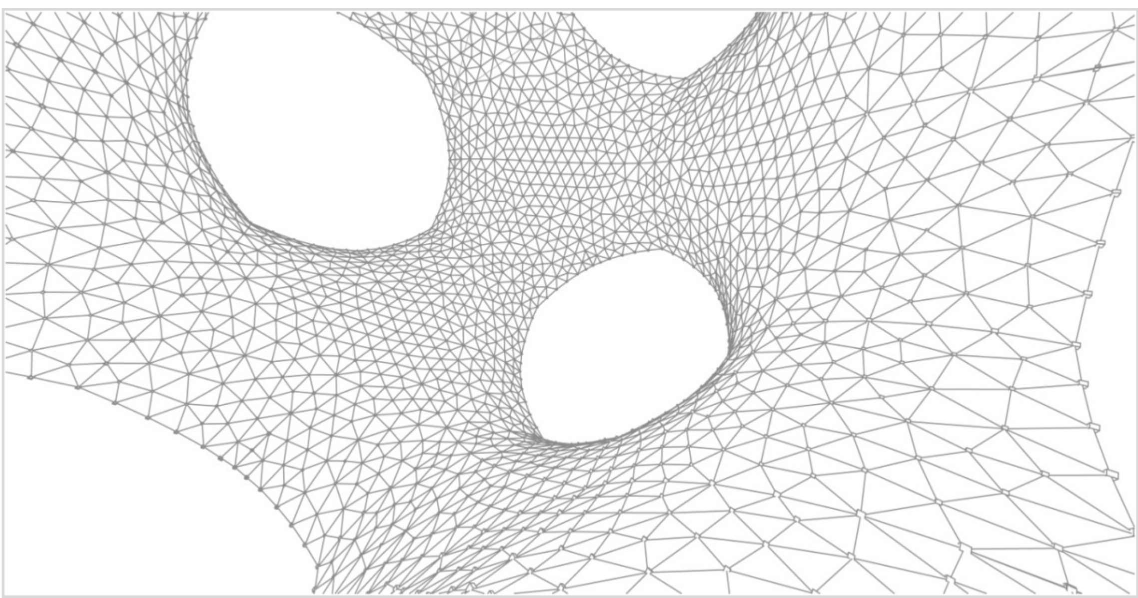
Minimal surfaces as self-organizing systems

A particle-spring system simulation for generating triply periodic minimal surface tensegrity structures

Vlad Tenu



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I, Vlad Tenu, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Signature:

Abstract

Minimal surfaces have gradually been translated from the field of mathematics into the architectural design research due to their remarkable geometric properties. The simulations of soap films or protein folding are only some of the many applications in various fields, while architecture and engineering have been applying them for tensile roof structures since the early 1960's. The research question relates to the problem of creating a computational generative tool which simulates a parametric minimal surface with a non-standard method using a self-organizing particle-spring system and achieving a controlled level of subdivision modularity of the surface for fabrication. The process is iterative and it has a different approach from a standard computational method such as the dynamic relaxation algorithm, because it does not start with a pre-given topology and it consists of two simultaneous processes: the one that is defining the minimal surface geometry and the one that is creating the subdivision of the surface to control the basic constituent modules. The method is tested on the case of triply periodic minimal surfaces and from the fabrication point of view it is focusing on defining a tensegrity modular system composed of interlocked rings with a unique dimension or a pre-given set of standard dimensions.

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Contents

List of Illustrations

1. Introduction

The general research question of the study is related to the design problem of minimal surface structures while the aim is to create an algorithmic alternative method for generating minimal surfaces, based on a particle-spring system. The main concept is based on the simulation of a potential tensioned membrane such as a virtual soap film within an pre-given boundary, by using an iterative algorithm which is performing on two directions simultaneously: the generative algorithm for the minimal curvature geometry and the subdivision technique for the triangulation of the surface in order to reach an optimal modular configuration for making possible the fabrication of the generated structure.

1.1. The Architectural Problem and Thesis Aims

The research is focused on both the form-finding and the fabrication related to the geometric properties of minimal surfaces, while the question that emerges is how the translation from the virtual three-dimensional space to the built artifact could be embodied into a computational process which would also solve the issues within the fabrication framework.

The study is of a common interest with a similar series of projects of **Loop.pH**, a design studio which is specialized in the conception, construction and fabrication of environmentally responsive textiles for the built environment. They have been developing cellular fabrication systems for a set of minimal surface objects, involving weaving to form complex patterns or creating tensegrity structures composed of interlocked fiber-glass rings.

Figure 1. Metabolic Media, Loop.pH ,London , 2008 , (Source: < ttp://loop.ph/bin/view/Loop/MetabolicMedia>)

Metabolic Media is a structure based on a modular tensegrity system composed of interlocked identical fibre-glass rings which defines a complex geometry with minimal surface characteristics forming a rounded dodecahedral toroid. The fabrication process is a case of a bottom-up cellular manufacturing process, similar to a textile hyperbolic surface crochet but at a larger scale.

The objective is to create a computational framework for developing a similar system for testing and fabrication of several minimal surface geometries of various sizes and degrees of complexity and controlling the fabrication method of such an assembly with one or multiple size modular components. The study is focused on the case of triply periodic minimal surfaces and on a similar bottom-up approach for both sides of the problem. Accordingly, the aim is to investigate the possibility of using a particle-spring system which would create a minimal surface while the springs could serve as the main geometrical base for defining the manufacturing modular pieces.

From the point of view of the generative process, the study involves a form-finding strategy considering the properties of infinitely periodic minimal surfaces. The problem is reduced to creating the basic minimal surface module, within a basic kaleidoscopic cell (usually a tetrahedron or a prism) characteristic to the specific potential geometric configuration. The concept is to simulate a tensioned membrane, defined by the particles and the springs and bounded by the faces of the kaleidoscopic cell. By defining a system of constraints and specific attributes to the particles, the hypothesis is that the behavior of the system would suggest a minimal surface membrane, a virtual soap film between the faces of the basic tetrahedron.

While the iterations of the algorithm are performing a self-organizing process of the particles so that they could define a surface with a local area curvature minimization, the springs are controlled by a Delaunay triangulation algorithm so that they reach an efficient topology made of linear elements with one stable length or a specified set of standard lengths in order to define standard sizes as fabrication components.

The investigation is meant to define a different approach to the problem of minimal surfaces computational simulation. While the numerical methods deal with the differential geometry algorithms, dynamic relaxation might be considered a similar method by generalizing the particle-spring characteristics and the algorithmic similarities in the case of starting with a pre-given topology. In this study, however, the algorithm does not start with an existing topology and the accent is put on the iterative growth process, controlled by the geometrical constraints of the growth environment (the kaleidoscopic cell) and the dynamic tessellation of the surface.

1.2. Computational design

Minimal surfaces are a specific example of complex geometries with diverse range of applications in fields such as nanotechnology or molecular engineering and are a primary element in the physical simulation of compound polymers, black holes, soap films or protein folding. Computation is playing an essential role in the simulation and modeling process of such complex phenomena. The methods used are developing different approaches coming from the mathematical field such as numerical methods, differential geometry or other algorithms such as shortest path segmentation or dynamic relaxation.

Dynamic relaxation is the most frequent method used in structural engineering, regarding the computation of network structures for optimizing predefined topologies. The algorithm is based on applying external forces to a system of nodes connected by linear elements and it performs an iterative relaxation process which modifies the position of the nodes in order to minimize the potential energy of the system. The disadvantage of the method from the point of view of the modularity would be that it does not allow a reconfiguration of the basic topology.

In contrast with the dynamic relaxation, the proposed method for creating a minimal surface is the one of using a form-finding generative algorithm which is based on particle-spring system dynamics. It does not start with a predefined topology as it is based on a growth process, by adding nodes according to the geometrical constraints and the subdivision rules applied. The aim is to find an optimized topology for fabrication while generating the minimal surface.

Particle systems are a famous technique in computer graphics, being used for realistic physical simulations of natural complex phenomena such as fire, smoke, clouds, water or complex behaviors of the dynamics and modeling of clothes, hair, fur or grass. In the case of particle-spring systems, the particles are connected with virtual springs which generate the forces to be applied to the particles in order to achieve an elastic state of equilibrium of the system, according to the masses of the particles, the lengths of the springs or external forces such as gravity.

The concept of self-organization is very often related to particle systems being a model of complex behavior found in the natural environment. Various examples such as the Beloussov-Zhabotinsky chemical reaction or flocking behavior of birds are implying the notion of self-organization and are exhibiting interesting properties leading to "*autonomous emergence and maintenance of structural order*", according to Michael Wheeler. (Wheeler, 2005)

1.3. Tensegrity Structures

Tensegrity structures are a special category of modular systems composed of basic structural cells that consist of compressed solid struts and tensioned cables. Because of their lightness and very interesting visual complexity they are very much appreciated and used by architects and designers. The term belongs to Richard Buckminster Fuller and comes from tensional integrity, a definition which characterizes the best the phenomenon within the interaction between the components. According to Antony Pugh, "*A tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space*." (Burckhardt, 2008)

Extrapolating from the definition made by Antony Pugh, tensegrity structures could be identified under many forms in the environment around us. For example, the skin of a balloon is considered the tensile component, while the air inside it is the compressed element. The same principle is to be found in nature, the cells of the green plants behaving in a similar way to the balloons while providing structural integrity. In a similar way, other human artifacts could be identified as systems composed of tensioned and compressed elements working together to reach a state of structural stability "*even if no external load is present*". (Burckhardt, 2008) A relevant architectural example from this point of view could be the pre-stressed concrete in which the internal steel armature is in tension while the concrete is in compression.

1.4. Structure of the Thesis

Within the next section of the thesis, a more detailed description of related subjects such as the triply periodic minimal surfaces and tensegrity structures will be presented. In the following section 3, the implemented method will be presented together with the parallel alternative tests. Section 4 will describe the outcome of the various tests and different implementations of the algorithm while in section 5 an analysis and a critical assessment of the results will be performed. The overall conclusion will cover also the future possible directions in the future development.

2. Background

2.1. Minimal surfaces

The study of minimal surfaces has experienced a highly increased level of interest in the latest decades due to questions such as: *"What are the possible shapes of natural objects in equilibrium and why? When a closed wire is dipped into a soap solution and afterward raised up from the solution, the surface spanning the wire is a soap film. The soap film is in a state of equilibrium. What are the possible shapes of soap films and why? Or why is DNA like a double spiral staircase? 'What' and 'why' are questions that, when answered, help us understand the world we live in. The answer to any question about the shape of natural objects is bound to involve mathematics."* (Colding and Minicozzi, 2005)

A minimal surface is a geometry concept which refers to a surface with zero mean curvature that has the property of being locally area-minimizing, in a sense of having the smallest area within a given boundary (Brakke). Soap films are classical examples of minimal surfaces. The special properties of the minimal surfaces were used in various fields from nanotechnology to architecture, leading to very interesting applications such as the light roof tensile structures in the case of Frei Otto, for which he used physical models and soap films as a form-finding tool.

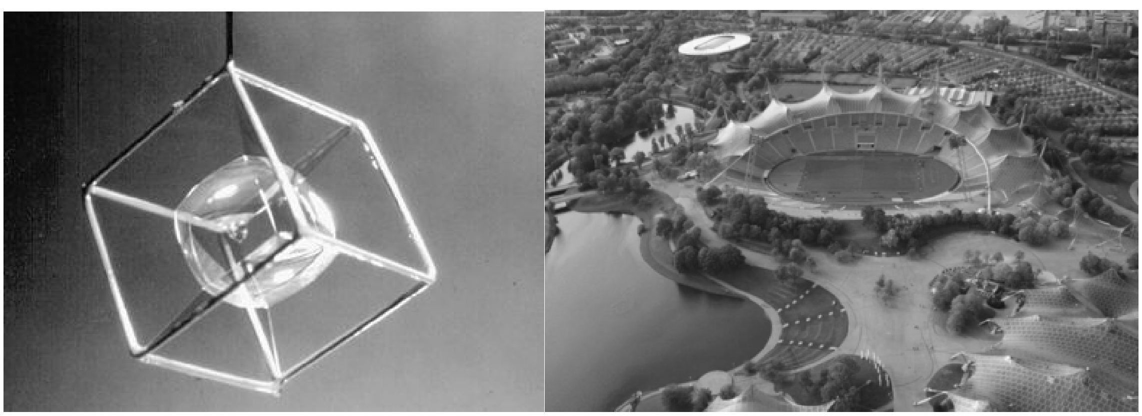


Figure 2. Left: Soap Bubble within a metallic frame (Source: <www.emis.de/.../NNJ/conferences/N1996-Emmer.html>) Right: Frei Otto's Munich Olympic Stadium 1972(Source: live.cgcu.net/editions/livic/1596>)

Starting with catenoids or helicoids which were the first ones to be identified around the beginning of the 18^{th} century, a very important moment in the research related to minimal surfaces was the discovery of the Costa Surface by Celso Costa in 1982, followed by different new classes of surfaces obtained by a series of rotational symmetries. Besides the most famous types such as the Enneper Surface or the Riemann Surface, the minimal surfaces can have a large amount of different configurations, reaching very high levels of complexity by forming infinite repetitive crystalline structures such as the doubly or triply periodic minimal surfaces.

Infinite periodic minimal surfaces started being investigated after the first published example described by H.A. Schwarz in 1865, in the same time that Riemann and Weierstrass were independently publishing memoirs on investigating the same typology of surfaces. They were called surfaces even if they presented symmetries related to the diamond crystal structures. (Schoen, 1970) Their geometry is based on a potential infinite degree of repetition of a surface region module based on an absolute symmetry in three dimensions.

