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Matching, sorting and wages ☆

Jeremy Lise [a,b], Costas Meghir [c,d,b], Jean-Marc Robin [e,a]

[a] University College London, United Kingdom
[b] IFS, United Kingdom
[c] Yale University, United States
[d] NBER, United States
[e] Sciences Po, Paris, France

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ABSTRACT

We develop an empirical search-matching model which is suitable for analyzing the wage, employment and welfare impact of regulation in a labor market with heterogeneous workers and jobs. To achieve this we develop an equilibrium model of wage determination and employment which extends the current literature on equilibrium wage determination with matching and provides a bridge between some of the most prominent macro models and microeconometric research. The model incorporates productivity shocks, long-term contracts, on-the-job search and counter-offers. Importantly, the model allows for the possibility of assortative matching between workers and jobs due to complementarities between worker and job characteristics. We use the model to estimate the potential gain from optimal regulation and we consider the potential gains and redistributive impacts from optimal unemployment benefit policy. Here optimal policy is defined as that which maximizes total output and home production, accounting for the various constraints that arise from search frictions. The model is estimated on the NLSY using the method of moments.

1. Introduction

Labor market imperfections may justify labor market interventions. Within a competitive framework regulation would be welfare reducing, as it would typically reduce employment and increase insiders' wages. By contrast, any friction constraining the allocation of workers to jobs inevitably allows some agents to appropriate a greater share of the rent than a central planner would deem fit. Indeed, if there are important complementarities in production, mismatch may produce substantial welfare losses relative to the first best or a constrained planner.

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E-mail addresses: j.lise@ucl.ac.uk (J. Lise), c.meghir@yale.edu (C. Meghir), jeanmarc.robin@sciencespo.fr (J.-M. Robin).

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We develop a search-matching model in which workers with different abilities are assigned to different tasks. Our model combines elements from key papers in the equilibrium search literature. Thus we allow for endogenous job destruction because of productivity shocks, drawing from the seminal paper of Mortensen and Pissarides (1994). To this framework we introduce two sided heterogeneity with potential complementarities between job and worker productivity following Shimer and Smith (2000). In this way we can investigate sorting in the labor market, one of our motivating interests. To account for job-to-job transitions and to better explain wage growth we allow for on-the-job search, which is not present in Shimer and Smith (2000). Wage determination is drawn from Postel-Vinay and Robin (2002), Dey and Flinn (2005) and Cahuc et al. (2006). In other words we combine bargaining over the surplus (for workers out of unemployment or moving to a new job) with Bertrand competition when a poaching firm is involved.

In this way, our model provides a bridge between an essentially theoretical literature on allocation of heterogeneous workers to jobs,[1] and the large microeconometric literature on job mobility and wage dynamics.[2] The framework that we develop allows for frictions and inefficiencies in the labor market. The frictions are due to the time it takes to locate jobs, which means that individuals will spend time searching for a job while unemployed and if working they are likely to be mismatched (if there are complementarities in production). One may argue that such frictions are inevitable; nevertheless it is important to understand the welfare loss that they cause relative to the benchmark of a frictionless economy, because this provides a measure of how dominant search frictions are in determining economic outcomes.[3] More importantly, our model can quantify the welfare loss from inefficiencies that can potentially be addressed by labor market regulation: first, the number of job seekers cause congestion making it harder for others to find jobs – this is a standard externality in models with endogenous arrival rates. Beyond that the potential complementarities between worker and firm productivities, the possibility of on-the-job search and the lack of commitment on the worker side allows for the possibility of another inefficiency: workers and jobs are sometimes willing to form matches whose flow output is lower than the combined cost of a vacancy and the lost out-of-work benefit. On the one hand the job, with its local monopsony power manages to extract sufficient surplus to make it worth hiring the worker; on the other hand the worker prefers the resulting current loss to the increased flow of income when out of work because being in a job provides a better outside offer to negotiate a wage once an alternative offer arrives. These features may be important in the labor market and our model can quantify their importance for welfare.

We estimate our model using panel data on workers and use the estimates to quantify how much sorting there is with respect to unobserved characteristics. A first set of studies interested in this question follow Abowd et al. (1999, AKM) and assess the degree of assortative matching in the labor market by calculating the correlation between worker and firm fixed effects estimated by a panel-data regression of individual wages on workers' and employers' indexes.[4] They typically find non-significant or negative correlations. Andrews et al. (2008) show that this estimated correlation is contaminated by a spurious statistical bias and propose a bias-corrected estimator. Using German IAB data, they find that the negative OLS estimate is turned into a positive number after applying the bias correction (0.23 instead of somewhere in the range $[-0.19, -0.15]$).

However, Eeckhout and Kircher (2011) strongly argue against the possibility of identifying assortative matching in labor markets by this approach, bias-corrected or not. This is because the surplus of a match is not a monotonic function of worker and firm characteristics in general. Depending on the distribution of matches around the optimal, Beckerian allocation, a positive or a negative AKM correlation can be estimated irrespective of the sign of the correlation between workers' and firms' true unobserved characteristics in the population of active matches. In the previous working paper we also find that the AKM correlation is misleading and demonstrate that positive sorting with respect to unobserved characteristics may induce a negative correlation in the worker and firm effects estimated on a panel of wages. Lopes de Melo (2009), Hagedorn et al. (2012), Bagger and Lentz (2014) reach similar conclusions with different models.

Note that this argument invalidating a structural interpretation of AKM is implicit in Gautier and Teulings (2006), who estimate a regression model of log wages on a quadratic function of worker and employer types. These types are calculated by projecting log wages separately on workers' and employers' observed characteristics. Gautier and Teulings's estimation rests on various parametric restrictions, but nevertheless convincingly claims 1) that the wage equation is nonlinear in workers' and employers' types, and 2) that they are positively correlated.

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1 See also Shi (2001), Teulings and Gautier (2004), Moscarini (2005) and Gautier et al. (2010) for alternative approaches to two-sided matching models without and with on-the-job search.

2 See amongst others MaCurdy (1982), Altonji and Shakotko (1987), Abowd and Card (1989), Topel (1991), Topel and Ward (1992), Meghir and Pistaferri (2004), Altonji and Williams (2005), Guvenen (2007), Bonhomme and Robin (2009), Guvenen (2009), Low et al. (2010), Lise (2013), Altonji et al. (2013).

3 See also Teulings and Gautier (2004), Gautier and Teulings (2015) for an approach to measuring the welfare cost of search frictions. Their approach differs both theoretically and empirically from ours: Their production function is a quadratic in the distance of the worker and job characteristics – it is not estimated. There are no shocks to firm productivity. The relative efficiency of on-the-job search is set at different values but not estimated from the data. Finally, wages are approximated with observed and unobserved determinants assumed to be orthogonal.

4 See Goux and Maurin (1999) and Abowd et al. (2009) who present results for French and U.S. matched employer–employee data, and Gruetter and Lalive (2009) for Austrian data.

Identification of complementarity and sorting, given that we use a panel of workers' wages and labor market transitions extracted from the NLSY,[5] is also hard to prove or disprove theoretically. However, we argue that sorting can be seen in the way wage and employment mobility vary as a function of the length of time spent working following an unemployment spell. Due to search frictions, the cohort of workers entering the labor market following a spell out of work will start off mismatched, but through on-the-job search they will become better and better matched with time spent in the labor market. If the tendency to sort in equilibrium is strong this will result in wages spreading out as workers sort themselves. Monte Carlo simulations for the Simulated Method of Moment estimator that we have implemented here seem to suggest that the set of moments we match do identify the degree of sorting. We find that the NLSY data are best fitted by our model assuming no complementarity and zero sorting for workers with high-school education or less and positive complementarity and sorting for college-graduates.[6]

Finally, our model offers an empirical framework for understanding employment and wage determination in the presence of firm–worker complementarities, search frictions and productivity shocks. As a result it offers a way for evaluating the extent to which regulation may be welfare improving and can evaluate the impact of specific policies such as unemployment benefit. In a search framework with match complementarities unemployment benefit can have ambiguous effects on employment and total output. On the one hand, it allows workers to be more picky and form better matches, which comes at the cost of longer unemployment spells and higher unemployment. On the other hand, the fact that higher quality matches will be formed may induce firms to create more jobs, increasing the contact rate and potentially reducing unemployment duration. Our framework allows this effect to be quantified (see also Acemoglu and Shimer, 2000). And, in addition, it allows us to analyze the effect of such policies on the distribution of welfare thus showing who pays and who benefits from such a policy in this non-competitive environment.

We find that the degree to which labor market interventions are justified depends on whether we are looking at the low or high skilled markets. Our finding that the market for unskilled labor (high school graduates or less) is characterized by an extremely low degree of complementarity implies that mismatch is not very costly. If we reallocated the employed workers in this group optimally the difference in (steady state) welfare would be 1.1 percent. On the other hand, among the college graduates, optimally reallocating the employed workers produces a difference in (steady state) welfare of 6.8 percent. We also find that search frictions are significant. If we do the same experiment with full employment the welfare changes are 7.8 and 19.6 for the high school and college groups respectively. The interaction of search frictions and the cost of mismatch (the degree of complementarity) differ between the two labor markets, implying that a social planner who is constrained by these friction could attain a welfare increase of 2 percent for the high school group but only 0.7 percent for the college group, largely by trading off the level of employment against the cost of creating vacancies. Finally, we find that if we limit the planner to an optimal unemployment benefit program, this can go a long way to realizing the potential welfare gains for the high school group. However, it is ineffectual for the college educated group as the resulting employment distortions outweigh the gains from improved match quality.

The paper proceeds as follows. Section 2 describes the model, Section 3 and Section 4 present the estimation procedure. Sections 5 and 6 describe the data and the choice of moments. Section 7 presents the results of estimation, analyses the fit and discusses estimation of the degree of sorting. Section 8 presents the welfare analysis, the cost of search frictions, and analyses in detail the optimal unemployment benefit policy. Section 9 concludes.

2. The model

We build a model of individual employment and wage dynamics, with heterogeneous workers and jobs and with productive complementarities at the match level. This model draws from Mortensen and Pissarides (1994), as far as the process of match creation and destruction is concerned, and from Postel-Vinay and Robin (2002), Dey and Flinn (2005) and Cahuc et al. (2006) in order to incorporate on-the-job search in the Mortensen–Pissarides model. In addition, we draw from Postel-Vinay and Turon (2010) the renegotiation mechanism for wages following firm-level productivity shocks. The wage dynamics follow from the process of search and matching and from firm-level productivity shocks, but entirely abstract from human capital accumulation and idiosyncratic ability shocks. While important, incorporating human capital accumulation would complicate matters beyond the scope of this paper.[7]

2.1. Workers, jobs and matches

In this economy, there is a fixed measure of infinitely lived individuals that is normalized to one. They differ from each other according to ability, and ability differences are permanent, continuously distributed across workers, and observable by

[5] We preferred to stick to standard worker panel data for two reasons. First, matched employer–employee data are less universal and not always easily accessible to researchers. Second, value-added per worker (used by Cahuc et al., 2006; Bagger et al., 2014; Bagger and Lentz, 2014) does not measure well the labor productivity of a single job isolated from the other jobs in a firm.

[6] There may be other reasons for increasing inequality over time for a cohort, including permanent shocks to productivity as in Meghir and Pistaferri (2004) and even heterogeneous accumulation of human capital as in Guvenen (2007). These alternative explanations have not been explored here and we believe that Identification may require richer information, such as matched employer–employee data.

[7] See Bagger et al. (2014) for a recent estimation of a search model with human capital and shocks to worker ability rendered tractable by assuming piece-rate contracts as in Barlevy (2008). Yet Bagger et al. assume away any possibility of sorting between workers and jobs.

all agents, but not by the researcher. Let $x \in [\underline{x}, \overline{x}]$ denote worker ability and let $\ell(x)$ be its PDF. We also denote by $u(x)$ the (endogenous) number of x among the unemployed, with $U = \int_{\underline{x}}^{\overline{x}} u(x)\,\mathrm{d}x$.

Workers are matched pairwise to jobs or tasks. Jobs are characterized by a technological factor, say labor productivity, denoted y, which is also continuous and observable by all agents, but not the researcher. There is no match-specific type; match specificity results from the combination of a worker type x with a job type y; these together with the current wage paid to the worker will form the relevant state space for the worker and the firm value. Jobs are subject to persistent idiosyncratic productivity shocks, which can trigger separations and wage renegotiations. So, if an employee quits, the job continues to exist and is available for another match. These idiosyncratic productivity shocks may reflect changes in the product market (shifts in demand) or events such as changes in work practices or investments (which are not modeled here). Specifically, we assume that y fluctuates according to a jump process: shocks arrive at a rate δ, and a new productivity level y' is drawn from a distribution with PDF $\gamma(y')$ on $[\underline{y}, \overline{y}]$. Thus the size of δ determines the persistence of the shocks.

The number of available type-y jobs in the economy is $n(y)$, and the total number of jobs is $N = \int_{\underline{y}}^{\overline{y}} n(y)\,\mathrm{d}y$. Jobs are either vacant or matched. The number of vacant type-y posts is $v(y)$ and the resulting total number of vacancies is denoted by $V = \int v(y)\,\mathrm{d}y$. Both distributions are endogenous due to a free entry mechanism that will be described later.

