ADAPTIVE TRAFFIC SIGNAL CONTROL USING APPROXIMATE DYNAMIC PROGRAMMING

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ABSTRACT

This paper develops an adaptive traffic signal control method that optimizes performance over time. The adaptive controller, instead of retrieving control parameters from a library according to changes in traffic, allows the value of control parameters to be slowly time-varying so that controller's behavior conforms to new circumstance. The adaptive controller is developed on Approximate Dynamic Programming (ADP). By replacing the look-up table containing real value of each state combination with an approximate function, ADP obviates the limit of Dynamic programming in cases of high dimensionality and of incomplete information. Approximation is progressively improved by employing online learning techniques. In this paper, we investigate ADP approach with reinforcement learning and perturbation learning. Through numerical examples, this paper shows that an adaptive controller based on ADP method is operational at isolated intersections while offering potentials for network control. It also shows that a simple linear approximation is sufficient to provide practical significance and to improve performance substantially towards theoretical limit.

1. INTRODUCTION

Operating traffic signal in urban area requires proper timings for signal indications so that varying demand could be effectively managed and control objectives being met. Conventional algorithms are often so designed that, with given input, they generate timings with fixed stage duration and sequence. Later development in online control systems allows timings to be responsive (or vehicle actuate) by adjusting green extension or early cut-off on the edge of stage change. Control parameters for responsive systems as such are usually preset and retrieved in a library that requires manual updates periodically. To optimize performance over time, however, is to progressively adapt control policy over time. In this regard, this paper we introduce an adaptive traffic signal control method. The adaptive controller, instead of retrieving control parameters from a preset library according to changes in traffic, allows the value of control parameters to be slowly time-varying so that controller’s behavior conforms to new circumstance. Signal timings produced by the adaptive controller is free of constraints on stage duration and stage orders, unless otherwise specified. The underlying theory for such an adaptive controller is Approximate Dynamic Programming (ADP). The technique supervising time-varying estimation of control parameters is reinforcement learning.

ADP is a theoretical framework that allows various approximation and learning techniques to be employed to overcome the restrictions of Dynamic Programming (DP) in real-time adaptive control. Dynamic programming, developed by Richard Bellman, traditionally, is the only exact and efficient way to solve problems in optimization over time, in the general case where noise and nonlinearity are present (Werbos, 2004). Key to the problem solving process in DP is the backward recursive estimation of the value of being in a state of the current stage. Subject to the probability distribution of exogenous information, an optimal decision is to optimize the overall utility in the process of transferring the state of current stage to the next state of the following stage. In doing so, DP guarantees global optimum in problem solving. Powerful as it is, DP’s usefulness in real-time control is severely limited. This is because of the exponential relationship between computational burden and the size of state space, the complexity of probability distribution of information, and the number of optional decisions. This relationship is often referred as ‘Three Curses of Dimensionality’ (Powell, 2007). Yet, this is not the end. Supplying complete information for evaluating policies, as required by DP, is unrealistic in real-time control. In the introduction to fundamentals of ADP, we will show that, using an approximate function to replace the exact values in DP, ADP drastically reduces computational burden, and by adopting a forward algorithmic strategy, ADP utilizes limited real-time information to evaluate control policy and update approximation.

The reason of introducing reinforcement learning is that parameters of the approximate function are not known a priori, and we need a technique to learn the proper values in a changing environment where a ‘teacher’ is not available. Such a learning process is therefore unsupervised. Among a number of unsupervised learning techniques, we select perturbation analysis and back-propagation neural network to provide the learning capability.

The rest of the paper is organized as follows: Section 2 provides a review of existing traffic signal control systems from which we identify the scope for development and the objective of this ADP approach. Section 3 shows the fundamentals of ADP technique, and how approximation can be progressively improved through learning techniques. Section 4 introduces the numerical experiments and analyzes ADP results in comparisons. Section 5 concludes this study and identifies the scope for future research in applying ADP to traffic control.

2. TRAFFIC SIGNAL CONTROL

The operation of traffic signal settings can be broadly classified into off-line and on-line approaches. Off-line methods usually take on historical traffic data as inputs for the design of signal plans. Whereas on-line approaches utilize real time traffic information collected from loop detectors to develop the responsive signal settings. Here we focus the review only on online control systems. A terminology frequently used in both signal control field and dynamic programming is *stage*. To avoid confusion between the terms we henceforth use *signal-stage* to denote a group of one or more traffic/pedestrian phases which receive a green signal during a particular period, i.e. the stage of signal control, and use *stage* to refer the term in dynamic programming.

Among online approaches, those who collect real-time traffic information from loop detectors and evaluated instantaneously to generate the most up-to-date signal settings for implementation pertains to responsive control; and those who also utilize real-time traffic information to select the pre-specified signal plans which are best matches with the detected traffic pattern pertains to actuated control. Several well-known traffic signal design packages are reviewed. Those important characteristics of the packages are summarized in Table 1.

Table 1 Summary of different design programs for traffic signal control

Program	Traffic data	Decision on signal settings	Signal cycle	Signal coordination	Origin country	Objective for optimization	Servomechanism
OPAC	On-line data from upstream detectors	Change of current signal settings Rolling forward	Acyclic	Nil	US	Delay	Decentralized
UTOPIA	On-line data from upstream detectors	Green start and duration times and offsets	Required	With offset optimization	ITALY	Delay and stop	Centralized
SCATS	On-line data from stop line (downstream)detector	Pre-sepcified signal plan replacement	Required	With offset optimization	Australia	Capacity*	Centralized
SCOOT	On-line data from upstream detectors	Change of whole signal plan	Required	With offset optimization	UK	Delay, stop, and congestion	Centralized
PRODYN	On-line data from pair of upstream detectors	Change of current signal settings	Acyclic	Possible	France	Total delay	Decentralized
MOVA	On-line data from pair of upstream detectors	Green extension or not	N/A	Nil	UK	Delay, stop and capacity	Decentralized
DYPIC	Off-line basis and perfect information	Complete signal settings	Acyclic	Nil	UK	Delay	Decentralized

* Vehicle discharge rates are monitored and signal will be changed if it is found less than the saturation flow rate

SCOOT (Hunt et al., 1981) and SCATS (Luk, 1984) are basically online variants of offline optimization strategies. Manual engineering work is required to update traffic data and feed them into offline optimizer, TRANSYT for example, for a preparation of a library of plans that applies to different time periods of a day. The ultimate performance of systems as those depends on the accuracy of database and its conformity to software requirements. The online capability then enables the selection of the most appropriate plan from the library according to detected traffic from sensors, adjusts offsets between adjacent intersections to facilitate coordination, and makes small adjustment to the signal plans. SCOOT reduces delay to vehicles by 12% when compared to up-to-date plans from TRANSYT. SCATS produces 23% reduction in travel time in comparison with uncoordinated operations.

DYPIC (Robertson and Bretherton, 1974), OPAC (Gartner, 1983) and PRODYN (Henry *et al*., 1983) are successively development in dynamic optimization of traffic signals. DYPIC is actually a backward dynamic programming algorithm serves only for analytical purpose, which is to create a series of look-up tables containing exact value associated with each state, along with performance indices serving as benchmarks. An empirical function of quadratic form is proposed for a heuristic solution aiming for practical uses. The heuristic solution adopts the concept of *rolling horizon*, which means that: first, a planning horizon is split into a ‘head’ period with detected traffic information and a ‘tail’ period with synthesized traffic information; second, an optimal policy is calculated for the entire horizon, but is implemented only for the ‘head’ period; finally, when next discrete time interval is arrived and newly detected information becomes available, the process rolls forward and repeats itself. Gartner (1983) gives a detailed description of rolling horizon approach in his introduction of OPAC. However, OPAC does not abide by the principle of optimality adherent to dynamic programming, rather it uses Optimal Sequential Constrained (OSCO) search to plan for the entire horizon, and employ terminal cost to penalize queues left after the horizon. OPAC in both simulation and field tests saves 5-15% from existing traffic-actuated methods, with most of the benefits coming from situation of high degree of saturation. The concerns with OPAC are that the restrictions in OSCO search reduces the flexibility of decision making, and a long planning horizon (60s) raises practical questions about the value of optimizing far into the future on the basis of synthesized information when the decisions planned may never the implemented. PRODYN, also adopting rolling horizon approach, optimizes timings via a forward dynamic programming (FDP). To avoid computing optimality equation at grid point that eventually poses the problem of dimensionality, the FDP is particularly designed that it aggregates state variables into a few subsets, and the value of being in a subset is only evaluated when it is actually being arrived at. Value function presenting the future cost in the FDP is directly adopted from Robertson and Bretherton’s work. By evaluating all the subsets that can be possibly arrived at, the FDP records the optimal trajectory of control policy in the planning horizon (75s). And the process rolls forward as it is defined in rolling horizon approach. Experiments (Henry 1989) show that PRODYN yields an average gain in total travel time of 10% with a 99.99% significance.

UPTOPIA (Urban Traffic Optimization by Integrated Automation, Mauro *et al*. 1984, 1989) is a hybrid control system that combines online dynamic optimization and offline optimization. This is achieved by constructing system hierarchy with an area level and a local level. Area controller generates reference plan, and local controllers adapt reference plan and dynamically coordinates signals in adjacent intersections. Rolling horizon approach is again used by local controller to optimize performance, and the planning horizon is 120s, with the process being repeated every 3 seconds. To automate the process of updating reference plans that are generated by TRANSYT, an AUT (Automatic Updating of TRANSYT) module is developed. AUT first collects traffic data continually by means of some of the detectors in the network. The data are processed to calculate traffic flow pictures for various parts of the day. The model predicts the traffic flow profiles to be used when calculating new reference plans. Afterwards AUT prepares the data to be used in the TRANSYT calculation and starts TRANSYT optimization for selected effects. The benefits recorded after UTOPIA's implementation show an increase of 15% in average speed for private vehicles and 28% for public transport with priority.

