Phipps and Burton (1994) suggest quite strongly that the data are not consistent with this hypothesis. As in those studies, the unitary model here is taken in the strict sense of a single objective function that is maximized under the usual constraints of a fixed price budget and nonnegative consumptions. Income pooling might not hold if these constraints are modified or other constraints are added. For example, including conditions ensuring the free participation of the individuals in the household may lead to a dependence of household demands on individual income as well as joint income. We do not consider this type of constraint as consistent with the unitary model since it implicitly introduces the individuality of household members into the picture.

Other disquieting findings can be found in demand studies on (micro) household data (see, e.g., Browning and Meghir 1991; Blundell, Pashardes, and Weber 1993). First, symmetry of the Slutsky matrix is usually rejected. Even though own substitution effects often prove to have the expected sign,[1] the rejection of symmetry suggests that choices cannot be rationalized as the outcome of a constrained maximization problem with a single utility function.

The final disquieting finding concerns the instrumenting of total expenditure in demand models. This is essential on micro data since there are several reasons why total expenditure might be endogenous (see Deaton [1985] or Blundell [1986] for a discussion). The obvious instrument to use is income, but other variables are also used to achieve some overidentification. Unfortunately, the overidentifying restrictions are usually rejected. Although no one has made a systematic investigation of this rejection, it is at least plausible that it is attributable to the invalid exclusion of some income variable.

Clearly the empirical support for the unitary approach is rather weak, to say the least. What recent empirical analysis points toward is that multiperson households cannot be treated as single decision

[1] This is true in a wider context than demand studies. For example, Keeley et al. (1978) find positive substitution effects in a labor supply model estimated on data from the Seattle and Denver Income Maintenance Experiment.

makers and that household allocations should probably rather be considered as the outcome of some interaction between household members with different preferences.[2] The goal of the present paper is precisely to develop and estimate a "collective" model of this kind.

The first task, of course, is to derive a formalized model of household behavior that follows the multiutility line of argument. Such a theoretical development, following the initial model of Chiappori (1988, 1992), is provided in a companion paper (Bourguignon, Browning, and Chiappori 1994). Browning and Chiappori (1994) derive (and test) the analogues of the Slutsky integrability conditions for the collective model considered here. The main implications of these results for our present empirical investigation are detailed in Section II.

Having discussed some of the issues that arise in modeling intrahousehold allocation, in Section III we present a parametric model of intrahousehold allocation decision making and show that, given one critical assumption, we can identify almost everything about such decision making using conventional family expenditure data. If a good is consumed by only one person, then we term such a good an exclusive good. Our critical assumption is that we have two such goods, one for each person.

In Section IV, we present some informal empirical analysis using Canadian family expenditure data. Our principal conclusion is that the conventional "single–decision maker" model fails for couples but not for single people. Although complementary to the evidence mentioned above, our finding is more focused in that it specifically identifies the failure of the conventional model with the presence of more than one person in the household.

In Section V, we use the Canadian data on couples with no children to estimate the parameters of our model. The goods we treat as exclusive are men's and women's clothing. Our principal finding is that "who gets what" in the household depends on the relative incomes and ages of the two partners and how wealthy the household is. We also present some estimates of exactly how these factors affect allocation. To our knowledge this is the first time such estimates have been presented in the literature.

[2] To be precise, we ought really to refer to multiadult households, possibly with young children. Thus we would not be averse to modeling lone-parent families in which all the children are young using a unitary assumption. In effect, it may reasonably be assumed that, at least until a certain age, children have no decision power in the household. Below we restrict attention to households containing two adults with no children.

II. The Theoretical Framework

A. *Modeling Issues*

To help in discussion of the modeling of intrahousehold allocation decisions, we consider four sets of issues: (i) the partitioning of goods into private and public goods, (ii) the nature of preferences, (iii) the mechanism used to reach decisions, and (iv) what the econometrician can observe.

i) The public/private issue is a familiar one. Although it may be reasonable to treat some goods as private (e.g., alcoholic beverages),[3] there are some goods that clearly have a strong public element (e.g., heating). Where to draw the line between public and private goods is not easy. For example, food is private in the sense that only one person can eat any piece of food, but there is clearly some public element in food preparation. In all that follows we assume that we can unambiguously designate goods to be private or public. To fix ideas, suppose that we are looking at a two-person (A and B) household. We let $\mathbf{q}^A$ and $\mathbf{q}^B$ represent vectors of private goods going to A and B, respectively, and $\mathbf{Q}$ represent a vector of public goods. If we denote total household expenditure on goods by y, then we have the budget constraint

$$\mathbf{p}'(\mathbf{q}^A + \mathbf{q}^B) + \mathbf{P}'\mathbf{Q} = y,$$

where $\mathbf{p}$ and $\mathbf{P}$ are price vectors for the private goods and the public goods, respectively.

ii) We turn now to preferences. The most general preference structure is

$$U^i = f^i(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}) \quad \text{for } i = A, B.$$

We refer to this as altruistic preferences. More restrictive forms for preferences have been suggested in the literature. These include the following:

same preferences:	$U^i = F(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$,	$i = A, B$;
caring or nonpaternalistic:	$U^i = F^i(\mathbf{v}^A(\mathbf{q}^A, \mathbf{Q}), \mathbf{v}^B(\mathbf{q}^B, \mathbf{Q}))$,	$i = A, B$;
egotistic:	$U^i = \mathbf{v}^i(\mathbf{q}^i, \mathbf{Q})$,	$i = A, B$.

Thus with "caring" preferences, each person cares about the other's allocation only insofar as it gives the other person some individualistic welfare. The aggregator functions $F^A(\cdot)$ and $F^B(\cdot)$ are assumed to have $F^i(\cdot)$ strictly increasing in both subutility functions. Note that the subutility functions are the same in both welfare functions even

[3] This, of course, ignores any externalities that may be caused.

though the aggregator functions do not have to be the same. Fairly obviously, "egotistic" is a special case of caring in which $F^A(v^A, v^B) = v^A$, and similarly for B.

The issues that arise for the public/private nature of goods and how we model preferences are not independent. For example, if we assume altruistic preferences, then the distinction between private and public goods becomes blurred since both $\mathbf{q}^A$ and $\mathbf{q}^B$ are public in the usual sense. Conversely, if all goods are public, then we cannot distinguish between altruism and egotism.

If we assume caring then, there is also another distinction between goods that we find convenient. If there is a good that only one person in the household cares about, then the distinction between private and public is not very well defined. We thus choose to categorize such a good separately as *exclusive* rather than public or private. For example, cigarettes are private, but if only one person smokes (and, again, there are no externalities!), then it is categorized as exclusive. Conversely, the presence of a telephone is public, but if only one person ever uses it, then it is exclusive. Note that exclusivity depends on the properties of the utility function. The need to distinguish between private, public, and exclusive goods will emerge below. For notational convenience we shall include the exclusive goods in $\mathbf{q}^A$ or $\mathbf{q}^B$ along with private goods; we term such goods nonpublic.

iii) The third set of issues concerns the mechanism the members of the household use to decide what to buy. Many procedures have been proposed in the literature. For example, if each partner has an income and the sum of these incomes is equal to household income, then we could assume that each makes a private decision about what to buy and then look at the Nash equilibria (if any exist) for this "game." More sophisticated versions would take account of the fact that this is a repeated game. Alternatively, we could look at bargaining models following the line initiated by Manser and Brown (1980) and McElroy and Horney (1981).