A match between a worker of type x and a job of productivity y produces a flow of output $f(x, y)$. We will specify this function to allow for the possibility that x and y are complementary in production, implying that sorting will increase total output. However, we wish to determine this empirically, as it is important both for understanding the labor market and for evaluating the potential effects of regulation. Matches can end both endogenously, as we characterize later, and exogenously. We denote by ξ the rate of exogenous job destructions, defined as an event that is unrelated to either worker or job characteristics.

Equilibrium will result in a joint distribution of x and y, the distribution of matches. We denote this by $h(x, y)$, which satisfies the balance equations:

$$\int h(x, y)\,\mathrm{d}y + u(x) = \ell(x), \tag{1}$$

and

$$\int h(x, y)\,\mathrm{d}x + v(y) = n(y). \tag{2}$$

2.2. Meetings and match formation

We relate the aggregate number of meetings between vacancies and searching workers through a standard aggregate matching function $M(\cdot,\cdot)$. This takes as inputs the total number of vacancies V and the total amount of effective job seekers $U + s(1 - U)$, where s is the relative search intensity of employed workers vis-a-vis unemployed. The matching function is assumed to be increasing in both arguments and exhibit constant returns to scale.

For the purpose of exposition it is useful to define

$$\kappa = \frac{M(U + s(1 - U), V)}{[U + s(1 - U)]\,V},$$

which summarizes the effect of market tightness in a single variable. In a stationary equilibrium κ is constant, but it is not invariant to policy, and it is important to allow it to change when evaluating interventions or counterfactual regulations. Note that while $M(\cdot,\cdot)$ governs the aggregate number of meetings, whether or not a meeting translates into a match will be determined by the decisions of individual workers and firms.

The matching parameter κ allows us to calculate all relevant meeting rates. The instantaneous rate at which an unemployed worker meets a vacancy of type y is $\kappa V \cdot \frac{v(y)}{V} = \kappa v(y)$. The instantaneous probability for any vacancy to make a contact with an unemployed worker of type x is $\kappa u(x)$. Employed workers are contacted by jobs of type y with instantaneous probability $s\kappa v(y)$. Finally, the rate at which vacancies are contacted by a worker x employed at a job y is $s\kappa h(x, y)$.

Individuals and jobs are risk neutral and we assume efficiency, in the sense that any match where the surplus is positive will be formed when the worker and the job meet. Under these conditions we can characterize the set of equilibrium matches and their surplus separately from the sharing of the surplus between workers and jobs.

Let $W_0(x)$ denote the present value of unemployment for a worker with characteristic x. This will reflect the flow of income when out of work (or home production) and the expected present value of income that will arise following a successful job match. Similarly $\Pi_0(y)$ denotes the present value to a job of posting a vacancy arising from the expected revenues of employing a suitable worker net of expected posting costs. Let also $W_1(w, x, y)$ (respectively $\Pi_1(w, x, y)$) denote the present value of a wage contract w for a worker x employed at a job y (respectively the firm's discounted profit).

The surplus of an (x, y) match is defined by

$$S(x, y) = \Pi_1(w, x, y) - \Pi_0(y) + W_1(w, x, y) - W_0(x). \tag{3}$$

For a pair (x, y) a match is feasible and sustainable if the surplus is nonnegative, $S(x, y) \geq 0$. We focus on equilibria such that for all x there is at least one y such that $S(x, y) \geq 0$.

2.3. Wages

Different wages are negotiated when leaving unemployment, upon poaching, or after a shock to the job productivity.

2.3.1. Wage negotiation with unemployed workers

The wage for a worker transiting from unemployment is $w = \phi_0(x, y)$ and we assume that it is set to split the surplus according to Nash bargaining with worker's bargaining parameter β[8]:

$$W_1(\phi_0, x, y) - W_0(x) = \beta S(x, y). \tag{4}$$

A simple assumption would be to set β to zero. However this may lead to the counterfactual implication that for some matches initial wages are negative implying workers pay to obtain a high value job with the potential of future wage increases. Although this is not implausible (unpaid internships) this is never observed in our survey data. By allowing β to be determined by the data we avoid this problem and allow greater flexibility in fitting the facts.

2.3.2. Poaching

Wages can only be renegotiated when either side has an interest to separate if they do not obtain an improved offer, assuming that the match remains viable for both parties. The events that can trigger renegotiation occur when a suitable outside offer is made, or when a productivity shock changes the value of the surplus sufficiently. We consider first the impact of an outside offer.

We assume that incumbent employers respond to outside offers: a negotiation game is then played between the worker and both jobs as in Dey and Flinn (2005) and Cahuc et al. (2006). If a worker x, currently paired to a job y such that $S(x, y) \geq 0$, finds an alternative job y' such that $S(x, y') \geq S(x, y)$, the worker moves to the alternative job. This is because the poaching firm can always pay more than the current one can match. Alternatively, if the alternative job y' produces less surplus than the current job, but more than the worker's share of the surplus at the current job, i.e. $W_1 - W_0(x) < S(x, y') < S(x, y)$ (where W_1 denotes the present value of the current contract), then the worker uses the outside offer to negotiate up her wage. Lastly, if $S(x, y') \leq W_1 - W_0(x)$, the worker has nothing to gain from the competition between y and y' because she cannot make a credible threat to leave, and the wage does not change.

In either one of the first two cases, the worker ends up in the higher surplus match. If the worker changes firms, she uses the surplus at the previous match as the outside option when bargaining. If the surplus at the current match exceeds the surplus at the poaching firm we assume the incumbent firm makes a take it or leave it offer and there is no bargaining. Assume $S(x, y) \geq S(x, y')$. The bargained wage when switching firms (from y' to y) in this case is $w = \phi_1(x, y, y')$ such that the worker obtains the entire surplus of the incumbent job plus a share of the incremental surplus between the two jobs, i.e.

$$\begin{aligned} W_1(\phi_1, x, y) - W_0(x) &= S(x, y') + \beta \left[S(x, y) - S(x, y') \right] \\ &= \beta S(x, y) + (1 - \beta) S(x, y'). \end{aligned} \tag{5}$$

The share of the increased surplus β accruing to the worker will be determined empirically. If the surplus in the current match is greater than with the poaching firm, the incumbent firm retains the worker by offering a wage $w = \phi_2(x, y, y')$ that delivers to the worker the total surplus at the poaching firm, precluding barraging, i.e.,

$$W_1(\phi_2, x, y) - W_0(x) = S(x, y'). \tag{6}$$

In our approach there is an asymmetry between workers and firms because the latter do not search when the job is filled. As a result they do not fire workers when they find an alternative worker who would lead to a larger total surplus, nor do they force wages down when an alternative worker is found whose pay would imply an increased share for the firm. We decided to impose this asymmetry because in many institutional contexts it is hard for the firm to replace workers in this way. Moreover, we suspect that even when allowed firms would be reluctant to do so in practice. We do, however, allow firms to fire a worker when the current surplus becomes negative and then immediately search for a replacement.

2.3.3. Productivity shocks

Another potential source of renegotiation is when a productivity shock changes y to y' thus altering the value of the surplus. If y' is such that $S(x, y') < 0$, the match is endogenously destroyed: the worker becomes unemployed and the job will post a vacancy.

[8] In what follows we will distinguish between wages obtained when moving from unemployment ϕ_0, when switching jobs ϕ_1 and when the firms retains the worker by responding to an outside offer ϕ_2.

Suppose now that $S(x, y') \geq 0$. The value of the current wage contract w becomes $W_1(w, x, y')$. Future pay negotiations whether due to productivity shocks or competition with outside offers will be affected by the new value of the match. However, the current wage may or may not change. If the wage w is such that the worker is still obtaining at least as much as her outside option, without taking more than the new surplus, $0 \leq W_1(w, x, y') - W_0(x) \leq S(x, y')$, neither the worker nor the job has a credible threat to force renegotiation: both are better off with the current wage w being paid to the worker than walking away from the match to unemployment and to a vacancy respectively. In this case there will be no renegotiation. If, however, $W_1(w, x, y') - W_0(x) < 0$ or $W_1(w, x, y') - W_0(x) > S(x, y')$ (with $S(x, y') \geq 0$) then renegotiation will take place because a wage can be found that keeps the match viable and each partner better off within the match relative to unemployment for the worker and a vacancy for the job.

To define how the renegotiation takes place and what is the possible outcome we use a setup similar to that considered by MacLeod and Malcomson (1993) and Postel-Vinay and Turon (2010). The new wage contract is such that it moves the current wage the smallest amount necessary to put it back in the bargaining set. Thus, if at the old contract $W_1(w, x, y') - W_0(x) < 0$, a new wage $w' = \psi_0(x, y')$ is negotiated such that

$$W_1(\psi_0, x, y') - W_0(x) = 0, \tag{7}$$

which just satisfies the worker's participation constraint. If at the new y', $W_1(w, x, y') - W_0(x) > S(x, y')$, a new wage $w' = \psi_1(x, y')$ is negotiated such that the firm's participation constraint is just binding:

$$W_1(\psi_1, x, y') - W_0(x) = S(x, y'). \tag{8}$$

Note that a firm hiring an unemployed worker offers the value of unemployment plus a share of the surplus, whereas after a positive shock to the surplus value there is no Nash bargaining. This may seem both ad hoc and inelegant. However, consider the other situation when the match surplus falls below the worker surplus at current wage. Under Nash bargaining the worker would get the same share of the new surplus as in the other triggering situation, wiping out any wage gains due to outside offers. That does not sound right. Intuitively, the worker should have more bargaining power in one case than in the other. Our assumption (zero bargaining power when the worker values falls below the value of unemployment; full bargaining power when the firm profit falls below the value of a vacancy) may seem a bit extreme, but it is motivated by the idea that the worker's bargaining power should depend on whether renegotiation is wanted by the worker or the employer, and it ensures that the value function of the worker is monotonically increasing in the wage.

Furthermore, wages respond to job specific productivity shocks, but not always in an obvious direction. Separations and pay changes may happen following both good shocks that increase the value of productivity y and bad shocks that decrease it. It is all about mismatch: what matters is what happens to the overall surplus. A positive productivity shock, for example, can imply that the quality of the match becomes worse and the surplus declines, since the outside option of the firm has changed and it may be worthwhile to separate from the current worker and post a vacancy to find a better worker. Conversely a negative productivity shock can improve the surplus if this means the job type is now closer to the optimal one sought by the worker. A shock that reduces the surplus can still lead to a wage increase to compensate the worker who is now matched with a job with fewer future prospects of wage increases. Thus what really matters as far as the viability of the match and the possible options for renegotiation is whether a shock improves or worsens a particular match, measured by whether it leads to an increase or a decrease, respectively, of the surplus.

2.4. Value functions

The next step in solving the model is to characterize the value functions of workers and jobs, which have been kept implicit up to now. These define the decision rules for each agent. We proceed by assuming that time is continuous.

2.4.1. Unemployed workers

Unemployed workers are always assumed to be available for work at a suitable wage rate. While unemployed they receive income or money-metric utility (home production) depending on their ability x and denoted by $b(x)$. Thus the present value of unemployment to a worker of type x is $W_0(x)$, which satisfies the option value equation

$$rW_0(x) = b(x) + \kappa\beta \int S(x, y)^+ v(y)\,\mathrm{d}y, \tag{9}$$

where the subjective discount rate is denoted by r and we define in general $a^+ = \max(a, 0)$. Thus the integral represents the expected value of the surplus of feasible matches given the worker draws from the distribution of vacant jobs $v(y)$. She contacts a job of type y at a rate $\kappa v(y)$ and the match is consummated if the surplus $S(x, y)$ is non-negative, in which case she gets a share β of the surplus.

2.4.2. Vacant jobs

Using (3), (4), and (5), the present value of profits for an unmatched job meeting a worker with human capital x from unemployment is

$$\Pi_1(\phi_0(x, y), x, y) - \Pi_0(y) = (1 - \beta)\, S(x, y),$$

where $1-\beta$ represents the proportion of the surplus retained by the firm and $\Pi_0(y)$ is the value of a vacancy. A job meeting a worker who is already employed will have to pay more to attract her. The value of a job with productivity y, meeting an employed worker in a lower surplus match with productivity y' is

$$\Pi_1(\phi_1(x, y, y'), x, y) - \Pi_0(y) = (1-\beta)\left[S(x, y) - S(x, y')\right].$$

Based on these considerations, the present value of a vacancy for a job with productivity y is

$$r\Pi_0(y) = -c + \delta \int \left[\Pi_0(y') - \Pi_0(y)\right]\gamma(y')\,\mathrm{d}y' + \kappa(1-\beta)\int S(x, y)^+ u(x)\,\mathrm{d}x$$
$$+ s\kappa(1-\beta)\iint \left[S(x, y) - S(x, y')\right]^+ h(x, y')\,\mathrm{d}x\,\mathrm{d}y'. \tag{10}$$

The c is a per-period cost of keeping a vacancy open. The second term reflects the impact of a change in productivity from y to y', assuming that productivity shocks continue to accrue at rate δ when the job is vacant and assuming that the distribution of productivity shocks is $\gamma(y')$. The third term is the expected gain from matching with a previously unemployed worker. The fourth term is the expected gain from poaching a worker who is already matched with another job. The notation $[\cdot]^+$ ensures that integration is over all possible x and y' that increase the current surplus.

