Structural models of the labour market and the impact and design of tax policies

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I, Andrew James Shephard, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Andrew James Shephard | Date

To my parents

Abstract

This dissertation is concerned with the estimation of structural models of the labour market and the application of these models in both evaluating policy reforms, and exploring their implications for taxation design. The programme that is at the centre of much of the empirical exploration in this thesis is the British Working Families' Tax Credit (WFTC), which during its lifetime, provided the main form of in-work support for lower income families with children.

The first chapter of this thesis estimates a discrete choice hours of work model using data from before and after the introduction of WFTC. To the extent that behavioural responses to tax reforms are informative about preferences, it uses the estimated model directly to explore problems related to the optimal design of the tax and transfer system. It derives new theoretical results and empirically explores the extent to which the tax authorities may wish to condition the tax schedule on age of children. Given the use of hours contingent payments in the UK tax credit system, it also investigates the desirability of including a measure of hours of work in the tax base.

The second and third chapters of this thesis firstly develop the methodology, and then consider how our view of programmes such as WFTC is affected once the presence of labour market frictions and the importance of job search activity is acknowledged. In doing so, it greatly extends the empirical equilibrium job search literature. By introducing the monopsonistic behaviour of firms, it considers how these firms may optimally adjust their wages following the introduction of programmes which encourage work, such as WFTC. The equilibrium impact of the reform on a range of outcomes for both WFTC-eligible and non-eligible workers is assessed.

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London, June 2010

Data Acknowledgements

Chapter 1 of this thesis makes use of Family Resources Survey and Family Expenditure Survey data. Chapters 2 and 3 of this thesis make use of Quarterly Labour Force Survey data. Material from the Family Resources Survey is crown copyright material, produced by the Department for Work and Pensions, and made available by the Economic and Social Data Service. Material from the Family Expenditure Survey data is crown copyright material, collected by the Office for National Statistics and made available through the Economic and Social Data Service. Material from the Quarterly Labour Force Survey is crown copyright material, and has been made available by the Office for National Statistics through the Economic and Social Data Service. Crown copyright material is reproduced with the permission of the Controller of HMSO and the Queen's Printer for Scotland. The Department for Work and Pensions and the Economic and Social Data Service bear no responsibility for the analysis or interpretation presented herein.

Conjoint work

Chapter 1 of this thesis is based on work conducted jointly with Richard Blundell. I thank Professor Blundell for allowing me to draw on this work in this thesis.

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Introduction

This dissertation is concerned with the estimation of structural models of the labour market, and the application of these models in both evaluating policy reforms, and exploring their implications for taxation design. It is motivated by the secular decline in the relative wages of low skilled workers and the growth in single-parent households, which has resulted in the emergence of a number of programmes which aim to enhance the labour market attachment of such workers. The particular programme that is at the centre of much of the empirical exploration in this thesis is the British Working Families' Tax Credit (WFTC), which during its lifetime, provided the main form of in-work support for lower income families with children.

Tax credit programmes like the British WFTC, and the similar Earned Income Tax Credit programme in the U.S., have been studied extensively in the economics literature and attract considerable attention among policy makers. The broad employment impacts of these programmes has been evaluated in a large number of empirical studies, and there is a general consensus in the literature that these programmes have been particularly effective in raising the employment of lone mothers, one of the main beneficiaries of these programmes. One objective of this thesis is to contribute to this literature.

This thesis is comprised of three self-contained chapters. The first chapter of this thesis, *Employment, Hours of Work and the Optimal Taxation of Low Income Families* (joint with Richard Blundell) estimates a stochastic discrete choice hours of work model using data from before and after the introduction of WFTC. To the extent that behavioural responses to tax reforms are informative about preferences, it uses the estimated model directly to explore problems related to the optimal design of the tax and transfer system. The use of a structural model to consider optimal design problems contrasts to the usual approach taken in the theoretical optimal tax research. In this literature, formulae for optimal marginal tax rates are typically derived in terms of quantities including labour supply elasticities, the distribution of worker types, and the preferences of the government. The advantage of this structural approach to optimal taxation is that it introduces a consistency between the labour supply model and the optimal tax model. Moreover, features which are emphasised in the micro-econometric labour supply literature – such as unobserved heterogeneity, fixed work related costs of work, child-care expenditure, and the detailed non-convexities of the tax and transfer system – are all allowed to influence the choice of the tax policy.

New theoretical results which are useful for solving this class of problem are derived, and the extent to which the tax schedule may vary with the age of children is explored. The paper shows that pure tax credits (that is, negative marginal effective tax rates) may be optimal for mothers with older children. Given the use of hours contingent payments both in the tax credit system of the UK and in other countries, the chapter also explores how the optimal schedule changes once the partial observability of hours of work is explicitly incorporated. In contrast to the design of the current UK system, a case for primarily rewarding full-time workers is presented, and the welfare gains from doing so are shown to be potentially large for families with older children. The paper then demonstrates that the realisation of appreciable welfare gains from using hours of work information is contingent upon this information being accurately observed.

The type of structural model developed in *Employment, Hours of Work and the Optimal Taxation of Low Income Families* ruled out the presence of any equilibrium effects. The second and third chapters of this thesis introduce a very different class of structural model to consider the impact of programmes like WFTC. They begin with the premise that labour market frictions and job search activity are important. Empirically they are considered important because it takes individuals time to find a suitable job, and because the largest adjustments in both wages and working hours typically occur when individuals change their employer. The presence of these frictions may also have implications for our understanding of reforms like WFTC. In particular, if firms set wages then the presence of search frictions tends to bestow some degree of monopsony power upon firms. Should labour supply increase following a reform like WFTC, then firms may respond to this by lowering the wages that they offer. This would effectively reduce the transfer given to eligible families, while non-eligible families may be made worse off if they compete within the same labour market. Understanding the quantitative importance of these equilibrium effects is therefore an essential part of any assessment of policies that are designed to encourage work.

Before applying these models to study the impact of WFTC, the second chapter of this thesis, *Wage Posting with Two-Sided Heterogeneity* provides a methodological review of empirical search models. It begins by constructing an equilibrium model of the labour market with heterogeneous workers and firms: workers differ in their unobserved work opportunity costs, while firms differ in their productivity. It provides a synthesis of the existing literature and extends it by allowing the arrival rates of job offers – one of the key structural parameters in these models – to vary with employment status. It also demonstrates how to introduce further within market heterogeneity, which both allows the model to better account for differences in labour market outcomes across individuals, and also permits differential responses following policy reforms. Theoretical properties of the model are derived, numerical simulations are presented, and the model is shown to be empirically tractable. In particular, the type of semi-parametric estimation technique that has been used in simpler job search models is generalized to this setting. Estimation results using UK data are presented, and the results obtained from the proposed semi-parametric procedure and alternative parametric specifications are compared. Illustrative simulations are then presented which use the estimated model to explore the equilibrium impact of actual UK minimum wage legislation.

The theoretical and empirical framework developed in the second chapter is extended and applied in the final chapter of this thesis, *Equilibrium Search and Tax Credit Reform*, which uses an equilibrium job search model with wage posting to analyse the labour market impact of the WFTC reform. The model allows for a rich characterisation of the labour market, by incorporating hours responses, accurate representations of the UK tax and transfer system, and both worker and firm heterogeneity. Here, workers may differ in their opportunity costs of work, as well as their transitional parameters and the tax and transfer system that they face. In addition to setting wages, firms also choose a recruiting effort intensity, which allows the arrival rate of job offers to be endogenized at the macroeconomic level, and thereby introduces a further equilibrium channel. The model is estimated using pre-reform longitudinal survey data using an extension of the semi-parametric estimation proposed in the previous chapter, and the impact of actual tax reform policies is then simulated.

The main simulation and estimation results are performed under the assumption that all family types (including those who are both eligible and ineligible for tax credits), are operating within a single integrated market. In this setting, the model predicts that WFTC (and the contemporaneous reforms) increased employment, with lone mothers experiencing the largest employment increase. While equilibrium effects do play a role in this reform, they are found to be small, with the changes in employment, earnings and working hours, being primarily due to the direct effect of the changing job acceptance behaviour of workers. Indeed, the predicted employment impacts are comparable with those obtained in the quasi-experimental literature, which is consistent with the view that these equilibrium effects are not large. One reason why these monopsonistic equilibrium effects are relatively small is because firms may be reluctant to reduce wages if doing so adversely affects the number of non-eligible workers that they may attract. It is demonstrated that the equilibrium effects of the same reforms may be much larger if the labour market is solely comprised of lone mothers, one of the main beneficiaries of WFTC.

The analysis of tax policy necessitates a detailed characterisation of the tax and transfer system. The UK system is particularly complicated due to the way that many elements of the tax and transfer system interact with each other. To facilitate the empirical analyses performed in this thesis, a very efficient and flexible micro-simulation tax library called *FORTAX* has been developed, which allows very detailed components of income to be simulated under a range of actual and hypothetical tax and transfer systems. This model was applied extensively in both *Employment, Hours of Work and the Optimal Taxation of Low Income Families* and *Equilibrium Search and Tax Credit Reform*, and is described in the appendix to this thesis. The entire code has been made freely available under the GNU General Public License version 3 (GPLv3), with the hope that it will encourage other researchers to perform detailed empirical analysis of tax policy.

Chapter 1

Employment, Hours of Work, and the Optimal Taxation of Low Income Families[1]

1.1 Introduction

The empirical analysis of labour supply behaviour has strong implications for the design of earnings taxation. Our aim here is to use a microeconometric labour supply model to assess the design of tax rate reforms for the low paid. In particular, to examine policies that aim at reducing the effective tax rates on work for low income families, as in the significant expansions of earned income tax credits in the UK and the US.[2]

Tax credit reforms have been evaluated extensively in the UK and elsewhere. The evidence that tax credit policies encourage work is compelling and the positive impact on employment has been found to be particularly strong for single mothers, see for example Eissa and Liebman (1996) and Blundell et al. (2000). These and other studies tell us about the labour supply impact of tax credit reforms. Given that such labour supply responses also help us to learn about preferences, it is possible to move beyond the evaluation of particular reforms, and consider problems related to the optimal design of the tax and transfer system. In the spirit of Mirrlees (1971), we shall ask: how should the government best allocate a fixed amount of revenue to the design of earnings taxation?

The analysis draws on the microeconometric and the optimal taxation literature. In the microeconometric literature certain common and robust features of estimated labour supply responses of the low paid have emerged. Specifically, the importance of distinguishing between the intensive margin of hours of work and the extensive margin where the work decision is made. Labour supply elasticities appear to be much larger at the extensive margin, at least for certain household demographic types, see Blundell and Macurdy (1999).

The optimal taxation literature explores consequences for design. In parallel with the empirical regularities, the literature on the design of tax and transfer systems has increasingly focussed on the extensive margin and the use of work conditions, see for example Beaudry et al.

[1] This chapter based on work conducted jointly with Richard Blundell.

[2] See Blundell and Hoynes (2004), for example.

(2009), Besley and Coate (1992), Choné and Laroque (2005), Laroque (2005), Moffitt (2006), Phelps (1994) and Saez (2001, 2002). Our approach is closest to that by Saez (2002) who, building on earlier work by Diamond (1980), examines the optimality of tax credit designs within a Mirrlees framework but one which acknowledges the distinction between the extensive margin and intensive margin of labour supply. Indeed, Saez (2002) derives approximate optimal tax formula in terms of representative labour supply elasticities at the extensive and intensive margin. Recently, Immervoll et al. (2007) implement this approach and suggest that for reasonable welfare weights, tax credits would be an optimal policy across a wide set of economies. As part of the Mirrlees Review, Brewer et al. (2009) use this approach to explore the taxation of families in the UK.

The contribution of this paper is threefold. First, we take the structural model of employment and hours of work seriously in designing the structure of taxes and transfers, allowing the distribution of earnings, fixed costs of work and demographic differences to influence the design of tax policy. Second, we consider the case where hours of work are partially observable to the tax authorities and consider the case for hours contingent reforms. Third, we assess the role of conditioning on the age of children in the rate schedule for earnings taxation.

Our exploration of hours contingent reforms is motivated by the common use of hours based eligibility in the tax credit systems of countries like the UK, Ireland and New Zealand. Hours information is also used in the design of work conditioned earnings supplements, for example in the Canadian Self-Sufficiency Project (Card and Robins, 1998) and in the TANF programme of welfare payment in the US (Moffitt, 2003). It has also been proposed as a mechanism for improving tax design, see Keane (1995), although not within an optimal tax framework. Given the likely difficulties in recording and monitoring hours of work, our analysis also considers scenarios where hours are subject to measurement error, or where individuals may directly misreporting their hours of work to the tax authorities.

The microeconometric analysis is based on a stochastic discrete choice labour supply model (Hoynes, 1996; Keane and Moffitt, 1998; Blundell et al., 2000; van Soest et al., 2002). This model allows for discrete choices over non-linear budget constraints and fixed costs of work to re-examine the optimal design problem. The optimal tax model is then derived directly from the labour supply model together with the estimated distribution of earnings, fixed costs of work, childcare costs, demographic differences and unobserved heterogeneity.[3]

The analysis is set in a static environment with fixed costs of work and stigma costs of accessing welfare benefits. We are therefore ignoring dynamic effects in both labour supply choices and in the design of the tax structure. Our focus is on the design of the tax schedule for low earners and the role of tax credits. Although an experience pay-off in earnings would change the optimal structure, we think our approach captures the most important aspects of design for this group. The evidence points to relatively low or negligible experience effects

[3] An alternative model which incorporates constraints on labour supply choices in an optimal design problem is developed in Aaberge and Colombino (2008).

for low earnings single parents, see Card and Hyslop (2005) and Gladden and Taber (2000). A more subtle dynamic effect may act through fertility decisions. Keane and Wolpin (2007) note that fertility effects may largely counteract the direct impact on labour supply. However, the effect of tax reform on fertility behaviour is generally found to be significant but small, see Hoynes (2009). A further key dynamic aspect of tax design is the interaction with savings taxation and the taxation of lifetime income. In certain circumstances, the taxation of saving can be used to relax the incentive compatibility constraint on earnings taxation (see Banks and Diamond, 2009). However, with fixed costs of work, credit constraints and earnings uncertainty there is likely to remain a strong role for nonlinear earnings tax design of the type described here.

The results of our analysis point to marginal tax rates that are broadly increasing in earnings, and that are lower than under the current UK system. Moreover, we show that heterogeneity is important. In particular, we present a case for pure tax credits at low earnings but only for mothers with school aged children. It is also found that hours contingent payments can improve design. Indeed, if hours can be accurately observed, we present an empirical case for using a full-time work rule rather than the part-time rule currently in place for parents in the UK. While this is found to be a more effective instrument, the welfare gains remain modest in size for all but parents with older children. These welfare gains are also shown to reduce significantly with moderate amounts of misreporting or measurement error.

The paper proceeds as follows. In the next section we develop the analytical framework for optimal design within a stochastic structural labour supply model. In Section 1.3 we outline the WFTC reform in the UK and its impact on work incentives. Section 1.4 outlines the structural microeconometric model, while in Section 1.5 we describe the data and model estimates. Section 1.6 uses these model estimates to derive optimal tax schedules. We provide evidence for lowering the marginal rates at lower incomes and also document the importance of allowing the tax schedule to depend on the age of children. We also discuss how introducing hours rules affects tax credit design, and how important these are likely to be in terms of social welfare. Finally, Section 1.7 concludes.

1.2 The Optimal Design Problem

The policy analysis here concerns the choice of a tax schedule in which the government is attempting to allocate a fixed amount of revenue R to a specific demographic group – single mothers – in a way which will maximise the social welfare for this group. Such a schedule balances redistributive objectives with efficiency considerations. Redistributive preferences are represented through the social welfare function defined as the sum of transformed individual utilities, where the choice of transformation reflects the desire for equality.

In this section we develop an analytical framework for the design of tax and transfer policy that allows for two scenarios. In the first only earnings are observable by the tax authority, in the second we allow for partial observability of hours of work. Rather than assuming that individuals are unconstrained in their choice of hours, we suppose that only a finite number of hours choices are available, with hours of work h chosen from the finite set $\mathcal{H} = \{h_0, \ldots h_J\}$. The formulation of the optimal tax design problem will depend upon what information is observable to the tax authorities. We always assume that the government can observe earnings wh and worker characteristics X, and we shall also allow for the possibility of observing some hours of work information. In much of our analysis we will assume that rather than necessarily observing the actual hours h that are chosen, the tax authorities is assumed to only be able to observe that they belong to some closed interval $\mathbf{h} = [\underline{h}, \overline{h}] \in \mathcal{H}$ with $\underline{h} \leq h \leq \overline{h}$. For example, the tax authorities may be able to observe whether individuals are working at least h_B hours per week, but conditional on this, not how many. Depending on the size of the interval, this framework nests two important special cases; (i) when hours are perfectly observable $h = \underline{h} = \overline{h}$ for all $h \in \mathcal{H}$; (ii) only earnings information is observed $\mathbf{h} = \mathcal{H}_{++}$ for all $h > 0$. In general this is viewed as a problem of partial observability since actual hours h are always contained in the interval $\mathbf{h}$. In our later analysis in Section 1.6.3 we will explore the effect that both random hours measurement error, and possible direct hours misreporting have upon the optimal design problem.

Work decisions by individuals are determined by their preferences over consumption c and labour hours h, as well as possible childcare requirements, fixed costs of work, and the tax and transfer system. Preferences are indexed by observable characteristics X, including the number and age of her children, and vectors of unobservable (to the econometrician) characteristics ϵ and ε; the distinction between these vectors will be made in Section 1.4. We let $U(c, h; X, \epsilon, \varepsilon)$ represent the utility of a single mother who consumes c and works h hours. We will assume that she consumes her net income which comprises the product of hours of work h and the gross hourly wage w plus non-labour income and transfer payments, less taxes paid, childcare expenditure, and fixed costs of work. In what follows we let F denote the distribution of state specific errors ε, and G denote the joint distribution of (X, ϵ).[4]

In our later empirical analysis individual utilities $U(c, h; X, \epsilon, \varepsilon)$ will be described by a parametric utility function and a parametric distribution of unobserved heterogeneity (ϵ, ε). Similarly, a parametric form will be assumed for the stochastic process determining fixed costs of work and childcare expenditure. To maintain focus on the optimal design problem, we delay this discussion regarding the econometric modelling until Section 1.4; for now it suffices to write consumption c at hours h as $c(h; T, X, \epsilon)$,[5] where $T(wh, \mathbf{h}; X)$ represents the tax and transfer system. Non-labour income, such as child maintenance payments, enter the tax and transfer schedule T through the set of demographics X, and for notational simplicity we abstract from the potential dependence of the tax and transfer system on childcare expenditure. Taking the schedule T as given, each single mother is assumed to choose her hours of

[4] Throughout our analysis we assume that ε is independent of both ϵ and X.

[5] The assumptions that we later make regarding the error term ε ensure that consumption will not depend on ε for given work hours h.

work $h^* \in \mathcal{H}$ to maximize her utility. That is:

$$h^* = \arg\max_{h \in \mathcal{H}} U(c(h; T, X, \epsilon), h; X, \epsilon, \varepsilon). \tag{1.1}$$

We assume that the government chooses the tax schedule T to maximize a social welfare function W that is represented by the sum of transformed utilities:

$$W(T) = \int_{X,\epsilon} \int_{\varepsilon} \Upsilon(U(c(h^*; T, X, \epsilon), h^*; X, \epsilon, \varepsilon)) dF(\varepsilon) dG(X, \epsilon) \tag{1.2}$$

where for a given cardinal representation of U, the utility transformation function Υ determines the governments relative preference for the equality of utilities.[6] This maximization is subject to the incentive compatibility constraint which states that lone mothers choose their hours of work optimally given T (equation 1.1) and the government resource constraint:

$$\int_{X,\epsilon} \int_{\varepsilon} T(wh^*, \mathbf{h}^*; X) dF(\varepsilon) dG(X, \epsilon) \geq \overline{T}(\equiv -R). \tag{1.3}$$

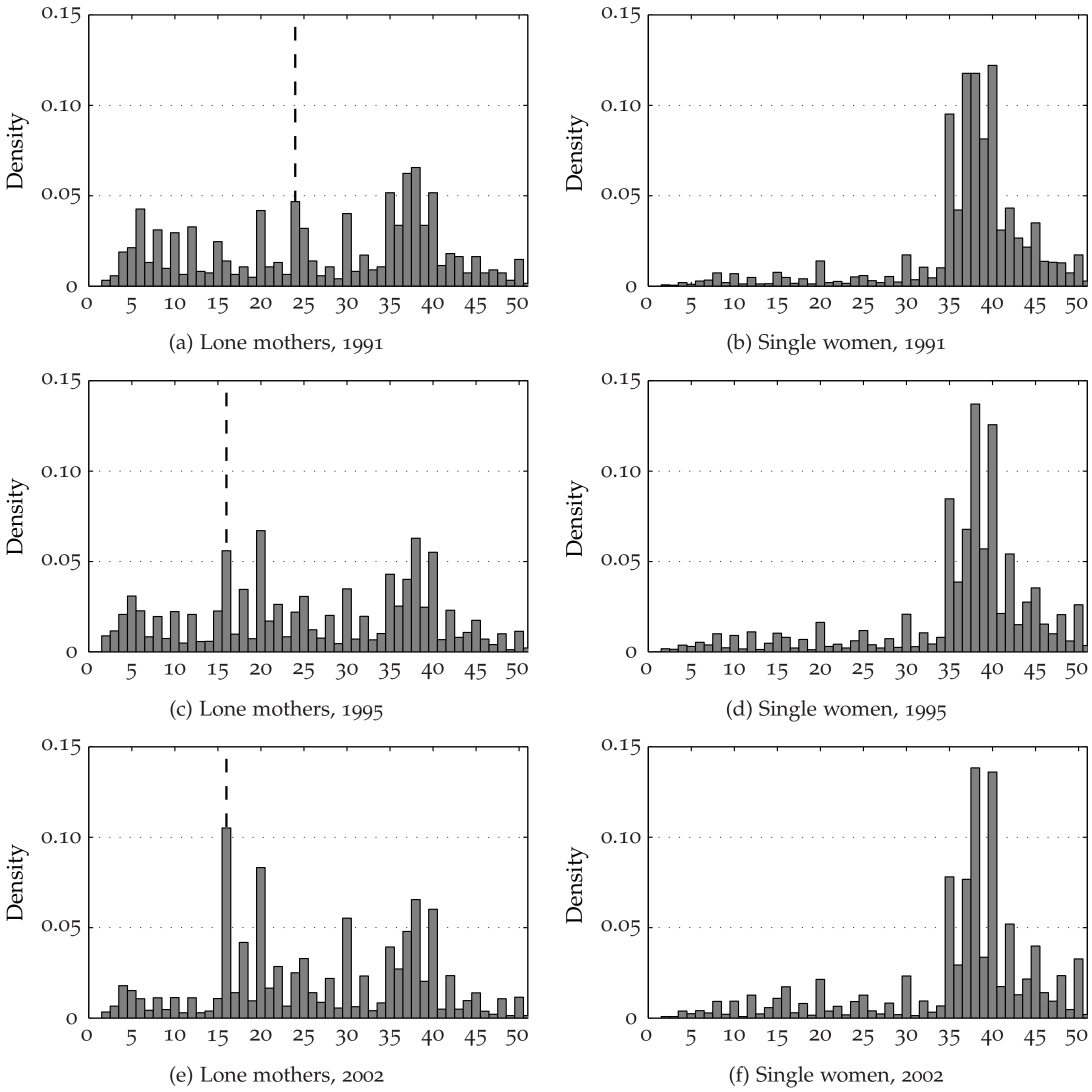
In our empirical application we will restrict T to belong to a particular parametric class of tax functions. This is discussed in Section 1.6 when we examine the optimal design of the tax and transfer schedule.

1.3 Tax Credit Reform

The increasing reliance on tax-credit policies during the 1980s and 1990s, especially in the UK and the US, reflected the secular decline in the relative wages of low skilled workers with low labour market attachment together with the growth in single-parent households (see Blundell, 2001, and references therein). The specific policy context for this paper is the Working Families' Tax Credit (WFTC) reform which took place in the UK at the end of 1999. A novel feature of the British tax credit system is that it makes use of hours conditions in addition to an earnings condition. Specifically WFTC eligibility required a working parent to record at least 16 hours of work per week. Moreover there was a further hours contingent bonus for working 30 hours or more.

As in the US, the UK has a long history of in-work benefits, starting with the introduction Family Income Support (FIS) in 1971. Over the years, these programmes became more generous, and in October 1999, Working Families' Tax Credit was introduced, replacing a similar, but less generous, tax credit programme called Family Credit (see Blundell et al., 2008, for example). As noted above, an important feature of British programmes of in-work support since their inception – and in contrast with programmes such as the US Earned Income Tax Credit – is that awards depend not only on earned and unearned income and family characteristics, but also on a minimum weekly hours of work requirement. In April 1992, the

[6] Given the presence of preference heterogeneity, a more general formulation would allow the utility transformation function Υ to vary with individual characteristics.



(a) Lone mothers, 1991
(b) Single women, 1991
(c) Lone mothers, 1995
(d) Single women, 1995
(e) Lone mothers, 2002
(f) Single women, 2002

Figure 1.1: Female hours of work by survey year. Figure shows the distribution of usual hours of work for women by year and presence of children. Sample is restricted to women aged 18–45. Calculated using UK Labour Force Survey data (for 1991) and UK Quarterly Labour Force Survey data (1995 and 2002). Horizontal axes measure weekly hours of work; the vertical line indicates the minimum hours eligibility.

minimum hours requirement fell from 24 to 16 hours a week. The impact of this reform on single parents' labour supply is ambiguous: those working more than 16 hours a week had an incentive to reduce their hours to (no less than) 16, while those previously working fewer than 16 hours had an incentive to increase their labour supply to (at least) the new cut-off. Figure 1.1 shows that the pattern of observed hours of work over this period strongly reflects these incentives. Single women without children were ineligible.[7]

The tax design problem we discuss here relates directly to the features of the WFTC. Indeed we assess the reliability of our labour supply model in terms of its ability to explain behaviour before and after the reform. There were essentially five ways in which WFTC

[7] In 1995, there was another reform to Family Credit, in the form of an additional (smaller) credit for those adults working full time (defined as 30 or more hours a week).

Table 1.1: Parameters of FC/WFTC

	April 1999 (FC)	October 1999 (WFTC)	June 2000 (WFTC)	June 2002 (WFTC)
Basic Credit	49.80	52.30	53.15	62.50
Child Credit				
under 11	15.15	19.85	25.60	26.45
11 to 16	20.90	20.90	25.60	26.45
over 16	25.95	25.95	26.35	27.20
30 hour credit	11.05	11.05	11.25	11.65
Threshold	80.65	90.00	91.45	94.50
Taper rate	70% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance
Childcare	Expenses up to £60 (£100) for 1 (more than 1) child under 12 disregarded when calculating income	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £135 (£200) for 1 (more than 1) child under 15

Notes: All monetary amounts are in pounds per week and expressed in nominal terms. Minimum FC/WFTC award is 50p per week in all years above.

increased the level of in-work support relative to the previous FC system: (i) it offered higher credits, especially for families with younger children; (ii) the increase in the threshold meant that families could earn more before it was phased out; (iii) the tax credit withdrawal rate was reduced from 70% to 55%; (iv) it provided more support for formal childcare costs through a new childcare credit; (v) all child maintenance payments were disregarded from income when calculating tax credit entitlement. The main parameters of FC and WFTC are presented in Table 1.1.

The WFTC reform increased the attractiveness of working 16 or more hours a week compared to working fewer hours, and the largest potential beneficiaries of WFTC were those families who were just at the end of the FC benefit withdrawal taper. Conditional on working 16 or more hours, the theoretical impact of WFTC is as follows: (i) people receiving the maximum FC award will face an income effect away from work, but not below 16 hours a week; (ii) people working more than 16 hours and not on maximum FC will face an income effect away from work (but not below 16 hours a week), and a substitution effect towards work; (iii) people working more than 16 hours and earning too much to be entitled to FC but not WFTC will face income and substitution effects away from work if they claim WFTC (see Blundell and Hoynes, 2004).

