To Match or Not To Match? Optimal Wage Policy with Endogenous Worker Search Intensity *

Fabien Postel-Vinay
INRA Paris-Jourdan, Paris
CREST-INSEE, Malakoff
CEPR, London†

Jean-Marc Robin
Université de Paris I,
CREST-INSEE, Malakoff
CEPR, London‡

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†Address: Laboratoire d'Economie Appliquée de l'INRA, Ecole Normale Supérieure, 48 bld Jourdan, 75014 Paris, France, e-mail: fpostel@java.ens.fr. Internet: www.inra.fr/Internet/Departements/ESR/UR/lea/fpostel.htm.

‡Address: EUREQua, Université de Paris I, 110 bld de l'Hôpital, 75013 Paris, France. Email: robin@ensae.fr

ABSTRACT

We consider an equilibrium search model with on-the-job search where firms set wages. If employers are perfectly aware of all workers' job opportunities, then when an employee receives an outside job offer, it is optimal for their employer to try to retain them by matching the offer, so long as the resulting wage doesn't exceed the worker's productivity. A Bertrand competition is thus triggered between the incumbent employer and the "poacher", which results in a wage increase for the worker.

However, if workers are able to vary their search intensity, then this "offer-matching" policy runs into a moral hazard problem. Knowing that outside offers lead to wage increases, workers are induced to search more intensively, which is costly for the firms. Assuming that firms can commit never to match outside offers, we examine the set of firm types for which it is preferable to do so. We derive sufficient conditions for the equilibrium to be of the sort "all firms match" or "no firm matches". Finally, computed examples show that, even though virtually any situation can be observed in equilibrium when the sufficient conditions are not met, a plausible pattern is one where a "dual" labor market emerges, with "bad" jobs at low-productivity, nonmatching firms and "good" jobs at high-productivity, matching firms.

Keywords: Labor market frictions, wage dispersion, search effort, moral hazard.

JEL codes: J31, J61, J64.

RESUME

Nous considérons un modèle de recherche d'emploi d'équilibre avec recherche pendant l'emploi où les firmes fixent les salaires. Sous l'hypothèse d'information complète et en l'absence d'effort de recherche, la stratégie optimale des firmes est de conditionner le salaire offert sur l'état du travailleur (employé ou non, à quel salaire) et de contrer autant que possible toute offre extérieure faite à un employé. Lorsque l'effort de recherche est endogène, il se peut qu'il soit dans l'intérêt de l'employeur de ne pas surenchérir sur une offre extérieure et retenir un employé, de façon à réduire l'incitation à chercher. Nous dérivons des conditions suffisantes pour qu'il existe un équilibre de marché dans lequel aucune firme ne répond à une telle agression extérieure ou au contraire toutes répondent. Nous montrons aussi différentes simulations numériques dans lesquelles apparait un marché dual où certaines firmes répondent aux offres extérieures et d'autres s'abstiennent.

Mots-clés : Frictions sur le marché du travail, dispersion des salaires, effort de recherche, hazard moral.

Classification JEL : J31, J61, J64.

1 Introduction

This paper considers an equilibrium job search model with on-the-job search where firms set wages. As was shown by Postel-Vinay and Robin (2002a, 2002b), if employers are perfectly aware of all workers' characteristics and job opportunities, then it is optimal for them to offer their reservation wage to any worker the firm comes in contact with. Also, when an employed worker receives an outside job offer, it is optimal for the incumbent employer to try to retain the worker by matching the outside offer, so long as the resulting wage doesn't exceed the worker's marginal productivity. This triggers a Bertrand game between the incumbent employer and the "poacher", which results in either a wage increase or a job change for the worker.

In this paper we examine the following point: If workers are able to vary their job search intensity, then the "offer-matching" behavior of firms runs into a moral hazard problem. Knowing that outside offers are matched by the employers and thus lead to wage increases, workers are induced to search more intensively, which can be very costly for the firms. The latter thus face a trade-off between loosing profitable workers at a relatively low frequency if they don't match offers, and being forced to grant relatively frequent wage increases if they do.

We look at a situation where heterogeneous firms can commit never to match outside offers, and examine the set of firm types for which it is preferable to do so. We derive sufficient conditions on the parameters for the equilibrium to be of the type "all firms match" or "no firm matches". We also show in computed examples that, when the sufficient conditions are not met, basically any situation can be observed in equilibrium. We argue that a plausible situation is one where high-productivity firms choose to match offers, while lower-productivity firms choose to commit not to match offers. The labor market then looks "dual", with "bad jobs" at low-productivity firms paying low wages and offering no within-firm career prospects, and "good jobs" at more productive firms that try to retain their workers from being poached by their competitors, thus regularly granting wage increases.

Our contribution is directly related to two strands of literature. One is obviously the "equilibrium job search" literature (see e.g. Mortensen and Pissarides, 1999 and Mortensen, 2002 for surveys and recent developments), and the other is the literature on individual employment contracts (see Malcomson, 1999, for a survey).

The former conveys the general idea that, even though market frictions are a source of employer monopsony power, the possibility that worker have to search for alternative job offers while employed is a limitation to that employer monopsony power, as it brings firms into some sort of imperfect Bertrand competition. However, the search literature puts relatively little direct emphasis on the strategic aspects of employees' search for a better job. Insights in this matter are provided in a series of papers by Mortensen and coauthors (see e.g. the two references cited earlier, and also Christensen et alii, 2001) where the job search effort put forth by workers—and in particular by employed workers—is made endogenous. Mortensen shows under reasonably (un)restrictive assumptions that, as one should expect, higher paid employees search less actively, with the satisfactory implication that higher paying firms have lower rates of labor turnover. He does so, however, in a particular context where firms are only allowed to post fixed wages, i.e. to use contracts where by assumption wages are constant over the duration of job matches.

Clearly, there are both empirical and theoretical objections to this restriction. Empirically, standard wage regressions always have a positive estimated coefficient on job tenure. On the theory side, the individual employment contract literature generically predicts that wages vary within an employment relationship in reaction to changes in the firms' or the workers' outside opportunities (see e.g. Malcomson's 1999 survey and the related evidence in Beaudry and DiNardo, 1991). In the context of a labor market affected by search frictions, the fact of finding an alternative employment opportunity certainly constitutes a change in the worker's outside option which is likely to cause a wage renegotiation. Now, even if renegotiation is not allowed for, as in the wage-posting setup, on-the-job search is nevertheless a source of moral hazard that renders fixed-wage contracts non optimal, as emphasized by two recent contributions of Stevens

(2000) and Burdett and Coles (2001). In a frictional labor market, firms will seek to retain their employees by offering wage-tenure contracts. Note that the critic addressed by Stevens, Burdett and Coles to the Burdett-Mortensen (1989) model is the same as the "bonding critic" made to Shapiro and Stiglitz (see for example Carmichael, 1990, and Cahuc and Zylberberg, chapter 5 of a forthcoming book). This is not a pure coincidence. Equilibrium search models and efficiency wage models belong to the same literature.

The papers by Stevens and Burdett and Coles do not yet fully address the moral hazard problem, as they do not endogenize worker search effort. They neglect to take into consideration that by nature wage-tenure posted contracts should limit employees' search. They thus certainly overstate the firms' reaction to employees' search on the job. Our contribution can be understood as an attempt to shed some light on the issue of why and when do firms commit not to match outside offers when search effort is endogenously chosen by employees. It does not help to understand why firms would post wages instead of conditioning wage contracts on worker states. In wage posting models, indeed, firms offer the same wage to all workers, whether they are employed or not, and whatever employees' current wage. Yet, it helps understanding why firms may not want to counter alternative offers made to their employees. We note, however, that our contribution is not completely general as we do not consider the possibility of indexing wages on tenure. It therefore stands beside those of Stevens, Burdett and Coles as one additional piece of argument in the complex discussion of the form of wage contracts in frictional labor markets.[1]

We end this discussion of the related literature by one last remark. What makes here the theory particularly difficult is that in order to understand the effect of search frictions on wage contracts it is necessary to describe the labor market as a whole, meaning in general

[1] It seems that the main focus of this literature is not so much the form of *optimal* contracts in general than the form of optimal contracts under certain restrictions. To cite Cahuc and Zyberberg (p. 268, chapter 5), "a general principle of the theory of incentives [...] is that a principal who has at his or her disposal a sufficiently wide range of strategies can always make the agent's participation constraint binding, and thus appropriate the entire surplus [...]. The existence of a rent for the agent in a model with moral hazard is thus grounded on restrictions—that require explanation—on the strategic options of individuals."

equilibrium. Our contribution is therefore related to the efficiency wage literature also by the fact that market externalities also play an important role (see Shapiro and Stiglitz, 1984, and MacLeod and Malcomson, 1989, 1998).

The paper has 6 Sections besides this Introduction. Section 2 describes the basic economic environment. Section 3 exposes the wage setting process and describes worker mobility patterns. Section 4 examines equilibrium firm profits. Section 5 looks at the equilibrium wage policies of firms according to their types. A series of computed examples is finally commented on in section 6. Section 7 concludes and discusses the main results.

2 The environment

Workers, firms and matches. We consider the market for a homogeneous occupation, at a steady state and on which a unit mass of atomistic workers face a unit mass of competitive firms that produce one unique multi-purpose good. Firms and workers live forever, and discount the future at rates r and ρ, respectively. As we shall see below, it is not anecdotal that we allow the discount rate of workers to differ from the one of firms.

Workers can either be employed or unemployed, and the unemployment rate of a given category of labor is denoted by u. The pool of unemployed workers is steadily fueled by job destructions that occur at the exogenous rate δ. Workers are homogeneous.

All firms are endowed with a constant-returns-to-labor technology. The marginal productivity of labor equals a firm-specific constant p. Firms exogenously differ in this productivity parameter p. Firm size is limited by search frictions, the existence of which allows low-productivity firms to survive on the market, along with higher-productivity firms.

Search frictions. We assume that firms and workers are brought together pairwise through a random and time-consuming search process. Specifically, unemployed workers sample job offers sequentially at a Poisson rate λ_0. As in the original paper by Burdett and Mortensen (1998), employees may also search for a better job while employed. The arrival rate of offers to on-the-job searchers is $(\lambda_1 + s)$, where s is the endogenous intensity of their search activity. Search effort s is bounded from above at $\overline{s}$ and the cost of search is a function $c(s) > 0$, with $c'(s) > 0$ and $c''(s) \geq 0$. The constant $\lambda_1 > 0$ is the "minimal" arrival rate of offers: even passive workers get offers falling on their lap every now and then. The type (per period output) p of the firm from which a given offer originates is assumed to be randomly selected from the interval $\left[\underline{p}, \overline{p}\right]$ according to the sampling distribution $\Gamma' = \gamma$, with a continuous density γ.

Wage setting. We make four fundamental assumptions about the wage setting mechanism:

1. Firms can vary their wage offers according to the characteristics of the particular worker they meet;

2. Firms can counter the offers received by their employees from competing firms;

3. Firms can commit not to match outside offers. They do so ex ante, i.e. before they meet any worker. Any subsequent deviation from such a commitment is observable by everyone on the market;

4. Wage contracts are long-term contracts that can be renegotiated by mutual agreement only.

Assumptions 2 and 3 are important. In particular, as the workers' search intensity is endogenous, assumption 3 enables firms to induce workers to search less.[2] It will turn out in equilibrium that some firms will choose to match offers, while some will choose to commit not to match offers. We shall denote by γ_{m} and γ_{n} the unnormalized sampling densities of productivities of firms who do and do not match, respectively. We thus have $\gamma_{\mathrm{m}}(p) + \gamma_{\mathrm{n}}(p) = \gamma(p)$ for all p. We let Γ_{m} and Γ_{n} denote the associated (defective) cdfs. We denote as $m = \Gamma_{\mathrm{m}}(\overline{p}) \geq 0$

[2] We shall only consider matching strategies of a 0-1 type. Matching strategies where an employer declares that they will match outsider offer with a probability θ are ruled out by assumption. This restriction will actually turn out to coincide with the employers' optimal strategy under the particular assumption that we are going to adopt about the workers' search behavior (see below for more on this point).

and $n = \Gamma_{\mathrm{n}}(\overline{p}) \geq 0$ respectively the total sampling probabilities of matching and nonmatching firms (with obviously $m + n = 1$).

Finally, we use the convention that workers prefer to leave unemployment when they are offered a wage contract yielding the same present value as the value of unemployment but that employees prefer not to move when they are offered by a poaching firm a wage contract yielding the same value as their wage in their current firm.

3 Worker behavior: wages and search effort

We now exploit the preceding series of assumptions to derive the precise values of the wage resulting from the various forms of firm-worker contacts. From there, we also characterize the optimal search behavior of workers. The next three sections contain step-by-step derivations of wages and search efforts. The results are then summarized in section 3.4.

3.1 Wage competition and worker value functions

The lifetime utility of an unemployed is denoted by U_0, and that of the same worker when employed at a firm of type p and paid a wage w is $U_{\mathrm{m}}(w, p)$ if the firm matches alternative offers, and $U_{\mathrm{n}}(w, p)$ if it doesn't. Four distinct situations may occur according to the respective types of the firms that compete over a given worker. We look at the four cases in a sequence.

Employees of nonmatching firms. Let us first consider the case of a worker earning w at a type p nonmatching firm, and suppose this worker receives an outside offer from a type q firm.

