# Computational Modeling of Bound and Radiation Mode Optical Electromagnetic Fields in Multimode Dielectric Waveguides

*David R. Selviah,*

*Ioannis Papakonstantinou and Richard James*

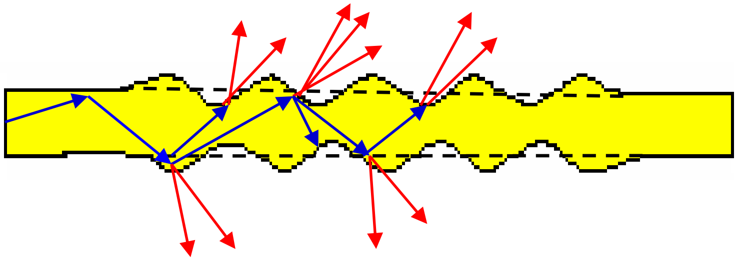
*d.selviah@ee.ucl.ac.uk, i.papakonstantinou@ee.ucl.ac.uk, r.james@ee.ucl.ac.uk*

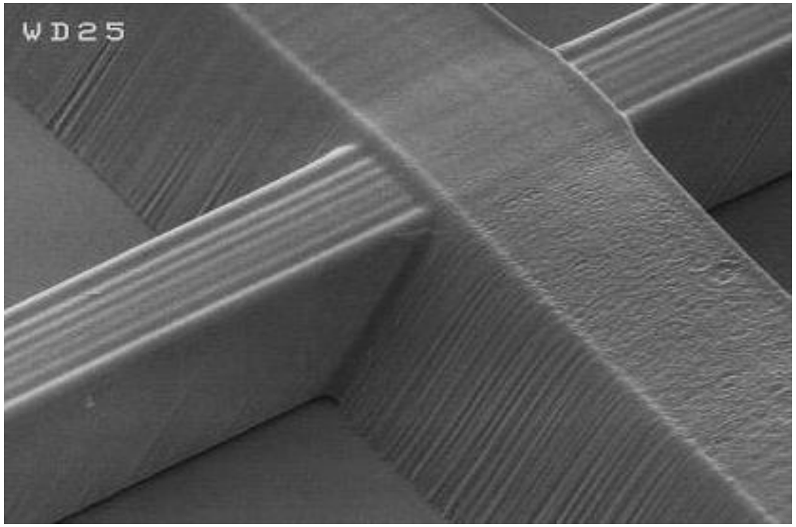
*Department of Electronic and Electrical Engineering,*
*University College London, UCL, UK*

*Invited Paper*

# Motivation

- Optical multimode waveguides are being investigated for board-to-board and chip-to-chip interconnects
- Rectangular core waveguides suffer from sidewall nano-roughness due to fabrication procedures
- Photolithographic fabrication replicates limited mask resolution roughness
- Polycrystalline materials etch to give rough side walls (high $T_g$ materials)
- Direct UV laser fabrication roughness due to stage minimum step size
- Roughness couples bound modes to each other and affects the modal equilibrium power distribution
- Roughness also couples bound modes to radiation modes scattering light from the core into the cladding resulting in loss and crosstalk



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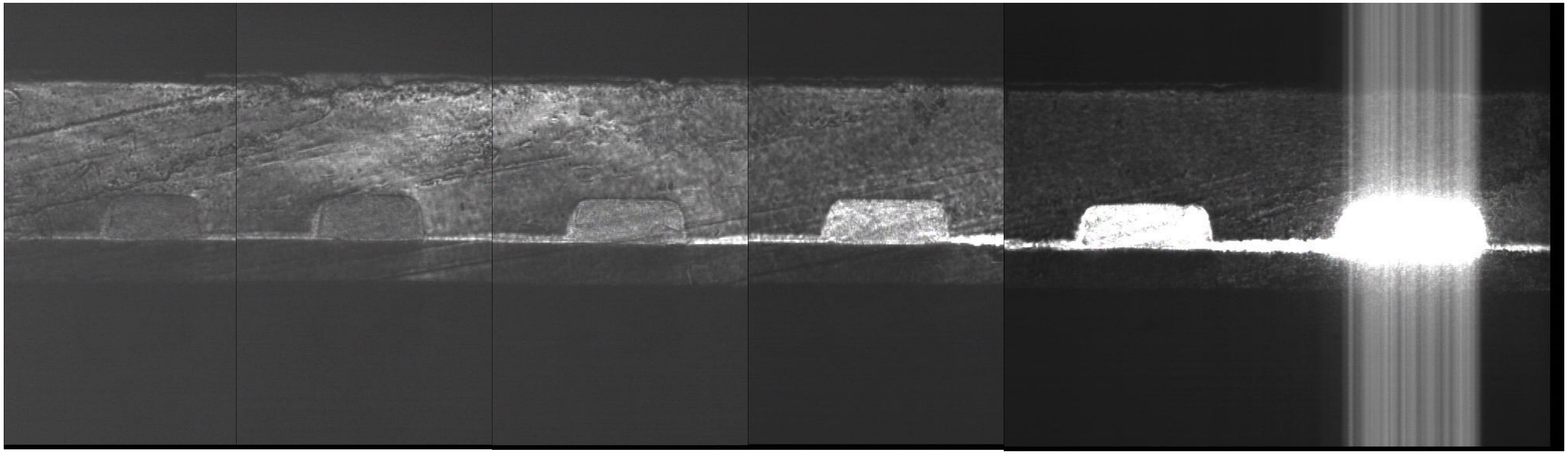
# Crosstalk in Chirped Width Waveguide Array

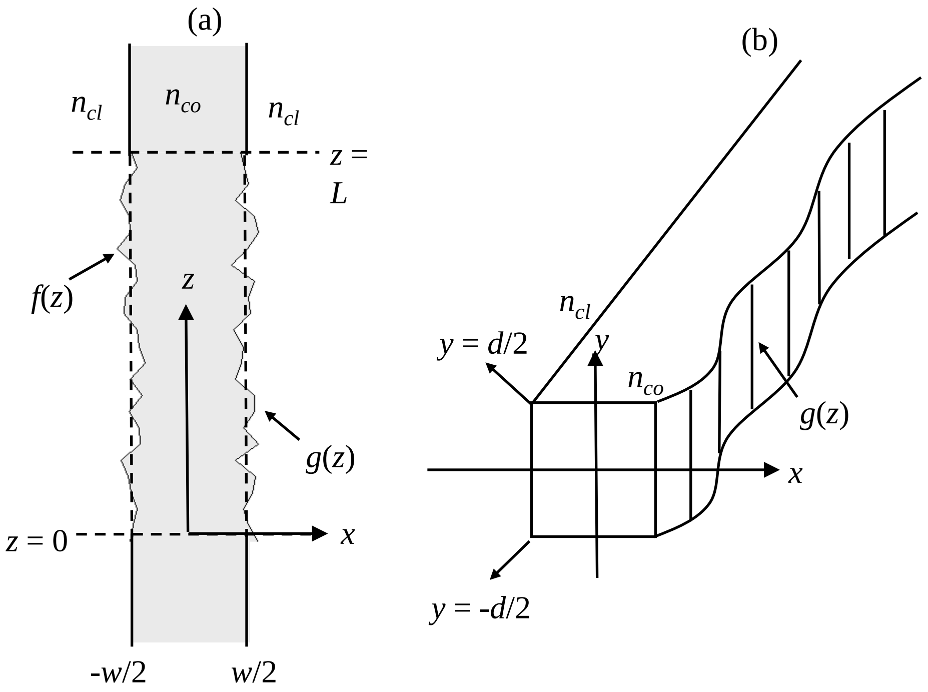


100 µm 110 µm 120 µm 130 µm 140 µm 150 µm

- Light launched from VCSEL imaged via a GRIN lens into 50 µm x 150 µm waveguide
- Photolithographically fabricated chirped width waveguide array
- Photomosaic with increased camera gain towards left