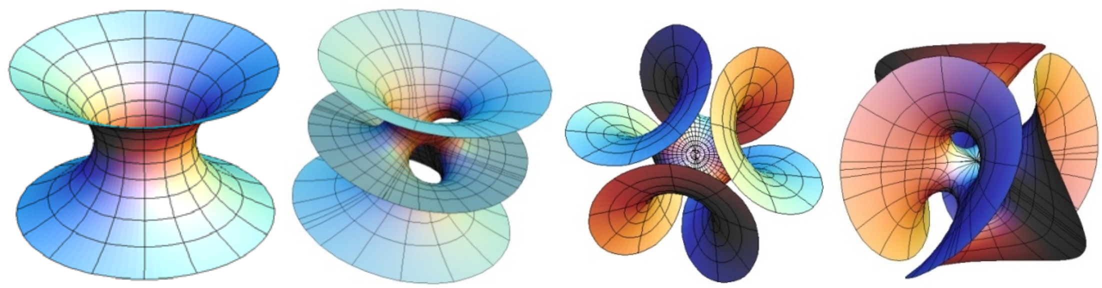


Figure 3. Minimal Surfaces : Catenoid , Costa, Ennepeer , Chen Gackstatter

(Source: < http://xahlee.org/surface/gallery_m.html>)

Kaleidoscopic cells are fundamental regions for groups of reflection in three dimensions for generating triply periodic minimal surfaces. They are "*convex polyhedra which provide plane boundaries for finite minimal surfaces which can be replicated by reflection to yield infinite periodic minimal surfaces without self-intersection."* (Schoen, 1970) There are two categories among the seven types of standard kaleidoscopic cells, represented by prisms and tetrahedral geometries. Together with the main rectangular parallelepiped, there are the triangular prisms and the tetragonal disphenoid, the trirectangular tetrahedron and the quadrirectangular tetrahedron. *"Many of the triply periodic minimal surfaces have embedded straight lines"* which are to be identified as symmetry axes (type C2) of 180 degree rotational symmetries. (Brakke)

One of the most famous triply periodic minimal surfaces, is the ***Shwarz P Surface*** which *"divides the space in two congruent labyrinths"* (Brakke) and it has a fundamental region which is the 48th part of a cube, a trirectangular tetrahedron. The ***Complementary P Surface Family*** is a set of surfaces complementary to the Shwarz P Surface. They all have the same kaleidoscopic cell as a fundamental region. Among them, ***Neovius' Surface*** is another cell with 'necks' towards the middle of every edge o the cube and ***Schoen's C Surfaces*** are examples of a similar process but with a 96-fold symmetry (having 96 copies of the same fundamental region). Some of the most interesting triply periodic minimal surfaces families having a *quadrirectangular tetrahedron* as a kaleidoscopic cell are the **Batwing Family**, with a correspondent ***Schoen's Batwing Surface***, which has the fundamental region defined as a half of the 48th part of a cube, the ***Disphenoid families*** or even **Hybrid Surfaces.**

From the point of view of the topology the surfaces could gain '*handles*' without losing their properties. The concept of a handle could be imagined as a sphere which becomes a torus after having attached one. Accordingly, the surfaces could be characterized by their *genus*, which reveals the number of handles that have been attached to the surface. (Colding and Minicozzi, 2005)

"*Minimal surfaces may be also characterized as surfaces of minimal surface area for given boundary conditions*", (Weisstein). From the mathematical point of view, the process of finding a minimal surface within a boundary with specific constraints is determined by the calculus of variations. The mathematical model of minimal surfaces is based on differential geometries and is strictly related to the standard computational method of visualization used in most of the cases. Dynamic relaxation is another method which is based on an iterative process that tends to an equilibrium state after a "pseudo-dynamic process in time". (Lewis, 2003)

Computational methods

Dynamic Relaxation started being developed as a numeric, differential technique. "*The method relies on a discretized continuum in which the mass of the structure is assumed to be concentrated(lumped) at given points(nodes) on the surface. The system of lumped masses oscillates about the equilibrium position under the influence of 'damping'. The iterative scheme reflects a process, in which static equilibrium of the system is achieved by simulating a pseudo-dynamic process in time.*" (Lewis, 2003)The use of damping differentiates various types of methods which include parameters such as viscous behavior or kinetic energy that help in the simulation of different categories of tensioned cables or membrane structures. It is a method that starting with a predefined topology, leads to a formation of a minimal surface, within the constraining boundaries or existing forces.

The most famous application is the Great Court Roof of the British Museum, designed by Foster and Partners Architects in collaboration with the Buro Happold, the structural engineers. Computation was an essential element in the design of the steel and glass roof, in order to define a *"spiraling geometry"* using a series of numerical and analytic methods, in order to *"satisfy architectural, structural and glazing constraints."* (Williams, 2001)

Ken Brakke's ***Surface Evolver*** is an interactive modeling software used for the modeling of liquid surfaces which are shaped by various forces and constraints. It is designed for simulating soap bubbles, foams, liquid solder, capillary shapes, and other liquid surfaces which would be shaped by reaching minimum energy from the point of view of the surface tension. The resulted surfaces are represented as triangular meshes in order to control the potential complicated topologies or topological changes, such as foam coarsening or quasi-static flow. It has a command prompt interface with interactive three-dimensional graphics. (Brakke)

It is an example of great relevance to the subject of the thesis for its different approach in defining standard or triply periodic minimal surfaces. The algorithm behind the software is taking the input of the user which defines an initial surface which is implemented as a "*simplicial complex. The program evolves the surface toward minimal energy by a gradient descent method. The aim can be to find a minimal energy surface, or to model the process of evolution by mean curvature, […] for surface tension energy in the context of varifolds and geometric measure theory."* (Brakke)

Evolver can simulate the energy as a combination of various factors such as surface tension, gravitation, squared mean curvature, volume or boundary constraints, knot energies, user defined surface integrals or prescribed mean curvature. The basic concepts are related to an iterative algorithm which simulates an evolution process. Starting with a basic geometry as an assembly of vertices, edges and faces and the system of constraints and forces, a total energy is calculated. The process of minimizing this energy is actually the evolution. One iteration defines on evolution step. The algorithm calculates *"the force on each vertex from the gradient of the total energy of the surface as a function of the position of that vertex. The force gives the direction of motion. Second, the force is made to conform to whatever constraints are applicable. Third, the actual motion is found by multiplying the force by a global scale factor."* (Brakke)

Several studies on the tessellation of surfaces and on the development of discrete differential geometry methods were being made in accordance to the increasing architectural design need of digital fabrication of free-form structures and surfaces. It is a principle according to for a given geometry there can be suggested different discretizations, while the question is which one is the most efficient for its application in the architectural field.

A relevant example related to the minimal surfaces efficient modular tessellation is a mathematical study made by Alexander I. Bobenco, a study which solves issues related to the curvature and the integrability for polyhedral surfaces. However, being related to the investigation subject of the present thesis research, it is a different approach in the sense of not aiming to achieve a polyhedral tessellation of a pre-given surface leading to non-equal polygonal quadrilaterals, which could be considered a top-down method, but to generate a surface with a finite number of identical triangular subdivisions as a bottom-up process.

Figure 4 . Discrete geometry of a *symmetric and asymmetric Schwarz P-surface(Sechelmann, Bucking) (Source: www.math.tu-berlin.de/~*bobenko*/Rio.pdf>)*

2.2. Tensegrity structures

"How to obtain 'non-column' space rationally and beautifully has been a dream of mankind for years. It is always our ambition to develop elegant, fascinating, unbelievable and even 'unreasonable' and 'impossible' forms. There seems no limit to architectural art. Meanwhile, a designer has to face a combination of different objectives and constraints such as safety, costs, aesthetics, manufacturing and functional requirements. Normally, rationality in structural point of view is required." (Bing, 2004)

Space structures are usually appreciated by architects and engineers for their lightness, visual aesthetic complexity or flexibility together with the efficiency in developing models for construction of architectural elements such as roofs or facades. The fascinating properties of the tensegrity structures are moving beyond the aesthetic reasons and give the possibility of building very-large scale structures, theoretically, in a case of a domical or spherical configuration, the size of a city. The economical aspect is a major one, as in a tensegrity structure the tensile members predominate while the compressed elements are reduced to the minimum. (Burkhardt, 2008)

The word tensegrity is a contraction of two words: "tensile" and "integrity" and it is attributed to Buckminster Fuller. It is meant to define the category of lightweight three-dimensional structures which are composed of both compressed rigid linear elements and tensioned cables and have as a characteristic the fact that the stability of the structure lies in the tension. (Skelton and Oliveira, 2009)

Following their discovery by sculptor Kenneth Snelson, while he was a student of Buckminster Fuller, in the same time the French architect Emmerich was developing similar structural models independently, several patents related to various aspects of tensegrity structures were attributed to all three of them. While Fuller and Emmerich were focusing on the development of architectural spherical and domical structures, Snelson was interested primarily in the artistic investigation and application of the principles. Fascinating examples of pioneering tensegrity structures were produced by all of them, such as Kenneth Snelson 's 'Needle Tower' at the Hirshhorn Museum of Modern Art, Washington,DC .

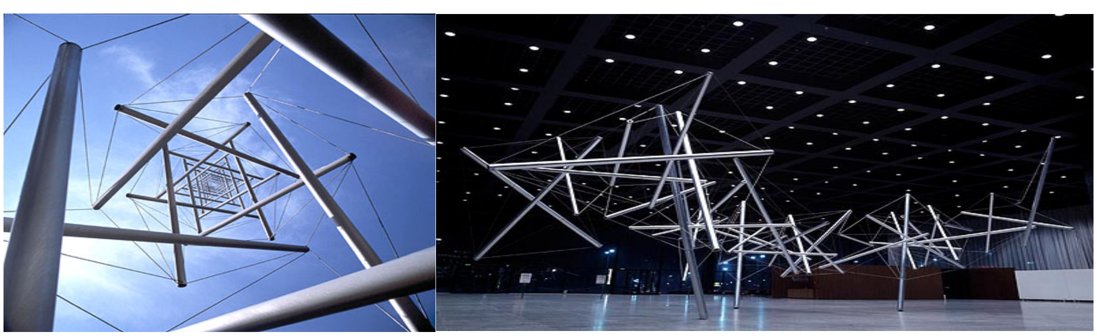


Figure 5. Kenneth Snelson's Needle Tower and a tensegrity sculpture (Source: < http://www.kennethsnelson.net/sculpture/outdoor/images/twer_vertical.jpg>, < http://swamiobryans.blogspot.com/2009/03/message-from-swami.html>)

Modularity is one of the great attributes of tensegrity structures which can reach a high level of structural complexity starting from the very basic modules, such as Snelson's X module, or the simple prismatic or the more advanced polyhedral cells called simplexes. At a lower level, the simplexes are made of basic struts and cables with predefined dimensional modules. Thus, the tensegrity structures are not reduced to only cable-strut systems. According to Burkhardt, the analogies to the balloons or plants are to be considered when thinking of any system in which tension and compression work together to reach stability within a structural system. Following this idea, other types of tensioned elements such as membranes or curved struts were successfully introduced within tensegrity systems. (Burkhardt, 2008)

A special case of tensegrity structures is the one of interlocked, connected or woven rings in which the basic elements, the rings are undertaking tension and compression forces simultaneously. Due to the continuous nature of material distribution within a ring, the investigation toward the distribution of forces and the structural properties or behavior of such a system becomes one of great potential. They could be considered as single base modules or as in three-dimensional assemblies forming tetrahedral, cubical or polyhedral geometries. Sculptor Bo Atkinson has made several studies on ring reinforced concrete, called *"ringforcements"*(Atkinson, 2000), and its application on several configuration of both regular and freeform structural elements. The study was focused on both connected and interlocked metallic strings and included several structural tests which have proven the high level of potential of such systems.

The use of fiberglass rings, as modules for constructing minimal surface geometries, has been developed extensively by Loop.pH, a design studio with a biomimmetic design approach which simulates self-supporting cellular structures embodied in artistic installations. Based on an analog cell to cell manufacturing process, similar to the crochet method of creating textile fabrics as cellular hyperbolic surfaces, the results are extremely interesting from both the aesthetic and the structural point of view.

3. Methodology

3.1. The concept and the algorithm

Aiming to obtain a simulation of a tensioned membrane by using a bottom-up generative approach in order to create a tool that could construct various types of triply periodic minimal surfaces, the methodology is based on the dynamic behavior of a particle-spring system. The hypothesis is that a particle-spring net which is defining a surface, due to the elastic properties of the springs will tend to behave like an elastic membrane, responding to forces and constraints. Accordingly, because of the elastic properties, it will tend to achieve a minimal surface area between the defining boundaries. The solution to generate infinite triply periodic minimal surfaces relies on establishing the system of constraints or forces that need to be applied in order to satisfy the mean curvature characteristic and the topological configuration of the surface obtained by the reflection of the basic kaleidoscopic cell.

The problem is reduced to creating the basic surface region within the kaleidoscopic cell, which is the basic tetrahedral fundamental region for the group of reflections in three dimensions, after which the surface could be reflected and form the triply periodic minimal surface. The example chosen for illustrating the methodology is the well known *Schwarz P-surface* which has as a kaleidoscopic cell a tri-rectangular tetrahedron which represents the 48th part of a cube. The principle is based on the rules defined by Schwarz in order to construct a periodic minimal surface:

"*1. If part of the boundary of a minimal surface is a straight line, then the reflection across the line, when added to the original surface, makes another minimal surface. 2. If a minimal surface meets a plane at right angles, then the mirror image of the plane, when added to the original surface, also makes a minimal surface.*" (Weisstein)

In order to create a minimal surface within the cell, a network of particles needs to be created which would connect the faces of the tetrahedron. The connection to the faces is realized by a series of new types of particles, limited in their behavior by constraints: fixed particles, particles constrained on the edges of the tetrahedron and particles constrained on the faces of the tetrahedron. The theory leading to generating a minimal surface is based on the principle that tension, acting within a spring between a constrained particle and a free one will tend to reach a minimum length for the spring and tend to become perpendicular to the constraining surface or edge, achieving a minimal distance in relation to it. Considering the bottom-up approach for the whole process, besides the boundary defining constrained ones, the algorithm starts with just one particle. As particles are added, a Delaunay adapted algorithm is optimally triangulating the surface while the boundary particles are maintaining their constrained relationship with the faces and the edges of the tetrahedron. Topologically, the aim is to obtain only one length or a set of standard lengths for the springs as edges of triangles, hence another iterative process is controlling the stable lengths of the springs to adapt to the morphological changes of the surface.

3.2. The particle-spring system

For the purpose of the research, the definition of the particle system was made simple enough to satisfy the premises for the other aspects of the simulation. It is not using mass, acceleration or viscous damping as it is not programmed for a realistic physical dynamic simulation of the particles behavior. Accordingly, every particle is defined by two vectors: position and velocity. The two vectors are updated at every iteration according to the previous instance and a temporary value is stored and calculated as a result of the current position and velocity vectors' sum. It is a method which controls the behavior of the particle by analyzing and confirming the next step to be taken.

The springs are defined by an ideal length which dictates the forces that need to be exerted to each of the particles according to the current length of the connection. In case of a smaller distance between the particles, the spring is in compression, meaning that it will exert a force that repels the particles. If the distance is bigger than the ideal length, the spring would be in tension and thus it needs to exert a force that would pull the particles closer to each other. In case the distance between the particles is equal to the ideal length of the spring it means that the spring is in equilibrium. In order to control the multiple spring connections between particles, each particle accumulates in a temporary force vector all the forces transmitted by the connected springs, which is applied after summing all of them.

3.3. The kaleidoscopic cell

The physical parameters of the space in which the particle system will perform, was created by using a category of vertices and face entities that had to be defined in order to control the boundary of the environment in which the self-organizing particle would simulate a tensioned membrane. The faces have attributes such as the normal on the plane vector or the centroid vector, which would be necessary for identifying the parameters needed for creating the constraints defining the relationship between a particle and a face.