As for matches, a post is destroyed if $\Pi_0(y) < 0$. However, we only consider stationary equilibria such that posts exist for ever, ruling out job creation and destruction as we have ruled out the birth and death of workers. As we discuss below our free entry condition is that the lowest productivity job in the support makes zero profits. Assuming that the production function increases in y, in Appendix A we show that $\Pi_0'(y) > 0$. So $\Pi_0(y) > 0$ for all $y \in [\underline{y}, \overline{y}]$ if $\Pi_0(\underline{y}) > 0$. The resulting equilibrium distribution of jobs is

$$n(y) = N\gamma(y), \tag{11}$$

and is thus exogenous up to the number of jobs N. We do this for simplicity so that we do not have to endogenize both the number of jobs and the profitability threshold at the equilibrium.

2.4.3. Employed workers

In order to derive the wage rates we need to define the value of a job to a worker $W_1(w, x, y)$. This is the present value to the worker of a wage contract w for a feasible match (x, y) (if $S(x, y) < 0$ there is no match and no wage). Any wage contract w delivers w in the first unit of time. The continuation value depends on competing events: the job may be terminated with probability ξ; the job may be hit by a productivity shock with probability δ; the worker may be poached with probability $s\kappa V$.

Given the above discussion, the flow value of working at wage rate w in a feasible match (x, y) is determined by the following Bellman equation,

$$[r + \delta + \xi + s\kappa v(\mathcal{A}(w, x, y))][W_1(w, x, y) - W_0(x)]$$
$$= \left(w - b(x) - \kappa\beta \int [S(x, y')]^+ v(y')\,\mathrm{d}y'\right) + \delta \int \left[\min\left\{S(x, y'), W_1(w, x, y') - W_0(x)\right\}\right]^+ \gamma(y')\,\mathrm{d}y'$$
$$+ s\kappa \int_{\mathcal{A}(w,x,y)} \left[\min\left\{S(x, y), S(x, y')\right\} + \beta\left[S(x, y') - S(x, y)\right]^+\right] v(y')\,\mathrm{d}y', \tag{12}$$

where

$$\mathcal{A}(w, x, y) = \left\{y' : W_1(w, x, y) - W_0(x) < S(x, y')\right\}$$

is the set of jobs that can lead to a wage change (either by moving or renegotiation) and $v(A) = \int_A v(y)\,\mathrm{d}y$, for any set A.

On the right hand side the first term is the wage net of the flow value of unemployment (all in parentheses). The second term is the expected excess value to the worker of a productivity shock (times the probability that it occurs): in this case the worker either ends up with the entire new surplus or the new value or indeed nothing if the match is no longer feasible (see equations (7) and (8)). The third line is the expected excess value following an outside offer as in equation (5). The integral is over all offers that can improve the value (whether the worker moves or not).

2.4.4. The match output and joint surplus

Having defined the non-employment value for the worker, the vacancy value for the firm, the value of a contract w to the worker (with a similar expression for the value to the firm), we can now calculate the surplus value $S(x, y)$ defined by equation (3). We show in Appendix A that the match surplus $S(x, y)$ is independent of the current wage contract and is defined by the fixed point in the following equation,

$$(r+\xi+\delta)\,S(x,y) = f(x,y) - b(x) + c - \kappa\beta \int S(x,y')^{+} v(y')\,\mathrm{d}y'$$
$$- \kappa(1-\beta)\int S(x',y)^{+} u(x')\,\mathrm{d}x' - s\kappa(1-\beta)\iint \left[S(x',y) - S(x',y')\right]^{+} h(x',y')\,\mathrm{d}x'\,\mathrm{d}y'$$
$$+ s\kappa\beta \int \left[S(x,y') - S(x,y)\right]^{+} v(y')\,\mathrm{d}y' + \delta \int S(x,y')^{+}\gamma(y')\,\mathrm{d}y'. \tag{13}$$

Note that the surplus of an (x, y) match never depends on the wage. This follows from the Bertrand competition between the incumbent and the poaching firm that is induced by on-the-job search and disconnects the poached employee's outside option from both the value of unemployment and her current wage contract. Thus the Pareto possibility set for the value of the worker and the job is convex in all cases, implying that the conditions for a Nash bargain are satisfied. This contrasts with Shimer (2006) where jobs do not respond to outside offers and where the actual value of the wage determines employment duration in a particular job. This feature also has a computational advantage since the equilibrium distribution of matches can be determined without simultaneously computing the wage rates for workers.

2.5. Steady-state equilibrium

The exogenous components of the model are the distribution of ability $\ell(x)$, the form of the matching function $M(\cdot,\cdot)$ as well as the arrival rate of shocks δ, the transition probability $\gamma(y)$, the job destruction rate ξ, the relative search intensity of employed workers s, the discount rate r, the value of home production (or leisure) $b(x)$, the cost of posting a vacancy c, bargaining power β, and the production function $f(x, y)$.

In equilibrium all agents follow their optimal strategy. The distribution of matches $h(x, y)$, is determined by the steady state flow equation, and the number of jobs is determined by a free-entry mechanism that we make explicit below. The unemployment distribution $u(x)$ and the distribution of vacancies $v(y)$ follow from the balance equations (1) and (2).

2.5.1. Steady-state flow equation

In a stationary equilibrium flows in and out of any worker stock must balance each other out. Let us consider the stocks of existing matches of type (x, y). They can be destroyed for a number of reasons. First, there is exogenous match destruction at a rate ξ; second, with probability δ, the job component of match productivity changes to some value y' different from y, and the worker may move to unemployment or may keep the job forming a new match (x, y'); third, the worker may receive an offer and quit to a higher surplus (x, y') match. On the inflow side, new (x, y) matches are formed when some unemployed or employed workers of type x match with vacant jobs y, or when (x, y') matches are hit with a productivity shock and exogenously change from (x, y') to (x, y).

Equating inflows to outflows, we have for all (x, y) such that the match is feasible (i.e. $S(x, y) \geq 0$),

$$\left[\xi+\delta+s\kappa v\left(\overline{\mathcal{B}}(x,y)\right)\right] h(x,y) = \left[u(x) + sh(x,\mathcal{B}(x,y))\right]\kappa v(y) + \delta\gamma(y)\int h(x,y')\,\mathrm{d}y', \tag{14}$$

where

$$\mathcal{B}(x,y) = \{y' : 0 \leq S(x,y') < S(x,y)\},$$
$$\overline{\mathcal{B}}(x,y) = \{y' : S(x,y') \geq S(x,y)\},$$

are the sets of jobs that would imply $(\overline{\mathcal{B}})$ or not imply $(\mathcal{B})$ an improvement of the match surplus for x. Thus, $s\kappa v\left(\overline{\mathcal{B}}(x,y)\right)$ is the probability of receiving an alternative offer, when employed, from a job that beats the current one.

This equation defines the steady-state equilibrium, together with the accounting equations (1) and (2).

2.5.2. Free entry

At this stage the only parameter that is not yet determined is the number of jobs N, as we only consider equilibria with infinitely lived jobs. Hence, $n(y) = N\gamma(y)$. Job creation is one-shot event and a number N of jobs is created as long as $\Pi_0(\underline{y})$ remains positive.[9]

At the equilibrium, $\Pi_0(\underline{y}) = 0$. Hence,

$$c = \delta\int \Pi_0(y')\gamma(y')\,\mathrm{d}y' + \kappa(1-\beta)\int S(x,\underline{y})^{+} u(x)\,\mathrm{d}x$$
$$+ s\kappa(1-\beta)\int \left[S(x,\underline{y}) - S(x,y')\right]^{+} h(x,y')\,\mathrm{d}x\,\mathrm{d}y'. \tag{15}$$

[9] We implicitly assume that $\Pi_0(\underline{y})$ is decreasing in the number of jobs N. However, this is not at all obvious as Π_0 depends on N in a complicated way.

This condition will determine the number of potential jobs N in the economy. This is a complicated equation as N enters κ via $V = N - L - U$ and U, and κ conditions all other equilibrium variables.

Appendix B provides a simple iterative algorithm that uses these equations and that of the surplus to compute the equilibrium objects.

3. Model specification

We complete the presentation of the model with its parametric specification. The central element of the model is the production function, which drives the potential gains from sorting. Here we specify this to be a CES in the individual and firm characteristic.

$$f(x, y) = A\left[\alpha x^{\rho} + (1-\alpha) y^{\rho}\right]^{\frac{1}{\rho}}.$$

This allows for various degrees of complementarity depending on the estimated value of ρ, including the extreme case of linearity, in which case there are no productivity gains from sorting. The question of nonparametric identification of the degree of complementarity is addressed in another paper (Lamadon et al., 2014). Following the data description we offer a heuristic argument on the how the parameters are identified here.

A key element of the model is the distribution of worker and firm types. We choose the flexible, yet parsimonious Beta distribution, each with support $\underline{x} = \underline{y} = 0$ and $\overline{x} = \overline{y} = 1$. The parameters of the distribution of x (a_x, b_x) and those of the distribution of y (a_y, b_y) are estimated together with the remaining parameters of the model. When x and y are unobservable, it is not possible to separate the production function from the distributions. In other words we can reparameterize the model for x and y to have uniform distributions and change the way these two characteristics enter the production function. Nevertheless, the complementarity parameter is identified in any case.

The final component of the model is the matching function. This regulates the number of matches depending on the number of vacancies and the number of unemployed and is important for evaluating counterfactual policies. However, without fluctuations in U and V it is a constant. Indeed the matching function can only be identified as a result of fluctuations in the aggregate vacancies and unemployment and consequently cannot be estimated within our environment, where no aggregate quantity is allowed to change. We thus take the matching function from the literature, for the purposes of simulating counterfactuals. We use an elasticity of 0.5 and the functional form

$$M(U + s(1-U), V) = \eta\sqrt{[U + s(1-U)]\,V},$$

where the search intensity for the unemployed has been normalized to one (see, Blanchard and Diamond, 1991 and Petrongolo and Pissarides, 2001); the search intensity of the employed s as well as the overall level of matches η are estimated from the data together with the remaining parameters of the model.

3.1. Measurement error

In our model wages are stochastic because of the type of firm that an individual may encounter, the outside offers or the layoffs that may occur and the productivity shocks. There will also be unexplained variation due to her productivity characteristic x. In the data an additional source of variation is measurement error that we need to account for, so as not to bias the other sources of variation.

We use the monthly records of wages in the NLSY. Thus, while it may be reasonable to assume that measurement error is independent from one year to the next it may not be so within the year, as all records are reported at the same interview with recall. Having experimented with a number of alternatives, including a common equi-correlated component across all months, we settled on a measurement error structure that is common within year and independent across years. The variance of measurement error, which is assumed to be lognormal, is estimated alongside the other parameters of the model.

4. Estimation method

To estimate the model we use the Simulated Method of Moments (SMM).[10] To implement the approach we calculate a vector of moments from the data $\widehat{m}_N = \frac{1}{N}\sum_{i=1}^{N} m_i$, where for example, $\widehat{m}_N$ may be the mean wage for those one year out of unemployment or wage growth t periods following a job move, etc. Counterparts to these moments, defined as $\widehat{m}_S^M(\theta) = \frac{1}{S}\sum_{s=1}^{S} m_s^M(\theta)$, are then constructed based on S simulated careers from the model, given a parameter vector θ. The procedure then is to iterate on the parameters so as to bring the moments constructed from the simulated data as close as possible to the ones estimated from the data. Specifically we find a value for θ to maximize

$$L_N(\theta) = -\frac{1}{2}\left(\widehat{m}_N - \widehat{m}_S^M(\theta)\right)^{\mathsf{T}} \widehat{W}_N^{-1}\left(\widehat{m}_N - \widehat{m}_S^M(\theta)\right),$$

[10] See for example McFadden (1989) and Pakes and Pollard (1989). Constructing the likelihood function for this model is intractable.

where $\widehat{W}_N$ is taken to be the diagonal of the estimated covariance matrix of $\widehat{m}_N$. To compute the variance of m_i in the actual sample we use the bootstrap.

The simulated moments are not necessarily a smooth function of the parameters, although they would become so as the number of simulations increased to infinity. However, with any finite and relatively small number of simulations derivative based methods are not appropriate for finding the minimum. We thus use a method developed by Chernozhukov and Hong (2003), which does not require derivatives of the criterion function. They construct a Markov chain that converges to a stationary process of which the ergodic distribution has a mode that is asymptotically equivalent to the SMM estimator. Appendix C describes this procedure in detail.

5. The data

We use the 1979 to 2002 waves of the National Longitudinal Survey of Youth 1979 (NLSY). The NLSY consists of 12,686 individuals who were 14 to 21 years of age as of January, 1979. It contains a nationally representative core random sample, as well as an over-sample of black, Hispanic, the military, and poor white individuals. For our analysis, we keep only white males from the core sample.

We only include data for individuals once they have completed their education. We also drop individuals who have served in the military, and follow workers up to the point of a non-employment spell of 36 months or longer. We consider these workers to have left the labor force. We subdivide the data into two education groups: high school degree or less, and college graduate. The model is estimated separately on these subgroups.

Individuals are interviewed once a year and provide retrospective information on their labor market transitions and their earnings. From this we construct histories at a monthly frequency aggregating the data as follows: we define a worker as employed in a given week if he worked more than 35 hours in the week. We define a worker's employment status in a month as the activity he was engaged in for the majority of the month, treating unemployment spells of two weeks or less as job-to-job transitions. After sample selection, we are left with an unbalanced panel of 2125 individuals (446,747 person months).