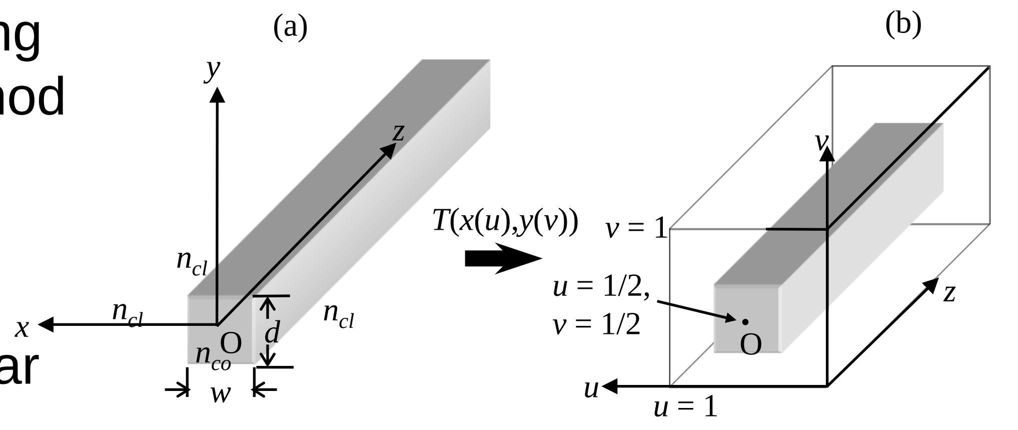
# Motivation

- The effect of sidewall roughness has not been analysed thoroughly in buried channel waveguides due to the lack of the form of the radiation modes
- Our aim is to calculate the radiation modes for rectangular core waveguides and to use these to find the effect of sidewall roughness on the propagation loss, equilibrium length and modal power distribution at equilibrium



# Bound Mode Calculation using the Fourier Decomposition Method

- Bound modes can be found using the Fourier Decomposition Method (FDM) as follows
- The infinite *xy* space is mapped onto a unit square in the *uv* domain by the use of a non-linear conformal transformation
- The wave equation is transformed




$$x = p\tan\left[\pi\left(u-\frac{1}{2}\right)\right],\ y = q\tan\left[\pi\left(v-\frac{1}{2}\right)\right].$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2n^2(x,y)-\beta^2\right)E_x(x,y)=0,$$

$$\Downarrow$$

$$\left\{\left(\frac{\partial u}{\partial x}\right)^2\cdot\frac{\partial^2}{\partial u^2}+\frac{\partial^2 u}{\partial x^2}\cdot\frac{\partial}{\partial u}+\left(\frac{\partial v}{\partial y}\right)^2\cdot\frac{\partial^2}{\partial v^2}+\frac{\partial^2 v}{\partial y^2}\cdot\frac{\partial}{\partial v}+k^2n^2(u,v)-\beta^2\right\}E_x(u,v)=0.$$

# Bound Mode Calculation using the Fourier Decomposition Method

- Bound modes are expressed as a Fourier series with sine wave terms that go to zero at the boundaries of the unit square to satisfy the Sommerfeld and Neumman boundary conditions

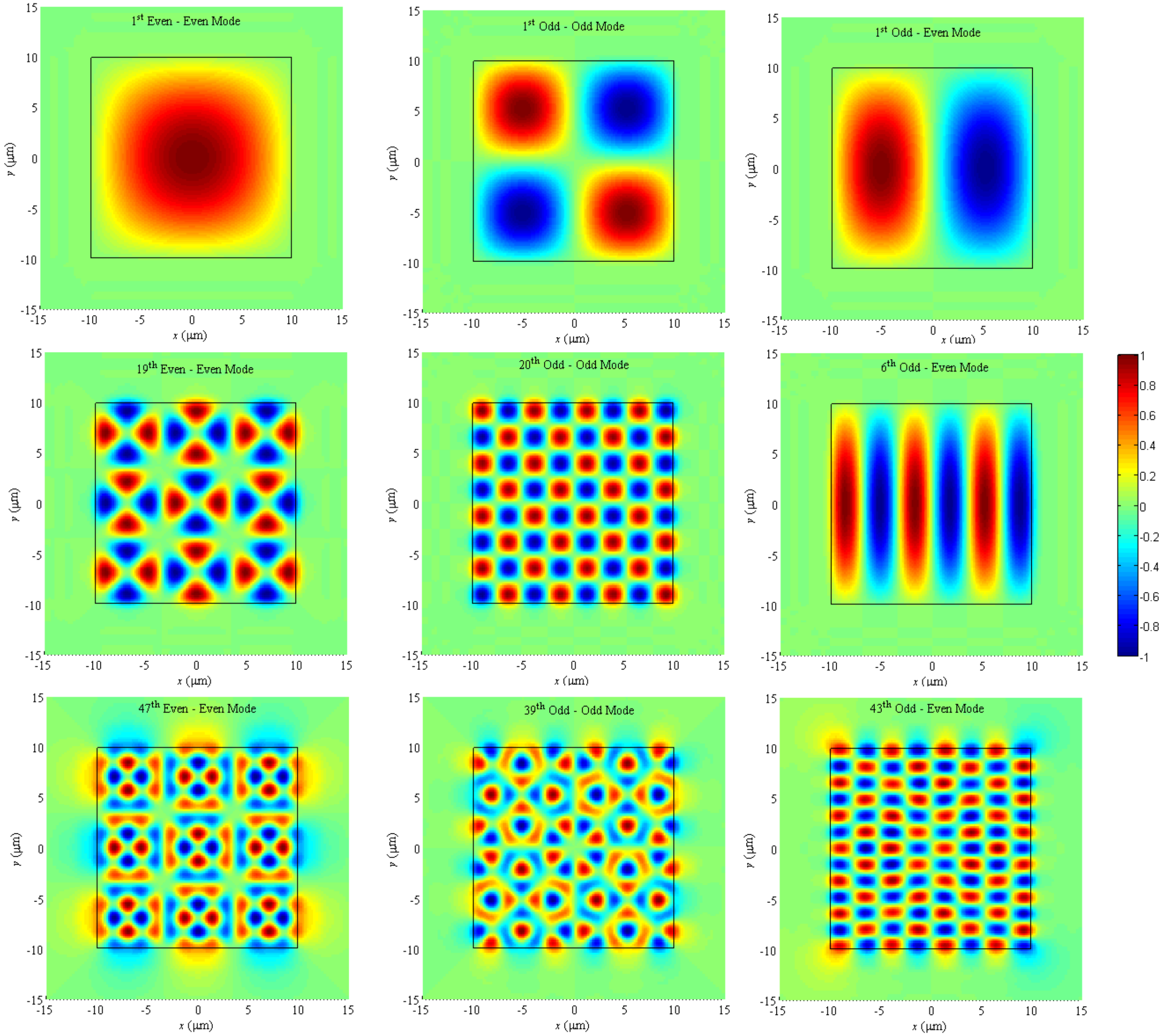
$$E_z(u,v) = 4\sum_{m=1}^{M}\sum_{n=1}^{N} c_{mn}\sin(m\pi u)\sin(n\pi v)$$

- The bound mode series is substituted into the wave-equation, multiplied by an elementary sine wave function and integrated to give an eigenvalue problem

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\left\{\mathbf{P}_{mn}^{m'n'}+\mathbf{Q}_{mn}^{m'n'}-(\beta^2-k^2n_{cl}^2)\delta_{mm',nn'}\right\}\cdot c_{mn}=0$$

- The real eigenvalues correspond to the propagation constants of the bound modes and the eigenvectors give the amplitude of the terms

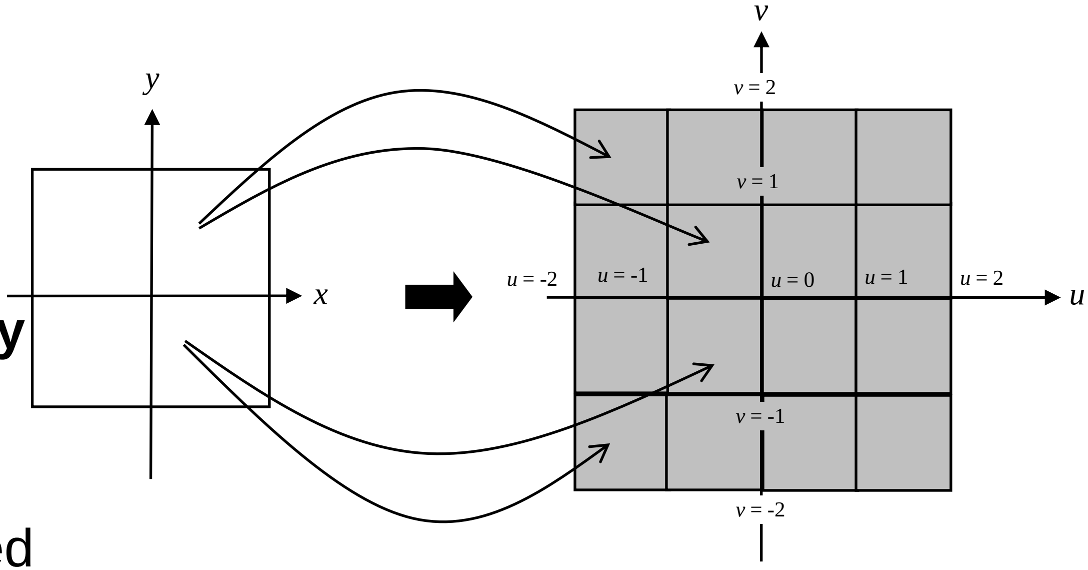
# Bound Modes found by the Fourier Decomposition Method