MOVA (Vincent and Peirce, 1988) is the only one in this review package that is purposely designed for isolated intersection. The system generates signal timings cycle-by-cycle that vary continuously according to the latest traffic condition. Upon signal-stage change, MOVA uses *vehicle gap* detected through pairs of upstream detectors to terminate green extension. The criterion for extension is whether the gap reaches certain critical values. There are two operational modes for uncongested and congested conditions in which delay and stop minimising and capacity maximising routines are adopted respectively. MOVA evaluates its control policy very half second.

From the reviews on online traffic signal control systems, a scope for new development in this field is perceived as the follows:

First, controller at a local intersection may dynamically adjust signal timings with a larger freedom so long as it is conforming to the prevailing traffic and facilitates the overall traffic flow in a network, and certain critical constraints such as minimum and maximum green time are not breached. This would allow controller to give green indication regardless of preset signal-stage order, during and cycle time, and hence a larger levy in controlling changing traffic. OPAC and PRODYN's local controllers are already capable of doing this, but a general formulation is yet to be shown for a multiple signal-stage intersection and DP's principal of optimality should be reaffirmed. The key issue here is how to build an algorithm on the principle of optimality in a forward rolling process (thus unitizing limited online information), while preventing computational burden from exploding towards intractable. As optimality in control is not particularly sensitive to variations in traffic arrivals beyond 25s into the future (Robertson and Bretherton, 1974), the rolling horizon does not have to be very long.

Second, controller may automatically adjust control policy so that it reflects the changes in prevailing traffic in time and online. If the mechanism defining control policy over time is largely a dependant on internal function parameters, the parameters then should be tuned intelligently and accordingly. Although UTOPIA is capable of automating the updating of TRANSYT plans at area level, its local controllers are still subjective to system-wide reference plans.

Third, a rapid review of control policy, i.e. a finer resolution in decision-making, is a definite advantage over a slower review frequency, i.e. a coarser resolution in decision-making. This is because what could be addressed in coarser resolution won't be left out in a finer resolution, but information in a finer case could be lost in a coarser case. MOVA's resolution level has reached half a second, but its timing plan is still cyclic.

The objective for this study therefore is to develop a practical approach based on ADP that adopts the principle of optimality, adaptively adjusts its control parameters and reviews its decision at a rate of per half second. Application of the new approach is limited to isolated intersection with multiple signal-stages in this study. However, this does not necessarily limit its potential in being implemented in decentralized network.

3. APPROXIMATE DYNAMIC PROGRAMMING

In principle, ADP systems should be able to approximate the solution to any problem in control or planning which can be formulated as an optimization problem (Werbos, 1996). In this study we seek an approximation of real cost-to-go in dynamic programming to suffice the designated objective. Discussion on ADP formulation starts from the fundamentals of DP. We will show in 3.2 and 3.3 how DP's usefulness is constrained by dimensionality and availability of information, and how ADP drastically reduces the computational burden while offers efficient usage of limited online information. The open framework of ADP allows various learning techniques to be adopted for online tuning of approximate function. Two techniques: reinforcement learning and perturbation learning, are discussed respectively in 3.4 and 3.5. System dynamics are discussed in 3.6.

3.1 Notations

$J(\mathbf{x})$ is the total future discounted cost starting from state $\mathbf{x}$, or the cost-to-go,

$\bar{J}(\mathbf{x})$ is the approximate function of $J(\mathbf{x})$,

ψ is the decision (*eg* initiate change of signal-stage or defer decision),

$\mathbf{x}$ is the system state,

ω is a vector of traffic arrival information,

$e^{-\gamma\tau(\psi)}$ is a exponential discount rate,

$\tau(\psi)$ is the time over which decision ψ is implemented,

γ is a discount rate for cost incurred in the future,

$f(\omega, \mathbf{x}, \psi)$ is the transition functions that transfer current state to state at time $\tau(\psi)$ after decision ψ is implemented,

$c(\omega, \mathbf{x}, \psi)$ is the immediate cost of implementing decision ψ,

$\mathbf{l}$ is a vector of traffic state,

$\mathbf{g}$ is a vector of controller state,

$O(\boldsymbol{\omega}, \mathbf{x}, \psi)$ is a function returns a vector of vehicle departures,

$\mathbf{w}$ is a vector of parameters in approximate value function,

$\phi(.)$ is a basis function for approximation, Φ is a vector of ϕ(.).

3.2 Dynamic Programming

Dynamic programming decomposes a problem into a set of sub-problems denoted as *stage*, with a policy decision required at each stage. Each stage has a number of *states* associated with the beginning of that stage. A polciy decision at each stage is to transform the currentstate to a state asscociated with the beginning of the next stage. To find an optimal policy for the overall problem, for example,

$$\underset{\psi}{\mathrm{Min}}\left[\underset{\omega}{\mathrm{E}}\left\{\sum_{t=0}^{T} e^{-\gamma\tau(\psi)} c_t\left(\mathrm{f}\left(\omega,\mathbf{x},\psi\right)\right)\right\}\right], \tag{1}$$

DP recursively calculates

$$\mathrm{J}_t(\mathbf{x}) = \underset{\psi}{\mathrm{Min}}\left[\underset{\omega}{\mathrm{E}}\left\{c_t\left(\omega,\mathbf{x},\psi\right) + e^{-\gamma\tau(\psi)}\mathrm{J}_{t+1}\left(\mathrm{f}\left(\omega,\mathbf{x},\psi\right)\right)\right\}\right], \tag{2}$$

to guarantee that the policy being found leads to global optimum. Eq. (2) is also denoted *the optimality equation*.

Upon applying DP to traffic signal control, a stage is a discrete time interval along the planning period, and a state of a stage is a combination of traffic state and controller state. A state of traffic at a intersection can be specified by the number of vehicles queuing in each of the approaching links; the state of controller can be specified by the signals which are green, any changes which are currently underway, the times at which they will be completed and the times of expiry of any minimum or maximum permitted durations. A decision at a stage can which to change the current indication, and if so, which stage of signal control is to change to. An optimal policy for the overall signal control problem is then a set of decisions of alternating indications that meet the objective in control.

Although (2) offers a simple and elegant way of problem solving, it can be computationally intractable even for very small problems. This is because to evaluate a decision at a stage the algorithm loops over the entire state space of the stage. As the number of state variable increases, the process has to consider all possible combinations of values of the several state variables. The number of combinations can be as large as the product of the number of possible values of the respective variables. The computational load therefore tends to explode when additional state variables are introduced. For example, in developing PRODYN, Henry *et al.* (1983) has found that the memory requirements for an intersection are:

$$2\left[\Delta l\left(\prod_{i=1}^{I} C_n^1\right) TU\right]\left(r_1 + r_2\right), \tag{3}$$

where 2 stands for the current and future tables of state values,

Δl: unit vehicle length,

I: number of links,

C_n^1: a combination with a subset size of 1, and a set size of *n*, *T:* green time already elapsed,

U: number of signal-stages

r_1 and r_2: memory requirement for traffic state and controller state respectively.

If an intersection has four links in two signal-stages (*I*=4 and *U*=2), and no more than 19 vehicles could possibly queue in a link (n=20, including 0 state), with Δl=1 and a maximum green time of 60s (*T*=13 at a rate of 5s per interval), r_1=2 bytes, r_2=2 bytes, the memory requirement is:

$$2\left(20^4 \times 13 \times 2\right)\left(2+2\right) = 33280000 \text{ bytes}.$$

If the complexity of the intersection rose to more combination of signal-stages and resolution refined to half a second, for example, the requirements in computation and memories can even be proven (as it will show in numerical experiment of this study) to be too costly for up-to-date computing facility with microprocessor. DP's vulnerability to increase in size of state space, the complexity of probability distribution of information, and the number of optional decisions are frequently referred as 'the Three Curse of Dimensionality' (Powell, 2007). Another concern restricts DP's application in real-time control is that information from detectors about traffic are normally of the next few (typically 10) seconds. In a rolling horizon approach, traffic models may supplement information for the rest of the planning horizon, which is usually more than 60s in previous studies. However, using DP means starting calculations at the end of the horizon where information on arrivals is least certain. This scenario puts difficulty on the justification of using a costly optimization algorithm in a process involving a large number of modeled information and decisions planned in the 'tail' period may never be implemented.

Since the crux of DP algorithm is to establish a look-up table of exact state values, we may replace the state values with an approximate function that is built on state variables. This would drastically reduce computational burden, and in the mean time, it offers an alternative of stepping forward through time. In a forward process, the more reliable part of information can be therefore utilized to produce decisions to accommodate the more imminent demand. These are the premises for the introduction of approximate dynamic programming.