We remove aggregate growth from wages based on average wage growth in years 10 to 20 from completion of education. At this point the main source of wage growth is due to aggregate productivity. Having removed this constant growth rate from wages we assume that the remaining growth is attributable to gains from job search.[11]

Finally, we trim the data to remove a very small number of outliers when calculating wage changes. When calculating month to month wage changes, we exclude observations where the wage falls by more than half or increases by more than a factor of five.

6. Choice of moments and identification

The model will be estimated based on worker level data recording transitions between jobs and between work and unemployment as well wages over time. We have no information on the firm side itself (such as for example productivity). As such identification is challenging. Lamadon et al. (2014) discuss the formal non-parametric identification of a model similar to this, albeit without productivity shocks and using matched employer employee data. Other insightful work with more formal discussion of identification is Hagedorn et al. (2012), who analyze a version of the model without on-the-job search, and Bagger and Lentz (2014), where sorting results from heterogeneous search strategies. The key result is that the complementarities in the worker–job match can be identified nonparametrically in their context, using job-side information on productivity and job duration. Here we present a heuristic description of the identification argument based on the simpler data at our disposal and using a specific parametric form for the match production function.

Transitions in and out of work and between jobs play a key role in parameters controlling labor market mobility. In particular η (matching efficiency) and s (relative search intensity) that help determine the arrival rate of offers are identified by exit from unemployment and by mobility between jobs. Exit from employment is governed by two components of the model: productivity shocks to the job and the exogenous destruction rate. However, productivity shocks are also related to job-to-job transitions and wage growth and cannot be set to fit perfectly the exit rate from employment. The remainder is captured by the exogenous match destruction rate ξ.

As noted earlier, the support of the distribution of unobserved productivity x and y have both been normalized to $(0, 1)$ and each have been assumed to follow a beta distribution with parameters (a_x, b_x) and (a_y, b_y) respectively; these are directly linked to the cross sectional distribution of wages. Through its effect on productivity shocks and job mobility, y is also linked to the variability of wage growth within and between jobs. Hence the identification of the weight of y in the match production function, α, is driven by the variances of wage growth, conditional on the various types of transition. Similarly the within and between-job variance of wage growth is informative about the rate of arrival of productivity shocks δ because (together with arrival of job offers) such shocks can lead to renegotiation of wages.

Central to the model is the complementarity parameter, which determines the amount of sorting. Key to identification with the type of data we have at our disposal is the fact that sorting, together with search frictions – identified using

[11] Our model does not allow for human capital accumulation. Any such growth in the data is accounted for by improved job matches through job search. Developing a model that allows for human capital accumulation is beyond the scope of this paper.

the information on transitions, control the way cross-sectional wage inequality increases for a particular cohort of workers as they sort into more suitable jobs following entry into the labor market. To capture this we simulate the evolution of wages and employment of an entry cohort – a group of individuals drawn from $\ell(x)$ and all starting out unemployed. Under the assumptions of our model these are comparable to an NLSY entry cohort starting its labor market career following completion of education. Because of search frictions, this cohort of workers will start off mismatched, but if there are production complementarities the process of on-the-job search will result in job reallocation improving sorting; this in turn will lead to an increase in wage inequality. In addition, if sorting is an important feature of the labor market, the average wage gains to changing jobs will decline with time in the labor market, as the cohort of workers move toward their ideal matches. Similarly, the variance of wage changes should decline as there are fewer offers that improve matches substantially and workers make fewer transitions. Alternatively, if there are no production complementarities (or these are very weak) then sorting will not be a feature of the equilibrium, and wage inequality will not increase with time for our cohort as there is no tendency for the economy to reallocate them across jobs. The degree of complementarity will affect the variance of wages *within* jobs (as well as between) because it affects the degree to which a firm can and wishes to respond to outside offers. For the same reasons as those described above the offers that will be capable of inducing wage growth will decline in frequency with time in the labor market. Thus the degree of complementarity in production can be identified using the profile of wage growth and its variance between jobs as well as within jobs.[12]

Changing the distribution of y and its shocks affects transitions both between jobs and in and out of unemployment. In addition, changing the distribution of x will affect the variance of wage growth in a very specific way, governed by the structure of pay setting and will also affect transitions in and out of work. Thus the distribution of these objects is intimately linked to observed transitions, which will limit the extent to which we can explain the variance of wages and their growth. We thus identify the variance of measurement error in wages from the variance of wages that the economic model is unable to reproduce.

The remainder of the parameters are identified as follows. The bargaining parameter β is primarily identified by wages following a job change, because it regulates the extent of a job-to-job wage increase. However, as we show below many model generated moments depend on this parameter. The parameter b in $b(x) = f(x, b)$, which reflects the income flow while out of work, is identified by the starting wage in new jobs following unemployment, because (for example) a very high value will not be consistent with low initial pay. The flow cost of vacancy posting c governs the profitability of posting vacancies, and is identified by matching the vacancy to unemployment ratio, which is observed from macroeconomic data.

Identification of the model in practice requires a careful choice of moments that will be sensitive to the parameters we need to estimate. In particular, to summarize the employment dynamics, we use the long-run employment rate and the transitions between employment states and between jobs, calculated on years 16–20. To summarize wage dynamics, we include the level of (log)wages and their cross sectional variance as well as wage growth and its variance both within and between jobs. Each of these moments is calculated separately by year in the labor force (year since leaving school for the cohort). All the moments we use, their values in intervals of five years and the value produced by the model are shown in Table 4. In addition, we target the mean vacancy to unemployment rate (based on the mean and standard deviation from Hagedorn and Manovskii, 2008).

Estimation and practical identification relies on the moments predicted by the model being sensitive to changes in the parameters: a model-predicted moment which is sensitive to a particular parameter is helpful in identifying it. In Tables 6 and 7 we show the elasticity of each of the moments we use with respect to each of the parameters (separately for each education group). Thus for example the complementarity parameter ρ has an important effect on moments to do with the distribution of wages and wage growth. The same is true for the parameters related to the distribution of x and y (a_x, b_x, a_y, b_y). However these parameters also change job finding and job switching rates. By contrast the variance of measurement error leaves transition rates completely unaffected. Examination of this table also indicates the our choice of moments is sufficient for identifying all parameters. An important lesson to be drawn from this table is that, with some exceptions, it is difficult to associate the identification of any one parameter with a single feature of the data. However, a complete analysis of identification would require an analysis of the rank of this matrix of derivatives, which is beyond the scope of this paper.

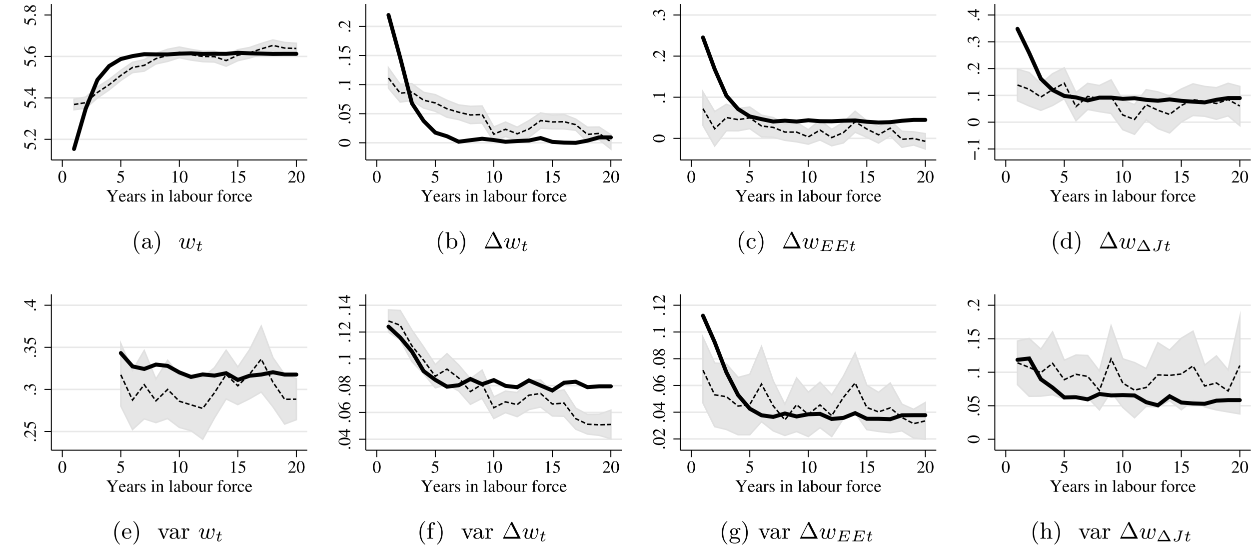
7. Estimation results

7.1. The fit of the moments

Summaries of the fit of the targeted moments by the model are presented in Table 4. As seen from the table the transitions rates are fitted remarkably well.

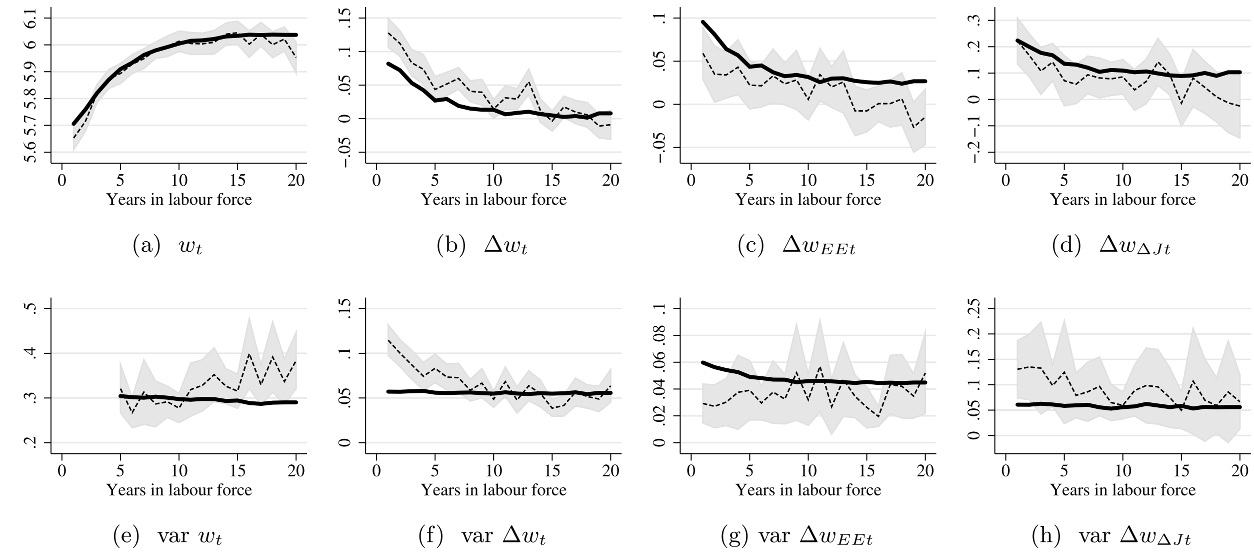
In Figs. 1 and 2 we summarize the fit of the model for wages, wage growth and the corresponding variances, both overall and by type of transition for the lower and higher education individuals respectively. The model generally fits these

[12] Our model can explain the increase in the cross sectional variance within a cohort by sorting. However, in principle other modeling choices, not explored here, could also explain the rise, such as permanent shocks to individual productivity or heterogeneous accumulation of human capital. Nevertheless, as we point out sorting has implications not only for the growth in the cross sectional variance of the wages of a cohort but also for the way the variance of wage growth evolves. Identification in such a richer model from worker level data is an open question.



Notes: w is the log wage, Δw_t is overall log wage growth, Δw_{EEt} is within job log wage growth, $\Delta w_{\Delta Jt}$ is between job log wage growth; var denotes variance. The solid line denotes the moments as predicted by the model, the dashed line is the equivalent data moment and the shaded area corresponds to a point-wise 95% confidence interval.

Fig. 1. Fit to wage moments: High school or less.



Notes: w is the log wage, Δw_t is overall log wage growth, Δw_{EEt} is within job log wage growth, $\Delta w_{\Delta Jt}$ is between job log wage growth; var denotes variance. The solid line denotes the moments as predicted by the model, the dashed line is the equivalent data moment and the shaded area corresponds to a point-wise 95% confidence interval.

Fig. 2. Fit to wage moments: College graduate.

patterns very well and certainly captures the qualitative features of the data.[13] Any wage growth generated by the model is due to the job search process and reflects mobility towards better jobs and (for the higher educated people) improvements in sorting. For the lower educated people as we shall see there are very few complementarities; however as the workers move to higher surplus jobs (because of improved firm productivity) and by receiving outside offers they can improve their wage. This process of outside offers is responsible for the observed wage growth for the low skilled.

[13] This comparison does not reflect estimation error and hence provides a narrower confidence interval for evaluating the difference between the model and the data moment.

Table 1
Parameters.

		High school or less	College graduate
Complementarity parameter	ρ	0.938	−0.895
		(0.088)	(0.274)
Worker bargaining power	β	0.188	0.272
		(0.027)	(0.020)
Probability of exogenous match destruction	ξ	0.011	0.004
		(0.004)	(0.0001)
Probability of a shock to y	δ	0.017	0.008
		(0.020)	(0.001)
Home production parameter $b(x) = f(x, b)$	b	0.122	0.000
		(0.033)	(0.032)
Vacancy cost (months of average output)	c	2.344	1.575
		(0.118)	(0.213)
Matching efficiency	η	0.650	0.425
		(0.031)	(0.032)
Relative search intensity (employed)	s	0.332	0.157
		(0.058)	(0.012)
Discount rate (annual)	r	0.05	

Notes: Monthly frequency. Standard deviation of MCMC chain in parentheses.