# Extension of FDM to calculate Radiation Modes

- Our extension of the Fourier Decomposition Method to enable it also to calculate radiation modes arises from two key observations:

1) The FDM Non-linear conformal transformation actually maps all of the *xy* domain into a **periodic array** of unitary squares in the *uv* domain and not just into a single unit square
2) Radiation modes do not need to satisfy boundary conditions at infinity in the *xy* domain and at the unit square boundaries in the *uv* domain



# Radiation Mode Calculation

- We express the radiation modes as a four term Fourier series, with the terms representing the various symmetries in the *u* (*x*), and *v* (*y*) directions

$$E_x^r(u,v) = 4\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\Big\{C_{mn}^{r,oo}\sin(2m\pi u)\sin(2n\pi v) + C_{mn}^{r,eo}\cos(2m\pi u)\sin(2n\pi v)$$
$$+ C_{mn}^{r,oe}\sin(2m\pi u)\cos(2n\pi v) + C_{mn}^{r,ee}\cos(2m\pi u)\cos(2n\pi v)\Big\}$$

- In addition, we express the radiation modes as a free-space field propagating in the cladding plus a response perturbation field due to the presence of the waveguide $E_x^r(u,v) = E_x^f(u,v) + E_x^R(u,v)$

$$E_x^r(u,v) = \overbrace{4\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\Big\{C_{mn}^{f,oo}S_{mn}^{oo}(u,v) + C_{mn}^{f,eo}S_{mn}^{eo}(u,v) + C_{mn}^{f,oe}S_{mn}^{oe}(u,v) + C_{mn}^{f,ee}S_{mn}^{ee}(u,v)\Big\}}^{E_x^f(u,v)}$$
$$+ \overbrace{4\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\Big\{C_{mn}^{R,oo}S_{mn}^{oo}(u,v) + C_{mn}^{R,eo}S_{mn}^{eo}(u,v) + C_{mn}^{R,oe}S_{mn}^{oe}(u,v) + C_{mn}^{R,ee}S_{mn}^{ee}(u,v)\Big\}}^{E_x^R(u,v)}.$$

# Radiation Mode Calculation

- By inserting radiation mode series into the wave equation, multiplying with an elementary set of harmonic functions and integrating, we obtain a system of linear equations

$$\sum_{m=0}^{M}\sum_{n=0}^{N}\left\{\mathbf{P}_{mn,m'n'}^{ij}+\mathbf{Q}_{mn,m'n'}^{ij}-(\beta^2-k^2n_{cl}^2)\delta_{mm',nn'}\right\}\cdot C_{mn}^{R,ij}$$

$$=\sum_{m=0}^{M}\sum_{n=0}^{N}\mathbf{P}_{mn,m'n'}^{ij}\cdot C_{mn}^{f,ij},$$

- As the multiplying elementary set of harmonic functions are orthogonal to some terms in our Fourier Series in the *uv* domain those terms vanish during the integration
- This leaves only one set of symmetries
- An eigenvalue problem is not formed as in the case of the bound modes as the radiation modes form a continuum

# Radiation Mode Calculation

- Each free-space mode in the cladding generates a response perturbation and thus a radiation mode via

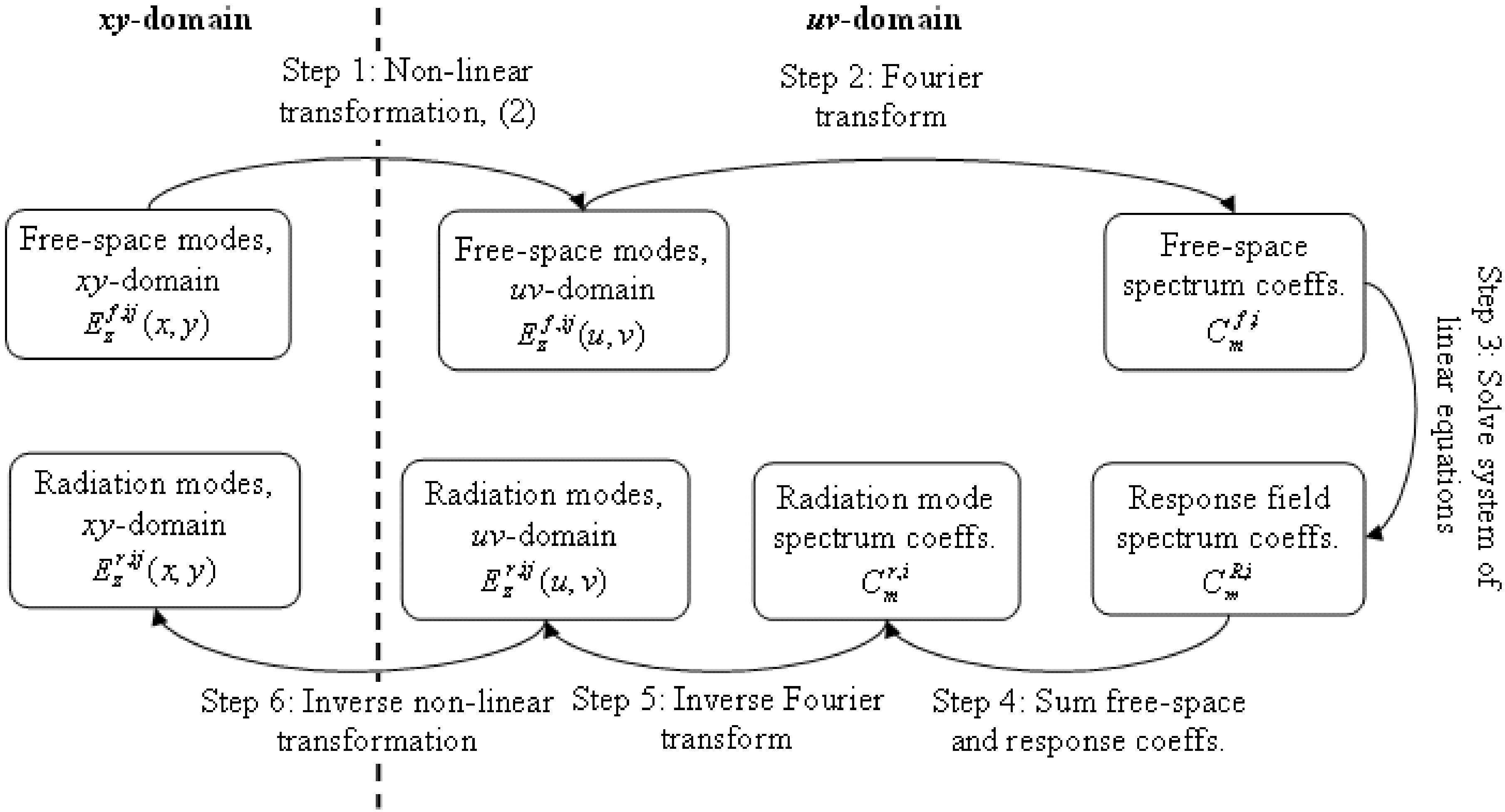
$$E_x^r(u,v) = E_x^f(u,v) + E_x^R(u,v)$$