We assume that nonmatching firms systematically offer their reservation wage to any worker that they meet on the search market.[3] Therefore, if the type q poacher is a nonmatching firm, then, the incumbent employer having also committed not to match outside offers, the challenging firm offers the worker her reservation wage (plus epsilon) which is equal to w since

[3] That offering the reservation be the optimal strategy of any nonmatching firm is not obvious, and not true in general, since offering a higher wage may be a means of limiting worker turnover, as in Burdett and Mortensen (1998). A restriction on the model parameters (notably the sampling distribution γ) is needed for that, and will be explicited later in the paper. For now, we merely *assume* that nonmatching firms will behave like this in equilibrium.

there is no other gain but a greater wage in changing employers. The only restriction on mobility is that q must be no less than w.

If the poaching firm is of the matching type, then it still offers the employee of the non-matching firm their reservation wage (plus epsilon). The novelty here is that the reservation wage is no longer necessarily equal to w and will be characterized later on.

It follows that an outside offer generates no capital gain to a worker employed at a non-matching firm, the value $U_{\mathrm{n}}(w,p)$ of a wage contract w in a non-matching firm of productivity p simply solves the following asset pricing equation:

$$\rho U_{\mathrm{n}}(w,p) = w - c(s) + \delta \cdot \left[U_0 - U_{\mathrm{n}}(w,p)\right].$$

As a result, the optimal search effort when employed at a nonmatching firm is obviously 0. We thus get from the last equation:

$$U_{\mathrm{n}}(w) = \frac{w + \delta U_0}{\rho + \delta}, \tag{1}$$

which has the additional implication that $U_{\mathrm{n}}(w,p) \equiv U_{\mathrm{n}}(w)$ is independent of p.

Employees of matching firms. We now turn to the case of a worker earning w at a type p matching firm, and that receives an offer from a type q firm. In this case, the two firms enter a Bertrand game, the winner of which is the firm with highest productivity. Such a game was already analyzed in detail in Postel-Vinay and Robin (2002a,2002b).

The most simple situation is the one where the worker is paid a wage exactly equal to her marginal productivity p. It then makes no difference to the worker whether her employer matches offers or not because her wage cannot be raised to exceed p anyway. It follows that the present value of being paid the marginal productivity at a type p matching firm is

$$U_{\mathrm{m}}(p,p) = \frac{p + \delta U_0}{\rho + \delta} = U_{\mathrm{n}}(p). \tag{2}$$

Now obviously, $U_{\mathrm{m}}(p,p)$ is the highest value that an employee of a type p matching firm can hope to get. It is as high as the incumbent type p employer is able to go in the Bertrand competition. In other words, it is the reservation value of the worker.

If the poacher's type q is greater than p, then the poacher can afford to offer the worker a value that strictly exceeds $U_{\mathrm{m}}(p,p)$ (for instance $U_{\mathrm{m}}(q,q)$ if the poacher is of the matching type, or $U_{\mathrm{n}}(q)$ if it is of the nonmatching type), and therefore wins the Bertrand competition and attracts the worker. The optimal wage offer made by a matching poacher of type q is $\phi(p,q)$, defined by the worker's indifference condition:

$$U_{\mathrm{m}}(p,p) = U_{\mathrm{m}}\left[\phi(p,q), q\right]. \tag{3}$$

Likewise, the optimal wage offer made by a nonmatching poacher of type q is the incumbent employer's marginal productivity p (plus epsilon), since $U_{\mathrm{m}}(p,p) = U_{\mathrm{n}}(p)$.

Now, if the poacher's type q is less than p, then it is not able to offer the worker a value of $U_{\mathrm{m}}(p,p)$, since the best it can do is to offer $U_{\mathrm{m}}(q,q) = U_{\mathrm{n}}(q)$ (depending on its matching type). The incumbent employer therefore successfully counters the poacher's offer by offering the wage $\phi(q,p)$ such that $U_{\mathrm{m}}\left[\phi(q,p), p\right] = U_{\mathrm{m}}(q,q) = U_{\mathrm{n}}(q)$.

Naturally, this only entails a wage increase for the worker if the latter's previous wage w was less than $\phi(q,p)$. In the opposite case, nothing happens: the poacher's best offer is beaten by the worker's *statu quo* option of staying at firm p with a wage w.

To sum up, starting from any wage w, firms bid up to the point where the firm with the second highest reservation price (productivity) cannot outbid the one with the highest productivity.

Second, let $q(w,p)$ be the maximal productivity of an employer from which a firm with productivity p can poach an employee by offering a wage w:

$$U_{\mathrm{m}}\left[q(w,p), q(w,p)\right] = U_{\mathrm{n}}\left[q(w,p)\right] = U_{\mathrm{m}}(w,p). \tag{4}$$

(Note that $q(p,p) = p$ and that $\phi(q(w,p),p) = w$. Knowing the function $q(w,p)$ is thus equivalent to knowing the function $\phi(q,p)$.) With this notation, we can now characterize the present value of a wage contract w at a matching firm of productivity p. $U_{\mathrm{m}}(w,p)$ solves the following asset value equation:

$$
\begin{aligned}
\rho U_{\mathrm{m}}(w,p) &= w - c(s) + \delta \cdot [U_0 - U_{\mathrm{m}}(w,p)] \\
&+ (\lambda_1 + s) \cdot \int_{q(w,p)}^{p} [U_{\mathrm{m}}(x,x) - U_{\mathrm{m}}(w,p)]\, d\Gamma_{\mathrm{m}}(x) + (\lambda_1 + s)\, \overline{\Gamma}_{\mathrm{m}}(p)\, [U_{\mathrm{m}}(p,p) - U_{\mathrm{m}}(w,p)] \\
&+ (\lambda_1 + s) \cdot \int_{q(w,p)}^{p} [U_{\mathrm{n}}(x) - U_{\mathrm{m}}(w,p)]\, d\Gamma_{\mathrm{n}}(x) + (\lambda_1 + s)\, \overline{\Gamma}_{\mathrm{n}}(p)\, [U_{\mathrm{n}}(p) - U_{\mathrm{m}}(w,p)], \quad (5)
\end{aligned}
$$

where $\overline{\Gamma}(\cdot) \equiv 1 - \Gamma(\cdot)$. The right hand side of this equation contains four terms: the instantaneous payoff $w - c(s)$, the job destruction risk term $\delta \cdot [U_0 - U_{\mathrm{m}}(w,p)]$, the specific risk related to drawing an outside offer from a matching firm and the specific risk related to drawing an outside offer from a nonmatching firm. In these last two cases, the offer may either result in a mobility (whenever the poaching firm's productivity is greater than the incumbent's) or not.

We can now replace $U_{\mathrm{n}}(x)$ and $U_{\mathrm{m}}(x,x)$ by their expressions from (1) and (2) into (5) and integrate by parts to get:

$$
(\rho + \delta)\, U_{\mathrm{m}}(w,p) = w - c(s) + \delta U_0 + \frac{\lambda_1 + s}{\rho + \delta} \cdot \int_{q(w,p)}^{p} \overline{\Gamma}(x)\, dx, \tag{6}
$$

Finally, from (4) and (6), we get the following definition of $q(w,p)$:

$$
q(w,p) - \frac{\lambda_1 + s(w,p)}{\rho + \delta} \cdot \int_{q(w,p)}^{p} \overline{\Gamma}(x)\, dx = w - c[s(w,p)], \tag{7}
$$

where $s(w,p)$ is the search intensity decided by a worker paid w at a matching firm of type p, which we now look at.

3.2 Search intensity

The optimal search effort of a worker employed at a type (m, p) firm earning a wage of w thus obeys the following rule:

$$
c'[s(w,p)] \leq \frac{1}{\rho + \delta} \cdot \int_{q(w,p)}^{p} \overline{\Gamma}(x)\, dx \quad \text{and} \quad s(w,p) \leq \overline{s}, \tag{8}
$$

with equality in one of the two inequalities. The following limiting case will be considered in the paper: if the marginal cost of search effort is a constant c, workers will put forth the effort

$\overline{s}$ if their wage is less than the threshold wage $w_S(p)$ solving $c = \int_{q(w_S(p),p)}^{p} \overline{\Gamma}(x)\,dx/(\rho+\delta)$, and will make zero effort as soon as their wage exceeds $w_S(p)$. As c becomes infinitesimal, $w_S(p) \to p$ and all employees of matching firms tend to search with the maximal intensity $\overline{s}$. The search intensity is therefore constant across matching firms (and equals $\overline{s}$ in these firms), as well as across nonmatching firms (where it equals 0).

3.3 Unemployed workers

We let b denote the constant flow earnings of an unemployed worker (net of search costs), which they have to forgo from the moment they find a job. To ensure that all existing firm types are able to hire a positive workforce, it is necessary that the infimum of Γ's support, $\underline{p}$, be no less than b, for a firm less productive than b would never attract any worker. Whenever that condition is met, any type-p firm will want to hire any unemployed worker upon meeting them on the search market. Since firms offer their reservation wages to the unemployed workers they come in contact with, the latter don't expect any capital gains from their first job. As a result, U_0 simply equals b/ρ.

Obviously, if b is an unconditional level of unemployment income, then unemployed workers don't have any incentive to put forth any search effort, implying $\lambda_0 = \lambda_1$. We can circumvent this restriction by assuming for instance that eligibility for unemployment benefits is conditional on a verifiable search effort, which ensures an arrival rate of offers of λ_0.

Finally, it is straightforward to check that the wage offered to an unemployed worker by a nonmatching firm is exactly equal to b, while that offered by a matching firm is equal to:

$$\phi_0(p) = b - \frac{\lambda_1 + \overline{s}}{\rho+\delta} \cdot \int_b^p \overline{\Gamma}(x)\,dx = \phi(b,p), \tag{9}$$

which is less than b.

3.4 Summary

We sum up the previous results for all possible cases in table 1. Each row of the table indicates a different worker state (employed/unemployed, employed at a firm of productivity p, matching

Worker's state	Offer arrival rate	Offer coming from a firm with productivity p' and matching type n or m		
Unemployed	λ_0	If $p' > b$,	mobility, no increase in value	$\begin{cases} \text{if n,} & w' = b^+ \\ \text{if m,} & w' = \phi(b,p)^+ \end{cases}$
Earns w at a non-matching type p firm	λ_1	If $p' \leq w$,	no mobility, no increase in value	no wage change
		If $p' > w$,	mobility, no increase in value	$\begin{cases} \text{if n,} & \text{no wage change} \\ \text{if m,} & w' = \phi(w,p')^+ < w \end{cases}$
Earns w at a matching type p firm	$\lambda_1+\overline{s}$	If $p' \leq q(w,p)$,	no mobility, no increase in value	no wage change
		If $q(w,p) < p' \leq p$,	no mobility, value increases	$w' = \phi(p',p)^+ > w$
		If $p' > p$,	mobility, value increases	$\begin{cases} \text{if n,} & w' = p^+ \\ \text{if m,} & w' = \phi(p,p')^+ < p \end{cases}$

Table 1: Mobility and Wage Setting

or not matching outside offers). The worker's state conditions a specific search intensity which determines the actual job offer arrival rate as displayed in column 2. The third column indicates which value of the productivity p' of the firm making an offer induces the worker to change employer. The fourth and last column shows the wage w' that results from the job offer according to the respective types of the current employer and the poaching firm. Finally, the functions $\phi(p,p')$ and $q(w,p)$ are defined as

$$\phi(p,p') = p - \frac{\lambda_1 + \overline{s}}{\rho + \delta} \cdot \int_p^{p'} \overline{\Gamma}(x)\,dx, \tag{10}$$

$$q(w,p) = w + \frac{\lambda_1 + \overline{s}}{\rho + \delta} \cdot \int_{q(w,p)}^{p} \overline{\Gamma}(x)\,dx. \tag{11}$$

In the table, we use a superscript $^+$ for w' to indicate that by convention a firm has to offer strictly more than the reservation wage to make a worker accept the offer (such amount plus epsilon).

4 Equilibrium analysis of matching commitments

The objective of this section is to provide some elements of characterization of the Nash equilibrium matching commitments, i.e. to address the question of which firm types decide to match outside offers in equilibrium and which firms decide to commit not to match. This decision is obviously based upon the profits accruing to the entrepreneur in both cases, which we shall precisely define and derive in the coming analysis. Before we do so, however, a preliminary characterization of the steady-state distributions of wages and firm types across workers is needed. This is what the following paragraph is about.