The vertices and the face entities are a very useful element in providing flexibility to the program, for generating, simulating and testing various types of kaleidoscopic cell configurations. By specifying the starting set of constrained particles, attached to the geometries of the basic fundamental regions, the different configurations and tests would be applied also to a variety of minimal surface solutions for the same kaleidoscopic cell.

3.4. Creating constraints

In order to achieve the expected behavior of the system, a set of constraints had to be created for the particles. The constraints are strictly geometric and related to the boundary of the fundamental region, the tri-rectangular tetrahedron. Except for the free particles, which have a total degree of freedom in all three directions of the Euclidean space, a new category of particles needed to be created. The fixed particles would be locked on an initial position, without having the possibility to be moved, but would interact with all the others through spring connections.

In relation with the kaleidoscopic cell, in order to provide the possibility of an optimal displacement for the particles defining the boundaries of the obtained membrane, a category of particles fixed on face was added. Considering the faces as separate entities, dictated by the position of the vertices of the tetrahedron, the principle behind this constraint was based on correcting the trajectory of a particle according to its position relative to the correspondent face. When created, according to the initial position, a specific face constraint would be added to the particle's attributes. With every iteration, the particle's projection on the face would create a vector with a magnitude equivalent to the distance between the particle and the plane, which would be subtracted from the position vector of the particle. This way, according to all the applied spring forces, after the new position of a particle fixed on a face is determined, the new position will be corrected in order to bring the particle back on the specified plane.

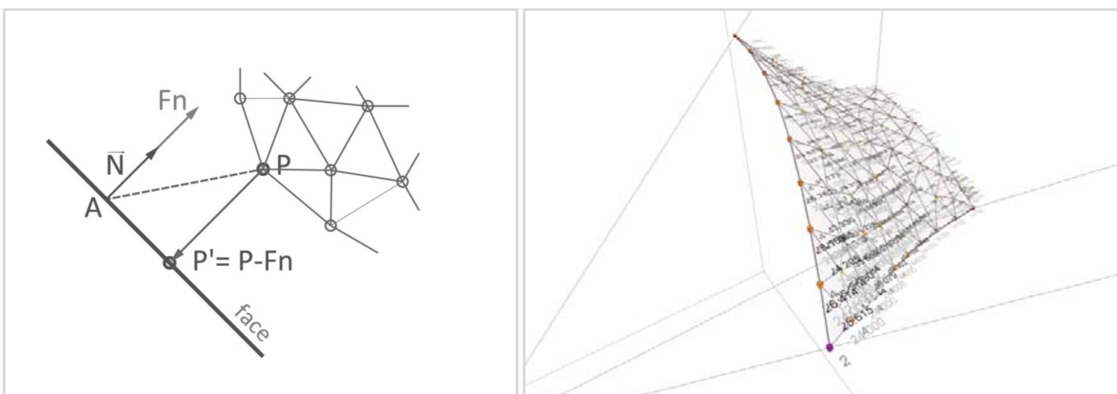


Figure 6. Principle of constraining a particle on a face

In order to achieve the last type of constraints, according to which the particles need to remain fixed on the edges of the tetrahedron, the concept was based on the application of the previous face constraint. When applied simultaneously, a particle kept on two faces will be repositioned on the edge defined by the intersection of the two planes. Accordingly, two correction vectors corresponding to the two faces were subtracted from the temporary position of the particle in order to place it on the edge defined by them for every new step of the algorithm.

3.5. The Delaunay Algorithm

According to the purpose of the self-organizing particle system, the one of forming a surface through spring connections, the problem of finding a criterion for choosing the right particles to be connected was reduced to solving a triangulation algorithm which would allow the springs to create a tessellation which could be mapped on a surface, without creating closed three-dimensional geometries.

Given a number of random particles in space, for connecting them with springs which will generate a triangulated surface mesh, a Delaunay algorithm was used. In order to avoid the formation of tetrahedral or polyhedral geometries, the solution was to adapt a two-dimensional Delaunay algorithm to a three-dimensional configuration, so that, in relation to a base plane, all the points would be projected on it, and the algorithm will map the two-dimensional solution to the three-dimensional configuration. For every iteration, after finding the center of the circumscribed circle of every potential triangular face defined by the projections of three particles on the base plane, the algorithm is checking whether there is another particle projected within the same circle. If no other particle is found then the corresponding particles are connected with springs.

3.6. The modular tessellation

The manufacturing side of the research is focused on generating a geometry composed of single or multiple standard size modular elements. From the algorithmic point of view, the elements are the springs that connect the particles. Having the property of tending to reach a defined standard length, the springs could form a homogenous system in which they achieve a state of equilibrium based on generating a geometry composed of only identical linear elements. Another step in the optimization of the surface could lead to researching the possibility of multiple size linear elements, a case in which the state of equilibrium would be reaching only a fixed number of predefined dimensional instances of the modules.

The tessellation algorithm applied is updating the configuration of the surface according to the current lengths of the springs. In case the existing springs are in tension and they are reaching a length above a pre-defined threshold, new particles are inserted in the system to compensate the tensional energy. In case a spring is in compression, and its length is below a minimal value admitted, one of the particles that are defining the spring is removed from the system. After a number of iterations the algorithm leads to a system in equilibrium, in which case the lengths of the springs have become equivalent throughout the surface. Within a kaleidoscopic cell, because of the geometrical constraints of the boundaries of the surface, the maximum level of accuracy is impossible to achieve, but the tolerances would be acceptable for a flexible manufacturing system. In order to increase the level of precision from the dimensional point of view, the multiple stable length springs would be an alternative to finding the optimal configuration of a series of identical modular elements.

3.7. Reflection

After a series of iterations and after the fundamental surface region has reached a minimal surface configuration, on order to obtain the corresponding triply periodic minimal surface module, a number of 48 reflections are needed, relative to the faces of the tetrahedron, in order to form a cube. The cube itself can be multiplied for creating two or three-dimensional arrays, acting as a cell of the theoretically infinite triply periodic minimal surface which would be obtained.

3.8. Alternative tested methods

Following a concept in a strict correlation to the reflection of the fundamental regions in order to obtain a triply periodic minimal surface, another method for defining a minimal surface geometry with a particle-spring system was tested. The idea of reflection was extrapolated to the idea of creating virtual particles which would be reflected projections of the particles within the boundary of the kaleidoscopic cell, in relation to the faces of the tetrahedron. In this case, the factor which would affect the curvature of the surface would be given by the spring connections between the ‘real’ particles and their reflected instances, the ‘virtual’ ones. The reflected clones would have the property of keeping the exact corresponding position of the original particles, a fact which is maintaining the reflective property of the basic surface.

Two alternatives were tested. One was realized by connecting the particles found close to a face within the tetrahedron to their correspondent projection in relation to the same face. The results were similar to the main method, with a satisfactory behavior from the point of view of the quality of the obtained curvature. The other alternative was to create connections between a particle in the proximity of a face and the clones of all the particles connected through springs with the same particle, in relation to the closest face. The second method proved to be very expensive computationally and not able to work with a considerable number of particles.

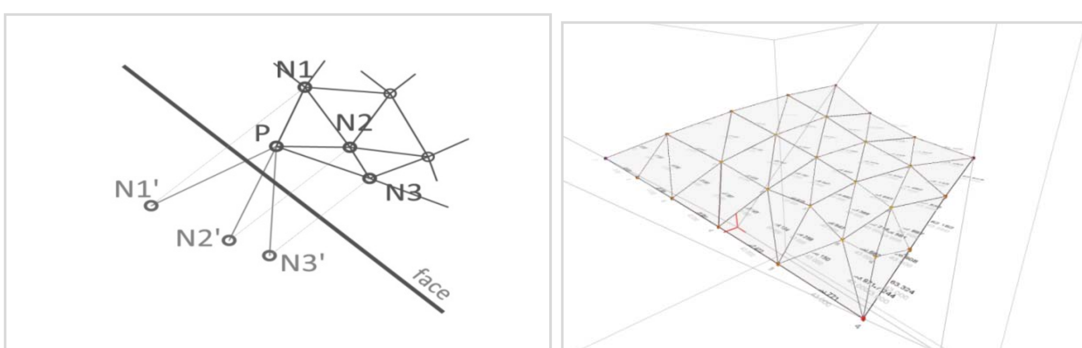


Figure 7. Principle of connection between a particle in the proximity of a face and the ‘virtual’ projections of the neighbors

4. Testing and Results

The testing strategy for the methods involved applying the algorithm to a standard triply periodic minimal surface and to identify the efficiency of the algorithm by analyzing the level of accuracy in generating the geometry and in subdividing the surface for a uniform triangulation given by the distribution of the particles on the surface. The algorithm was initially tested on a classical Schwarz P Surface, following the effect of the basic rules and constraints to the quality of the generated surface.

After identifying the position of the basic surface region within the tetrahedral kaleidoscopic cell, the algorithm would start with one free particle besides the four constrained ones that would define the initial boundary (two fixed and two on edges that would be positioned in the middle of the four defining edges). The strategy also consisted in starting with a considerably big ideal length for the springs so that the triangulation and the subdivision algorithm could perform by affecting gradually the geometrical and the topological configuration of the surface. By decreasing the ideal length new particles and springs are born and while self-organizing, the shape of the surface starts to gain curvature and become smoother according to the degree of tessellation. The results for the illustrated 6 initial different tests involving from 5 to 92 particles that generated from 8 to 240 springs, were showing explicitly the transition between the angular faceted configuration of the surface to a more densely tessellated surface with a smoother curvature.

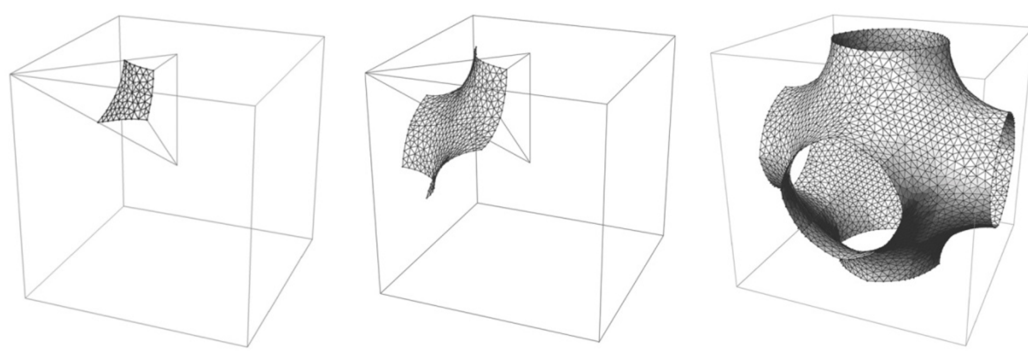


Figure 8. The fundamental region, the result after 6 reflections and the complete module of the Schwarz P Surface

The gradual process of generating the surface, given by the decrease of the ideal length for the springs, is based on a correlation between the value of the ideal length and the length of all springs. If a disequilibrium occurs, particles are added or removed from the system in order to rearrange the connections between them. This correlation between the linear decrease of the value of the ideal length of the springs and the increase in number of the particles and the connections between them is illustrated in the graph below. The performance of the algorithm is linear from this point of view, considering that a small decrease of the ideal length creates a new generation of particles and the corresponding springs would tend to reach a state of equilibrium through their dynamic behavior.

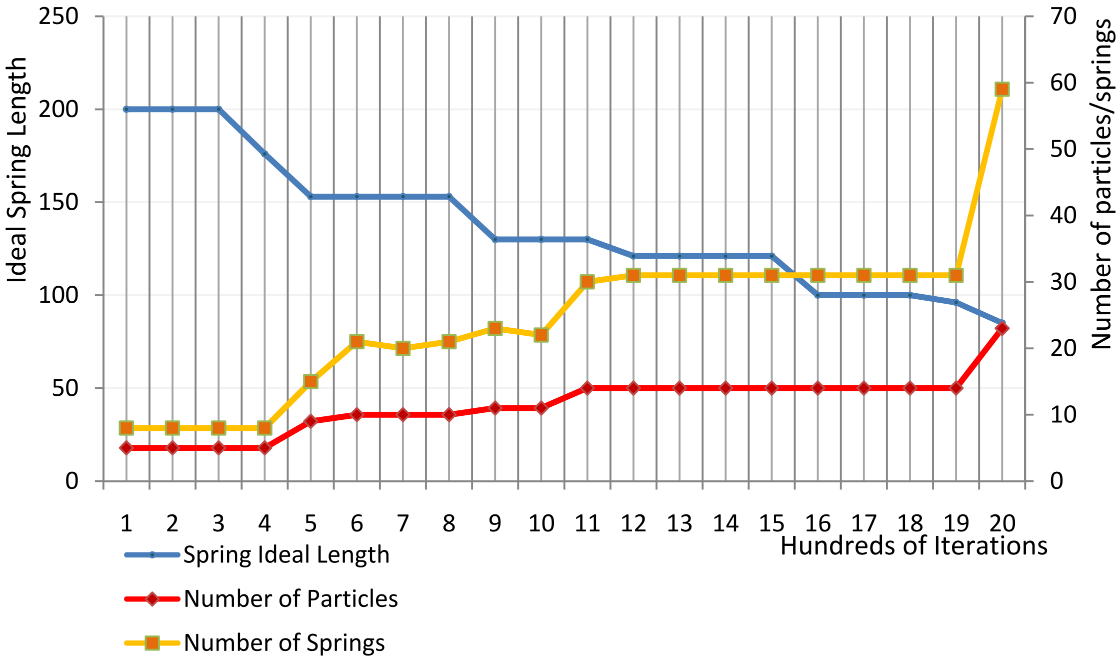


Figure 9. Graph illustrating the increase in number of the particles in relation with the decrease of the Ideal Spring Length

From the point of view of the equilibrium of the system, after several runs, the conclusion was that there are some more stable configurations than others in relation to the number of particles and the ideal length. It is a phenomenon related to the lengths of the springs and the dimensions of the boundary they need to fill in. The self-organizing process, with a strong dynamic behavior caused loss of stability in some intermediary instances which was instantly solved by decreasing the ideal length of the springs, hence increasing the tension within the surface. Certain combinations of spring lengths and number of particles are always stable, *usually* in the initial steps, while the ideal spring length is big enough. As it gets smaller and the number of the particles gets bigger during the intermediary steps, the dynamism of the system is increased but after a set of iterations the surface gets more stable in the end.