When analyzing the effect of the WFTC programme it is necessary to take an integrated view of the tax system. This is because tax credit awards are counted as income when calculating entitlements to other benefits, such as Housing Benefit and Council Tax Benefit. Families in receipt of such benefits would gain less from the WFTC reform than otherwise equivalent families not receiving these benefits; Figure 1.2 illustrates how the various policies impact

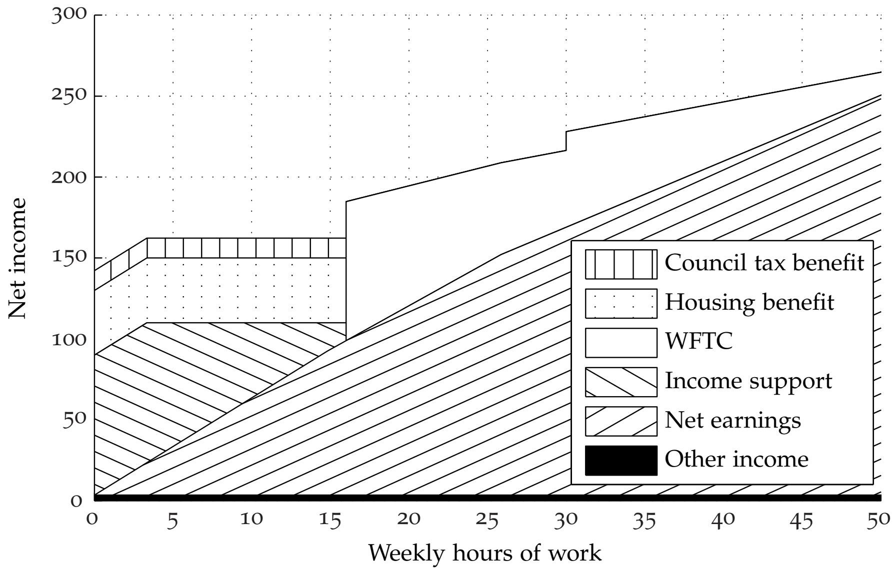


Figure 1.2: Tax and transfer system interactions. Figure shows interaction of tax and transfer system under April 2002 system for a lone parent with a single child aged 5, average band C council tax, £40 per week housing costs, and no childcare costs. All incomes expressed in April 2002 prices. Calculated using FORTAX.

on the budget constraint for a low wage lone parent. Moreover, there were other important changes to the tax system affecting families with children that coincided with the expansion of tax credits, and which make the potential labour supply responses considerably more complex. In particular, there were increases in the generosity of Child Benefit (a cash benefit available to all families with children regardless of income), as well as notable increases in the child additions in Income Support (a welfare benefit for low income families working less than 16 hours a week).[8]

1.4 A Structural Labour Supply Model

The labour supply specification develops from earlier studies of structural labour supply that use discrete choice techniques and incorporate non-participation in transfer programmes, specifically Hoynes (1996) and Keane and Moffitt (1998). Our aim is to construct a credible model of labour supply behaviour that adequately allows for individual heterogeneity in preferences and can well describe observed labour market outcomes. As initially discussed in Section 1.2, lone mothers have preferences defined over consumption c and hours of work h. Hours of work h are chosen from some finite set $\mathcal{H}$, which in our empirical application will correspond to the discrete weekly hours points $\mathcal{H} = \{0, 10, 19, 26, 33, 40\}$.[9] We augment the model discussed in Section 1.2 to allow the take-up of tax-credits to have a direct impact

[8]For many families with children, these increases in out-of-work income meant that, despite the increased generosity of in-work tax credits, replacement rates remained relatively stable. There were also changes to the tax system that affected families both with and without dependent children during the lifetime of WFTC: a new 10% starting rate of income tax was introduced; the basic rate of income tax was reduced from 23% to 22%; there was a real rise in the point at which National Insurance (payroll tax) becomes payable.

[9]These hours points correspond to the empirical hours ranges 0, 1–15, 16–22, 23–29, 30–36 and 37+ respectively.

on preferences through the presence of some stigma or hassle cost (discussed further below), and we use P (equal to one if tax credits are received, zero otherwise) to denote the endogenous programme participation decision.[10] These preferences may vary with observable demographic characteristics X (such as age, region, the number and age of children), and vectors of unobservable (to the econometrician) characteristics ϵ and ε. Here ε is used specifically to denote the additive state specific errors which are attached to each discrete hours point and are assumed to follow a standard Type-I extreme value distribution so that:

$$U(c,h,P;X,\epsilon,\varepsilon) = u(c,h,P;X,\epsilon) + \varepsilon_h.$$

While we will later consider alternative preference specifications, our results will largely assume Box-Cox preferences of the form:

$$\begin{aligned} u(c,h,P;X,\epsilon) = \alpha_y(X,\epsilon)\frac{c^{\theta_y}-1}{\theta_y} + \alpha_l(X,\epsilon)\frac{(1-h/H)^{\theta_l}-1}{\theta_l} \\ + \alpha_{yl}(X)\frac{c^{\theta_y}-1}{\theta_y}\frac{(1-h/H)^{\theta_l}-1}{\theta_l} - P\eta(X,\epsilon) \end{aligned} \tag{1.4}$$

where $H = 168$ denotes the total weekly time endowment, and where the set of functions $\alpha_y(X,\epsilon)$, $\alpha_l(X,\epsilon)$, $\alpha_{yl}(X)$ and $\eta(X,\epsilon)$ capture observed and unobserved preference heterogeneity. The function $\eta(X,\epsilon)$ is included to reflect the possible disutility associated with claiming in-work tax credits ($P = 1$), and its presence allows us to rationalize less then complete take-up of tax credit programmes. In each case we allow observed and unobserved heterogeneity to influence the preference shifter functions through appropriate index restrictions. We assume that $\alpha_{yl}(X) = X'_{yl}\beta_{yl}$, $\log\alpha_y(X,\epsilon) = X'_y\beta_y + \epsilon_y$ and $\log\alpha_l(X,\epsilon) = X'_l\beta_l + \epsilon_l$, with programme participation costs also assumed to be linear in parameters, $\eta(X,\epsilon) = X'_\eta\beta_\eta + \epsilon_\eta$. We do not impose concavity on the utility function.

The choice of hours of work h affects consumption c through two main channels: firstly, through its direct effect on labour market earnings and its interactions with the tax and transfer system; secondly, working mothers may be required to purchase childcare for their children which varies with maternal hours of employment. Given the rather limited information that our data contains on the types of childcare use, we take a similarly limited approach to modelling, whereby hours of childcare use h_c is essentially viewed as a constraint: working mothers are required to purchase a minimum level of childcare $h_c \geq \alpha_c(h,X,\epsilon)$ which varies stochastically with hours of work and demographic characteristics. Since we observe a mass of working mothers across the hours of work distribution who do not use any childcare, a linear relationship (as in Blundell et al., 2000) is unlikely to be appropriate. Instead, we assume the presence of some underlying latent variable that governs both the selection mechanism and the value of required childcare itself. More specifically, we assume that the total childcare

[10] All other transfer programmes are assumed to have complete take-up.

hours constraint is given by:

$$\alpha_c(h, X, \epsilon) = \mathbf{1}(h > 0) \times \mathbf{1}(\epsilon_{c_X} > -\beta_{c_X} h - \gamma_{c_X}) \times (\gamma_{c_X} + \beta_{c_X} h + \epsilon_{c_X}) \tag{1.5}$$

where $\mathbf{1}(\cdot)$ is the indicator function, and where the explicit conditioning of the parameters and the unobservables on demographic characteristics X reflects the specification we adopt in our estimation, where we allow the parameters of this stochastic relationship to vary with a subset of observable characteristics X_c (specifically, the number and age composition of children). Total weekly childcare expenditure is then given by $p_c h_c$ with p_c denoting the hourly price of childcare. Empirically, we observe a large amount of dispersion in childcare prices, with this distribution varying systematically with the age composition of children. This is modelled by assuming that p_c follows some distribution $p_c \sim F_c(\cdot; X_c)$ which again varies with demographic characteristics. We approximate this distribution by discretizing the empirical childcare price distribution including zero price and conditional on X_c.

Individuals are assumed to face a budget constraint, determined by a fixed gross hourly wage rate (assumed to be generated by a log-linear relationship of the form $\log w = X_w' \beta_w + \epsilon_w$) and the tax and transfer system. We arrive at our measure of consumption by subtracting both childcare expenditure $p_c h_c$ (which also interacts with the tax and transfer system) and fixed work-related costs from net-income. These fixed work-related costs help provide a potentially important wedge that separates the intensive and extensive margin. They reflect the actual and psychological costs that an individual has to pay to get to work. We model work-related costs as a fixed, one-off, weekly cost subtracted from net income at positive values of working time: $f = \alpha_f(h; X, \epsilon) = \mathbf{1}(h > 0) \times (X_f' \beta_f + \epsilon_f)$. It then follows that consumption at a given hours and programme participation choice is given by:

$$c(h, P; T, X, \epsilon) = wh - T(wh, \mathbf{h}, P; X) - p_c h_c - f \tag{1.6}$$

where non-labour income, such as child maintenance payments, enter the tax and transfer schedule T through the set of demographic characteristics X, and with the explicit conditioning of T on childcare expenditure suppressed for notational simplicity.

In order to fully describe the utility maximization problem of lone mothers, we denote $P^*(h) \in \{0, E(h; X, \epsilon)\}$ as the optimal choice of programme participation for given hours of work h, where $E(h; X, \epsilon) = 1$ if the individual is eligible to receive tax credits at hours h, and zero otherwise. Assuming eligibility, it then follows that $P^*(h) = 1$ if and only if the following condition holds:

$$u(c(h, P = 1; T, X, \epsilon), h, P = 1; X, \epsilon) \geq u(c(h, P = 0; T, X, \epsilon), h, P = 0; X, \epsilon) \tag{1.7}$$

where $c(h, P; X, \epsilon)$ is as defined in equation 1.6. It then follows that the optimal choice of hours $h^* \in \mathcal{H}$ maximizes $U(c(h, P^*(h); T, X, \epsilon), h, P^*(h); X, \epsilon, \varepsilon)$ subject to the constraints as detailed above.

1.5 Data and Estimation

1.5.1 Data

We use six repeated cross-sections from the Family Resources Survey (FRS), from the financial year 1997/8 through to 2002/3, which covers the introduction and subsequent expansion of WFTC. The FRS is a cross-section household-based survey drawn from postcode records across Great Britain: around 30,000 families with and without children each year are asked detailed questions about earnings, other forms of income and receipt of state benefits. Our sample is restricted to lone mothers who are aged between 18 and 45 at the interview date, not residing in a multiple tax unit household, and not in receipt of any disability related benefits. Dropping families with missing observations of crucial variables, and those observed during the WFTC phase-in period of October 1999 to March 2000 inclusive, restricts our estimation sample to 7,110 lone mothers.

1.5.2 Estimation

The full model (preferences, wages, and childcare) is estimated simultaneously by simulated maximum likelihood; the likelihood function is presented in Appendix 1.A.[11] We incorporate highly detailed representations of the tax and transfer system using FORTAX (see Appendix A.1 in this thesis). The budget constraints vary accurately with individual circumstances, and reflect the complex interactions between the many components of the tax and transfer system. To facilitate the estimation procedure, the actual tax and transfer schedules are modified slightly to ensure that there are no discontinuities in net-income as either the gross wage or child care expenditure vary for given hours of work. We do not attempt to describe the full UK system here, but the interested reader may consult Adam and Browne (2009) and O'Dea et al. (2007) for recent surveys; see Appendix A.1 in this thesis for a discussion of the implementation of the UK system in FORTAX.

For the purpose of modelling childcare, we define six groups by the age of youngest child (0–4, 5–10, and 11–18) and by the number of children (1 and 2 or more). The stochastic relationship determining hours of required childcare $\alpha_c(h, X, \epsilon)$ varies within each of these groups, as does the child care price distribution $F_c(\cdot; X_c)$. Using data from the entire sample period, the childcare price distribution is discretized into either four price points (if the youngest child is aged 0–4 or 5–10) or 2 points (if the youngest child is aged 11-18). In each case, the zero price point is included, and the probability that lone mothers face each of these discrete price points is estimated.

[11] This simultaneous estimation procedure contrasts with existing UK-centric labour supply studies that have used discrete choice techniques. Perhaps largely owing to the complexity of the UK transfer system, these existing studies (such as Blundell et al., 2000) typically pre-estimate wages which allows net-incomes to be computed prior to the main preference estimation. In addition to the usual efficiency arguments, the simultaneous estimation here imposes internal coherency with regards to the various selection mechanisms.

The unobserved wage component ϵ_w and the random preference heterogeneity terms $(\epsilon_y, \epsilon_l, \epsilon_f, \epsilon_\eta, \epsilon_{c_X})$ are assumed to be normally distributed. Given the difficulty in identifying flexible correlation structures from observed outcomes (see Keane, 1992), we allow ϵ_y to be correlated with ϵ_w, but otherwise assume that the errors are independent. In the later results presented we additionally restrict the standard deviation of both ϵ_l and ϵ_f to be zero as we found them to be both very small in magnitude and imprecisely estimated. The integrals over ϵ in the log-likelihood function are approximated using simulation methods (see Train, 2003); we use 400 quasi-random draws generated using Neiderreiter's method. The model is estimated using a sequential quadratic programming method.

1.5.3 Specification and Structural Parameter Estimates

The estimates of the parameters of our structural model are presented in Table 1.2. The age of the youngest child has a significant impact on the estimated fixed costs of work α_f; fixed work related costs are higher by around £15 per week if the youngest child is of pre-school age. The presence of young children also has a highly significant effect on the interacted leisure-consumption parameter α_{yl}, but does not have any quantitatively large or significant effect on the linear preference terms α_y and α_l. Whilst the age of the youngest child is important, the actual number of children does not have a significant effect upon the preference parameters.

Lone mothers who are older are estimated to have a lower preference for both consumption and leisure, but higher costs of claiming in-work support. Meanwhile, the main impact of education comes primarily on the preference for leisure α_l; mothers who have completed compulsory schooling have a lower preference for leisure. Ethnicity enters the model through both fixed costs of work and programme participation costs η; we find that programme participation costs are significantly higher for non-white lone mothers. Programme participation costs are found to fall significantly following the introduction of WFTC, although the reduction in the first year is small (as captured by the inclusion of a variable equal to one in the first year of WFTC).

Both the intercept γ_c and the slope coefficient β_c in the child care equation are lower for those with older children. This reflects the fact that lone mothers with older children use child care less, and that the total childcare required varies less with maternal hours of work. To rationalize the observed distributions, we require that the standard deviation σ_c is also larger for those with older children. The price distribution of childcare for each group was discretized in such a way that amongst those mothers using paid childcare, there are equal numbers in each discrete price group. Our estimates attach greater probability on the relatively high childcare prices (and less on zero price) than in our raw data. Individuals who do not work are therefore more likely to face relatively expensive childcare were they to work.

The hourly log-wage equation includes years of education completed (which enters positively), and both age and age squared (potential wages are increasing in age, but at a diminishing rate). Lone mothers who reside in the Greater London area have significantly higher

Table 1.2: Simulated maximum likelihood estimation results

Preference parameters

	constant	youngest child 0-4	youngest child 5-10	number of children-1	age	compuslory schooling	non-white	London	WFTC period	year 2000
α_y	1.566	-0.104	-0.029	-0.010	-0.010	-0.027	–	–	–	–
	(0.131)	(0.119)	(0.108)	(0.031)	(0.005)	(0.083)	–	–	–	–
α_l	2.781	0.030	0.024	0.057	-0.047	-(0.407)	–	–	–	–
	(0.187)	(0.168)	(0.157)	(0.044)	(0.007)	(0.085)	–	–	–	–
α_{yl}	4.112	7.578	3.587	–	–	–	–	–	–	–
	(1.630)	(2.065)	(1.849)	–	–	–	–	–	–	–
θ_y	0.302	–	–	–	–	–	–	–	–	–
	(0.111)	–	–	–	–	–	–	–	–	–
θ_l	2.813	–	–	–	–	–	–	–	–	–
	(0.816)	–	–	–	–	–	–	–	–	–
α_f	0.284	0.151	0.043	0.044	0.006	0.081	-0.035	0.228	–	–
	(0.083)	(0.084)	(0.068)	(0.032)	(0.005)	(0.063)	(0.053)	(0.046)	–	–
η	0.760	–	–	–	0.028	-0.058	0.328	–	-0.475	0.394
	(0.177)	–	–	–	(0.008)	(0.146)	(0.153)	–	(0.102)	(0.114)

Continued . . .

Table 1.2: (continued)

Childcare parameters

	1 child youngest age 0-4	1 child youngest age 5-10	1 child youngest age 11-18	2 children youngest age 0-4	2 children youngest age 5-10	2 children youngest age 11-18
γ_c	4.481	-7.767	-27.833	5.035	-25.872	-58.522
	(2.041)	(1.494)	(5.354)	(3.646)	(3.319)	(11.016)
β_c	0.701	0.672	0.309	1.163	1.308	0.639
	(0.066)	(0.049)	(0.157)	(0.133)	(0.115)	(0.323)
σ_c	13.171	11.783	24.814	26.944	27.420	42.667
	(0.466)	(0.312)	(2.274)	(0.905)	(0.868)	(3.757)
$\Pr(p_{cc}^1)$	0.181	0.172	0.153	0.159	0.133	0.178
	(0.019)	(0.018)	(0.036)	(0.019)	(0.016)	(0.049)
$\Pr(p_{cc}^2)$	0.205	0.179	–	0.194	0.146	–
	(0.021)	(0.019)	–	(0.023)	(0.018)	–
$\Pr(p_{cc}^3)$	0.240	0.194	–	0.267	0.164	–
	(0.023)	(0.020)	–	(0.028)	(0.020)	–
p_{cc}^0	0.000	0.000	0.000	0.000	0.000	0.000
p_{cc}^1	0.972	0.810	1.820	0.541	0.570	1.658
p_{cc}^2	2.172	1.594	–	1.555	1.474	–
p_{cc}^3	3.436	2.576	–	2.942	2.474	–

Wage equation

constant	education	age	age squared	London	non-white	1998	1999	2000	2001	2002	σ_w
0.250	0.081	0.052	-0.054	0.191	-0.030	-0.013	0.028	0.130	0.138	0.146	0.406
0.043	(0.002)	(0.012)	(0.017)	(0.024)	(0.027)	(0.025)	(0.029)	(0.025)	(0.023)	(0.024)	(0.005)

Notes: All parameters estimated simultaneously by simulated maximum likelihood, using FRS data and with sample selection as detailed in Section 1.5. Incomes are expressed in hundreds of pounds per week in April 2002 prices. Age and age squared are defined in terms of deviations from the median value; age squared is divided by one hundred. Compulsory schooling is equal to 1 if the individual completed school at age 16 or above. Education measures years of education completed. London is equal to one if resident in the Greater London area. WFTC period is equal to one if individual is interviewed post-October 1999. Standard errors are presented in parentheses.

Table 1.3: Predicted and empirical frequencies by age of youngest child

	All		0-4		5-10		11-18	
	Predicted	Empirical	Predicted	Empirical	Predicted	Empirical	Predicted	Empirical
0 hours	0.551	0.550	0.709	0.708	0.491	0.488	0.319	0.320
	(0.005)	(0.006)	(0.007)	(0.008)	(0.008)	(0.010)	(0.012)	(0.013)
10 hours	0.069	0.068	0.053	0.050	0.080	0.082	0.080	0.081
	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.005)	(0.006)	(0.007)
19 hours	0.101	0.121	0.085	0.099	0.114	0.139	0.114	0.130
	(0.003)	(0.004)	(0.003)	(0.005)	(0.003)	(0.007)	(0.004)	(0.009)
26 hours	0.081	0.070	0.056	0.044	0.093	0.084	0.113	0.098
	(0.002)	(0.003)	(0.002)	(0.004)	(0.002)	(0.005)	(0.003)	(0.008)
33 hours	0.092	0.077	0.051	0.042	0.105	0.087	0.157	0.136
	(0.002)	(0.003)	(0.002)	(0.004)	(0.003)	(0.005)	(0.004)	(0.009)
40 hours	0.106	0.115	0.046	0.058	0.117	0.120	0.217	0.235
	(0.003)	(0.004)	(0.003)	(0.004)	(0.005)	(0.006)	(0.009)	(0.012)
Take-up rate	0.766	0.765	0.822	0.788	0.767	0.783	0.709	0.715
	(0.007)	(0.005)	(0.010)	(0.007)	(0.009)	(0.008)	(0.011)	(0.012)

Notes: Empirical frequencies calculated using FRS data with sample selection as detailed in Section 1.5. The discrete points 0, 10, 19, 26, 33 and 40 correspond to the hours ranges 0, 1–15, 16–22, 23–29, 30–36 and 37+ respectively. Empirical take-up rates calculated using reported receipt of FC/WFTC with entitlement simulated using FORTAX. Predicted frequencies are calculated using FRS data and the maximum likelihood estimates from Table 1.2. Standard errors are in parentheses, and calculated for the predicted frequencies by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

wages, and the inclusion of time dummies track the general increase in real wages over time. Unsurprisingly, there is considerable dispersion in the unobserved component of log-wages.

The within sample fit of the model is presented in Tables 1.3 and 1.4. We match the observed employment states and the take-up rate over the entire sample period very well (see the first column of Table 1.3). We slightly under predict the number of lone mothers working 19 hours per week, and slightly over predict the number working either 26 or 33 hours per week, but the difference is not quantitatively large. Similarly, we obtain very good fit by age of youngest child. The fit to the employment rate is particularly good, and the difference between predicted and empirical hours frequencies never differs by more than around two percentage points.

The fit of the model over time is presented in Table 1.4. Fitting the model over time is more challenging given that time only enters our specification in a very limited manner - through the wage equation and via the change in the stigma costs of the accessing the tax credit. Despite this we are able to replicate the 9 percentage point increase in employment between 1997/98 and 2002/03 reasonably well with our model, although we do slightly under predict the growth in part-time employment over this period.

To understand what our parameter estimates mean for labour supply behaviour we simulate labour supply elasticities under the actual 2002 tax system across a range of earnings and household types. The results of this exercise are presented in Table 1.5. Participation elasticities are lowest for single mothers whose youngest child is under 4 (an elasticity of 0.57), while they are significantly higher for mothers with school aged children (0.82 if youngest child is aged 5-10; 0.72 if the youngest child is aged 11-18). Across all child age groups, ex-

Table 1.4: Predicted and empirical frequencies: 1997-2002

	1997		2002	
	Predicted	Empirical	Predicted	Empirical
0 hours	0.592	0.600	0.507	0.508
	(0.007)	(0.014)	(0.007)	(0.013)
10 hours	0.071	0.080	0.069	0.062
	(0.003)	(0.008)	(0.003)	(0.006)
19 hours	0.092	0.100	0.114	0.140
	(0.003)	(0.009)	(0.003)	(0.009)
26 hours	0.072	0.052	0.091	0.079
	(0.002)	(0.006)	(0.002)	(0.007)
33 hours	0.080	0.064	0.103	0.093
	(0.002)	(0.007)	(0.002)	(0.008)
40 hours	0.094	0.104	0.115	0.120
	(0.003)	(0.009)	(0.003)	(0.009)
Take-up rate	0.716	0.688	0.817	0.838
	(0.011)	(0.013)	(0.008)	(0.010)

Notes: Empirical frequencies calculated using FRS data with sample selection as detailed in Section 1.5. The discrete points 0, 10, 19, 26, 33 and 40 correspond to the hours ranges 0, 1–15, 16–22, 23–29, 30–36 and 37+ respectively. Empirical take-up rates calculated using reported receipt of FC/WFTC with entitlement simulated using FORTAX. Predicted frequencies are calculated using FRS data and the maximum likelihood estimates from Table 1.2. Standard errors are in parentheses, and calculated for the predicted frequencies by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

tensive elasticities are higher than intensive elasticities at low earnings, but at higher earnings levels the intensive elasticities dominate.[12] Intensive elasticities are typically higher for lone mothers with older children, as are the extensive elasticities except at low earnings levels; extensive elasticities are very similar for lone mothers whose youngest child is aged 5-10 or aged 11-18. The individual behaviour that these summary elasticity measures reflect will have implications for the optimal design of the tax and transfer system (see Section 1.6).

1.5.4 Simulating the WFTC Reform

Before we proceed to consider optimal design problems using our structural model, we first provide an evaluation of the impact of the WFTC reform discussed in Section 1.3 above on single mothers. This exercise considers the impact of replacing the actual 2002 tax systems with the April 1997 tax system on the 2002 population. This exercise is slightly different to simply examining the change in predicted states over this time period as it removes the influence of changing demographic characteristics.

The results of this policy reform simulation are presented in Table 1.6. Overall we predict that employment increased by 4 percentage points as a result of these reforms, with the increase due to movements into both part-time and full-time employment. Comparing with Table 1.4 we find the reform explains a little under half of the rise in employment over this period. The predicted increase in take-up of tax credits is also substantial, with this increase driven both by the changing entitlement and the estimated reduction in programme participation costs.

[12] See the note accompanying Table 1.5 for a precise definition of these elasticities.

Table 1.5: Simulated elasticities by age of youngest child

Weekly	0-4		5-10		11-18	
Earnings	Extensive	Intensive	Extensive	Intensive	Extensive	Intensive
50	0.168	0.025	0.205	0.085	0.144	0.130
	(0.017)	(0.003)	(0.020)	(0.009)	(0.018)	(0.016)
100	0.128	0.055	0.178	0.177	0.151	0.269
	(0.012)	(0.008)	(0.012)	(0.018)	(0.011)	(0.030)
150	0.100	0.077	0.155	0.239	0.153	0.387
	(0.010)	(0.012)	(0.009)	(0.025)	(0.008)	(0.042)
200	0.067	0.076	0.112	0.231	0.116	0.394
	(0.006)	(0.012)	(0.005)	(0.024)	(0.005)	(0.041)
250	0.043	0.066	0.074	0.194	0.077	0.340
	(0.004)	(0.010)	(0.004)	(0.020)	(0.004)	(0.035)
300	0.027	0.051	0.046	0.147	0.045	0.252
	(0.003)	(0.007)	(0.002)	(0.014)	(0.003)	(0.024)
350	0.016	0.035	0.028	0.102	0.025	0.170
	(0.002)	(0.005)	(0.002)	(0.009)	(0.002)	(0.015)
400	0.024	0.034	0.039	0.094	0.028	0.140
	(0.002)	(0.004)	(0.003)	(0.008)	(0.002)	(0.011)
Participation	0.566		0.820		0.720	
elasticity	(0.047)		(0.042)		(0.036)	

Notes: All elasticities simulated under actual 2002 tax systems with complete take-up of WFTC. Earnings are in pounds per week and are expressed in April 2002 prices. Participation elasticities simulated by increasing consumption at all positive hours choices by 1%. Extensive and intensive earnings elasticities simulated by increasing consumption at the hours point closest to the respective earnings point. Extensive elasticities measure the increase in the employment rate following a 1% increase in consumption at the respective level of earnings. Intensive elasticities measure the increase in the proportion of individuals at each earnings point from any positive hours point following a 1% increase in consumption at the respective level of earnings. Standard errors are in parentheses, and calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

Table 1.6: Impact of reforms: 1997-2002

	2002 system	1997 system	change
0 hours	0.507	0.547	-0.039
	(0.006)	(0.007)	(0.003)
10 hours	0.069	0.072	-0.002
	(0.003)	(0.003)	(0.001)
19 hours	0.114	0.098	0.015
	(0.003)	(0.004)	(0.002)
26 hours	0.091	0.078	0.013
	(0.002)	(0.002)	(0.001)
33 hours	0.103	0.089	0.014
	(0.002)	(0.002)	(0.001)
40 hours	0.115	0.117	-0.001
	(0.004)	(0.004)	(0.001)
Take-up	0.817	0.683	0.134
rate	(0.008)	(0.019)	(0.015)

Notes: impact of tax and transfer system reforms on hours of work and take-up simulated using FRS 2002 data by replacing actual 2002 tax systems with the April 1997 tax system. Standard errors are in parentheses and are calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

1.6 The Optimal Design of the Tax and Transfer Schedule

In this section we use our structural model to examine the design of the tax and transfer schedule. We show the importance of allowing the schedule to depend on the age of children. One of the key results is that marginal rates should be lower for low earnings families with older children. Given the use of a minimum hours condition for eligibility in the British tax credit system, we also consider the design in the case of a minimum hours rule. We show that if hours of work are partially (but otherwise accurately) observable, then there can be non-trivial welfare gains from introducing an hours rule for lone mothers with older children. However, accurately observing hours of work is crucial for this result. Our results suggest that if hours of work are subject to measurement error – whether this be random or due to direct misreporting – then the welfare gains that can be realised may be much reduced. Our analysis here therefore supports the informal discussion regarding the inclusion of hours in the tax base in Banks and Diamond (2009). Before detailing these results, we first turn to the choice of social welfare transformation and the parameterisation of the tax and transfer schedule.

1.6.1 Optimal Tax Specification

We have shown that using parameter estimates from a structural model of labour supply, the behaviour of individuals can be simulated as the tax and transfer system is varied. With these heterogeneous labour supply responses allowed for, the structural model provides all the necessary information to maximise an arbitrary social welfare function, subject to a government budget constraint. Note that our analysis here integrates that tax and transfer system.

To implement the optimal design analysis we approximate the underlying non-parametric optimal schedule by a piecewise linear tax schedule that is characterized by a level of out-of-work income (income support), and seven different marginal tax rates. These marginal tax rates, which are restricted to lie between -100% and 100%, apply to weekly earnings from £0 to £300 in increments of £50, and then all weekly earnings above £300. We do not tax any non-labour sources of income, and do not allow childcare usage to interact with tax and transfer schedule unless explicitly stated. When we later allow for partial observability of hours we introduce additional payments that are received only if the individual fulfills the relevant hours criteria.

The optimal tax schedule is solved separately for three different groups on the basis of the age of youngest child: under 4, aged 5 to 10 and 11 to 18. For these illustrations, we have also conditioned upon the presence of a single child. For each of these groups we set the value of government expenditure equal to the predicted expenditure on this group within our sample.[13] Conditioning upon this level of expenditure we calculate the tax and transfer schedule that maximizes social welfare in each of these groups. We adopt the following utility

[13] To date we have made no attempt to calculate what the optimal division of overall expenditure is between these three groups. This therefore makes an implicit assumption regarding the value that the government attaches on the welfare of these groups.

transformation in the social welfare function:

$$\Upsilon(U;\theta) = \frac{(\exp U)^{\theta} - 1}{\theta} \tag{1.8}$$

which controls the preference for equality by the one dimensional parameter θ and also permits negative utilities which is important in our analysis given that the state specific errors ε can span the entire real line. When θ is negative, the function (1.8) favours the equality of utilities; when θ is positive the reverse is true. By L'Hôpital's rule $\theta = 0$ corresponds to the linear case. We solve the schedule for a set of parameter values $\theta = \{-0.4, -0.2, 0.0\}$ and then derive the social weights that characterise these redistributive preferences. We do not consider cases where $\theta > 0$. The presence of state specific Type-I extreme value errors, together with our above choice of utility transformation has some particularly convenient properties, as the follow Proposition now demonstrates.