4.1 Steady-state worker and wage distributions

Even without knowing which firm types choose to match and which don't in equilibrium, we can define $\ell_{\mathrm{n}}(p)$ as the steady-state equilibrium unnormalized density of nonmatching firm productivities across employed workers. In other words, $\ell_{\mathrm{n}}(p)$ equals the steady-state measure of workers employed at type p nonmatching firms. We can further define $G_{\mathrm{n}}(w|p)$ (resp. $g_{\mathrm{n}}(w|p)$) the cdf (resp. density) of wages in the population of such workers. Likewise, we introduce similar definitions for matching firms: $\ell_{\mathrm{m}}(p)$ and $G_{\mathrm{m}}(w|p)$ (resp. $g_{\mathrm{m}}(w|p)$). The following Proposition characterizes those four distributions.[4]

Proposition 1 (Equilibrium wage and productivity distributions) *Let $\varphi_{\mathrm{m}}(p)$ and $\varphi_{\mathrm{n}}(p)$ be the pair of functions solving the system of differential equations:*

$$\left\{\begin{array}{l} \dfrac{\varphi_{\mathrm{m}}'(p)}{\varphi_{\mathrm{m}}(p)} = \dfrac{\lambda_1+\overline{s}}{\delta+(\lambda_1+\overline{s})\,\overline{\Gamma}(p)} \cdot \left[\gamma(p) + \dfrac{\delta+\lambda_1\overline{\Gamma}(p)}{\delta+\lambda_1\overline{\Gamma}_{\mathrm{m}}(p)} \cdot \gamma_{\mathrm{m}}(p)\right] \\ \varphi_{\mathrm{n}}'(p) = (\lambda_1+\overline{s})\,\dfrac{\varphi_{\mathrm{m}}(p)\,\gamma_{\mathrm{m}}(p)}{\delta+\lambda_1\overline{\Gamma}_{\mathrm{m}}(p)} \end{array}\right. \tag{12}$$

[4] Again, the densities ℓ_{m} and ℓ_{n} are not normalized, and we have:

$$\int_{\underline{p}}^{\overline{p}} \left[\ell_{\mathrm{m}}(x)+\ell_{\mathrm{n}}(x)\right] dx = 1-u.$$

over $\left[\underline{p},\overline{p}\right]$ *with initial conditions*

$$\left\{\begin{array}{l}\varphi_{\mathrm{m}}\left(\underline{p}\right)=\dfrac{\lambda_0 u\left(\delta+\lambda_1\right)}{\left(\delta+\lambda_1+\overline{s}\right)\left(\delta+\lambda_1 m\right)} \\ \varphi_{\mathrm{n}}\left(\underline{p}\right)=\dfrac{\lambda_0 u}{\delta+\lambda_1 m}\end{array}\right. . \tag{13}$$

Then:

1. *For all* p*, the distribution of wages in* matching firms *with productivity* p *has support included in* $\{\phi_0(p)\} \cup (\phi(\underline{p}, p), p]$*. The cdf of wages within these firms is such that*

$$G_{\mathrm{m}}\left(\phi_0(p)|p\right)\ell_{\mathrm{m}}\left(p\right)=\frac{\lambda_0 u}{\delta+\lambda_1+\overline{s}}\gamma_{\mathrm{m}}\left(p\right), \tag{14}$$

   and

$$G_{\mathrm{m}}\left(w|p\right)\ell_{\mathrm{m}}\left(p\right)=\varphi_{\mathrm{m}}\left[q\left(w,p\right)\right]\gamma_{\mathrm{m}}\left(p\right), \tag{15}$$

   over $(\phi(\underline{p}, p), p]$.

2. *For all* p*, the distribution of wages across employees of* nonmatching firms *with productivity* p *has support included in* $[b, b+\varepsilon) \cup (\underline{p}, p]$.[5] *The corresponding cdf is such that*

$$G_{\mathrm{n}}\left(b|p\right)\ell_{\mathrm{n}}\left(p\right)=\frac{\lambda_0 u}{\delta+\lambda_1}\gamma_{\mathrm{n}}\left(p\right), \tag{16}$$

   and

$$G_{\mathrm{n}}\left(w|p\right)\ell_{\mathrm{n}}\left(p\right)=\varphi_{\mathrm{n}}\left(w\right)\gamma_{\mathrm{n}}\left(p\right), \tag{17}$$

   over $(\underline{p}, p]$.

3. *Finally, for all* p*, the densities of firm types across employed workers* $\ell_{\mathrm{m}}(p)$ *and* $\ell_{\mathrm{n}}(p)$ *are defined by:*

$$\ell_{\mathrm{m}}\left(p\right)=\varphi_{\mathrm{m}}\left(p\right)\gamma_{\mathrm{m}}\left(p\right) \tag{18}$$

   and

$$\ell_{\mathrm{n}}\left(p\right)=\varphi_{\mathrm{n}}\left(p\right)\gamma_{\mathrm{n}}\left(p\right). \tag{19}$$

[5] With ε an arbitrarily small positive real number.

The proof of this Proposition is confined to the Appendix. It mainly rests on the various firm- and worker-level flow-balance equations implied by the steady-state assumption.

The contents of this Proposition are essentially technical and will be used in the simulations below. Two things are worth emphasizing, however. First, while $\ell_{\mathrm{m}}(p)$ and $\ell_{\mathrm{n}}(p)$ represent the *densities* of workers employed at firms of types (m, p) and (n, p) respectively, the functions $\varphi_{\mathrm{m}}(p)$ and $\varphi_{\mathrm{n}}(p)$ are interpreted as the (mean) *sizes* of such firms.

Second, the Proposition formalizes the intuition that wage dispersion degenerates to an atom at the monopsony wage b on a market where all firms choose the nonmatching strategy.[6] Intuitively, there is no reason indeed why wages would ever depart from b on a market where no firm responds to the outside job offers received by their workers. Unemployed workers are hired by any of those nonmatching firms at a wage equal to b, and employed workers are successfully poached by competitors offering a wage equal to b plus an infinitesimal amount.

Contrary to what happens in the Burdett and Mortensen (1998) world, on-the-job search and between-firm competition for hiring and retaining workers does not entail any equilibrium wage dispersion among ex-ante similar workers in this case. Even though we haven't explicitly modelled it here, the only potential source of wage dispersion in a situation like this is heterogeneity in the unemployed workers' reservation wages, as described by Albrecht and Axell (1984) and Albrecht and Vroman (2001).

Now, in the general case where some firm types choose to match offers and some choose not to, Proposition 1 predicts non degenerate within- and between-firms wage distributions, the shape of which in turn depend on which firm types choose to match offers and which choose not to. This is the question addressed in the following paragraphs.

[6] **Proof of this claim:** Since in that situation we have $\gamma_{\mathrm{m}}(p) = 0$ for all p, system (12) implies that $\varphi_{\mathrm{n}}(p)$ remains constant over $\underline{p}, \overline{p}$. Using this constancy property in (17) and (19) then shows that $G_{\mathrm{n}}(w|p) \equiv 1$ for all (w, p) with $b < w \leq p$.

4.2 Firm behavior

As we already argued, the decision of whether to match or not to match offers hinges on the comparison between the ex-ante profits made under either strategy—i.e. the expected profits at the time when the commitment is made, which is before any worker is hired by the firm. What we have to do first is therefore to come up with a definition and of course an expression of those ex-ante profits.

We follow Mortensen (2002) in our specification of firm behavior by assuming that firms maximize *expected profit per contacted worker*.[7] This quantity has two components: the expected present discounted value of a job occupied by a worker paid some wage w, and the density of wages accepted by contacted workers. The next two paragraphs looks at the former (under both matching strategies), and the paragraph after that determines the latter and finally derives the criterion that entrepreneurs maximize when they choose either to match or not to match offers.

The following notation is adopted for the rest of the paper: The discounted sum of future expected profit flows from a filled job at a firm with productivity p and matching type m (n) that pays a current wage of w is designated by $J_{\mathrm{m}}(w,p)$ ($J_{\mathrm{n}}(w,p)$).

Value of a filled job to a matching firm: $J_{\mathbf{m}}(w,p)$. In order to write down the Bellman equations characterizing $J_{\mathrm{m}}(w,p)$, we should remember two things: first, firms discount the future at rate r, possibly different from the workers' ρ, and second, matching commitments are assumed to hold through the entire lifetime of a job. With that in mind, the results established

[7]Much of the job search literature follows the initial assumption of Burdett and Mortensen (1998) that firms maximize their steady-state profit flows. As those two authors have emphasized, both criteria are equivalent if one assumes that firms don't discount the future, i.e. if one assumes $r = 0$. We do not want to make this assumption in the present analysis, as we shall see that the difference between r and ρ—i.e. the firms' and the workers' discount rates—plays an interesting role in equilibrium determination. This result would be obliterated if we forced $r = 0$. Treatment of the moral hazard problem analyzed in this paper in the context of the original Burdett and Mortensen assumption was carried out by us in an earlier version of this paper, which is available upon request.

in the previous sections directly imply that, for any (w,p) with $q = q(w,p)$:

$$\begin{aligned}
\left[r+\delta+(\lambda_1+\bar{s})\,\overline{\Gamma}\,[q(w,p)]\right] J_{\mathrm{m}}(w,p) &= p-w+(\lambda_1+\bar{s})\int_{q(w,p)}^{p} J_{\mathrm{m}}[\phi(x,p),p]\,d\Gamma(x)\\
&\Updownarrow\\
\left[r+\delta+(\lambda_1+\bar{s})\,\overline{\Gamma}(q)\right] J_{\mathrm{m}}[\phi(q,p),p] &= p-\phi(q,p)+(\lambda_1+\bar{s})\int_{q}^{p} J_{\mathrm{m}}[\phi(x,p),p]\,d\Gamma(x)\,.
\end{aligned}$$

Take the first line in the above equation. $(p-w)$ is the profit flow accruing to the type p firm from a job paying a wage w. In addition, the worker leaves the match (either to unemployment or to a more attractive job) with instantaneous probability $\left[\delta+(\lambda_1+\bar{s})\,\overline{\Gamma}(p)\right]$, thus leaving the firm with zero residual value. Finally, with a probability flow of $(\lambda_1+\bar{s})\{\Gamma(p)-\Gamma[q(w,p)]\}$, the worker receives an outside offer from a firm with type within $[q(w,p),p]$, which the employer successfully counters. The latter thus loses the value $J_{\mathrm{m}}(w,p)$, and recovers a job worth $J_{\mathrm{m}}[\phi(x,p),p]$, where x is the unsuccessful poacher's type. Integrating over $[q(w,p),p]$ and adding those three terms, we get the above Bellman equation.

Differentiation w.r.t. q of the last definition of $J_{\mathrm{m}}[\phi(q,p),p]$ shows that:

$$\frac{\partial}{\partial q}\{J_{\mathrm{m}}[\phi(q,p),p]\} = -\frac{1}{\rho+\delta}\cdot\frac{\rho+\delta+(\lambda_1+\bar{s})\,\overline{\Gamma}(q)}{r+\delta+(\lambda_1+\bar{s})\,\overline{\Gamma}(q)},$$

which can be integrated to finally yield:[8]

$$J_{\mathrm{m}}[\phi(q,p),p] = \frac{1}{\rho+\delta}\int_{q}^{p}\frac{\rho+\delta+(\lambda_1+\bar{s})\,\overline{\Gamma}(x)}{r+\delta+(\lambda_1+\bar{s})\,\overline{\Gamma}(x)}dx. \tag{20}$$

Note in passing that this expression becomes remarkably simple in the special case where workers and firms have equal discount rates ($r=\rho$).

Value of a filled job to a nonmatching firm: $J_{\mathrm{n}}(w,p)$. The value of a nonmatching job, $J_{\mathrm{n}}(w,p)$, is much simpler to characterize as the worker leaves the job upon receiving an offer from any firm whose productivity exceeds their current wage. As a consequence,

$$\left[r+\delta+\lambda_1\overline{\Gamma}(w)\right] J_{\mathrm{n}}(w,p) = p-w. \tag{21}$$

[8] This uses the fact that $J_{\mathrm{m}}(p,p)=0$.

Comparing with the initial Bellman equation for $J_{\mathrm{m}}(w,p)$, we see that outside offers are more likely to cause the worker to leave the firm in the nonmatching case, with the counteracting advantage that such offers are less frequent in that case (rate λ_1 instead of $\lambda_1+\overline{s}$), since workers have no incentive to incur the cost of active search.[9]

Expected profits per worker contacted. Matching commitments are made ex ante, i.e. they are "posted" together with the job offer. Thus, to make its decision of whether to match or not to match outside offers, a firm does not only compare the filled job values J_{n} and J_{m}, but rather the corresponding expected values of contacting a potential job applicant, π_{m} and π_{n}.

The number (measure) of workers contacted by a type p firm per unit time is given by $n_{\mathsf{c}}\cdot\gamma(p)$ where:

$$n_{\mathsf{c}}\cdot\gamma(p)=\left\{\lambda_0 u+(\lambda_1+\overline{s})\int_{\underline{p}}^{\overline{p}}\ell_{\mathrm{m}}(x)\,dx+\lambda_1\int_{\underline{p}}^{\overline{p}}\ell_{\mathrm{n}}(x)\,dx\right\}\cdot\gamma(p).$$

The first term in curly brackets is the number of contacted unemployed workers, the second term counts employed job seekers from matching firms, and the third corresponds to employed job seekers from nonmatching firms. The constant n_{c} is the average number of contacts per firm and unit of time.