3.3 Approximation of Dynamic Programming

Let $\bar{\mathrm{J}}(\mathbf{x})$ be a function that approximates the cost-to-go of DP, ADP is to meet objetive in (1) by forwardly calculating

$$\hat{\mathrm{J}}(\mathbf{x}) = \underset{\psi}{\mathrm{Min}}\left[\underset{\omega}{\mathrm{E}} \left\{ \ell\left(\omega, \mathbf{x}, \psi\right) + e^{-\gamma\tau(\psi)} \bar{\mathrm{J}}^{n-1}\left(\mathbf{f}\left(\omega, \mathbf{x}, \psi\right)\right) \right\} \right], \quad (4)$$

where $\hat{\mathrm{J}}(\mathbf{x})$ is a new observation of the cost-to-go at state **x**, and *n* denotes the number of updates to the approximation function. How well ADP approach works largely depends on how good the approximation is, and how much discount we give to cost further into the future. We may set the course to find a close-form solution for (4), but this has been proven to be extremely difficult (Werbos, 2004). Without knowing the approximate function *a priori*, we may hypothesis any structure that we deem as appropriate and employ a machine learning technique to tune the functional parameters progressively. There are a large number of approximation techniques available, including aggregation, regression and most frequently neural networks. Let us assume that the approximate function is piecewise linear, which can be expressed as:

$$\bar{J}(\mathbf{x}, \mathbf{w}) = \mathbf{w}^T \phi(\mathbf{x}). \tag{5}$$

or

$$\bar{J}(\mathbf{w}) = \mathbf{w}^T \Phi, \tag{6}$$

where Φ is a $|S| \times K$ matrix whose kth column is equal to ϕ_k; that is, $\Phi = (\phi_1 \cdots \phi_k)$.

Using (5) to replace real cost-to-go J(**x**), the memory requirement for a four-link two signal-stages intersection described above is

$$2(4 \times 13 \times 2)(2 + 2) = 832 \text{ bytes.}$$

This amount is one 2.5x10^{-5}th of what would have been in classic DP. The efficiency in computing allows a controller configured with average computing capacity of this day to work with comprehensive phase combination and operate at rather fine resolution. And because it calculates forward, it starts decision making with the actual data detected from online sensors, and implements decisions only for the 'head' period in a rolling horizon approach. The head period, depending on the resolution in decision-making, is less or equal to the period of actual data. The concern of starting planning backward from where data is least certain is then circumvented.

We may further assume that a certain learning technique is supervising the adaptation of parameter vector **w** whenever new observation $\hat{J}(\mathbf{x})$ is available. This can be expressed as:

$$\bar{J}^k(\mathbf{x}, \mathbf{w}) = \mathbf{F}\left(\hat{J}(\mathbf{x}), \bar{J}^{k-1}(\mathbf{x}, \mathbf{w}), \mathbf{w}\right) \tag{7}$$

Depending upon the specific technique in use, the objective of parameter tuning could be different, although a least square approach for minimizing the error between estimation and observation is quite common.

Now we may summarize the general ADP algorithm, which is shown in Fig. 1.

3.4 Reinforcement learning

Reinforcement learning is a paradigm of learning without a 'teacher'. It is a 'behavioral' learning problem: It is performed through *interaction* between the learning system and its environment, in which the system seeks to achieve a specific goal despite the presence of uncertainties (Barto *et al*., 1983; Sutton and Barto, 1998). Trail and error search are important distinguishing characteristics of reinforcement learning.

A reinforcement learning agent generally consists of four basic components: *a policy, a reward function, a value function,* and *a model of the environment*. In a problem defined by (1) and (2), the *policy* is the principal of optimality shown in (2). The policy is the ultimate determinant of behavior and performance. The *reward function* is shown as c(ω, **x**, ψ), which returns the immediate and defining feature of the problem faced by the agent. *Value function* is presented by J(**x**), which predicts the rewards in the long run. And the *model of the environment* is function f(ω**, x,** ψ), which predicts (in stochastic environment) or determines (in deterministic environment) the next state. In an approximation that seeks to replace the exact J(**x**) with a function approximator shown in (4), reinforcement learning technique updates the parameters of the function approximator upon each observation of a state transition and the associated cost, with the objective of improving approximations of long-term future cost as more and more state transitions are observed. The relationship between the components of a reinforcement learning agent is shown in Fig. 2. The specific technique employed in this study is a variant of *temporal-difference learning* (TD learning).

(TD) learning, originally proposed by Sutton (1988), is a method for approximating long-term future cost as function of current state. Let us consider an irreducible aperiodic Markov process of infinite horizon whose states lie in a finite or countable infinite space X. The true cost-to-go $J^{*}: \mathrm{X} \mapsto \Re$ associated with this Markov process is defined by:

$$J^{*}(\mathbf{x}) \cong \mathop{\mathrm{E}}_{\omega}\left\{\sum_{t=0}^{\infty} e^{-\gamma\tau(\psi)} c\left(\mathrm{f}\left(\omega_t, \mathbf{x}_t, \psi_t\right)\right) \middle| \mathbf{x}_0 = \mathbf{x}\right\}, \tag{8}$$

assuming that this expectation is well-defined. We consider a linear approximation of $J^{*}: \mathrm{X} \mapsto \Re$ using a function defined by (4), which is $\bar{\mathrm{J}}(\mathbf{x}, \mathbf{w}): \mathrm{X} \times \Re^{K} \mapsto \Re, \text{with } \mathbf{w} \in \Re^{K}$. The temporal difference d_t corresponding to $\mathbf{x_t}$ to $\mathbf{x_{t+1}}$ is then defined as:

$$d_t = c\left(\omega_t, \mathbf{x}_t, \psi_t\right) + e^{-\gamma\tau(\psi)} \bar{J}\left(\mathrm{f}\left(\omega_t, \mathbf{x}_t, \psi_t\right), \mathbf{w}_t\right) - \bar{J}\left(\mathbf{x}_t, \mathbf{w}_t\right). \tag{9}$$

Then, for t=0,1, …, the temporal difference learning method updates $\mathbf{w}_t$ according to the formula

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \theta_t d_t \sum_{k=0}^{t} \left(e^{-\gamma\tau(.)}\lambda\right)^{t-k} \nabla_{\mathbf{w}} \bar{J}\left(\mathbf{x}_t, \mathbf{w}_t\right), \tag{10}$$

or taking account of our case of linear approximation

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \theta_t d_t \sum_{k=0}^{t} \left(e^{-\gamma\tau(.)}\lambda\right)^{t-k} \phi\left(\mathbf{x}_t\right). \tag{11}$$

where θ_t is a sequence of scalar step sizes that satisfy the following terms for convergence

$$\sum_{t=0}^{\infty} \theta_t = \infty, \tag{12}$$

and

$$\sum_{t=0}^{\infty} \theta_t^2 < \infty. \tag{13}$$

Parameter λ of (11) is known as *trace eligibility* factor, which takes value in [0,1]. Since temporal difference learning is actually a continuum of algorithm parameterized by λ, it is often referred as TD(λ). A more convenient representation of TD(λ) can be obtained by defining a sequence of *eligibility vectors* z_t by

$$z_t = \sum_{k=0}^{t} \left(e^{-\gamma\tau(.)}\lambda \right)^{t-k} \phi(\mathbf{x}_t). \tag{14}$$

With this notation, Eq.(11) can be rewritten as

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \theta_t d_t z_t. \tag{15}$$

For a better understanding of the nature of *trace eligibility* factor λ and the underlying dynamics of the TD algorithm, we may define a TD(λ) operator by

$$\left(T^{(\lambda)}J\right)(\mathbf{x}) = (1-\lambda)\sum_{m=0}^{\infty} \lambda^m E\left[\sum_{t=0}^{m} e^{-\gamma t} c(\omega_t, \mathbf{x}_t, \psi_t) + e^{-\gamma(m+\tau(\psi_t))} J\left(\mathbf{x}_{t+\tau(\psi_t)}\right) | \, \mathbf{x}_0 = \mathbf{x} \right], \tag{16}$$

for $\lambda \in (0,1)$, and

$$\left(T^{(1)}J\right)(\mathbf{x}) = E\left[\sum_{t=0}^{\infty} e^{-\gamma t} c(\omega_t, \mathbf{x}_t, \psi_t) | \, \mathbf{x}_0 = \mathbf{x} \right] = J^*(\mathbf{x}), \tag{17}$$

for λ=1, and

$$\left(T^{(0)}J\right)(\mathbf{x}) = E\left[c(\omega_t, \mathbf{x}_t, \psi_t) + e^{-\gamma\tau(\psi_t)} J\left(\mathbf{x}_{t+\tau(\psi_t)}\right) \right], \tag{18}$$

for λ=0. It can be seen that TD(1) is a true and unbiased estimation of $J^*(\mathbf{x})$, and TD(0) is an equivalent to *single-pass algorithm* as all the calculations including the update of approximation are finished at the end of each forward pass (Powell, 2007). In the case of TD(0) only the most recent observation matters, which gives it an overall simplicity in a family of TD algorithms. Although TD(1) gives a quicker convergence than TD(0) in general, in the cases where states are frequently visited, TD(0), i.e. the single-pass algorithm, will work fine. Van Roy *et al.* (1996) developed an online approximation of inventory holding cost using TD(0). The approach reduces inventory costs by about ten percent in comparison to optimized “order-up-to” policies. In this paper we employ TD(0) too.

With operator $T^{(\lambda)}J$, we can rewrite (11) in the form

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \theta_t D\left(T^{(\lambda)}\left(\mathbf{w}_t^T \Phi\right) - \mathbf{w}_t^T \Phi \right) \phi(\mathbf{x}_t), \tag{19}$$

where D is a $n \times n$ diagonal matrix with diagonal entries $\pi(1), \ldots, \pi(\mathrm{n})$, and $\pi(i) > 0$ for all $I \in S$. In the case of $\lambda \in [0,1)$, we can think (16) as a steepest descent method for a weighted linear least-square problem as

$$\sum_{\mathbf{x} \in X} \pi(\mathbf{x}) \left(\left(T^{(\lambda)}\left(\mathbf{w}_t^T \Phi\right)\right)(\mathbf{x}) - \bar{J}(\mathbf{x}, \mathbf{w}) \right)^2 \tag{20}$$

with respect to $\mathbf{w}$, given a fixed $\mathbf{w}_t$. The convergence prove of linear value function approximation defined by (16) is shown in Tsitsiklis and Van Roy (1996), while a meticulous prove on the convergence of TD(0) is shown in Sutton (1988).