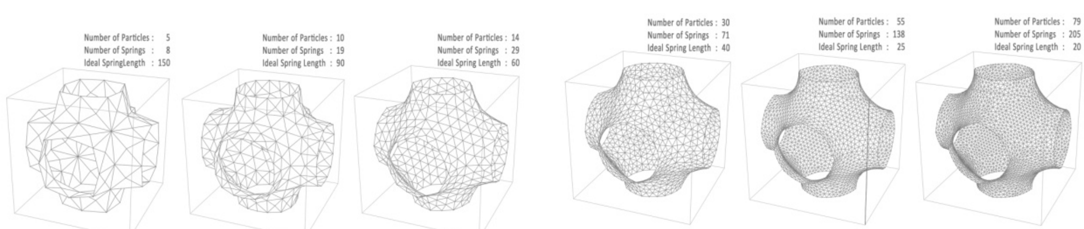


Figure 10. Different degrees of tessellation of the Schwarz P Surface according to the number of particles

4.1. Geometrical analysis

From the overall geometrical point of view, before the reflection of the fundamental regions, the result was satisfactory, showing that the main hypothesis was proved in a sense that the network of particles, due to the springs between them, and the geometrical constrains leaded to an apparent minimization of the mean curvature of the surface behaving as expected in order to generate the basic region of a Schwarz P Surface.

After the reflection, by looking more into detail at the obtained curvatures in relation to the faces of the cube and the relationship between the mirrored instances of the surface, there was a certain deviation that was noticed which was influencing the continuity of the surface. In order to evaluate the obtained curvature, the analysis was made on the planar circles formed on every face of the cube. In accordance to the change in the number of particles, even if the curvature became smoother, the deviation from the circular boundary that should have been obtained on every face of the containing cube was consistent enough to get visible as a small break of the surface continuity after the reflection. Theoretically, the perfect curvature should be obtained by an infinite number of particles, which would lead to an indefinite number of segments of the circle. Following this idea, in accordance with an increased number of particles, a smaller curvature deviation was expected. The hypothesis is that the more particles the surface has, the more accurate the curvature of the surface would be.

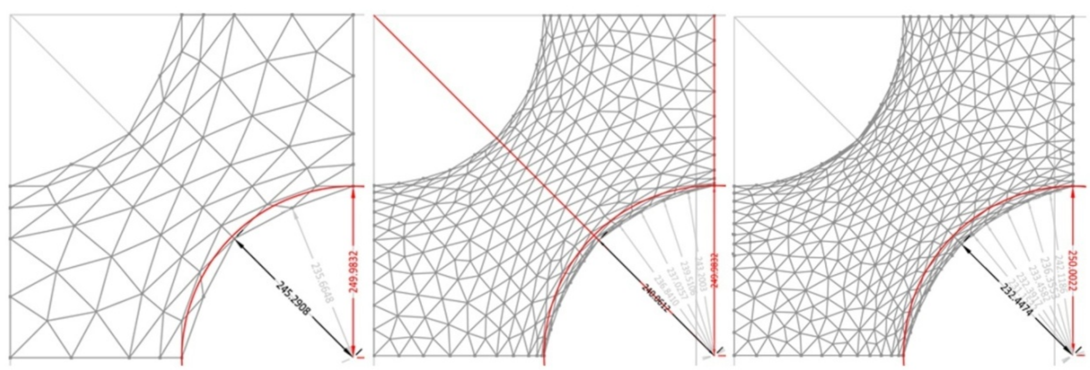


Figure 11. Radius deviation situations on the face of the cube

For a more accurate evaluation of the curvature of the surface, a deeper analysis of the phenomenon was initiated. The average of the measurements of the radii deviations were mapped onto a graph in relation to the increase of the number of particles. As the number of particles was increased the deviation from the main curvature oscillated, but tending to reach a lower value, according to the graph below.

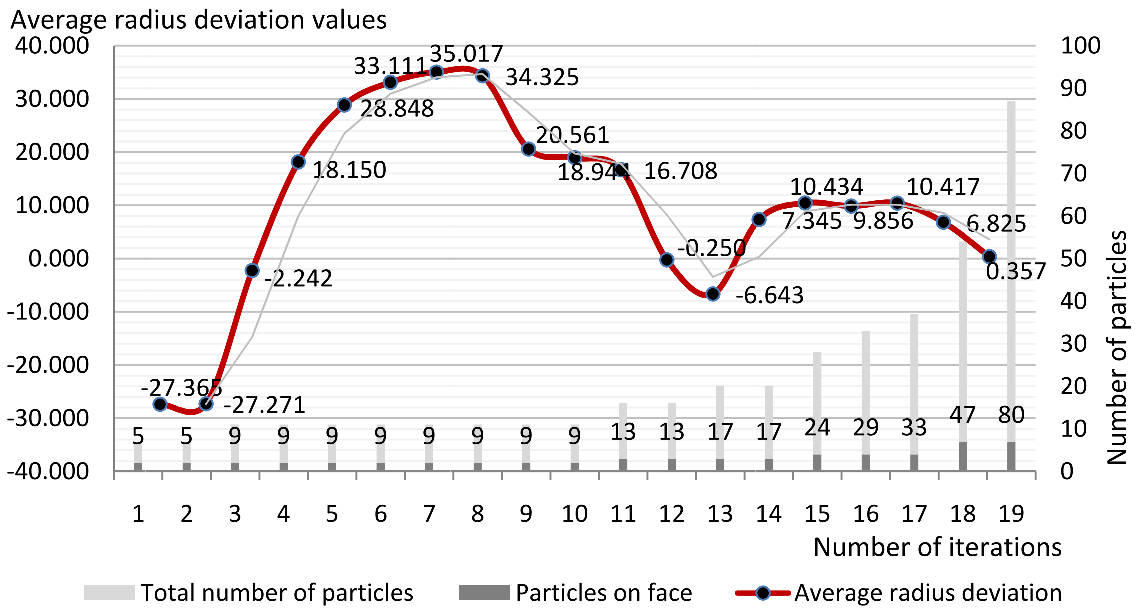


Figure 12. ***The average deviation from the radius of the circle on the face of the cube.***

The other factor which influences the curvature of the surface is given by the properties of the springs. According to their strength, the tension within the surface could vary and affect the overall curvature. In order to obtain a minimal surface, the ratio between tension and compression within the springs should be reduced to a minimum, but balanced towards the tensional factor. Simulating the tension requires mainly attraction within the springs, or stronger attraction then repulsion, as the compression will break the continuity of the surface by generating unwanted extrusions and tetrahedral geometries which would definitely change the appearance of the resulted geometry.

The relation between attraction and repulsion was given by a coefficient k, meant to increase the value of the attraction coefficient active within the spring's force. The tested values were 1, 10 and 100 which leaded to the conclusion that the highest value was corresponding to the higher level of stability from the point of view of the dynamics, but with high oscillations and bigger absolute values for the average deviation from the radius of the circle on the face of the cube.

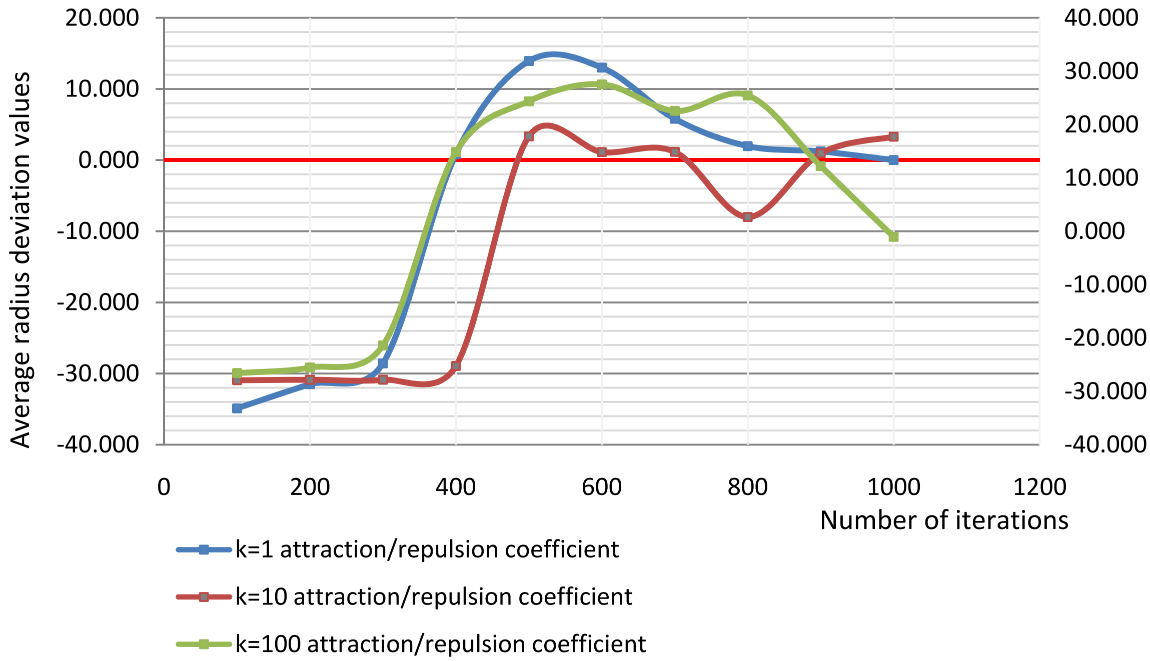


Figure 13. ***The average radius deviation values for the different spring conditions***

4.2. Topological analysis

From the topological point of view, the efficiency of the algorithm could be assessed by analyzing the tessellation of the surface related to the uniform distribution of particles and to their correspondent valence (number of neighbors) and by the level of accuracy in reaching the ideal lengths of the springs within the triangulation process. The valence distribution within the system's particles is controlled by the Delaunay algorithm which is constraining the number of neighbors from 2 to 7 or potentially, as in very few cases, 8 neighbors.

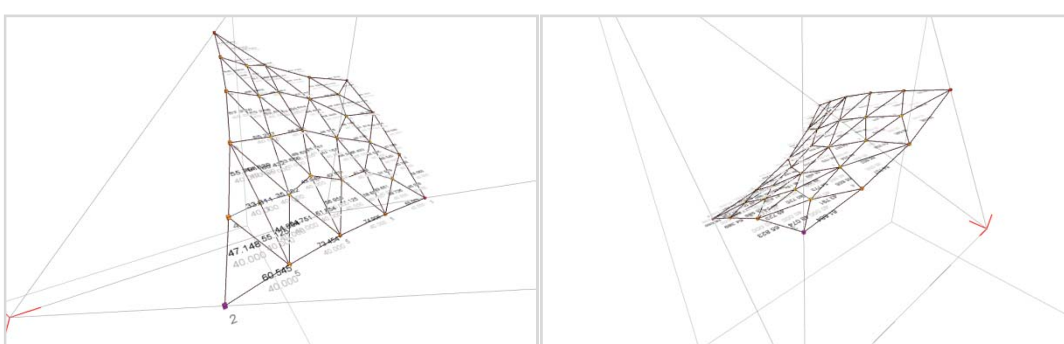


Figure 14. Stills from the running algorithm with the numerical display of the spring length deviations and the nodes valences

Following the increase of the number of particles in strict correlation with their valences, the corresponding bigger number of connections was creating a temporary instability of the system in relation to the dynamic behavior of the particles and the predisposition of connectivity through springs. In some of the cases, the subdivision algorithm leaded to the formation of clusters of particles with tetrahedral or polyhedral geometries. Even if the Delaunay algorithm was the main factor in preventing this type of phenomena, after repeated experiments, a series of additional conditions for connecting the springs were added. The connection filters were preventing the inefficient connections between similar constrained particles or the formation of tetrahedrons. The relationship between the number of particles and the corresponding number of springs is presented in the following graph which covers a number of 2000 iterations.

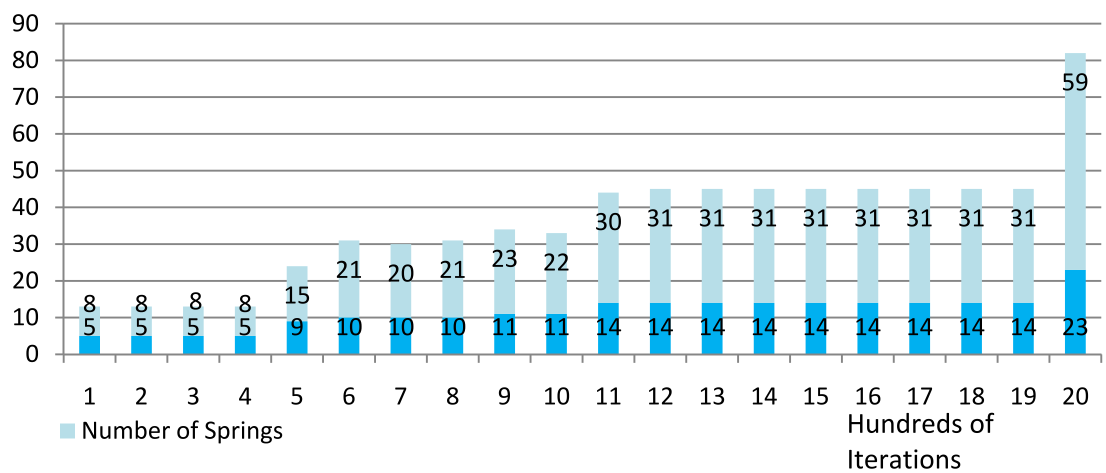


Figure 15. ***Number of particles versus number of connections between them***

Due to the configuration of the fundamental region and the constraints defining the boundaries of the surface, the problem of the precision consists in obtaining equal size linear elements within the triangulated surface. In strict correlation with the attributes of the springs, which provide the tension necessary for defining the curvature of the surface, the length deviation of the springs relating to the ideal lengths was relatively high in most of the cases and has been analyzed in different situations, on a different number of particles and different values for the attraction/repulsion coefficient k.