Let us first consider the expected value of contacting a worker for a matching firm of type p, which we denote as $\pi_{\mathrm{m}}(p)$. Since the origin of future applicants (i.e. their wage and incumbent

[9] Now that the value $J_{\mathrm{n}}(w,p)$ is defined, we can return to the question of a nonmatching firm's optimal wage offer policy (see foonote 3). Given its precommitment not to respond to poachers, a nonmatching firm might indeed find it more profitable to offer more than its applicant's reservation wages to reduce the chance that its worker leaves to a better paying firm, i.e. to limit its turnover. Clearly, equation (21) alone does not imply that $\partial J_{\mathrm{n}}/\partial w<0$ for all (w,p), as:

$$\frac{\partial J_{\mathrm{n}}}{\partial w}(w,p)=\frac{(p-w)\,\lambda_1\gamma(w)-\left[r+\delta+\lambda_1\overline{\Gamma}(w)\right]}{\left[r+\delta+\lambda_1\overline{\Gamma}(w)\right]^2}.$$

We thus have to impose the additional restriction on Γ that for all (p,w) with $w<p$:

$$(p-w)\,\lambda_1\gamma(w)<r+\delta+\lambda_1\overline{\Gamma}(w).$$

This restriction ensures that if all nonmatching firms implement the same "minimal wage offer policy", then it is optimal for the marginal nonmatching firm to do so as well, i.e. this reservation wage policy is an equilibrium strategy.

employer's type) is not known in advance to a firm, the latter has to integrate over all possible origins. Accordingly, the expected value of meeting an applicant solves:

$$\pi_{\mathrm{m}}(p) = \frac{\lambda_0 u}{n_{\mathrm{c}}} J_{\mathrm{m}}\left[\phi(b,p),p\right] + \frac{\lambda_1 + \overline{s}}{n_{\mathrm{c}}} \int_{\underline{p}}^{p} J_{\mathrm{m}}\left[\phi(q,p),p\right] \ell_{\mathrm{m}}(q)\, dq$$
$$+ \frac{\lambda_1}{n_{\mathrm{c}}} \left(\int_{\underline{p}}^{p} J_{\mathrm{m}}\left[\phi(w,p),p\right] \int_{w}^{\overline{p}} g_{\mathrm{n}}(w|q)\, \ell_{\mathrm{n}}(q)\, dq dw + J_{\mathrm{m}}\left[\phi(b,p),p\right] \int_{\underline{p}}^{\overline{p}} G_{\mathrm{n}}\left(\underline{p}|q\right) \ell_{\mathrm{n}}(q)\, dq \right),$$

The first term in the above equation corresponds to the case where the firm contacts an unemployed worker. This happens with probability $\lambda_0 u / n_{\mathrm{c}}$ and the corresponding offered wage is the unemployed workers' reservation wage for working at a type p-matching firm $\phi(b,p)$, which was determined in the last section (equation (9) and Table 1). The second term is the expected value of a contact with an employee of a matching firm. The flow probability of meeting an employee of a type q matching firm is equal to $(\lambda_1 + \overline{s})\, \ell_{\mathrm{m}}(q) / n_{\mathrm{c}}$. The offer needed to attract such a worker is $\phi(q,p)$, hence the corresponding job value $J_{\mathrm{m}}\left[\phi(q,p),p\right]$. Attracting the worker is only possible if $q \leq p$, which defines the bounds of the integral in the second term above. The last term gives the expected value of an application from an employee of a nonmatching firm. The probability with which an employee of a type q nonmatching firm earning a wage w is met equals to $\lambda_1 g_{\mathrm{n}}(w|q)\, \ell_{\mathrm{n}}(q) / n_{\mathrm{c}}$ (employees of nonmatching firms search less intensively). The offer needed to attract such a worker is a function $\phi(w,p)$ of their current wage w (see Table 1 again), thus yielding the value $J_{\mathrm{m}}\left[\phi(w,p),p\right]$. The feasible (w,p) pairs are defined by $w \leq p$ (otherwise the type p firm is not able to attract the worker), and $w \leq q$ (the worker obviously earns less than their current marginal productivity). Hence the integral in the third term above.

Using the results of Proposition 1, the above expression can be transformed into:

$$\pi_{\mathrm{m}}(p) = \frac{\lambda_0 u}{n_{\mathrm{c}}} \frac{(\delta + \lambda_1)}{\delta + \lambda_1 m} J_{\mathrm{m}}\left[\phi(b,p),p\right]$$
$$+ \frac{\lambda_1 + \overline{s}}{n_{\mathrm{c}}} \int_{\underline{p}}^{p} \frac{\delta + \lambda_1 \overline{\Gamma}(q)}{\delta + \lambda_1 \overline{\Gamma}_{\mathrm{m}}(q)} J_{\mathrm{m}}\left[\phi(w,p),p\right] \varphi_{\mathrm{m}}(q) \gamma_{\mathrm{m}}(q)\, dq. \quad (22)$$

Similarly, for nonmatching firms:

$$\pi_{\mathrm{n}}(p) = \frac{\lambda_0 u}{n_{\mathsf{c}}} \frac{(\delta + \lambda_1)}{\delta + \lambda_1 m} J_{\mathrm{n}}(b,p) + \frac{\lambda_1 + \overline{s}}{n_{\mathsf{c}}} \int_{\underline{p}}^{p} \frac{\delta + \lambda_1 \overline{\Gamma}(q)}{\delta + \lambda_1 \overline{\Gamma}_{\mathrm{m}}(q)} J_{\mathrm{n}}(q,p)\, \varphi_{\mathrm{m}}(q) \gamma_{\mathrm{m}}(q)\, dq. \tag{23}$$

This expression is exactly similar to (22), with $J_{\mathrm{m}}[\phi(w,p), p]$ changed into $J_{\mathrm{n}}(q,p)$. Clearly, the meeting rates of all types of workers are equal in both cases, and the only thing that differs between the matching and nonmatching strategies is the wage that has to be offered to attract a worker. As is summarized in Table 1, nonmatching firms meeting the employee of a matching firm must offer the incumbent job's marginal productivity q (as opposed to $\phi(q,p)$ in the matching case), and a nonmatching firm meeting the employee of another nonmatching firm simply offers the worker's current wage w plus epsilon (as opposed to $\phi(w,p)$ in the matching case). Hence the correspondence between equations (22) and (23).

4.3 Matching commitments

The correspondence between firm types and matching commitments hinges on the sign of the difference between the above two values. Specifically, what we have to look at is $\Delta\pi(p) = \pi_{\mathrm{m}}(p) - \pi_{\mathrm{n}}(p)$: firm types for which this difference is positive (negative) in equilibrium will choose to match (not to match) offers.

Taking the difference between (22) and (23), we obtain the following:

$$n_{\mathsf{c}} \cdot \Delta\pi(p) = \frac{\lambda_0 u (\delta + \lambda_1)}{\delta + \lambda_1 m} \Delta J(b,p) + (\lambda_1 + \overline{s}) \int_{\underline{p}}^{p} \frac{\delta + \lambda_1 \overline{\Gamma}(q)}{\delta + \lambda_1 \overline{\Gamma}_{\mathrm{m}}(q)} \Delta J(q,p)\, \varphi_{\mathrm{m}}(q) \gamma_{\mathrm{m}}(q)\, dq, \tag{24}$$

where we define $\Delta J(q,p)$ as $J_{\mathrm{m}}[\phi(q,p), p] - J_{\mathrm{n}}(q,p)$. Using (20) and (21), we thus get:

$$\Delta J(q,p) = \int_q^p \frac{\lambda_1 \overline{\Gamma}(q) \left[\rho + \delta + (\lambda_1 + \overline{s}) \overline{\Gamma}(x)\right] - (\rho - r)(\lambda_1 + \overline{s}) \overline{\Gamma}(x)}{(\rho + \delta)\left[r + \delta + (\lambda_1 + \overline{s}) \overline{\Gamma}(x)\right]\left[r + \delta + \lambda_1 \overline{\Gamma}(q)\right]} dx. \tag{25}$$

The rest of the paper will thus be devoted to examining the sign of the right hand side of (24).

5 Equilibrium properties

5.1 A first set of equilibrium properties

Segmentation. Equations (24) and (25) have important immediate implications. The first one is that, under the assumption of a continuous density $\gamma(\cdot)$, the function $p \mapsto \Delta\pi(p)$

is continuous over the productivity interval $\left[\underline{p}, \overline{p}\right]$. This implies that in equilibrium, the set of firm productivity levels $\left[\underline{p}, \overline{p}\right]$ can be divided into a sequence of adjacent intervals, say $\{[p_i, p_{i+1}]; i = 0, \ldots, I\}$, where $p_0 = \underline{p}$, $p_I = \overline{p}$, and where if firms with productivity levels within $[p_i, p_{i+1}]$ choose e.g. to match offers, then firms with productivity levels within $[p_{i+1}, p_{i+2}]$ choose not to match offers (and vice-versa). In the sequel, we shall refer to productivity intervals where firms match offers as *matching segments*, and to intervals where firms choose not to match offers as *nonmatching segments*. Note that $\gamma_{\mathrm{m}}(p) = \gamma(p)$, $\gamma_{\mathrm{n}}(p) = 0$ over any matching segment, while $\gamma_{\mathrm{m}}(p) = 0$, $\gamma_{\mathrm{n}}(p) = \gamma(p)$ over any nonmatching segment. This property is helpful for solving the system of differential equations (12) in Proposition 1 (see the Appendix).

Simple sufficient conditions. The second important implication is that a sufficient condition for all firm types to be willing to match offers in equilibrium is that workers be at least as patient as firms are ($\rho \leq r$). In that case, equation (25) shows that $\Delta J(q, p) \geq 0$ for all (q, p), which together with (24) implies that $\Delta\pi(p) \geq 0$ for all p. Relatively patient workers are inclined to trade low wages today for higher wages in the future, and it takes smaller wage increases to retain them, as appears from the definition of the mobility wage (10). This clearly makes offer-matching more attractive an option for firms. Note that the above sufficient condition admits the limiting case where firms and workers have equal discount rate. Standard though it may be, this assumption nonetheless appears to be somewhat unrealistic. Estimations of this model under the assumption that all firms prefer to match offers were conducted by Postel-Vinay and Robin (2002b), who found values of $\rho - r$ ranging from 30 to 55% annual discount (depending on the category of labor considered).[10]

A third, more intuitive implication is that all firm types choose to match offers in equilibrium

[10] Also, as we already noted, another common practice is to follow Burdett and Mortensen (1998) and assume that firms maximize their steady-state profit flows, which amounts to assuming that $r = 0$. Clearly in this case, the sufficient condition is unlikely to be satisfied.

in the limiting case where $\overline{s} \to 0$.[11] It then makes no difference to the firm whether workers search actively or not and the moral hazard problem vanishes. It is then obviously always optimal to adopt the matching behavior, as matching offers becomes costless. Conversely, as $\lambda_1 \to 0$, keeping a positive $\overline{s}$, all firms tend to find the nonmatching option more profitable whenever $\rho \geq r$.[12]

5.2 What low-p firms do in equilibrium.

Under what condition all firms with productivity less than a given value p optimally decide not to match outside offers? Examination of equations (24) and (25) lead to the following statement:

Proposition 2 *Let $p_{\mathrm{n}} \in \left[\underline{p}, \overline{p}\right]$. In equilibrium, all firms with productivity $p \leq p_{\mathrm{n}}$ prefer not to match outside offers if and only if*

$$\int_b^{p_{\mathrm{n}}} \frac{(\rho - r - \lambda_1)(\lambda_1 + \overline{s})\overline{\Gamma}(x) - \lambda_1(\rho + \delta)}{r + \delta + (\lambda_1 + \overline{s})\overline{\Gamma}(x)} dx > 0. \tag{26}$$

$$\Updownarrow$$

$$\Delta J(b, p_{\mathrm{n}}) < 0.$$

The Appendix contains a formal proof of this Proposition. The intuition, however, is clear enough. As we saw from Proposition 1, if all firms with productivity within some interval $\left[\underline{p}, p_{\mathrm{n}}\right]$ choose the nonmatching strategy, then the wage distribution among the employees of those firms degenerates to a mass point at the monopsony wage, b. As a consequence, the only term "that counts" in the comparison of ex-ante job values $\Delta\pi(q)$ (equation (24)), is $\Delta J(b, p)$. Condition (26) ensures that this term be negative for all p in $\left[\underline{p}, p_{\mathrm{n}}\right]$.

[11] **Proof of this claim:** The numerator of the integrand in (25) can be rewritten as
$$\rho\left[\lambda_1\overline{\Gamma}(q) - (\lambda_1 + \overline{s})\overline{\Gamma}(x)\right] + \lambda_1\overline{\Gamma}(q)\left[\delta + (\lambda_1 + \overline{s})\overline{\Gamma}(x)\right] + r(\lambda_1 + \overline{s})\overline{\Gamma}(x),$$
which, as $x \geq q \Rightarrow \overline{\Gamma}(x) \leq \overline{\Gamma}(q)$, is clearly positive when $\overline{s} = 0$. This in turn makes $\Delta J(q, p)$ positive for all (q, p). Hence the positive sign of $\Delta\pi(p)$ for all p.

[12] Remark that in the—admittedly unrealistic—case where firms are less patient than workers ($\rho < r$), then all firms prefer to match offer *even if* $\lambda_1 = 0$, *i.e. even if passive workers never get* any *job offers.* Intuitively, firms can charge very high "entry fees" to very patient workers in the form of low initial wages in exchange for higher wages in the future. This kind of low-intercept, high-slope wage profile is more attractive to relatively impatient entrepreneurs than the flat profiles implied by the nonmatching strategy.

Note that Proposition 2 implies that the least productive firms (those with productivity $\underline{p}$) choose not to match outside offers if and only if

$$(\rho - r - \lambda_1)\,\overline{s} > \lambda_1\,(r + \delta + \lambda_1)\,. \tag{27}$$

This condition is interesting because it becomes necessary and sufficient as $\Gamma(\cdot)$ degenerates to a mass at $\underline{p}$. It thus fully characterizes the equilibrium of a homogeneous firm model. A point worth noting about this result is the following: parameter configurations exist where the unique firm type prefers to match outside offers in the homogeneous model—it is the case when (27) is violated. In this situation, workers are induced to actively search on-the-job. As firms are homogeneous, such employed job search is a pure rent-seeking activity that has no social value. This clearly exhibits the inefficiencies associated with the strategic aspects of employed job search.