3.5 Perturbation Learning

Regression models mapping state to observation iteratively over time such as TD(λ) offers one way of approximating cost-to-go when the number of parameters are not to large. Otherwise the inverse of matrix in a least-square problem would be too expensive for real-time operation. An alternative to regression model is to explore the structure of the cost-to-go function, and the monotonicity in particular. In traffic signal control we have a monotonic increase problem. This is to say that the cost-to-go value of a state increases as the number of vehicles remaining in the system increases. Additionally, the value increases more rapidly when more vehicles are remained in red links than in green. In real-time operation we may not have the knowledge of the exact cost-to-go function, but we can approximate the property of monotonicity. Since monotonicity can be easily maintained by a linear approximation, we may define a piecewise learning approximator on the basis of (5)

$$\bar{J}(\mathbf{x}, \mathbf{w}) = \sum_{k=1}^{K} \mathbf{w}(k)\phi_k(\mathbf{x}), \tag{21}$$

where $\mathbf{w}(k)$ is the k^{th} element of parameter vector $\mathbf{w}$ and each ϕ_k is a fixed scalar function defined on the state space X. Since the partial differentiation of the linear approximator with respect to $\phi_k(\mathbf{x})$ yields

$$\frac{\partial \bar{J}(\mathbf{x}, \mathbf{w})}{\partial \phi_k(\mathbf{x})} = \mathbf{w}(k),$$

we can easily show that a new observation of $\mathbf{w}$(k) can be obtained through

$$\ddot{\mathbf{w}}(k) = \frac{\ddot{J}(\mathbf{x}+e_k) + \ddot{J}(\mathbf{x}-e_k)}{2\|\Delta_k\|}, \tag{22}$$

where e_k is a K-dimension vector of zeroes with a unit increment in the k^{th} element, and $\ddot{J}(.)$ calculated from (4). With the new observation, we then smooth to obtain an updated estimate of the functional parameter

$$\mathbf{w}^n(k) = (1-\theta_{n-1})\mathbf{w}^{n-1}(k) + \theta_{n-1}\ddot{\mathbf{w}}(k), \tag{23}$$

where n here denotes the number of updates.

Papadaki and Powell (2003) develop a similar ADP approach to (21) for stochastic batch dispatch problem, in which results from both the linear and nonlinear (concave) approximation are compared. Although nonlinear approximation works better than linear ones as iteration goes on, a large-scale problem may make nonlinear approximation extremely difficult or impossible in implementation. For such problems, a linear approximation is the best.

3.6 System dynamics

In this section we will show how the dynamics of a signal control system is formulated into the ADP algorithm. Recalling that a state $\mathbf{x}$ of traffic control system is a combination of traffic state and controller state, we denote traffic state by $\mathbf{L}$ and controller state by $\mathbf{G}$. We further define that traffic state $\mathbf{L}$ by the number of vehicles queuing in each of the approaching links, and controller state $\mathbf{G}$ by the state of signal (i.e. red or green, amber state is not considered in this study) of each link. For an intersection having I links, for i=1, … , I, we define

$$\phi\left(\mathbf{x}\left(\mathbf{L}(i),\mathbf{G}(i)\right)\right)=\begin{cases}\begin{bmatrix}\mathbf{L}(i)\\0\end{bmatrix} & \text{for } \mathbf{G}(i)=1,\\ \begin{bmatrix}0\\\mathbf{L}(i)\end{bmatrix} & \text{for } \mathbf{G}(i)=0;\end{cases}$$

where g=1 stands for green indication and g=0 for red. In such a manner, the basis function $\phi(\mathbf{x})$ transforms state $\mathbf{x}$ to a column vector of dimension $K=2\times I$

$$\phi(\mathbf{x})=\begin{bmatrix}\phi(\mathbf{x}(1))\\\vdots\\\phi(\mathbf{x}(I))\end{bmatrix}.$$

The parameter $\mathbf{w}$ is defined as a K-dimension column vector, and the piecewise linear approximator shown by (21) can be interpreted as a weighted sum of queues remaining in the system. Depending on the specific link and the state of signal on that link, the weight assigned to the queue may vary.

Function $c(\omega,\mathbf{x},\psi)$ returns the immediate cost of implementing decision ψ, providing state $\mathbf{x}$ and information ω. Information ω is a vector of future arrivals on each traffic link

$$\omega=\left[\omega(1),\ldots,\omega(I)\right]^T.$$

Decision ψ is a $I\times\tau(\psi)$ vector, where $\tau(\psi)$ spans over a period that is long enough to supervise both inter-green and minimum-green. This is because once a 'change' decision is implemented, the controller state can not be altered until both inter-green and mini-green are clear for safty and engineering reasons. For example, a decision ψ of 'change from link i=I to link i=1' can be expressed as

$$\psi=\begin{bmatrix}\underbrace{\overbrace{\begin{matrix}0&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&&\vdots\\1&0&\cdots&0\end{matrix}}^{\text{Inter-green}}\;\overbrace{\begin{matrix}1&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&&\vdots\\0&0&\cdots&0\end{matrix}}^{\text{Mini-green}}}_{\tau(\psi)}\end{bmatrix},$$

wheras upon a 'no change' ψ is a $I\times\tau(\psi)$ zero matrix. An actuall change in signal indication is implemented when $\psi(i,t)$=1. A decision is always formed and implemented at the *beginning* of each time interval, and the changes in signal indication can be modeled by

$$\mathbf{G}_t(i)=\left(\mathbf{G}_{t-1}(i)+\psi(i,t)\right)\text{mod}_2. \tag{24}$$

The immediate cost c is then calculated through

$$\mathrm{c}\left(\omega_t, \mathbf{x}_t, \psi_t\right)=\sum_{t'=0}^{\tau(\psi)} e^{-\gamma(t')} \sum_{i=1}^{I}\left(\mathbf{L}_{t'}(i)+\omega_{t'}(i)-\mathrm{O}\left(\omega_t(i,t'), \mathbf{x}_{t'}(i), \psi_t(i,t')\right)\right) \tag{25}$$

where O(ω(*i*), **x**(i), ψ) is the outflow function that returns

$$\mathrm{O}\left(\omega_t(i), \mathbf{x}_t(i), \psi_t(i)\right)=\begin{cases}\mathrm{Min}\left(s(i),\ \mathbf{L}_t(i)+\omega_t(i)\right) & \text{for } g(i)=1, \\ 0 & \text{for } g(i)=0.\end{cases} \tag{26}$$

$s(i)$ is the saturation departure rate for link i.

The transfer function f(ω, **x**, ψ) transfers the system from current state to the next state

$$\mathrm{f}\left(\omega_t, \mathbf{x}_t, \psi_t\right)=\begin{cases}\mathbf{L}_{t'}(i)=\mathbf{L}_{t'-1}(i)+\omega_{t'-1}(i)-\mathrm{O}_{t'-1}\left(\omega_t(i,t'-1), \mathbf{x}_{t'-1}(i), \psi_t(i,t'-1)\right), \\ \mathbf{G}_{t'}(i)=\left(\mathbf{G}_{t'-1}(i)+\psi_t(i,t'-1)\right)\mathrm{mod}_2,\end{cases} \tag{27}$$

for $t'=t,\ldots,t+\tau(\psi_t)$, and $i=1,\ldots,I$.

In the following section we will discuss the testing environment for numerical experiment and its realization in computer simulation.

4. EXPERIMENT AND RESULTS

In this section is organized as the following: we first introduce how the data for numerical experiment are generated in computer simulation in 4.1, followed the design of testing bed in 4.2, the configuration of decision set of ADP controller in 4.3 and the traffic flow setting that present different operation environment in 4.4. Section 4.5 introduces the general simulation environment. Comparison between DP, ADP, and Fixed-time performances in different circumstances are shown in 4.6. From these comparisons, we are able to discern the advantages of ADP controller in online operation, and the difference between reinforcement learning and perturbation learning.

4.1 Traffic Data

Traffic data is assumed to be detected from inductive loops imbedded upstream. Detectors provide arriving traffic of the next 10 seconds for each traffic link, which is known at the beginning of each time interval. In the coarser resolution, time is divided into short intervals of 5 seconds each, and traffic rate takes integer value of 0, 1, or 2 vehicles only; in the finer resolution, intervals are 0.5 seconds each, and traffic rate is a binary variable taking either 0 or 1. Other assumptions for approaching traffic are:

1. Approaching traffic is homogenous.
2. Queue lengths are calculated at the end of each interval, neglecting the detail of vehicle behavior during the interval. Signals may only be switched at the boundary between intervals.

The random arrival process for the coarser resolution is generated by binomial distribution. The maximum rate of 2 is set so that the arrival rate could not exceed the capacity. Learning R be the random traffic rate, we denote the cumulative distribution function by

$$F(r) = P\{R \le r\}$$

and probability of $R=m$, where m takes integer value of 0, 1, or 2, is

$$P\{R=m\} = \binom{n}{m} p^m (1-p)^{n-m}, m = 0,1,...,n. \tag{28}$$

Simulation of random arrival process is then realized by using *Inverse Transformation Method* (ITM), which first generate a *uniform random number z* between 0 and 1, and then set $F(r)=z$ and solve for r to produce the desired random observation from the probability distribution.