Chiappori (1988, 1992) has analyzed the case in which we assume only that outcomes are efficient. This is particularly attractive in the context of the household since the "players" have a long-term relationship and are in an environment that does not change much from period to period.

iv) Finally, we have to consider what the econometrician can observe. Typically we observe household purchases of goods only within a certain period. Even if we equate these purchases with consumption (which is what investigators generally do for nondurables), we do not observe the individual consumptions of private goods. Sometimes, however, we may have a private good for which we can observe individual consumptions; we term such a good *assignable*. The distinction between exclusive and assignable when there is no price variation is not always very precise. For example, the individual consumptions of an assignable good can be thought of as two exclusive goods (one for each person). An example here would be clothing. If this is private and the husband consumes only men's clothing and the wife consumes only women's clothing, then we can either think of the total clothing commodity as an assignable good or think of men's and women's clothing as two exclusive goods.

B. The Sharing Rule

In all that follows we shall assume that, however allocation decisions are made, they lead to efficient outcomes. We refer to this as the *collective* setting.

We can consider two types of questions about this setting. On the one hand, we may try to test for it; this implies first deriving testable implications. Surprisingly enough, not much is needed for that purpose (see Bourguignon et al. 1994). The only requirement is the presence of variables that may safely be assumed to influence the decision process but not preferences. An example of such variables, used throughout the literature, is different income sources within the household. The presence of such variables gives rise to testable restrictions on demands. For these tests no specific assumption on either the nature of goods (i.e., whether they are public, private, or exclusive) or the form of preferences (altruistic, egotistic, or caring) is necessary (see Bourguignon et al. 1994). Tests of these restrictions are presented in Bourguignon et al. (1993). We find that, though income pooling is strongly rejected, the "collective" restrictions are not.

A more ambitious purpose is to estimate the structural model—that is, individual demands and the decision process—from observed behavior. That is the goal of the present paper. Specifically, we shall be interested in investigating how final outcomes depend on the income each person brings into the household.

Not surprisingly, additional assumptions are needed to achieve identification. Here, we first set out some assumptions: (i) some goods are nonpublic; (ii) preferences are caring; (iii) each member's subutility function is separable with respect to nonpublic consumptions:

$$v^i(\mathbf{q}^i, \mathbf{Q}) = V_i(u^i(\mathbf{q}^i), \mathbf{Q});$$

and (iv) one private good is assignable or we can identify two exclusive goods (one for each person).

Conditions i and ii exclude the most general preference structure in which each member's nonpublic consumption directly influences the spouse's utility (say, through consumption externalities). Condition iii guarantees that each member's marginal rate of substitution between nonpublic goods does not depend on the level of public consumption in the household.[4] Finally, condition iv states that we can observe something about the intrahousehold allocation.

We can now state the two basic results underlying the remainder of the paper. First, let $\hat{\mathbf{q}}^A$, $\hat{\mathbf{q}}^B$, and $x = \mathbf{p}'(\hat{\mathbf{q}}^A + \hat{\mathbf{q}}^B)$ denote, respectively, each member's equilibrium vector of nonpublic consumption and the household's total expenditures on nonpublic goods. Then efficiency has the following consequence.

PROPOSITION 1. *Existence of a sharing rule.*—Under assumptions i–iii and efficiency, there exist scalars x_A and x_B, with $(x_A + x_B) = x$, such that $\hat{\mathbf{q}}^A$ and $\hat{\mathbf{q}}^B$ are solutions of

$$\max u^i(\mathbf{q}^i) \quad \text{subject to } \mathbf{p}'\mathbf{q}^i = x_i \quad \text{for } i = A, B. \tag{P}$$

Proof. Define $x_i = \mathbf{p}'\hat{\mathbf{q}}^i$ and assume that $\hat{\mathbf{q}}^A$ and $\hat{\mathbf{q}}^B$ are not solutions of (P). Then there exist $\tilde{\mathbf{q}}^A$ and $\tilde{\mathbf{q}}^B$ that cost no more but provide a higher private utility for one member without making the other member worse off. Given the strict monotonicity of the aggregator functions $F^i(\cdot)$, the allocation $(\hat{\mathbf{q}}^A, \hat{\mathbf{q}}^B)$ is not a Pareto-efficient bundle since it is dominated by $(\tilde{\mathbf{q}}^A, \tilde{\mathbf{q}}^B)$, a contradiction. Q.E.D.

In words, given assumptions i–iii and efficiency, it is as though allocations in the household are made using a two-stage allocation procedure. At the top stage, total household income is allocated to saving, public goods, and each of the partners for expenditure on nonpublic goods. At the bottom stage each of the partners spends his or her individual total expenditure on nonpublic goods. Given the caring assumption, partner i of course chooses $\hat{\mathbf{q}}^i$. Two things are worth noting. First, at the bottom stage, allocation is independent of the choice of public goods since we have assumed separability. Second, we do not need any assignability for this result (i.e., we do not assume condition iv here).

It is important to emphasize that we are not suggesting that this is the actual procedure followed but simply that the allocation decisions can be seen as though they were generated by such a two-stage procedure if preferences are caring and outcomes are efficient. This as if distinction is familiar: we do not assume that individual agents actually maximize a utility function but rather that they behave as though they do.

Following Becker (1981), we term the division of total expenditure

[4] Without condition iii it is still possible to define and identify a sharing rule as below. However, one has to everywhere consider demand functions for private goods that are conditional on the consumption of public goods.

on nonpublic goods between the two partners a "sharing rule." Let $\mathbf{z}$ be a vector of exogenous variables that affect the decision process but do not influence preferences, the budget constraint, or the consumption set. Typically, $\mathbf{z}$ will contain each member's personal income, plus a range of "extra-environmental parameters" (EEPs in McElroy's [1990, 1992] terminology). They may include sex ratios in marriage markets, laws concerning alimony and child support, changes in tax status associated with different marital states, and, in developing countries, the ability of women to return to their natal homes and prohibitions on women working outside the home. Now let the share of person A in total expenditure on nonpublic goods be given by the function $\rho(\mathbf{z}, x)$; we refer to this function as the sharing rule. In proposition 1 we have $x_A = \rho(\mathbf{z}, x)x$ and $x_B = [1 - \rho(\mathbf{z}, x)]x$.

This idea of a sharing rule is central to all that follows. If preferences are caring and outcomes are efficient, then any allocation of nonpublic expenditures can be rationalized as the outcome of a sharing rule procedure. These are sufficient conditions; it may be that other household decision processes and classes of preferences also give outcomes that could be the result of a sharing rule for expenditures on nonpublic goods (at least locally).

The sharing rule reflects the outcome of the decision process; it can be seen as a "reduced form" of the actual procedure. Additional structure could be introduced with the help of more specific assumptions (e.g., Nash bargaining). We do not follow this path; rather, we only assume efficiency.

C. *Estimation*

We assume below that we observe at least one factor that affects sharing; that is, $\mathbf{z}$ is nonempty. Our goal is to estimate the sharing rule $\rho(\mathbf{z}, x)$. This is possible if there is an assignable good or two exclusive goods as stated in the following proposition.

PROPOSITION 2. *Identifying the sharing rule.*—Under the assumptions of proposition 1 and condition iv and

$$\frac{\partial q_1^A/\partial z_k}{\partial q_1^A/\partial x} \neq \frac{\partial q_1^B/\partial z_k}{\partial q_1^B/\partial x} \quad \text{for at least one } k,$$

each member's share ρx and $(1 - \rho)x$ is identified up to a (unique) additive constant.