Table 2
Implications for wage growth and job mobility.

	High school or less	College graduate
Probability of shock to job productivity y	0.017	0.008
Probability of endogenous job destruction	0.000	0.002
Mean separation rate conditional on shock to y	0.000	0.307
Probability of job contact when unemployed	0.171	0.231
Mean matching rate given a contact	1.000	0.603
Probability of job contact when employed	0.057	0.036
Mean matching rate given a contact	0.249	0.207

Note: Monthly frequency.

7.2. Parameter estimates

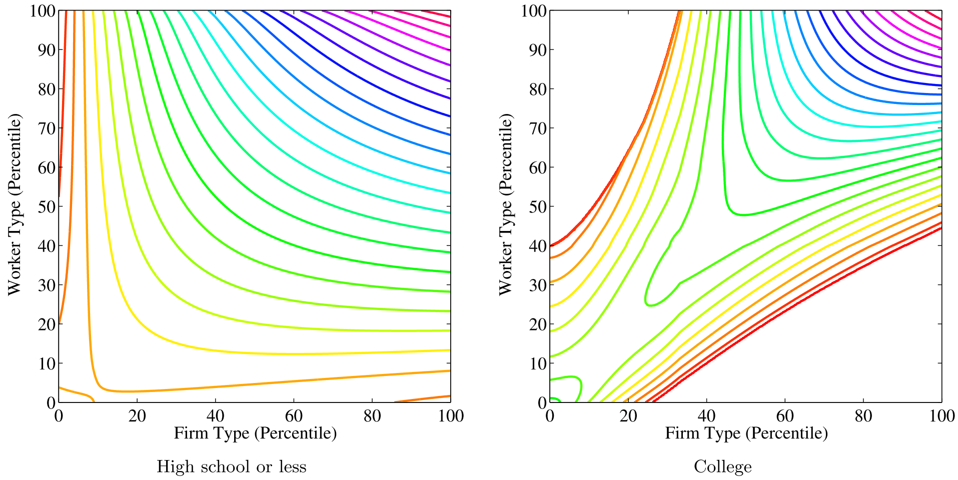
The complete set of parameter estimates is presented in Table 5. Here we focus on a subset that have a direct economic interpretation; these are presented in Table 1.

The key parameter of interest is the complementarity parameter (ρ). For lower skill worker this is close to and indeed not significantly different from one. This implies that there are practically no complementarities between worker and job characteristics, which appear to be perfect substitutes. However, for college graduates the elasticity of substitution is about 0.53 implying a high degree of complementarity and hence large gains from sorting. The gains from moving up the job ladder are thus much more important for college graduates than for those with lower levels of education. However, as mentioned above, the unskilled can still gain from outside offers: first, search frictions will imply a surplus, since the departure of the worker will mean the job will remain idle for some time and hence the firm will have an incentive to match outside offers from jobs with lower surplus. Second, the surplus is increasing in job productivity y but (almost) not in worker productivity x. This is because the cost of a vacancy is constant but the flow of out-of-work income is increasing in x (see equation (13)). Hence higher productivity firms can afford higher wages. The way the surplus is split is driven by the Nash bargaining parameter β. This is slightly higher for college graduates than for unskilled workers. The former obtain 27% of the surplus while the unskilled about 18%.

Exogenous layoffs occur about once every 7.5 years for the unskilled and effectively never for college graduates.[14] However the endogenous layoffs are more important for the latter. Endogenous productivity shocks occur every five years for the low educated and every 10 for the college graduates.[15] But conditional on occurring the separation rate is much higher for the latter. This is because an endogenous separation will occur either because the match became totally unviable (i.e. the job cannot produce enough to pay both outside options) or because of resulting mismatch, making it better for the job to remain vacant and the worker unemployed while they each wait for a better match; however, the latter is not an issue for the unskilled since there are no complementarities in production. The probability of endogenous job destruction overall and conditional on receiving a shock is given in Table 2. This is very low (surprisingly so), with the model implying that almost all separations are either due to exogenous layoffs or due to moving to better matched jobs following an outside offer.

Turning now to the parameters governing search, the search intensity (s) for the employed workers is a third of that for those out of work among the low education group and half that among the college graduates. Effectively this means that the rate of arrival of job offers is much higher for those out of work and this has an implication on what jobs the unemployed

[14] This is calculated as $\frac{1}{12\xi}$.
[15] Calculated as $\frac{1}{12\delta}$.



Note: We plot the contours of the Surplus function $S(x, y)$. No matches occur in the NW and SE part of the graph for College Graduates, where no contours are shown.

Fig. 3. Contours of the Surplus function over the Equilibrium matching sets.

are willing to take. The implications of the parameter estimates of the search technology (η and s) are better understood by calculating the probabilities of a job contact when employed and unemployed, which are displayed in Table 2. The contact rates for the unskilled when unemployed are lower than for the higher skilled (17.1% compared to 23.1%). However the unskilled accept all offers when unemployed. Both the contact rate when working and the rate at which alternative offers are accepted declines with skill.

7.3. Sorting in the labor market

The complementarities between worker and job characteristics for the higher education group imply perfect assortative matching in a first best world. However, the extent of sorting that occurs in practice depends on the importance of frictions.

In Fig. 3 we summarize the sorting patterns in the decentralized equilibrium, as implied by our point estimates of the parameters. The lines in the figure are contours of the surplus function. On each contour the surplus is constant and non-negative, for various combinations of worker and job productivities. In the left hand panel, corresponding to the low education workers, such lines cover the entire support of the distribution of worker and firm characteristics. This reflects the fact that all matches are viable, leading to a non-negative surplus, because of the almost complete absence of complementarities.[16] In the right hand panel, relating to college graduates, the upper-left and lower-right sections have no contours: these areas represent combinations of worker and job types where the surplus is negative and no matches ever occur.[17] Waiting for a better match has higher value than starting to produce.

For low skill workers over most of the space the contours are downward sloping. This points to a trade off between firm and job characteristics at a fixed match surplus. However, for college graduates the contours are mostly upward sloping (except at the highest levels of skills and job characteristics). This is because, when complementarities are very important an increase in the job productivity requires an increase in the human capital of the worker if the surplus is to remain constant, rather than decline.

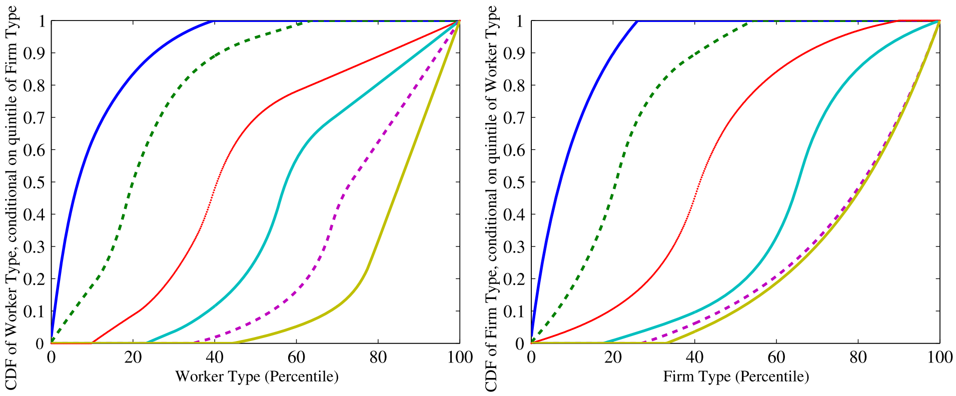
As an example of the matching process consider a college educated worker at the 50th percentile of the x distribution. The surplus initially increases in the type of the job, is maximized when matched to a job at the 50th percentile, and then declines again. This worker will initially match with any job above the 15th percentile, but will always move to a job that is closer to the 50th percentile, which may involve moving up or down the quantiles of y.

Fig. 4 illustrates sorting within the College educated group by plotting the distribution of worker (job) characteristics conditional on various percentiles of the job (worker) productivity they are matched with. Distributions conditional on higher values of productivity of the counterpart to the match stochastically dominate those that are conditional on lower values. Moreover, the support of these distributions is limited to a strict subset of the entire support, reflecting the fact that some matches never occur.

Finally, matches are not uniformly distributed over this support, even for the low educated. We illustrate the density of matches over the matching set in Fig. 5. For the college group, as we approach the main diagonal, of perfect sorting the

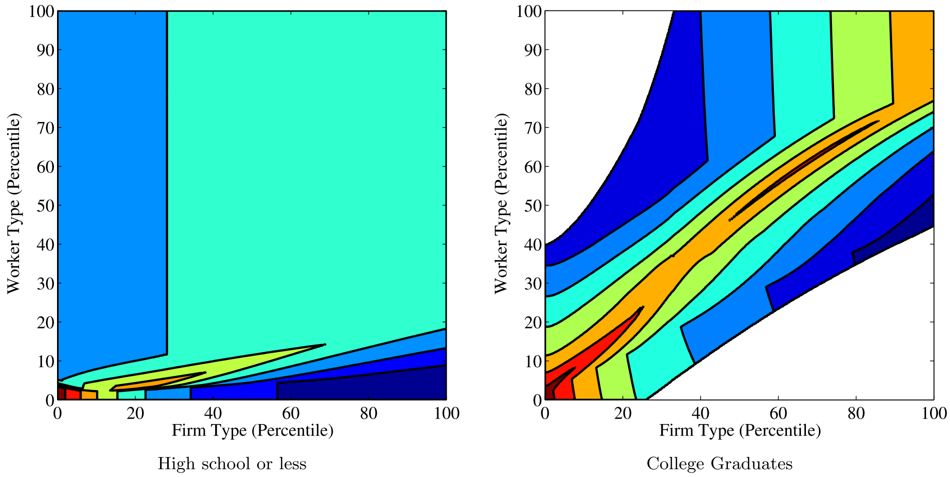
[16] Although our estimate of ρ is not statistically significantly different than one for the high school or less group, all descriptions made here use the point estimate presented in the table above.

[17] The asymmetry of the matching set is due to the fact that only workers can search for alternative matches when employed, as well at to the asymmetry in the estimated type distributions.



Note: We plot the CDF of worker (firm) type conditional on the quantile of the firm (worker) type. The lines moving from left to right are the lowest, 20, 40, 60, 80 and highest quantiles.

Fig. 4. Stochastic dominance and sorting of workers to firms – College Graduates.



Note: The Graph illustrates the density of matches over the matching set. The blue colors denote low density. Colors in the red part of the spectrum imply higher density. There is a clear indication of positive sorting among college graduates with orange colors along the main diagonal.

Fig. 5. Density of Matches over the Equilibrium matching sets. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

density of matches increases. For the high school group, all workers above the 20th percentile move up toward the highest firm type at the same rate. Below the 20th percentile there is some sorting induced by the mild complementarity (for low x-types the surplus decreases when moving to higher y-type firms, although it is still positive. A caveat here is that our estimate is not distinguishable from linearity, in which case there would be no sorting at all for this group.

8. Welfare analysis

To provide a sense of the potential gains from policy, we consider three thought experiments. First we take the estimated search frictions as given and look at the Planner's constrained efficient solution. This experiment provides an upper bound on what can be achieved by policies that work to eliminate congestion externalities, taking the frictions as given. Second, we consider the thought experiment of ignoring the search frictions and solving the frictionless assignment problem. We do this both keeping the number of employed worker fixed to the estimated level, as well as for the full employment case, allowing us to separate the employment from the mismatch effects of frictions. This experiment provides an estimate of the upper bound to the benefits of finding technological solutions around the frictions (such as improved centralized matching) but has nothing to say about either the feasibility or costs of such a program. Finally, we consider an optimal Unemployment benefit program. This experiment provides an estimate of the potential gain from a feasible policy.

8.1. The planner's constrained efficient solution

The potential for welfare-enhancing labor market regulation arises from the job search frictions and the externalities they cause during the job allocation process. The externalities arise from the classic issue of "overcrowding" among job seekers, i.e. when an extra person or vacancy seeks a match it reduces the arrival rate for others as implied by the matching function. An extra dimension arises in our model because of heterogeneity and sorting: by having low quality jobs compete for workers they lengthen the time it takes to fill higher productivity ones, without adding much when they are filled (because they have zero or near zero surplus).[18] This implies that because of complementarity, cutting some low productivity jobs may increase welfare, even if this means that some very low productivity workers never work.

An additional inefficiency comes from the fact that workers can seek outside offers to increase their share of the match surplus. As a result, they are willing to form matches even when the current output of the match is below what they could produce at home, allowing also for the cost of the vacancy: $f(x, y) < b(x) - c$. Even though the worker and firm are producing less together than they would separately, the fact that the firm has monopsony power up front means it is willing to hire the worker and extract most of the match surplus during the early periods of the match. The worker is willing to be in the match as it provides a better outside option so as to extract surplus via poaching firms.

Any regulatory intervention in the labor market will improve welfare only to the extent that it can address the externalities discussed above, and to the extent to which they are significant. Thus, to provide a measure of the potential welfare gains from labor market regulation (such as in-work benefits, unemployment benefit, minimum wages, severance pay etc.) we solve the planners problem respecting the constraints arising from search frictions.