To extend the random process to the finer resolution, a shifted Bernoulli process is adopted. With probability P, an arrival is generated and the trial has duration $T_g=n_b\Delta t$; with probability (1-P), no arrival is generated and the trial has duration $T_g=\Delta t$. Thus the mean number of vehicles that are generated in a single trial is E(N)=P, and the mean duration of the trial is E(T)=[Pn_b+(1-P)] Δt. An estimate of the mean rate of traffic generation is given:

$$q = \frac{E(N)}{E(T_g)} = \frac{P}{\left[1+P(n_b-1)\right]\Delta t}, \tag{29}$$

The probability required to generate a certain mean arrival rate can be deduced by inversing the expression:

$$P = \frac{q\Delta t}{1-(n_b-1)q\Delta t}. \tag{30}$$

The rest of the process for generating random traffic at finer resolution is identical to that of the coarser resolution. Specifications for generating traffic data in both cases are summarized in Table 2.

Table 2 Generating Traffic Data

	Time Interval	Traffic Rate	Random Process	Simulation
Coarser Resolution	5 seconds	0, 1, or 2	Binomial	ITM
Finer Resolution	0.5 seconds	0 or 1	Shifted Bernoulli	ITM

It is worth noting that because queue lengths are calculated at the end of each interval. The delays measured in the coarser resolution are not directly comparable to delays measured in the finer resolution. In particular, in the case of the coarser resolution, no delay is attributed to either vehicle in a time increment during which two depart, whilst in the finer resolution, delay would occur to one the vehicles for the whole of the departure time of the other. To facilitate the comparison between the two resolutions, we record the decision series in the coarser case and implement them in the finer case. The result thus obtained is then compared with performance measure of the finer case.

4.2 Traffic Intersection

To demonstrate the readiness of the control approach for operation, we design an isolated intersection having three signal-stages, as shown in Fig. 2. In designing the control environment, we also assume that:

1. Signal phases are composed of effective greens and effective reds only, and no amber interval is considered.
2. There are no constraints on the maximum duration of a green period. Both inter-green and minimum green are set as 5 seconds.
3. The saturation flow on all lanes is 2 vehicles per interval. This is equivalent to 1440 vehicles per hour, a rate that is sufficiently close to the saturation flow of a single traffic lane.

Because we only consider an isolated intersection, there is no treatment of the physical length of traffic queue, and 1therefore the queue length won't affect approaching traffic or the detectors that are assumed to be laid sufficiently upstream. For simplify the traffic state combination, especially to facilitate comparison with other techniques such as DP, traffics on each signal-stage are grouped into a single link, such Link-A for signal-stage 1, Link-B for signal-stage 2, and Link-C for signal-stage 3.

It is worth mentioning the treatment of *lost time*. Lost time in traffic engineering pertains to the time (in seconds) during which vehicles receiving green signal are unable to pass through an intersection. The total lost time is the sum of *Start-up lost time* and *Clearance lost time*. If no specific observations were made for the lost times, the two elements of total lost time may either assumed to be 2 seconds (Roger P. Roess *et al.*, 2004). In the formulation and experiment presented here, lost time is not modeled for reasons of simplicity. It would argue that without treating lost time, the optimal signal control is straightforward —— serving one traffic stream till queue is cleared, then switch to non-empty signal-stages. Since such a policy is not a feature in our ADP approach, it would serve as a measurement of the performance.

4.3 Decision Set

The basic decision set is 'change now' (ψ=1) or 'change later' (ψ=0). However, if a change at any later time that is within the 'head' of the planning horizon produces less delay than 'change now', the system will remain the current signal indication. In the case of 'change now', the system will also decide which signal-stage to call. To facilitate this decision-making policy, we have a planning horizon of 20s. The first 10s forms the 'head', which is fed with detected information, and the second 10s forms the 'tail', which is fed with modeled data. The complete decision set is illustrated in general form in Fig. 4. In the numerical experiment, however, we artificially set the time-span of a decision as 10s, which is identical to the period of detected information, with 5s inter-green and 5s minimum green. This design is to facilitate comparison between different control strategies. In the coarser resolution, decision is implemented for the first 5s, whereas in the finer resolution, decision is only implemented for the first half-second, and then the system rolls forward.

4.4 Traffic Flows

We consider two scenarios of traffic inflows in experiments. Scenario A has a constant hourly rate for each traffic link, and the specifications are 432 v/h for Link-A, 252 v/h for Link-B, and 432 v/h for Link-C. In the second scenario, a profile of traffic is generated for each link. Each profile of traffic is composed of three periods: a pre-peak period, a peak period, and a post-peak period, as shown in Table 3 and Fig.5 respectively. Comparisons among ADP, DP and the best fixed-time timings are made on the basis of the first scenario. Scenario B has a profile of traffic, results obtained from which provides an insight on the performance and learning pattern of two different learning techniques, i.e. Reinforcement learning and perturbation analysis.

Table 3 Scenario B: a Profile of Traffic

	Link A (v/h)	Link B (v/h)	Link C (v/h)
Pre-Peak	250	150	250
Peak	500	250	500
Post-Peak	350	200	350

4.5 Computer Simulation

All the numerical experiments are completed in computer simulation. The simulation program is built on MATLAB 7.4.0, and runs on a PC configured with Pentium® Dual Core 3.60GHz and 3.50GB of RAM. Function ‘rand’ of MATLAB is called to generate uniformly distributed random number, which is the core for simulating traffic flows. The reinforcement learning technique is built by calling the *neural network toolbox* (V5.0.2). We use a linear neuron to represent a piecewise linear function approximator. Adaptation of neuron weights (i.e. the parameter vector **w**) is performed by ‘adapt’ function, which is set to minimize the least mean square between neural network output $\bar{J}(\mathbf{x}, \mathbf{w})$ and the *target*. By setting *target* as the new observation of state value $\ddot{J}(\mathbf{x})$ at each time increment t, the neural network approach completes TD(0) learning. This form of achieving TD(0) learning has been suggested by Betsekas and Tsitsiklis (1995) who call it *Neuro-Dynamic Programming*. A concern of using neural network to provide learning capacity is how to specify the stepsize θ_t, or the learning rate. A common approach in stochastic approximation is to use a time-varying rate taking the form of

$$\theta_t = \frac{a}{t},$$

where a is a constant. It can be easily shown that this time-varying form satisfies the terms of convergence set by (12) and (13). However, the large learning rate at the beginning of the learning process may make the system prone to ‘overshooting’, which blows up the parameters. Moreover, in TD(0) learning, all the elements of the parameter vector are updated at once, and there may not be a single rule that is going to be right for all of the parameters. For these concerns, we use a constant learning rate θ=0.005 for neural network, at the expense of slow convergence. In the case of perturbation analysis, because of the design of the algorithm, we are relieved from the concern of overshooting and each parameter can be assigned a specific stepsize. Therefore, for each parameter, we use

$$\theta_n = \frac{a}{a+n-1},$$

where a=40, and n counts the number of times the parameter has been updated (note that we use n rather than t here).

4.6 Results and Comparison

With traffic scenario A, in which the hourly traffic rate on each link is constant and the overall degree of saturation is high, we compare the results from different control strategies, including DP, ADP, and Fixed-time. DP provides the benchmark on the higher end of performance, whereas Fixed-time provides the benchmark on the lower end. In evaluating ADP performance, we obtain results from both the coarser resolution at 5s per interval and the finer resolution at half-second per interval. It is also of our interest to see whether ADP with different learning technique performances differently.

With traffic scenario B, in which a profile of traffic is generated, we compare the learning patterns from different learning techniques in a noisy and changing environment, along with the respective performance over time. In the rest of this section, we denote ADP with reinforcement learning by ADP_RL, and ADP with perturbation learning by ADP_PL.

4.6.1 Traffic Scenario A

Since that both DP and ADP algorithms discount the delay incurred in the future by $e^{-\gamma}$ per time increment, and there is no specific method to determine the best γ*, we employ exhaustive search to find the optimum value. Fig. 5 plots the average performances of ADP_RL and ADP_PL over an hour at the finer resolution (equivalent to 7200 time increments) against an array of discount rates. ADP_RL yields the best performance at γ=0.10, while ADP_PL at γ=0.11. Since the actual difference between the best performances of each learning technique is negligible, we round discount rate at γ=0.10 per time increment for later experiments on ADP. This is equivalent to say that delay incurred one second later will be discounted by $(1-e^{-2\gamma})\times 100\%$=18%, and so on. As for each decision ψ, $\tau(\psi)$=20 increments, the magnitude of $\bar{J}(.)$ is discounted by $(1-e^{-\gamma\tau(\psi)})\times 100\%$=86%.

In similar manner we find that the best discount rate for ADP_PL at the coarser resolution is γ=0.07. The computation burden and the time it takes to run a simulation prohibits us from searching discount rate for DP exhaustively. Rather, we directly use γ=0.10 for DP. It is worth noting that the computer in use can hardly run DP with more than 6 minutes' real data in a single run of simulation, otherwise the computer reports 'out of memory'. The result of DP obtained is actually an average performance over 10 independent simulations, with each of 6 minutes. The best fixed-time timing at the coarser resolution is found through exhaustive search, which has a cycle time of 120s and the same setting in inter-green and minimum-green. The fixed-timed timing's signal-stage sequence and durations are shown in Fig. 7. The averaged performances of each control strategy in comparison are summarized in Table 4.