Proof. We give here the proof if there is an assignable good; the proof for two exclusive goods follows the same lines. Let good 1 be assignable (i.e., we observe the individual demands q_1^A and q_1^B). Let

$q_1^i = f_1^i(x_i)$ be i's Engel curve for good 1. Define $x^A(\mathbf{z}, x) = \rho(\mathbf{z}, x)x$ and $x^B(\mathbf{z}, x) = [1 - \rho(\mathbf{z}, x)]x$. Thus $q_1^i = f_1^i(x^i(\mathbf{z}, x))$. Now define

$$L_k^i = \frac{\partial q_1^i/\partial z_k}{\partial q_1^i/\partial x} = \frac{\partial x^i/\partial z_k}{\partial x^i/\partial x}, \quad i = A, B \text{ and } k = 1, \ldots,$$

where $\partial x^i/\partial z_k$ and $\partial x^i/\partial x$ are the partials of x^i with respect to environmental factors z_k and total expenditure x, respectively. Note that these L_k^i's are observable. Also,

$$\frac{\partial x^A}{\partial x} + \frac{\partial x^B}{\partial x} = 1 \quad \text{and} \quad \frac{\partial x^A}{\partial z_k} + \frac{\partial x^B}{\partial z_k} = 0.$$

Thus

$$\frac{\partial x^A}{\partial x} = \frac{L_k^B}{L_k^B - L_k^A} \quad \text{and} \quad \frac{\partial x^A}{\partial z_k} = L_k^A \frac{\partial x^A}{\partial x}.$$

Note that the first of these expressions is defined for at least one k, by assumption. Hence the partials of the x^A function are all identified, which in turn implies that the partials of $\rho(\mathbf{z}, x)$ are also identified. Q.E.D.

The previous result shows that, for recovering the intrahousehold allocation of nonpublic consumption, we need only to observe how the allocation of one private good (or two exclusive goods) reacts to exogenous changes in the economic environment. This provides the basis for the estimation procedure we present in Section III below.

Two points should be stressed. First, the allocation of private expenditures is identified only up to a constant. It can actually be shown that this is the best we can do unless we can observe the allocation of all nonpublic goods (see Bourguignon et al. 1994). Second, as suggested by the proof, there will be testable restrictions on demand functions. Indeed, once the partials of $\rho(\mathbf{z}, x)$ have been identified, cross-derivative conditions will have to be checked. This will generate partial differential equations on the L_k^i's and second-order conditions on observable consumption behavior; tests based on this are developed in Bourguignon et al. (1994).

What proposition 2 shows is that the existence of an assignable good (or two exclusive goods) is sufficient for identification of the sharing rule (up to a constant). It is not necessary. Given some specific functional form, it may be enough simply to have only one exclusive good. This will be the case for the nonlinear model we derive in the next section. Indeed, under some circumstances we can identify the sharing rule with no information about who gets what of any good (see Bourguignon et al. 1994). This identification, however, relies on estimating second-cross-partials of the (household) demand functions for private goods. The advantage of assuming assignability (or exclusivity) is that the sharing rule is identified from first-order effects, and hence the identification may be more robust. This is conditional, of course, on the validity of the assignability assumption.

We assume that clothing is assignable (or, as discussed at the end of Sec. II*A* above, men's clothing and women's clothing are exclusive). It is important to be clear about the implications of this assumption since it is the critical identifying assumption in our work below. Since we are maintaining that preferences are caring, our assumption that each person's consumption of clothing is nonpublic implies that wives care about their husband's clothing only inasmuch as it contributes to the welfare of their husband (and vice versa).[5] Many readers will be thoroughly skeptical of this implication. In particular, it may be that either husbands or wives do care about how their spouses dress; this is a rejection of nonpublicness. An important point to note in this regard is that our assumptions impose testable restrictions on demands. A rejection can be viewed narrowly as evidence that clothing is a public good or more broadly as evidence that the caring version of the collective model is invalid. Of course, if we had other assignable goods, then we could use them to derive the sharing rule. Comparing the sharing rules obtained with different supposedly assignable goods would in fact provide additional tests of the actual nature of these goods and of the general collective framework used throughout this paper.

III. A Parametric Model

To minimize heterogeneity we shall be considering only married[6] couples with no one else in the household. Further, we shall consider only couples in which both partners work full-time. This restriction is necessary to remove any substitution effects between commodity demands and labor supply (see Browning and Meghir 1991). We also assume that the selection into this group is exogenous for all the processes we deal with below.

We denote the wife by A and the husband by B. As above, we let $\mathbf{z}$ denote variables that enter the sharing function $\rho(\cdot)$ but do not otherwise affect individual demands. We let $\mathbf{y}^A$ ($\mathbf{y}^B$) denote variables that directly enter the demand function for women's (men's) clothing

[5] A weaker assumption is that each cares about the other's clothing only up to some minimum and not thereafter. Thus men's (or women's) clothing could be public for low levels of expenditures but exclusive above some threshold.

[6] In the data set we use, "married" includes both legally married and common-law.

but do not enter the sharing function.[7] The vectors $\mathbf{y}^A$ and $\mathbf{y}^B$ may have elements in common (e.g., the region of residence). There may also be variables in $\mathbf{y}^A$ that do not appear in $\mathbf{y}^B$; for example, $\mathbf{y}^A$ might include the age of A but not the age of B.

The vector of sharing factors $\mathbf{z}$ might include differences of variables that appear in one set of $\mathbf{y}^i$ but not the other; for example, if $\mathbf{y}^A$ includes A's age but not B's (and vice versa), then $\mathbf{z}$ might include the difference in ages. However, the most important candidates for inclusion in $\mathbf{z}$ are the incomes of the two partners. They may affect how the partners share expenditures, but they should not affect individual demands once we condition on the total expenditure by each person. This is conditional on taking account of the dependence of demands on labor supply (which is obviously highly correlated with individual income). Since we consider only agents who work full-time in our empirical work, the dependence of demands on labor supply is taken care of automatically.

As discussed in Section II, we assume the existence of a sharing rule. This gives the division of expenditure on nonpublic, nondurable goods conditional on savings and public goods and durables purchases. Formally, let x_A and x_B be the amounts of money for expenditure on nonpublic goods that each partner receives, and let $x = (x_A + x_B)$ be total expenditure on these goods. We do not observe x_A and x_B, but we have

$$x_A = \rho(\mathbf{z}, x)x \tag{1a}$$

and

$$x_B = [1 - \rho(\mathbf{z}, x)]x, \tag{1b}$$

where $\rho \in [0, 1]$.[8]

The household demand for good j is given by

$$\begin{aligned} q_j &= \alpha^j(\mathbf{y}^A, x_A) + \beta^j(\mathbf{y}^B, x_B) \\ &= \alpha^j(\mathbf{y}^A, x\rho(\mathbf{z}, x)) + \beta^j(\mathbf{y}^B, x[1 - \rho(\mathbf{z}, x)]), \end{aligned}$$

where $\alpha^j(\cdot)$ and $\beta^j(\cdot)$ are the demand functions for good j by A and B, respectively, with either $\alpha^j(\cdot)$ or $\beta^j(\cdot)$ equal to zero for an exclusive good. If good j is not assignable, then we observe only the response of q_j to changes in $(\mathbf{y}^A, \mathbf{y}^B, \mathbf{z}, x)$. As discussed in Section II, we assume that clothing is an assignable good. Since we shall be concerned only with men's and women's clothing, below we drop the j superscript.

[7] We could also allow for variables that enter preferences directly and enter the sharing rule. As we shall see below, the parameters associated with such variables are not identified, and so we choose to exclude them a priori.

[8] Formally, the (0, 1) bounds are derived from a model with egotistic preferences. With caring preferences we have $\rho \in [\epsilon, \sigma]$, where $0 \leq \epsilon \leq \sigma \leq 1$.