Proposition 1. *Suppose that the utility transformation function is as specified in equation 1.8. If $\theta = 0$ then conditional on X and ϵ the integral over (Type-I extreme value) state specific errors ε in equation 1.2 is given by:*

$$\log\left(\sum_{h\in\mathcal{H}} \exp(u(c(h;T,X,\epsilon),h;X,\epsilon))\right) + \gamma$$

where $\gamma \approx 0.57721$ is the Euler-Mascheroni constant. If $\theta < 0$ then conditional on X and ϵ the integral over state specific errors is given by:

$$\frac{1}{\theta}\left[\Gamma(1-\theta) \times \left(\sum_{h\in\mathcal{H}} \exp(u(c(h;T,X,\epsilon),h;X,\epsilon))\right)^{\theta} - 1\right]$$

where Γ is the gamma function.

Proof. The result for $\theta = 0$ follows directly from an application of L'Hôpital's rule, and the well known result for expected utility in the presence of Type-I extreme value errors (see McFadden, 1978). See Appendix 1.B for a proof in the case where $\theta < 0$. □

This proposition, which essentially generalizes the result of McFadden (1978), facilitates the numerical analysis as the integral over state specific errors does not require simulating. Moreover, the relationship between the utilities in each state, and the contribution to social welfare for given (X, ϵ) is made explicit and transparent.

1.6.2 Implications for the Tax Schedule

The underlying properties from the labour supply model, together with the choice of social welfare weights, are the key ingredients in the empirical design problem. We have seen from Table 1.5 that the intensive and extensive labour supply responses differ substantially. They also vary with the age of the youngest child. As expected this is reflected in the optimal

Table 1.7: Social welfare weights under optimal system by age of youngest child

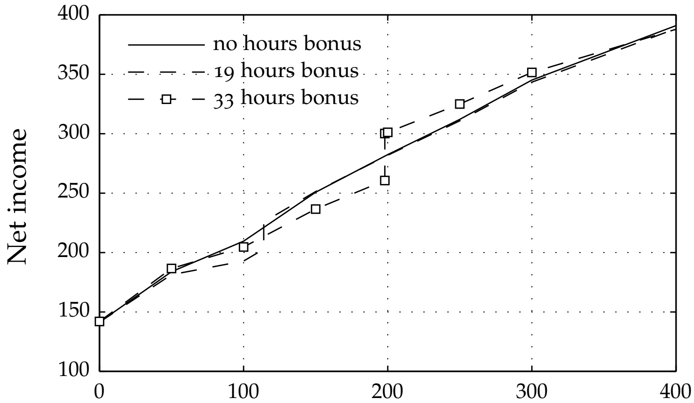
Weekly	0-4			5-10			11-18		
Earnings	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$
0	1.226	1.208	1.143	1.493	1.418	1.228	1.701	1.539	1.238
0–50	1.034	0.966	0.856	1.381	1.282	1.076	1.680	1.497	1.174
50–100	0.838	0.837	0.784	1.103	1.092	0.968	1.352	1.284	1.047
100–150	0.643	0.714	0.802	0.886	0.950	0.952	1.119	1.140	1.016
150–200	0.524	0.647	0.851	0.704	0.828	0.969	0.883	0.980	1.015
200–250	0.423	0.563	0.842	0.562	0.707	0.929	0.705	0.834	0.971
250–300	0.335	0.483	0.883	0.440	0.595	0.912	0.549	0.702	0.948
300+	0.202	0.331	0.775	0.253	0.397	0.860	0.323	0.479	0.905

Notes: Table presents social welfare weights under optimal structure of marginal tax rates and out-of-work income by age of child and under range of distributional taste parameters θ as presented in Table 1.8. All incomes are in pounds per week and are expressed in April 2002 prices. Social weights are normalized so that the sum of weights multiplied by earnings density under optimal system is equal to unity.

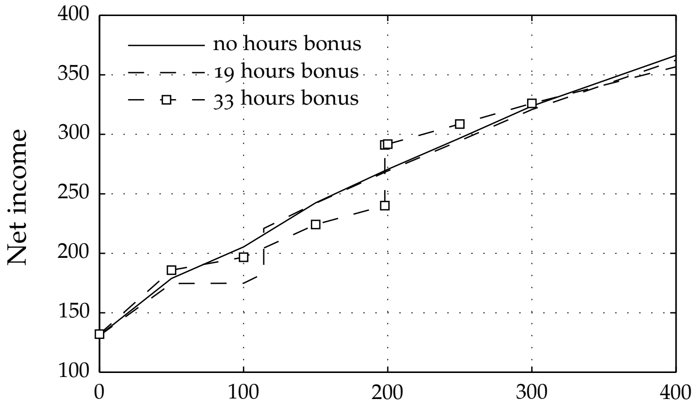
tax results. For the choice of utility transformation function in equation 1.8 we examine the impact of alternative θ values. In Table 1.7 we present the underlying social welfare weights evaluated at the optimal schedule (discussed below) across the different child age groups according to these alternative θ values. For all three values of θ considered here the weights are broadly downward sloping. For the most part we focus our discussion here on the -0.2 value, although we do provide a sensitivity of our results to the choice of θ and find the broad conclusions are robust to this choice.

In Table 1.8 we present the optimal tax and transfer schedules across the alternative θ values and for all child age groups (also see Figure 1.3(a)–(c) for $\theta = -0.2$). In all the simulations performed here, the structure of marginal tax rates is broadly progressive with lower rates at lower earnings levels. In particular, marginal rates are typically much lower in the first tax bracket (earnings up to £50 per-week) and for lone mothers with a child aged between 11 and 18 we obtain pure tax credits (negative marginal tax rates) in this bracket. Marginal tax rates are typically much higher in the second bracket (weekly earnings between £50 and £100), but then fall before proceeding to generally increase with labour earnings. As we increase the value of θ (corresponding to less redistributive concern), we obtain reductions in the value of out-of-work income. This is accompanied by broad decreases in marginal tax rates, except in the first tax bracket where marginal tax rates increase. The social welfare weights presented in Table 1.7 reflect these changes.

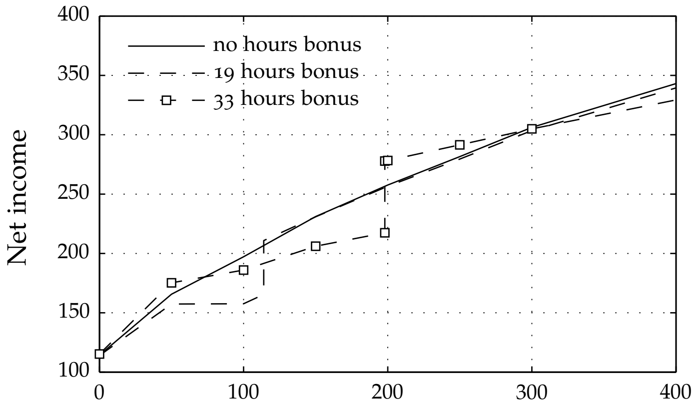
Our optimal tax simulations reveal some important differences by the age of children. In particular, marginal tax rates tend to be higher at low earnings for lone mothers with younger children, but lower at high earnings. There are two important observations to make here. Firstly, there are far fewer lone mothers with young children who obtain high earnings under the respective optimal tax and transfer systems: only around 25% of lone mothers whose child is aged 0–4 have earnings that exceed £100 per week; in contrast, around 70% of lone mothers with children in the oldest age group have earnings exceeding this amount. Secondly, the



(a) Youngest child aged 0–4



(b) Youngest child aged 5–10



(c) Youngest child aged 11–18

Figure 1.3: Optimal tax schedules with hours bonuses. All schedules are calculated with $\theta = -0.2$ and assuming an hourly wage of £6. All incomes are measured in April 2002 prices. Horizontal axis measures earnings in pounds per week.

Table 1.8: Optimal marginal tax schedules by age of youngest child

Weekly	0-4			5-10			11-18		
Earnings	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$
0–50	0.107	0.150	0.241	0.020	0.043	0.120	-0.045	-0.028	0.060
	(0.026)	(0.021)	(0.029)	(0.034)	(0.023)	(0.020)	(0.056)	(0.040)	(0.028)
50–100	0.618	0.486	0.205	0.631	0.470	0.154	0.552	0.369	0.101
	(0.045)	(0.046)	(0.057)	(0.036)	(0.037)	(0.040)	(0.044)	(0.042)	(0.034)
100–150	0.239	0.177	-0.024	0.325	0.259	0.043	0.407	0.322	0.080
	(0.023)	(0.025)	(0.037)	(0.024)	(0.023)	(0.027)	(0.025)	(0.024)	(0.028)
150–200	0.424	0.367	0.144	0.513	0.437	0.127	0.565	0.468	0.098
	(0.014)	(0.016)	(0.030)	(0.012)	(0.016)	(0.034)	(0.015)	(0.019)	(0.040)
200–250	0.444	0.407	0.136	0.523	0.476	0.202	0.582	0.522	0.219
	(0.010)	(0.012)	(0.029)	(0.009)	(0.012)	(0.035)	(0.011)	(0.013)	(0.041)
250–300	0.384	0.338	0.118	0.517	0.461	0.096	0.580	0.507	0.094
	(0.015)	(0.016)	(0.038)	(0.011)	(0.015)	(0.038)	(0.014)	(0.018)	(0.044)
300+	0.559	0.542	0.343	0.602	0.575	0.298	0.663	0.631	0.335
	(0.010)	(0.010)	(0.032)	(0.008)	(0.009)	(0.040)	(0.008)	(0.009)	(0.044)
Out-of-work	142.545	141.401	133.762	135.548	131.041	108.591	123.733	114.296	79.458
income	(1.273)	(1.188)	(1.270)	(1.833)	(1.752)	(3.200)	(3.579)	(3.451)	(4.651)

Notes: Table presents optimal structure of marginal tax rates and out-of-work income by age of child and under range of distributional taste parameters θ. All incomes are in pounds per week and are expressed in April 2002 prices. Standard errors are in parentheses and are calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

childcare requirements of mothers with young children are considerably higher (see Table 1.2). As such, the marginal rates presented in Table 1.8 understate the effective marginal tax rates that mothers with young children face. If we explicitly allow the tax system to subsidize childcare expenditure (we consider a 70% subsidy, which corresponds to the formal childcare subsidy rate under WFTC), then the level of out-of-work income remains effectively unchanged (since non-working mothers do not require childcare in our structural model), while marginal tax rates increase across the entire distribution of earnings for mothers with very young children. There are small increases for mothers with children aged 5–10, and effectively no change for mothers with children aged 11–18. Full results are available upon request.

In the simulation results in Table 1.8 we also present standard errors for the parameters of the optimal tax schedule. We obtain these by sampling 500 times from the distribution of parameter estimates and re-solving for the optimal schedule conditional on the sample distribution of covariates. The standard errors that we obtain are typically quite small, but this does raise some concern that our results may be sensitive to our particular specification of the utility function. Before proceeding further, we consider the robustness of our main results to the utility function parameterization by estimating our labour supply model with different preference representations, and then exploring the implications for design under each of these. We consider two alternative representations: (i) modify the utility function presented in equation 1.4 by adding squared Box-Cox transformations of consumption and leisure (henceforth referred to as *utility 2*); (ii) preferences that are quadratic in leisure and consumption[14]

[14]That is: $u(c, h, P; X, \epsilon) = \alpha_y c^2 + \alpha_l l^2 + \alpha_{ly} cl + \beta_y c + \beta_l l - P\eta$, with observable heterogeneity X influencing the coefficients through linear index restrictions, and with unobserved preference heterogeneity ϵ entering the model

Table 1.9: Optimal marginal tax schedules by age of youngest child (robustness exercise)

Weekly	0-4			5-10			11-18		
Earnings	Utility 1	Utility 2	Utility 3	Utility 1	Utility 2	Utility 3	Utility 1	Utility 2	Utility 3
0–50	0.150	0.181	0.125	0.043	0.019	0.015	-0.028	0.006	0.014
	(0.021)	(0.029)	(0.032)	(0.023)	(0.025)	(0.030)	(0.040)	(0.063)	(0.060)
50–100	0.486	0.596	0.335	0.470	0.439	0.257	0.369	0.327	0.247
	(0.046)	(0.062)	(0.044)	(0.037)	(0.042)	(0.042)	(0.042)	(0.093)	(0.063)
100–150	0.177	0.170	0.261	0.259	0.220	0.271	0.322	0.298	0.309
	(0.025)	(0.055)	(0.015)	(0.023)	(0.025)	(0.021)	(0.024)	(0.064)	(0.027)
150–200	0.367	0.362	0.361	0.437	0.413	0.374	0.468	0.453	0.432
	(0.016)	(0.027)	(0.022)	(0.016)	(0.020)	(0.019)	(0.019)	(0.069)	(0.023)
200–250	0.407	0.411	0.410	0.476	0.461	0.452	0.522	0.510	0.512
	(0.012)	(0.016)	(0.019)	(0.012)	(0.015)	(0.016)	(0.013)	(0.070)	(0.020)
250–300	0.338	0.353	0.353	0.461	0.447	0.416	0.507	0.495	0.477
	(0.016)	(0.022)	(0.021)	(0.015)	(0.019)	(0.016)	(0.018)	(0.026)	(0.020)
300+	0.542	0.564	0.557	0.575	0.570	0.583	0.631	0.622	0.646
	(0.010)	(0.014)	(0.025)	(0.009)	(0.012)	(0.016)	(0.009)	(0.013)	(0.020)
Out-of-work	141.401	141.276	140.637	131.041	129.398	125.817	114.296	113.329	111.085
income	(1.188)	(1.407)	(1.217)	(1.752)	(1.954)	(2.292)	(3.451)	(6.336)	(4.966)

Notes: Table presents optimal structure of marginal tax rates and out-of-work income by age of child and range of utility function specifications (*utility 1*, *utility 2*, and *utility 3* – see Section 1.6 for details) with $\theta = -0.2$. All incomes are in pounds per week and are expressed in April 2002 prices. Standard errors are in parentheses and are calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

as in Blundell et al. (2000) (referred to as *utility 3*). The results of this robustness exercise are presented in Table 1.9 in the case when $\theta = -0.2$. Across all the different age groups, we find that the schedules are very similar to those arrived at using our original utility representation (referred to as *utility 1* in the table). This therefore suggests that the results we present are not too dependent upon our choice of utility function.

1.6.3 Introducing an Hours Rule

For several decades the UK's tax credits and welfare benefits have made use of rules related to weekly hours of work. As discussed in Section 1.3, individuals must work at least 16 hours a week to be eligible for in-work tax credits, and receive a further smaller credit when working 30 or more hours. While many theoretical models rule out the observability of any hours information, this design feature motivates us to explore the optimal structure of the tax and transfer system when hours can be partially observed as set out in Section 1.2. We begin by assuming that the tax authority is able to observe whether individuals are working 19 hours or more, which roughly corresponds to the placement of the main 16 hours condition in the British tax-credit system, and for now we do not allow for any form of measurement error. In this case the tax authority is able to condition an additional payment on individuals working such hours. When the tax authority is only able to observe earnings, it is unable to infer whether an individual with a given level of earnings is low wage-high hours, or high wage-low hours. Since the government may value redistribution more highly in the former case, it

similarly.

may be able to better achieve its goals by introducing an hours rule into the system.

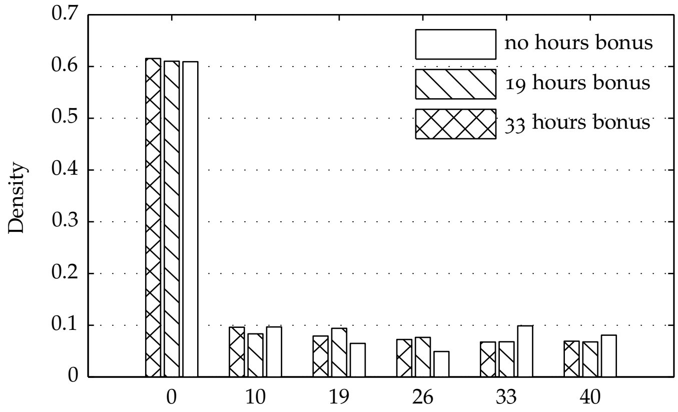
The results of this exercise are presented in Figure 1.3(a)–(c) with $\theta = -0.2$ and assuming an hourly wage rate of £6 for all child age groups. The figures show that the size of the hours bonus exhibits a very pronounced age gradient; we obtain a weekly hours bonus equal to £23, £38 and £45 for lone mothers with children aged 0–4, 5–10 and 11–18 respectively.[15] It therefore appears that there is a much smaller requirement for a part-time hours bonus for families with children aged below 5. But as the children age the optimal schedule changes quite dramatically with a strong move towards an hours bonus.

Relative to the optimal system when such a rule is not implementable, the hours bonus increases marginal rates in the part of the earnings distribution where this hours rule would roughly come into effect (particularly in the £50 to £100 earnings bracket) while marginal rates further up the distribution, as well as the level of out-of-work support, are essentially unchanged. As a result of this, some non-workers with low potential wages may be induced to work part-time, while some low hours individuals will either not work or increase their hours. Similarly, some high earnings individuals will reduce their hours to that required for the bonus. The hours bonus is sufficiently large for lone mothers with school aged children, that it implies a negative participation tax rate at 19 hours when earning the minimum wage rate.

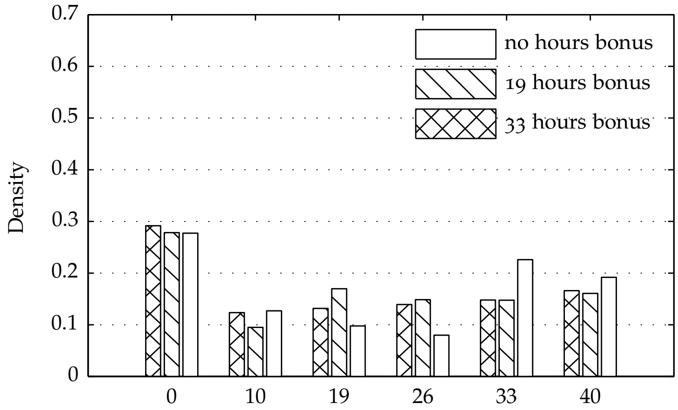
Although there are some notable changes in the structure of the constraint when hours information is partially observable (particularly for lone mothers with older children), it does not follow that it necessarily leads to a large improvement in social welfare. Indeed, in the absence of the hours conditioning, there are only few individuals working less than 19 hours (see Figure 1.4(a)–(c)) so the potential that it offers to improve social welfare appears limited. We now attempt to provide some guidance concerning the size of the welfare gain from introducing hours rules. The exact experiment we perform is as follows: we calculate the level of social welfare under the optimal schedule with hours contingent payments, and then determine the increase in expenditure per-person that is required to obtain the same level of social welfare in the absence of such hours conditioning. In conducting this experiment we allow all the parameters of the (earnings) tax schedule to vary so this is obtained at least cost.

The results of this analysis are presented in Table 1.10. Unsurprisingly, when children are aged less than 5 the increased expenditure required to achieve the level of social welfare obtained under the 19 hour rule is negligible. However, even when children are of school age, the required increased expenditure is found to be small (and is clearly negligible when the less redistributive preferences are considered). Even without allowing for any form of measurement error, it follows that unless the costs of partial hours observability is sufficiently low, it would appear difficult to advocate the use of a 19 hour rule based upon this analysis.

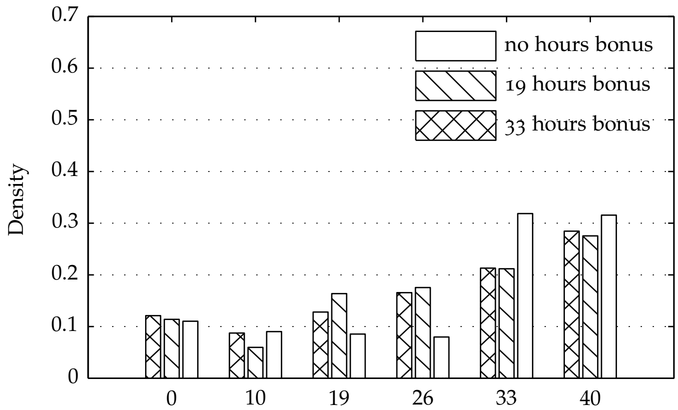
[15] We also explore the impact that varying the redistributive taste parameter θ has on the size of the hours bonus at 19 hours and on the overall structure of the budget constraint: when $\theta = -0.4$ there is little change in the size of the bonus; when $\theta = 0.0$ the optimal bonus is approximately halved for all child age groups.



(a) Youngest child aged 0–4



(b) Youngest child aged 5–10



(c) Youngest child aged 11–18

Figure 1.4: Hours distributions under optimal schedules. Hours distributions are calculated under the respective optimal tax systems with $\theta = -0.2$. Horizontal axis measures hours of work per week.

Table 1.10: Quantifying the welfare gain of hours rules

	19 hours			optimal hours		
	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$
0–4	0.250	0.213	0.05	0.782	0.854	0.956
	(0.2%)	(0.2%)	(0.0%)	(0.7%)	(0.7%)	(0.8%)
5–10	1.118	0.884	0.130	2.760	2.711	1.476
	(1.3%)	(1.0%)	(0.2%)	(3.2%)	(3.2%)	(1.7%)
11–18	1.592	1.083	0.08	5.016	4.471	1.720
	(3.0%)	(2.1%)	(0.2%)	(9.5%)	(8.5%)	(3.3%)

Notes: Table shows the additional expenditure requirement per person by age of child and under range of distributional taste parameters θ that is necessary to achieve the same level of social welfare as under the respective hours rules with a schedule that varies only with earnings. All incomes are in pounds per week and are expressed in April 2002 prices. Figures in parentheses correspond to the proportional increase in required expenditure.

This has very important policy implications given that the UK tax credit system makes heavy use of very similar hours conditions.[16]

An Optimal Hours Rule?

The social welfare gains from introducing a 19 hours rule appear to be only very modest in size at best. In this section we explore whether there are potentially larger gains by allowing the choice of the point at which the hours rule becomes effective to be part of the optimal design problem. The optimal schedules with $\theta = -0.2$ are also shown in Figure 1.3(a)–(c). In all cases, we get an optimal hours rule at the fifth (out of six) discrete hours point, which corresponds to 33 hours per week.[17] We also note that the size of the optimally placed hours bonus always exceeds that calculated when the hours rule became effective at 19 hours per week. The age gradient that we observed previously is still preserved. Introducing an hours rule further up the hours distribution allows the government to become more effective in distinguishing between high wage/low effort and high effort/low wage individuals than at 19 hours to the extent that few higher wage individuals would choose to work very few hours. Relative to the schedule when the hours rule is set at around 19 hours, this alternative placement tends to make people with low and high earnings better off, while people in the middle range lose. While we again find that very little happens to the level of out-of-work income, there are much more pronounced changes to the overall structure of marginal rates. In particular, there are large reductions in the marginal tax rate in the first tax bracket for all groups (there is now a tax credit of -0.20 for lone mothers with children aged 11–18, and -0.08 for lone mothers with children aged 5–10), while marginal rates now become higher at higher earnings (especially in the presence of older children). Figure 1.4(a)–(c) show the

[16]This finding contrasts with Keane and Moffitt (1998) which considered introducing a work subsidy in a model with three employment states (non-workers, part-time and full-time work) and multiple benefit take-up. Even small subsidies were found to increase labour supply and to reduce dependence on welfare benefits. In contrast to our application (where we are moving from a base with marginal rates well below 100% at low earnings), their simulations considered introducing the subsidy in an environment where many workers faced marginal effective tax rates which often exceeded 100%.

[17]As was the case with the 19 hours rule, we find that with $\theta = -0.4$ there is essentially no change in either the size or placement of the hours bonus. However, when $\theta = 0.0$ we find that the size of the optimal bonus is approximately halved for all child age groups, whilst the optimal placement shifts to 40 hours per-week.

resulting impact on the hours distribution.

As before, we attempt to quantify the benefits from allowing for hours conditioning. Performing the same experiment as we conducted under the 19 hours rule we find that the required increase in expenditure is considerably larger than that obtained previously (again, see Table 1.10). For lone parents with children aged 11–18, an 8.5% increase in expenditure would be required to achieve the same level of social welfare when $\theta = -0.2$. We believe that if hours can be accurately observed (as this analysis so far assumes), then this represents a non-trivial welfare gain. For lone mothers with younger children, the welfare gains are far more modest. In any case, if the government wishes to maintain the use of hours conditional eligibility, the analysis here suggests that it may be able to improve design by shifting towards a system that primarily rewards full-time rather than part-time work.[18]

1.6.4 Measurement Error and Hours Misreporting

The results presented so far have not allowed for any form of measurement error. While earnings may not always be perfectly measured, it seems likely that there is more scope for mismeasurement of hours as they are conceivably harder to monitor and verify. Indeed, the presence of hours rules in the tax and transfer system presents individuals with an incentive to not truthfully declare whether they satisfy the relevant hours criteria. Relative to when hours are always accurately reported, this would seem to weaken the case for introducing a measure of hours in the tax base. In this section we quantify the importance of such measurement error by considering two alternative scenarios: firstly, we consider the case where hours are imperfectly observed due to random measurement error; secondly, we allow individuals to directly misreport their hours of work to the tax authorities.

In performing this analysis it is necessary to modify our analytical framework from Section 1.2 to distinguish between actual hours of work h, and reported hours of work h_R. While actual hours continue to determine both leisure and earnings, reported hours of work directly affect consumption through the tax schedule, with $T = T(wh, \mathbf{h_R}; X)$. They will also have a direct impact on utility when we allow for individual hours misreporting (discussed below).

Measurement Error

We allow for random measurement error by adding an independent and normally distributed error term ν to work hours h to form a pseudo reported hours measure, $\tilde{h}_R = h + \nu$. Actual reported hours h_R are then given by the nearest discrete hours point in the set of hours $\mathcal{H}_{++}$. We assume that ν has zero mean, and in Table 1.11 we show how the size of the hours bonus and the associated welfare gain, vary as the standard deviation of the measurement error term σ_ν increases in value. A clear pattern emerges. Across all groups, the optimal size of the hours bonus declines as reported hours become less informative. Furthermore, the

[18] We also considered alternative social welfare functions where the government places an explicit weight on employment. In these simulations we obtained lower out-of-work income, together with lower marginal tax rates at low earnings. However, such considerations did not have a large impact on either the size or placement of the optimal hours bonus.

Table 1.11: The effect of random measurement error on the optimal hours bonus

Standard	0–4			5–10			11–18		
Deviation	bonus	hours	welfare	bonus	hours	welfare	bonus	hours	welfare
0	39.54	33	0.7%	51.02	33	3.2%	60.42	33	8.5%
2	37.90	33	0.7%	49.42	33	3.0%	58.87	33	8.2%
4	33.87	33	0.6%	43.38	33	2.5%	52.07	33	6.9%
6	29.13	33	0.5%	36.99	33	2.0%	43.52	33	5.4%
8	23.88	33	0.3%	29.91	33	1.4%	33.42	33	3.7%
10	19.24	33	0.3%	23.83	33	1.1%	30.44	26	2.8%
12	15.06	33	0.2%	20.13	26	0.8%	24.26	26	2.1%
14	13.07	33	0.1%	17.49	26	0.6%	20.76	26	1.7%
16	11.70	26	0.1%	15.73	26	0.6%	18.24	26	1.4%

Notes: Table shows how the optimal placement and size of hours contingent payments varies with random hours measurement error by age of youngest child and with $\theta = -0.2$. Standard Deviation refers to the standard deviation of the additive independent normally distributed hours measurement error term. The columns "welfare" refer to the percentage increase in required expenditure to achieve the same level of social welfare compared to when no hours conditioning is performed. All incomes are in pounds per week and are expressed in April 2002 prices.

placement of the optimal hours rule is reduced from 33 to 26 hours for relatively high values of σ_V. In the simulations where the standard deviation of the error term is equal to 8 (so that a single standard deviation results in reported hours differing from actual hours by a single category), the welfare gain from using hours information is more than halved relative to no measurement error. The presence of random measurement error clearly reduces the desirability of conditioning upon hours, and if it is modest or large in size, then the welfare gains that are achievable are only small, even amongst lone mothers with older children.

Hours Misreporting

We have shown that random measurement error reduces the extent to which the government may wish to condition upon hours of work, and it also diminishes the welfare gains that are achievable. In the case of hours conditioning, it is plausible that the form of misreporting is likely to be more systematic than random measurement error. Here we modify our setup to allow individuals to directly misreport their reported hours of work. We let h_B be the required hours of work to receive a bonus (received if $h \geq h_B$), and we continue to let h_R denote reported hours of work. Misreporting is only possible if $h > 0$, so that the tax authorities can always accurately observe employment status. If individuals misreport their hours of work then they must incur a utility cost, which is assumed to depend upon the distance $h_R - h$. Since misreporting hours is costly, it is only necessary to consider the cases when hours are truthfully revealed $h_R = h$, or when $h_R = h_B > h$.

We therefore modify the individual utility function by including $h_R - h$ as an explicit argument, so that $U = u(c, h, h_R - h; X, \epsilon) + \varepsilon_h$. This modified utility function is as in equation 1.4 but now with the additional cost term $b \times (h_R - h)$ subtracted from u whenever $h_R > h$.[19] If misreporting is not possible, then this is equivalent to $b = \infty$. We do not allow individuals

[19] In practice misreporting costs are likely to vary with both observed and unobserved worker characteristics. While it is sufficient to model this as a single cost for the purpose of our discussion and simulations here, our framework can easily be extended to incorporate such heterogeneity.