Specifically the planner chooses total vacancies, the distribution of matches and the distribution of unemployed workers to maximize total output and home production subject to the flow constraints implied by the frictions and the costs of vacancies (both of which we view as a fixed element of the environment), i.e.

$$\max_{h^{SP}, u^{SP}, V^{SP}} \int f(x, y) h^{SP}(x, y)\, dx\, dy + \int b(x) u^{SP}(x)\, dx - cV^{SP} \tag{16}$$

subject to the estimated matching and production technologies and the stationary transition equations (14), the accounting identities (1) and (2), and $V = \int v(y) dy$. In the above the superscript *SP* is used to distinguish the endogenous objects chosen by the planner as opposed to those arising in a decentralized economy.

A mathematically equivalent formulation to directly choosing the distribution of matches is to have the planner choose the set of admissible matches $\mathcal{M}^{SP}(x, y)$ and the measure of jobs N^{SP}. We approximate the solution to the planner's problem by specifying the boundaries of the matching set as polynomials: $\mathcal{M}^{SP}(x, y) = \left\{x, y | x > \sum_{i=1}^{I} \tau_i y^{i-1}, y > \sum_{j=1}^{J} \tau_j x^{j-1}\right\}$. We find that polynomials of order $I, J = 4$ provide a good approximation as no increase in steady state output is found by further increasing the degree of polynomial.

Table 3 shows the breakdown of contributions to total welfare under different scenarios. The first column relates to the fully decentralized economy we observe from the data. The second column shows the results of the planner maximizing welfare as in (16). For the lowest education group the constrained planner is able to improve on the decentralized outcome by two percent. For the higher education group the planner can attain an increase in welfare of only 0.71 percent. The planner increases unemployment and reduces the number of jobs. To understand what is going on, note that increased match quality contributes nothing to the welfare increase. Output per match does not change. The planner increases welfare by reducing vacancies (which are very costly) and allowing more workers to engage in home production. For the high school or less group, where there are no complementarities in production, this reallocation achieves a relatively large increase in welfare. For college graduates, the welfare gains are much more modest because by reducing the number of vacancies some high surplus jobs are also eliminated and home production is less effective for this group. Nevertheless, the reduction of jobs for them is much lower.

8.2. The costs of search frictions

Search frictions prevent positive assortative matching and cause unemployment. Indeed there are varying degrees of complementarity in production for both education groups and the extent to which mismatch causes welfare losses will vary. One way of gauging this is to ignore search frictions and sort workers and jobs optimally, imposing perfect assortative matching as in Becker (1973).[19] Column (3) runs the counterfactual experiment where $u(x)$ and $v(y)$ are kept the same as estimated but $h(x, y)$ is changed such that workers and jobs are perfectly positively sorted. Match quality (output per employee) increases by 1.0 percentage point for unskilled workers and by 6.0 percentage points for the college graduates groups. The increases in steady state welfare (output plus home production less vacancy costs) are quite similar at 1.09 and 6.84, percentage points respectively.

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[18] See also Sattinger (1995).

[19] Note that these frictions are taken as technological constraints when estimating the decentralized economy and when solving the Planner's constrained efficient problem. Eliminating these frictions should be viewed as a thought experiment to gauge the extent of their effect on output. We have no means of assessing the feasibility or costs of their removal. See also Gautier and Teulings (2015) for an interesting approach to measuring the cost of search frictions in the presence of sorting, albeit in the context of a simpler model.

Table 3
Output and employment.

	Decentralized frictional (1)	Constrained planner (2)	Frictionless with actual employment (3)	Frictionless with full employment (4)	Optimal unemployment benefit (5)
			High school or less		
Welfare	100.00	102.02	101.09	107.78	101.39
Match Output	102.70	97.95	103.79	107.29	100.89
Home Production	3.87	6.41	3.87	0.49	4.78
Recruiting Costs	−6.56	−2.34	−6.56	0.00	−4.28
E/L	94.01	90.06	94.01	96.59	92.60
N/L	96.59	90.98	96.59	100.00	94.29
V/U	43.12	9.27	43.12	–	22.75
Match quality	1.00	1.00	1.01	1.02	1.00
Corr(x, y)	0.14	0.29	1.00	1.00	0.24
			College graduate		
Welfare	100.00	100.71	106.84	119.57	100.00
Match Output	108.02	106.56	114.87	119.57	108.01
Home Production	0.00	0.00	0.00	0.00	0.00
Recruiting Costs	−8.02	−5.85	−8.02	0.00	−8.00
E/L	95.73	94.37	95.73	100.00	95.80
N/L	101.46	98.54	101.46	100.00	101.51
V/U	134.19	74.10	134.19	–	136.06
Match quality	1.00	1.00	1.06	1.06	1.00
Corr(x, y)	0.81	0.82	1.00	1.00	0.81

Notes: Column (1) is the estimated economy. In column (2) the constrained planner chooses admissible matches to maximize steady state output. Here we restrict the planner to choosing a reservation type for x and y, approximated by a fourth order polynomial: $\mathcal{M}^{SP}(x, y) = \left\{x, y | x > \sum_{i=1}^{4} \tau_i y^{i-1}, y > \sum_{j=1}^{4} \tau_j x^{j-1}\right\}$, and the measure of firms active in the market N. The frictionless benchmarks of columns (3) and (4) hold N/L at the estimated decentralized level and force the positive assortative allocation $\{x, y(x)\}$. Column (3) keeps the same unemployment and vacancy distributions $u(x)$ and $v(y)$ as in the benchmark. In column (4) we optimally reallocate workers across firms and employment states, filling all possible jobs with a worker. In column (5), optimal unemployment benefit policy is modeled as proportional to expected output: $b_0(x) = b_0 \int f(x, y)h(y|x)\,dy$ and we impose the balanced budget $\int b_0(x)u(x)\,dx = \tau \int \int f(x, y)h(x, y)\,dx\,dy$. The implied policy parameters for the group with high school or less are a replacement rate of 11.09 percent funded by a tax on output of 0.95 percent. For college graduates, the replacement rate is 0.56 percent funded by a tax on output of 0.02 percent. Match quality is defined as output per match, relative to the decentralized equilibrium.

In column (4), we run a similar counterfactual in which we assign all jobs to workers. Unemployment is drastically reduced and welfare increases a lot, largely because there are no recruiting costs, but also because of reduced unemployment, and a slightly higher match quality for the lowest education groups (see also the discussion in Subsection 8.3.2).

This experiment indicates that there may be substantial gains from reducing search frictions: for the lowest skill group policies that improve search technology could improve welfare by up to 7.8% (19.6% for the college educated group). Part of this increase comes from reducing unemployment. But part also comes from improving sorting if we take the point estimate of the elasticity of substitution, which is 16.2. This can be seen from the third column of Table 3 where unemployment is kept equal to the level in the benchmark economy and we observe a rise in match quality.

8.3. Optimal unemployment benefit

The final column of Table 3 presents the welfare (output) gains from an optimal unemployment benefit policy: here optimal refers to the policy that maximizes total output and home production, subject to the constraints that arise from the existence of search frictions.[20] We approximate the unemployment benefit as a payment proportional to a worker's type specific expected market production: $b_{UI}(x) = b_0 \int f(x, y)h(y|x)\,dy$. Benefits are funded by a proportional tax on match output: $\int b_{UI}(x)u(x)\,dx = \tau \int f(x, y)h(x, y)\,dx\,dy$.

8.3.1. Overall efficiency gains

For the low education group, optimal unemployment benefit can deliver 1.4% of improved welfare, corresponding to 68.8 percent of the potential gains attainable by the planner working under the same constraints. This involves increasing the baseline flow utility of being out of work (home production) for each individual by 11.09 percent of their expected output if employed, and financing this by a tax on output of 0.95 percent. The gain is effectively zero for the high education group.

It is worth noting again that the improvement in steady state output comes from very different sources when comparing the elimination of frictions to the constrained planner or optimal unemployment benefit scheme. With the removal of

[20] In a model with risk aversion unemployment benefit would also mitigate the effects of risk. Here agents are risk neutral and the insurance role of unemployment benefit is not accounted for – hence the use of the word benefit, instead of insurance.

Table 4
Model fit.

	High school or less				College graduate		
	Years	Model	Data	S.D.	Model	Data	S.D.
Employment rate	16–20	0.941	0.934	0.004	0.958	0.959	0.005
Job finding rate (*UE*)	16–20	0.171	0.157	0.017	0.141	0.134	0.033
Job losing rate (*EU*)	16–20	0.011	0.011	0.001	0.006	0.006	0.001
Job changing rate (ΔJ)	16–20	0.014	0.014	0.001	0.008	0.007	0.001
log wage	1–5	5.426	5.429	0.013	5.813	5.790	0.019
	6–10	5.609	5.584	0.013	5.974	5.973	0.018
	11–15	5.614	5.599	0.013	6.024	6.021	0.021
	16–20	5.613	5.637	0.013	6.037	6.004	0.024
wage growth	1–5	0.098	0.085	0.007	0.055	0.088	0.010
	6–10	0.006	0.044	0.007	0.018	0.041	0.008
	11–15	0.004	0.027	0.006	0.007	0.025	0.008
	16–20	0.004	0.020	0.006	0.005	0.003	0.009
wage growth on the job	1–5	0.128	0.048	0.017	0.068	0.039	0.015
	6–10	0.043	0.018	0.011	0.036	0.022	0.014
	11–15	0.042	0.020	0.011	0.028	0.013	0.014
	16–20	0.042	0.005	0.009	0.026	−0.007	0.014
wage growth at job change	1–5	0.197	0.124	0.029	0.181	0.142	0.042
	6–10	0.089	0.073	0.027	0.116	0.079	0.035
	11–15	0.083	0.042	0.031	0.098	0.066	0.042
	16–20	0.083	0.075	0.031	0.098	0.020	0.054
variance log wage	1–5	0.343	0.317	0.019	0.304	0.321	0.025
	6–10	0.326	0.293	0.017	0.301	0.287	0.022
	11–15	0.316	0.295	0.015	0.296	0.328	0.025
	16–20	0.318	0.308	0.015	0.289	0.368	0.031
variance wage growth	1–5	0.104	0.110	0.005	0.057	0.092	0.008
	6–10	0.082	0.080	0.005	0.056	0.064	0.007
	11–15	0.080	0.069	0.005	0.055	0.055	0.007
	16–20	0.081	0.055	0.004	0.055	0.052	0.007
variance wage growth on the job	1–5	0.074	0.053	0.012	0.054	0.033	0.010
	6–10	0.038	0.045	0.009	0.047	0.037	0.011
	11–15	0.037	0.048	0.009	0.045	0.038	0.011
	16–20	0.037	0.037	0.007	0.045	0.038	0.010
variance wage growth at job change	1–5	0.094	0.105	0.019	0.061	0.124	0.035
	6–10	0.064	0.093	0.017	0.057	0.077	0.022
	11–15	0.058	0.088	0.021	0.059	0.082	0.029
	16–20	0.056	0.091	0.023	0.055	0.078	0.039
vacancy to unemployment ratio		0.431	0.540	0.054	1.342	0.540	0.054

Note: We target the long run employment and transition rates and the entire profile of the wage moments, with the exception of the variance of log wage where we exclude the first 5 years.

frictions there is a direct increase in market production and a gain when netting out the costs of vacancy creation from home production. For the low education group, where the constrained planner can improve steady state output, this is implemented largely by raising unemployment and reducing the number of vacancies, resulting in lower vacancy creation costs and higher levels of home production, but without improving the average quality of productive matches.

8.3.2. The redistributive effects of the policy

As discussed above, we find that there is a potential aggregate gain from an optimal unemployment benefit scheme (although negligible for college graduates). In addition to overall efficiency gains, we are also interested in the redistributive effects of policy. In an environment with heterogeneous workers it is not necessarily the case that an increase in steady state output will benefit all workers the same, indeed it may harm some. In Fig. 6 we plot the difference in value, by worker type x, between being unemployed in an economy with and without the optimal unemployment benefit scheme. While the value of unemployment is higher for all worker types in the low education group, it is effectively zero for college educated workers above the second quintile. Thus, there are no losers from this optimal UI policy, but the gains are concentrated among the low educated as well as the lower productivity individuals among the college graduates.

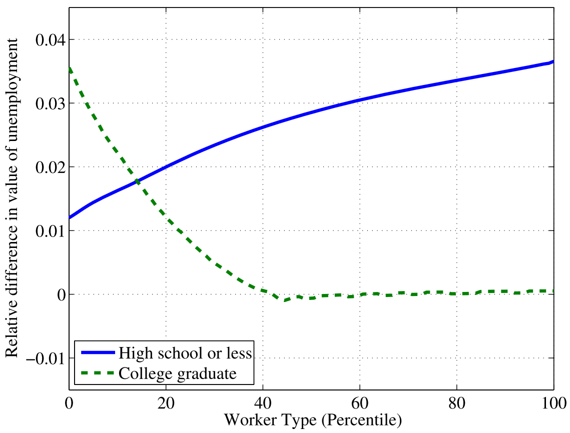
9. Concluding remarks

We develop an equilibrium model of employment and wage determination, which builds on the work of Mortensen and Pissarides (1994), Shimer and Smith (2000) and Postel-Vinay and Robin (2002). In our model both workers and firms are heterogeneous and their productivity characteristics are potentially complementary in production creating the possibility of sorting. However, firms are subject to productivity shocks. Workers can search both on and off the job. This creates an environment where there may be potential for welfare improving labor market regulation. Moreover our framework is well

Table 5
Parameters.