Note that all the variables on the right-hand side of the demand equations above are observable. We ought, however, to make some allowance for unobservable heterogeneity. There are three potential sources of such heterogeneity: the sharing rule and the two individual preferences. The most satisfactory treatment would be to allow for each of them and then to develop a full stochastic model that would also allow us to take account of the possible endogeneity of the sample selection on married couples in full-time employment. Thus one could then allow that in households in which the sharing rule is highly dependent on relative incomes, there is more incentive for each individual to participate in the labor force. We regard this as a most important area of future research, but here we adopt a much more conventional approach of simply adding error terms to each demand equation and ignoring the possible sample selection bias.

To parameterize our demand functions, we let the log demand for women's clothing ($\ln a$) be

$$\ln a = \alpha_0 + \boldsymbol{\alpha}_A' \mathbf{y}^A + \alpha_x \ln x_A + \alpha_q (\ln x_A)^2, \tag{2a}$$

where we denote $\ln(x)$ as $\ln x$. In the same way, the log demand for men's clothing is given by

$$\ln b = \beta_0 + \boldsymbol{\beta}_B' \mathbf{y}^B + \beta_x \ln x_B + \beta_q (\ln x_B)^2. \tag{2b}$$

Now, let

$$\rho(\mathbf{z}, x) = \frac{e^{\psi(\mathbf{z},x)}}{1 + e^{\psi(\mathbf{z},x)}} \tag{3}$$

to bound ρ between zero and one. Combining (1)–(3) gives the demand equations

$$\ln a = \alpha_0 + \boldsymbol{\alpha}_A' \mathbf{y}^A + \alpha_x \left(\ln \frac{xe^{\psi}}{1 + e^{\psi}}\right) + \alpha_q \left(\ln \frac{xe^{\psi}}{1 + e^{\psi}}\right)^2 \tag{4a}$$

and

$$\ln b = \beta_0 + \boldsymbol{\beta}_B' \mathbf{y}^B + \beta_x \left(\ln \frac{x}{1 + e^{\psi}}\right) + \beta_q \left(\ln \frac{x}{1 + e^{\psi}}\right)^2. \tag{4b}$$

Finally, let

$$\psi(\mathbf{z}, x) = 2(\delta_0 + \boldsymbol{\gamma}'\mathbf{z} + \theta \ln x) \tag{5}$$

(the reason for the scaling by two will become clear below).

The $\psi(\cdot)$ function controls the share of total expenditure each partner receives as we vary the $\mathbf{z}$ variables and total expenditure. The constant δ_0 "centers" the shares; the lower it is, the lower the share of person A. The θ parameter controls how "luxurious" A's purchases are. To see this, note that

$$\frac{\delta \ln x_A}{\delta \ln x} = 1 + 2\theta(1 - \rho).$$

If θ is positive, then A's total expenditure (x_A) is rising proportionately more quickly than total expenditure, with the $\mathbf{z}$ variables held constant (since $\rho < 1$). Changes in the $\mathbf{z}$ variables affect what each person gets through changes in total expenditure and how much of that each person gets:

$$\frac{\delta \ln x_A}{\delta z_k} = [1 + 2\theta(1 - \rho)]\frac{\delta \ln x}{\delta z_k} + 2\gamma_k(1 - \rho).$$

For example, let z_1 be the wife's income and suppose that γ_1 is positive. Then an increase in the wife's income will (probably) lead to an increase in her total expenditure since it will (probably) increase total expenditure and her share in total expenditure.

As already discussed, we shall only assume efficiency in decision making; this is a relatively weak assumption. One motivation for this assumption would be that the two partners engage in some bargaining with no asymmetric information. Although we do not use such an assumption, we can provide an informal test of whether it is appropriate if we decide a priori how particular factors in the sharing rule affect bargaining positions. The obvious example here is the incomes of the two partners: we may expect that an increase in the relative income of one person increases his or her share of total expenditure on private goods.[9] In any case, if increases in some variable z^A increase A's bargaining strength and some other variable z^B decreases it, then z^A and z^B must have opposite effects on demands. We shall return to this issue in the empirical section below.

An interesting remark is that all the parameters in the sharing rule are formally identified from either of (4a) and (4b) alone. However, this identification is dubious since some of it is achieved simply by the nonlinearity in $\psi(\cdot)$. To check this, we develop a linearized version

[9] Note the "may" here. We do not have any model to this effect and it is not necessarily true in general. This will certainly be the case if there are unobservable (to the econometrician) factors at play in the bargaining, in which case an unobservable increase in bargaining power for one partner may lead to a *decrease* in income since income is not now so important for maintaining a bargaining position. This also abstracts from a consideration of public goods. It is possible that an increase in A's bargaining strength leads to an increase in her welfare, but a decrease in her share of private expenditures if there is a change in public goods purchases that offsets this decrease.

of our model that has a "linear in parameters" unrestricted form. This linearized form serves two purposes. First, we use it to show our concerns regarding the identification of some of the parameters. Second, the linearized model is a good deal easier to estimate than (4); as we shall see, the nonlinear estimation can be quite tedious. Thus we use conventional diagnostics and model selection techniques on our linearized model to help in the selection of a preferred but parsimonious unrestricted nonlinear model.[10]

The details of the derivation of the linearized model are given in Appendix A. For the linearized model we can show that of the parameters in (4a) and (5), only $\boldsymbol{\alpha}_A$ is always identified from the women's clothing equation. Of the other parameters, our interest centers on the parameters of the sharing rule in (5), $(\delta_0, \theta, \boldsymbol{\gamma})$, and the Engel curve in (4a), $(\alpha_0, \alpha_x, \alpha_q)$. In Appendix A we show that these parameters are identified with two degrees of freedom. That is, we can arbitrarily fix two of them, and then this serves to identify the rest. Of course, when we consider both demands simultaneously—that is, (4b) as well as (4a) and (5)—then we can identify all the parameters except for one. If we take the unidentified parameter to be δ_0, then this is a restatement of proposition 2: we can identify the sharing rule only up to a constant. In our empirical work below we fix the constant δ_0 so that the share of each person at the mean of the data is one-half.

Finally, we know that our assumption of a collective setting implies restrictions on demands. To test them we proceed as follows. First we estimate the parameters of (4a) and (5) from our nonlinear model of women's clothing. Then we estimate the parameters of the following variant of (4b) and (5) for the men's clothing equation:

$$\ln b = \beta_0 + \boldsymbol{\beta}_B' \mathbf{y}^B + \beta_x\left(\ln\frac{x}{1+e^{\lambda}}\right) + \beta_q\left(\ln\frac{x}{1+e^{\lambda}}\right)^2, \tag{6}$$

where

$$\lambda(\mathbf{z}, x) = 2(\delta_0 + \boldsymbol{\gamma}_B'\mathbf{z} + \theta_B \ln x). \tag{7}$$

We then test

$$(\gamma, \theta) = (\gamma_B, \theta_B). \tag{8}$$

We interpret this as a joint test of our assumption that clothing is assignable and that the caring version of the collective model holds.

[10] In the empirical section we shall present some results that show that our linearized model is a good approximation to the nonlinear one and that our concerns about identification are well founded.

IV. An Informal Look at the Data

We begin with an informal look at the data. This informal investigation uses data on couples and on singles. It has three goals. First, we wish to highlight some of the principal features of the data. Second, we examine whether our data for couples exhibit some of the same failures of the single-utility model that we discussed in Section I. Finally, we wish to address directly one important objection to the use of clothing to identify "who gets what" in the household.