Table 1.12: The effect of hours misreporting on the optimal hours bonus

Misreporting	0–4			5–10			11–18		
Cost	bonus	hours	welfare	bonus	hours	welfare	bonus	hours	welfare
∞	39.54	33	0.7%	51.02	33	3.2%	60.42	33	8.5%
0.64	39.54	33	0.7%	51.01	33	3.2%	60.41	33	8.5%
0.32	38.54	33	0.7%	49.03	33	3.1%	57.92	33	8.4%
0.16	29.85	33	0.6%	34.12	33	2.6%	41.38	33	7.0%
0.08	17.35	33	0.4%	19.50	33	1.7%	23.44	33	4.6%
0.04	8.58	33	0.2%	11.04	33	1.0%	12.14	33	2.6%
0.02	5.30	33	0.1%	6.16	33	0.6%	6.73	33	1.5%
0.01	2.75	33	0.1%	3.22	33	0.3%	3.77	33	0.8%

Notes: Table shows how the optimal placement and size of hours contingent payments varies with the utility cost of hours misreporting by age of youngest child and with $\theta = -0.2$. "Misreporting Cost" refers to the additive utility cost associated with misreporting, and is measured per-hour overstated and relative to standard deviation of the state specific error ε. The columns "welfare" refer to the percentage increase in required expenditure to achieve the same level of social welfare compared to when no hours conditioning is performed. All incomes are in pounds per week and are expressed in April 2002 prices.

to manipulate their earnings wh. At a given actual hours of work $h < h_B$ individuals will report their hours as $h_R = h_B$ if and only if the utility gain exceeds the cost. That is:

$$u(c(h, T(wh, \mathbf{h_B}; X), X, \epsilon), h, h_B - h; X, \epsilon) > u(c(h, T(wh, \mathbf{h}; X), X, \epsilon), h, 0; X, \epsilon).$$

We refer to the parameter b as the misreporting cost, and in the results presented in Table 1.12 this is measured relative to the standard deviation of the state specific error ε. With an hours bonus payable at 33 hours per week (for example), a value of $b = 0.16$ would mean that the utility cost of reporting 33 hours when actual hours are 26 is equivalent to a $0.16 \times (33 - 26) = 1.12$ standard deviation change in the realisation of the state specific error. The table illustrates that as the utility cost of misreporting becomes very low, the welfare gain from using reported hours of work effectively disappears (but the optimal placement remains at 33 hours for all values considered). Again, this suggests that the welfare gains from using hours of work information may be small unless the scope for misreporting hours of work is limited.

1.7 Conclusions

The aim of this paper has been to examine the optimal schedule of marginal tax rates and design of earned income tax credits. The context for this design problem has been the tax and transfer schedule for lone parents in Britain. To address this tax design problem we developed a structural labour supply model which incorporated unobserved heterogeneity and the non-convexities of the tax and welfare system as well as allowing for childcare costs and fixed costs of work. We also explicitly allow for different labour supply responses at the intensive and extensive margins.

To mirror the hours contingent nature of the British tax credit system we developed an analytical framework that explicitly allowed for the tax authorities to have partial observability of hours of work. We contrasted this to the standard case in which only earnings (and employment) are revealed to the tax authority.

The structural labour supply model appeared reliable and the estimated model suggested that lone parents with very young children are much less responsive to changes in financial work incentives than are lone parents with children of school age. This has implications for tax design. For those with very young children – where the marginal value of leisure is high – the optimal policy design suggests it is better to offer high levels of income support together with higher marginal tax rates when in work. In contrast, for those with school age children, where leisure is valued less highly, the results suggest a move to a lower level of income support but also lower marginal tax rates, increasing the incentives to work.

Our results highlight a role for conditioning effective tax rates on the age of children. Tax credits being found to be most important for low earning families with school age children. Hours contingent payments, as feature in the British tax credit system, are also found to lead to improvements in the tax design at least for those parents with school age children. If the tax authorities are able to choose the lower limit on working hours that trigger eligibility for such families, then we find an empirical case for using a *full-time* work rule rather than the part-time rule currently in place for parents in the UK. While this is found to be a more effective instrument, the welfare gains remain modest in size for all but parents with older children. These welfare gains are also shown to reduce significantly with moderate amounts of misreporting or measurement error.

Chapter 1 Appendix

1.A Likelihood Function

In what follows let $\mathcal{P}_j(X, p_{c_k}, \epsilon) \equiv \Pr(h = h_j | X, p_{c_k}, \epsilon)$ denote the probability of choosing hours $h_j \in \mathcal{H}$ conditional on demographics X, the childcare price p_{c_k}, and the vector of unobserved preference heterogeneity $\epsilon = (\epsilon_w, \epsilon_{c_X}, \epsilon_y, \epsilon_l, \epsilon_f, \epsilon_\eta)$. Given the presence of state specific Type-I extreme value errors, this choice probability takes the familiar conditional logit form. We also use $\pi_k(X) \equiv \Pr(p_c = p_{c_k} | X)$ to denote the probability of the lone mother with characteristics X facing childcare price p_{c_k}. In the case of non-workers ($h = h_0$), neither wages nor childcare are observed so that the likelihood contribution is simply given by:

$$\sum_k \pi_k(X) \int_\epsilon \mathcal{P}_0(X, p_{c_k}, \epsilon) dG(\epsilon).$$

Now consider the case for workers when both wages and childcare information is observed so that h_c is not censored at zero. Using $E_h \equiv E(h; X, p_c, \epsilon)$ to denote eligibility for in-work support we define the indicator $\mathcal{D}(e, p) = \mathbf{1}(E_h = e, P = p)$. We also let $\Delta u(h_j | p_{c_k}, X, \epsilon_{|\epsilon_\eta = 0})$ denote the (possibly negative) utility gain from claiming in-work support at hours h_j, conditional on demographics X, the childcare price p_{c_k}, and the vector of unobserved preference heterogeneity ϵ with $\epsilon_\eta = 0$. Suppressing the explicit conditioning for notational simplicity, the likelihood contribution is given by:

$$\prod_k \pi_k(X)^{\mathbf{1}(p_c=p_{c_k})} \iiint_{\epsilon_y,\epsilon_l,\epsilon_f} \Bigg\{ \mathcal{D}(1,1) \int_{\epsilon_\eta<\Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)}$$
$$+ \mathcal{D}(1,0) \int_{\epsilon_\eta>\Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} + \mathcal{D}(0,0) \int_{\epsilon_\eta} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} \Bigg\}$$
$$dG(\epsilon|\epsilon_w = \log w - X_w'\beta_w, \epsilon_c = h_c - \gamma_{c_X} - \beta_{c_X} h)$$
$$g_{w,c}(\log w - X_w'\beta_w, h_c - \gamma_{c_X} - \beta_{c_X} h).$$

If working mothers are not observed using childcare, then h_c is censored at zero and the childcare price also unobserved. If $\overline{\epsilon}_c = -\gamma_{c_X} - \beta_{c_X} h$, then the likelihood contribution is given by:

$$\sum_k \pi_k(X) \iiiint_{\epsilon_c<\overline{\epsilon}_c,\epsilon_y,\epsilon_l,\epsilon_f} \Bigg\{ \mathcal{D}(1,1) \int_{\epsilon_\eta<\Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)}$$
$$+ \mathcal{D}(1,0) \int_{\epsilon_\eta>\Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} + \mathcal{D}(0,0) \int_{\epsilon_\eta} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} \Bigg\}$$
$$dG(\epsilon|\epsilon_w = \log w - X_w'\beta_w) g_w(\log w - X_w'\beta_w).$$

Our estimation also allows for workers with missing wages. This takes a similar form to the above, except that it is now necessary to also integrate over the unobserved component of wages ϵ_w.

1.B Proof of Proposition

For notational simplicity we abstract from the explicit conditioning of utility on observed and unobserved preference heterogeneity and let $u(h) \equiv u(c(h), h; X, \epsilon)$. We then define V as the integral of transformed utility over state specific errors conditional on (X, ϵ):

$$V \equiv \int_\varepsilon \Upsilon\left(\max_{h\in\mathcal{H}} [u(h) + \varepsilon_h]\right) dF(\varepsilon). \tag{1.9}$$

To prove this result we first differentiate V with respect to $u(h)$:

$$\begin{aligned} \frac{\partial V}{\partial u(h)} &= \int_\varepsilon \left(\frac{\partial \Upsilon\left(\max_{h\in\mathcal{H}} [u(h)+\varepsilon_h]\right)}{\partial u(h)} \right) dF(\varepsilon) \\ &= \int_\varepsilon \Upsilon'\left(u(h)+\varepsilon_h\right) \times \mathbf{1}\left(h = \arg\max_{h'\in\mathcal{H}} \left[u(h') + \varepsilon_{h'}\right] \right) dF(\varepsilon). \end{aligned}$$

Given our choice of utility transformation function in equation 1.8 and our distributional assumptions concerning ε the above becomes:

$$\begin{aligned}\frac{\partial V}{\partial u(h)} &= \int_{\varepsilon_h=-\infty}^{\infty}\left\{e^{(u(h)+\varepsilon_h)}\right\}^{\theta}\left(\prod_{h'\neq h} e^{-e^{-\{\varepsilon_h+u(h)-u(h')\}}}\right)\times e^{-\varepsilon_h}e^{-e^{-\varepsilon_h}}d\varepsilon_h \\ &= \left\{e^{u(h)}\right\}^{\theta}\int_{\varepsilon_h=-\infty}^{\infty}\{e^{\varepsilon_h}\}^{\theta}\times\exp\left(-e^{-\varepsilon_h}\sum_{h'\in\mathcal{H}} e^{-(u(h)-u(h'))}\right)e^{-\varepsilon_h}d\varepsilon_h.\end{aligned}$$

We proceed by using the change of variable $t=\exp(-\varepsilon_h)$ so that the above partial derivative becomes:

$$\frac{\partial V}{\partial u(h)} = \left\{e^{u(h)}\right\}^{\theta}\int_{t=0}^{\infty} t^{-\theta}\times\exp\left(-t\sum_{h'\in\mathcal{H}} e^{-(u(h)-u(h'))}\right)dt.$$

By defining $z\equiv t\times\sum_{h'\in\mathcal{H}} e^{-(u(h)-u(h'))}$ we can once again perform a simple change of variable and express the above as:

$$\begin{aligned}\frac{\partial V}{\partial u(h)} &= \left\{e^{u(h)}\right\}^{\theta}\left\{\textstyle\sum_{h'\in\mathcal{H}} e^{-(u(h)-u(h'))}\right\}^{\theta-1}\int_{z=0}^{\infty} z^{-\theta}e^{-z}dz \\ &= e^{u(h)}\left\{\textstyle\sum_{h'\in\mathcal{H}} e^{u(h')}\right\}^{\theta-1}\int_{z=0}^{\infty} z^{-\theta}e^{-z}dz \\ &= e^{u(h)}\left\{\textstyle\sum_{h'\in\mathcal{H}} e^{u(h')}\right\}^{\theta-1}\Gamma(1-\theta)\end{aligned}\tag{1.10}$$

where the third equality follows directly from the definition of the Gamma function $\Gamma(\cdot)$. Note that this integral will always converge given that we are considering cases where $\theta<0$. Integrating equation 1.10 we obtain:

$$V=\frac{1}{\theta}\left[\Gamma(1-\theta)\times\left(\sum_{h'\in\mathcal{H}}\exp\left\{u(h')\right\}\right)^{\theta}-1\right]\tag{1.11}$$

where the constant of integration is easily obtained by considering the case of a degenerate choice set and directly integrating 1.9. This completes our proof of the Proposition.

Chapter 2

Wage Posting with Two-sided Heterogeneity

2.1 Introduction

Partial job search models provide a natural framework for studying many features of modern labour markets. While these models may be useful in describing a number of stylized facts, the appropriateness of such models when performing policy analysis is limited because equilibrium responses by firms are ruled out by assumption. A logical extension of this literature is provided by the theoretical and empirical research on equilibrium job search. In this literature, the behaviour of firms is explicitly modelled. Here, the competition between firms acts as the fundamental determinant of wages, with the extent of this competition limited by the presence of informational frictions in the economy.

This paper considers an equilibrium job search model with a specific form of wage determination: firms post wage offers prior to meeting potential employees, which workers may then either accept or reject without any possibility of bargaining. This is known as *wage posting*. We contribute to this literature by first developing and estimating a wage posting model with on-the-job search and unobserved worker and firm heterogeneity. Firms may differ with respect to their productivity whilst workers differ with respect to their opportunity costs of employment. While such a model was analysed by Bontemps et al. (1999), this was conducted under the restriction that the arrival rate of job offers was independent of employment status. In their empirical analysis, this over identifying restriction lead to a poor fit of unemployment durations, which is perhaps why this model had been little exploited in the literature. The model we consider here is one which has been referred to as either being intractable, or particular difficult to estimate (see, for example, van den Berg, 1999). While many of the theoretical properties of this model are indeed difficult to characterise analytically, this paper demonstrates that it remains empirically tractable and provides a useful benchmark model for conducting many types of policy experiment.

While the importance of on-the-job search as a source of wage growth for employees is well documented (see, for example, Topel and Ward, 1992), simultaneously allowing for worker and firm heterogeneity allows the model to explain both the dispersion of wages, and individual employment histories. Similarly, allowing the arrival rates of job offers to vary by employment status also allows durations to be better explained. Of course, simply fitting empirical distributions and durations is a very limited objective of an equilibrium job search model. The value of such models is perhaps greatest when they are applied as tools for understanding the impact of policy reform. In these models the existence of search frictions gives firms some degree of monopsony power, which means that labour market outcomes can differ from the those of a competitive model. In particular, reforms such as minimum wage legislation (which we shall consider in our application in this paper) and tax programmes designed to encourage labour market participation (see Chapter 3), can all have potentially very rich effects. To better understand the value that is introduced by these features, we first provide a brief survey of the literature.

The starting point of this literature is Diamond (1971). In this model all workers are assumed identical and there is no possibility of searching for a better job once employed. Unemployed workers sequentially sample job offers from some wage offer distribution. Since workers are homogeneous they will all follow the same *reservation wage* strategy, accepting any wage that is greater than the the common reservation wage. In such a model no firm would set a wage below the common reservation wage as they would attract no workers. By the same logic, no firm would offer a wage above the reservation wage as doing so will not attract any more workers. In this case the distribution of wages would collapse to a degenerate distribution equal to the reservation wage. The Diamond result generated much criticism of the basic search model, and this ultimately led to the emergence of the equilibrium job search literature.

Within this class of job search model, there are two standard ways of generating wage dispersion as an equilibrium outcome. While the mechanisms are very different, both approaches generate an upward sloping labour supply curve at the level of the firm. Since, in Diamond (1971), equilibrium wages are equal to the reservation wage of unemployed workers, it may be possible to generate wage dispersion should workers differ in their reservation wages. Albrecht and Axell (1984) maintain the same core assumptions as in Diamond, but allow workers to differ in their opportunity costs of employment. In this model, workers with higher employment opportunity costs will be those with higher reservation wages. This creates the possibility of wage dispersion in equilibrium as it means that firms face a trade-off between profits per work, and the number of workers employed by the firm. Thus, even amongst identical firms, some may choose to offer a high wage (earning a small profit margin per worker, but with a large workforce), whilst others may offer a lower wage (earning a higher profit per worker, but with a smaller workforce). Therefore there is an upward sloping labour supply curve at the level of the firm.

The second approach maintains the assumption that workers (and firms) are homogeneous, but allows workers to continue searching for a better job when they are employed (Burdett and Mortensen, 1998). For employed workers, their reservation wage will equal their current wage. Firms now have an incentive to increase their wage offer because by doing they are able to attract workers from lower paying jobs. Thus, at the level of the firm there is again an upward sloping labour supply curve which allows firms with the same productivity level to offer distinct wages in equilibrium.

While both of the above approaches are successful in the sense that they generate wage dispersion as an equilibrium outcome, they provide an unsatisfactory fit to empirical wage distributions. In Albrecht and Axell (1984) each point of support in the wage distribution must correspond to a point of support in the distribution of reservation wages,[1] while in Burdett and Mortensen (1998) the theoretical distributions of wage offers and wage earnings both have increasing densities. The apparent empirical failings of Burdett and Mortensen can be overcome to some extent by introducing firm level heterogeneity. Building upon the original demonstration by Mortensen (1990) that more productive firms offer higher wages in equilibrium, Bontemps et al. (2000) provided a comprehensive theoretical analysis of the Burdett and Mortensen model with a continuous distribution of firm productivity. They showed that the model may induce empirical wage distributions by allowing for an appropriately skewed distribution of firm productivity, and proposed a simple three-step semi-parametric estimation procedure for this model.

Both the original Burdett and Mortensen (1998) model and the extension analysed in Bontemps et al. (2000) imply a constant exit rate out of unemployment. While the model of Albrecht and Axell (1984) clearly does not provide any role for job-to-job transitions as a source of wage growth, it does display negative unemployment duration dependence at the aggregate.[2,3] Bontemps et al. (1999) combine both approaches by allowing for continuous distributions of both firm productivity and work opportunity costs under the restriction that the arrival rate of job offers is the same for unemployed and employed workers. This over identifying restriction simplifies the analysis as it implies that the optimal strategy of workers is independent of the equilibrium wage offer distribution (see the later discussion in Section 2.2). However, in their empirical application this ultimately led to a poor fit to the unemployment duration data.

Simultaneously allowing for heterogeneity in both firm productivity and reservation wages, together with potentially different search efficiency on- and off-the-job, is therefore desirable as it allows us to explain the dispersion of wages, durations, and heterogeneity in unemployment histories. Moreover, it is particularly useful in terms of performing policy

[1] To understand this, note that by definition of the reservation wage, if a firm was offering a wage strictly between two reservation wages, then it could always increase its profits by reducing the wage it offered to the lower of the two reservation wages: there would be no change in employment, but profits per worker would increase.

[2] See Machin and Manning (1999) for European evidence on duration dependence.

[3] Note that there is no true duration dependence, but rather this is a purely compositional effect due to workers differing in the wage offers that they are willing to accept. Negative duration dependence will be observed regardless of the shape of the distribution of reservation wages.

analysis as reservation wage heterogeneity essentially introduces a labour supply dimension. To understand this, note that in the homogeneous worker model of Bontemps et al. (2000) all workers accept all jobs. This means that the imposition of a legal minimum wage (for example) will either have no effect on unemployment, or will have a negative effect if the number of firms active in the labour market is reduced. In contrast, if some workers are selective in the wages that they are willing to accept, then higher wage offers will, other things equal, increase job acceptance behaviour. The effect of a minimum wage on employment outcomes is therefore potentially much richer, and is *a priori* ambiguous. Similarly, wage subsidy or tax credit type programmes designed to boost employment would have no effect on employment in the basic homogeneous worker model, but could have sizeable impacts in a model with worker heterogeneity (see Chapter 3 of this thesis).

The starting point of this paper is therefore to extend the analysis of Bontemps et al. (1999) to cases where no restrictions are placed on the relative efficiency of on- and off-the-job search. In Section 2.2 we describe the model, characterise the equilibrium, and perform some simple comparative static exercises. Section 2.3 discusses the identification of the model using longitudinal survey data, and proposes alternative estimation techniques. In Section 2.4 we present some estimation results, and compare and contrast the results obtained under the proposed parametric and semi-parametric estimators. An extension of the model which introduces further within market heterogeneity is described in Section 2.5, and in Section 2.6 we apply this model in our analysis of the introduction of actual UK minimum wage legislation. Finally, in Section 2.7 we conclude.

2.2 Basic Model

In this section we set out a model with wage posting and on-the-job search, where workers are heterogeneous with respect to their opportunity cost of employment. The model presented here is a direct extension of Bontemps et al. (1999) to allow for the arrival rate of job offers to potentially vary with employment status. As such, the exposition follows that of Bontemps et al. closely. We begin by setting out the core assumptions and describe the optimal job acceptance strategy of workers. We then characterise the steady state flows, describe the optimal wage setting behaviour of firms, and discuss properties of the equilibrium. We end this section by performing some comparative static exercises.

2.2.1 Model Assumptions

The economy consists of a continuum of individuals with a population size normalized to unity. Time is continuous and individuals live forever with the constant discount rate $\rho > 0$. These individuals (or workers) can be either employed or unemployed, and both search for jobs. While workers are assumed to be equally productive at a given firm (see below), they differ in their opportunity cost of employment b which has the cumulative distribution function H on support $[\underline{b}, \overline{b}]$, with $-\infty \leq \underline{b} < \overline{b} \leq \infty$. To simplify some of the exposition we shall always assume that $\underline{b}$ is sufficiently low, so that in the absence of any binding minimum wage, all firms are active in the labour market. We assume that H is continuous with strictly positive density on the entire support.

Jobs are completely characterised by a wage rate w that is assumed to be constant throughout an individuals employment spell within a given firm. Individuals sequentially sample these job offers at the exogenous Poisson rate $\lambda_u > 0$ when unemployed, and $\lambda_e \geq 0$ when employed. We place no restrictions on the relative magnitude of these quantities. Workers sample these jobs from a wage offer distribution F with support $[\underline{w}, \overline{w}]$ and we let $f \equiv F'$ denote the corresponding density function.[4] Upon receiving a job offer, workers may choose to either accept or reject it. There is no bargaining. Regardless of the current wage, all employment spells end at the Poisson rate $\delta > 0$ and there is no recall of past job offers. In the following we let $\kappa_e = \lambda_e/\delta$ and $\kappa_u = \lambda_u/\delta$. As emphasized by van den Berg and Ridder (2003), the parameters κ_e and κ_u can be thought of as labour market friction parameters. In particular, κ_e is the number of job offers an individual can expect to receive when employed, before exiting to unemployment.

Worker Strategies

The flow utility of an employed worker earning wage w is given by $v_e(w) = w$. Similarly, the flow utility of an unemployed worker is simply equal to their work opportunity cost: $v_u = b$.[5] The value of b (which is the only source of worker heterogeneity) should be interpreted as representing both the value of unemployment benefit and other non-pecuniary costs/benefits associated with unemployment. It is straightforward to verify that the value from employment is strictly increasing in the wage w, which implies the existence of a *reservation wage* policy: employed workers will accept any wage that is strictly greater than their current wage w; unemployed workers with opportunity cost b will accept any job offer w that is greater than or equal to some value $\phi(b)$. As demonstrated by Mortensen and Neumann (1988), the reservation wage for unemployed workers $\phi(b)$ is implicitly defined as:

$$\phi(b) = b + (\kappa_u - \kappa_e) \int_{\phi(b)}^{\overline{w}} \frac{\overline{F}(w)}{1 + \rho/\delta + \kappa_e \overline{F}(w)} dw \tag{2.1}$$

where $\overline{F} \equiv 1 - F$. Note that in the case that $\kappa_u = \kappa_e$ we have $\phi(b) = b$ so that the optimal strategy of workers is independent of the equilibrium wage offer distribution. The significance of this assumption is that it implies that there is no feedback from the strategy of firms to the strategy of workers. This was the model analysed in Bontemps et al. (1999).

2.2.2 Steady State Flows

This section characterises the steady state of the labour market by using flow equations. For now, we treat the wage offer distribution F as given. In Section 2.2.3 we describe how this

[4] To avoid some technical complications, we shall assume from the outset that the distribution of wage offers is continuous. See Bontemps (1998) and Bontemps et al. (1999, 2000) for further details.

[5] We are implicitly assuming that there is no access to either saving or borrowing technology.

distribution emerges in equilibrium.

Distribution of Reservation Wages

In deriving the steady state flows it is necessary to consider the distribution of reservation wages. This distribution in the whole population (that is, among both unemployed and employed workers) is denoted A and it is related to the distribution of work opportunity costs through the inverse reservation wage equation ϕ^{-1}:[6]

$$A(x) = H(\phi^{-1}(x)) = H\left(x - (\kappa_u - \kappa_e)\int_x^{\overline{w}} \frac{\overline{F}(w)}{1+\rho/\delta+\kappa_e\overline{F}(w)}dw\right). \quad (2.2)$$

The distribution of reservation wages amongst the stock of unemployed and employed workers are similarly defined as A_u and A_e respectively. Since workers who are unemployed (employed) will have higher (lower) reservation wages on average, it must be true that $A_u(x) \leq A(x) \leq A_e(x)$. These distributions are related to A according to:

$$A(x) = uA_u(x) + (1-u)A_e(x) \quad (2.3)$$

where u is the steady state unemployment rate and $1-u$ the steady state employment rate. It is necessary to consider these distributions when describing the equilibrium of the labour market. In particular, the distribution of reservation wages amongst the unemployed A_u is required to describe the flows from the unemployment pool into employment at a given wage. In steady state we require that the flow of individuals with a reservation wage less than or equal to ϕ who exit the employment pool due to their job being exogenously destroyed must exactly the flow of such workers who enter employment. That is,

$$\delta(1-u)A_e(\phi) = \begin{cases} \lambda_u u A_u(\phi) & \text{if } \phi \leq \underline{w} \\ \lambda_u u A_u(\underline{w}) + \lambda_u u \int_{\underline{w}}^{\phi} \overline{F}(w)dA_u(w) & \text{if } \phi > \underline{w}. \end{cases} \quad (2.4)$$

Differentiating equation 2.4 using Leibniz's rule we obtain $\delta(1-u)A'_e(x) = \lambda_u\overline{F}(x)uA'_u(x)$, which when combined with equation 2.3 implies $A'(x) = uA'_u(x)(1+\kappa_u\overline{F}(x))$ so that we may then write:

$$uA_u(\phi) = \begin{cases} \frac{1}{1+\kappa_u}A(\phi) & \text{if } \phi \leq \underline{w} \\ \frac{1}{1+\kappa_u}A(\underline{w}) + \int_{\underline{w}}^{\phi}\frac{dA(w)}{1+\kappa_u\overline{F}(w)} & \text{if } \phi > \underline{w} \end{cases} \quad (2.5)$$

and where we note that the density of the reservation wage distribution (obtained by differ-

[6] In the special case that $\kappa_u = \kappa_e$ these distributions coincide. That is: $A(x) = H(\phi^{-1}(x)) = H(x)$.

enetiating equation 2.2) is given by:

$$A'(x) = H'\left(x - (\kappa_u - \kappa_e)\int_x^{\overline{w}} \frac{\overline{F}(w)}{1+\rho/\delta+\kappa_e\overline{F}(w)}dw\right)\left[\frac{1+\rho/\delta+\kappa_u\overline{F}(x)}{1+\rho/\delta+\kappa_e\overline{F}(x)}\right]. \qquad (2.6)$$

Remark An alternative way to derive $uA_u(\phi)$ is to note that the unemployment rate of a worker, conditional on their reservation wage w is given by $(1+\kappa_u\overline{F}(w))^{-1}$. Integrating this expression over the (whole population) distribution of reservation wages no greater than ϕ then yields the result in equation 2.5.

Between Jobs and the Distribution of Wages

In what follows we let $G(w)$ denote the fraction of employed workers with a wage no greater than w, and denote the density of wages amongst the employed as $g \equiv G'$. This cumulative distribution will dominate the distribution of wage offers F both because of on-the-job search (workers gravitate to higher paying jobs) and because of reservation wage heterogeneity (workers are selective in the jobs that they are willing to accept). In a steady-state equilibrium, the number of individuals who leave jobs paying a wage no greater than w (either by their job being destroyed at Poisson rate δ or by gravitating to a higher paying job) must exactly equal the number of individuals who exit the unemployment pool and receive such wages. We therefore have:

$$\left[\delta+\lambda_e\overline{F}(w)\right](1-u)G(w) = \lambda_u F(w)uA_u(\underline{w}) + \lambda_u u\int_{\underline{w}}^{w}(F(w)-F(x))dA_u(x). \qquad (2.7)$$

To proceed further we note that the RHS of equation 2.7 above may be written as:

$$\begin{aligned}
&\lambda_u F(w)uA_u(\underline{w}) + \lambda_u u\int_{\underline{w}}^{w}(\overline{F}(x)-\overline{F}(w))dA_u(x)\\
&= \lambda_u uA_u(\underline{w}) + \lambda_u u\int_{\underline{w}}^{w}\overline{F}(x)dA_u(x) - \lambda_u\overline{F}(w)uA_u(\underline{w})\\
&= \delta(1-u)A_e(w) - \lambda_u\overline{F}(w)uA_u(\underline{w})\\
&= \delta A(w) - uA_u(w)[\delta+\lambda_u\overline{F}(w)]
\end{aligned}$$

where the second equality follows from equation 2.4 and the third equality from equation 2.3. Substituting this expression into equation 2.7 and dividing through by δ we then obtain:

$$\left[1+\kappa_e\overline{F}(w)\right](1-u)G(w) = A(w) - \left[1+\kappa_u\overline{F}(w)\right]uA_u(w) \qquad (2.8)$$

or equivalently,

$$G(w) = \frac{A(w) - (1+\kappa_u\overline{F}(w))\left[\frac{1}{1+\kappa_u}A(\underline{w}) + \int_{\underline{w}}^{w}\frac{dA(x)}{1+\kappa_u\overline{F}(x)}\right]}{\left[1+\kappa_e\overline{F}(w)\right](1-u)} \qquad (2.9)$$

which expresses the distribution of wage earnings in terms of the distribution of reservation wages, wage offers, and transitional parameters. Note that the distribution of wage earnings presented above in equation 2.9 is essentially the same as that which appear in Proposition 2 of Bontemps et al. (1999). They only differ to the extent that we have the reservation wage distribution appearing in place of the underlying opportunity cost distribution (these are the same in their model) and that differential arrival rates appear where relevant.