Table 4 Performance Comparisons with Traffic Scenario A

	DP	**ADP_RF**	**ADP_PL**	**ADP_PL**	**Fixed-time**
Resolution	Finer	Finer	Finer	Coarser	Coarser
Traffic Scenario	A	A	A	A	A
Simulation Time (minutes)	6	60	60	60	60
Run Time (minutes)	720	5.5	12	0.3	0.2
Discount Rate γ	0.10	0.10	0.10	0.07	–
Averaged Performance (v.s/s)	4.27	5.67	5.80	9.98	19.17

In case of the coarser resolution, ADP_PL reduces delay in terms of vehicle-second per second by 48% from the best fixed-time timing. In the case of the finer resolution, in which control policy is 10 times more frequently reviewed, ADP_PL further reduces delay by 42% from its counterpart in the coarser case. We here recall that direct results from different resolution are not comparable, as what has been discussed in Section 5.1. Only by recording decisions produced in the coarser case and implementing them in the finer case did the measured performance comparable. The advantage of finer resolution is illustrated by Fig. 8, from which we see that during the same period of time finer controller adjusts timings far more frequently than the coarser controller, thus keeping queues in system little over time (queues on any link hardly exceed 8 vehicles at a time). Also shown is Fig. 8 is that no matter in finer or coarser case ADP controller always clears the queue on green link before switching to others. This echoes the argument associated with lost-time in 4.2.

In finer cases, ADP_RL performs almost as good as ADP_PL, but takes less than half the time in computing. The major contributor to the different in computing speed may be the that upon updating parameters, ADP_RL calculates a single observation $\ddot{J}(.)$ for all the parameters, whereas for every single parameter ADP_PL calculates two observations. However, the computing speed of both make them fully capable for online operation. In general, with the finer resolution, ADP controllers incurr one and half vehicle-second more than the DP controller per second. DP controller takes an enomous amount of time (12 hours or so) to finish a simluation of 6-mintus operation at an intersection of only three confliting links.

The large ratio in discounting future delay also means that the approximator's contribution to operation at the finer resolution is limited. Fig. 5 shows that ignoring what's to come 10s later only worsens performance by half a vehicle-second, whereas taking a bigger account of it has strong negative impact on performance. Therefore, it is of practical interest to remain the approximation function simple and easy to update.

5.6.2 Traffic Scenario B

In this scenario of traffic input, we would like to show the learning patterns of ADP in a changing and noisy environment and in the mean time to test whether different learning techniques yield distinguishable results. The examples of evolution of functional parameters of ADP_PL and ADP_RL are shown in Fig. 9 and Fig. 10 respectively. Series in both figures can be divided into a group of higher values, which represents the value of parameters assigned to red states, and a group of lower values, which represents parameters assigned to green states. Because ADP_PL has time-varying learning rates that are large in the beginning, parameters of ADP_PL start with strong oscillation and only begin to level off when the system enters post-peak time, partly due to the diminishing learning rate and partly to the steadier environment. The ADP_RL, which by using neural network adopts a constant learning of small magnitude, has more stable parameters over time.

The rising traffic volume in the middle of the controlling period gives sharp rise to parameters assigned to green states, but not much to the other group of parameters. This pattern reflects the near saturation traffic condition at the intersection, in which the amount of arriving traffic is close to the maximum departure rate. Queues on green therefore dissipate much slower. Despite of the different learning technique, it is interesting to see how similar the estimation of parameter is when system enters post-peak.

The paired performances of the two learning techniques in 10 independent simulations are recorded in Table 5. The discount rate is set at γ=0.10 for both. We use two-sample *t*-test to test the hypothesis that the two means are equal. At the degree of significance α=0.05, and degree of freedom n=9, the two means are equal, as shown in Table 5. Recalling that ADP_RL is actually a regression method that minimizes weighted square errors and ADP_PL estimates gradients directly regardless of errors in prediction, the equality of the two means suggests that learning techniques have limited effects on the performance when the approximation function is piecewise linear, discount rate is large and decision is frequently reviewed.

The general conclusion we can draw from the numerical examples is that the ADP approach is fully capable of online operation at an isolated intersection. It saves nearly half of the delays from the best fixed-time timings at a resolution of 5s, and a further substantial reduction can be achieved by improving resolution to 0.5s. For those benefits, the ADP controller only takes 10s data of future arrivals with a competitive computation advantage. When arriving traffic changes over time, the learning pattern of the function parameters captures the alternating trend in the environment. However, we also notice that the best performance of ADP is yielded at a relatively large discount rate, and that different learning techniques make no difference in performance, and little in learning. It suggests that a simple piecewise linear approximator is sufficient for online operation, and the benefit of exploring complex approximation, such as nonlinear approximator, may not be well justified by the cost in computation.

Table 5 Performance Comparisons with Traffic Scenario B

Sample	ADP_PL	ADP_RL
1	3.159583	3.376944
2	3.089722	2.974583
3	2.729583	2.623472
4	2.853194	2.985972
5	3.223472	3.3225
6	3.205278	3.184444
7	3.20375	3.286667
8	4.049861	4.266806
9	3.658611	3.522222
10	3.239028	3.215
Mean	3.2412082	3.275861
t Stat		-0.819958009
P(T<=t) one-tail		0.216708989
t Critical one-tail		1.833112923
P(T<=t) two-tail		0.433417978
t Critical two-tail		2.262157158

6. CONCLUSIONS

ADP is an emerging technique under intensive development in the fields of Artificial Intelligence. Researches on its application have been pursued in power plant management, aerospace technology, robotic control and environment protection. This study investigates ADP application to the field of traffic signal control, aiming to develop a self-sufficient adaptive controller for online operation. Through the review on existing traffic signal control systems and theories, we identify the objectives for this investigation in ADP controller as providing dynamic control for real-time operation, being adaptive through online learning, and frequently reviewing control policy. ADP controller achieves the objectives by replacing real cost-to-go in DP with a simple piecewise linear approximation function, and uses specific learning techniques to tune the function parameters online. We have shown that operating at a resolution of 5s, the ADP controller saves 48% of delays from the best fixed-time timing, and that if operating at resolution of 0.5s, a further 42% reduction in delay can be gained. In comparing with the result from DP, which is theoretical limit, ADP incurs only one and a half second more delay per second while keeping computational requirement at manageable level for online operation. These findings suggest ADP as a sufficient and practical approach to adaptive real-time operation at an isolated intersection.

Two learning techniques —— reinforcement learning and perturbation learning respectively —— are investigated in this study. The crux of the reinforcement learning technique presented here is TD(0) method, whereas perturbation learning directly estimates each parameter of the approximate function by perturbing system state. Although each learning technique approaches to parameter estimation differently, the learning effects are quite similar and no difference can be discerned from numerical experiments with a profile of traffic flows. ADP controller equipped with either learning technique yields the best performance by discounting future delay at about 18% per second, suggesting limited influence of approximated cost-to-go function in evaluating optimum decision. Combining the equality between two learning techniques together with the limited influence of the approximated cost-to-go, it suggests that a simple linear approximation is good enough to sustain the superiority of ADP controller in real-time operation. Exploring more complex approximation may not be proven as cost effective.

The general formation of the ADP controller presented by this study can be extended to more complex intersection without difficulty. However, the norm of contemporary traffic signal operation in urban area is a coordinated network system, and achieving system-wide optimality is a common objective. This study has so far focused on isolated intersection where coordination is not considered. Given the comparative advantage of ADP controller show in this study, we identify the decentralized network operation presented in OPAC and PRODYN as the starting point for introducing ADP to system-wide optimization. A challenging issue here is how to build traffic and controller state of adjacent intersections into the objective function of a local controller, and how the local controller could explore the power of ADP to learn the pattern of combination.

There are a few previous attempts to bring intelligent features into network signal control. Mikami and Kakazu (1994) introduce generic reinforcement learning to this problem, followed by Nakamiti and Gomide (1996), who propose a fuzzy logic approach. Both methods are built on case-based mechanism, which is to retrieve plan from library according to recognized state patterns. Spall and Chin (1997) employ neural network to learn cyclic plans according to pattern recognition, and use perturbation (Simultaneous Perturbation Stochastic Approximation, SPSA) to adapt neural weights. An update study by Srinivasan *et al*. (2006) on SPSA descries a method using a cost function associated with traffic state, but leaves little detail on the cost function itself or on system dynamics. In general, an adaptive and dynamic controller such as that presented by this paper is yet to be developed for network operation. For future research aiming system-wide intelligent traffic signal control, the ADP approach presented by this study might be a competitive candidate.

ACKNOWLEDGEMENTS

The first author would like to thank Rees Jeffreys' Road Fund for the financial support for the work presented here. The second author would like to thank The Croucher Foundation for the support of the Croucher Fellowship period 2006-08. The work described in this paper was jointly supported by grants from the City

University of Hong Kong (Project Number: 7001967 and 7200040) and the Research Grants Council of the Hong Kong Special Administrative Region, China (Project Number: 9041157).

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Step 0. Initialization:

a. Initialize $\bar{J}(.)$.

b. Choose an initial state $\mathbf{x}_0$.

c. Set t=0, k=0.

Step 1. Receive random information $\omega_{\mathbf{t}}$.

Step 2. While t<=T,

a. Solve Eq. (4) to find $\psi_{\mathbf{t}}$*.

b. Update approximation through Eq. (6).

c. Implement $\psi_{\mathbf{t}}$*, and compute $\mathbf{x_{t+1}}=f(\omega_{\mathbf{t}}, \mathbf{x_t}, \psi_{\mathbf{t}}*)$.

Step 3. Let t=t+1. If t<T, go to step 1, otherwise stop.