Our initial interest centers on how clothing expenditures are affected by variables that are not usually included in demand equations. In particular, in the usual demand model the incomes of the husband and wife should not enter the demand equation if we condition on total expenditure. One immediate objection arises to this statement. Suppose that, even within particular occupations, higher-paid jobs require more expensive work clothing.[11] Then the incomes of the husband and wife will enter the demand equations for clothing even if we condition on total expenditure, occupation, and education. To check this we shall use single people as a control: if such an effect is present, then it should show up for singles as well as for couples.

The data we use in this study are drawn from the four Canadian family expenditures (FAMEX) surveys conducted in 1978, 1982, 1984, and 1986. Among other things, these surveys give annual expenditures by households on a comprehensive collection of goods. The fact that expenditures for the whole year are taken means that these data have less of an "infrequency of purchase" problem than, say, the U.K. Family Expenditure Survey or the U.S. Consumer Expenditure Survey. This is important for us since we concentrate on expenditures on men's and women's clothing. Of course, there is still some "lumpiness" in these expenditures even when we take annual expenditures; we shall return to controlling for this below.

We begin with two subsamples: single males in full-time employment and single females in full-time employment. The sample sizes are 1,312 and 1,353, respectively. For each subsample we estimate a clothing demand equation. The dependent variable is the log of clothing expenditures divided by the price of clothing.[12] On the right-hand side we have age variables, the log of the price of clothing, the log of the price of other nondurables, and dummies for region of residence,

[11] Below we consider only people who work full-time (see the remarks at the beginning of Sec. III).

[12] In fact, here and for other "log" variables we take the inverse hyperbolic sine. The virtue of this transformation is that it is defined for the whole real line, and it is "log" for high values of the argument and "linear" for values close to zero. Thus we can use this transformation to remove the "left skewness" that taking logs often induces (see Burbidge, Magee, and Robb 1988).

car ownership, city living, home ownership, education (a dummy for more than high school), language spoken (dummies for French and other than English or French), and occupation (a dummy for professional and managerial). We also include total nondurables expenditure variables, where nondurables comprise food, alcohol, tobacco, services, recreation, and clothing. Specifically, we include the log and log squared of this variable deflated by a nondurables price index.

As we have already mentioned, clothing expenditures tend to be lumpy (even when we take annual expenditures). This may induce an endogeneity in the total expenditure variables since the purchase of an expensive coat, say, will lead to abnormally high clothing expenditure and total expenditure. To instrument the two total expenditure variables we use income variables; specifically, we include the log and log squared of deflated income in the instrument set.[13] In the instrument set we also include all the right-hand-side variables listed in the previous paragraph (except for the total expenditure variables, of course), year dummies (to capture intertemporal allocation effects), and (log) prices of durables and cars (to capture within-period substitution effects). Thus we have some overidentification; this is important since we focus mainly on whether (once we condition on total expenditure and other variables such as occupation and education) the income variables enter the demand equation. With some overidentification we can test whether the exclusion restrictions for the income variables are valid.

We estimated the clothing equations for single men and single women separately by instrumental variables. We tested for exogeneity of total expenditure and found that it is decisively rejected in both cases. On the other hand, the overidentifying restrictions are not rejected. This indicates that we do not need income variables in the demand equations once we condition on total expenditure. Thus, for single people, the exclusion restrictions needed to use the income variables as instruments are valid. Another finding that will be used later is that we do need to include the log squared of total expenditure. We also subjected our estimates to a battery of diagnostic tests (discussed in more detail in App. B); we found these qualitative results to be robust.

Turning to couples, in this informal investigation we shall focus on the difference between (log) household demands for men's and

[13] The income variable is total gross income. Although it would be desirable to break income into its components, this is not possible for the first two years of the survey (1978 and 1982). It might also be argued that net income is the appropriate concept. Although we do have net income for singles, we do not have such a variable for the husband and wife in the sample we use below. Consequently, we use gross income here to maintain comparability.

women's clothing. This is to allow us to use single-equation methods; clearly if some variable is "significant" in the log difference equation, then it must be "significant" in either the men's or women's clothing demand equation.[14] We have 1,520 couples in our subsample. In figure 1 we plot the difference in log clothing demands against the difference in the (log deflated gross) incomes of the two partners.[15] We also plot the simple ordinary least squares (OLS) line (with no other covariates) and a cubic spline nonparametric fit. Three important features of the data are apparent in this figure. First, the majority of the points are in the northwest quadrant. Thus the husband's income tends to be higher than the wife's, but the expenditure on women's clothing tends to be higher than expenditures on men's clothing. Second, there is a pronounced positive linear association between the two variables (the slope coefficient for the OLS line is .13 with a standard error of .04). Finally, it looks as though the OLS line and the nonparametric curve correspond closely (at least for the bulk of the data).

To allow for the influence of other variables, we ran a regression with the difference in clothing demands on the left-hand side, and on the right-hand side we include the demographic variables given above (with the husband- and wife-specific components for those variables such as education and age that can vary between the partners). We use the same instruments as for singles except that now we include the log and log squared of the wife's gross income and the log and log squared of the ratio of gross incomes. In our first estimated equation we included these four income variables on the right-hand side; thus we are identifying the parameters from the other instruments (year dummies and the prices of nondurables and cars). We find that the four income variables are jointly significant: an $F(4, 1{,}496)$ statistic of 5.06, which has a 0.05 percent probability under the null. Further investigation indicated that the only one of the four variables that had any real significance was the difference in log gross incomes: the $F(3, 1{,}496)$ statistic for excluding the other three variables is 1.67. We conclude that the difference in income variable is needed in the difference in clothing demand equation.

To check the robustness of the finding in the last paragraph, we ran a number of checks; they are presented in Appendix B. As before, we conclude that our finding is robust.

It may be worth stressing that the preceding findings are not necessarily inconsistent with the unitary model. But then one must be will-

[14] In the formal analysis in Sec. V below, we model the two demands separately in a simultaneous system.

[15] Here and below differenced variables are always the wife's value minus the husband's value.

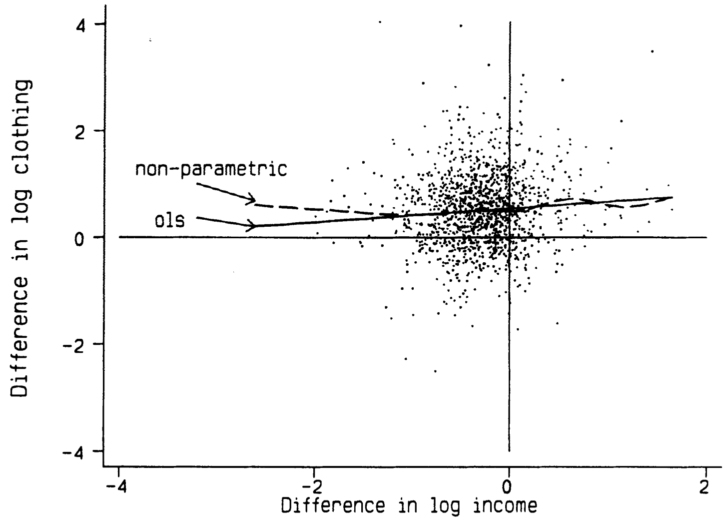

FIG. 1.—Raw data

ing to accept that some of the unobservable variables that explain individual incomes—or wage rates—also affect the common preferences of the household. Our finding is that for a given level of total income, less is spent on women's clothing in households with relatively better off husbands. We believe that this finding is difficult to rationalize in a unitary model.