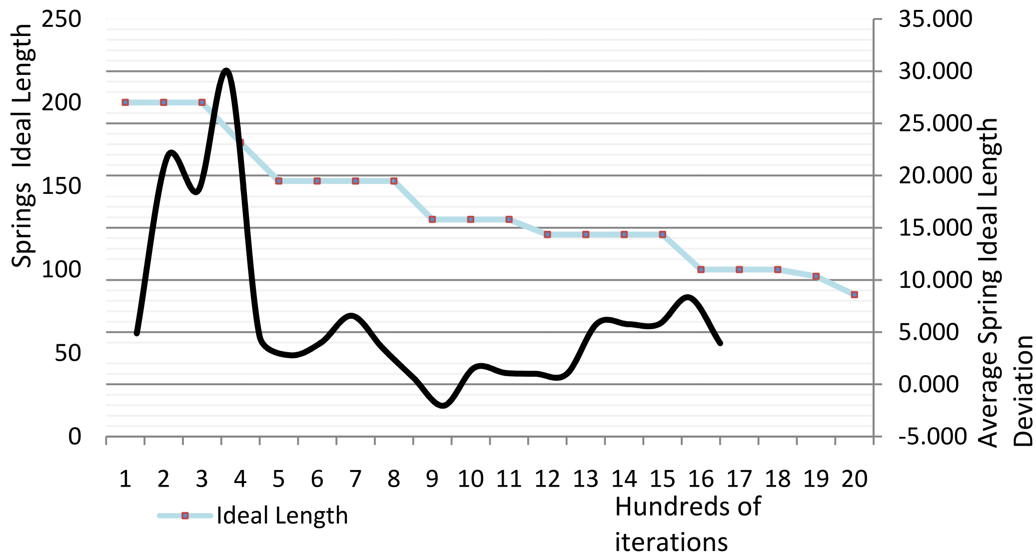


Figure 16. ***Graph of average deviation from the spring ideal length for the attraction/repulsion coefficient K=1***

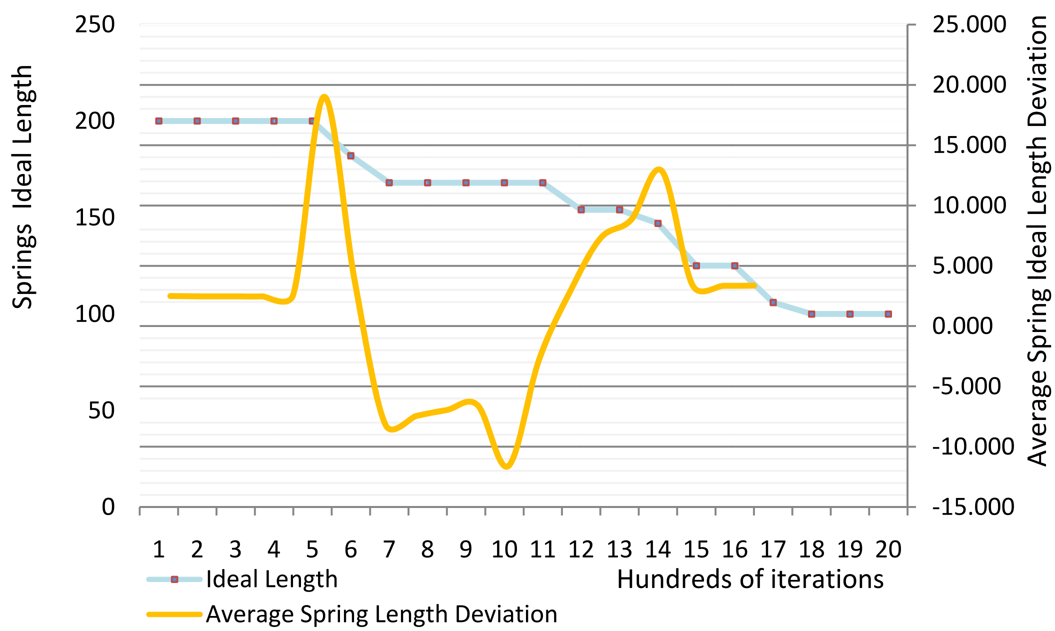


Figure 17. ***Graph of average deviation from the spring ideal length for the attraction/repulsion coefficient K=10***

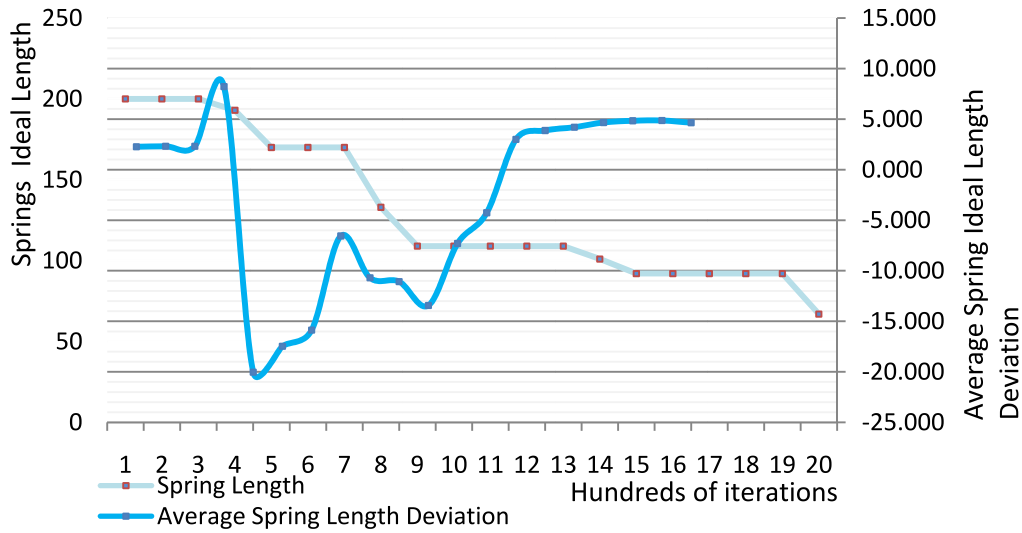


Figure 18. ***Graph of average deviation from the spring ideal length for the attraction/repulsion coefficient K=100***

The results of the graphic analysis are showing that the high values for the k coefficient are reaching to a more unstable system, with higher values for the absolute deviation but with an approximately constant level after a considerable amount of iterations. The conclusion is that the higher tension within the final surface, from the point of view of the tessellation, by leading to a bigger but constant deviation from the ideal length is creating an optimized configuration for the fabrication. From the point of view of the fabrication not the value of the deviation is essential, but its constant level of deviation, obtained by the homogenous distribution of it within the surface's springs, in order to reach equal lengths for the elements.

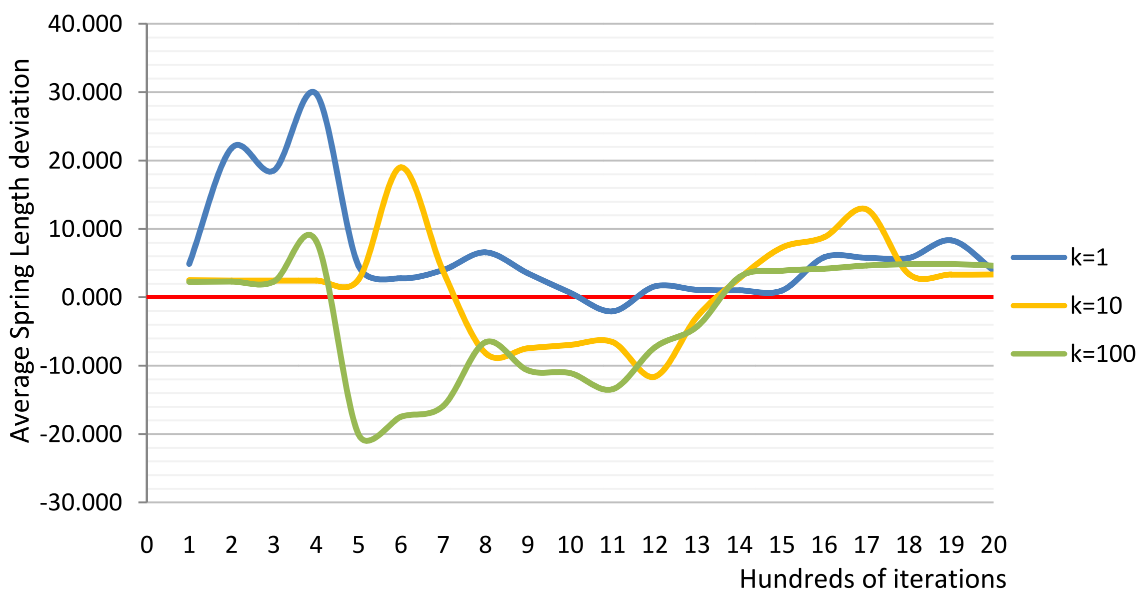


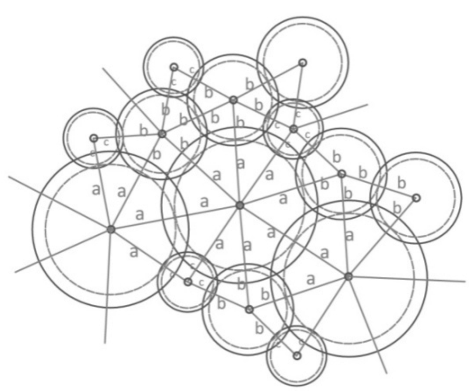
Figure 19. ***Average deviation from the spring ideal length for different values of the attraction/repulsion coefficient K***

Considering the manufacturing system involves a relatively low degree of accuracy, by focusing on the level of precision of the algorithm in obtaining the final geometry, the analysis shows that the strong attraction within the springs leads to a more feasible system for fabrication. Because of the irregular geometry boundaries, reaching a uniform distribution of the particles by obtaining equal spring lengths is not giving a satisfactory level of precision in obtaining a minimal level for the values of deviation from the average spring length within the system. The problem of the impossibility of reconstructing the surface from identical component configurations leaded to investigating the possibility of using a set of multiple equal sizes for the elements.

4.3. Multiple lengths

The previous geometrical and topological tests were repeated for the modified algorithm for multiple ideal lengths. In the case of the multiple lengths the problem increased in complexity, but the hypothesis was that there would be better chances for the modules to reach different equal lengths while adapting to the irregular boundaries of the surface. The system was based on defining a set of ideal lengths as ratios of a main rest length.

The number of the ideal lengths was related to the problem of circle packing onto the surface, generated by the architectural problem of the research. The concept behind the multiple size circle packing was based on the analogy with the soap bubble arrays, aiming to simulate a similar two-dimensional cellular system with a uniform triangular order, following the similar natural laws that govern the phenomenon. (Pearce, 1980) The objective was to generate an optimized distribution of the particles and the springs in order to form a triangulated surface which could be reconstructed from circles of 3 standard sizes. Accordingly, in order to have the positions of the particles as the centers of the circles, the ideal lengths of the springs should be composed of different combinations of the three radii of the circles R_a, R_b, R_c, by taking into consideration the valence (number of neighbors) of each particle. In relation to the number of neighbors of one particle, its proportional value affecting the spring ideal length will be attributed to the spring together with the correspondent value of the other particle at the other end of the connection. The algorithm was modified in order to control the 6 possible resulted combinations of the three radii as attributed ideal lengths for the springs.



$$R_a = \frac{5L}{10} \qquad R_b = \frac{3L}{10} \qquad R_c = \frac{2L}{10}$$

$$L = \frac{2L}{10} + \frac{3L}{10} + \frac{3L}{10}$$

$$R_{aa} = \frac{10L}{10} \qquad R_{ab} = \frac{8L}{10} \qquad R_{ac} = \frac{7L}{10}$$

$$R_{bb} = \frac{6L}{10} \qquad R_{bc} = \frac{5L}{10} \qquad R_{cc} = \frac{4L}{10}$$

Figure 20. Principle for the circle packing proposal for three circle sizes

The tested modules were defined as incremental ratios of an absolute ideal length such as 2, 3 and 5 tenths of the main ideal length. It was considered a theoretically flexible configuration able to adapt to various dimensional boundaries. Accordingly, the final six ideal lengths were formed by the different combinations of the three ratios of the absolute ideal length L.

After defining the algorithm for attributing the different ideal lengths according to the valences of the particles, the surface had a similar behavior from the geometrical point of view but with higher values for the radius deviation. The overall stability of the system was considerably lower initially due to the behavior of the particles but the average spring length deviation reached a more stable level after a certain amount of iterations.

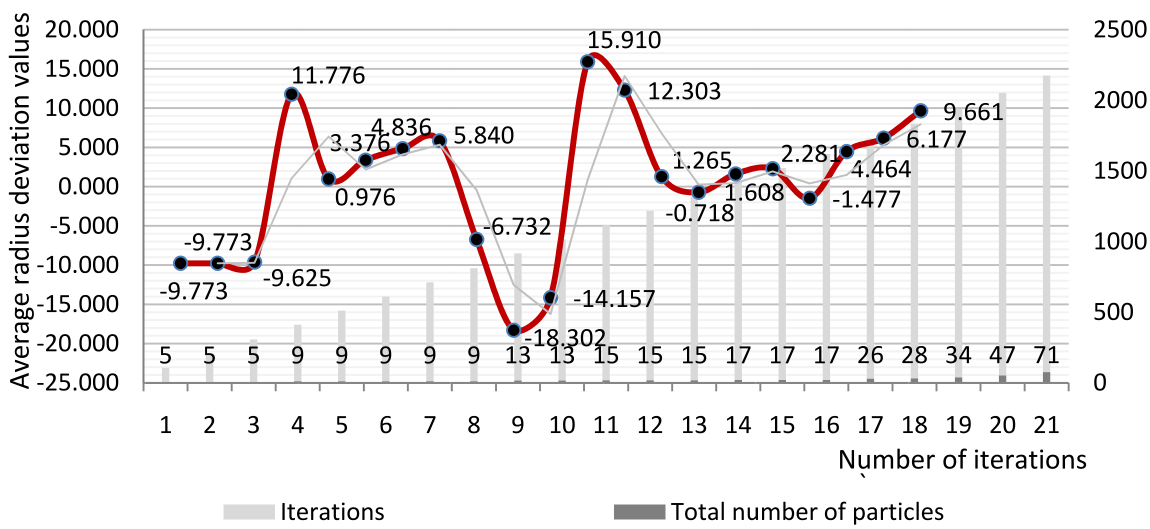


Figure 21. ***Curvature analysis for the multiple lengths***

From the topological point of view, the tessellation of the surface started to develop irregular configurations, with a similar tendency to push the bigger size elements towards the centre of the surface, while the small sized elements were concentrating near the boundaries. Another phenomenon was the appearance of particles with eight neighbors temporarily due to the movements caused by the triangulation process.