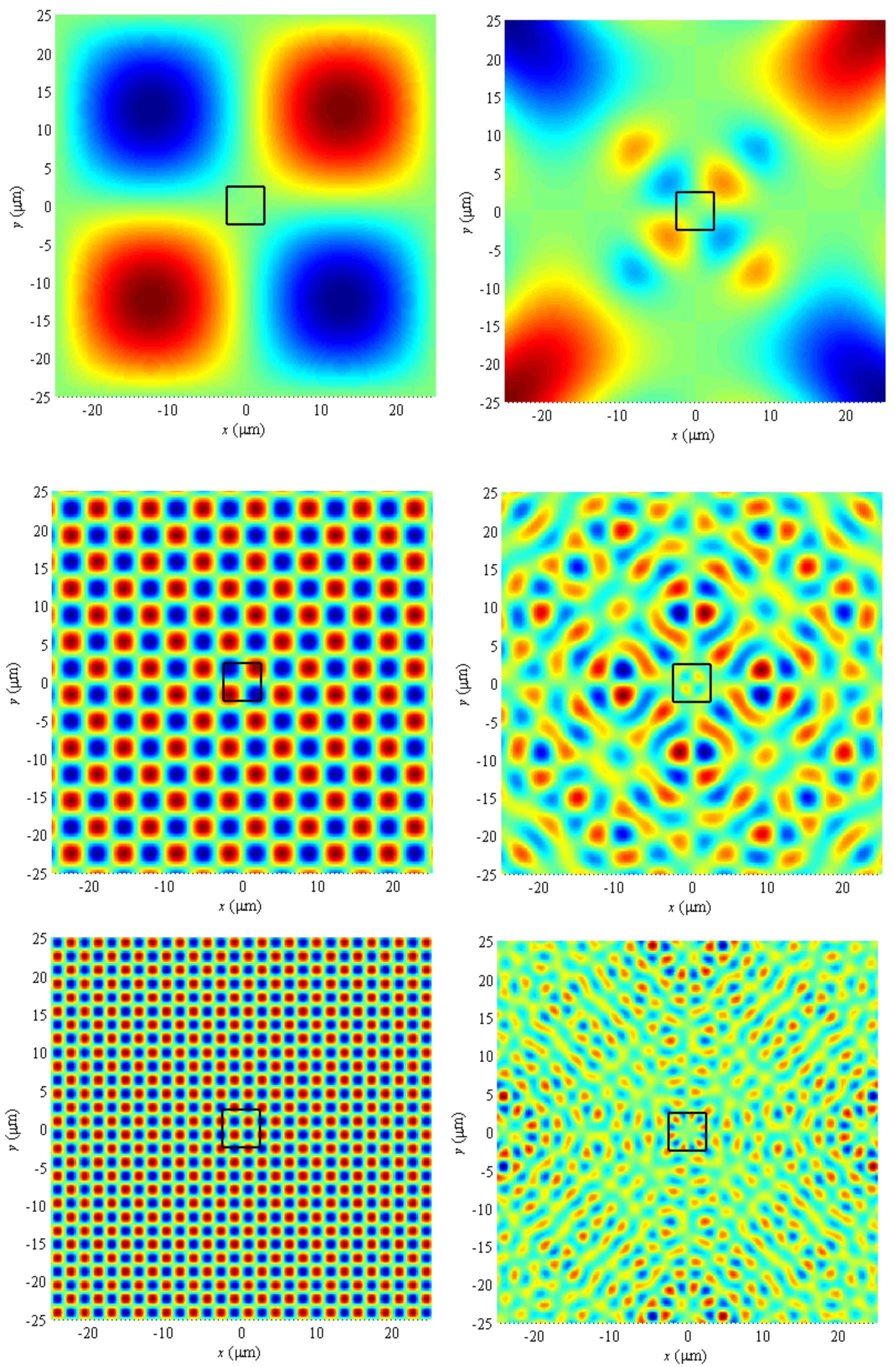
- By scanning through all free space fields with propagation constants $-kn_{cl.} < \beta_r < kn_{cl.}$ we can thus generate all radiation modes in a waveguide

- Normalization of the entire radiation mode results from normalizing the free-space part only

- Our semi-analytical method is not restricted to waveguides with rectangular cross-section but can be applied to arbitrary waveguides if we divide them into sufficiently thin rectangular elements

# Radiation Mode Calculation

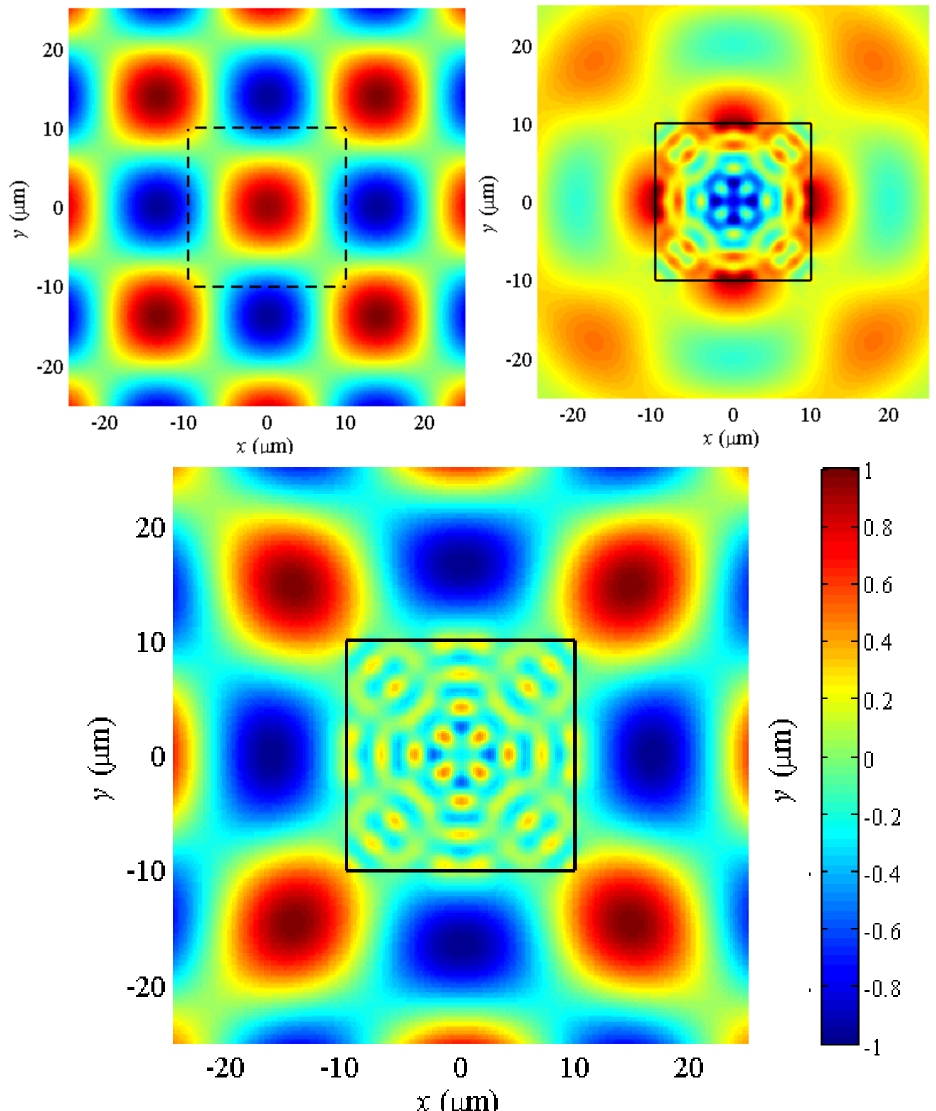





# Free Space Fields and Corresponding Calculated Response fields

Small square shows square waveguide location

# Radiation Mode Calculated




Free Space Field and Corresponding Calculated Response field

Radiation Mode composed of combined Free Space Field and Calculated Response Field