Figure 1 A General Approximate Dynamic Programming Algorithm

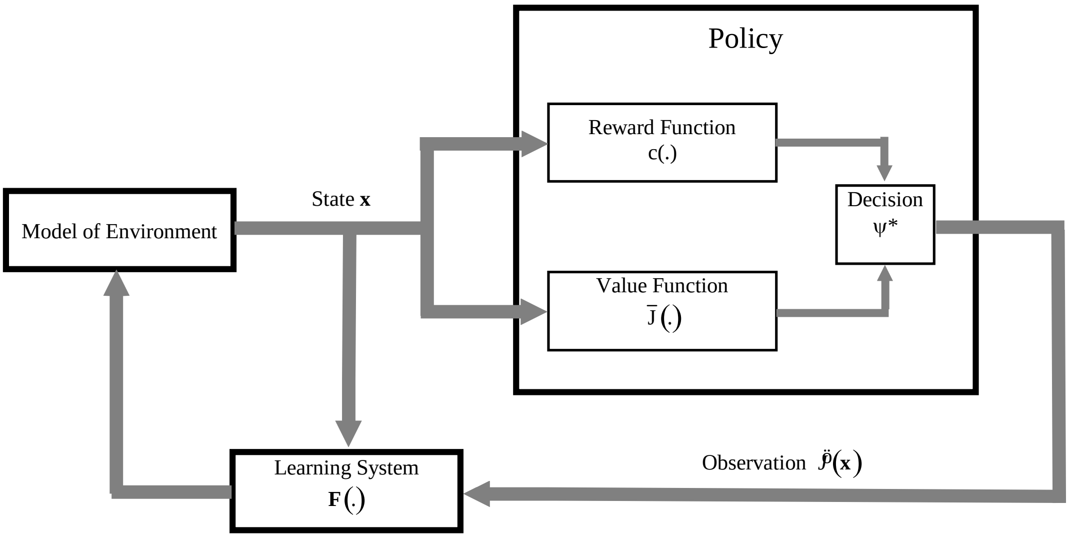


Figure 2 Main Components of a Reinforcement Learning Agent

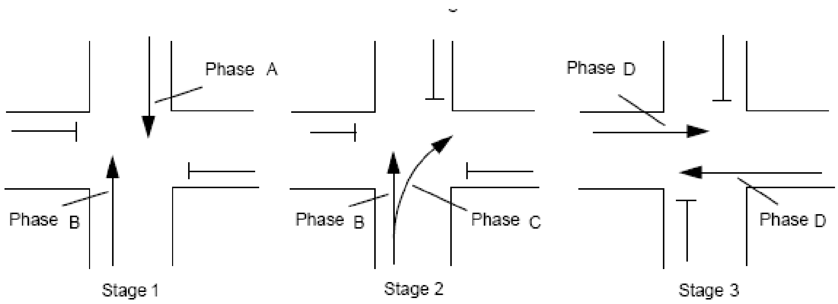


Figure 3 Intersection Layout

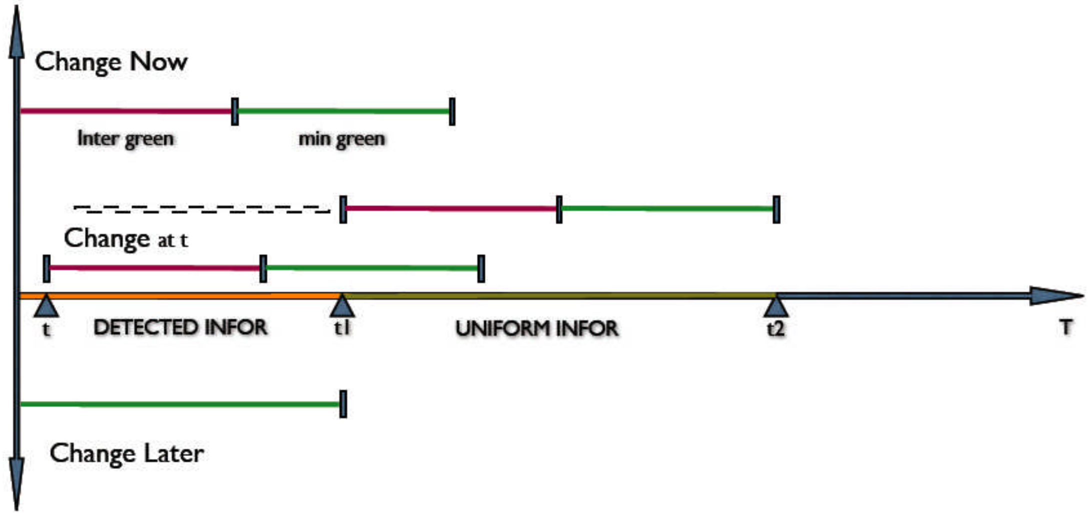


Figure 4 Decision Set for ADP in Signal Control

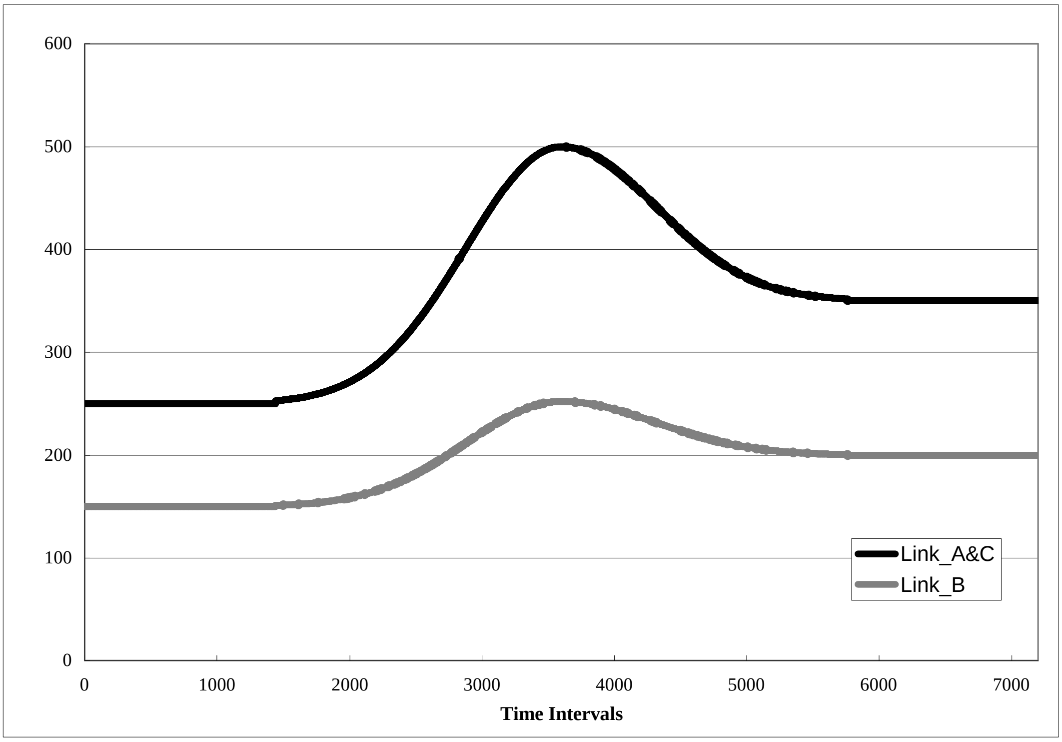


Figure 5 Profile of Traffic

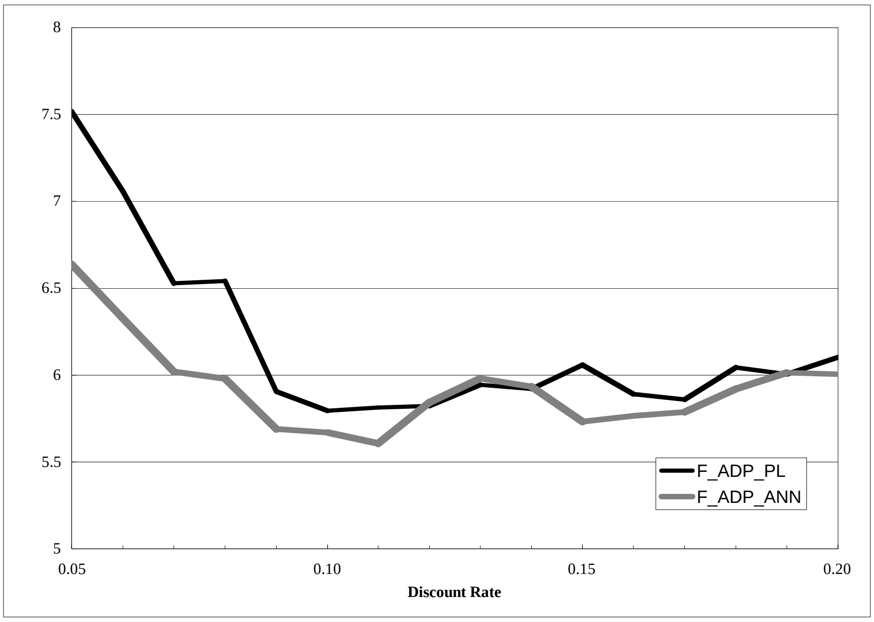


Figure 6 Discount Rates and ADP Performance

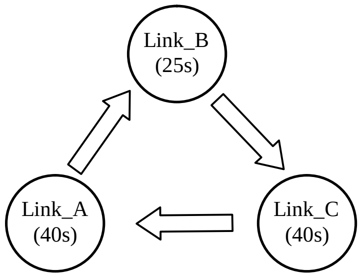