Note also that the integrand in (26) is decreasing and negative in the vicinity of $x = \overline{p}$. Proposition 2 therefore states that for a given set of parameters ρ, r, δ, λ_1 and $\overline{s}$ the right tail of the distribution of productivities must not be too long (small enough kurtosis) for all firms to be of the nonmatching type in equilibrium. As typically wage distributions have long tails and estimated distributions of productivities even longer tails, it is thus unlikely that condition (26) be verified over the entire support of the distribution of productivities.

More generally, Proposition 2 suggests that high-productivity firms have an intrinsic advantage in adopting the matching behavior. Intuitively, this is because the cost for high-productivity firms in terms of wage increase of responding to the offers received by their workers from low-p firms is relatively small as a share of total profits per job for two reasons: The average wage increase needed to retain their worker is relatively small as a share of the profit flow for high-p firms, and the high-p firms are also more likely to be successful in countering the outside offers received by their workers.

δ	$\lambda_1 + \overline{s}$	ρ	r
.1	.7	.3	0

Table 2: Baseline parameter values (annual)

5.3 Further equilibrium characterization.

Equations (24) and (25), together with system (12), the solution of which is explicited in the Appendix, allow in principle to derive the exact condition for $\Delta\pi(p) > 0$ for all $p \in \left[\underline{p}, \overline{p}\right]$, just as we did in Proposition 2 for the left end of this interval. Unfortunately, all parameters and the distribution of productivities interact in this condition in a very intricate way which makes it difficult to get any clear intuition about the separate influence of each parameter. This condition is therefore pretty useless, save for simulating the model, which we now do.

6 Computed examples

A calibration. This section contains a series of computed examples of our model labor market, showing that a large variety of patterns can be observed in equilibrium. A similar model was estimated on French data by Postel-Vinay and Robin (2002b) under the assumption that all firms match offers in equilibrium. The estimations were conducted separately for seven categories of workers. Even though the estimates vary a bit across worker categories, the set of values gathered in Table 2 can be considered reasonable as a rough baseline calibration of the model.[13] Because of the assumption that all firms match offers, the arrival rate of offers actually estimated in Postel-Vinay and Robin (2002b) is $\lambda_1 + \overline{s}$, not λ_1. What remains undetermined here is the relative value of λ_1 and $\overline{s}$. We shall therefore examine a series of different values. Finally, we simulate the model under the limiting assumption that $\underline{p} = b$.[14]

[13] The value of the discount rate ρ reported in Table 2 may seem quite high. In fact, Postel-Vinay and Robin (2002b) found values between .35 (executives and managers) and .80 (unskilled manual workers), under the assumption of a linear instantaneous utility function with $r = 0$. The latter assumption is relatively innocuous, as what really counts in the profit differential $\Delta\pi(p)$ is the discount rate differential, $r - \rho$.

[14] The estimates in Postel-Vinay and Robin (2002b) indicate that $\underline{p} > b$. However, the estimated gap between $\underline{p}$ and b is relatively small compared to the range of firm productivities, $\overline{p} - \underline{p}$.

Baseline cases. We start with an "agnostic" assumption about productivity dispersion in this first series of simulations, i.e. we assume that offers are uniformly sampled from $\left[\underline{p}, \overline{p}\right] = [1, 2]$.[15] Figures 1 to 3 plot the relative gain of matching offers $\Delta\pi(p)/\pi_{\text{n}}(p)$ against p for values of $\overline{s}$ corresponding to "active" searchers receiving offers 3, 4, and 7 times more frequently than "passive" workers, respectively. The necessary condition (27) is met in the last two cases only.

<**Figures 1, 2 and 3 about here**>

We see from Figure 1 that the first case is an "all firms match" equilibrium: $\Delta\pi(p)$ is positive at all productivity levels. As the effectiveness of "active search" ($\overline{s}$) increase (Figs. 2 and 3), the equilibrium moves to a situation where low-productivity firms choose not to match offers, while high-productivity firms match offers (the percentage of nonmatching firms in the case of Figure 2 equals 37%). We thus observe a "dual" labor market in this case, with "bad jobs" at low-productivity firms that only offer b as a wage with no within-firm career prospects, and "good jobs" at more productive firms that try to retain their workers from being poached by their competitors. Finally, as $\overline{s}$ increases, it becomes too costly for any firm type to match and we end up in a "no firm matches" equilibrium with an earnings distribution that collapses to an atom at b and no job-to-job turnover.

It is obvious from all three Figures that it is relatively more beneficial for high-productivity firms than for low-productivity firms to match outside offers, which confirms the intuition that we had from the previous section. This advantage of high-p firms in matching, however, also depends on the assumed shape of the sampling distribution of productivities, as we now show.

Firm concentration and wage policy. We now take a brief look at how the shape of the productivity distribution affects the equilibrium. We start from the situation of Figure 2 above, where "good" (matching) and "bad" (nonmatching) firms coexist on the market. The cutoff

[15] A uniform γ over $[1, 2]$, together with the adopted parameter values, is compatible with the necessary condition that appears in footnote 9, which ensures that nonmatching firms offer their reservation wages to every worker they meet.

productivity between good and bad firms turns out to be $p_c = 1.37$ with the chosen calibration (firms less productive than $p_c = 1.37$ choosing the nonmatching behavior). We then look at changes in the "concentration" of good and bad firms, and see how it changes the equilibrium strategies.

<Figures 4 to 7 about here>

Figure 5 exemplifies a situation where there is a relatively high concentration of "bad" firms, the initially uniform sampling density $\gamma(\cdot)$ being changed to the mixture of uniforms depicted on Figure 4. (The new $\gamma(p)$ is equal to $\frac{.55\times \mathrm{I}\{\underline{p}\leq p\leq p_c\}}{p_c-\underline{p}} + \frac{.45\times \mathrm{I}\{p_c\leq p\leq \overline{p}\}}{\overline{p}-p_c}$. All other parameters keep the same value as on Figure 2.) What we see from Figure 5 is that some firms with productivities below p_c, which used not to match offers in the uniform case, now switch to the matching strategy (the new cutoff productivity being $p_c' \simeq 1.25$ in this new example). This is obviously driven by new the shape of the productivity sampling density $\gamma(\cdot)$, which is now more concentrated at low values of p. Firms with high p's are consequently more "isolated", in the sense that the chances for one of their workers to contact a firm with a comparable or higher productivity are more limited. The cost to those firms of high offer arrival rates is therefore limited as well, since their workers are relatively more likely to contact unattractive poachers, thus only costing a modest or even no wage increase.[16]

This feature can obviously change somewhat as the shape of $\gamma(\cdot)$ is altered. Figure 7 depicts the converse case, where high-p offers are relatively more frequent (the corresponding sampling density being plotted on Figure 6). The result of this shift in the sampling density exactly mirrors that obtained on Figure 5: some jobs more productive than p_c now find it optimal not to match offers, whereas they chose the matching behavior in the benchmark case of a uniform sampling density.

[16]Note that, although the *productivity range* over which firms prefer not to match offers is clearly narrowed toward $\underline{p}$, nothing general can be said about the *total equilibrium share* of nonmatching firms on the market, as the density of firms with low p's is also increased. It turns out in the example that this share is slightly higher in the case of Figure 5 (37.42%) than in that of Figure 2 (37%).

As we already argued, a typical estimation of the productivity density in a job search model like the one presented in this paper would be steeply decreasing at low productivities, with a relatively long tail (see e.g. Postel-Vinay and Robin, 2002b). This is best proxied by the case depicted on Figure 5, where $\gamma(\cdot)$ is concentrated at low productivities.[17] As a conclusion, even though the model leaves room for virtually any equilibrium pattern of matching commitments across firm types, it nonetheless strongly suggests that a plausible situation would be one where the labor market is dual, with "bad" jobs at low-p, nonmatching firms, and "good" jobs at high-p, matching firms that offer upward sloping career paths.

7 Conclusion and discussion

In this paper we have addressed the issue of a firm's optimal wage policy when employed workers can strategically use the search for outside job offers to put employers into competition, thus forcing the latter to raise their wages. Specifically, we show that it may be profitable for some firms to "refuse competition", i.e. to ex ante commit not to match the outside job offers received by their employees, even when it is ex post optimal to match those offers.

Even though general equilibrium characterization is rather complicated in this context, we are able to derive sufficient conditions for the equilibrium to be of the sort "all firms match" or "no firm matches". More importantly, we find that matching and nonmatching firms generically coexist in equilibrium. We argue in fact from calibrated examples that a plausible situation is one where the labor market is "dual", with "bad" jobs at low-productivity, nonmatching firms offering stagnant within-firm career profiles and "good" jobs at high-productivity, matching firms in which wage-tenure profiles are (on average) upward sloping.

Our results should be qualified by the following series of remarks, which we also view as ideas to be pursued in future research. Firstly, an issue that we do not address in the paper is the nature of the commitment device that could be available for nonmatching firms to use.

[17] A simulation with a more "realistic" density $\gamma(\cdot)$—e.g. a Pareto—is naturally possible (available upon request), and delivers a picture which is qualitatively similar to Figure 5.

Clearly, reputation effects have to be the answer. If an employer claiming to be nonmatching were to renege on its initial commitment as one of its employees receives an outside job offer, then if the rest of its employees can observe the deviation, one would expect them to start searching actively, which would negatively affect the employer's profit.

Secondly, some of our results depend on the restrictions that we impose on equilibrium. Some of those restrictions are merely conditions on the parameters (see footnote 9) ensuring that the situation we look at is an equilibrium. Others are more fundamental restrictions on the class of equilibria considered: in particular, we do not allow firms to pursue more sophisticated wages policies, such as pre-commiting to unconditional tenure-dependent wage profiles (as in Stevens, 2000, or Burdett and Coles, 2001) or simply to the same flat, unconditional wage profile for every worker (as in the original Burdett and Mortensen model). In the particular context of our model we can always argue that, from the firms' point of view, such "wage-posting" behavior never beats the "offer-matching" strategy, since workers hired at nonmatching, "wage-posting" firms would still have an incentive to search actively in the hope to find a better posted offer which would strictly increase their value function. However, this argument is only valid under the restriction that the workers' choice of a search effort is essentially 0/1, i.e. to either search actively or not at all. In the more general case where effort is a continuous variable, then the firms' arbitrage would be less simple, and the result less clear-cut.[18] Unfortunately, mathematical tractability limits the possibility that we have to address those questions more formally in the context of this particular model. Hopefully, more satisfactory answers will come in the future as the output of further research.

A final issue that was only quickly mentioned in the paper[19] is that of social efficiency. Even though it may be profit-increasing for some private firms, the strategy of matching offers encourages employed job search as a rent-seeking activity. Since not all employer-worker contact

[18] Other strong assumptions could be changed in a way that would alter the results, e.g. that of complete information and perfect verifiablility of outside offers.

[19] See the discussion of equation (27) in Section 5.2.

have positive social value, the decentralized equilibrium of our model economy probably features excessive employed job search and excessive job-to-job worker reallocation. However, a proper assessment of efficiency would require a careful modelling of labor demand, again something that we leave to future research.

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Appendix

A Proof of Proposition 1

In a steady-state, outflows of workers leaving an employment stock because of lay-off or poaching are exactly compensated by corresponding inflows of workers coming either from unemployment or poached from other firms.

Nonmatching firms. Let us first consider the distribution of wages within nonmatching firms. Unemployed workers are offered a wage equal to b by those firms. If all firms were nonmatching there would be no reason why wages would ever depart from b. A nonmatching firm poaching a worker paid b in another nonmatching firm would attract the worker for b plus an infinitesimal amount (say b^+). Conversely, a nonmatching firm poaching the employee of a matching firm would attract the worker by offering their marginal productivity in that matching firm. It follows that in general the support of wages in a nonmatching firm of productivity p should be a subset of $\{b\} \cup (\underline{p}, p]$.

In any such firm there is a mass $G_{\mathrm{n}}(b|p)\,\ell_{\mathrm{n}}(p)$ of employees paid b. In order to maintain that stock constant, it must be the case that

$$(\delta + \lambda_1)\, G_{\mathrm{n}}(b|p)\,\ell_{\mathrm{n}}(p) = \lambda_0 u \gamma_{\mathrm{n}}(p)\,.$$

The fraction $(\delta + \lambda_1)\, G_{\mathrm{n}}(b|p)\,\ell_{\mathrm{n}}(p)$ of employees of firms with productivity p paid b leaving the stock is equal to the fraction of the unemployed population u who are contacted by a firm of type p, which occurs with probability $\lambda_0 \gamma_{\mathrm{n}}(p)$.

Next, consider the stock of employees paid a wage w in $(\underline{p}, p]$. The steady-state outflow is the fraction δ who is laid off plus the fraction $\lambda_1 \overline{\Gamma}(w)$ who gets a better offer from a competing firm. The inflow is entirely made of workers who have been poached out of other firms. Equating inflows and outflows yields

$$\left[\delta + \lambda_1 \overline{\Gamma}(w)\right] g_{\mathrm{n}}(w|p)\,\ell_{\mathrm{n}}(p) = \left[(\lambda_1 + \overline{s}) \cdot \ell_{\mathrm{m}}(w) + \lambda_1 \cdot \int_w^{\overline{p}} g_{\mathrm{n}}(w|x)\,\ell_{\mathrm{n}}(x)\,dx\right] \gamma_{\mathrm{n}}(p) \tag{28}$$

since only the workers of a matching firm of productivity w are poached by a nonmatching firm of productivity p at wage w^{+}, and since all workers paid w in a non matching firm of any productivity $x \geq w$ are poached by a nonmatching firm of productivity p at wage w^{+}.