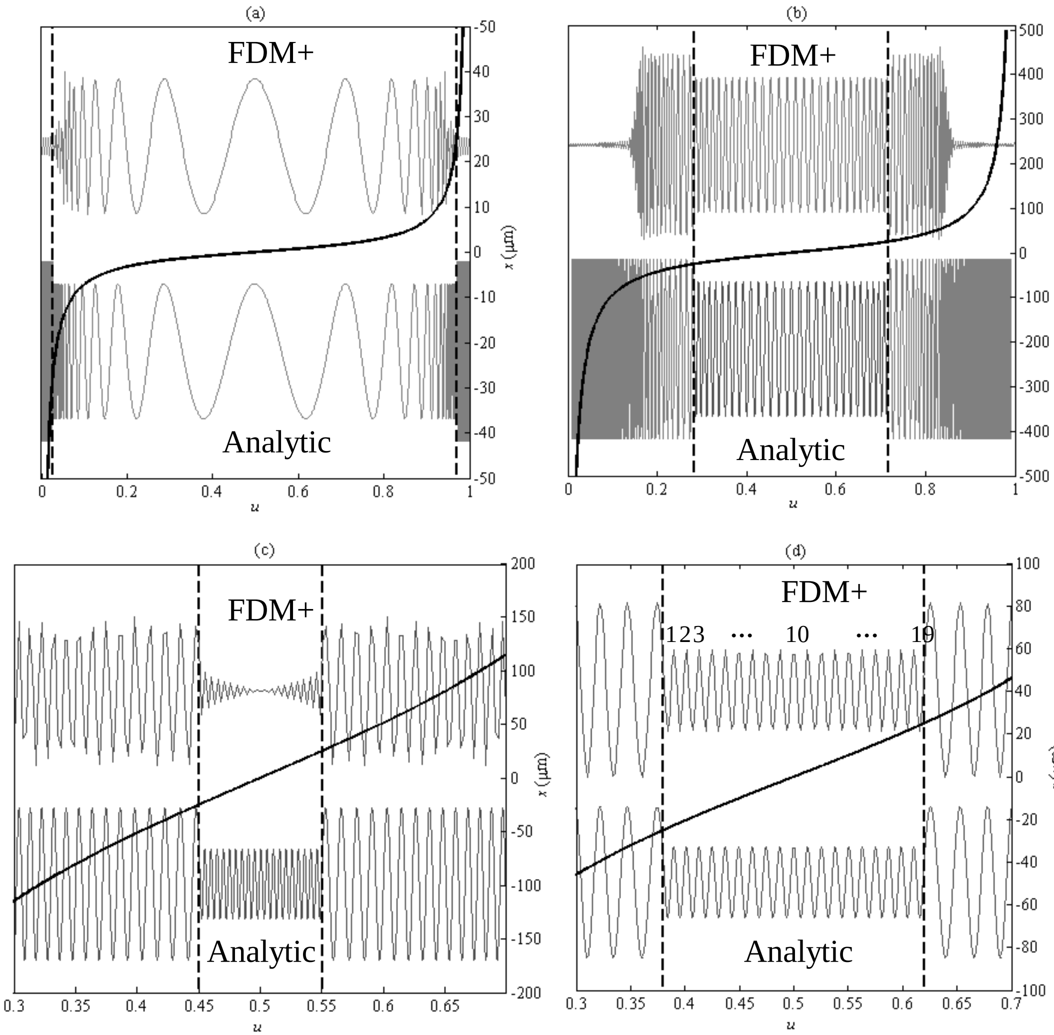
Square outline shows square waveguide location

# Method Verification, Algorithm Optimization

- To verify the calculation method the radiation modes calculated for 2D slab waveguides are compared with analytic solutions
- Dashed lines mark the position of the waveguide walls
- Good agreement depends on an optimum choice of the conformal transformation scaling parameters *p* and *q*

$$x = p\tan\left[\pi\left(u-\frac{1}{2}\right)\right], y = q\tan\left[\pi\left(v-\frac{1}{2}\right)\right]$$

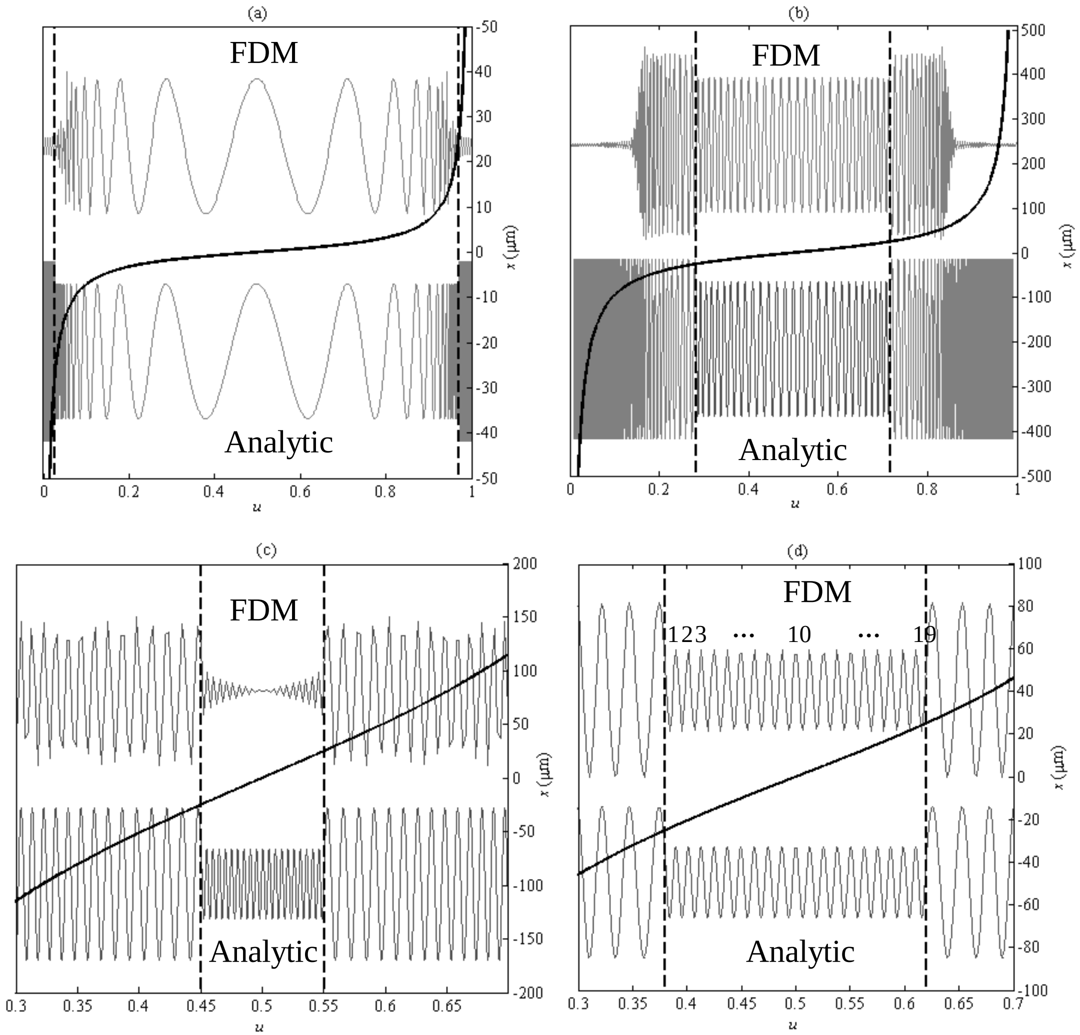
$w$ = 20 μm, $n_{co}$ = 1.556, $n_{cl}$ = 1.5249 and $\lambda$ = 0.85 μm.



# Method Verification, Algorithm Optimization

- If “small” scaling parameters are used, the FDM method fails to resolve the very fast oscillating field close to the waveguide boundaries because we operate in the non-linear regime of the transformation

- If “too large” scaling parameters are used, waveguide boundaries are brought too close to each other in the *uv*-domain and fields become highly oscillatory inside the core



# Algorithm Optimization

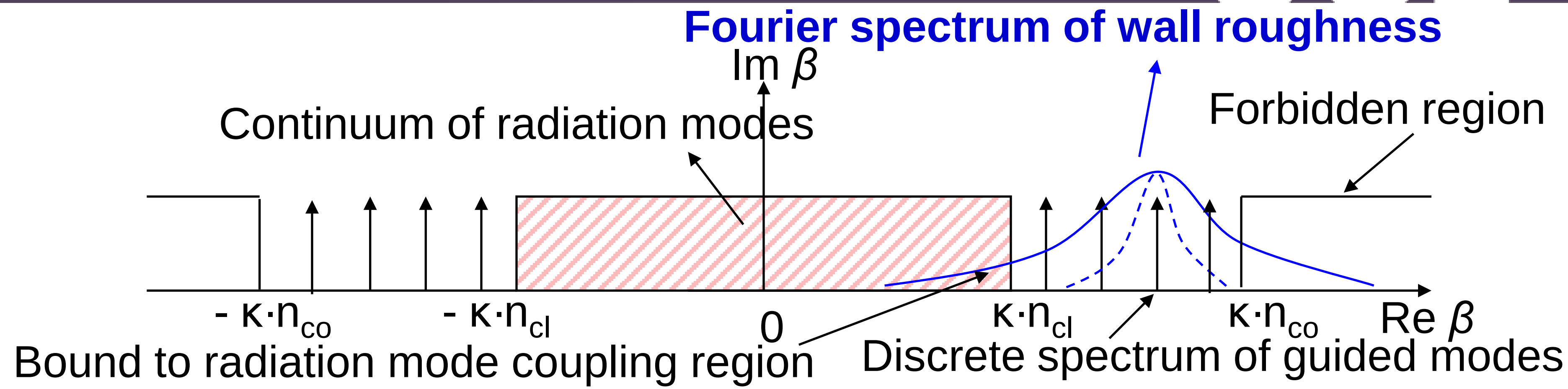
- In both cases of small and large scaling constants the high frequency oscillations require a very large number of Fourier components *M*, *N* to adequately describe them