The principal conclusion we draw from the informal analysis in this section is that incomes matter in demand equations for couples in a way that they do not for singles. There is no obvious way to rationalize the inclusion of income variables in demand equations for couples and not for singles if we assume that these demands are in each case the outcome of the constrained maximization of a single utility function. A very obvious alternative is that husband and wife have differing preferences and that the final allocation decisions made depend on, among other things, who gets the income. All of this leads us to conclude that it is worthwhile to go on and use the structural model of the within-household allocation process developed in Section III. It is to this that we now turn.

V. Empirical Results

We start with the linearized model derived in Appendix A. Even when we do this we are somewhat restricted in how general a model we can begin with since the unrestricted linear form includes quadratics in the candidate variables in the sharing function $\rho(\cdot)$. We include in the demand equations the variables given in the informal section. The most general sharing rule variables we use are the differences in age, education, occupation, and (log) income and the levels of the (log) income of the wife and (log) total expenditure. Note that although own age, education, and occupation are included in each individual's demand, the spouse's age, education, and occupation variables are not. Thus we can include differences of these variables in the sharing rule. Note as well that by including the difference in log incomes and the log of one individual income, we can readily test whether the two incomes have equal (in absolute value) and opposite (in sign) coefficients: the coefficient on the log of the wife's income should be insignificant. As discussed above, the two incomes having opposite signs in the sharing rule will be consistent with many bargaining models.

We begin with a model that allows for the endogeneity of total expenditure. For instruments we use the variables used in the informal section and quadratics in the sharing function variables. When one looks at the exogeneity of total expenditure, the results are very much like those in the informal section. If we include the income variables in the sharing rule, then exogeneity is not rejected. If we exclude the income variables, then the overidentifying restrictions and exogeneity are rejected. We interpret this to mean that we need to include some income variables in the sharing function, but if we do this then total expenditure is exogenous or we have no "good" instruments. We consequently take the total expenditure variable to be exogenous for the sharing process. Formally, this means that we are assuming that the errors on the determination of total expenditure are uncorrelated with the errors on the sharing rule and the clothing equations; that is, we have a recursive set of equations.

Our next tests are designed to reduce the number of variables in the sharing function. In the linearized version we always include total expenditure in the sharing rule and test for the validity of the other variables listed above. To do this we apply a sequence of F-tests on the unrestricted linear model. We found only the differences in age and income to be significant. Consequently, we include only these variables in the sharing function (and total expenditure) when we estimate the nonlinear model.

The estimation of the nonlinear model proceeds in a number of stages. First we estimate each of unrestricted forms (4a)/(5) and (6)/(7) separately. We then estimate them as a seemingly unrelated regression equation (SURE) system (using Telser's SURE estimates[16] to

[16] That is, we take the residuals from each equation estimated on its own, put them in the other equation, and reestimate the two equations.

generate starting values). Then we test the constraints (8). In all of this we found that the likelihood function for the SURE system had many local maxima. Since all the equations are linear if we fix the $\psi(\cdot)$ and $\lambda(\cdot)$ parameters in (5) and (7), respectively, we can use grid searches over these parameters to ensure that we have found the global maximum. It is this need to grid search over the parameters of the sharing function that makes it virtually impossible to begin with the nonlinear model with more than a very few variables in the $\psi(\cdot)$ and $\lambda(\cdot)$ functions. Hence the use of the linearized model to cut down on the **z** variables. Note that since we cannot use Wald tests for the significance of particular variables in a nonlinear system (see, e.g., Gregory and Veall 1985), we have to use likelihood ratio tests. Using likelihood ratio tests on systems that have multiple local maxima is also very time consuming, which is yet another reason for doing some of the initial model selection on the linearized model. When estimating the nonlinear model, we also set δ_0 to zero so that each person's share is one-half at the mean of the data (see the discussion toward the end of Sec. III).

In table 1 we present the estimates of the sharing rule parameters from equations (4a) and (6). Columns 1 and 2 refer to women's and men's clothing, respectively; column 3 refers to the system with the equality of the sharing rule parameter estimates set equal (i.e., with [8] imposed). The $\chi^2(3)$ statistic for constraining the parameters equal is 4.11, which has a probability of 25 percent under the null that they are equal. We interpret this to mean that we can treat clothing as assignable in a caring/collective model. If we impose this restriction, then all the parameters are relatively well determined.

As can be seen, the coefficients on the differenced variables are

TABLE 1

System Estimates of Sharing Rule Parameters

Sharing Rule Variable	Unrestricted: Women's Clothing (4a) (1)	Unrestricted: Men's Clothing (6) (2)	Restricted (8) (3)
Wife's age − husband's age	.170	.139	.144
	[49.1%]	[3.80%]	[3.31%]
Wife's log(income) − husband's log(income)	.111	.019	.026
	[.95%]	[.29%]	[.005%]
Log(total expenditure)	1.495	.631	.618
	[28.6%]	[.32%]	[.28%]

Note.—Figures in brackets are probabilities of likelihood ratio statistics under the null that the parameter is zero.

positive so that older and higher-income partners receive more of total expenditure. The coefficient on total expenditure in the sharing rule is also positive. This implies that the wife receives proportionately more as expenditure goes up. Since total expenditure is increasing with lifetime wealth, this implies that women receive more in wealthier households, with relative income shares and age differences held constant.

Figures 2 and 3 give some idea of the magnitudes implied by these estimates. In figure 2 we plot the predicted share of the wife in total expenditure against the wife's share in household (gross) income. We set the ages equal and set total expenditure to the first, second, and third quartiles in our sample. Once again we remind the reader that we have fixed the constant in the sharing function so that the share is one-half when there is no difference in incomes or ages and the household has mean total expenditure. Before discussing the substantive implications of our estimates, we note that all the curves in figures 2 and 3 are close to linear. This gives us some confidence that the initial investigation on the linearized model is not too biased.

Figure 2 shows clearly the two main features of our parameter estimates. First, the share in total expenditure is increasing in the share of income but only modestly. Going from supplying 25 percent of household income to supplying 75 percent (holding total expenditure constant) raises the share in total expenditure by about 2.3 percent. On the other hand, the effect of total expenditure, when the wife's share in income is held constant, is quite substantial. A 60

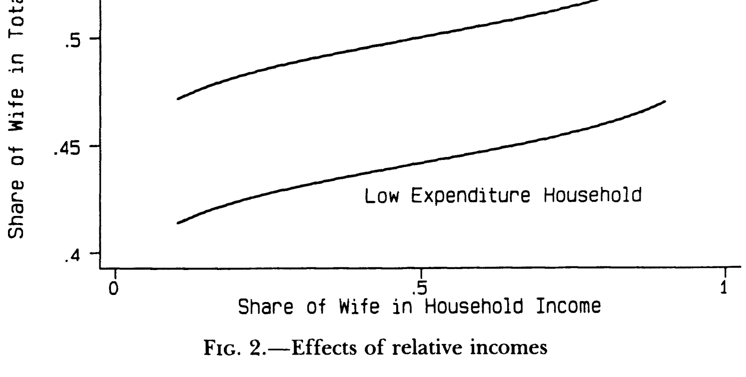

FIG. 2.—Effects of relative incomes

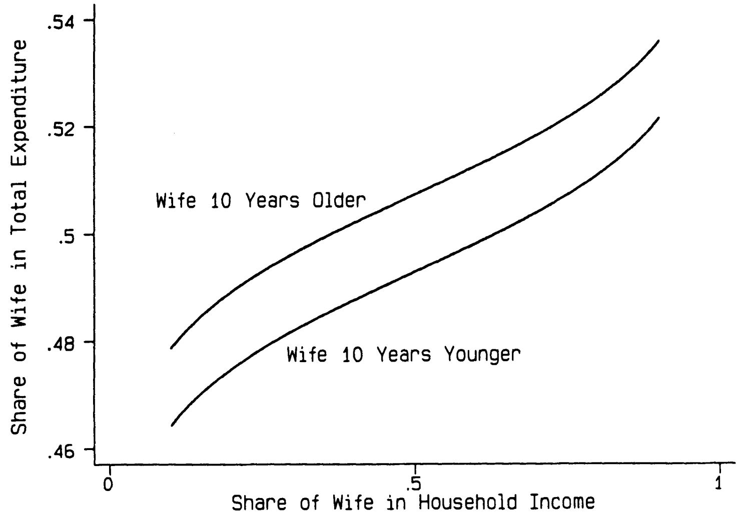

FIG. 3.—Effect of age differences

percent increase in total expenditure (going from the low curve to the high curve) increases the wife's share by about 12 percent. Thus the two effects together can lead to quite sizable changes in the intrahousehold allocation of expenditure. The share of the wife can range from about 40 percent to about 60 percent. Finally, figure 3 indicates that, while statistically significant, the effect of age differences is small: going from being 10 years younger to 10 years older raises the share by less than 2 percent.