		High school or less		College graduate	
Matching efficiency	η	0.650	[0.031]	0.425	[0.032]
Relative search intensity	s	0.332	[0.058]	0.157	[0.012]
Probability of exogenous job destruction	ξ	0.011	[0.004]	0.004	[0.000]
Probability of a shock to y	δ	0.017	[0.020]	0.008	[0.001]
Home production parameter $b(x) = f(x, b)$	b	0.122	[0.033]	0.000	[0.032]
Vacancy cost (months of average output)	c	2.344	[0.118]	1.575	[0.213]
Production function	A	6.964	–	7.541	–
$f(x, y) = A\left(\alpha x^{\rho} + (1-\alpha) y^{\rho}\right)^{\frac{1}{\rho}}$	α	0.607	[0.010]	0.606	[0.053]
	ρ	0.938	[0.088]	−0.895	[0.274]
Worker type distribution	a_x	0.797	[0.021]	0.891	[0.085]
$x = \text{Beta}^{-1}(U, a_x, b_x)$	b_x	0.987	[0.089]	0.616	[0.167]
Firm type distribution	a_y	1.988	[0.007]	1.034	[0.063]
$y = \text{Beta}^{-1}(U, a_y, b_y)$	b_y	0.967	[0.012]	1.147	[0.286]
Worker bargaining power	β	0.188	[0.027]	0.272	[0.020]
Variance of measurement error	σ^2	0.006	[0.087]	0.019	[0.006]
Elasticity of substitution	$(1-\rho)^{-1}$	16.158		0.528	
Mean x		0.447		0.591	
Variance x		0.089		0.096	
Skewness x		0.258		−0.469	
Kurtosis x		−1.211		−1.200	
Mean y		0.673		0.474	
Variance y		0.056		0.078	
Skewness y		−0.740		0.116	
Kurtosis y		−0.572		−1.149	

Note: monthly frequency. Standard deviation of MCMC chain in square brackets.



Note: We calculate the distributional impact as $\frac{W_0^{UI}(x) - W_0(x)}{W_0(x)}$.

Fig. 6. Welfare difference in value of unemployment by worker type.

suited to consider the redistributive (as well as efficiency) implications of policy. The scope for and impact of policy is thus an empirical issue in our model.

We estimate the model based on NLSY data and find strong evidence of complementarities between worker and firm characteristics, leading to sorting for the college educated workers. For this education group these complementarities imply large efficiency losses due to mismatch between job and worker productivities caused by search frictions. The complementarities are much weaker for the low education workers, where the production function is effectively linear in individual an job productivities.

Mismatch is a source of inefficiency that labor market regulation cannot correct; this would require changing the job search technology, improving job finding rates and enabling more mobility following shocks. However, we show that the potential welfare gains from eliminating mismatch and frictions can be as high as 20% for college graduates and 8% for the lower educated.

Policies such as unemployment benefit can improve efficiency to the extent that they address the externalities induced by search frictions. We establish that optimal labor market regulation can improve welfare by up to 2% for low skill workers and 0.71% for college graduates. Some 70% of the improvement can be achieved with optimal unemployment benefit alone for the lower educated individuals, but such a policy can achieve nothing for the college graduate group.

Table 6
Elasticity of moments with respect to parameters: High school or less.

	Year	η	s	ξ	A	α	ρ	δ	β	b	c	σ	a_x	b_x	a_y	b_y
Employment	16–20	0.08	−0.01	−0.06	0.00	−0.01	0.01	0.03	−0.03	−0.01	−0.03	0.00	−0.01	−0.02	0.01	0.00
Job finding	16–20	0.86	−0.13	0.30	0.00	−0.24	0.23	0.27	−0.47	−0.23	−0.60	0.00	−0.17	−0.25	0.17	0.01
Job losing	16–20	0.00	0.00	0.99	0.00	0.00	0.00	−0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Job changing	16–20	0.71	0.16	0.38	0.00	−0.02	−2.14	0.08	−0.18	−0.02	−0.14	0.00	0.01	−0.25	−0.07	0.16
log wage	1–5	0.02	−0.05	0.02	1.07	−0.01	0.05	0.02	0.13	0.05	−0.02	0.00	0.12	0.11	−0.09	−0.07
	6–10	0.02	−0.01	−0.01	1.06	−0.03	0.05	0.03	0.04	0.02	−0.01	0.00	0.09	0.07	−0.08	−0.06
	11–15	0.01	−0.01	−0.02	1.06	−0.03	0.05	0.03	0.04	0.02	−0.01	0.00	0.09	0.07	−0.08	−0.06
	16–20	0.01	−0.01	−0.02	1.06	−0.03	0.05	0.03	0.04	0.02	−0.01	0.00	0.09	0.07	−0.08	−0.06
wage growth	1–5	−0.34	0.72	−0.94	0.00	−0.38	0.12	0.41	−1.46	−0.86	0.41	−0.04	−0.53	−0.99	0.32	0.20
	6–10	−1.83	0.85	−2.32	0.00	2.07	0.27	−0.13	5.04	−0.26	1.28	0.14	−0.08	−0.29	−0.07	0.15
	11–15	−1.95	1.13	−1.16	0.00	0.72	−3.08	−0.18	−0.32	−0.27	1.58	−0.78	−0.16	−0.31	−0.02	0.09
	16–20	−1.70	1.46	−0.87	0.00	−0.17	7.89	−0.46	−1.97	0.96	1.64	1.56	−2.91	0.22	−5.53	0.09
wage growth	1–5	0.01	0.64	−0.15	0.00	−0.52	0.29	0.48	−1.70	−0.93	0.18	−0.02	−0.63	−1.06	0.40	0.20
on the job	6–10	−0.26	0.54	0.31	0.00	−0.50	0.32	0.42	−1.62	−0.86	0.24	0.01	−0.59	−0.96	0.36	0.18
	11–15	−0.14	0.55	0.64	0.00	−0.54	0.38	0.45	−1.66	−0.91	0.18	0.00	−0.63	−1.03	0.41	0.18
	16–20	−0.09	0.55	0.69	0.00	−0.54	0.27	0.46	−1.69	−0.93	0.14	0.01	−0.63	−1.06	0.42	0.19
wage growth	1–5	−0.42	0.54	−0.42	0.00	−0.44	−0.09	0.35	−1.50	−0.84	0.37	−0.01	−0.55	−0.95	0.31	0.20
at job change	6–10	−0.66	0.46	−0.08	0.00	−0.46	0.40	0.28	−1.48	−0.83	0.33	−0.02	−0.57	−0.95	0.35	0.17
	11–15	−0.54	0.46	0.38	0.00	−0.51	1.34	0.35	−1.55	−0.91	0.23	−0.02	−0.65	−0.96	0.44	0.13
	16–20	−0.55	0.45	0.48	0.00	−0.52	1.05	0.37	−1.58	−0.89	0.22	0.02	−0.65	−0.93	0.44	0.13
var log wage	1–5	−0.12	1.19	−0.44	0.00	0.28	−0.44	0.08	−2.12	−1.26	0.17	0.06	−0.57	−1.69	−0.05	0.63
	6–10	−0.23	0.91	−0.11	0.00	0.35	−0.60	−0.05	−1.85	−0.94	0.17	0.08	−0.47	−1.33	−0.11	0.57
	11–15	−0.21	0.88	0.11	0.00	0.33	−0.66	−0.07	−1.84	−0.90	0.16	0.09	−0.47	−1.27	−0.12	0.56
	16–20	−0.21	0.89	0.19	0.00	0.35	−0.65	−0.07	−1.83	−0.88	0.17	0.08	−0.46	−1.26	−0.13	0.56
var wage growth	1–5	0.39	1.91	−0.23	0.00	−0.87	0.46	0.71	−2.85	−2.09	−0.04	0.46	−1.21	−2.59	0.63	0.51
	6–10	0.24	1.74	0.17	0.00	−0.85	0.47	0.66	−2.80	−1.94	0.03	0.55	−1.12	−2.37	0.59	0.47
	11–15	0.25	1.74	0.39	0.00	−0.88	0.49	0.67	−2.80	−1.95	0.01	0.56	−1.15	−2.35	0.62	0.45
	16–20	0.23	1.75	0.41	0.00	−0.86	0.47	0.65	−2.80	−1.91	0.04	0.55	−1.13	−2.30	0.60	0.46
var wage growth	1–5	0.15	1.75	−0.60	0.00	−0.73	0.32	0.61	−2.73	−1.95	0.11	0.63	−1.04	−2.30	0.51	0.49
on the job	6–10	−0.02	1.47	−0.19	0.00	−0.67	0.30	0.49	−2.62	−1.63	0.14	0.93	−0.89	−1.75	0.43	0.37
	11–15	0.03	1.44	0.21	0.00	−0.69	0.34	0.50	−2.61	−1.65	0.10	0.94	−0.89	−1.80	0.45	0.37
	16–20	0.09	1.49	0.31	0.00	−0.71	0.27	0.50	−2.63	−1.67	0.06	0.93	−0.92	−1.79	0.47	0.37
var wage growth	1–5	0.05	1.79	−0.99	0.00	−0.67	−0.40	0.58	−2.72	−2.09	0.22	0.51	−1.03	−2.60	0.44	0.59
at job change	6–10	−0.14	1.52	−0.87	0.00	−0.67	−0.62	0.48	−2.63	−1.80	0.27	0.66	−0.94	−2.27	0.43	0.48
	11–15	−0.14	1.46	−0.12	0.00	−0.72	0.82	0.55	−2.67	−1.90	0.17	0.70	−1.03	−2.29	0.54	0.45
	16–20	−0.10	1.50	0.09	0.00	−0.74	0.32	0.55	−2.68	−1.86	0.14	0.71	−1.01	−2.14	0.53	0.44
V/U		0.35	−0.03	−0.41	0.00	−0.63	0.59	0.60	−1.33	−0.63	−1.73	0.00	−0.49	−0.69	0.47	0.02

Table 7
Elasticity of moments with respect to parameters: College graduate.

	Year	η	s	ξ	A	α	ρ	δ	β	b	c	σ	a_x	b_x	a_y	b_y
Employment	16–20	0.03	0.00	−0.01	0.00	0.27	0.00	0.00	0.03	−0.52	0.12	0.00	−0.05	0.24	0.05	−0.08
Job finding	16–20	0.21	−0.02	0.56	0.00	0.34	0.03	0.29	0.17	−2.44	0.11	0.00	−0.02	0.32	0.03	−0.24
Job losing	16–20	0.18	−0.01	0.57	0.00	−0.11	0.04	0.30	0.11	0.10	−0.11	0.00	0.05	−0.09	−0.05	−0.08
Job changing	16–20	0.50	0.76	0.16	0.00	−0.03	0.02	0.33	−0.29	−0.86	−0.21	0.00	−0.05	−0.42	−0.09	0.50
log wage	1–5	0.00	−0.01	0.01	1.05	−0.09	−0.03	0.01	0.06	0.04	0.03	0.00	0.20	0.02	−0.13	0.02
	6–10	0.01	0.00	0.01	1.05	−0.08	−0.02	0.01	0.04	0.04	0.02	0.00	0.18	0.02	−0.13	0.01
	11–15	0.00	0.00	0.01	1.05	−0.08	−0.02	0.01	0.03	0.04	0.02	0.00	0.18	0.02	−0.13	0.01
	16–20	0.00	0.00	0.00	1.04	−0.08	−0.02	0.01	0.03	0.04	0.03	0.00	0.18	0.02	−0.12	0.01
wage growth	1–5	0.07	1.32	−0.08	0.00	0.20	0.20	0.14	−1.96	−1.29	0.03	−0.49	−0.82	−0.41	0.00	0.49
	6–10	−0.32	0.79	−0.05	0.00	0.35	0.13	−0.01	−1.34	−0.66	0.32	0.08	−0.82	0.11	0.21	0.15
	11–15	−0.47	0.32	−0.09	0.00	0.40	0.03	−0.24	0.06	1.73	0.40	0.45	−0.76	0.42	0.29	−0.33
	16–20	−1.25	1.44	−0.68	0.00	0.36	4.25	0.27	−1.80	−0.79	4.77	2.71	49.28	1.34	−1.79	0.30
wage growth on the job	1–5	0.24	1.50	0.31	0.00	−0.11	0.30	0.36	−2.31	−1.62	−0.25	−0.54	−0.67	−0.79	−0.08	0.81
	6–10	0.14	0.86	0.14	0.00	0.21	0.19	0.09	−1.61	−1.02	0.05	0.07	−0.65	−0.20	0.01	0.39
	11–15	0.08	−3.34	1.09	0.00	−0.08	−0.75	0.53	−2.37	−1.57	0.71	0.67	−4.01	−1.23	0.22	2.16
	16–20	0.23	1.28	0.22	0.00	0.28	0.71	0.29	−2.60	−1.57	0.09	16.89	−1.74	−0.65	0.25	0.86
wage growth at job change	1–5	−0.41	0.16	0.01	0.00	0.42	0.15	0.07	−1.20	−0.20	0.32	0.15	−0.74	0.26	0.25	−0.11
	6–10	−0.49	0.09	0.00	0.00	0.44	0.21	0.00	−1.07	−0.13	0.24	0.05	−0.69	0.20	0.12	−0.12
	11–15	−0.70	−0.10	0.12	0.00	0.52	0.18	0.06	−0.94	−0.15	0.36	−0.01	−0.83	0.32	0.18	−0.23
	16–20	−0.75	0.01	0.37	0.00	0.26	0.27	0.07	−1.03	−0.44	0.31	−0.33	−0.84	0.18	0.18	−0.18
var log wage	1–5	−0.26	0.34	0.13	0.00	0.87	0.25	0.11	−2.10	−0.01	−0.16	0.42	−1.75	0.68	0.57	−0.91
	6–10	−0.31	0.13	0.12	0.00	0.96	0.25	0.06	−1.49	−0.06	−0.13	0.42	−1.85	0.64	0.64	−1.00
	11–15	−0.29	0.08	0.12	0.00	0.97	0.25	0.04	−1.25	−0.06	−0.15	0.43	−1.86	0.61	0.65	−1.02
	16–20	−0.29	0.05	0.14	0.00	0.97	0.25	0.04	−1.09	−0.06	−0.16	0.44	−1.85	0.60	0.66	−1.04
var wage growth	1–5	−0.01	0.16	0.02	0.00	0.02	0.02	0.05	−1.59	−0.17	0.02	1.50	−0.09	−0.02	0.00	0.03
	6–10	0.00	0.11	0.03	0.00	0.02	0.02	0.05	−1.19	−0.18	0.01	1.50	−0.09	−0.01	0.01	0.02
	11–15	−0.02	0.09	0.02	0.00	0.02	0.02	0.04	−0.97	−0.17	0.02	1.51	−0.09	−0.01	0.01	0.01
	16–20	−0.01	0.09	0.04	0.00	0.03	0.02	0.04	−0.89	−0.17	0.01	1.50	−0.07	−0.01	0.00	0.02
var wage growth on the job	1–5	−0.01	0.15	0.03	0.00	0.03	0.03	0.05	−1.50	−0.13	0.02	1.49	−0.11	−0.01	0.01	0.03
	6–10	−0.02	0.07	0.01	0.00	0.03	0.01	0.05	−0.88	−0.09	0.02	1.51	−0.07	0.01	0.01	0.01
	11–15	−0.03	0.05	0.01	0.00	0.01	0.01	0.04	−0.61	−0.08	0.02	1.52	−0.07	0.00	0.02	−0.01
	16–20	−0.03	0.04	0.01	0.00	0.02	0.01	0.03	−0.47	−0.08	0.01	1.52	−0.04	0.00	0.00	0.01
var wage growth at job change	1–5	0.05	0.36	0.08	0.00	0.16	0.11	0.09	−2.27	−0.09	0.15	1.40	−0.35	0.12	0.08	−0.04
	6–10	−0.23	0.18	−0.01	0.00	0.16	0.05	0.11	−1.87	−0.04	0.15	1.41	−0.30	0.12	0.08	−0.08
	11–15	−0.25	0.19	−0.06	0.00	0.07	0.02	0.07	−1.61	−0.10	0.07	1.43	−0.26	0.05	0.04	−0.05
	16–20	−0.17	0.08	0.03	0.00	0.04	0.08	0.05	−1.12	−0.10	0.07	1.43	−0.15	0.05	0.00	−0.05
V/U		−0.42	0.04	0.31	0.00	−1.24	0.10	0.18	−0.52	−2.68	−1.33	0.00	0.72	−1.74	−0.71	1.09