Note that equation (28) implies that $g_{\mathrm{n}}(w|p)\,\ell_{\mathrm{n}}(p)$ has the form $\varphi_{\mathrm{n}}'(w) \cdot \gamma_{\mathrm{n}}(p)$, where $\varphi_{\mathrm{n}}'(w)$ is some continuous function of w, and is independent of p. As $G_{\mathrm{n}}(b|p)\,\ell_{\mathrm{n}}(p) = \lambda_0 u \gamma_{\mathrm{n}}(p) / (\delta + \lambda_1)$ also has that same form, it follows that

$$G_{\mathrm{n}}(w|p)\,\ell_{n}(p) = \int_b^w g_{\mathrm{n}}(x|p)\,\ell_{\mathrm{n}}(p)\,dx + G_{\mathrm{n}}(b|p)\,\ell_{\mathrm{n}}(p) = \varphi_{\mathrm{n}}(w) \cdot \gamma_{\mathrm{n}}(p).$$

The function $\varphi_{\mathrm{n}}(\cdot)$ is thus an increasing function, the same for all p. Since we also know that $G_{\mathrm{n}}(p|p) = 1$, it must be the case that $\varphi_{\mathrm{n}}(p) \cdot \gamma_{\mathrm{n}}(p) = \ell_{\mathrm{n}}(p)$ for all p such that $\gamma_{\mathrm{n}}(p) \neq 0$.

We finally turn to the initial value of φ_{n}, i.e. $\varphi_{\mathrm{n}}\left(\underline{p}\right)$. There is a measure $G_{\mathrm{n}}\left(\underline{p}|p\right)\ell_{\mathrm{n}}(p)$ of employees paid less than $\underline{p}$ by nonmatching firms of type p. Equating inflows and outflows for this stock yields:

$$(\delta + \lambda_1)\, G_{\mathrm{n}}\left(\underline{p}|p\right)\ell_{\mathrm{n}}(p) = \left[\lambda_0 u + \lambda_1 \int_{\underline{p}}^{\overline{p}} G_{\mathrm{n}}\left(\underline{p}|x\right)\ell_{\mathrm{n}}(x)\,dx\right]\gamma_{\mathrm{n}}(p).$$

The reasoning here is exactly similar to the one that yielded $G_{\mathrm{n}}(b|p)\,\ell_{\mathrm{n}}(p)$. Only here, a fraction $\lambda_1 \gamma_{\mathrm{n}}(p)$ of all workers employed at *nonmatching* firms at a wage less than $\underline{p}$ has to be added to the inflow. The latter are indeed willing to move to a different nonmatching firm for an offer only infinitesimally more generous than their current wage, which is still less than $\underline{p}$. Solving for $\varphi_{\mathrm{n}}\left(\underline{p}\right)$ in the last equation gives the second equation in (13).

Matching firms. Matching firms with productivity p attract unemployed workers by offering their reservation wage $\phi_0(p) = \phi(b,p)$. They poach employees paid w at nonmatching firms by offering them $\phi(w,p)$ and employees of matching firms with productivity $q < p$ by offering them $\phi(q,p)$. Since wages in nonmatching firms are in the set $\{b\} \cup (\underline{p}, \overline{p}]$ it follows that wages in matching firms with productivity p belong to the set

$$\{\phi(b,p)\} \cup (\phi(\underline{p},p), \phi(p,p)] = \{\phi_0(p)\} \cup (\phi(\underline{p},p), p].$$

Equating inflows and outflows for the stocks $G_{\mathrm{m}}(w|p)\,\ell_{\mathrm{m}}(p)$ in matching firms of productivity p yields:

$$\begin{aligned}&\left\{\delta+(\lambda_1+\overline{s})\cdot\overline{\Gamma}\left[q(w,p)\right]\right\}\cdot G_{\mathrm{m}}(w|p)\,\ell_{\mathrm{m}}(p)\\ &\quad=\left[\lambda_0 u+(\lambda_1+\overline{s})\cdot\int_{\underline{p}}^{q(w,p)^-}\ell_{\mathrm{m}}(x)\,dx+\lambda_1\cdot\int_{\underline{p}}^{\overline{p}}G_{\mathrm{n}}\left[q(w,p)^-|x\right]\ell_{\mathrm{n}}(x)\,dx\right]\cdot\gamma_{\mathrm{m}}(p).\end{aligned}\qquad(29)$$

The LHS is the outflow: a fraction is laid off and all workers paid less than w in a matching firm of productivity p accept a job in another firm (matching or nonmatching) if its productivity exceeds $q(w,p)$. The RHS is the inflow: a fraction comes from the unemployment stock; another fraction comprises former employees of matching firms with productivity less than $q(w,p)$;[20] the last share is made of former employees of all nonmatching firms who were paid less than $q(w,p)$.

This equation implies that $G_{\mathrm{m}}(w|p)\,\ell_{\mathrm{m}}(p)$ has the form $\varphi_{\mathrm{m}}[q(w,p)]\cdot\gamma_{\mathrm{m}}(p)$. Setting $w=p$, which implies $q(w,p)=q(p,p)=p$ and $G_{\mathrm{m}}(w|p)=G_{\mathrm{m}}(p|p)=1$, we finally get that $\ell_{\mathrm{m}}(p)=\varphi_{\mathrm{m}}(p)\cdot\gamma_{\mathrm{m}}(p)$ for all p.

Finally turning to the initial conditions, it is easy to let $w=\phi_0(p)$ or $w=\phi\left(\underline{p},p\right)$ in (29) to get

$$G_{\mathrm{m}}[\phi_0(p)|p]\,\ell_{\mathrm{m}}(p)=\frac{\lambda_0 u}{\delta+\lambda_1+\overline{s}}\cdot\gamma_{\mathrm{m}}(p),$$

and the first equation in (13) for $\varphi_{\mathrm{m}}(\underline{p})$.

Proof of the Proposition. Substitution of $\varphi_{\mathrm{n}}'(w)\cdot\gamma_{\mathrm{n}}(p)$ for $g_{\mathrm{n}}(w|p)\,\ell_{\mathrm{n}}(p)$, $\varphi_{\mathrm{n}}(w)\cdot\gamma_{\mathrm{n}}(p)$ for $G_{\mathrm{n}}(w|p)\,\ell_{\mathrm{n}}(p)$, $\varphi_{\mathrm{m}}[q(w,p)]\cdot\gamma_{\mathrm{m}}(p)$ for $G_{\mathrm{m}}(w|p)\,\ell_{\mathrm{m}}(p)$, and $\varphi_{\mathrm{m}}(p)\cdot\gamma_{\mathrm{m}}(p)$ for $\ell_{\mathrm{m}}(p)$ in equations (28) and (29) leads to the system of differential equations (12). The results in the above two paragraphs then provide the associated initial conditions and prove points 1, 2 and 3 of the Proposition altogether. ∎

B Solution of system (12)

System (12) of Proposition 1 shows that $\varphi_{\mathrm{n}}(p)$ is constant over non matching segments, and strictly increasing over matching segments. Recalling that $\overline{\Gamma}_{\mathrm{m}}(p)=\overline{\Gamma}(p)-\overline{\Gamma}_{\mathrm{n}}(p_{\mathrm{i}})$ over any matching segment starting at p_{i}, (12) solves as:

$$\varphi_{\mathrm{m}}(p)=\varphi_{\mathrm{m}}(p_{\mathrm{i}})\cdot\left[\frac{\delta+\lambda_1\overline{\Gamma}_{\mathrm{m}}(p_{\mathrm{i}})}{\delta+\lambda_1\overline{\Gamma}_{\mathrm{m}}(p)}\right]^{\frac{\lambda_1\overline{\Gamma}_{\mathrm{n}}(p_i)(\lambda_1+\overline{s})}{\lambda_1\overline{\Gamma}_{\mathrm{n}}(p_i)(\lambda_1+\overline{s})-\delta\overline{s}}}\cdot\left[\frac{\delta+(\lambda_1+\overline{s})\,\overline{\Gamma}(p_{\mathrm{i}})}{\delta+(\lambda_1+\overline{s})\,\overline{\Gamma}(p)}\right]^{\frac{\lambda_1\overline{\Gamma}_{\mathrm{n}}(p_i)(\lambda_1+\overline{s})-2\delta\overline{s}}{\lambda_1\overline{\Gamma}_{\mathrm{n}}(p_i)(\lambda_1+\overline{s})-\delta\overline{s}}}\qquad(30)$$

[20] Since $\phi(p',p)\le w$ if and only if $p'\le q(w,p)$.

over any such segment, and as:

$$\varphi_{\mathrm{m}}(p) = \varphi_{\mathrm{m}}(p_{\mathsf{i}}) \cdot \frac{\delta + (\lambda_1 + \bar{s})\,\overline{\Gamma}(p_{\mathsf{i}})}{\delta + (\lambda_1 + \bar{s})\,\overline{\Gamma}(p)} \tag{31}$$

over a non matching segment starting at p_{i}.

C Proof of Proposition 2

First implication. The assertion that condition (26) is implied by the fact that all firms with $p \in \left[\underline{p}, p_{\mathrm{n}}\right)$ choose not to match outside offers is quite obvious. Indeed, this fact has the following two immediate implications:

1. $\gamma_{\mathrm{m}}(p) \equiv 0$ over $\left[\underline{p}, p_{\mathrm{n}}\right)$,
2. $\Delta\pi(p) < 0$ over $\left[\underline{p}, p_{\mathrm{n}}\right)$.

Now the definition (24) of $\Delta\pi(p)$ and, together with point 1 above, in turn imply that

$$\Delta\pi(p) = \frac{\lambda_0 u}{n_{\mathsf{c}}} \frac{(\delta + \lambda_1)}{\delta + \lambda_1 (1 - n)} \Delta J(b, p)$$

for all $p \in \left[\underline{p}, p_{\mathrm{n}}\right)$. Point 2 is then implies condition (26).

Second implication. The reverse implication is less straightforward. We first need to show that condition (26) implies that the same inequality holds for all $p \leq p_{\mathrm{n}}$, that is, for all $q \leq p_{\mathrm{n}}$:

$$\int_{\underline{p}}^{q} \frac{(\rho - r - \lambda_1)(\lambda_1 + \bar{s})\,\overline{\Gamma}(x) - \lambda_1(\rho + \delta)}{r + \delta + (\lambda_1 + \bar{s})\,\overline{\Gamma}(x)} dx > 0.$$

This is easily proved by contradiction. Suppose there exists a $q \in \left(\underline{p}, p_{\mathrm{n}}\right)$ such that the above integral be strictly negative. Then, since the numerator of the integrand is decreasing in x,[21] it has to be the case that

$$(\rho - r - \lambda_1)(\lambda_1 + \bar{s})\,\overline{\Gamma}(x) - \lambda_1(\rho + \delta) < 0$$

for all $x \geq q$. As a consequence, the function

$$p \mapsto \int_{\underline{p}}^{p} \frac{(\rho - r - \lambda_1)(\lambda_1 + \bar{s})\,\overline{\Gamma}(x) - \lambda_1(\rho + \delta)}{r + \delta + (\lambda_1 + \bar{s})\,\overline{\Gamma}(x)} dx$$

[21] This uses the fact that $\rho - r - \lambda_1$ has to be positive, otherwise condition (26) could not hold.

is decreasing for all $p \geq q$, which implies in particular that it takes a negative value at p_{n}, thus contradicting condition (26).