VI. Conclusions

There is clear evidence from many sources that households do not behave as though they are maximizing a single criterion. Section IV of this paper adds to that evidence: individual incomes matter for clothing demands for couples in a way that income does not for singles. If we accept that we need to go beyond the single-utility model, then the next step is to try to sort out what goes on inside the household. This is no mean task given the sort of data we currently have. Indeed many will feel that it is impossible.

In this paper we have invoked a number of assumptions (efficiency, caring preferences, and the assignability of clothing) in order to identify some of what goes on. As we have seen, under these assumptions we can identify everything about how intrahousehold sharing is affected by factors such as relative ages, incomes, and household total expenditure. On the other hand, we have also shown that the location of the sharing rule (how much each person receives at the mean of the data) is not identified.

The only factors that seem to affect sharing within the household are the differences in ages and incomes of the members and the wealth (strictly, the total expenditure) of the household. These effects are highly significant in a statistical sense. They also suggest that the influence of differential incomes and wealth on intrahousehold allocation can be fairly substantial. To illustrate, in a poor household in which the wife's share in income is only 25 percent of the total household income, she receives 42 percent of total expenditure. At the other extreme, in a wealthy household in which she receives 75 percent of the income, she has a 58 percent share in total expenditure. A final point is that our identifying assumptions generate testable restrictions on behavior. We found that they were not rejected by the data, which seems to confirm ex post the relevance of our approach.

Our work should clearly be seen as a first step in what we believe to be a fruitful direction. Household economics (with the obvious exception of Becker [1981]) has not taken individualism seriously enough. We believe that individuals, not households, are the basic decision units and that, as argued in Chiappori et al. (1993), the burden of proof should shift onto those who would claim that the unitary model is the rule and the collective model the exception.

Appendix A

Linearization and Identification

Linearization

Our linearization rests on two familiar approximations. First, for the logistic given in (3), we have

$$\rho \simeq \frac{1 + (\psi/2)}{2}$$

for ψ around zero. The other approximation is the very familiar $\ln(1 + \epsilon) = \epsilon$ for ϵ close to zero. From these approximations and (3) and (5) we have

$$\begin{aligned}\ln(\rho) &= \ln(0.5) + \ln\left(1 + \frac{\psi}{2}\right)\\ &= \ln(0.5) + \delta_0 + \boldsymbol{\gamma}'\mathbf{z}^d + \theta \ln x \qquad \text{(A1)}\\ &= \delta_A + \boldsymbol{\gamma}'\mathbf{z}^d + \theta y\end{aligned}$$

if ψ is close to zero (i.e., if each partner's share is close to 0.5).

Using this approximation, we can rewrite (4a) as

$$\begin{aligned}\ln a = (\alpha_0 + \alpha_x\delta_A + \alpha_q\delta_A^2) + \boldsymbol{\alpha}_A'\mathbf{y}^A + (\alpha_x + 2\alpha_q\delta_A)\boldsymbol{\gamma}'\mathbf{z} \\ + (1+\theta)(\alpha_x + 2\alpha_q\delta_A)\xi + \alpha_q(1+\theta)^2\xi^2 \\ + 2\alpha_q(1+\theta)\boldsymbol{\gamma}'(\xi^*\mathbf{z}) + \alpha_q(\boldsymbol{\gamma}'\mathbf{z})^2,\end{aligned} \tag{A2}$$

where $\xi = \ln x$. Corresponding to this nonlinear equation there is an unrestricted equation linear in quadratics of $(\xi, \mathbf{z})$:

$$\ln a = \pi_0 + \boldsymbol{\pi}_A'\mathbf{z}^A + \boldsymbol{\pi}_d'\mathbf{z} + \pi_x\xi + \pi_q\xi^2 + \boldsymbol{\pi}_c'(\xi^*\mathbf{z}) + \boldsymbol{\pi}_e'(\mathbf{z})^+, \tag{A3}$$

where $(\mathbf{z})^+$ denotes the vectorized outer product of $\mathbf{z}$ with redundant components removed. Thus if $\mathbf{z}$ is a k vector, then $(\mathbf{z})^+$ is the $k(k-1)/2$ vector $(z_1^2, z_1z_2, \ldots, z_1z_k, z_2^2, \ldots, z_k^2)$.

Using the same approximations, we can also derive linearized forms for the men's clothing equation:

$$\begin{aligned}\ln b = (\beta_0 + \beta_x\delta_B + \beta_q\delta_B^2) + \boldsymbol{\beta}_B'\mathbf{z}^B - (\beta_x + 2\beta_q\delta_B)\boldsymbol{\gamma}_B'\mathbf{z} \\ + (1-\theta_B)(\beta_x + 2\beta_q\delta_B)\xi + \beta_q(1-\theta_B)^2\xi^2 \\ - 2\beta_q(1-\theta_B)\boldsymbol{\gamma}_B'(\xi^*\mathbf{z}) + \beta_q(\boldsymbol{\gamma}_B'\mathbf{z})^2\end{aligned} \tag{A4}$$

with unrestricted form

$$\ln b = \tau_0 + \boldsymbol{\tau}_B'\mathbf{z}^B + \boldsymbol{\tau}_d'\mathbf{z}^d + \tau_x\xi + \tau_q\xi^2 + \boldsymbol{\tau}_c'(\xi^*\mathbf{z}) + \boldsymbol{\tau}_e'(\mathbf{z})^+. \tag{A5}$$

Identification

If we use just the women's clothing equation, then we have the following question: Given consistent estimates of the parameters of (A3), can we identify the parameters of (A2)? To look at this, we equate coefficients in (A2) and (A3):

$$\pi_0 = \alpha_0 + \alpha_x\delta_A + \alpha_q\delta_A^2, \tag{I1}$$

$$\pi_A = \alpha_A, \tag{I2}$$

$$\boldsymbol{\pi}_d = (\alpha_x + 2\alpha_q\delta_A)\boldsymbol{\gamma}, \tag{I3}$$

$$\pi_x = (1+\theta)(\alpha_x + 2\alpha_q\delta_A), \tag{I4}$$

$$\pi_q = \alpha_q(1+\theta)^2, \tag{I5}$$

$$\boldsymbol{\pi}_c = 2\alpha_q(1+\theta)\boldsymbol{\gamma}, \tag{I6}$$

$$\boldsymbol{\pi}_e = \alpha_q(\boldsymbol{\gamma})^+, \tag{I7}$$

where, as before, $(\boldsymbol{\gamma})^+$ denotes the vectorized outer product of $\boldsymbol{\gamma}$ with redundant elements ($\gamma_i\gamma_j$ for $i < j$) removed.