Unemployment

Finally, we derive the steady state unemployment rate u. This may be obtained by letting $\phi \to \infty$ in equation 2.5, which yields:

$$u = \frac{1}{1+\kappa_u} A(\underline{w}) + \int_{\underline{w}}^{\overline{w}} \frac{dA(x)}{1+\kappa_u \overline{F}(w)} + (1 - A(\overline{w})). \tag{2.10}$$

This equation decomposes the steady state unemployment rate into the contribution by three (endogenously determined) groups of workers: those who accept all offers; those who accept some and reject others; and those who reject all. The unemployment rate u is bounded below by $(1+\kappa_u)^{-1}$, which is the unemployment rate that would prevail in the absence of any reservation wage heterogeneity. In contrast to the homogeneous worker model, note that κ_e affects u through two channels: the direct effect through changes in worker reservation wages and the indirect effect through its potential impact on the equilibrium wage offer distribution F.

2.2.3 Firm Behaviour

In order to make this an equilibrium model we specify the behaviour of firms. As will become clear, it is the profit maximising behaviour of firms, taking as given the optimal strategies of both workers and other firms, that determines the equilibrium distribution of wage offers F. Firms may differ with respect to their (exogenously determined) productivity level p. The fraction of firms with productivity no greater than p is given by the cumulative distribution function $\Gamma(p)$ on support $[\underline{p}, \overline{p}]$. We normalize the total number of firms to unity, and assume that Γ is continuous, with productivity density $\gamma \equiv \Gamma'$ that is strictly positive on its entire support.

We assume that there is wage posting: each productivity p firm posts a single wage w prior to forming matches with potential employees, who can then either accept or reject the wage offer.[7] Matching is assumed to be random, so that the probability of encountering a given firm is independent of firm size and depends only upon the distribution of firm productivity types. This means that $F(w)$ also measures the number of firms offering a wage less than w.

The production technology of firms is characterized by constant returns to scale so that the interpretation of p is the (firm specific) marginal product of labour. Firms chooses a single wage w and maximize steady state profit flow subject to its production technology, and taking

[7] See Bontemps et al. (1999) for a discussion on the assumption that firms offer a single wage to all employees.

as given the behaviour of both other firms and workers. If we let $l(w)$ denote the steady state firm size at wage w, then the steady state profit flow may be written as:

$$\pi(p,w) = (p-w)l(w). \tag{2.11}$$

We may determine $l(w)$ by using flow equations. In particular, the number of workers who leave a firm paying wage w (either by their employment spell being destroyed at rate δ, or by a job-to-job transition to a higher wage firm) must exactly equal the number who accept such a wage (from either the unemployment or employment pools). That is:

$$l(w)[\delta + \lambda_e \overline{F}(w)] = \lambda_u u A_u(w) + \lambda_e (1-u) G(w)$$

so that $l(w)$ is clearly non-decreasing in w. This arises because firms which pay higher wages attract more workers from both the unemployment pool (the mechanism in Albrecht and Axell, 1984) and lower wage firms (the mechanism in Burdett and Mortensen, 1998). Using equations 2.5 and 2.9 we may then rewrite this flow equation as:

$$l(w) = \frac{\kappa_e A(w)}{(1+\kappa_e \overline{F}(w))^2} + \frac{(\kappa_u - \kappa_e)}{(1+\kappa_e \overline{F}(w))^2} \left[\frac{1}{1+\kappa_u} A(\underline{w}) + \int_{\underline{w}}^{w} \frac{dA(x)}{1+\kappa_u \overline{F}(x)} \right]. \tag{2.12}$$

In contrast to models without reservation wage heterogeneity, the absence of on-the-job search (that is, $\kappa_e = 0$) does not imply that employment is uniformly distributed across firms when matching is random. This is intuitive because low wage firms are only able to attract low reservation wage workers (it is straightforward to show that $l(w)$ is proportional to $A_u(w)$ in this case). More generally, note that the denominator in equation 2.12 may be written as $\kappa_u u A_u(w) + \kappa_e (1-u) A_e(w)$ so that steady state firm size $l(w)$ depends on the number of unemployed and employed workers with a reservation wage no greater than w weighted by the respective arrival rates. When $\kappa_u = \kappa_e = \kappa$ the denominator reduces to $\kappa A(w) = \kappa H(w)$.

2.2.4 Characterization of Equilibrium

Wage Policy Function

To characterise the equilibrium we denote the optimal *wage policy* of a productivity p firm as $w = K(p)$, with $K(p) = \arg\max_w \pi(p,w)$. First, we note that more productive firms must offer higher wages in equilibrium[8] so that $F(K(p)) = \Gamma(p)$. A corollary of this is that more productive firms also have larger firm size and earn higher profits. Maximising equation 2.11 subject to equation 2.12, the optimal wage w for a type p firm must satisfy the first order

[8] To see this suppose that $p_2 > p_1$ and let $w_2 = K(p_2)$ and $w_1 = K(p_1)$ denote the optimal wage policy of these firms. By definition of profit maximization it must be true that $(p_2 - w_2)l(w_2) \geq (p_2 - w_1)l(w_1) > (p_1 - w_1)l(w_1) \geq (p_1 - w_2)l(w_2)$ which implies that $(p_2 - p_1)l(w_2) > (p_2 - p_1)l(w_1)$. Since $l(w)$ is increasing in w it then follows that $w_2 > w_1$.

condition:

$$\kappa_e A(w) + (\kappa_u - \kappa_e)uA_u(w)$$
$$= (p - w)\left[\frac{2\kappa_e f(w)(\kappa_e A(w) + (\kappa_u - \kappa_e)uA_u(w))}{1 + \kappa_e \overline{F}(w)} + \frac{\kappa_u A'(w)(1 + \kappa_e \overline{F}(w))}{1 + \kappa_u \overline{F}(w)}\right] \tag{2.13}$$

which we will refer to in subsequent derivations. Noting that the profits of a productivity p firm are given by $\pi(p) = \pi(p, K(p)) = (p - K(p))l(K(p))$, from the envelope theorem it immediately follows that $\pi'(p) = l(K(p))$. Since it has been established that $l(w)$ is increasing in w, and that $K(p)$ is also increasing in p, it must be true that the equilibrium steady state profit flow of firms' is a convex function of p. Moreover, using this envelope result we are able to express these equilibrium profit flows as:

$$\begin{aligned}\pi(p) &= \pi(\underline{p}) + \int_{\underline{p}}^{p} l(K(x))dx \\ &= \frac{\kappa_u(\underline{p} - \underline{w}^*)A(\underline{w}^*)}{(1+\kappa_u)(1+\kappa_e)} + \int_{\underline{p}}^{p} l(K(x))dx \end{aligned} \tag{2.14}$$

where $\underline{w}^* = \arg\max_w \pi(\underline{p}, w)$ is the optimal wage policy of the least productive firm $\underline{p}$. By equating equation 2.14 to equation 2.11 (evaluated at $w = K(p)$) and then rearranging terms we obtain the following implicit equation for the optimal wage policy function:

$$K(p) = p - \left[\frac{\kappa_u(\underline{p} - \underline{w}^*)A(\underline{w}^*)}{(1+\kappa_u)(1+\kappa_e)} + \int\limits_{\underline{p}}^{p} l(K(x))dx\right] \times \frac{1}{l(K(p))} \tag{2.15}$$

which is a form that we later exploit when solving for the equilibrium of the model. By differentiating equation 2.15 it can be shown that whenever $\kappa_e > 0$ the optimal wage policy of firms evolves according to:

$$K'(p) = 2\kappa_e\gamma(p) \times \left[\frac{1 + \kappa_e\overline{\Gamma}(p)}{p - K(p)} - \frac{\kappa_u A'(K(p))}{(1 + \kappa_u\overline{\Gamma}(p))l(K(p))}\right]^{-1} \tag{2.16}$$

which we will refer to in some of the later theoretical results.[9] Once the wage policy function $K(p)$ has been solved, it is straightforward to derive a number of interesting equilibrium

[9] Equation 2.16 suggests an alternative way of solving for the equilibrium of the model. In particular, it is possible to express the solution of the wage policy function as a boundary value problem. The equilibrium may then be solved by providing an initial guess of the initial values (since $A(\underline{w})$ depends on the entire distribution of wage offers and is unknown) and then solving as an initial value problem. If the relevant boundary conditions are satisfied then the initial values are consistent with an equilibrium of the wage posting game, otherwise update the guess of the initial values.

objects. In particular, whenever $\kappa_e > 0$ the wage offer density is given by:

$$f(K(p)) = \frac{1+\kappa_e\overline{F}(K(p))}{2\kappa_e}\left[\frac{1}{p-K(p)} - \frac{\kappa_u A'(K(p))(1+\kappa_e\overline{F}(K(p)))}{(1+\kappa_u\overline{F}(K(p)))(\kappa_e A(K(p)) + (\kappa_u-\kappa_e)uA_u(K(p)))}\right] \tag{2.17}$$

which when combined with the observation that $F(w) = \Gamma(K(p))$ allows all the objects introduced in sections 2.2.1–2.2.3 to easily be computed.

Properties at $w = \underline{w}$

Understanding the properties of the model at $\underline{w}$ are very important both when solving the model numerically and when performing estimation (see the later discussion in Section 2.3). The following proposition describes the properties of $f(w)$ and $g(w)$ at $w = \underline{w}$.

Proposition 2. *Suppose that $A(\underline{w}) < 1$ and that $\kappa_e > 0$. If an equilibrium exists and there is no binding minimum wage then $f(\underline{w}) = g(\underline{w}) = 0$; conversley if $A(\underline{w}) = 1$ then $f(\underline{w}) = (1+\kappa_e) \times [2\kappa_e(\underline{p} - \underline{w})]^{-1} > 0$ and $g(\underline{w}) = f(\underline{w})[\kappa_u u/(1-u) + \kappa_e] \times (1+\kappa_e)^{-1} > 0$.*

The proof of this proposition is provided in Appendix 2.A. The proposition states that if some workers have reservation wages below the lowest equilibrium wage offer ($A(\underline{w}) < 1$), then it must be the case that the density of wage offers and earnings at $\underline{w}$ are both zero. By setting $f(\underline{w}) = 0$ in equation 2.13 it must be true that the lowest wage offer satisfies the first order condition:

$$(\underline{p} - \underline{w}^*)A'(\underline{w}^*) = A(\underline{w}^*) \tag{2.18}$$

which from equation 2.16 also implies that $K'(\underline{p}) = +\infty$. The implication of this is that if the empirical earnings density $\hat{g}(\underline{w}) > 0$ then this distribution of wage earnings is only implementable as an equilibrium when all workers accept all equilibrium wages ($A(\underline{w}) = 1$). Thus, reservation wage heterogeneity imposes strong restrictions on the set of admissible wage distributions. These conditions need not be true in the presence of a minimum wage, however, and we discuss this case in Section 2.6.

Definition of Equilibrium

We now proceed to define equilibrium of the labour market in the following definition:

Definition 1. *A labour market equilibrium in the economy is defined by a distribution of wage offers F and reservation wage functions ϕ such that simultaneously:*

1. *Workers follow a reservation wage strategy: unemployed workers accept any wage offer at least as high as $\phi(b)$ (where $\phi(b)$ is defined in equation 2.1); employed workers accept any wage strictly greater than their current wage.*

2. *The stratgey of each productivity p firm is to choose a wage w that maximizes profits given the strategies of other firms' and workers':*

$$K(p) = \arg\max_{w} \pi(p, w)$$

where $\pi(p, w)$ is as defined in equation 2.11.

3. *The distribution of wage offers in the economy satisfies: $F(K(p)) = \Gamma(p)$.*

Solving for the Labour Market Equilibrium

We solve for the equilibrium of the model by determining the wage policy function $K(p)$. Once this has been determined we are able to calculate all the relevant equilibrium functions. The feedback that the strategy of firms has on the job acceptance behaviour of workers whenever $\kappa_u \neq \kappa_e$ complicates the solution to the model. The numerical algorithm that is used to solve for the equilibrium of the economy is presented in Appendix 2.B. Essentially this involves discretizing the distribution of firm productivity and iterating on the wage policy function using equation 2.15. At each iteration step we also update the guess of the lowest wage $\underline{w}^*$ by using equation 2.18. While we have not developed any formal existence or uniqueness proof, such problems have never appeared during extensive numerical simulations.

2.2.5 Comparative Static Exercises

In wage posting models with two-sided heterogeneity as developed here, it is generally not possible to determine analytically how the equilibrium functions vary with the structural parameters. In order to provide some insight regarding their dependency, Bontemps et al. (1999) performed a number of interesting comparative static exercises under the restriction that $\kappa_u = \kappa_e$. Here we conduct a similar exercise. With the exception of the value taken by κ_e we adopt the same specific parameter values and distributional assumptions as used in their baseline model: we set $\kappa_u = 20$ and assume that work opportunity costs H are normally distributed with mean 2500 and standard deviation 1000; the productivity distribution is assumed Pareto with $\underline{p} = 3000$ and a Pareto parameter 2.8. We impose the additional parametrization $\rho/\delta = 1$ in all of the following simulations (the solution to the model is invariant to the value of this ratio whenever $\kappa_u = \kappa_e$).

In Figure 2.1a we show how the wage policy function changes as we vary κ_e. In the figure we consider three values: $\kappa_e = 10$, $\kappa_e = \kappa_u = 20$ and $\kappa_e = 30$. Since we are implicitly holding the job destruction rate δ fixed, this is equivalent to changing the arrival rate λ_e. As κ_e increases the figure shows that there is increased wage dispersion: $K(p)$ increases for high p and decreases for low p. The intuition for this result is essentially the same as in the model without employee heterogeneity: when κ_e is high, high productivity type firms have a much larger incentive to offer a higher wage since doing attracts a larger flow of workers. Meanwhile, low productivity type firms have little incentive to offer high wages since workers will exit to higher paying jobs more quickly. Figure 2.1b recasts these wage policy functions into the monopsony power index $1 - K(p)/p$. Note that this index is not necessarily monotone in p. Figure 2.1c shows directly that as we increase κ_e there is increasing dispersion of wage offers. The distribution of wage earnings mirrors the increased dispersion of wage offers and is shown in Figure 2.1d.

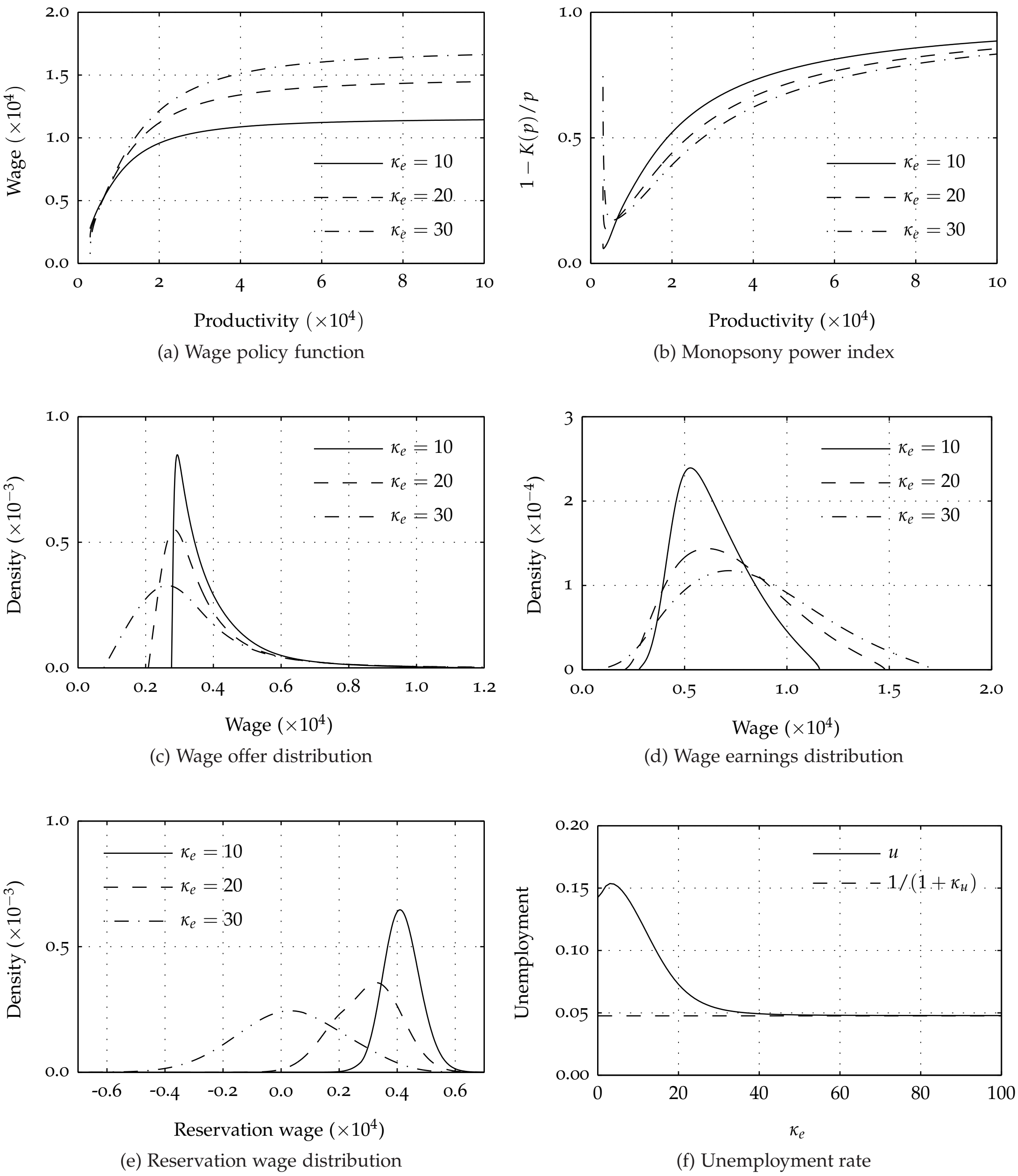


Figure 2.1: Impact of varying κ_e on equilibrium functions. Figure shows the impact of varying κ_e on the wage policy function, the monopsony power index, wage offers and earnings, reservation wages for unemployed workers, and unemployment. The broken line in panel (f) corresponds to the unemployment rate if workers accept all job offers. All other structural parameters are held constant. See Section 2.2.5 for the parametrizations.

Changes in κ_e not only affect the rate at which employed workers gravitate to higher paying jobs, but also affect the initial job acceptance decision of workers. For given F, when the rate at which jobs arrive when employed increases, forward looking individuals are more willing to accept a job paying a given wage since they now expect more wage progression via job-to-job transitions before exiting employment and so value employment more. Hence increasing κ_e shifts the reservation wage distribution to the left. In Figure 2.1e we show how the distribution of reservation wages amongst the unemployed A_u changes as κ_e is varied. Note that this figure also incorporates for the feedback effect of changing job acceptance behaviour on equilibrium wage offers F. The distribution of reservation wages amongst employed workers (not shown) also experiences a similar leftward shift as κ_e increases.

Finally we note that for the three values of κ_e considered in the above figures, we obtain unemployment rates of 12.8%, 7.3% and 5.3% respectively. Figure 2.1f also shows how the unemployment rate varies continuously as we increase the value of κ_e from $\kappa_e = 0$ (a version of the Albrecht and Axell (1984) model with a continuous distribution of work opportunity costs) to $\kappa_e = 100$. The figure demonstrates that as κ_e increases in value, the unemployment rate converges to the unemployment rate in the absence of reservation wage heterogeneity $(1 + \kappa_u)^{-1}$. While for a fixed wage offer distribution F an increase in κ_e necessarily lowers reservation wages and decreases unemployment, this is not necessarily true once the changes in F (described above) are allowed for. The effect of this can be seen in the figure, which shows that increases in κ_e cause an initial increase in unemployment.

Bontemps et al. (1999) also considers the impact of changes in the job destruction rate, increases in the mean value of work opportunity costs, and imposition of a legal minimum wage. The qualitative impact of these changes when $\kappa_e \neq \kappa_u$ is the same as when the arrival rates are equal, so the same exercises are not repeated here. The interested reader should consult Bontemps et al. for details.

2.3 Estimation

In this section we discuss the estimation and identification of wage posting models with two-sided heterogeneity using longitudinal survey data. In all cases we consider maximum likelihood estimation. We do however, consider different approaches to obtaining an estimate of the wage offer distribution F. Before doing this, we present the likelihood function.

2.3.1 Likelihood Function

We now derive the likelihood contribution for individuals in different labour market positions, and with different initial transitions. The derivation closely follows that of Bontemps et al. (1999), and here we continue to use u and e to index the respective states of unemployment and employment. Note that we do not use any information beyond the first observed transition. In what follows elapsed and residual durations are given by:

$$\begin{aligned}
t_{ub} &= \text{elapsed unemployment duration}\\
t_{uf} &= \text{residual unemployment duration}\\
d_{ub} &= 1 \text{ if unemployment duration left-censored, otherwise } 0\\
d_{uf} &= 1 \text{ if unemployment duration right-censored, otherwise } 0\\
t_{eb} &= \text{elapsed employment duration}\\
t_{ef} &= \text{residual employment duration}\\
d_{eb} &= 1 \text{ if employment duration left-censored, otherwise } 0\\
d_{ef} &= 1 \text{ if employment duration right-censored, otherwise } 0
\end{aligned}$$

while earned and accepted wages are denoted as follows:

$$\begin{aligned}
w_u &= \text{wage accepted by unemployed individuals}\\
d_u &= 1 \text{ if } w_u \text{ unobserved, otherwise } 0\\
w_e &= \text{wage of employees at date of first interview}\\
d_e &= 1 \text{ if } w_e \text{ unobserved, otherwise } 0
\end{aligned}$$

and employed worker initial transitions are indexed by:

$$v_e = 1 \text{ if employed experience a job-to-job transition, otherwise } 0.$$

In constructing the initial conditions in the likelihood function we have used the steady state distributions of earnings and unemployment rates. Implicitly this is assuming that all individuals have been operating in the labour market for infinite time. Obviously, this is not a true description of the labour market, but is somewhat less objectionable if individuals with only a few years of potential labour market experience are excluded from the constructed sample (see the sample selection used in Section 2.4.1).

Unemployed Workers

Using the above notation, we now derive the likelihood contribution for unemployed workers. The exact form this will take will depend upon whether an accepted wage is observed, and whether or not the unemployment durations are subject to censoring. If the accepted wage w_u is observed (so that we have $d_u = 0$ and $d_{uf} = 0$), the likelihood contribution is given by:

$$\lambda_u^{2-d_{ub}} \exp\left[-\lambda_u(t_{ub}+t_{uf})\right] \frac{A(\underline{w})}{1+\kappa_u} f(w_u) \\ + \int_{\underline{w}}^{w_u} \left[\lambda_u \overline{F}(b)\right]^{2-d_{ub}} \exp\left[-\lambda_u \overline{F}(b)(t_{ub}+t_{uf})\right] \frac{f(w_u)}{\overline{F}(b)} \frac{dA(b)}{1+\kappa_u \overline{F}(b)}$$

where we have integrated over the range of possible reservations wages among unemployed workers using equation 2.5.

If we do not observe a wage accepted by the unemployed ($d_u = 1$), but we nonetheless have $d_{ub} + d_{uf} < 2$, then it still must be the case that the reservation wage of such an individual is no greater than $\overline{w}$. It therefore follows that their likelihood contribution is:

$$\lambda_u^{2-d_{ub}-d_{uf}} \exp\left[-\lambda_u(t_{ub}+t_{uf})\right] \frac{A(\underline{w})}{1+\kappa_u} + \int_{\underline{w}}^{\overline{w}} \left[\lambda_u \overline{F}(b)\right]^{2-d_{ub}-d_{uf}} \exp\left[-\lambda_u \overline{F}(b)(t_{ub}+t_{uf})\right] \frac{dA(b)}{1+\kappa_u \overline{F}(b)}.$$

Finally, if we have both $d_u = 1$ and $d_{ub} + d_{uf} = 2$, then individuals are never observed in the employment state so we must also consider the probability that such individuals have a reservation wage that is greater than $\overline{w}$. The likelihood contribution then becomes:

$$\exp\left[-\lambda_u(t_{ub}+t_{uf})\right] \frac{A(\underline{w})}{1+\kappa_u} + \int_{\underline{w}}^{\overline{w}} \exp\left[-\lambda_u \overline{F}(b)(t_{ub}+t_{uf})\right] \frac{dA(b)}{1+\kappa_u \overline{F}(b)} + \left[1 - A(\overline{w})\right].$$

Employed Workers

The likelihood contribution of an individual working at wage w_e is given by:

$$(1-u)g(w_e)\left[\delta + \lambda_e \overline{F}(w_e)\right]^{2-d_{eb}-d_{ef}} \times \exp\left\{-\left(\delta + \lambda_e \overline{F}(w_e)\right)(t_{eb}+t_{ef})\right\} \times \left[\frac{\delta^{1-v_e}(\lambda_e \overline{F}(w_e))^{v_e}}{\delta + \lambda_e \overline{F}(w_e)}\right]^{1-d_{ef}}.$$

Note that in the above we do not use any information on the wage accepted following a job-to-job transition. The reason for adopting such a limited information approach is that the model we have developed does not permit job-to-job transitions associated with lower job values.[10] Finally, if the wage of an employed worker were missing ($d_e = 1$), then the likelihood contribution is simply given by the employment rate $1 - u$.

2.3.2 Estimation Procedure

Semi-parametric Estimation

Bontemps et al. (1999) considered the model with $\kappa_u \neq \kappa_e$ to be intractable. Here we describe how to generalise the three step semi-parametric estimator that was proposed in Bontemps et al. (1999, 2000) to this setting. While we no longer have a simple inversion between the observed earnings distributions and the unobserved wage offer distributions, it nonetheless remains possible to perform an inversion by iterating on the relevant flow equations. More specifically:

[10] This could be incorporated by allowing for wage measurement error in the estimation, or by introducing "reallocation shocks" as considered by Jolivet et al. (2006) and van den Berg and Ridder (1993, 1997). Reallocation shocks are more complicated in a model with reservation wage heterogeneity as individuals may wish to exercise an option to quit their reallocated job if it pays a wage below their reservation wage $\phi(b)$.

1. We estimate $\{\underline{w}, \overline{w}\}$ as the sample minimum and maximum values of w_e. We then calculate an estimate of the earnings density using non-parametric (kernel) techniques. We denote this estimated density as $\hat{g}$ and the estimated cumulative distribution function as $\hat{G}$. If no binding legal minimum wage was operational during the sample period, then the theoretical restriction that $g(\underline{w}) = 0$ can be imposed by modifying the standard kernel estimator in the neighbourhood of the estimated boundary (see the later discussion in Section 2.4).

2. We assume a parametric form for the distribution of work opportunity costs H with the finite parameter vector $\vec{\theta}_H$. Conditional on the set of structural parameters $\{\vec{\theta}_H, \lambda_u, \lambda_e, \delta\}$ we wish to recover the wage offer distribution F that induces the estimated empirical distribution $\hat{g}(w)$. First, we note that by differentiating equation 2.9 and rearranging terms we may express the wage offer density as:

$$f(w) = \frac{(1-u)g(w)\left[\delta + \lambda_e \overline{F}(w)\right]}{\lambda_u u A_u(w) + \lambda_e (1-u) G(w)}. \tag{2.19}$$

   To recover the wage offer distribution that induces our estimates of the empirical earnings distributions, we replace g and G in equation 2.19 with our non-parametric estimates $\hat{g}$ and $\hat{G}$. We provide an initial guess of f, integrate this to obtain F, and then iterate on this equation, exploiting the conditional linearity seen above. Note that both the distribution of reservation wages amongst the unemployed A_u and the unemployment rate u depend upon the entire distribution of wages offers. At each iteration step we scale the wage offer density by a normalization factor to ensure that we have a proper distribution function, and then verify that this normalization factors converge to unity. Conditional on the set of transitional parameters and distribution of work opportunity costs, we then obtain consistent estimates of the wage offer distribution and its density which we denote as $\hat{F}$ and $\hat{f}$ respectively. These estimates, together with the estimated wage earnings density $\hat{g}$ and the implied unemployment rate u and reservation wage distribution A, are then substituted into the likelihood function presented in Section 2.3.1.

3. Given the non-parametric estimate of wage offers $\hat{F}$ and the other structural parameters $\vec{\theta}$, we obtain an estimate of the productivity level associated with each wage offer by rewriting the first order condition given in equation 2.13 to obtain:

$$K^{-1}(w) = w + \left[\frac{\kappa_u A'(w)(1+\kappa_e \overline{F}(w))}{(1+\kappa_u \overline{F}(w))(\kappa_e A(w) + (\kappa_u - \kappa_e) u A_u(w))} + \frac{2\kappa_e f(w)}{1+\kappa_e \overline{F}(w)}\right]^{-1} \tag{2.20}$$

   The productivity density γ may then be obtained either by numerically differentiating Γ or by exploiting the relationship $F(w) = \Gamma(K^{-1}(w))$. In particular, since $F(w) = \Gamma(K^{-1}(w))$ it follows that $\gamma(K^{-1}(w)) = f(w)/(K^{-1})'(w)$. Differentiating equation 2.20

this can be shown to be given by:

$$\gamma(K^{-1}(w)) = f(w)\Bigg\{1 - \Big(K^{-1}(w) - w\Big)^2 \Bigg[\frac{2\kappa_e[f'(w)(1+\kappa_e\overline{F}(w)) + 2\kappa_e f(w)^2]}{(1+\kappa_e\overline{F}(w))^2} + \frac{\kappa_u(1+\kappa_e\overline{F}(w))(A''(w) - A'(w)/(K^{-1}(w)-w))}{(1+\kappa_u\overline{F}(w))(\kappa_e A(w) + (\kappa_u - \kappa_e)uA_u(w))} + \frac{\kappa_u A'(w) f(w)[\kappa_u(1+\kappa_e\overline{F}(w)) + \kappa_e(1+\kappa_u\overline{F}(w))]}{(1+\kappa_u\overline{F}(w))^2(\kappa_e A(w) + (\kappa_u - \kappa_e)uA_u(w))}\Bigg]\Bigg\}^{-1} \tag{2.21}$$

which depends on the derivative of the wage offer density. However, given an estimate of $g'(w)$ this can (by differentiating equation 2.19) be written as:

$$f'(w) = \frac{(1-u)g'(w)(1+\kappa_e\overline{F}(w)) - f(w)[2\kappa_e(1-u)g(w) + \kappa_u A'(w)/(1+\kappa_u\overline{F}(w))]}{\kappa_u u A_u(w) + \kappa_e(1-u)G(w)} \tag{2.22}$$

We construct confidence intervals by bootstrapping the entire three stage estimation procedure. The advantages of this three step procedure versus a completely parametric approach, are essentially threefold. Firstly, it is considerably easier to perform this numerical inversion than it is to solve the full model at every evaluation of the likelihood function. Second, it permits greater flexibility than simple parametric forms for the productivity distribution. Thirdly, since this semi-parametric estimation procedure does not make assumptions regarding the determination of F, both the estimate of F and the transitional parameters λ_u, λ_e and δ are valid under a range of possible models.