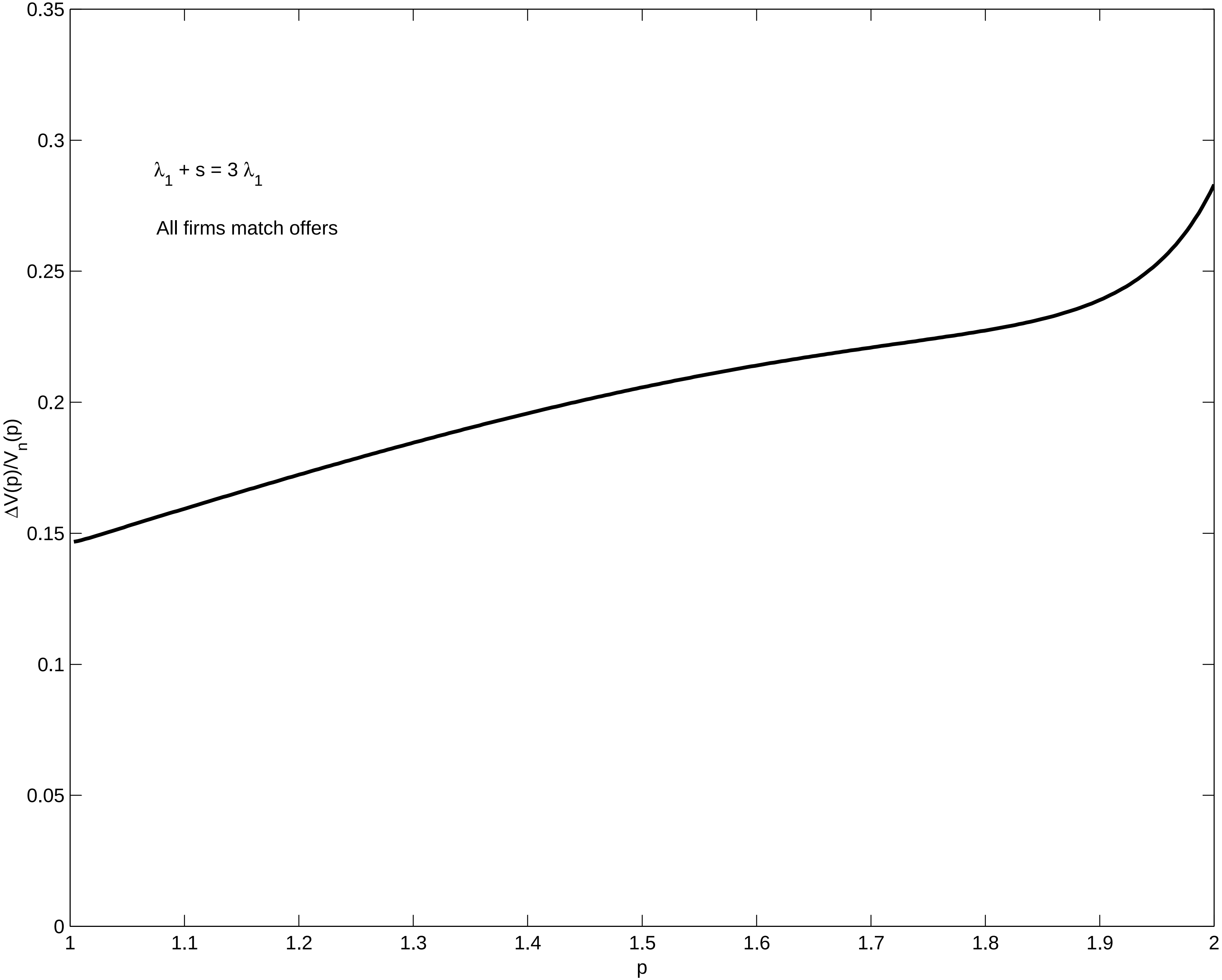
A particular implication of condition (26) is therefore that $\Delta J\left(b, \underline{p}\right) < 0$, which clearly implies that $\Delta\pi\left(\underline{p}\right) < 0$. By the continuity of $p \mapsto \Delta\pi(p)$, this shows that there exists a $q > \underline{p}$ such that $\Delta\pi(p) < 0$ over $\left[\underline{p}, q\right]$. Firms with productivities within this last interval thus choose not to match offers.

Now let $\overline{q}$ be the supremum of all such q's. If $q \geq p_{\mathrm{n}}$, then the Proposition is proved. In the reverse case, the continuity of $\Delta\pi(p)$ over $\left[\underline{p}, \overline{q}\right)$ implies that $\lim_{q \nearrow \overline{q}} \Delta\pi(q) = 0$. But, by the definition of q, this is equivalent to saying that $\lim_{q \nearrow \overline{q}} \Delta J(b, q) = 0$. We thus have found a $\overline{q} < p_{\mathrm{n}}$ such that

$$\int_b^{\overline{q}} \frac{(\rho - r - \lambda_1)(\lambda_1 + \overline{s})\,\overline{\Gamma}(x) - \lambda_1(\rho + \delta)}{r + \delta + (\lambda_1 + \overline{s})\,\overline{\Gamma}(x)} dx = 0,$$

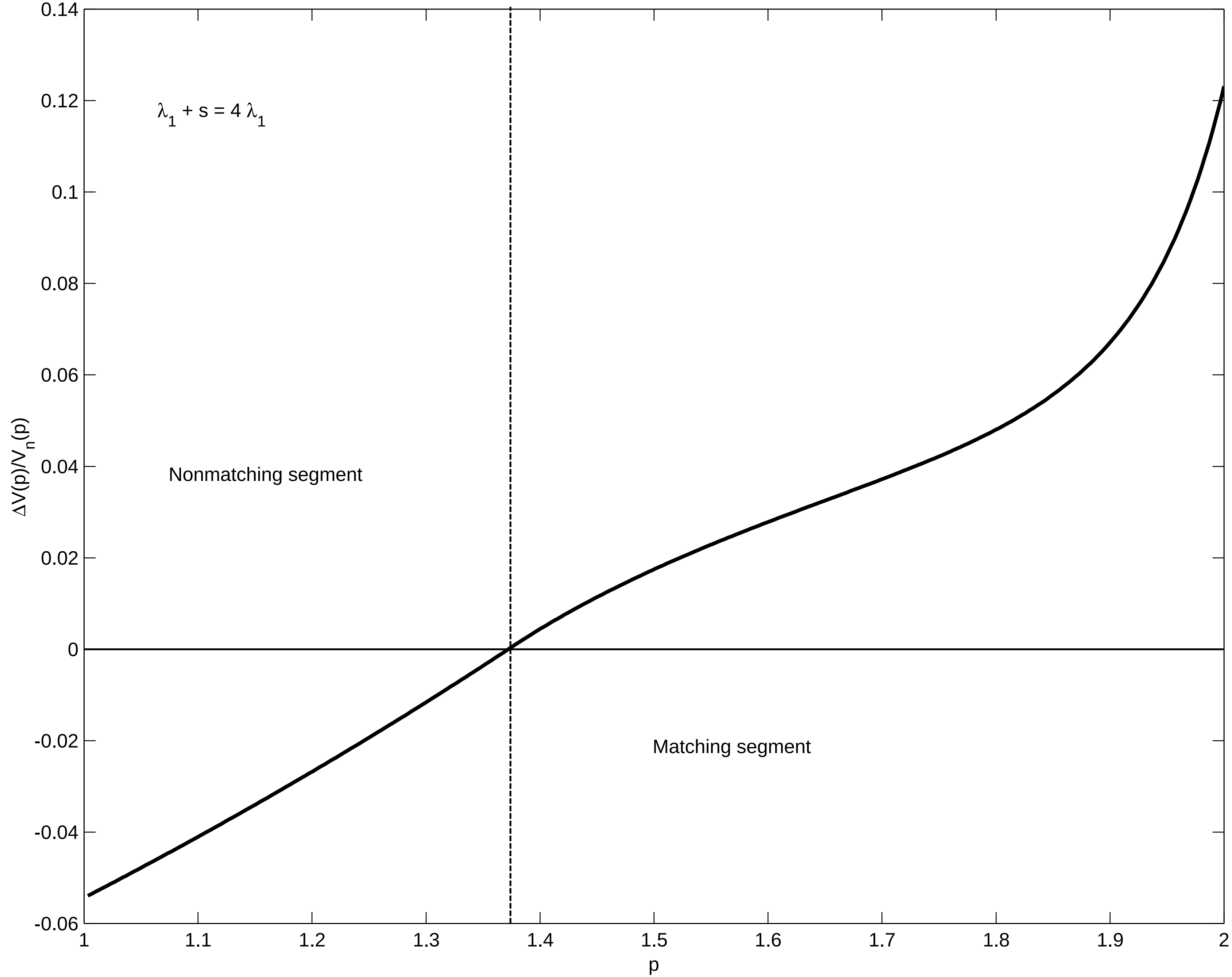
which, as we saw, violates condition (26). $\square$

Figure 1.



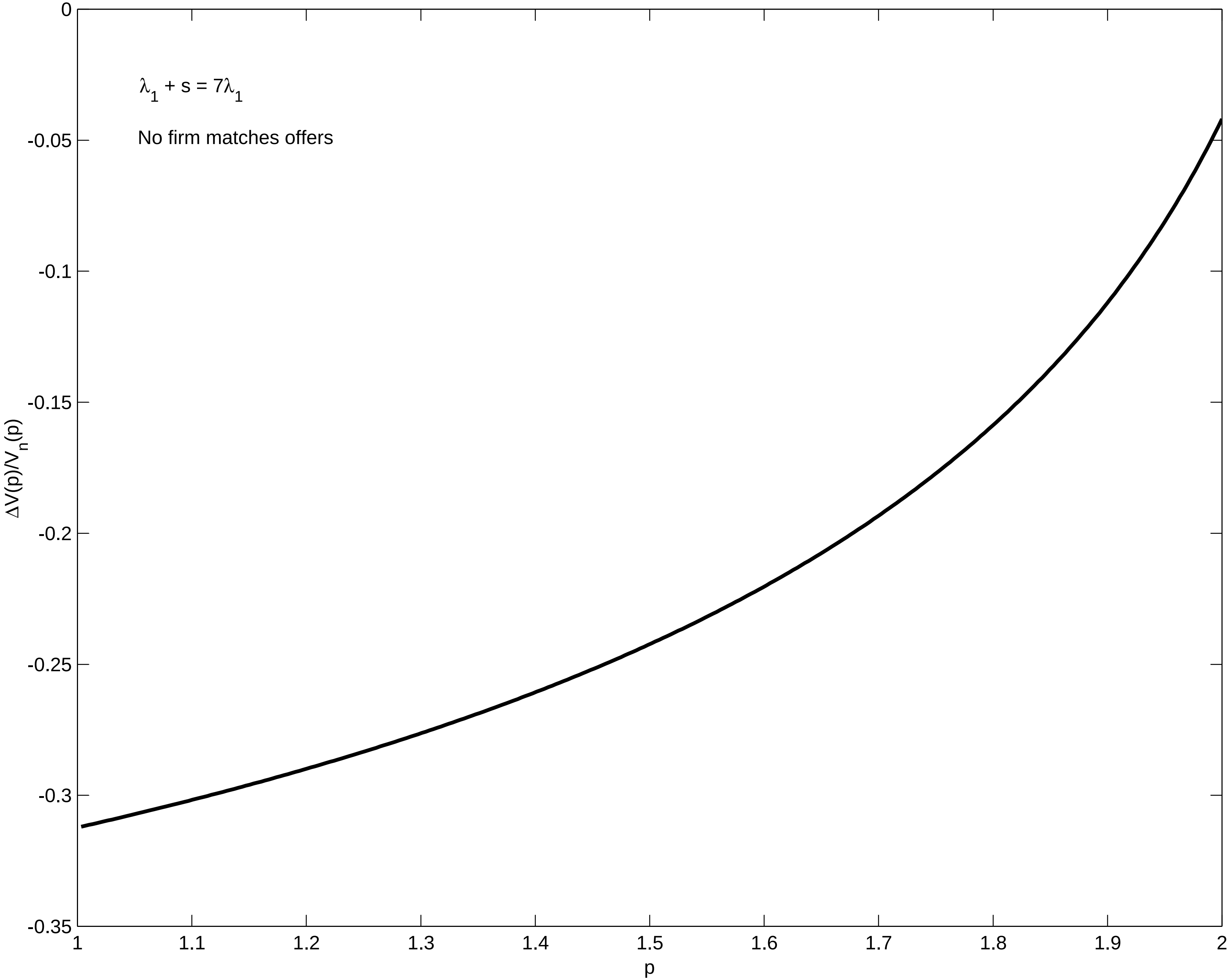
$\lambda_1 + s = 3\,\lambda_1$
All firms match offers
$\Delta V(p)/V_n(p)$
p
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

Figure 2.



λ1 + s = 4 λ1
Nonmatching segment
Matching segment
ΔV(p)/Vn(p)
p
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

Figure 3.



$\lambda_1 + s = 7\lambda_1$
No firm matches offers
$\Delta V(p)/V_n(p)$
p
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

Figure 4.