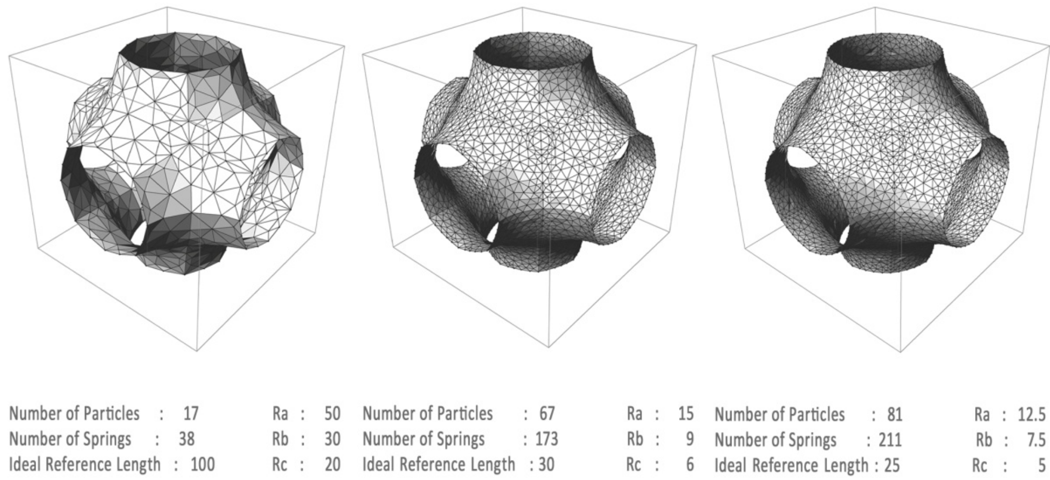


Figure 22. Multiple Ideal Spring Length instances of the Schwarz P Surface

From the point of view of the ideal length deviation the system proved to be more efficient in reaching a constant level of deviation relative to the ideal lengths of the springs which proved that the hypothesis was correct. Without achieving a high level of accuracy, the efficiency in reaching the different ideal lengths does not consist on the average value of the deviations which remains relatively high, but in the constant level of deviation that the system converges to. A constant level of deviation means a small change in the ratio between the lengths of the springs, which is the most important from the manufacturing point of view.

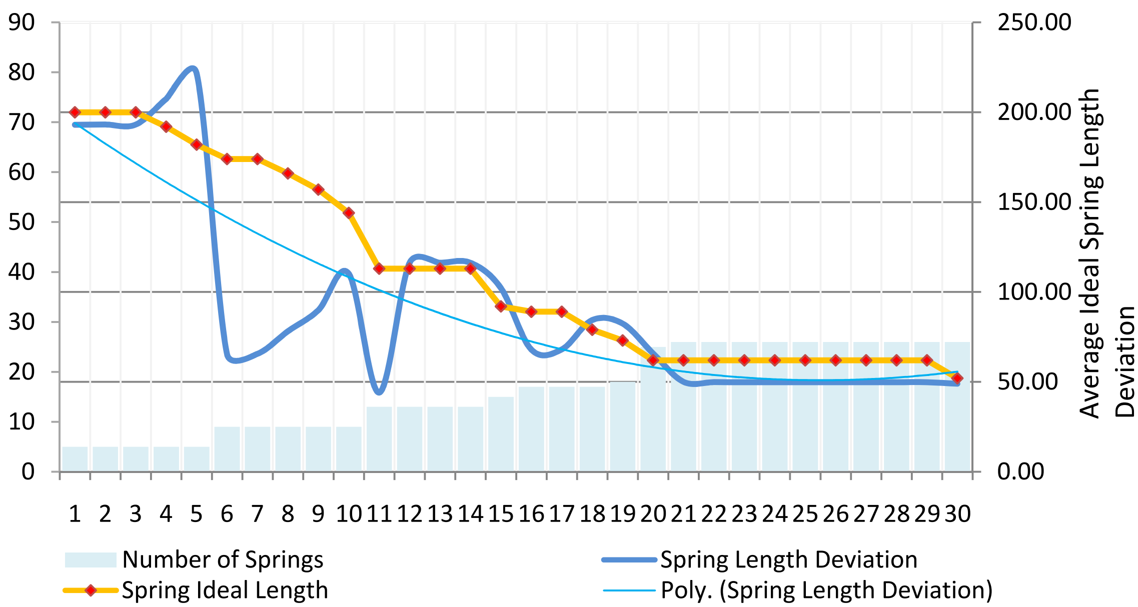


Figure 23. ***Average ideal spring length deviation for multiple lengths – decrease of values/increase of nr of springs***

5. Discussion

The previous tests and the results are showing that by using particle-spring systems it is possible to generate minimal surfaces. Based on the achieved tensional properties of the generated surfaces, the analogy with the natural systems that have a similar behavior is a fact that could be linked to the formation and the dynamics of soap films. From a cellular point of view, if we were to consider the particles as cells or molecules and the springs as the connections between them, the proposed system reaches an emergent quality of self-organization similar to one found in nature.

The algorithm is materialized through a similar concept which comes from the questions raised by Colding and Minicozzi regarding the equilibrium of natural organisms, which relates to the conservation of energy (Colding and Minicozzi, 2005). Every iteration is programmed to update the relationships between the components of the system, reapply the defined rules and minimize the energy, in our case the tensional energy in order to achieve a state of equilibrium.

5.1. Results overview

Following the tests and the experiments applied on the Schwarz P Surface, the findings are the ones to assess to what extent the methodology has achieved its goals in testing the main hypothesis. The geometrical analysis was pointing out the fact that the curvature of the obtained surface, according to the total number of particles and connections and to the properties of the springs is variable and it can adapt to various settings. The final results were showing that together with the increase in number of the particles and with a higher attraction factor, the curvature came closer to the ideal one, confirming the fact that with an infinite number of particles the system could converge to an absolute minimal surface. The discrete geometrical characteristic of the obtained surface is one of the main key points of the method, the level of approximation being the object of analysis of the results and not the absolute values. Including the architectural design factor into the analysis, the question that appears is how much precision in reaching the absolute zero mean curvature is needed from the aesthetic or functional point of view? The answer might involve both the aesthetic and the fabrication issue in a sense that the best solution would be an optimal combination of the two, in which the absolute values could leave room for the perfect balance between the design quality and the modular fabrication optimization.

According to the topological analysis, the triangulation of the surface was assessed according to the modularity issues, with direct relation to the manufacturing process. The efficiency in reaching modular dimensional values for the linear elements of the surface proved to be higher in obtaining constant values of the deviations more that minimal ones. In both of the cases, for one size modules or multiple size ones, according to the number of connections and the relationship between attraction and repulsion within the system, a similar convergence of the values of deviations was obtained. The multiple sizes system proved to be the more efficient than the single size one, due to the multiple possibilities of filling the geometrical boundaries for the surface. Together with reaching high values for the deviations, the overall ratio between the different ideal lengths of the springs was kept constant. While the flexibility of the proposed fabrication method is allowing this level of tolerances between the interlocked rings, the question unanswered is how far could the level of accuracy go? As a potential tool that could generate accurate geometries, the method could be optimized at a basic level of particle and spring settings and a more precise system of constraints could be defined, in order to reach a higher level of accuracy through a minimal tolerance process of fabrication.

5.2. Critical assessment

One of the most important features of the overall methodology is that it illustrates a different approach within the computational generative framework regarding the modeling of minimal surfaces, which includes the design factor through the optimization for manufacturing. It is more than a modeling process, consisting in a real-time simulation of a virtual tensioned membrane in order to reach the properties of a minimal surface. The algorithm is based on a bottom-up approach that simulates a growth process, which could reach emergent valences in relation to the behavior of the particles at the level of the tessellations of the obtained surface.

Even though the results of the simulation are reaching the appearance and the properties of a minimal surface, there are a series of potentially critical aspects about the precision of the method in achieving the absolute characteristics of a minimal mean curvature. The average deviation analysis showed that the method is not completely accurate from the point of view of the mean curvature, neither from the point of view of the modularity of the tessellation. This level of precision defines the tolerances admitted for the fabrication process. For high values of the standard deviation from the ideal modular dimensions, the manufacturing system could be compromised. From the point of view of the circle packing algorithm, the results might also be questionable as they depend on the same level of accuracy in achieving the ideal lengths. Due to the elastic properties of the proposed fabrication system, the tolerances could handle the expected deviations within the surface, from a theoretical point of view. Therefore, a new series of questions could arise related to the efficiency of the method in being not just a generative tool, but a precise tool for providing accurate information and drawings for fabrication.

5.3. Potential advantages of the method

The mainly architectural starting point of the research was related to the problem of the relationship between form-finding methods and the fabrication issue. Due to their properties, the minimal surfaces present a high level of interest for the architectural design field. The research of minimal surfaces in relation to the design is relatively a new phenomenon, but with several attempts of implementing minimal surface geometries in architectural structures. The problems that were experienced in all cases were related to the separation of the two processes: the computational modeling and the materialization of the geometry.

The advantage of the proposed method lies in the ability of combining the two processes within a parametric system that could include a lot more coefficients to be taken into consideration during the generative process. While a standard method would provide a 'rigid' strictly geometric or mathematical framework in defining a minimal surface without including the design factor, the generative proposed method could involve a lot of new parameters besides the current geometrical and the modular ones, in order to solve spatial, morphological, social or structural design problems. For example, the particles and the springs between them could include several structural parameters and constraints in order to generate optimized lattice structures, or they could simulate computational agents which could take into consideration several space syntax parameters.

The study focused on a specific case of minimal surfaces and a tensegrity structural system as a manufacturing method. However, the potential of the proposed generative tool is not limited to these solutions, as the geometries could reach higher levels of complexity by exploring the design possibilities of the known periodic minimal surfaces or even to explore new types of surfaces or hybrid typologies.

From the point of view of the fabrication, the modularity is the main subject of the research. As a general concept, the Delaunay triangulation of the surface is the principal framework for developing modular systems. Without ignoring the potential of the triangulation as it is, an infinite variety of different configurations could be derived from it, which could involve more complex tessellations of linear or irregular geometries. The resulted configurations could cover honeycombs, Voronoi polygons, or other types of ornamental or structural modular patterns.

The purpose of the research is to open a new direction within the computational design methodology, as part of the architectural design process, involving a multiple purpose design strategy which takes into consideration various categories of factors and constraints, as part of a parametric system. The modularity of the subdivisions of the resulted free-form surfaces could be extrapolated to various architectural applications such as facades, roofs, structural tensioned membranes or other types of architectural structures. The design framework is not limited to architecture, as the scale of the objects could reach the level of industrial design artifacts, furniture or installations. Due to the cellular logical structure of the system, in correlation to the interlocked circles used as a fabrication method, a feasible field of applications could include even fashion and textiles design.

5.4. Further investigations

The main research subject of this thesis covers just one of the many potential directions that could be taken in the investigation of the computational methods exploring the possible applications of minimal surfaces within a generative design framework. Following the tests and the results of the proposed method, some of the aspects of the algorithm that were found problematic, have already been identified. One of the aspects that proved to be essential in the behavior of the particle-spring system in generating the minimal surface was the attraction/repulsion coefficient. As an alternative from the linear coefficient defining the relationship between them, could involve a more advanced polynomial function using Lagrange interpolation coefficients. (Kanellos, 2007) A more advanced step would be to include factors such as acceleration, viscous drag or damping in the particle system in order to obtain a more accurate physical simulation. In this direction, another possibility worth exploring would be using solvers or integrators such as Euler or Runge-Kutta for the approximations of the solutions of the differential equations involved in the relationship between the new factors included in the simulation.

Another aspect to be taken into consideration for future investigation would be to reconsider the Delaunay triangulation algorithm. A three-dimensional Delaunay algorithm could be implemented as well as defining new constraints such as angle coefficients between the facets of the surface.

Following the same technique used in generating triply periodic minimal surfaces, the system of constraints could be extended to circular geometries in order to be able to focus on the generative process of other types of minimal surfaces, such as Costa or Enneper.

A future work on the proposed method could be focused on developing a more advanced parametric design tool in order to create a framework for developing more complex parametric minimal surfaces, taking into consideration a larger amount of coefficients in the generative process. The fabrication aspect, including different types of configurations or subdivision algorithms could be embedded in the algorithm in order to control the modularity of the surface and the export of the resulted geometries in relation to the digital manufacturing techniques.

6. Conclusions

The research was developed around the design problem of minimal surface structures, in order to create an alternative algorithmic design method for generating minimal surfaces, as well as for the construction of the surfaces from modular components. The contribution to knowledge of the study comes from the different approach of the project, in relation to the existing ones in the field, by using a simulation of tensioned surfaces to generate minimal surface geometries, moving forward in the direction of the final application in design, from the point of view of digital fabrication. While generating the surface, the method is optimizing the geometry for a modular fabrication system. The main difference in approach would come from the bottom-up algorithmic strategy of not starting with a predefined topology, as in the case of the dynamic relaxation method, but simulating an iterative growth process, optimized to reach a state of tensional equilibrium of the system.

Based on various analogies with natural systems, the algorithm is developing a process of self-organization with potential emergent qualities, following a specific set of rules and constraints which are active at a basic cellular level of the system, but with a great impact on the overall behavior of the generated surface. Accordingly, the aim of reaching the minimal properties of the resulted geometries is based mainly on the effects of the defined rules within the process of physical simulation, in order to achieve a behavior of a soap film as a virtual tensioned membrane. The optimization process of the subdivisions of the surface is embedded in the algorithm, controlling the relationships between the particles which would define the uniform microstructure of the surface.

The architectural problem which launched the investigations of this subject of research was essential in structuring a dual process methodology, involving the form-finding algorithm simultaneously with the modular dynamic tessellation of the surface. Using the particle-spring system as a framework for the simulation process, the potential of the proposed method could open new directions in the computational design field, by having the ability to involve more parameters in the generative design process. Together with aiming minimal surface properties and an optimal modular triangulation of the resulted geometry, the system could be programmed to reach a multiple objective optimization character which could include spatial, social or structural parameters.

7. Appendices

Appendix I: Illustrations of generated minimal surfaces

Complete set of illustrations for the Schwarz P Surface stages of evolution (pages 40 -43)

The set of images include the most important the stages of triangulation of the surface. The planar views and the axonometric views are providing an overall visualization of the process. The multiple Ideal Spring Length tests are also included. There is to be noticed a more perceptible irregular tessellation of the same surface, comparing to the previous tests for one single length.

Illustrations of the testing of the method on the Schwarz D Surface Typology (pages 44-45)

The Schwarz D Surface was tested with a similar algorithm, the only difference being given by the change in the coordinates of the parameters of the basic kaleidoscopic cell.