As can be seen from (I2), the α_A parameters are just identified; we can ignore (I2) from now on. To look at the identification of the other parameters, denote the number of $\mathbf{z}$ variables (i.e., the dimension of the $\boldsymbol{\gamma}$ vector) by n_d. We consider three cases, depending on whether $n_d = 0$, $n_d = 1$, or $n_d \geq 2$.

Case 1: $n_d = 0$

We have three equations—(I1), (I4), and (I5)—in five unknowns (α_0, α_x, δ_A, α_q, and θ); we cannot identify the latter uniquely.

Case 2: $n_d = 1$

We have six equations—(I1) and (I3)–(I7)— in six unknowns (α_0, α_x, δ_A, α_q, $\boldsymbol{\gamma}$, and θ). The Jacobian of the partials of the right-hand sides of these equations with respect to the parameters can be shown to be singular; hence the parameters are not uniquely identified. On the other hand, the conditions (I3)–(I7) do imply some restrictions. For example (note that $n_d = 1$ implies that $\boldsymbol{\gamma} \neq 0$),

$$\frac{\pi_x}{\pi_d} = \frac{2\pi_q}{\pi_c} = \frac{\pi_c}{2\pi_e} = \frac{1 + \theta}{\boldsymbol{\gamma}}.$$

Case 3: $n_d > 1$

We shall consider $n_d = 2$, so that $\boldsymbol{\gamma} = (\gamma_1, \gamma_2)'$ and $(\boldsymbol{\gamma})^+ = (\gamma_1^2, \gamma_1\gamma_2, \gamma_2^2)$. Rewriting (I7), we have

$$\pi_{e1} = \alpha_q \gamma_1^2, \tag{I7a}$$

$$\pi_{e2} = \alpha_q \gamma_1 \gamma_2, \tag{I7b}$$

and

$$\pi_{e3} = \alpha_q \gamma_2^2. \tag{I7c}$$

There are three equations in three unknowns. Since there are overidentifying restrictions ($[\pi_{e2}]^2 = \pi_{e1}\pi_{e3}$), at best we can identify only two parameters. In fact it is trivial to show that the Jacobian of the right-hand side has at most rank 2.

Appendix B

Diagnostics for the Informal Investigation

We present our investigation only for the couples. Similar examinations were made of the single male and single female results.

In figure B1 we plot the residuals from regressing the two variables in figure 1 on the other right-hand-side variables (including the total expenditure variables). This added variable (or partial regression) plot is useful since it gives some idea of the relationship between the two variables to hand, conditioning on the other right-hand-side variables (see Chaterjee and Hadi [1988] for an account of the diagnostics we use here and below). If income is not significant in the demand equations, then there should not be any significant relationship between the two sets of residuals plotted. In fact, though, we see that there is evidence of a positive relationship (the OLS slope coefficient is .16 with a standard error of .04). More important, the

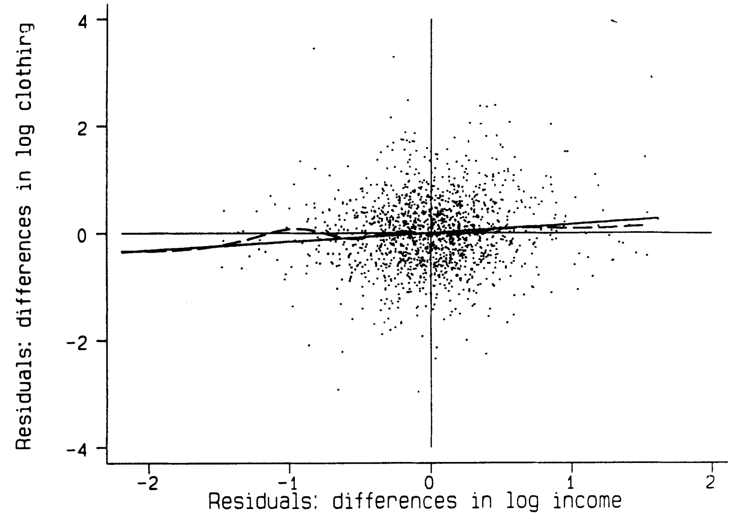

FIG. B1.—Residuals

nonparametric fit is very close to the OLS line. This is consistent with our finding that it is only the difference in log incomes that is important. It also indicates that our positive result is not being driven by outliers.

To investigate the robustness of our findings further, in figures B2 and B3 we present some more diagnostics. The first figure is a leverage-residual plot, that is, a plot of the squared standardized residuals against the "hat" statistic for each observation. Observations with a large residual (e.g., observa-

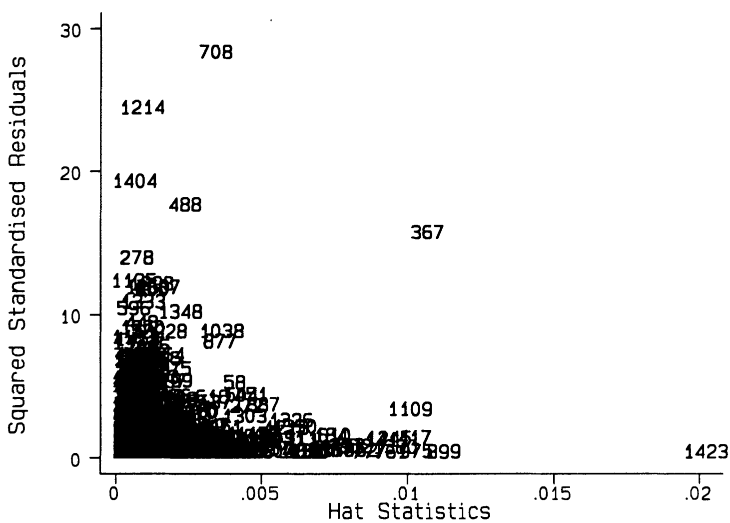

FIG. B2.—Diagnostics

FIG. B3.—DF-betas

tion 708) are ones that are fit badly; those with a large hat statistic (e.g., observation 1,423) are influential observations. Portmanteau diagnostic statistics such as the Cook statistic and the DFFITs statistic can informally be seen as weighted products of these two statistics so that observations that are influential and that fit badly (e.g., observation 367) are likely to give high values for these statistics. The point of identifying outliers is *not* to find points to throw out[17] but rather to identify observations that might be unduly influencing our qualitative inferences so that we can look more closely at these observations to see whether they are in any way anomalous. In this case we could not find anything "wrong" with observations 367, 708, or 1,423 (or other similar points). Excluding various subsets of high-influence/poorly fitted observations did not change the qualitative results.

The final figure presents a rather more focused diagnostic: the DF-betas statistic. The DF-betas value for a particular observation and coefficient shows how much influence that observation has on the estimated coefficient value. In figure B3 we plot the DF-betas statistic for the coefficient on the difference in log incomes against that variable. For example, if we remove observation 367, then the estimated coefficient on the difference in log incomes falls; conversely, observation 708 is pulling the coefficient toward zero. Note that in this figure extreme values for the difference in log incomes variable tend to have higher DF-betas statistics; this is simply reflecting the fact that extreme values have more influence. Once again we use this diagnostic plot to identify which points are particularly influential for our finding that relative

[17] Throwing out valid influential points has an obvious and deleterious effect on efficiency. Hence it is very bad practice to "trim" the sample by removing observations with very high or very low values for particular variables.

incomes seem to matter in the demand equations. Removing the more positive DF-betas observations, of course, reduces the coefficient on the differences in income variable, but we have to remove an implausibly high proportion of the data to reduce this variable to insignificance.

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