- Optimization (minimum number of Fourier components) is achieved if *p* and *q* are selected to balance the frequency components needed to describe the field inside the waveguide and just outside of it

# Mode Equilibrium and Equilibrium Length

- Waveguide sidewall roughness couples bound modes to each other and couples the bound modes to the radiation modes resulting in loss.
- Bound Modes propagate inside the waveguide exchanging energy with each other and transferring energy to the radiation modes until they reach an equilibrium or steady state
- The average power, over the waveguide length, for each mode remains constant at equilibrium (fluctuations in the instantaneous power in each mode still occur)
- When equilibrium is reached the ratio of the average power in one mode compared to the average power in another mode is constant.
- The equilibrium modal power distribution remains the same but scales down in magnitude with distance travelled due to loss to radiation modes and material loss.
- The distance from the launch point to the establishment of the equilibrium distribution is known as the “equilibrium length”
- The equilibrium length is important to determine whether cascaded waveguide bends or crossings can be treated as independent entities or whether they must be considered as a whole combination

# Waveguide Roughness: Coupling Mechanism




- Each guided mode is coupled to other guided modes and radiation modes via the Fourier spectrum of the roughness function
- A broad spectrum (short correlation lengths) couples **all** bound modes to each other and to the radiation modes. However, higher order modes (close to radiation spectrum) are coupled more strongly to the radiation modes and so have higher loss. At equilibrium the lower order modes remain.
- A narrow spectrum (long correlation lengths) couples only the higher order modes to the radiation spectrum and the guided modes are only coupled to their neighbours. So, the loss is determined by the coupling of higher order modes to radiation modes. At equilibrium many modes remain.
- Waveguides typically used for optical interconnects belong in the latter category

# Coupled Power Theory

- Coupled power theory expresses coupling between modes due to wall roughness by means of a system of first order differential equations
- Wall roughness is assumed to be a non-stationary process (random) which follows either Gaussian or exponential statistics

$$\frac{dP_p}{dz} = -2a_p P_p + \sum_{q=1}^{Q} h_{pq}(P_q - P_p)$$

$$h_{pq} = T_{pq}^2 \left\langle \left| F(\beta_q - \beta_p) \right|^2 \right\rangle$$

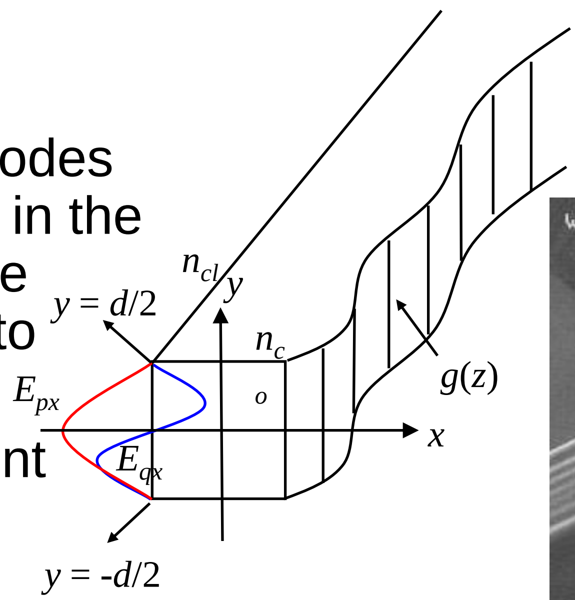
$$T_{pq} = \frac{\omega \varepsilon_o}{4i} \int_{-d/2}^{d/2} (n_{cl}^2 - n_{co}^2) \left[ (n_{cl}^2 / n_{co}^2) \mathbf{E}_{px}^* \cdot \mathbf{E}_{qx} + \mathbf{E}_{pz}^* \cdot \mathbf{E}_{qz} \right]_{x=w/2} dy$$

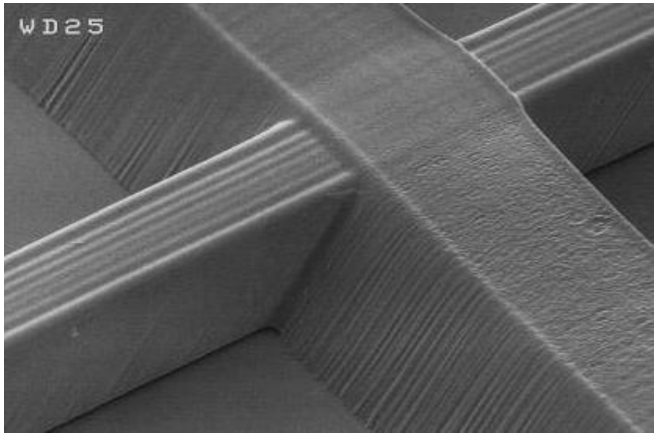
# Two Independent Equilibrium Mode Distributions

- Coupling coefficients between modes depend on an integral in the direction of the waveguide thickness

$$T_{pq} = \frac{\omega\varepsilon_o}{4i}\int_{-d/2}^{d/2}(n_{cl}^2 - n_{co}^2)\left[(n_{cl}^2/n_{co}^2)\mathbf{E}_{px}^* \cdot \mathbf{E}_{qx} + \mathbf{E}_{pz}^* \cdot \mathbf{E}_{qz}\right]_{x=w/2} dy$$

- Since many fabricated waveguides exhibit 1D roughness only (in the propagation direction), modes with different symmetries in the direction of the waveguide thickness cannot couple to each other
- Therefore two independent modal distributions must coexist at equilibrium

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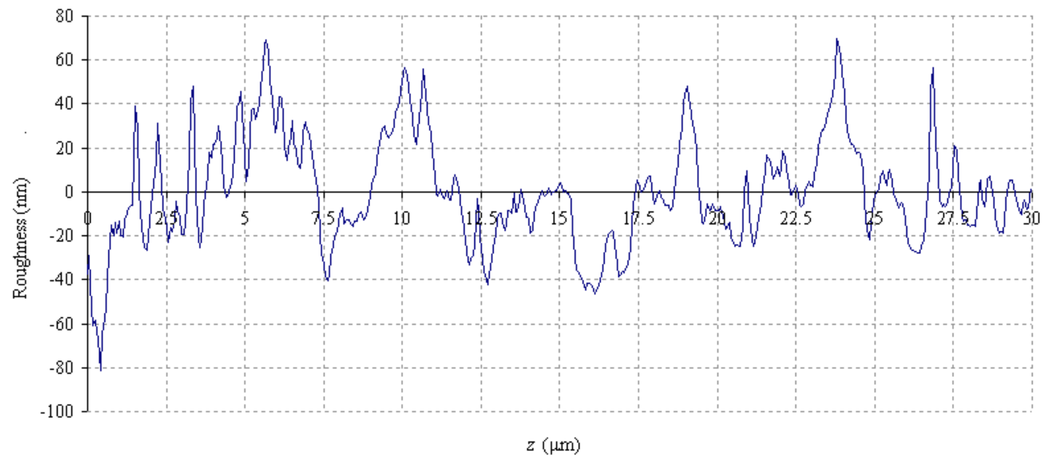
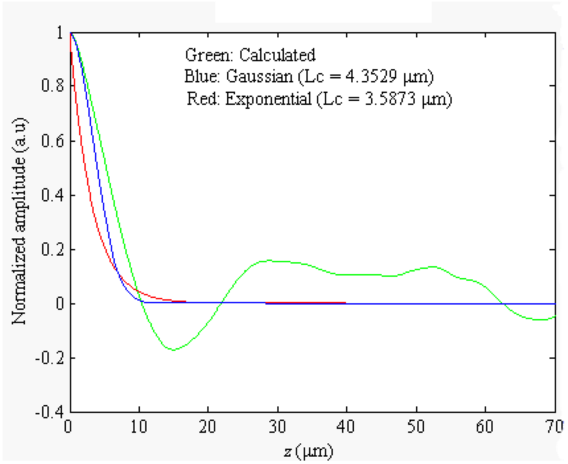
# AFM Measurements of Waveguide Sidewall Roughness