Our model opens up an empirical research agenda on which to build and address important issues. We demonstrate the importance of heterogeneity, sorting and search frictions. Among these are the welfare and labor market effects of risk and the role of assets in determining the wage offer distribution and the role of investment in human capital. Similarly, an important extension of such a model is considering investment decisions by firms and how this can affect productivity y which we took as given. Finally, this kind of model is well suited to interpreting matched employer employee data. Indeed such data could aid identification by providing direct information on firm level productivity, and on the distribution of worker types employed at the same firm type (see Lamadon et al., 2014). However, when we move to such data new, important and difficult questions arise when defining wage setting in an environment with sorting and multiple workers per firm. This is of course an important future area of research.

Appendix A. Mathematical

A.1. The continuation value and the equation for the surplus

Let $P(x, y)$ be the value of joint production of an (x, y) match. Then the surplus is defined by $P(x, y) - W_0(x) - \Pi_0(y) = S(x, y)$. Then we have that

$$
\begin{aligned}
rP(x, y) &= f(x, y) + \xi\left[W_0(x) + \Pi_0(y) - P(x, y)\right] \\
&\quad + s\kappa \int \left[\max\left\{P(x, y), \Pi_0(y) + W_0(x) + S(x, y) + \beta\left[S(x, y') - S(x, y)\right]\right\} - P(x, y)\right] v(y')\,\mathrm{d}y' \\
&\quad + \delta \int \left[\max\left\{P(x, y'), W_0(x) + \Pi_0(y')\right\} - P(x, y)\right] \gamma(y')\,\mathrm{d}y' \\
&= f(x, y) - \xi S(x, y) \\
&\quad + s\kappa \int \left[\max\left\{0, \beta\left[S(x, y') - S(x, y)\right]\right\}\right] v(y')\,\mathrm{d}y' \\
&\quad + \delta \int \left[\max\left\{P(x, y') - W_0(x) - \Pi_0(y'), 0\right\} - P(x, y) + W_0(x) + \Pi_0(y')\right] \gamma(y')\,\mathrm{d}y' \\
&= f(x, y) - \xi S(x, y) \\
&\quad + s\kappa\beta \int \left[S(x, y') - S(x, y)\right]^+ v(y')\,\mathrm{d}y' \\
&\quad + \delta \int S(x, y')^+ \gamma(y')\,\mathrm{d}y' - \delta S(x, y) + \delta \int \left[\Pi_0(y') - \Pi_0(y)\right] \gamma(y')\,\mathrm{d}y'.
\end{aligned}
$$

Substituting out $rW_0(x)$ and $r\Pi_0(y)$ from $rS(x, y) = rP(x, y) - rW_0(x) - r\Pi_0(y)$, we have $S(x, y)$ defined by the fixed point

$$
\begin{aligned}
(r + \xi + \delta)\, S(x, y) &= f(x, y) - b(x) + c - \kappa\beta \int S(x, y')^+ v(y')\,\mathrm{d}y' \\
&\quad - \kappa\,(1 - \beta) \int S(x', y)^+ u(x')\,\mathrm{d}x' - s\kappa\,(1 - \beta) \iint \left[S(x', y) - S(x', y')\right]^+ h(x', y')\,\mathrm{d}x'\,\mathrm{d}y' \\
&\quad + s\kappa\beta \int \left[S(x, y') - S(x, y)\right]^+ v(y')\,\mathrm{d}y' + \delta \int S(x, y')^+ \gamma(y')\,\mathrm{d}y'. \qquad (17)
\end{aligned}
$$

Note that

$$
\left[r + \xi + \delta + s\kappa\, v(\overline{\mathcal{B}}(x, y))\right] \frac{\partial S(x, y)}{\partial y} = \frac{\partial f(x, y)}{\partial y} - (r + \delta)\Pi_0'(y), \qquad (18)
$$

with

$$
\overline{\mathcal{B}}(x, y) = \{y' : S(x, y') \geq S(x, y)\},
$$

and $v\,(A) = \int_A v(y)\,dy$, for any set A. Therefore, for any x, the set of y's maximizing the surplus $S(x, y)$ is the set of y's maximizing $f(x, y) - (r + \delta)\Pi_0(y)$.

A.2. The expected profits of a vacant job increase with productivity

From the previous expression note also that

$$
(r + \delta)\Pi_0'(y) = \kappa\,(1 - \beta) \int\limits_{S(x, y) \geq 0} \frac{\partial S(x, y)}{\partial y} u(x)\,\mathrm{d}x + s\kappa\,(1 - \beta) \int h(x, \mathcal{B}(x, y)) \frac{\partial S(x, y)}{\partial y}\,\mathrm{d}x, \qquad (19)
$$

where $\mathcal{B}(x, y)$ is the set of jobs with a productivity y' leading to a lower surplus than the pair (x, y),

$$\mathcal{B}(x, y) = \{y' : 0 \leq S(x, y') < S(x, y)\}.$$

Hence, plugging the above expression for $\partial S(x, y)/\partial y$ in equation (19) shows that $\Pi_0'(y)$ is positive if $\partial f(x, y)/\partial y$ is positive.

Appendix B. Computing the equilibrium

The equilibrium is characterized by knowledge of the number of jobs N, the labor market tightness $\kappa(U, V)$, the joint distribution of active matches $h(x, y)$, and the surplus function $S(x, y)$. A fixed point iterative algorithm operating on (κ, h, S) can be constructed as follows.

First, with inputs κ, $h(x, y)$ and $S(x, y)$,

1. Calculate $u(x)$ using (1) and calculate $U = \int u(x)\, dx$.
2. Solve for V in equation

$$\kappa = \frac{M\left(U + s(1-U), V\right)}{\left[U + s(1-U)\right] V}.$$

3. Calculate $N = V + 1 - U$ and calculate $v(y)$ with equation (2).

Second,

1. Update h using equation (14) as

$$h(x, y) \leftarrow \frac{\delta \int h(x, y')\, dy' + \left[u(x) + sh\left(x, \mathcal{B}(x, y)\right)\right] \kappa v(y)}{\delta + \xi + s\kappa v\left(\overline{\mathcal{B}}(x, y)\right)} \mathbf{1}\{S(x, y) \geq 0\}. \tag{20}$$

2. Update S using equation (13).
3. Update κ using the free entry equation (15)

$$\kappa \leftarrow \frac{1}{1-\beta} \frac{c - \delta \int \Pi_0(y')\gamma(y')\, \mathrm{d}y'}{\int S(x, 0)^+ u(x)\, \mathrm{d}x + s \int \left[S(x, 0) - S(x, y')\right]^+ h(x, y')\, \mathrm{d}x\, \mathrm{d}y'}. \tag{21}$$

Alternatively, one can solve for (h, S) for a given κ and search for the κ that satisfies the free entry condition. The full iterative fixed point algorithm does not indeed guaranty positive updates for κ.

Appendix C. Chernozhukov and Hong's algorithm for SMM

The estimation procedure of Chernozhukov and Hong (2003) consists of simulating a chain of parameters that (once converged) has the quasi-posterior density

$$p(\theta) = \frac{e^{L_N(\theta)} \pi(\theta)}{\int e^{L_N(\theta)} \pi(\theta)\, d\theta}.$$

A point estimate for the parameters is obtained as the average of the N_S elements of the converged MCMC chain:

$$\hat{\theta}_{MCMC} = \frac{1}{N_S} \sum_{j=1}^{N_S} \theta^j,$$

and standard errors are computed as the standard deviation of the sequence of θ^j. To simulate a chain that converges to the quasi posterior, we use the Metropolis–Hastings algorithm. The algorithm generates a chain $\left(\theta^0, \theta^1, \ldots, \theta^{N_S}\right)$ as follows. First, choose a starting value θ^0. Next, generate ψ from a proposal density $q(\psi|\theta^j)$ and update θ^{j+1} from θ^j for $j = 1, 2, \ldots$ using

$$\theta^{j+1} = \begin{cases} \psi & \text{with probability} \quad d(\theta^j, \psi) \\ \theta^j & \text{with probability} \quad 1 - d(\theta^j, \psi), \end{cases}$$

where

$$d(\theta, \psi) = \min\left(\frac{e^{L_N(\psi)} \pi(\psi) q(\theta|\psi)}{e^{L_N(\theta)} \pi(\theta) q(\psi|\theta)}, 1\right).$$

This procedure is repeated many times to obtain a chain of length N_S that represents the ergodic distribution of θ. Choosing the prior $\pi(\theta)$ to be uniform and the proposal density to be a random walk ($q(\theta|\psi) = q(\psi|\theta)$), results in the simple rule

$$d(\theta, \psi) = \min\left(e^{L_N(\psi) - L_N(\theta)}, 1\right).$$

The main advantage of this estimation strategy is that it only requires function evaluations, and thus discontinuous jumps do not cause the same problems that would occur with a gradient based extremum estimator. Additionally, the converged chain provides a direct way to construct valid confidence intervals or standard errors for the parameter estimates if the optimal weighting matrix is used.[21] The drawback of the procedure is that it requires a very long chain, and consequently a very large number of function evaluations, each requiring the model to be solved and simulated. In practice, we simulate 100 chains in parallel, each of length 10,000, and use the last 1000 elements (pooled over the 100 chains) to obtain parameter estimates and the standard errors.[22]

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[21] To avoid the problem similar to that pointed out by Altonji and Segal (1996) we decided against using the optimal weight matrix. Indeed using it did not give sensible results. In this case the chain converges to a stationary process where the variance is a consistent estimator of the inverse of the Hessian J^{-1} (Theorem 1 of Chernozhukov and Hong, 2003) and the sandwich estimator ($J^{-1} I J^{-1}$) has to be used to calculate standard errors appropriately. Because of the lack of precision in the numerical approximation of the gradient $G(\theta) = \nabla_\theta \widehat{m}_S^M(\theta)$ (with $\widehat{I} = G(\widehat{\theta})^\intercal \widehat{W}_N^{-1} G(\widehat{\theta})$), we report the variance of the MC chain ($\widehat{J}^{-1}$). This is still informative. (Discussions with Han Hong and Ron Gallant helped us to understand this.)

[22] Details pertaining to tuning the MCMC algorithm, a parallel implementation, and related methods in statistics can be found in Robert and Casella (2004), Vrugt et al. (2009), Sisson and Fan (2011).

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