The main potential difficulty with the semi-parametric approach detailed above, is that the empirical distribution of wages $\hat{g}$ may not be implementable as an equilibrium of the wage-posting model. This problem would manifest itself with an improper distribution of firm productivity from the third step (that is, $K^{-1}(w)$ not being strictly increasing in w). This would seemingly prevent the estimated model from being used for policy analysis, which is often the ultimate objective of such an estimation. If the degree of non-monotonicity is not "large", then it may be possible to replace the third step productivity distribution estimate with one which is "close", subject to the requirement that it is a proper distribution function. Of course, this proper distribution will not induce the first step estimate $\hat{g}$ in equilibrium, but it may still be sufficiently close for practical purposes.

Remark An alternative to step 2 in the above involves parametrically specifying a wage offer distribution. If this distribution is described by the finite parameter vector $\vec{\theta}_F$ then the set of structural parameters is now given by $\{\vec{\theta}_F, \vec{\theta}_H, \lambda_u, \lambda_e, \delta\}$. Under this alternative approach, the theoretical distribution of wage earnings given the parametric offer distribution (and other structural parameters) will be substituted in place of the empirical distributions. Relative to the above, this has the advantage that it is not necessary to numerically invert the flow equation given in equation 2.19. However, parametrically specifying F does not remove the problem of implementability, which is the main disadvantage of the semi-parametric estimation approach.

Parametric Estimation

The potential problem of implementability can be avoided by estimating the complete equilibrium wage posting model parametrically. We now discuss this approach. This estimation procedure involves first specifying some parametric distribution of firm productivity (as characterised by some finite parameter vector $\vec{\theta}_\Gamma$), and then estimating the model by maximum likelihood as before. At each evaluation of the likelihood function, we solve for the equilibrium of the wage posting game. The equilibrium wage offer and wage earnings distributions, together with the unemployment rate, are then substituted in the likelihood function. The numerical algorithm that is used to solve the model in estimation is presented alongside the main algorithm in Appendix 2.B. The basic idea is to treat the empirical support of wages $[\underline{\hat{w}}, \hat{\overline{w}}]$ as fixed, and then solve for the inverse of the wage policy function $K^{-1}(w)$ that is consistent with these. In contrast to the semi-parametric estimation approach, the empirical distribution of wage earnings will not be matched perfectly by construction. However, the ability of the model to fit the empirical distribution of wages can be assessed by comparing the equilibrium and empirical distributions and we do this in Section 2.4.

2.3.3 Non-parametric Identification

Before presenting some results obtained using the proposed estimation procedures, we first explore the identification of the model. An important advantage of wage posting models in terms of empirical analysis is that they are structurally identified with only worker data (that is, data on wages, transitions and durations). To better understand identification of our model, we first consider the model of Bontemps et al. (2000). In this case, identification of the wage offer distribution (conditional on the set of transitional parameters) follows from a simple steady-state relationship between the distribution of earnings and the distribution of wage offers.

$$(1-u)G(w)\left[\delta + \lambda_e \overline{F}(w)\right] = \lambda_u u F(w) \tag{2.23}$$

which is a simpler form of equation 2.9 from earlier. This equation allows us to move from the known G to the unknown F. Moreover, in such a setting all job offers will be accepted by all unemployed workers so that the accepted wage distribution will also coincide with the wage offer distribution. This special case of the model presented in Section 2.2 is therefore over identified. Regardless of its source, once we allow for heterogeneity in the reservation wage of unemployed workers the distribution of accepted wages will no longer equal the wage offer distribution. This is because workers are selective in the wages that they are willing to accept, so that the distribution of accepted wages will dominate F. We are still able to establish non-parametric identification in this context because we observe as many distributions (starting wages and cross-sectional earnings) as distributions that we wish to recover. We now present this argument more formally.

In what follows, we let G^U denote the cumulative distribution function of wages first accepted by unemployed workers. Since individuals will accept any wage offer that is at least as high as their reservation wage, it follows that:

$$\begin{aligned} G^U(w) &= \int_{-\infty}^{w} \Pr(W < w | W > x) dA_u(x) = \int_{-\infty}^{w} \frac{F(w) - F(x)}{\overline{F}(x)} dA_u(x) \\ &= A_u(w) - \overline{F}(w) \left[\int_{\underline{w}}^{w} \frac{dA_u(x)}{\overline{F}(x)} + A_u(\underline{w}) \right]. \end{aligned}$$

If we combine the above expression with the density function of accepted wages $g^U \equiv G^{U\prime}$, we can write:

$$A_u(w; F) = G^U(w) + \frac{\overline{F}(w) g^U(w)}{f(w)} \tag{2.24}$$

which therefore demonstrates that the distribution of reservation wages amongst the unemployed on support $[\underline{w}, \overline{w}]$ is identified given knowledge of the wage offer distribution function F. To demonstrate identification of F we can use the between job flow equation from equation 2.19, eliminating the unobserved reservation wage distribution function to obtain the following differential equation that governs the evolution of the wage offer distribution function:

$$f(w) = \frac{(1-u)g(w)(1+\kappa_e \overline{F}(w)) - u g^U(w) \kappa_u \overline{F}(w)}{\kappa_u u G^U(w) + \kappa_e (1-u) G(w)}. \tag{2.25}$$

Given these functions we are able to separately identify the transitional parameters by using information on durations and transitions: λ_u is identified from unemployment durations; δ is identified by transitions to the unemployment pool; λ_e is identified from job-to-job transitions. We may identify A on the support of wages by using equation 2.5. In particular, we have:

$$A(w) = (1+\kappa_u) u A_u(\underline{w}) + u \int_{\underline{w}}^{w} (1 + \kappa_u \overline{F}(x)) dA_u(x)$$

on $[\underline{w}, \overline{w}]$. The structure of the model then permits identification of the leisure flow distribution H, and distribution of firm productivity Γ.

Identification Through Wage Growth

The discussion above suggests that we may again be able to establish over-identification if further distributions of wages are observed. We now demonstrate this to be the case by showing that it is possible to recover the distribution of wage offers by considering the distribution of wages that employed workers receive in their next job. This argument is valid regardless of whether or not there is any heterogeneity in worker reservation wages. We denote the cumulative distribution of upgraded wages as G^E, which is given by:

$$G^E(w) = \frac{\kappa_e}{\eta} \int_{\underline{w}}^{w} \frac{F(w) - F(x)}{1 + \kappa_e \overline{F}(x)} dG(x) \tag{2.26}$$

where η measures the probability that the next transition for an employed worker selected at random, is to another job. It is given by:

$$\eta = \int_{\underline{w}}^{\overline{w}} \frac{\kappa_e \overline{F}(x)}{1 + \kappa_e \overline{F}(x)} dG(x). \tag{2.27}$$

To demonstrate identification we first differentiate equation 2.26 using Leibniz's rule, and rearranging terms to obtain:

$$f(w) = \eta g^E(w) \times \left[\int_{\underline{w}}^{w} \frac{\kappa_e dG(x)}{1 + \kappa_e \overline{F}(x)} \right]^{-1} \tag{2.28}$$

where $g^E \equiv G^{E\prime}$ is the density function of upgraded wages. While this depends on the unknown η we can easily eliminate this term by using equation 2.26 so that we have a differential equation for $f(w)$ with initial condition $F(w) = 0$. This establishes identification of the wage offer distribution F and this over-identification result could, in principle, be used to test our model. The useful of this further identification result may be limited in many applications. Unless there is substantial job-to-job mobility, then it will be necessary to follow individuals for a considerable period of time to estimate the upgraded wage distribution. This is problematic not just in terms of suitable data availability, but also because it may be difficult to defend the assumption that these data are generated from the same steady state. Furthermore, the model as has been presented does not permit any job-to-job movements that are associated with wage cuts.[11]

2.4 Estimation Results

In this section we present some illustrative estimation results that have been obtained using UK survey data. We shall compare and contrast the estimation results that are derived under both the semi-parametric and parametric estimation procedures as described in Section 2.3.2. Before detailing these, we provide a brief description of our data.

2.4.1 Data

All the estimation results are obtained using a sub-sample of the UK Labour Force Survey (LFS). The LFS is a quarterly survey of around 60,000 households in Great Britain, with these households followed for five successive quarters or "waves". When individuals first enter the survey they are in wave one, so that in any given quarter, there are roughly equal proportions of individuals in each interview wave. This rolling panel structure means that there is approximately an 80% overlap in the samples for successive quarters. The LFS provides us with very rich information concerning the respondents labour market status. Crucially, we observe employment status and spell durations, together with earnings information (in the first and fifth waves since 1997). In the application here we follow individuals who are observed in the

[11] The idea of using wage growth to achieve identification is also explored by Barlevy (2008) and Barlevy and Nagaraja (2006) who using record-value theory demonstrate identification of the wage offer distribution by tracking the wage growth of workers as a function of past mobility.

first quarter of 1997 until (at the latest) the first quarter of 1998.

We classify individuals as being employed if they have a job, and non-employed if they do not. The sample is restricted to males who are aged between 22 and 55 at the initial interview date.[12] Individuals who are self-employed are excluded from the sample, as are part-time workers, and those in full-time education. Given the assumption that workers are equally productive at any given firm, we additionally restrict our sample to those individuals whose highest qualification is O-level (or equivalent) or below, and assume that any higher educated individuals operate in a separate labour market. After sample selection, we have almost 9,800 observations.

2.4.2 Estimation Specification

In all of the estimation results reported, worker opportunity costs are assumed to be normally distributed with mean μ and standard deviation σ. When we perform semi-parametric estimation, we obtain our first step estimate of the wage earnings distribution by using a Gaussian kernel. Since no minimum wage operated during our sample period, we impose the theoretical restriction that $g(\underline{w}) = 0$ on our estimate by scaling the density estimate towards zero in the neighbourhood of the estimated boundary.[13] Specifically, we set:

$$\hat{g}(w) = \left[\frac{1}{nh}\sum_{i=1}^{n}\phi\left(\frac{w-w_i}{h}\right)\right]\left[1-\exp\left(\frac{\underline{\hat{w}}-w}{\alpha}\right)\right] \tag{2.29}$$

where w_i, $i = 1, \ldots, n$, is the sample of wages amongst the employed; h is the kernel bandwidth; ϕ is the standard normal density; and α is a smoothing parameter that determines how quickly the wages near the lower support are scaled towards zero. In the results presented here we set $\alpha = 0.25$ and $h = 0.40$. The estimation results are not particularly sensitive to these values.

When estimating the model parametrically we consider alternative parameterizations of the productivity distribution. In all cases, the truncation points of these distributions are not parameters to be estimated using maximum likelihood, but are rather determined by the requirement that the equilibrium support of wages given the structural parameter vector $\{\vec{\theta}_\Gamma, \vec{\theta}_H, \lambda_u, \lambda_e, \delta\}$ coincides with the empirical support of wages.[14] The first distribution that we consider is the Pareto distribution. In addition to the truncation points this distribution is characterised by a single shape parameter, $\vec{\theta}_\Gamma = a$. The productivity density is given by:

$$\gamma(p) = \frac{a\underline{p}^a}{p^{a+1}[1-(\underline{p}/\overline{p})^a]}.$$

[12]By restricting the sample to those individuals who have at least six years of potential labour market experience, the use of steady state values when constructing the initial conditions in the likelihood function is somewhat less objectionable.

[13]In the absence of a binding legal minimum wage, modifying the first stage estimate in this way is a necessary (but not sufficient) condition for this estimate to be implementable as an equilibrium of the model. If this condition is ignored during estimation, then the equilibrium wage distribution obtaining using the third step productivity estimates will not coincide with the empirical distribution. However, provided that the uncorrected density estimate $\hat{g}(\underline{\hat{w}})$ is "close" to zero, the differences between these distributions is unlikely to be large.

[14]See both the algorithm and discussion in Appendix 2.B.

which is monotonically decreasing on its support. The second distribution we consider is one which potentially offers much greater flexibility. Specifically, we shall be considering the Pearson IV distribution (Pearson, 1895). In addition to the truncation points, this distribution is characterized by four parameters $\vec{\theta}_\Gamma = \{k_1, k_2, k_3, k_4\}$ which allows for varying degrees of skewness and kurtosis. It is therefore ideally suited for many economic applications where the distributions of interest are often asymmetric with extensive tails. Given the sparsity of references to this distribution in the literature, Appendix 2.C discusses the properties of the distribution and our implementation of it. The density of the distribution is given by:

$$\gamma(p) \propto \left[1 + \left(\frac{p - k_4}{k_3}\right)^2\right]^{-k_1} \exp\left[-k_2 \tan^{-1}\left(\frac{p - k_4}{k_3}\right)\right] \tag{2.30}$$

where k_1 and k_2 jointly determine the degree of skewness and kurtosis, while k_3 and k_4 are scale and location parameters.

2.4.3 Results

We now present the results obtained from the three sets of estimations performed: we refer to these different estimation specifications as *semi-parametric*, *Pareto*, and *Pearson*. In all cases, the parameter confidence intervals are constructed by bootstrapping the estimation procedures using 500 replications. In Table 2.1 we present the transitional parameter estimates, as well as the estimated parameters of the work opportunity cost distribution. The estimate of the job destruction rate δ is remarkably similar across specifications. The estimates imply that jobs are destroyed on average around every 160 months ($\approx 1/\hat{\delta}$). Across all specifications we obtain $\hat{\lambda}_u > \hat{\lambda}_e$ so that job offers arrive more frequently for unemployed than employed workers.[15] While the *semi-parametric* and *Pearson* specifications produce very similar arrival rate estimates (both implying that jobs for unemployed workers arrive on average every 30 months, with jobs for employed workers arriving on average about every 45 months), the estimates obtained from the *Pareto* specification are very different (14 and 25 months respectively). The estimated distribution of work opportunity costs differs slightly across the specifications: the mean of the distribution μ is lowest (highest) under the *semi-parametric* (*Pareto*) specification, while the reverse pattern is true for the estimated standard deviation of the distribution σ. All specifications imply that workers are selective in the wages that they are willing to accept.[16]

In Figure 2.2 we show what these parameter estimates imply for the distributions of wage offers F and wage earnings G. Figure 2.2a demonstrates that very similar wage offer distributions are obtained under both the *semi-parametric* and *Pearson* specifications. Results from the *Pareto* specification are very different, with a much higher concentration of low wage offers. The observation that the *Pearson* specification then induces a distribution of wage

[15] We reject the null hypothesis that $\lambda_u = \lambda_e$ across all specifications.

[16] Accounting for reservation wage heterogeneity is important for our estimates of the structural parameters. In particular, if we neglect it then the semi-parametric estimation procedure yields a much lower estimate of λ_u (0.014 compared to 0.032 with heterogeneity). Note however, that the latter "average job acceptance rate" for unemployed workers $\lambda_u \int \overline{F}(\phi) dA_u(\phi) = 0.017$ is very close to the no-heterogeneity estimate.

Table 2.1: Maximum likelihood estimation results

	$1/\delta$	$1/\lambda_u$	$1/\lambda_e$	μ	σ
Semi-parametric	169.31	31.37	47.22	2.06	2.53
	[161.75,177.27]	[24.43,37.70]	[39.73,54.21]	[1.22,2.56]	[2.02,3.25]
Pearson IV	167.55	28.72	44.49	2.41	2.13
	[160.29,175.08]	[23.50,33.36]	[37.61,50.72]	[2.12,2.68]	[1.83,2.45]
Pareto	163.72	14.32	24.60	2.75	1.27
	[158.06,170.04]	[13.68,15.46]	[22.30,28.03]	[2.62,2.90]	[1.15,1.38]

Notes: All durations are monthly. Incomes are measured in pounds per hour in April 1997 prices. The distribution of work opportunity costs H is assumed to be Normal, with mean μ and variance σ^2. The 5^{th} and 95^{th} percentiles of the bootstrap distribution of parameter estimates are presented in brackets, and are calculated using 500 replications.

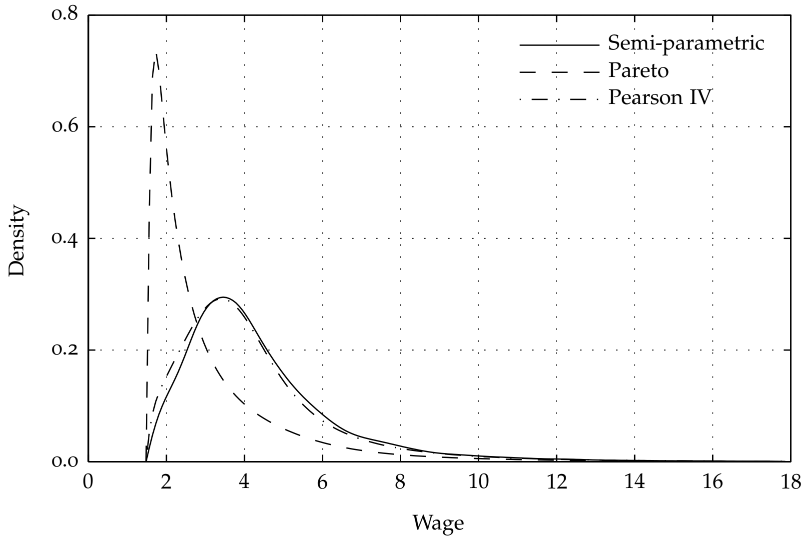
earnings that is very close to the non-parametric estimate is unsurprising given the similarity of both the wage offer distribution and estimated transitional parameters to those obtained from the semi-parametric approach. The *Pareto* specification also provides a reasonable fit to the cross-sectional wage earnings distribution, although the fit in the tails of the distribution is considerably less good. With a higher concentration of low wage offers it is fitting the distribution of wage earnings by forcing the arrival rates of job offers to be much higher (see Table 2.1).

When estimating the model using the semi-parametric approach, we recover the distribution of firm productivity by using the first order conditions from the firms' maximisation problem (see step 3 as described in Section 2.3.2). The point estimates from the *semi-parametric* model imply a monotonically increasing relationship between firm productivity p and wages w which means that the first step estimate of the empirical distribution of wages $\hat{g}$ is implementable as an equilibrium outcome of the model.[17] The resulting distribution of firm productivity is shown in Figure 2.3a, which also shows the distribution obtained from the two parametric specifications.[18] The reason why the *Pareto* specification provides a poorer fit to the data than does *Pearson* can easily be seen from the figure. The single parameter Pareto distribution has a probability density that is declining throughout its support which is ultimately too restrictive. Distributions that simultaneously allow for the possibility of an interior mode and a long right tail are much more likely to be successful in fitting empirical distributions and durations. Table 2.2 provides the estimates of the productivity parameters $\vec{\theta}_\Gamma$, while Figure 2.3b displays the wage policy function from the alternative specifications.

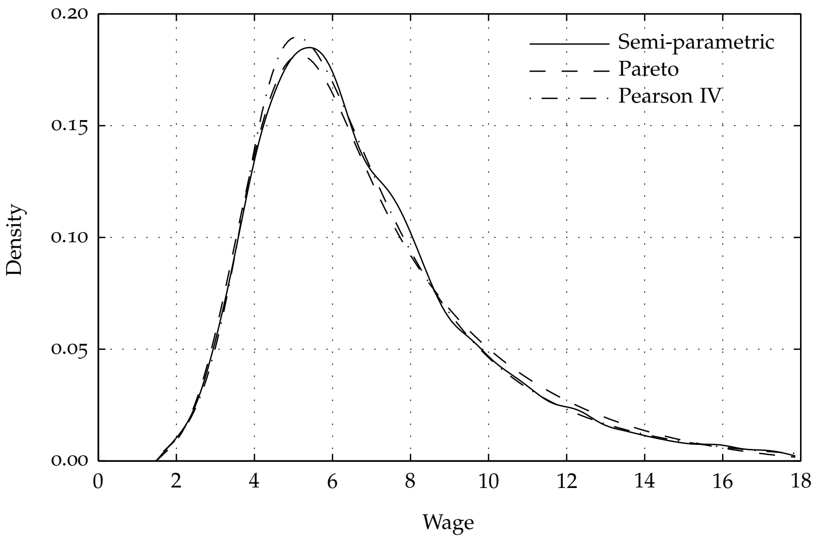
This exercise suggests that, in the case of parametric estimation, the shape of the assumed productivity distribution *does* matter, and it may matter a lot. The theory developed provides little guidance regarding suitable parametrizations of this distribution which is why the generalization of the semi-parametric estimation procedure proposed by Bontemps et al. (1999, 2000) is particularly appealing. As we have discussed, there may be situations where parametric estimation is either necessary or desirable, and in these cases it is judicious to compare

[17] This is certainly not true for all the bootstrap sample, so there remain implementability issues if attempting to perform inference using the estimated model.

[18] In order to aid the visual comparison between the distributions, this figure has been truncated at productivity levels exceeding $p = 30$.

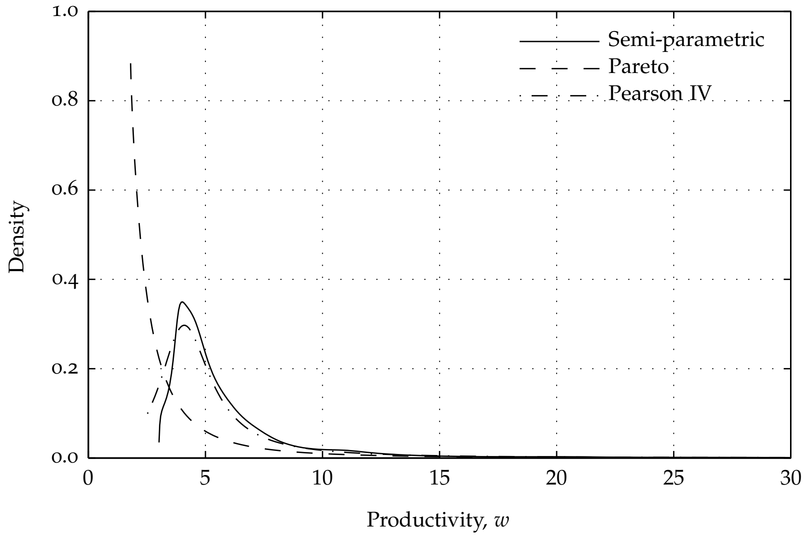


(a) Wage offer distribution

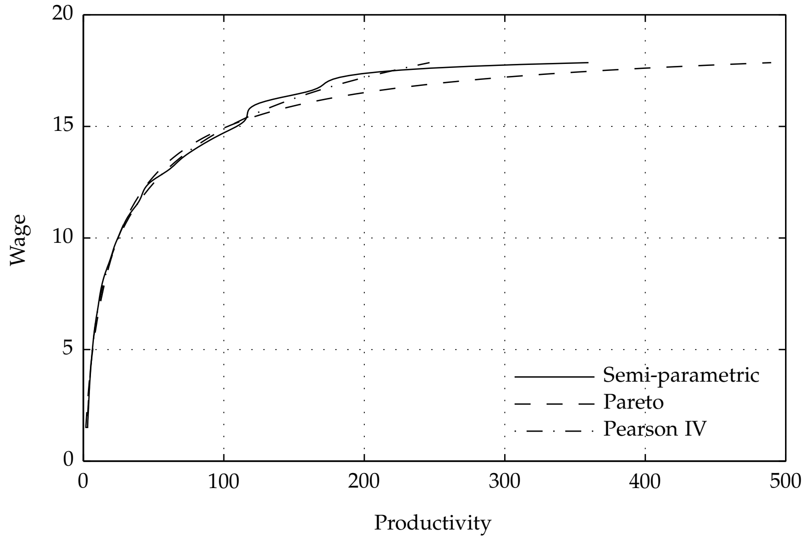


(b) Wage earnings distribution

Figure 2.2: Comparison of wage offer- and wage earnings distributions when model is estimated under the *semi-parametric*, *Pareto*, and *Pearson* specifications.



(a) Productivity distribution



(b) Wage policy function

Figure 2.3: Comparison of productivity distributions and wage policy functions when model is estimated under the *semi-parametric*, *Pareto*, and *Pearson* specifications. The productivity distribution is truncated at $p = 30$ in panel (a).

Table 2.2: Maximum likelihood estimation results (productivity distribution)

	$\underline{p}$	$\overline{p}$	a	k_1	k_2	k_3	k_4
Semi-parametric	3.02	359.57	–	–	–	–	–
	[2.43,4.33]	[231.39,496.54]	–	–	–	–	–
Pearson IV	2.53	246.43	–	1.07	0.39	-1.34	3.89
	[2.29,2.83]	[196.38,303.18]	–	[1.02,1.12]	[-0.20,0.83]	[-1.82,-1.08]	[3.59,4.26]
Pareto	1.80	489.62	0.06	–	–	–	–
	[1.78,1.88]	[351.40,698.94]	[0.06,0.07]	–	–	–	–

Notes: Incomes are measured in pounds per hour in April 1997 prices. The parameter a is the Pareto shape parameter; parameters $\{k_1, k_2, k_3, k_4\}$ are parameters of the Pearson IV distribution. k_1 and k_2 jointly determine the degree of skewness and kurtosis, while k_3 and k_4 are scale and location parameters. The 5^{th} and 95^{th} percentiles of the bootstrap distribution of parameter estimates are presented in brackets, and are calculated using 500 replications.

the empirical and equilibrium wage distributions, as well as the estimated transitional parameters to those obtained from the semi-parametric approach. If these differ substantially, then a distribution which affords more flexibility may be warranted. While we have considered relatively parsimonious specifications of the productivity distribution in our application here, approximating the distribution with say, a finite-dimensional sieve, is also possible.

2.5 Demographic Heterogeneity

In this section we consider an extension of the model presented in Section 2.2. Specifically we shall discuss how demographic heterogeneity in worker structural parameters may be introduced. The additional set of complications that this introduces will depend upon ones view of the labour market. If workers of different types were to operate in distinct segmented labour markets (between market heterogeneity), then both the model and available estimation procedures are exactly the same as described in Sections 2.2–2.3, with the qualification that all the analysis is performed conditional on demographic type. The more interesting case arises when workers of different types operate within a single labour market (within market heterogeneity). This setting provides an alternative to the segmented markets approach of van den Berg and Ridder (1998), and we now discuss this case.

2.5.1 Within Market Demographic Heterogeneity

Suppose that workers differ by their observable type, which we index by i. There are a total of I observable types. Let n_i denote the fraction of workers of a given type, with $\sum_i n_i = 1$. While the set of worker structural parameters $\{\vec{\theta}_{H_i}, \lambda_{ui}, \lambda_{ei}, \delta_i\}_{i \leq I}$ may vary with i, all workers are assumed to sample wage offers from the common wage offer distribution F. This reflects the notion that all individuals are operating within the same labour market. From the perspective of the worker, the problem is essentially the same as before. In particular, equations 2.1–2.10 continue to hold conditional on demographic type i (see also the later exposition in Chapter 3 of this thesis). The problem faced by firms is similar. They will continue to maximize their steady state profit flow $(p - w)l(w)$, with the qualification that firm size is now given by $l(w) = \sum_i n_i l_i(w)$ with $l_i(w)$ is defined as:

$$l_i(w) = \frac{\kappa_{ei} A_i(w)}{(1 + \kappa_{ei}\overline{F}(w))^2} + \frac{(\kappa_{ui} - \kappa_{ei})}{(1 + \kappa_{ei}\overline{F}(w))^2} \left[\frac{1}{1 + \kappa_{ui}} A_i(\underline{w}) + \int_{\underline{w}}^{w} \frac{dA_i(x)}{1 + \kappa_{ui}\overline{F}(x)} \right]. \tag{2.31}$$

The characterisation of the equilibrium is very similar to that as discussed in Section 2.2. As before, it is necessary to calculate the wage policy function $K(p)$ to determine the other equilibrium functions of interest.[19] In particular, the density of wage offers can be shown to

[19]It is straight forward to modify the algorithm presented in Appendix 2.B in the presence of within market demographic heterogeneity.

be given by:

$$f(w) = \left[\frac{1}{p-w} \sum_i n_i \frac{\kappa_{ei} A_i(w) + (\kappa_{ui} - \kappa_{ei}) u_i A_{ui}(w)}{(1+\kappa_{ei}\overline{F}(w))^2} - \sum_i \frac{n_i \kappa_{ui} dA_i(w)}{(1+\kappa_{ei}\overline{F}(w))(1+\kappa_{ui}\overline{F}(w))} \right] \times \left[\sum_i n_i \frac{2\kappa_{ei}(\kappa_{ei} A_i(w) + (\kappa_{ui} - \kappa_{ei}) u_i A_{ui}(w)}{(1+\kappa_{ei}\overline{F}(w))^3} \right]^{-1}$$

where $w = K(p)$.