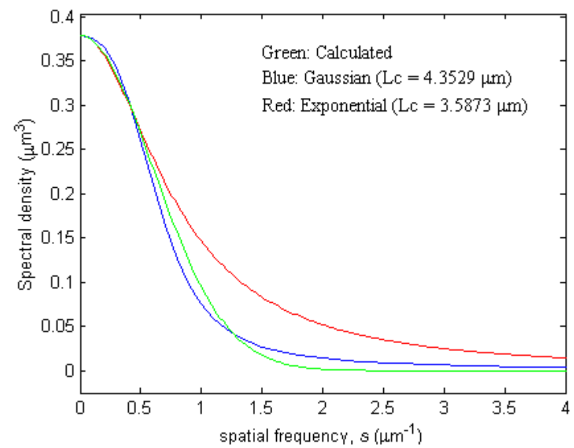
- Measurements of the waveguide sidewall roughness using Atomic Force Microscope on multimode polymer waveguides fabricated photolithographically

- Standard deviation of wall roughness is in the range
  9 nm < $\sigma$ < 74 nm

- Roughness autocorrelation length is in the range
  2 µm < $L_c$ < 8 µm



# Integration region in k-space

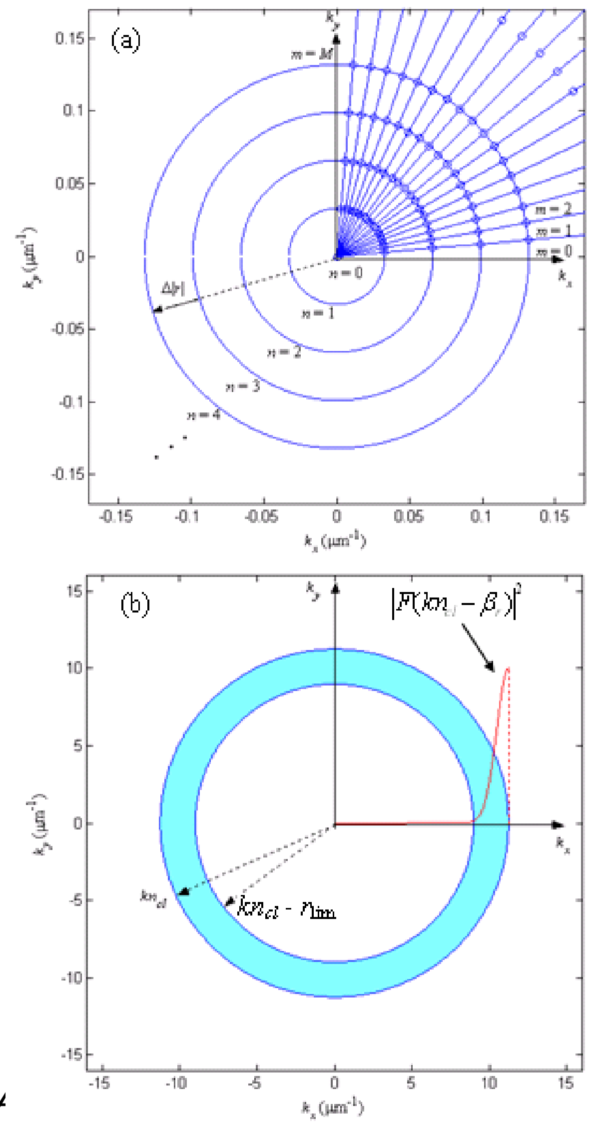
- Free-space wavenumbers satisfy the condition

$$k_x^2 + k_y^2 + \beta_r^2 = r^2 + \beta_r^2 = k^2 n_{cl}^2$$

$$r^2 = k_x^2 + k_y^2$$

$$k_y = r \sin\theta \qquad k_x = r \cos\theta$$

- The locus of $k_x$ and $k_y$ in the wavenumber space is a disk with radius $0 < r < kn_{cl}$.
- Radiation modes with $r \sim 0$ oscillate very rapidly and need a large number of Fourier components in order to resolve them
- However, as the measured autocorrelation length corresponds to a small distance in k-space, we need only consider radiation modes in a thin annulus with $r \sim kn_{cl}$



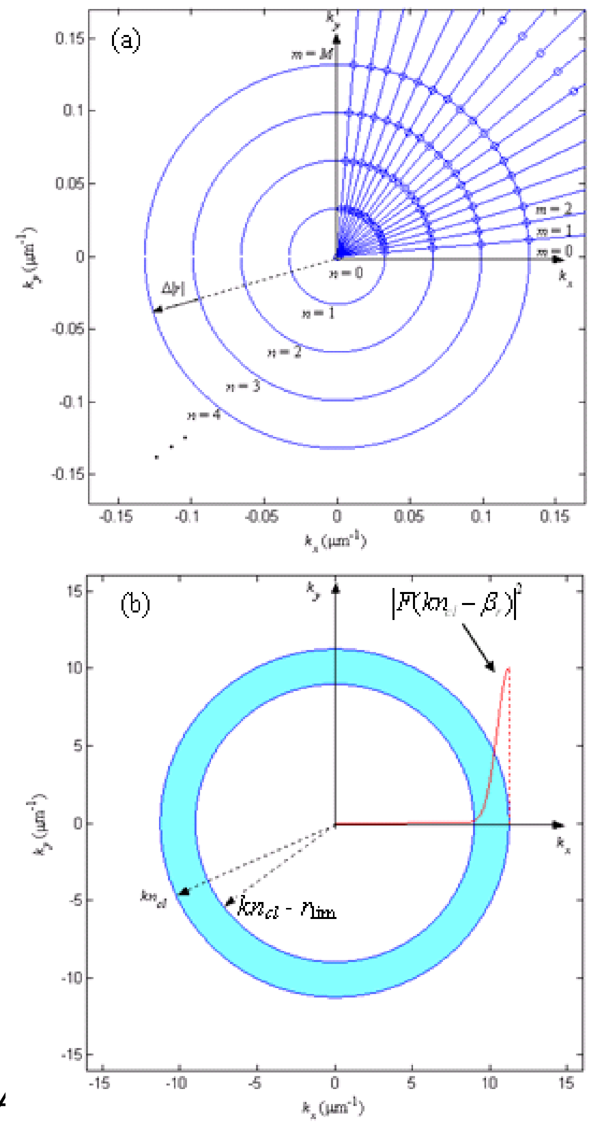
# Segmentation of radiation modes

- To numerically solve the coupled power equations the radiation mode spectrum is digitized

- For a 50 μm  50 μm waveguide 2880 radiation modes are needed to accurately calculate attenuation and equilibrium distribution for $L_c$ = 2 μm

- For the same waveguide we require $M$ = $N$ = 192 Fourier components for each radiation mode leading to 2880 systems of linear equations with $192^2$  $192^2$ equations. Parallel computing had to be used

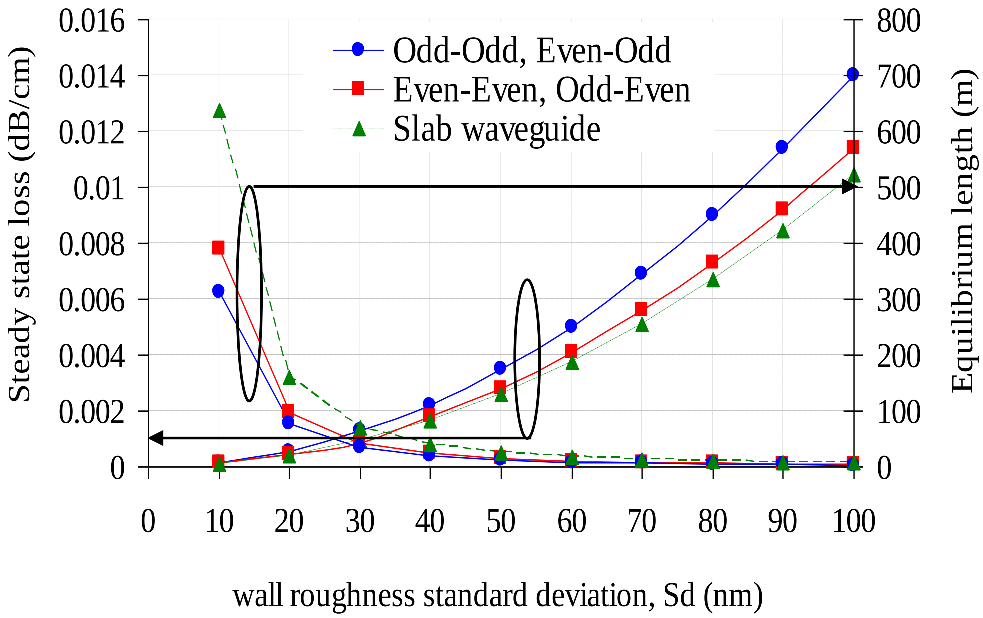
-  Each radiation mode calculation took ~17 min on a 4  4 processor grid with 900 MHz processors, 4GByte RAM nodes