Figure 24. Plan and axonometric views of the 6 recorded states of evolution of the Schwarz P Surface

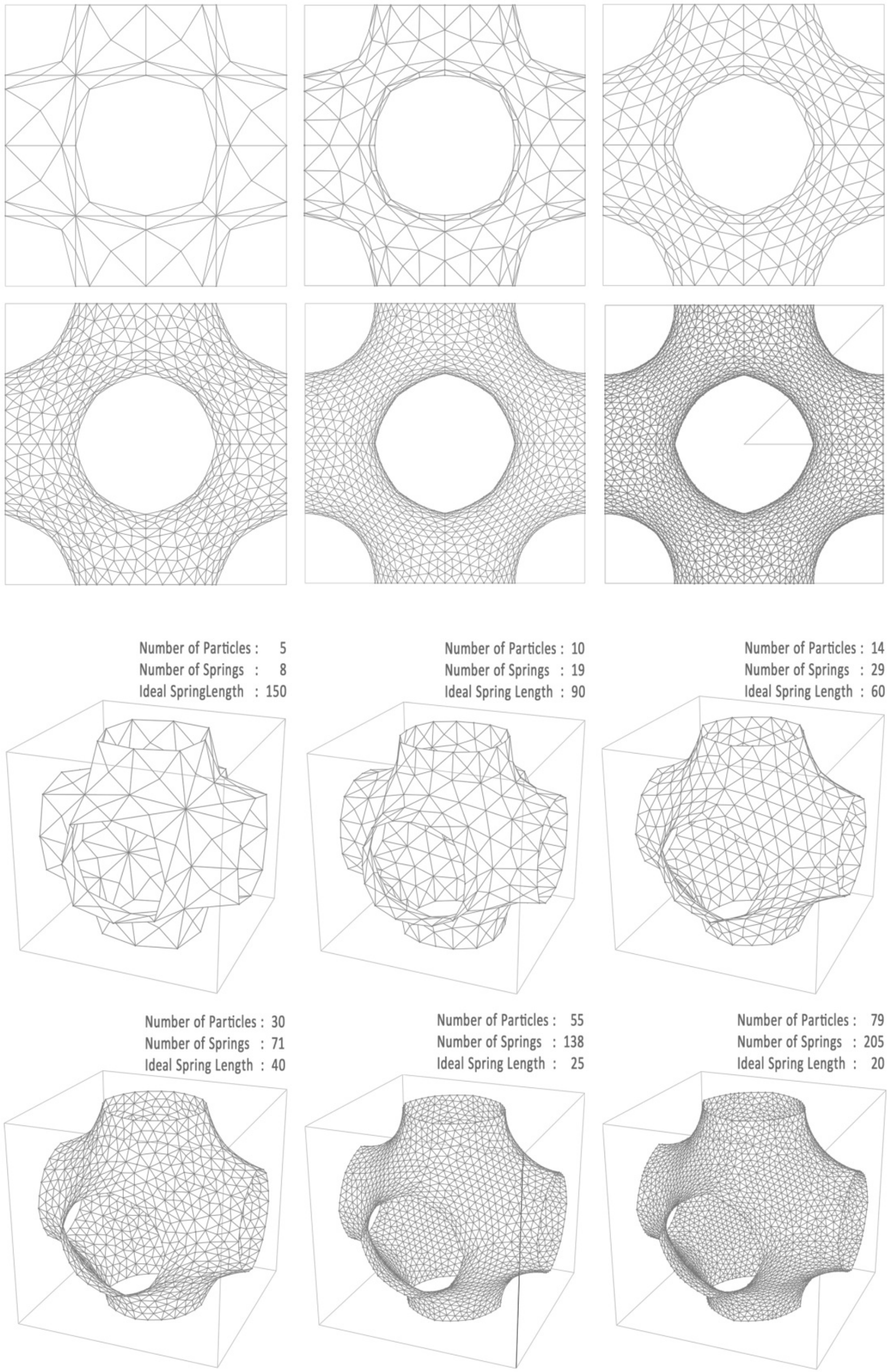
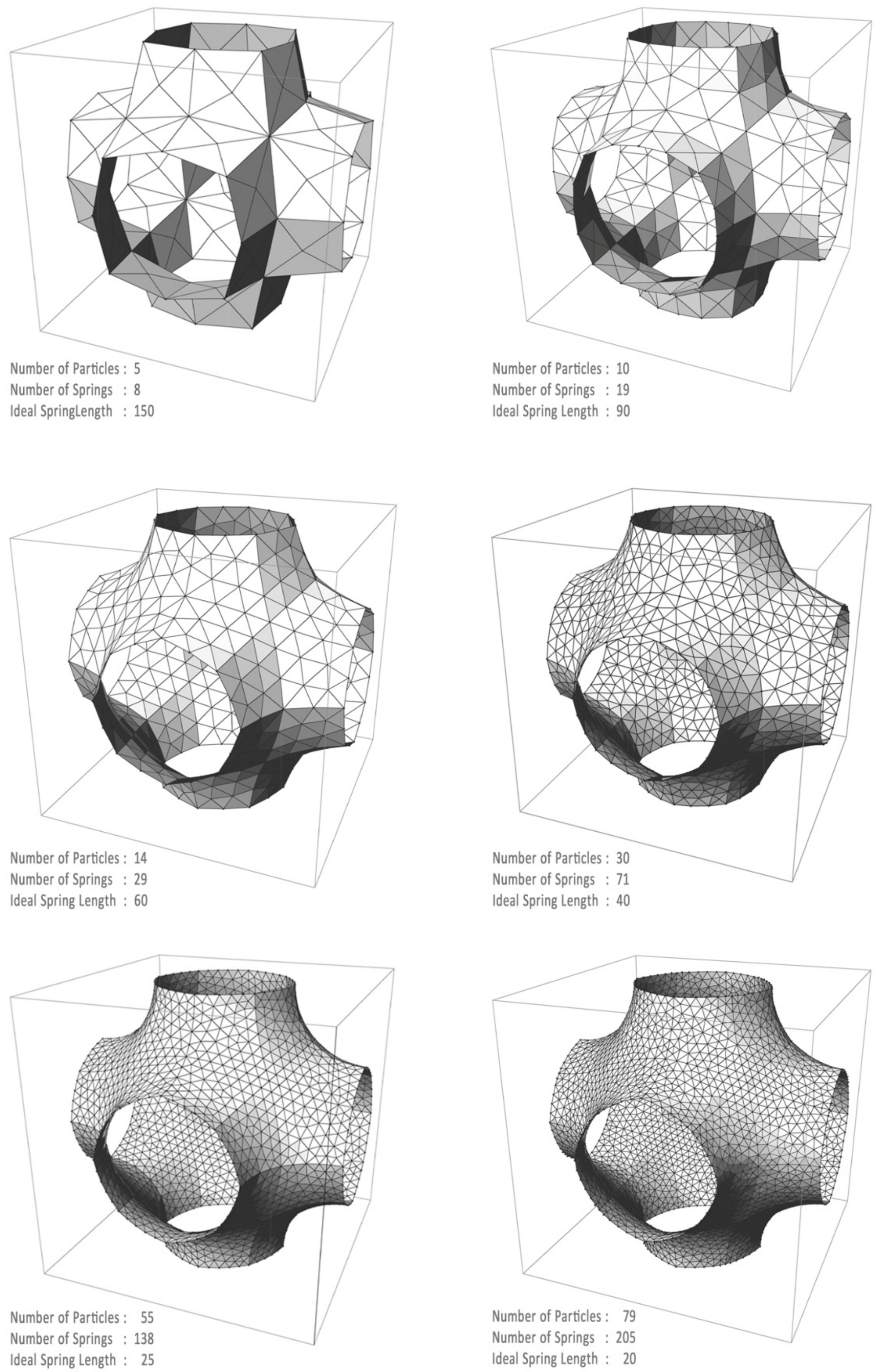


Figure 25. Illustrations of the gradual evolution process for the Schwarz P Surface



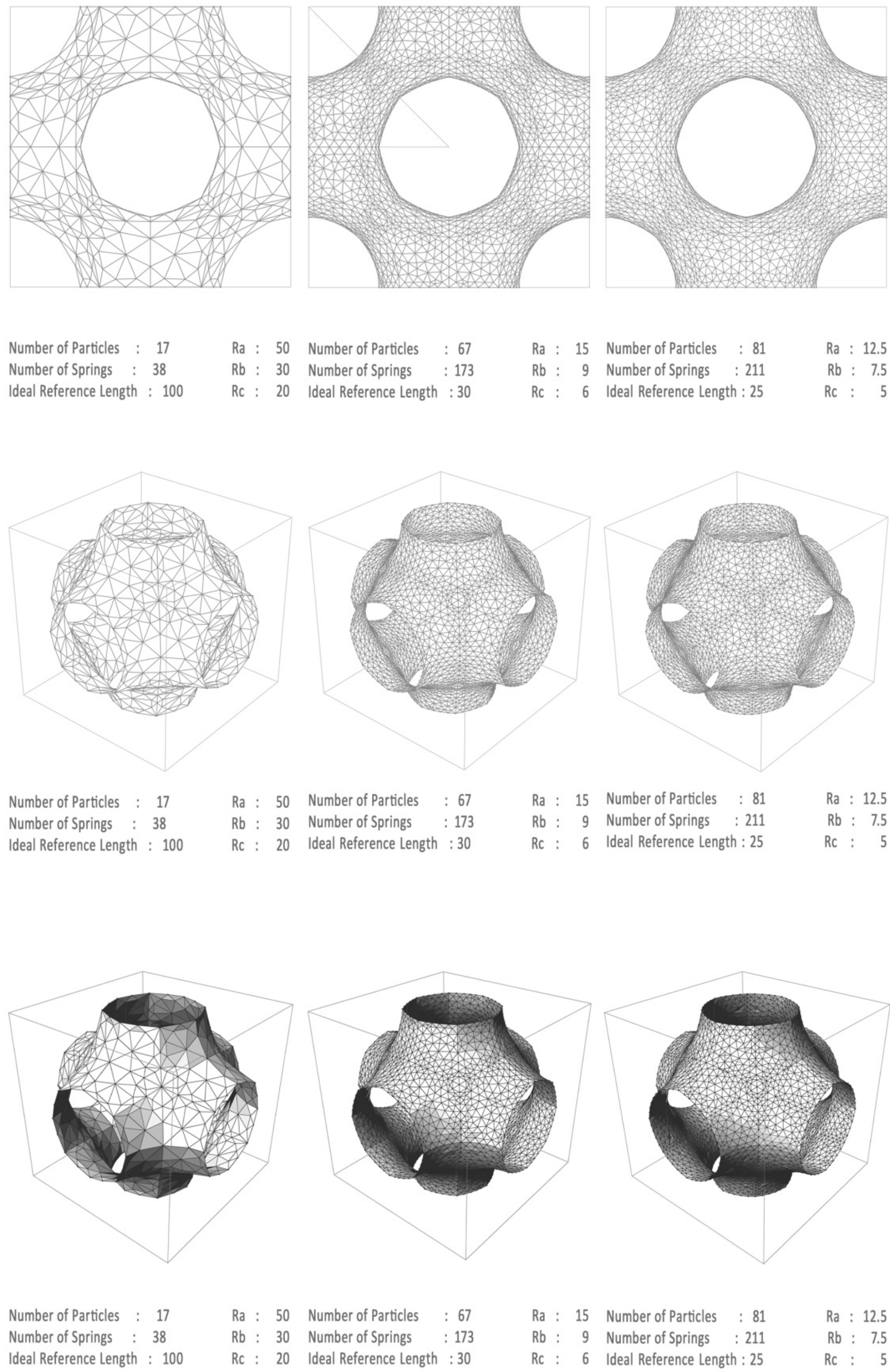


Figure 26. Multiple Ideal Spring Lengths test for the Schwarz P Surface

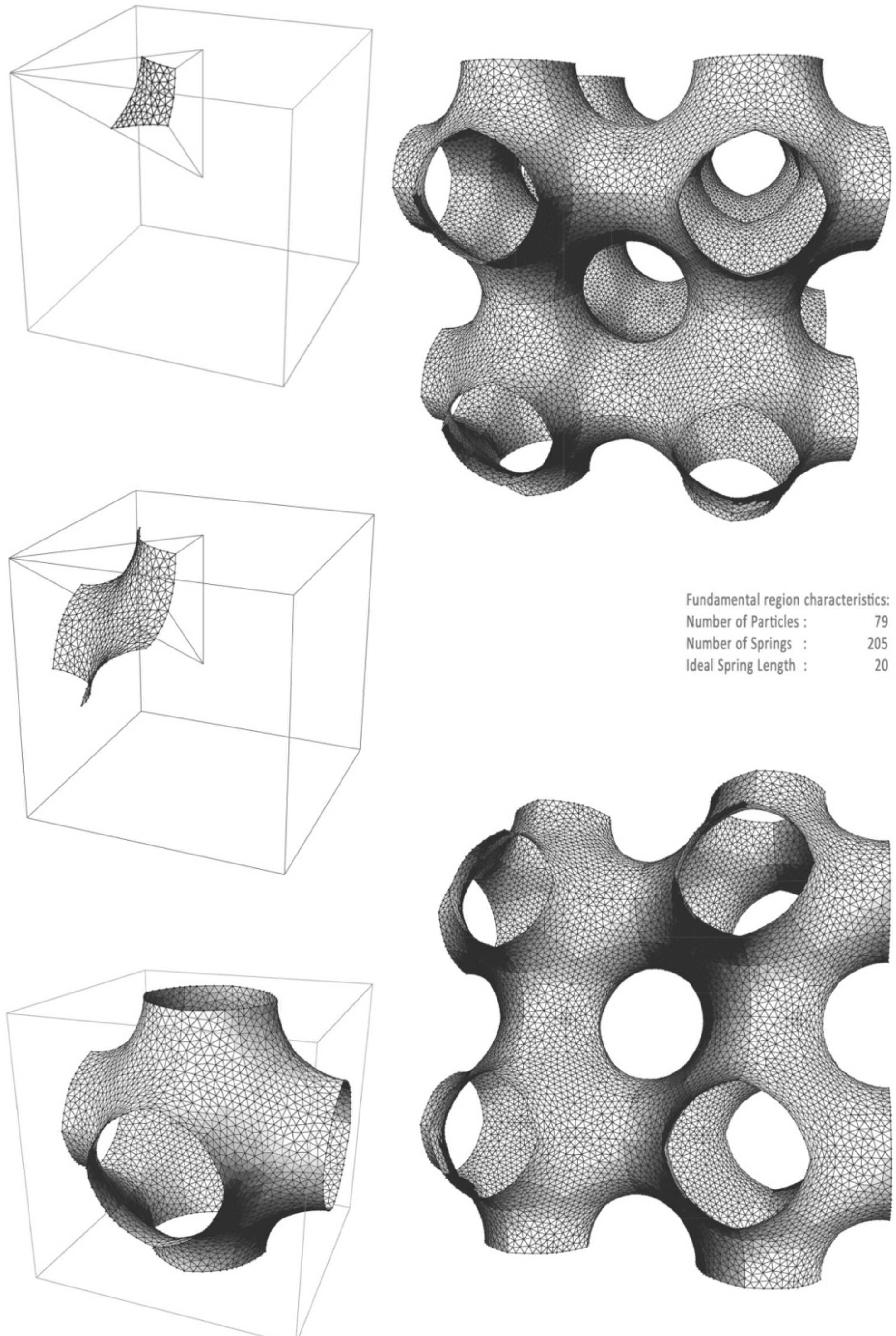
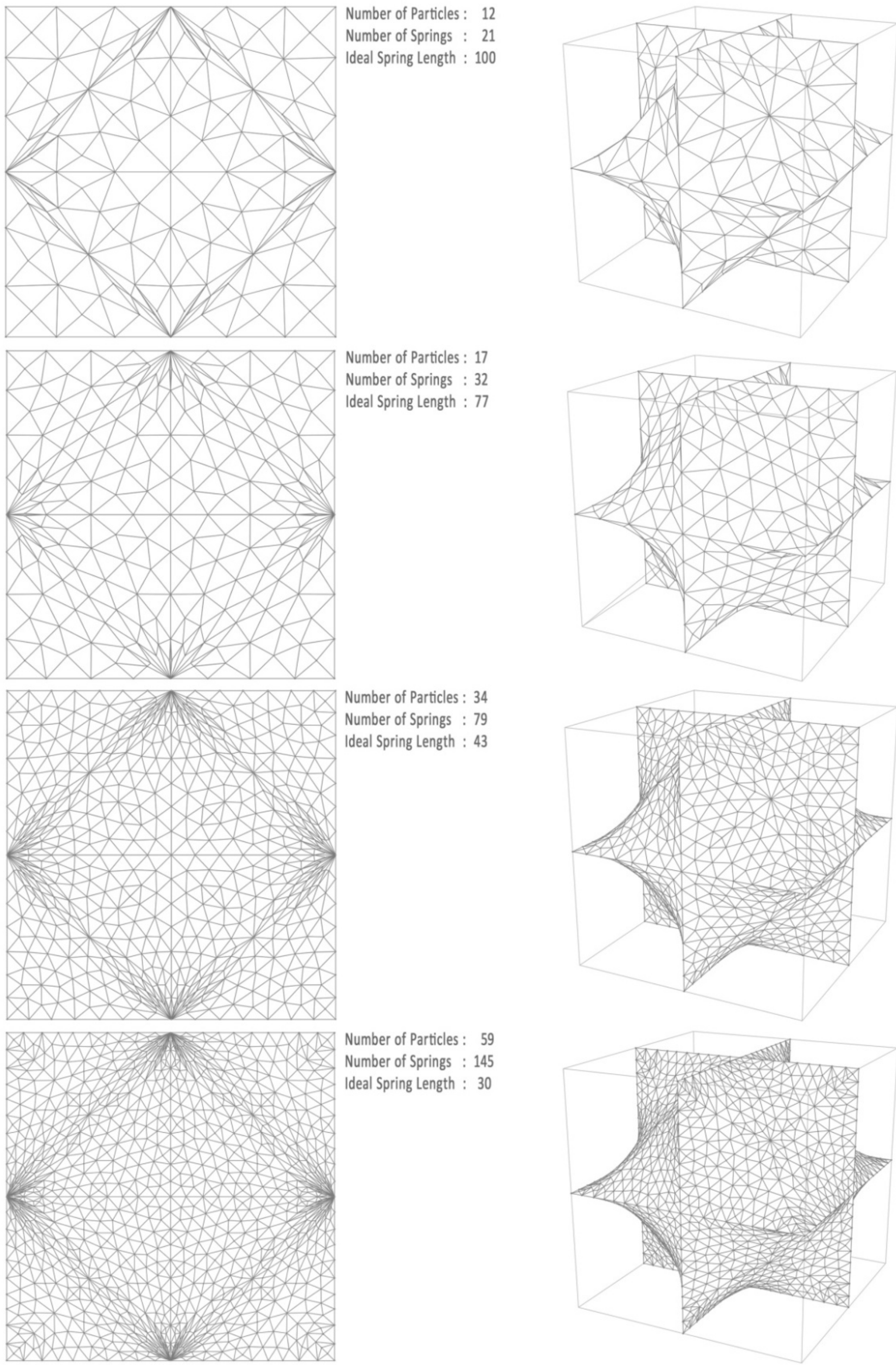