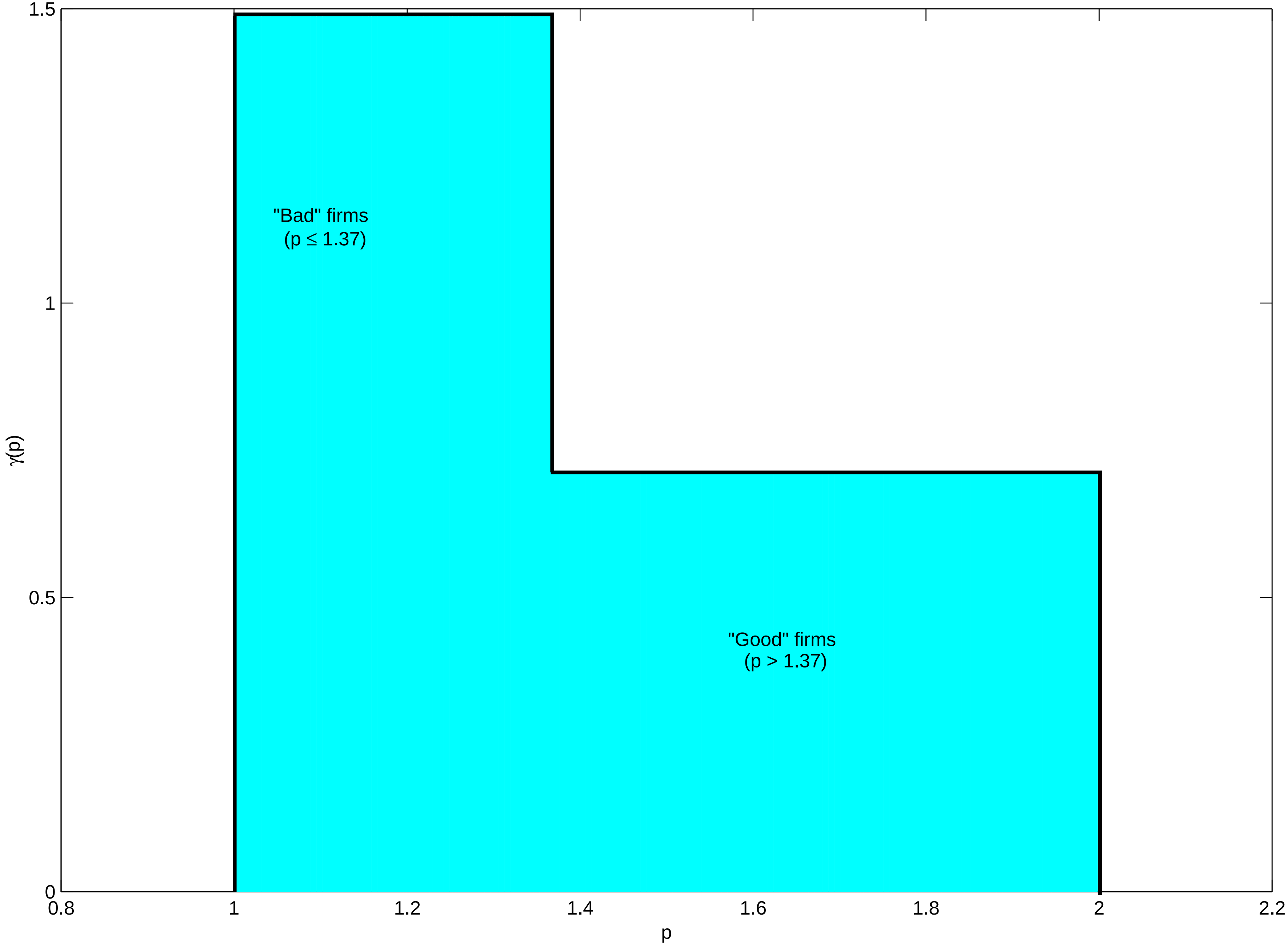
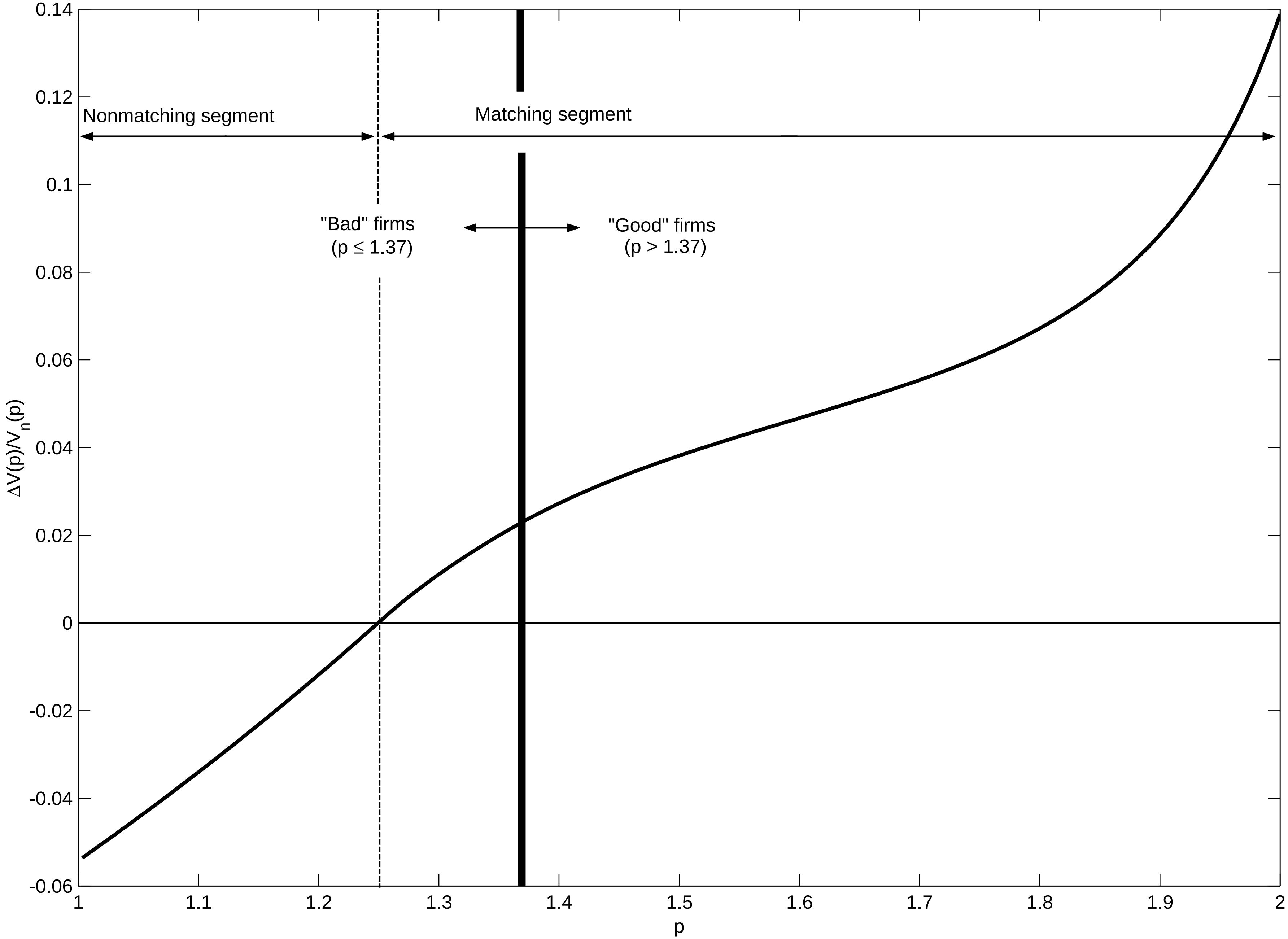


Figure 5.



Nonmatching segment
Matching segment
"Bad" firms
(p ≤ 1.37)
"Good" firms
(p > 1.37)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
ΔV(p)/V_n(p)
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
p

Figure 6.

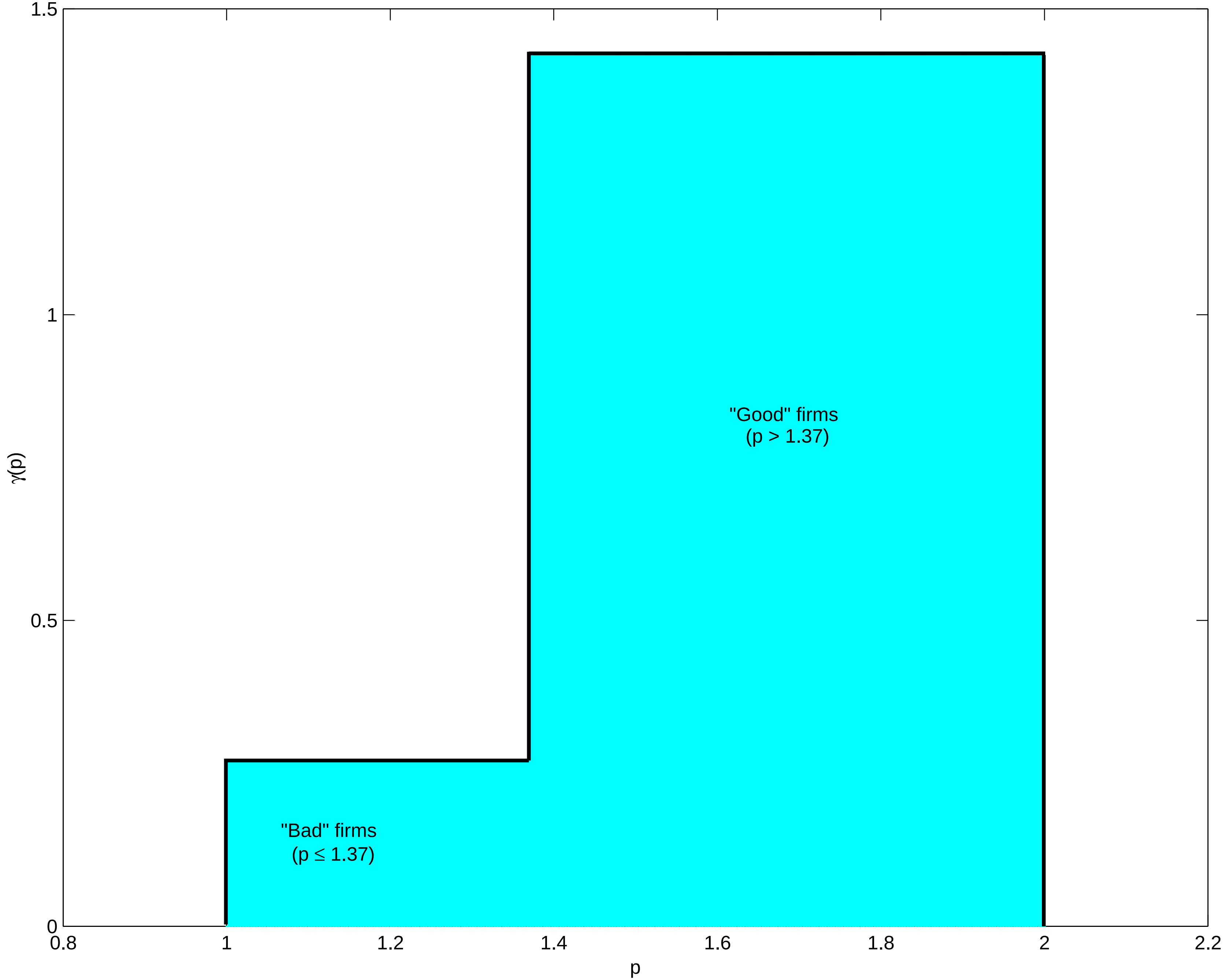
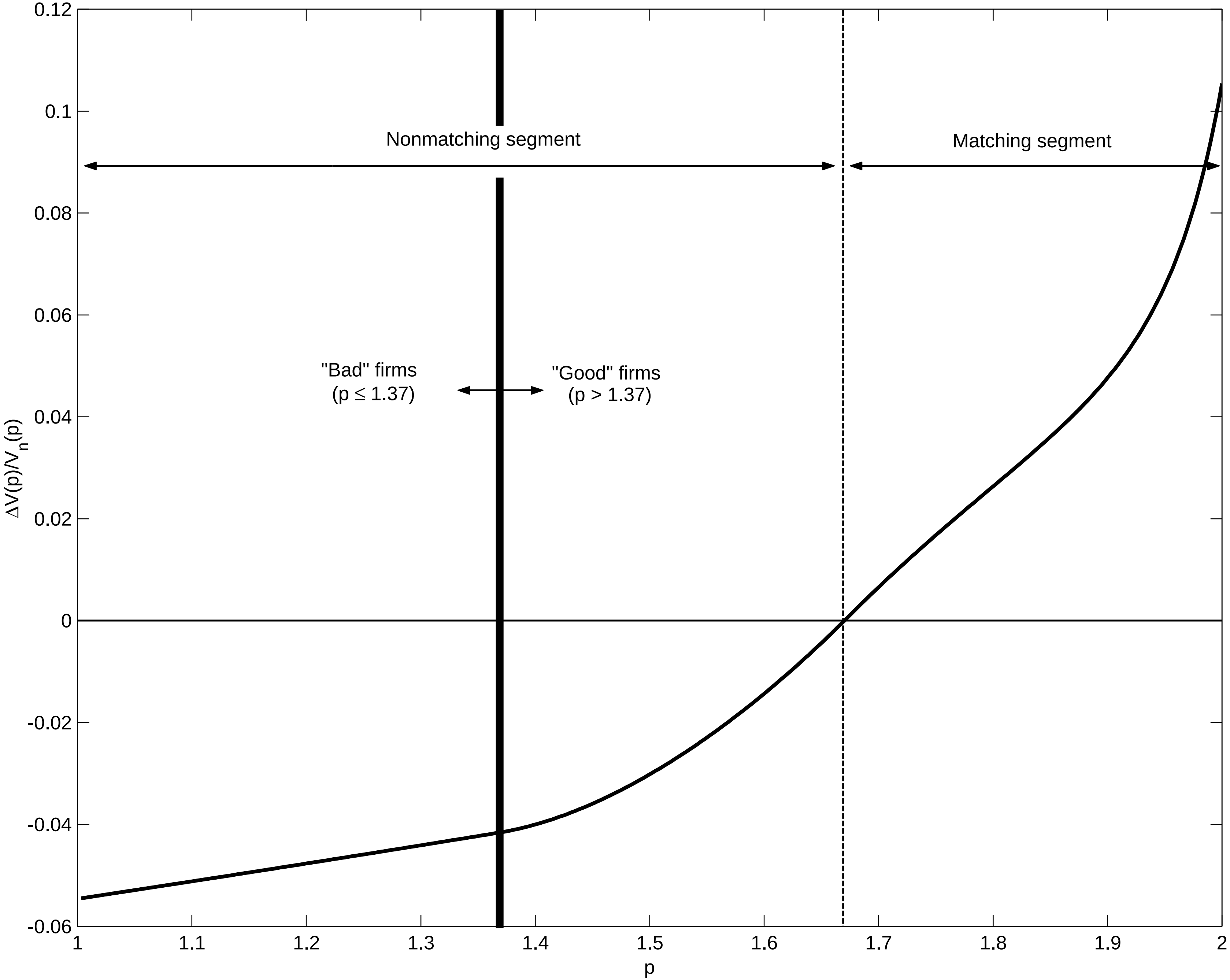


Figure 7.



Nonmatching segment
Matching segment
"Bad" firms
(p ≤ 1.37)
"Good" firms
(p > 1.37)
ΔV(p)/V_n(p)
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
p