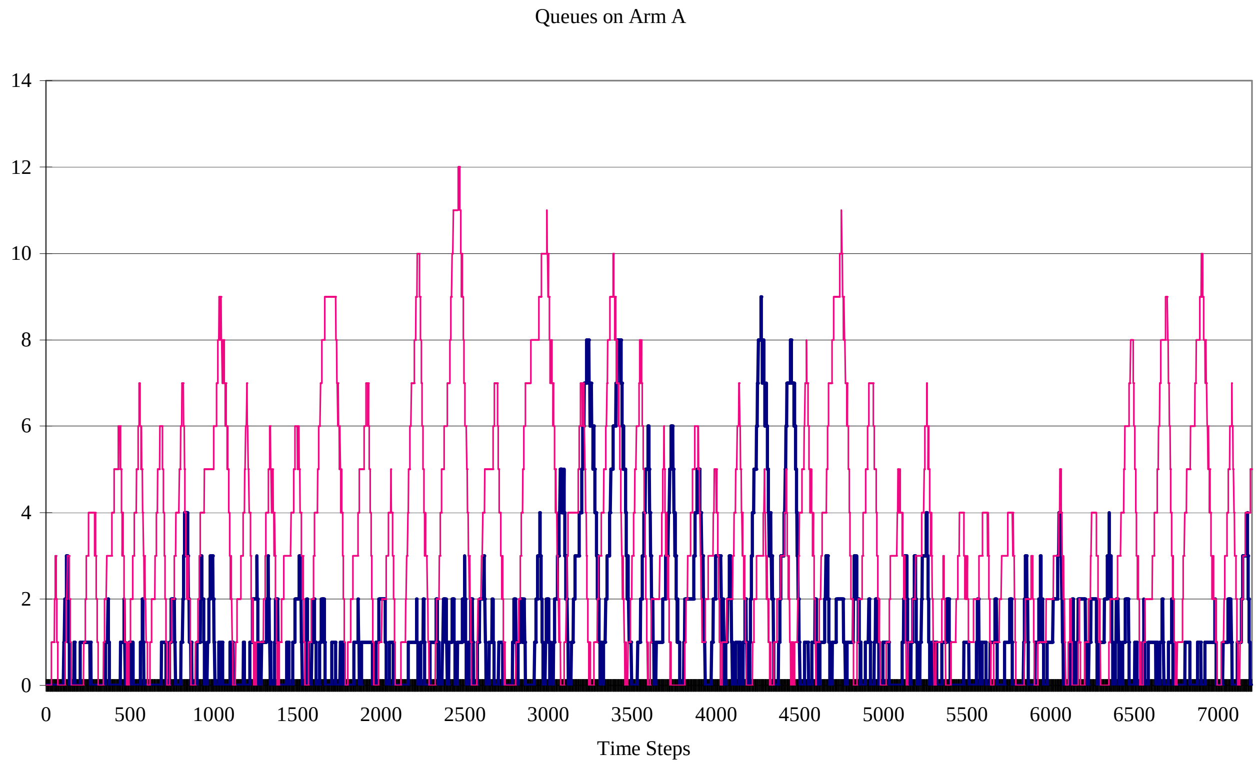


Figure 7 The Best Fixed-time Timing



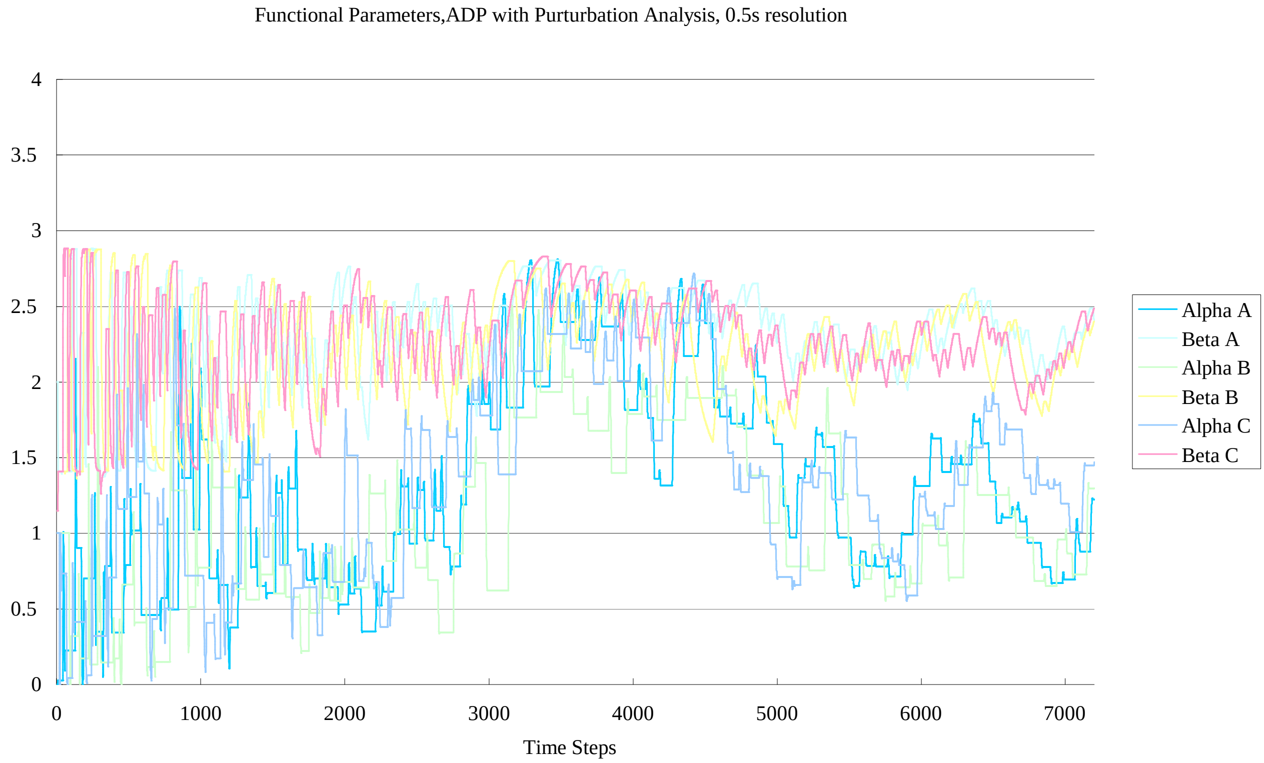

Functional Parameters,ADP with Purturbation Analysis, 0.5s resolution
4
3.5
3
2.5
2
1.5
1
0.5
0
0
1000
2000
3000
4000
5000
6000
7000
Time Steps
Alpha A
Beta A
Alpha B
Beta B
Alpha C
Beta C

Figure 9

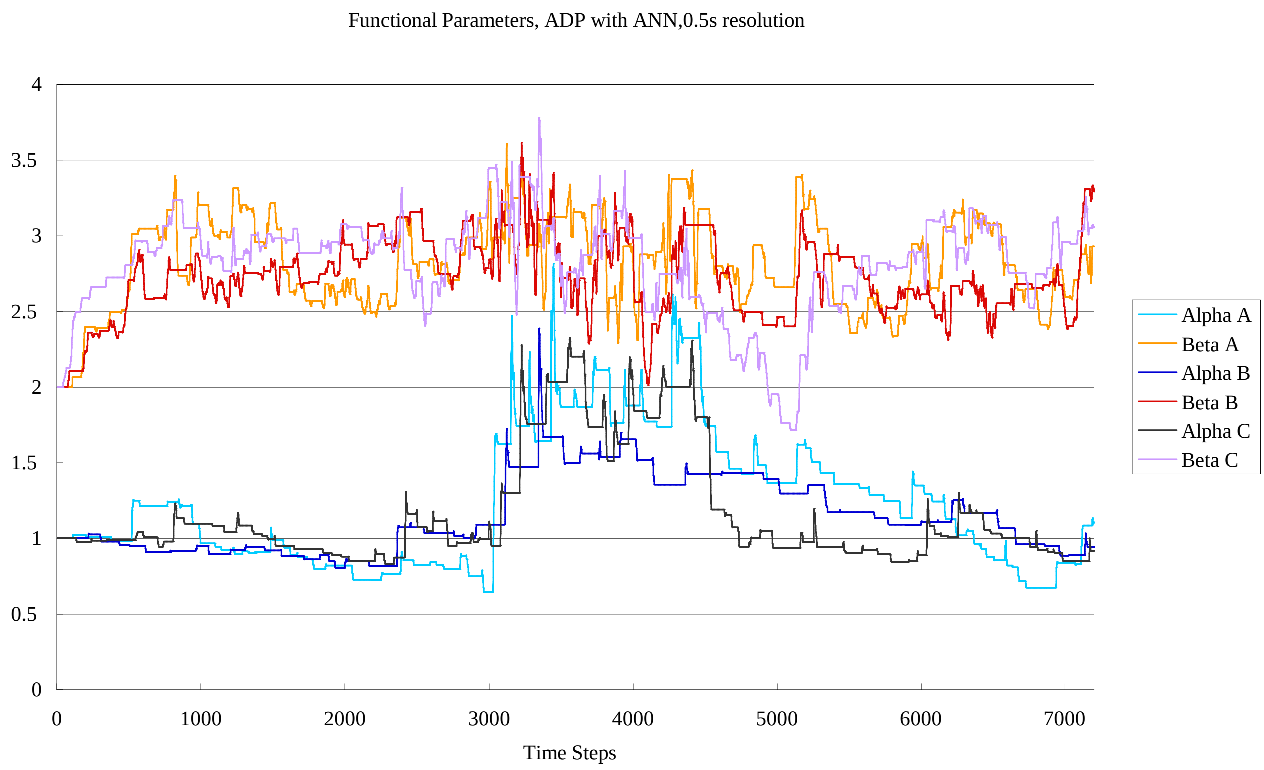

Functional Parameters, ADP with ANN,0.5s resolution
4
3.5
3
2.5
2
1.5
1
0.5
0
0
1000
2000
3000
4000
5000
6000
7000
Time Steps
Alpha A
Beta A
Alpha B
Beta B
Alpha C
Beta C

Figure 10