Figure 27. Schwarz P Surface evolution from the fundamental region to a two-dimensional group of reflections of the module

Figure 28. Plan and axonometric views of the Schwarz D Surface



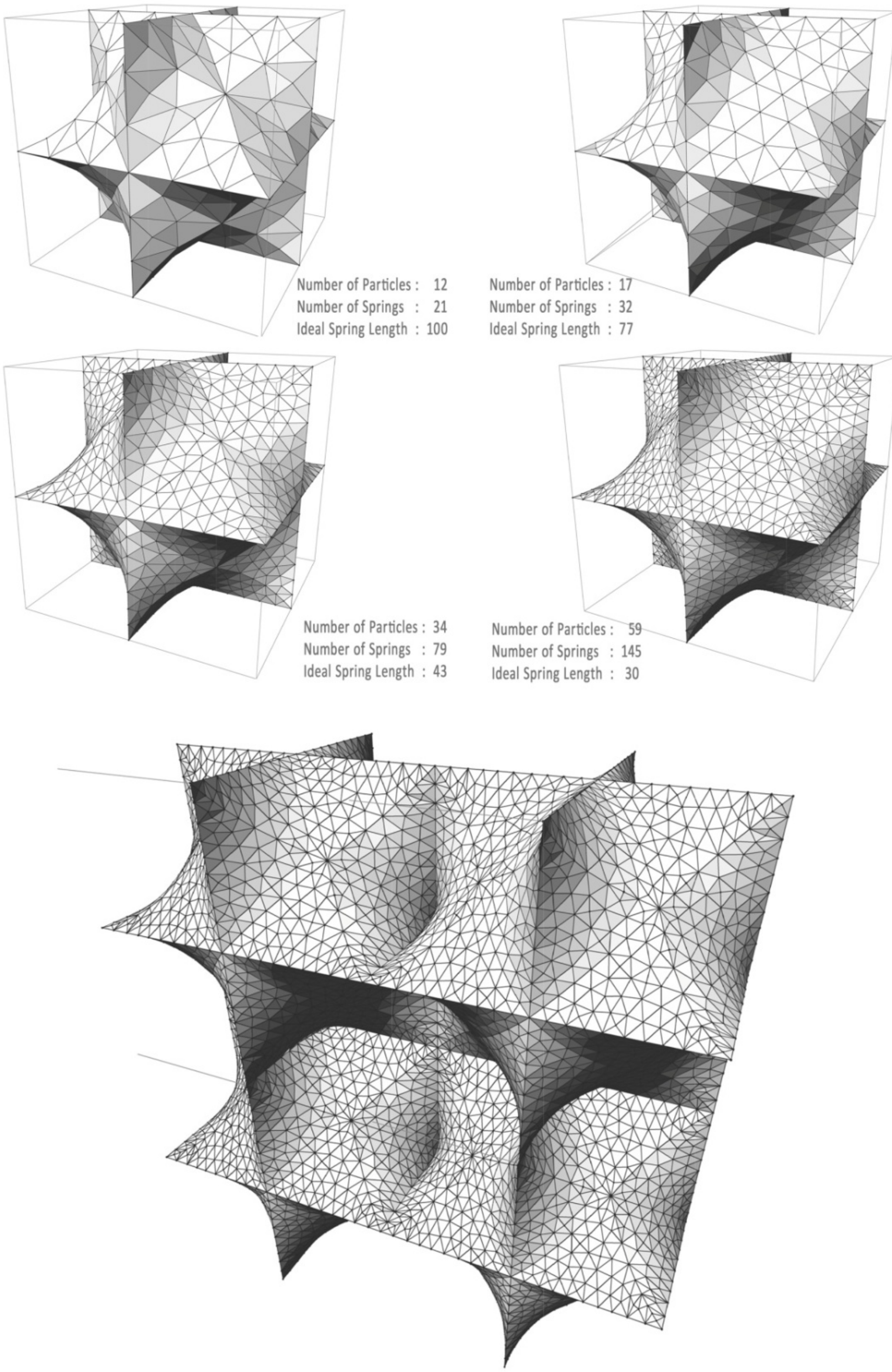


Figure 29. Illustrations of the gradual evolution process of the Schwarz D Surface, together with a reflection group of basic modules

Appendix II : Pseudocode (after Processing)

Basic adapted Delaunay triangulation function :

```
void triangulateSprings() //defines a Delaunay triangulation of the particles through springs
{ Vector3D  center = new Vector3D(0,0,0);
  float rad=1;
  springs.removeAllElements();
    for (int i = 0; i < Particles.size()-2 ; i++)
     { for (int j = i+1; j < Particles.size()-1 ; j++)
       { for (int k = j+1; k < Particles.size() ; k++)
         { Particle a = (Particle) Particles.get(i);
           Particle b = (Particle) Particles.get(j);
           Particle c = (Particle) Particles.get(k);
           boolean found=false;
           for (int m = 0; m < Particles.size() ; m++)
             { Particle pm = (Particle) Particles.get(m);
               center = center(a , b , c ); // calculates the center of the circle intersecting the three points
               rad = dist( center.x , center.y , 0 , a.pos.x , a.pos.y ,0 );
               float ddd = dist( center.x , center.y ,0 , pm.pos.x, pm.pos.y , 0 );
               if ((m!=i)&&(m!=j)&&(m!=k)&&( ddd <= rad ) ) // if there is another point within the radius of the circumscribed circle , the three particles are not valid to be connected
                    {found=true; }
             }

           if ((!found) && (angles(a,b,c)==true)) //If there is no other point within the circle and one of the angles of the triangle is not smaller than 15 degrees, it creates the springs between the three points
```

```
        { if ((isSpring(a,b)==false) )     //creates a new spring
                    {Spring a2b = new Spring (a,b,iLength);
                     springs.addElement(a2b);
                    }
        if ((isSpring(a,c)==false)) //creates a new spring
                    {Spring a2c = new Spring (a,c,iLength);
                     springs.addElement(a2c);
                    }
        if ((isSpring(b,c)==false) //creates a new spring
                     {Spring b2c = new Spring (b,c,iLength);
                     springs.addElement(b2c);
                     }     }
  }
}
```

Basic functions of the particle class:

```
class Particle implements  Comparable
{ Vector3D pos;
  Vector3D dir;
  Vector3D tdir;
  Face f; //  the face constraint
  Face f1,f2; // the faces corresponding to  the edge constraint
  ArrayList neighbors; // the list with all the connected particles
  boolean fixed=false;
  boolean onFace=false;
  boolean onEdge=false;
```

```
Particle(float x, float y, float z, float x_dir, float y_dir, float z_dir)//Main particle constructor

{ pos = new Vector3D(x,y,z);

  dir = new Vector3D(x_dir, y_dir,z_dir);

  tdir = new Vector3D();

  neighbours= new ArrayList();

}

 Particle(float x, float y, float z, float x_dir, float y_dir, float z_dir, Face fac) //Constructor for particles constrained on a face

{ pos = new Vector3D(x,y,z);

  dir = new Vector3D(x_dir, y_dir,z_dir);

  tdir = new Vector3D();

  onFace = true;

  f=fac;

  neighbours= new ArrayList();

}

 Particle(float x, float y, float z, float x_dir, float y_dir, float z_dir, Face fac1,Face fac2) //Constructor for particles constrained on edge

{ pos = new Vector3D(x,y,z);

  dir = new Vector3D(x_dir, y_dir,z_dir);

  tdir = new Vector3D();

  onEdge = true;

  f1=fac1;

  f2=fac2;

  neighbours= new ArrayList();


}

void move()

{ pos = add(pos,dir);
```

```
 }
 void draw()
 {   if (!fixed) {pushMatrix(); translate(pos.x, pos.y, pos.z); fill(250,200,50); box(2);  popMatrix();}
    if (onFace) {pushMatrix(); translate(pos.x, pos.y, pos.z); fill(250,150,50); box(2);  popMatrix();}
    if (onEdge) {pushMatrix(); translate(pos.x, pos.y, pos.z); fill(250,50,50); box(2);  popMatrix();}
    if (fixed)  {pushMatrix(); translate(pos.x, pos.y, pos.z); fill(150,50,150); box(2); popMatrix();}
    }
 }


void keepOnFace( Face fac ) //Constrains the particle on a face
 { onFace = true;
   f= fac;
  Vector3D fn = new Vector3D(f.startpoint.x, f.startpoint.y, f.startpoint.z );
   fn.normalise();
   float dp = dot(pos,fn);
   fn.scale(dp);
   pos = sub(pos,fn);
 }
  void keepOnEdge( Face fac1, Face fac2 )//Constrains the particle on an edge
 { onEdge = true;
   f1= fac1;
   f2= fac2;
   Vector3D fnorm = new Vector3D(0,0,0);
   Vector3D fn1 = new Vector3D(f1.startpoint.x, f1.startpoint.y, f1.startpoint.z );
   fn1.normalise();
   float dp1 = dot(pos,fn1);
   fn1.scale(dp1);        //projection on the first face
```

```
  Vector3D fn2 = new Vector3D(f2.startpoint.x, f2.startpoint.y, f2.startpoint.z );

  fn2.normalise();

   float dp2 = dot(pos,fn2);

  fn2.scale(dp2);   //projection on the second face

  fnorm = add(fn1,fn2);

  pos =sub(pos,fnorm);  //the resulted position on the edge

  clear(dir);

  tdir = new Vector3D();

 }

float distToFace(Face fac) //Returns the distance from the particle to a face

{   Face facet_= fac;

  Vector3D tpos= pos;

  Vector3D tdir= dir;

  Vector3D fn = facet_.f_norm;

  fn.normalise();

  float dp = dot(tpos,fn);

 return dp;

}


boolean angles(Particle pa, Particle pb, Particle pc) //Returns false if one of the angles of the triangle abc is smaller than 15 degrees

 {boolean angle=true;

  Vector3D a = new Vector3D ( pa.pos.x, pa.pos.y, pa.pos.z );

  Vector3D b = new Vector3D ( pb.pos.x, pb.pos.y, pb.pos.z  );

  Vector3D c = new Vector3D ( pc.pos.x, pc.pos.y, pc.pos.z  );

   Vector3D  p=  new  Vector3D(  (pa.pos.x+pb.pos.x+  pc.pos.x)/3,  (pa.pos.y  +  pb.pos.y  +  pc.pos.y)/3, (pa.pos.z + pb.pos.z + pc.pos.z)/3  );

   Vector3D  ab= sub(a,b);   Vector3D  cb= sub(c,b);   ab.normalise(); cb.normalise();
```

```
 Vector3D  bc= sub(b,c);   Vector3D  ac= sub(a,c);   bc.normalise(); ac.normalise();
 Vector3D  ca= sub(c,a);   Vector3D  ba= sub(b,a);   ca.normalise(); ba.normalise();
  float a1= dot(ba,ca);
  float b1= dot(ab,cb);
  float c1= dot(ac,bc);
  float anglea = degrees(acos(a1));
  float angleb = degrees(acos(b1));
  float anglec = degrees(acos(c1));
 if ((anglea < 15) || (angleb < 15) || (anglec < 15))
      {angle= false;
      }
 return(angle);
}
```

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