2.5.2 Identification and Estimation

Before proceeding to discuss estimation procedures in the presence of within market demographic heterogeneity, we note that exactly the same identification argument as presented in Section 2.3.3 continues to apply because the demographic type i is observed by the econometrician. Similarly, very little changes when estimating the model parametrically: the set of structural parameters $\{\vec{\theta}_\Gamma, \vec{\theta}_{H_i}, \lambda_{ui}, \lambda_{ei}, \delta_i\}_{i \leq I}$ induces an equilibrium distribution of wage offers F which is then substituted in the likelihood function, together with the *conditional* distribution of earnings g_i, unemployment u_i, and reservation wages A_i.

While semi-parametric estimation remains feasible, it takes a slightly modified form. In particular, if we were to perform the numerical inversion for each group i as described in Step 2 of Section 2.3.2, then we will typically obtain a different estimated F for each group even if the data were generated using common F. While there are conceivably different approaches that can be pursued in this context,[20] the approach we propose essentially modifies the three step procedure so that we recover a non-parametric distribution of wage offers $\hat{F}$ that induces the *unconditional* distribution of wages amongst the employed (that is, not conditional on worker demographic type i). The procedure is as follows:

1b. The first step is the same as before: $\{\underline{w}, \overline{w}\}$ are estimated as the sample minimum and maximum values of the wages of employed workers; a non-parametric estimate of the earnings density is similarly obtained and is denoted $\hat{g}$. None of these first stage estimates are calculated conditional on demographic type i.

2b. Conditional on the set of structural parameters $\{\vec{\theta}_{H_i}, \lambda_{ui}, \lambda_{ei}, \delta_i\}_{i \leq I}$ we wish to recover the wage offer distribution that induces the estimated empirical distribution $\hat{g}(w)$. For each demographic group i we have:

$$(1-u_i)g_i(w) = f(w) \left[\frac{\kappa_e A_i(w) + (\kappa_{ui} - \kappa_{ei}) u_i A_{ui}(w)}{(1+\kappa_{ei}\overline{F}(w))^2} \right]$$

which by averaging over the distribution of demographic types and then rearranging

[20]One such alternative could involve estimating the model on each group i in turn, and then combining the set of estimates $\{\hat{F}_i\}_{i \leq I}$ into a single $\hat{F}$. This single estimate of F can then be substituted into the first order conditions of firms to obtain an estimate of the productivity distribution as before.

terms yields:

$$f(w) = \left(\sum_i n_i(1-u_i)g_i(w) \right) \times \left[\sum_i n_i \frac{\kappa_e A_i(w) + (\kappa_{ui} - \kappa_{ei})u_i A_{ui}(w)}{(1+\kappa_{ei}\overline{F}(w))^2} \right]^{-1}.$$

This second step then essentially proceeds as before: we replace the term in parentheses by $(1-u)\hat{g}(w)$ where the aggregate equilibrium employment rate is given by $1-u = \sum_i n_i(1-u_i)$. Conditional of the structural parameter vector $\vec{\theta}$, an initial guess of f and F is provided. We then iterate on f until we obtain a fixed point. Our estimates of the wage offer distribution and its density (denoted as $\hat{F}$ and $\hat{f}$ as before) are then substituted into the likelihood function. These estimates are also used to determine the conditional wage earnings distribution g_i, unemployment rate u_i and reservation wage distribution A_i, which are all then substituted into the likelihood function.

3b. Given the non-parametric estimate of wage offers $\hat{F}$ and the vector of structural parameters $\vec{\theta}$, we obtain an estimate of the productivity level associated with each wage offer by rewriting the first order condition of the firms maximization problem as:

$$K^{-1}(w) = w + \frac{1}{l'(w)} \sum_i n_i \frac{\kappa_{ei} A_i(w) + (\kappa_{ui} - \kappa_{ei})u_i A_{ui}(w)}{(1+\kappa_{ei}\overline{F}(w))^2}$$

where $l'(w)$ is given by:

$$l'(w) = \sum_i n_i \frac{2\kappa_{ei} f(w)[\kappa_{ei} A_i(w) + (\kappa_{ui} - \kappa_{ei})u_i A_{ui}(w)]}{(1+\kappa_{ei}\overline{F}(w))^3} + \sum_i n_i \frac{\kappa_{ui} A_i'(w)}{(1+\kappa_{ui}\overline{F}(w))(1+\kappa_{ei}\overline{F}(w))}.$$

2.5.3 Data and Estimation Results

As an illustrative example of how demographic heterogeneity may be introduced into the model and estimation we continue to use the same data as described in Section 2.4.1, but now allow the worker structural parameters to vary with age of the worker (at the initial interview date). We allow for three age groups: age 22–30, age 31–40, and age 41–55. In both the estimation and simulation the group that the individual belongs to is considered exogenous and time invariant (so that the worker environment remains stationary), so it may be useful to interpret these as being cohort rather than age categorizations.

We estimate the model under the same three specifications (*semi-parametric*, *Pareto*, and *Pearson*) as we considered before. The transitional parameter estimates are presented in the first three columns of Table 2.3, and we again find that the *semi-parametric* and *Pearson* specifications yield very similar estimates. Across all specifications, a very pronounced age gradient is apparent: younger workers both encounter job offers less frequently (lower λ_{ui} and λ_{ei}) and also exit to unemployment more often (higher δ_i). Of course, since all groups of workers are (by assumption) sampling wage offers from the same distribution, the estimation may be using the transitional parameters to explain differences in both the employment rate and the cross-sectional distribution of earnings across groups. Indeed, an important between group difference is that the youngest age group has somewhat lower and less dispersed wages.[21] We are able to assess the extent that this is likely to be a concern by comparing these estimates to those which we obtain when estimating the model on each group separately. Intuitively, if the model is correctly specified then we are able to obtain consistent estimates of $\{\vec{\theta}_{H_i}, \lambda_{ui}, \lambda_{ei}, \delta_i\}_{i \leq I}$ and F by either estimating the models on each subgroup i, or by estimating the model where all workers are restricted to sample from the same wage offer distribution F. The extent to which they may or may not be similar can form the basis of a test.

Before we proceed we note that the extent to which the market may actually be considered "integrated" is very much a maintained assumption, and is one that should typically be justified by institutional features of the labour market (such as equal pay legislation). It is not possible to test the assumption that the labour market is integrated *per se* when using data from a single steady state, but only the weaker assertion that workers of different demographic types i face the same wage offer distribution F. We follow this approach here. While these distinctions are not important in terms of estimating the model, they have the potential to be important when the model is used for counter-factual policy experiments (see the later discussion in Chapter 3 of this thesis).

We now consider the extent to which the data supports the assumption that workers of different demographic types i face the same wage offer distribution. In the final three columns of Table 2.3 we present the estimated transitional parameters from the subgroup estimations, and while there are some small differences, none of them are large relative to the standard error of the estimates. In Figure 2.4 we compare the estimated wage offer and wage earnings distribution from the *semi-parametric* specification under the subgroup and common F assumptions. A visual comparison of the figures suggests that the model is indeed capable of explaining differences in the distribution of wages across groups through variation in the other structural parameters of the model.

2.6 An Application to UK Minimum Wage Legislation

Structural models are arguably most valuable when applied as a tool for understanding the impact of policy reforms. In this section we provide an illustrative application by considering how the equilibrium solution of the model changes as we introduce a minimum wage. Specifically, we shall use our empirical estimates from the previous sections to consider the effect of actual minimum wage legislation. The UK national minimum wage was introduced on April 1 1999 at a rate of £3.60 for workers aged 22 years and older and £3 for those aged 18 to 21 years. This was the first time that a national minimum wage operated in the UK. Given

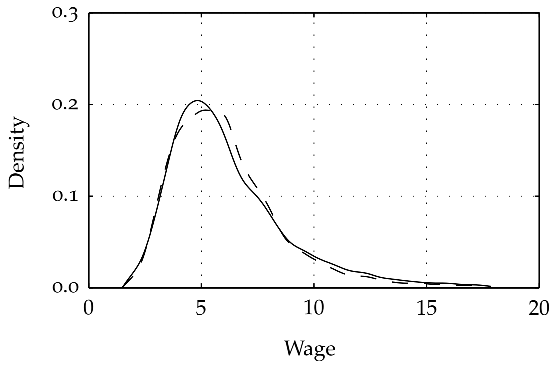
[21] Mean average wages amongst employed workers aged 22–30, 31–40, and 41–55, are £6.08, £6.96, and £7.10 respectively.

Table 2.3: Maximum likelihood estimation results with demographic heterogeneity

		Common F			Group specific F_i		
		$1/\delta_i$	$1/\lambda_{ui}$	$1/\lambda_{ei}$	$1/\delta_i$	$1/\lambda_{ui}$	$1/\lambda_{ei}$
Semi-parametric	Age 22–30	97.99	25.02	45.99	100.39	26.97	44.43
		[90.88,105.82]	[19.59,31.86]	[36.57,55.53]	[92.61,108.31]	[19.81,33.19]	[34.49,55.12]
	Age 31–40	178.46	30.30	46.06	174.06	32.90	55.88
		[163.60,191.32]	[21.34,38.57]	[38.93,54.17]	[159.45,186.72]	[17.80,42.45]	[34.82,70.73]
	Age 41–55	247.70	50.75	64.91	241.34	38.30	56.86
		[227.05,272.73]	[35.27,70.98]	[50.69,79.53]	[223.46,261.85]	[20.24,59.08]	[34.67,72.25]
Pearson IV	Age 22–30	96.78	25.83	49.03	98.05	23.90	42.84
		[91.20,104.14]	[22.12,32.13]	[40.49,62.72]	[91.40,107.81]	[16.42,32.21]	[29.78,51.83]
	Age 31–40	175.37	31.04	49.45	171.12	28.32	51.94
		[162.20,188.27]	[25.11,39.67]	[42.53,60.40]	[157.52,184.75]	[20.92,35.68]	[39.68,64.74]
	Age 41–55	251.55	59.39	65.66	239.83	39.78	60.94
		[232.53,281.26]	[43.21,93.70]	[56.90,75.82]	[222.42,260.26]	[23.66,55.96]	[42.95,76.74]
Pareto	Age 22–30	93.16	8.90	22.23	91.26	9.39	26.02
		[87.00,99.21]	[7.82,10.53]	[18.85,26.92]	[85.88,97.78]	[8.65,10.92]	[21.30,34.71]
	Age 31–40	168.97	11.90	22.63	168.32	13.96	27.32
		[157.45,180.44]	[10.37,13.71]	[19.58,27.31]	[157.24,179.49]	[12.92,16.76]	[23.58,35.06]
	Age 41–55	236.02	20.05	31.00	238.14	20.46	30.04
		[222.06,254.08]	[18.22,22.79]	[26.26,37.24]	[223.84,255.63]	[19.00,25.61]	[24.75,41.74]

Notes: All durations are monthly. The 5^{th} and 95^{th} percentiles of the bootstrap distribution of parameter estimates are presented in brackets, and are calculated using 500 replications.

(a) Wage offer, age 22–30



(b) Wage earnings, age 22–30

(c) Wage offer, age 31–40

(d) Wage warnings, age 31–40

(e) Wage offer, age 41–55

(f) Wage earnings, age 41–55

Figure 2.4: Figure compares the distribution of wage offers and wage earnings by group. Solid lines correspond to the distributions obtain with a common wage offer distribution F; broken lines correspond to those distributions with group specific F_i. All figures calculated using estimates from the *semi-parametric* specification.

our sample selection criteria (see Section 2.4.1), all workers in our sample will be eligible for the higher rate. Since our sample pertains to a slightly earlier period than was the minimum wage introduced, we reduce its introductory value in line with the growth in nominal Gross Domestic Product over this period.[22] This implies a main rate value of £3.21 in April 1997.

2.6.1 Theoretical Extensions

The model we presented in Section 2.2 and extended in Section 2.5 did not explicitly incorporate a minimum wage. In what follows we shall denote the value of this minimum wage as $w_{\min}$, and first consider the case where $w_{\min} \leq \underline{p}$. In this case all firms are able to remain active in the market, so that the equilibrium conditions remain the same as before with the qualification that the lowest wage offer must now satisfy:

$$\underline{w} = \arg\max_{w \geq w_{\min}} (\underline{p} - w) l(w).$$

If instead we have $w_{\min} > \underline{p}$ then fraction $\Gamma(w_{\min})$ of firms will not be active in the presence of a minimum wage, and the distribution of productivity amongst active firms will be given by: $(\Gamma(p) - \Gamma(w_{\min}))/(1 - \Gamma(w_{\min}))$. In contrast to the model without binding a minimum wage where $f(\underline{w}) = 0$ (see Proposition 2), by setting $w = \underline{w} = w_{\min}$ in equation 2.17 it is straightforward to verify that $f(\underline{w}) > 0$. Moreover, whenever $w_{\min} \geq \underline{p}$ the productivity of the least productive active firm will coincide with the minimum wage so that we have $f(\underline{w}) = +\infty$.

2.6.2 Simulated Impact of the Reform

In the simulations we present in this section we allow the arrival rate of job offers to depend upon the number of active firms in a very simple way: we set $\lambda_{ei}^{w_{\min}} = \lambda_{ei} \times \overline{\Gamma}(w_{\min})$ and $\lambda_{ui}^{w_{\min}} = \lambda_{ui} \times \overline{\Gamma}(w_{\min})$, where $\lambda_{ui}^{w_{\min}}$ and $\lambda_{ei}^{w_{\min}}$ denote the arrival rates of job offers for unemployed and employed workers in the presence of a legal minimum wage. In Table 2.6 we present the simulated impact of the introduction of the UK national minimum wage on unemployment. We present results under the same three estimation specifications, both with and without demographic heterogeneity incorporated. In the table we present bootstrapped confidence intervals of the policy impact. Note that while the empirical distribution of wages under the *semi-parametric* central estimates is implementable as an equilibrium of the model, this is *not* true for all the bootstrap samples. This highlights the potential problems that the semi-parametric estimation technique may introduce. To proceed with this exercise, we have constructed a proper distribution of productivity from the third step estimates by applying a rearrangement procedure on the estimate of $K{-}1(w)$.

Under the *semi-parametric* specification with demographic heterogeneity the model predicts a decrease in unemployment equal to around one percentage point. The impact is very similar for the different age groups, and is also very close to what is obtained in the absence

[22] This is obtained using Gross Domestic Product at current prices, UK National Statistics series YBEU.

Table 2.4: Impact of minimum wage on unemployment

	Demographic heterogeneity				No demographic
	Age 22–30	Age 31–40	Age 41–55	Age 22–55	Age 22–55
Semi-parametric	-0.009	-0.007	-0.012	-0.009	-0.009
	[-0.010,0.001]	[-0.009,0.002]	[-0.017,-0.004]	[-0.011,-0.001]	[-0.011,-0.003]
Pearson IV	-0.008	-0.010	-0.016	-0.012	-0.005
	[-0.009,-0.004]	[-0.014,-0.007]	[-0.030,-0.010]	[-0.018,-0.008]	[-0.008,-0.002]
Pareto	0.008	0.006	0.001	0.004	0.005
	[0.005,0.011]	[0.003,0.008]	[-0.001,0.002]	[0.003,0.005]	[0.004,0.006]

Notes: Incomes are measured in pounds per hour in April 1997 prices. The parameter a is the Pareto shape parameter; parameters $\{k_1, k_2, k_3, k_4\}$ are parameters of the Pearson IV distribution. k_1 and k_2 jointly determine the degree of skewness and kurtosis, while k_3 and k_4 are scale and location parameters. The 5^{th} and 95^{th} percentiles of the bootstrap distribution of parameter estimates are presented in brackets, and are calculated using 500 replications.

of any within market demographic heterogeneity. The results from the *Pearson* specification predict a similar decrease in unemployment, but a slight age gradient now emerges with a larger decrease in unemployment (in absolute terms) for the older groups. If we neglect demographic heterogeneity then this specification produces a somewhat smaller decrease in unemployment. In contrast, the impact simulated under the *Pareto* specification is very different: it predicts an increase in unemployment for the youngest two age groups, with essentially no employment impact for oldest group.

We show the impact of the imposition of the minimum wage on some equilibrium functions in Figure 2.5. All these figures were obtained using the central estimates from the *semi-parametric* specification with within market demographic heterogeneity. Since $w_{\min} > \underline{\hat{p}}$ there is a mass point at the minimum wage which is visible in both the wage offer distribution (Figure 2.5a) and the wage earnings distribution (Figure 2.5b). In Figure 2.5c we show the impact on the wage policy function. From this figure we can see that there are *spill-over* effects on wages above the minimum wage, although these do tend to be quite localized. Finally, in Figure 2.5d we show how the overall unemployment rate (plotted with the point-wise 90% confidence band) varies as we increase the value of the minimum wage. At very low values, the minimum wage is not-binding so there is no effect on unemployment, as it increases, unemployment first falls (due to increased job acceptance), and then it begins to increase sharply as dominant effect is coming through the number of active firms is declining. Interesting, the value of the introductory minimum wage (under this specification) is very close to level which minimises the level of unemployment.[23]

2.7 Conclusion

This paper has provided a synthesis of the existing literature on empirical equilibrium job search with wage posting. We develop and estimate a wage posting model with on-the-job search and unobserved worker and firm heterogeneity; firms may differ with respect to their productivity whilst workers differ with respect to their opportunity costs of employment. In

[23]For a discussion of the quasi-experimental evidence of the impact of the UK minimum wage see, amongst others, Dickens and Manning (2004), Metcalf (2008) and Stewart (2004a,b).

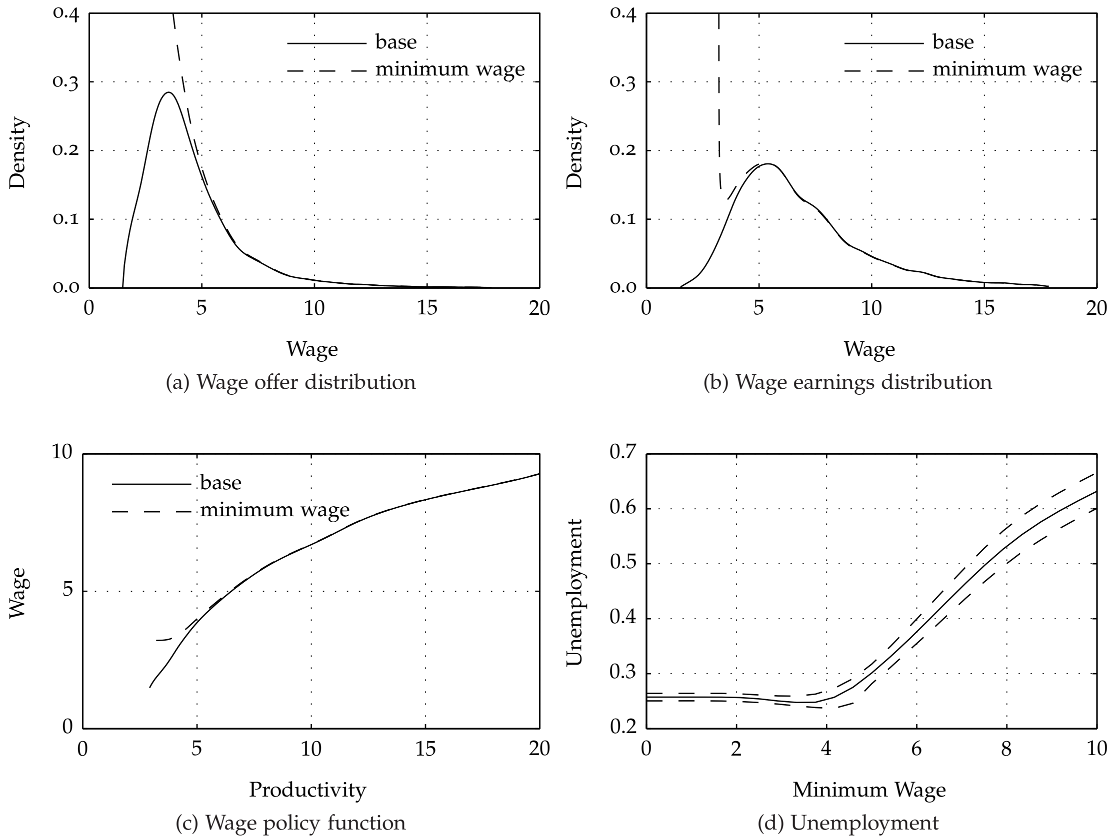


Figure 2.5: Simulated impact of the UK minimum wage on equilibrium functions under *semi-parametric* specification with within market heterogeneity. Broken line in panel (d) plots the 90% confidence bands.

contrast to Bontemps et al. (1999), we conduct this analysis without imposing restrictive assumptions on job offer arrival rates. We have provided a characterization of the equilibrium, presented illustrative numerical simulations, and shown how the model can be further extended to incorporate within market demographic heterogeneity. We have demonstrated that this model remains empirically tractable, and have described how the type of semi-parametric estimation technique that has been applied in simpler job search models may be generalized to this setting. We provide estimation results using UK data, and compare the results obtained from the semi-parametric procedure to alternative parametric specifications.

The model developed provides a useful benchmark model for conducting many types of policy experiment, and we provide some illustrative simulations which use our estimated model to explore the equilibrium effect of actual UK minimum wage legislation. In Chapter 3 of this thesis, we extend this model further and use it to provide a detailed equilibrium assessment of a UK tax credit programme.

Chapter 1 Appendix

2.A Proof of Proposition

Suppose first that $A(\underline{w}) < 1$, $\kappa_e > 0$, and that there is no binding minimum wage. In any equilibrium of the labour market, the first order conditions to the firms' maximization problem may be written as $(p - w)l'(w) = l(w)$. This formulation equates the marginal benefit of a small increase in the wage (in terms of the increased firm size) to its cost (a higher wage bill for all existing workers). Firm size $l(w)$ is as defined in equation 2.12 in the main text; the derivative of firm size with respect to the wage w is given by:

$$l'(w) = \frac{2\kappa_e f(w)(\kappa_e A(w) + (\kappa_u - \kappa_e)uA_u(w))}{(1+\kappa_e\overline{F}(w))^3} + \frac{\kappa_u A'(w)}{(1+\kappa_u\overline{F}(w))(1+\kappa_e\overline{F}(w))}. \tag{2.32}$$

Now, suppose that an equilibrium of the wage posting game exists with $\underline{w} = K(\underline{p})$ and $f(\underline{w}) > 0$. Since $f(w) = 0$ for all $w < \underline{w}$, it is straightforward to verify from equations 2.12 and 2.32 that $\lim_{\epsilon\downarrow 0} l'(\underline{w} - \epsilon) < l'(\underline{w})$ and $\lim_{\epsilon\downarrow 0} l(\underline{w} - \epsilon) = l(\underline{w})$ by the continuity of both F and H (and therefore A). This means that if $(\underline{p} - \underline{w})l'(\underline{w}) = l(\underline{w})$ then if the least productive firm deviates by offering wage $w - \epsilon$ (for some small $\epsilon > 0$) then the marginal benefit from a wage reduction must exceed the marginal cost: $(\underline{p} - \underline{w} + \epsilon)l'(\underline{w} - \epsilon) < l(\underline{w} - \epsilon)$. This contradicts $\underline{w}$ being the optimal wage for the productivity $\underline{p}$ firm. Hence, only $f(\underline{w}) = 0$ is compatible with a wage posting equilibrium of the labour market. Since $K'(p) = \gamma(p)/f(K(p))$ this result also implies that $K'(\underline{p}) = +\infty$. Differentiating equation 2.9 we have:

$$(1-u)g(w)[1+\kappa_e\overline{F}(w)] = f(w)[\kappa_u uA_u(w) + \kappa_e(1-u)G(w)] \tag{2.33}$$

so clearly $f(\underline{w}) = 0$ implies $g(\underline{w}) = 0$.

In the case that $A(\underline{w}) = 1$ in equilibrium (all unemployed workers accept all wage offers), it is straightforward to show that $l(w) \propto (1+\kappa_e\overline{F}(w))^{-2}$ so that the firms' first order condition reduces to:

$$2\kappa_e f(w)(p-w) = (1+\kappa_e\overline{F}(w)).$$

This implies that the offer density must satisfy:

$$f(\underline{w}) = \frac{1+\kappa_e}{2\kappa_e(\underline{p}-\underline{w})} > 0$$

and by setting $w = \underline{w}$ in equation 2.33 this yields:

$$g(\underline{w}) = f(\underline{w})\frac{\kappa_u u + \kappa_e(1-u)}{(1+\kappa_e)(1-u)} > 0,$$

which completes the proof of the proposition.

2.B Numerical Algorithm

This appendix sketches the numerical algorithm that is used when solving for the equilibrium of the model as presented in Section 2.2. It can be appropriately modified for the extensions that are later considered. The equilibrium of the model is solved by iterating on the wage policy function $K(p)$. Here we perform a change of variable so that the integration is always performed over p, although this may not always be necessary in practice. The algorithm proceeds as follows:

1. Discretize p on support $[\underline{p}, \overline{p}]$ and calculate $\overline{\Gamma}(p)$ and $\gamma(p)$ for all p.

2. Provide an initial guess of the wage policy function: $w_0 = K_0(p) < p$ for all p on support $[\underline{p}, \overline{p}]$, with w_0 strictly increasing in p.

3. Given the current guess of the wage policy function $w_n = K_n(p)$ calculate the inverse reservation wage equation:

$$\phi_n^{-1}(K_n(p)) = \frac{1+\rho/\delta+\kappa_u\overline{\Gamma}(p)}{1+\rho/\delta+\kappa_e\overline{\Gamma}(p)}K_n(p) - (\kappa_u-\kappa_e)(1+\rho/\delta)\int_p^{\overline{p}} \frac{K_n(x)d\Gamma(x)}{(1+\rho/\delta+\kappa_e\overline{\Gamma}(x))^2}.$$

4. Calculate the reservation wage distribution in the whole population as $A_n(K_n(p)) = H_n(\phi_n^{-1}(K_n(p))$.

5. Calculate the firm size at given wage policy guess $K_n(p)$ as:

$$l_n(K_n(p)) = \kappa_u\left[\frac{A_n(K_n(p))}{(1+\kappa_u\overline{\Gamma}(p))(1+\kappa_e\overline{\Gamma}(p))} - \frac{(\kappa_u-\kappa_e)}{(1+\kappa_e\overline{\Gamma}(p))^2}\int_{\underline{p}}^{p}\frac{A_n(K_n(x))d\Gamma(x)}{(1+\kappa_u\overline{\Gamma}(x))^2}\right].$$

6. For some step size $0 < s_n \leq 1$ update the guess of the lowest wage offer $\underline{w}_n^*$ using:

$$\underline{w}_n^* = K_n(\underline{p}) + s_n \times \left[\underline{p} - \frac{A_n(K_n(\underline{p}))}{A_n'(K_n(\underline{p}))} - K_n(\underline{p})\right]$$

where $A_n'(K_n(\underline{p})) = H_n'(K_n(\underline{p})) \times (1 + \rho/\delta + \kappa_u) \times (1 + \rho/\delta + \kappa_e)^{-1}$.

7. Obtain the updated guess of the wage policy function $K_{n+1}(p)$ using:

$$K_{n+1}(p) = K_n(p) + s_n \times \left\{ p - \left[(\underline{p} - \underline{w}_n^*) l_n(K_n(\underline{p})) + \int_{\underline{p}}^{p} l_n(K_n(x))dx \right] \frac{1}{l_n(K_n)} - K_n(p) \right\}.$$

8. If for some distance metric $d(K_{n+1}(p), K_n(p)) < \epsilon_{\text{tol}}$ then the wage policy function has converged and we have an equilibrium of the labour market. Otherwise, return to step 3. If the distance measure is exploding, then reduce the step size s_{n+1}.

Parametric Estimation

The algorithm developed above can be modified easily for the estimation procedure. Here, we take the support of wages as fixed and solve for the inverse of the wage policy function $K^{-1}(w)$. The modified algorithm proceeds as follows:

1. Discretize w on support $[\underline{w}, \overline{w}]$.

2. Provide an initial guess of the inverse wage policy function: $p_0 = K_0^{-1}(w) > w$ for all w on support $[\underline{w}, \overline{w}]$, with p_0 strictly increasing in w.

3. Given the finite parameter vector characterising the distribution of firm productivity $\vec{\theta}_\Gamma$ and the current guess of the inverse wage policy function, calculate $\overline{\Gamma}_n(p)$ and $\gamma_n(p)$ on support $[K_n^{-1}(\underline{w}), K_n^{-1}(\overline{w})]$. Set $\overline{F}_n(w) = \overline{\Gamma}(K_n^{-1}(w))$.

4. Calculate the inverse reservation wage equation:

$$\phi_n^{-1}(w) = w - (\kappa_u - \kappa_e) \int_w^{\overline{w}} \frac{\overline{F}_n(x)}{1 + \rho/\delta + \kappa_e \overline{F}_n(x)} dw.$$

5. Given $\phi_n^{-1}(w)$ calculate the reservation wage distribution in both the whole population $A_n(w) = H_n(\phi_n^{-1}(w))$, and among unemployed workers:

$$u_n A_n(w) = \frac{1}{1+\kappa_u} A_n(\underline{w}) + \int_{\underline{w}}^{w} \frac{dA_n(x)}{1 + \kappa_u \overline{F}_n(x)}$$

where $A_n'(w) = H_n'(\phi_n^{-1}(w)) \times (1 + \rho/\delta + \kappa_u \overline{F}_n(w)) \times (1 + \rho/\delta + \kappa_e \overline{F}_n(w))^{-1}$.