# Equilibrium Loss and Length

- First keeping the roughness autocorrelation length, $L_c$, constant and ignoring material loss
- As the sidewalls become rougher in terms of roughness amplitude standard deviation, $\sigma$,

1) The propagation loss increases
2) The equilibrium length decreases

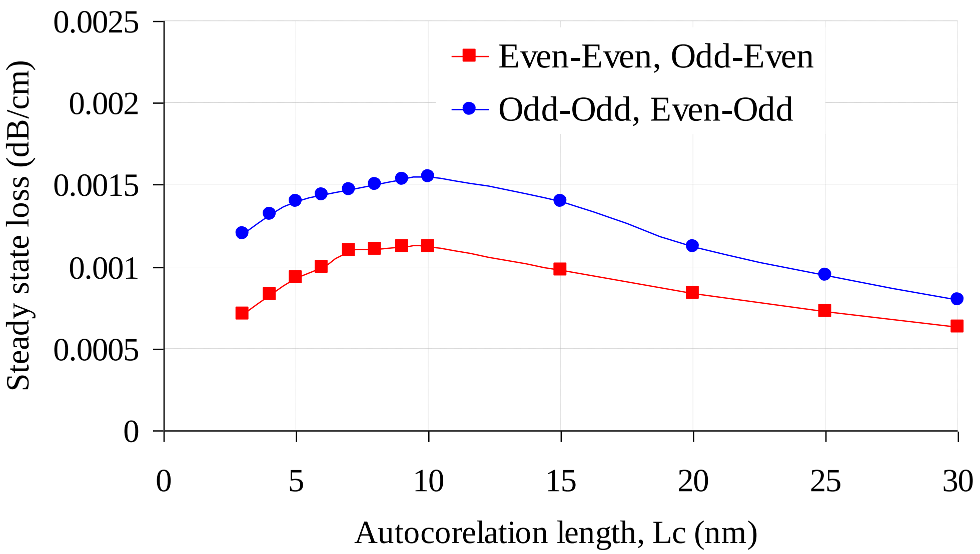



- Rougher waveguide sidewalls couple the bound modes more strongly to the radiation modes giving more loss
- Rougher waveguide sidewalls couple the bound modes more strongly to each other resulting in a faster establishment of the equilibrium distribution

# Equilibrium Loss and Length

- Now keep the standard deviation of the sidewall roughness amplitude, $\sigma$, constant
- As the autocorrelation length, $L_c$ of the sidewall roughness increases

1) The propagation loss decreases
2) The equilibrium length increases



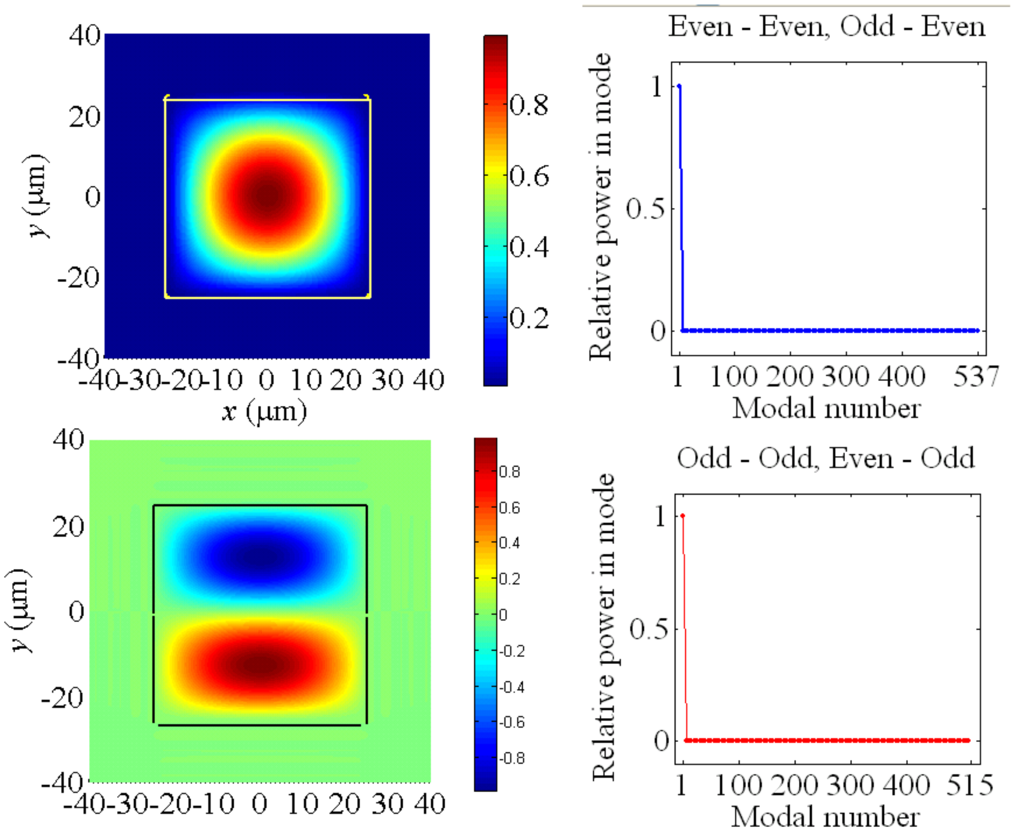

- Increasing the autocorrelation length of the sidewall roughness results in less random roughness in a narrower spatial frequency range
- The coupling to the radiation modes is less giving less loss.
- The coupling between the bound modes is less so it takes longer to establish an equilibrium distribution

For $L_c$ = 4 µm, $\sigma$ = 30 nm,
Even-even, odd-even modes:
$\alpha_{eeoe}$ = 0.0069 dB/cm
equilibrium length $L_{eeoe}$ = 10.3 m.
Odd-odd, even-odd modes:
$\alpha_{ooeo}$ = 0.0062 dB/cm
equilibrium length $L_{ooeo}$ = 18 m

# Ultimate Modal Equilibrium Distribution

- Finally after a long propagation distance or strong wall roughness, for all practical roughness standard deviation and autocorrelation lengths, only two modes remain.

- The two modes are the lowest order even and odd modes in the *y* direction.



# Conclusions

- A new algorithm using a non-linear transformation together with an extended Fourier Decomposition Method and a perturbation approach has been developed to calculate radiation modes.
- The radiation modes of highly multimode rectangular waveguides have been calculated for the first time.
- A relatively large number of Fourier components is required to calculate the radiation modes and parallel computing had to be used
- The rank of the linear systems can be significantly reduced by utilizing sparse matrix methods
- Modes with different symmetries in the waveguide thickness cannot interact with each other for 1D waveguide sidewall roughness
- A new result is that two modal sets propagate independently from each other inside the waveguide and gradually converge to two uncorrelated distributions

# Conclusions

- Propagation loss increases as either the standard deviation of the roughness increases or the autocorrelation length reduces
- Propagation loss has a maximum value when the autocorrelation length is about 0.2 times the width of the waveguide for any standard deviation
- The concept of convergence to an equilibrium modal distribution which just scales down with distance was not found to be supported except in the case of two single modes of different symmetry finally remaining.
- These equilibrium lengths, for all waveguides under investigation, were found to be much larger than the size of optical backplanes

# Acknowledgements


- Thanks to Dr. Jeremy Yates and UCL research computing services for use of the Keter SUN cluster for the parallel computing calculations

- Thanks for funding by the UK EPSRC via a DTA and an IeMRC OPCB Flagship Project grant. Ioannis Papakonstantinou was additionally supported by Xyratex Technology Ltd.

- Thanks to Kai Wang, UCL, Dave Milward, Ken Hopkins, and Richard Pitwon, Xyratex Technology Ltd., Navin Suyal, Exxelis Ltd., Aongus McCarthy, Prof. Andy Walker and Frank Tooley, Photonix for fruitful discussions

THANK YOU FOR YOUR ATTENTION