6. Calculate the firm size at given inverse wage policy guess $K_n^{-1}(w)$ as:

$$l_n(w) = \frac{\kappa_e A_n(w)}{(1 + \kappa_e \overline{F}_n(w))^2} + \frac{(\kappa_u - \kappa_e)}{(1 + \kappa_e \overline{F}_n(w))^2} \left[\frac{1}{1+\kappa_u} A_n(\underline{w}) + \int_{\underline{w}}^{w} \frac{dA_n(x)}{1 + \kappa_u \overline{F}_n(x)} \right].$$

7. For some step size $0 < s_n \leq 1$ update the guess of the lowest firm productivity level $\underline{p}_n^*$ using:
$$\underline{p}_n^* = K_n^{-1}(\underline{w}) + s_n \times \left[\underline{w} + \frac{A_n(\underline{w})}{A_n'(\underline{w})} - K_n^{-1}(\underline{w})\right].$$

8. Obtain the updated guess of the inverse wage policy function $K_{n+1}^{-1}(w)$ using:
$$\begin{aligned} K_{n+1}^{-1}(w) &= K_n^{-1}(w) \\ &\quad + s_n \times \left\{ w + \left[(\underline{p}_n^* - \underline{w}) l_n(w) + \int_{\underline{w}}^{w} l_n(x) K_n^{-1}(w)' dx \right] \frac{1}{l_n(w)} - K_n^{-1}(w) \right\}. \end{aligned}$$
where:
$$\begin{aligned} K_n^{-1\prime}(w) = &\frac{(1+\kappa_e \overline{F}_n(w))}{\kappa_e \gamma(K_n^{-1}(w))} \frac{1}{K_n^{-1}(\underline{w}) - \underline{w}} \\ &- \frac{\kappa_u (1+\kappa_e \overline{F}_n(w)) A_n'(w)}{(1+\kappa_u \overline{F}_n(w))(\kappa_e A_n(w) + (\kappa_u - \kappa_e) u_n A_{u_n}(w))} \end{aligned}$$

9. If for some distance metric $d(K_{n+1}^{-1}(w), K_n^{-1}(w)) < \epsilon_{\text{tol}}$ then the inverse wage policy function has converged and we have an equilibrium of the labour market. Otherwise, return to step 3. If the distance measure is exploding, then reduce the step size s_{n+1}.

Remark The main potential difficulty when estimating the model parametrically is that for a given parametrization of the productivity distribution $\vec{\theta}_p$, there may not exist a productivity level that is consistent with the empirical support of wages. Good initial values are essential, and this also necessitates a solver that can deal with the occasional discontinuities in the objective function that this may introduce.

2.C The Pearson Type IV Distribution

The Pearson (Pearson, 1895) family of distributions were developed to approximate all unimodal distributions.[24] The Pearson Type IV distribution allows for varying degrees of skewness and kurtosis, and so is ideally suited for many economic applications where distributions of interest are often asymmetric with extensive tails. The distribution is characterized by four parameters, $\vec{\theta}_k = \{k_1, k_2, k_3, k_4\}$, and these uniquely determine the first four moments of the distribution (see Stuart and Ord, 1994).

Despite these desirable properties, the distribution has been little exploited in the economics literature. Its sparsity in the literature is perhaps due to the difficulty in calculating the density and distribution functions. For $k_1 > 1/2$, Nagahara (1999) showed that the probability density function of the Pearson Type IV distribution can be expressed by:

$$p(x) = \mathcal{K}(k_1, k_2, k_3) \times \left[1 + \left(\frac{x - k_4}{k_3}\right)^2\right]^{-k_1} \exp\left[-k_2 \tan^{-1}\left(\frac{x - k_4}{k_3}\right)\right]. \quad (2.34)$$

[24]This section draws heavily on Heinrich (2004), which should be consulted for further details.

The parameters k_3 and k_4 in (2.34) are scale and location parameters, whereas the parameters k_1 and k_2 are shape parameters that jointly determine the degree of skewness and kurtosis of the distribution. The kurtosis of the distribution is decreasing in the parameter k_1, and the distribution is negatively (positively) skewed whenever $k_2 > 0$ ($k_2 < 0$). Note that when $k_2 = 0$ the distribution reduces to Student's t-distribution with $2k_1 - 1$ degrees of freedom (and so the Normal distribution when additionally $k_1 \to \infty$). More generally, the Pearson Type IV distribution can be considered as an asymmetric version of Student's t-distribution.

Before describing how the normalization factor and distribution function may be calculated in the general case, note that in our application we can calculate $p(x)$ up to a scale parameter on $[\underline{p}, \overline{p}]$ by first setting the normalization factor $\mathcal{K} = 1$, and then numerically integrate $p(x)$ on its support to obtain the cumulative distribution function $P(x)$ (again, up to a scale parameter). Dividing both the density and distribution function by $P(\overline{p})$ will produce the required quantities. More generally, to calculate the density function in equation (2.34) it is first necessary to calculate the normalization factor $\mathcal{K}(k_1, k_2, k_3)$ which is given by:

$$\mathcal{K}(k_1, k_2, k_3) = \frac{\Gamma(k_1)}{\sqrt{\pi} k_3 \Gamma(k_1 - 1/2)} \left| \frac{\Gamma(k_1 + ik_2/2)}{\Gamma(k_1)} \right|^2 \tag{2.35}$$

where $i = \sqrt{-1}$ and Γ is the Gamma function (not to be confused with the productivity distribution in the main text). While the normalization factor (2.35) can be calculated by evaluating the complex Gamma function directly, Heinrich (2004) proposed calculating the squared module in (2.35) by exploiting its relationship with the ${}_2F_1$ hypergeometric function, and by using a recursion relationship. This then allows $\mathcal{K}$ to be calculated to machine precision very efficiently.

Heinrich (2004) demonstrated that the cumulative distribution function itself can also be expressed in terms of the hypergeometric function. Specifically, it can be shown that:

$$\frac{P(x)}{p(x)} = \frac{k_3}{2k_1 - 1} \left(i - \frac{x - k_4}{k_3} \right) {}_2F_1 \left(1, k_1 + \frac{i\nu}{2}; 2k_1; \frac{2}{1 - i\frac{x - k_4}{k_3}} \right) \tag{2.36}$$

which converges absolutely when $x < k_4 - k_3\sqrt{3}$. When $x > k_4 + k_3\sqrt{3}$ we apply the symmetry identity $P(x|k_1, k_2, k_3, k_4) \equiv 1 - P(-x|k_1, -k_2, k_3, -k_4)$, and in the case that $|x - k_4| < k_3\sqrt{3}$ we apply a "linear transformation" as described in Abramowitz and Stegun (1965). Fortran and MATLAB numerical routines to calculate the distribution and density functions are available form http://www.ucl.ac.uk/ uctpajs/software.htm. These numerical routines evaluate the hypergeometric function in equation (2.36) using the method described in Michel and Stoitsov (2008).

Chapter 3

Equilibrium Search and Tax Credit Reform

3.1 Introduction

Over the past two decades earned income tax credit programmes have grown substantially in the UK, US and many other countries.[1] These programmes are typically motivated by a desire from policy makers to increase labour market participation among target groups, and to alleviate in-work poverty. While the effect of these policies on labour supply has been studied extensively, much less is known regarding the incidence of these tax credit programmes and the broader general equilibrium impact. The objective of this paper is to develop an empirical equilibrium job search model that provides us with an appropriate framework to address these issues, and to apply it in our analysis of the British Working Families' Tax Credit (WFTC) reform.

Tax credit programmes, such as EITC in the U.S. and WFTC (together with its predecessors and recent successor) in the UK, have received considerable attention in the economics literature. The employment impacts of these programmes have been evaluated in a number of empirical studies, which have typically adopted either a difference-in-differences (see for example, Eissa and Liebman, 1996) or structural discrete choice model approach (for example, Blundell et al., 2000). While the magnitudes of the estimated or simulated impacts do differ somewhat across these studies, there appears to be a general consensus that tax credit programmes like WFTC have had positive effects on the employment of lone mothers, while there is somewhat less agreement concerning the effect of these reforms on men and women in couples.[2]

Despite the wealth of labour supply studies, there is surprisingly little evidence regarding the equilibrium impact of these reforms. Exceptions include the recent tax credit incidence studies of Azmat (2006b), Leigh (2009), and Rothstein (2008, 2009). Using data from the federal expansion in the mid-1990s, Rothstein (2008) examined the effect of the US EITC on labour market participation and equilibrium wages. Using his central elasticity estimates,

[1] See Hotz and Scholz (2003) for EITC in the US, and Blundell and Hoynes (2004) for the British WFTC and its predecessors.

[2] Studies which have evaluated the employment impact of WFTC include Azmat (2006a), Blundell et al. (2004a), Brewer et al. (2006), Francesconi and van der Klaauw (2004), Gregg and Harkness (2003), and Leigh (2007). See also Section 1.5.4 from Chapter 1 in this thesis. These will be discussed in more detail in Section 3.5.5.

and assuming a distinct labour market for single women, he simulated a \$1 expansion in EITC payments and found that pre-tax earnings of eligible workers fell by \$0.30 while the earnings of ineligible workers fell by \$0.43. Leigh (2009) exploited state variation in EITC supplements over the period 1989–2002 to examine the effect of the tax credit on wages, and found that a one percent increase in the generosity of the programme reduced gross hourly wages by 0.5% for high school dropouts, 0.2% for those with a high school diploma, and had no effect on college graduates. Azmat (2006b) examined the incidence of the British WFTC and found evidence to suggest that firms discriminate by cutting the wage of claimant workers relative to similarly skilled non-claimant workers, and that there are spill-over effects on the wages of similarly skilled non-eligible workers.

The policy context of this paper is the UK earned income tax credit reform in October 1999, when the government replaced the existing Family Credit (FC) programme – the main form of in- work support for lower income families with children – with Working Families' Tax Credit. While WFTC represented a continued expansion of in-work programmes of support, it was considerably more generous than its predecessor, offering higher credits and a lower withdrawal/taper rate. Since the presence of dependent children is a central eligibility criteria for the receipt of WFTC, families without children (who act as a control group in the quasi-experimental employment impact studies) may also find themselves affected by the reform, if say, wages were to adjust. Consequently, the government's view regarding the desirability of such reforms – and more generally how such programmes should be designed – may depend crucially upon the nature and quantitative importance of these equilibrium effects.

In order to address these important issues we require a model in which both employment and the distribution of wages emerges as an equilibrium outcome. While a competitive model of the labour market is able to generate wage dispersion if individuals differ in their marginal productivity of labour, and may also generate equilibrium price effects following adjustments in labour supply (see for example, the model presented in Rothstein, 2008), there is empirical evidence that suggests that labour markets are characterised by substantial search frictions (see for example, van den Berg and Ridder, 2003). Departing from the competitive paradigm may have important implications for our understanding of tax credit reforms. In particular, if firms set wages then the presence of search frictions gives firms some degree of monopsony power. Thus, one possible equilibrium effect of the reform is that firms may attempt to extract a greater share of the rent by lowering the wages that they offer. If this mechanism is important, then the effective transfer to eligible individuals may be reduced, while non-eligible workers may be made worse off.

The equilibrium job search literature allows us to capture these and other effects in a dynamic and imperfectly competitive economy that is characterized by search frictions.[3] Here

[3]Our analysis remains partial equilibrium to the extent that capital is ignored, as is the product market and the possible effect of the reform on outcomes such as education and fertility. However, the model is equilibrium in the sense that employment, the distribution of wages, and the arrival rate of job offers are determined within the model.

it is the competition between firms that is the fundamental determinant of wages, with the extent of this competition limited by the presence of search frictions. We consider a model with ex-ante wage posting – firms set wages before meeting potential workers, which workers can then either accept or reject. Manning (2003) argues that while wage posting is not appropriate in all contexts, it is likely to provide a good characterisation of wage determination in many applications. This is likely to be particularly true when we are focussing on low-skilled labour markets, as we examine in this paper. Hall and Krueger (2008) present recent US survey evidence which suggests that while other forms of wage formation are also important, wage posting is much more prevalent in less skilled occupations (see also the discussion in Manning, 2003, chapter 5).[4]

It is instructive to consider briefly the conditions under which a dispersed offer distribution may be supported in an equilibrium with wage posting.[5] Diamond (1971) showed that if workers are homogeneous, and workers are unable to search while employed, then the equilibrium wage offer distribution would collapse to a degenerate distribution equal to the common reservation wage. The literature offers two standard ways of generating wage dispersion as an equilibrium outcome. The first is to assume that workers themselves are heterogeneous in their opportunity cost of employment (Albrecht and Axell, 1984; Eckstein and Wolpin, 1990). Since this ex-ante heterogeneity translates into heterogeneity in reservation wages, firms now face a trade-off between offering a low wage and just attracting workers with low reservation wages, or offering a high wage and attracting workers with both low and high reservation wages. In equilibrium firms will be indifferent between offering these wages, so that a dispersed wage distribution may emerge.

An alternative approach that generates wage dispersion as an equilibrium outcome maintains the homogeneous worker assumption, but allow workers to continue searching for jobs when employed (Burdett and Mortensen, 1998). For employed workers, their reservation wage will equal their current wage, so that firms are now able to attract more workers (through job-to-job transitions) by offering a higher wage. Hence, on-the-job search generates ex-post heterogeneity among ex-ante identical agents so that wage dispersion can be supported in equilibrium. If all firms are equally productive, then the model implies increasing wage offer and earnings densities. This apparent empirical failing can be overcome to some extent by allowing for heterogeneity in firm productivity. Building upon the analysis of Mortensen (1990), which demonstrated that more productive firms offer higher wages, Bontemps et al. (2000) allowed for a continuous distribution of firm productivity and showed that the model can induce empirical wage distributions by allowing for an appropriately

[4]Lise et al. (2005) simulate the general equilibrium effect of a wide scale implementation of the Canadian Self-Sufficiency Project (SSP) in a model with ex-post worker-firm bargaining. They find substantial general equilibrium effects, which reverse the positive cost-benefit conclusions of their partial equilibrium evaluation. Kolm and Tonin (2006) consider the impact of introducing an in-work benefit in an analytical framework using a Pissarides (2000) search model. Here general equilibrium effects reinforce the employment impact of the programme through job creation.

[5]Also see the more detailed review provided in Chapter 2.

skewed distribution of firm productivity.

To our knowledge, this is the first time that an empirical equilibrium job search model has been used to analyse a policy reform of this kind. Our analysis advances the existing literature in several dimensions, with our model designed to capture key features of the UK labour market and to allow for the possibility of rich equilibrium effects following reforms such as WFTC. Firstly, since one of the objectives of the reform was to increase labour supply, it is necessary to have a model with reservation wage heterogeneity so that at least some workers may accept some wage offers and reject others. Such a model was presented in Bontemps et al. (1999) which allowed for continuous distributions of firm productivity and worker opportunity costs with the restriction that the job offer arrival rate is independent of employment status. This over-identifying restriction simplifies the analysis as it implies that the optimal strategy of unemployed workers is independent of the equilibrium wage offer distribution (see Section 3.3.2). This restriction led to a poor fit of the duration data in their application, as empirically job arrival rates for unemployed workers are often estimated to exceed that of the employed. Building a model which relaxes this arrival rate restriction, similar to that which we developed in Chapter 2 of this thesis, will be an important part of our analysis.

A pertinent feature of the UK labour market is that certain groups of individuals – most notably women with children – work part-time at some point in their lifetime. Since the presence of children is a central eligibility criteria for receipt of WFTC, it is therefore necessary to incorporate hours into our model. While the use of the canonical labour supply model may be pervasive, there is a body of empirical work that challenges the view that individuals are able to freely choose their hours of work at a fixed hourly wage (Altonji and Paxson, 1988; Lundberg, 1985; Martinez-Granado, 2005; Moffitt, 1984; Stewart and Swaffield, 1997). Blundell et al. (2008) use British Household Panel Survey data to study the impact of a series of in-work benefit reforms in the UK during the 1990s, and found that the introduction of WFTC had positive effects on hours worked, but this increase was largely driven by women who changed job. That jobs sequentially arrive as wage-hours packages is an assumption that will be maintained throughout this paper. This is important not only in its ability to describe that apparent lack of hours flexibility within jobs, but also in its ability to potentially capture the so-called part-time penalty. Manning and Petrongolo (2008) document recent British evidence for female workers. We do not attempt to provide rich micro-foundations for this, but rather assume it is a purely technological description of firms – firms are able to offer either full-time jobs, or part-time jobs, but not both.

Within this framework we incorporate detailed representations of the UK tax and transfer system. Since we are interested in how WFTC affected different groups of workers, we build upon the model developed in Chapter 2 allow for demographic heterogeneity to influence the key structural parameters of our model as well as the tax and transfer schedules. Crucially

– and in contrast to the segmented markets approach adopted by van den Berg and Ridder (1998) – we assume that workers of all types operate within the same labour market. This allows us to capture rich equilibrium effects and to study possible unintended consequences of the tax credit reform within our model.

We develop a three step semi-non-parametric estimation technique similar to that proposed by Bontemps et al. (1999, 2000) and extended in Chapter 2 of this thesis, and estimate our model using UK Labour Force Survey data shortly before WFTC was introduced. Using the structural parameters we then simulate the equilibrium impact of WFTC, holding the structural parameters and the distribution of firm productivity fixed. This allows us to investigate the impact of the reform on employment, hours of work, unemployment durations, and the entire distribution of wages. Recognizing the role that firms may play as both wage setters and job creators, we follow Mortensen (2000) by complementing our model with aggregate matching functions, which then allows us to endogenize rate at which workers encounter job offers at the macroeconomic level. We find that the introduction of WFTC, together with other accompanying changes to the tax and transfer system between April 1997 and April 2002, increased employment for most groups, with lone mothers experiencing the largest increase. Our main simulations suggest that while equilibrium considerations do play a role in this particular reform, the changes in employment and earnings are dominated by the direct effect of changing job acceptance behaviour. We also show how the type of equilibrium effects considered have the potential to be much more important for tax reforms which are less targeted. We demonstrate that the equilibrium effects of the same reforms may be much larger if we consider a labour market solely comprised of lone mothers, one of the main beneficiaries of WFTC.

The paper proceeds as follows. Section 3.2 briefly describes the WFTC reform, while Section 3.3 presents our model and describes the optimal strategies of firms and workers. In Section 3.4 we describe the estimation and identification of our model, derive the likelihood function and discuss our data and estimation results. In Section 3.5 we present our main simulation exercises and compare our predictions to the actual employment changes observed over the relevant period, as well as the findings from other studies. Finally, Section 3.6 concludes.

3.2 The Working Families' Tax Credit Reform

The UK has a long history of in-work benefits, starting with the introduction Family Income Support (FIS) in 1971. Over the years, these programmes became more generous, and in October 1999, Working Families' Tax Credit was introduced, replacing a similar, but less generous, tax credit programme called Family Credit. Both WFTC and FC shared a similar eligibility structure, requiring recipients work for at least 16 hours per week, and with the credit tapered away with household earnings above a threshold. Both also offered a further credit when recipients worked at least 30 hours a week. There were essentially five ways in which

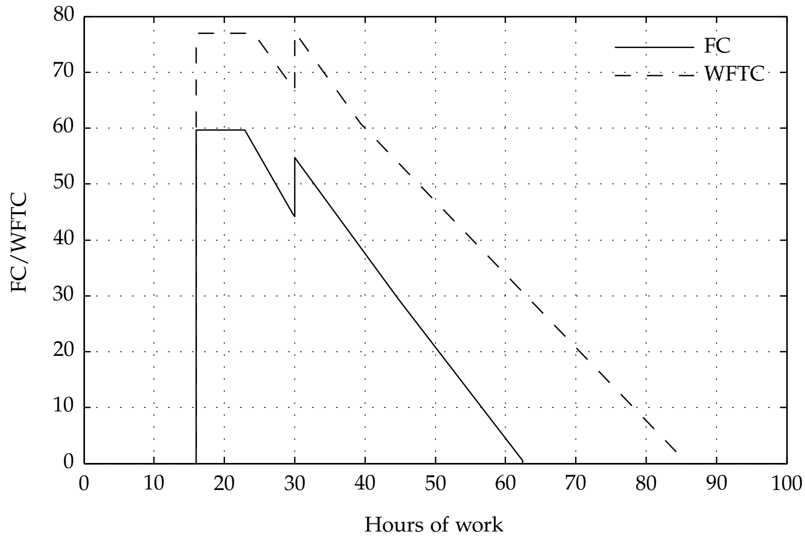


Figure 3.1: Tax credit awards under FC and WFTC. FC refers to Family Credit as of April 1997; WFTC refers to Working Families' Tax Credit as of April 2002. Figure assumes a lone parent with a single child aged 10, and a constant hourly wage rate of £3.50. All incomes expressed in April 1997 prices.

WFTC increased the level of in-work support relative to the FC system: (i) it offered higher credits, especially for families with younger children; (ii) the increase in the threshold meant that families could earn more before it was phased out; (iii) the tax credit withdrawal rate was reduced from 70% to 55%; (iv) it provided more support for formal childcare costs through a new childcare credit; (v) all child maintenance payments were disregarded from income when calculating tax credit entitlement. These changes meant that the largest potential beneficiaries of WFTC were those families who were just at the end of the FC benefit withdrawal taper (see Figure 3.1). The combined increases in generosity and caseload resulted in nominal spending on WFTC rising to £4.6 billion by 2000/1 and £6.3 billion by 2002/3 compared to £2.4 billion on FC in 1998/9. Table 3.1 presents the main parameters of FC and WFTC.[6]

When analysing the effect of programmes such as WFTC it is necessary to take an integrated view of the tax system. This is because tax credit income is counted as income when calculating entitlements to other benefits. Families in receipt of such benefits would gain less from the WFTC reform than otherwise-equivalent families not receiving these benefits. Furthermore, there were other notable changes to the tax system affecting families with children that coincided with the expansion of tax credits, therefore making the overall labour market impact more difficult to predict. In particular, there were increases in the generosity of Child Benefit (a cash benefit available to all families with children regardless of income), as well as notable increases in the child additions in Income Support (a welfare benefit for low income families working less than 16 hours a week). For many families with children, these increases

[6] There were also important administrative changes between FC and WFTC. In particular, while FC was paid direct to recipients as a cash benefit, WFTC was typically paid by employers through the wage packet.

Table 3.1: Parameters of FC/WFTC

	April 1999 (FC)	October 1999 (WFTC)	June 2000 (WFTC)	June 2002 (WFTC)
Basic Credit	49.80	52.30	53.15	62.50
Child Credit				
under 11	15.15	19.85	25.60	26.45
11 to 16	20.90	20.90	25.60	26.45
over 16	25.95	25.95	26.35	27.20
30 hour credit	11.05	11.05	11.25	11.65
Threshold	80.65	90.00	91.45	94.50
Taper rate	70% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance
Childcare	Expenses up to £60 (£100) for 1 (more than 1) child under 12 disregarded when calculating income	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £135 (£200) for 1 (more than 1) child under 15

Notes: All monetary amounts are in pounds per week and expressed in nominal terms. Minimum tax credit award is 50p per week in all years above.

in out-of-work income meant that despite the increased generosity of in-work tax credits, replacement rates remained relatively stable. There were also changes to the tax and transfer system that affected families both with and without dependent children during the lifetime of WFTC: a new 10% starting rate of income tax was introduced; the basic rate of income tax was reduced from 23% to 22%; there was a real rise in the point at which National Insurance (payroll tax) becomes payable.[7]

3.3 The Model

This section lays out the theoretical model that we use to study the impact of the WFTC reform. It is an extension of the model presented in Chapter 2, which itself built upon the analysis of Bontemps et al. (1999). We begin by setting out the core assumptions, and derive the optimal job acceptance strategies of workers. We then characterise the steady state flows and close the model by deriving the optimal wage setting and job creating behaviour of firms. A summary of the subsequent notation used is presented in Appendix 3.C.

3.3.1 Model Assumptions

The economy consists of a continuum of individuals with a population size normalized to unity. These individuals may differ with regards to observable demographic characteristics (for example, gender, marital status and the presence of children) that are finite in number

[7]There were also important non-tax related reforms over this period, which we do not consider in our analysis. Various "New Deal" programmes were introduced which aimed to improve both the incentives and the ability of the long-term unemployed to obtain employment (see Blundell et al., 2004b). Furthermore, a national minimum wage was introduced in April 1999, at a rate of £3.00 per hour for those aged 18-21 (the development rate) and at the higher rate of £3.60 per hour for those aged 22 and over. Since their introduction, both the main and the development rates have been subject to a number of above-inflation increases. The equilibrium impact of the introduction of the minimum wage was considered in Chapter 2 of this thesis.

and indexed by $i \in \mathcal{I}$. In our analysis these will be treated as being fixed and time invariant, and we let n_i denote the fraction of type i individuals, with $\sum_i n_i = 1$. As shall become clear, while all such individuals will operate in the same labour market, these demographic characteristics will be allowed to influence both the tax and transfer system (reflecting the conditioning performed by the UK tax authorities) as well as the transitional parameters of the model. Time is continuous and individuals live forever with the constant discount rate $\rho_i > 0$. There is no access to either saving or borrowing technology. These individuals (or workers) can be either employed or unemployed, and we use e and u to index these respective states. Jobs are characterised by a wage rate w and required hours of work h. We assume that the required hours of work within a job is a technological characterisation of firms, with firms offering either part-time jobs (hours h_0) or full-time jobs (hours $h_1 > h_0$), but not both.[8] The particular hours sector that firms operate in is exogenously determined.[9] Mirroring the conditioning that is performed by the UK government, hours can directly affect the tax schedule, with $T_i^h(wh)$ denoting the (potentially negative) net taxes paid by a employed worker with observable characteristics i at earnings wh and hours sector $h \in \{0, 1\}$. Similarly, the net transfer paid to an unemployed worker with characteristics i is given by $-T_i^u$. We assume that the tax schedule $T_i^h(wh)$ is continuously differentiable.

There is an exogenous distribution of firm productivity in both part-time and full-time sectors. The cumulative distribution of part-time firm productivity is given by Γ_0 on support $[\underline{p}_0, \overline{p}_0]$ while the cumulative distribution of full-time firm productivity is Γ_1 on support $[\underline{p}_1, \overline{p}_1]$. While workers are assumed to be equally productive at a given firm, they differ in their unobserved opportunity cost of employment b which has the cumulative distribution function H_i on support $[\underline{b}_i, \overline{b}_i]$. To simplify some of the exposition we assume that $\underline{b}_i$ is sufficiently low so that in equilibrium all firms are active in the labour market. In the presences of taxes, the flow utility of an unemployed individual is given by $b - T_i^u$, whereas for employed workers flow utility is assumed linear in net-income and a monetary dis-utility of work: $wh - T_i^h(wh) - C_i^h$, with C_i^h denoting the monetary dis-utility of work at hours h. From the outset we shall impose the location normalization $C_i^0 = 0$ for all i.

Individuals encounter part-time (full-time) job offers at the exogenous (to the worker) rate λ_{ui}^0 (λ_{ui}^1) when unemployed, and λ_{ei}^0 (λ_{ei}^1) when employed. To maintain focus on the decisions faced by workers, we postpone any discussion concerning how these arrival rates may depend upon the overall state (or tightness) of the labour market, but return to this issue in Section 3.3.5.[10] Employment spells end at rate δ_i regardless of whether individuals are employed in

[8] The framework we develop generalizes to more than two hours choices, and can also be applied in the context of other non-wage amenities. See Hwang et al. (1998) for an analysis of non-wage amenities in an equilibrium search framework.

[9] Note that we are implicitly assuming an indivisibility in the production technology; two part-time workers are not a substitute for a single full-time worker.

[10] A more realistic approach would also endogenize the job offer arrival rates λ_{ji}^h at the micro-level by relating them directly to an endogenously determined worker search effort, as in Christensen et al. (2005). In our analysis we do not allow the search effort of workers to vary with their current wage or to directly respond to any changes in the tax system. Non-hours labour supply responses to tax reforms, such as search effort, may be quantitatively important as

part-time of full-time jobs. Here and throughout this paper, we place no restrictions on the relative magnitude of these quantities, but note that the assumption that the job destruction and job arrival rates when employed are independent of whether individuals are currently engaged in part-time or full-time work should be considered an over-identifying restriction.

Regardless of observed characteristics i or unobserved characteristics b, all workers are assumed to sample part-time (full-time) wage offers from the distribution F_0 (F_1), with the lowest wage offer denoted $\underline{w}_0$ ($\underline{w}_1$) and the highest wage offer $\overline{w}_0 \geq \underline{w}_0$ ($\overline{w}_1 \geq \underline{w}_1$). The density functions of these distributions are given by $f_0 \equiv F_0'$ and $f_1 \equiv F_1'$ respectively. Since these do not depend on worker type this implies that, while the government may be able to condition taxes and transfers on demographic characteristics i, firms are unable to do so. We justify this assumption by the presence of anti-discrimination laws, such as the Equal Pay Act 1970, Sex Discrimination Act 1975, and various Employment Equality Regulations, which outlaw such practices. Note that this assumption does not imply that workers of all types will have the same distribution of earnings. This is because workers may differ in their job acceptance behaviour, and may also gravitate towards higher paying jobs at different rates. As noted in the introduction, assuming that workers operate in the same labour market permits interesting spill-over effects, so that workers who are not targeted by a particular tax reform (in our case, childless families) can still be indirectly affected by it.

We assume that there is wage posting: employers post wages prior to forming matches with potential employees, who can then either accept or reject the wage offer. Furthermore, firms may post job vacancies that affect the rate at which these firms encounter workers. Wages are assumed constant throughout an individual's employment spell within a given firm. In contrast to the papers surveyed above, jobs are no longer completely characterized by the wage that they offer, so the behaviour of individuals will now be summarized by a slightly more complicated reservation wage strategy which will depend on the required hours of work. These strategies are discussed in the following section.

3.3.2 Worker Strategies

In this section we describe the optimal strategies of unemployed and employed workers; formal derivations are presented in Appendix 3.A. To proceed we define $q_i(w)$ such that the value to a type i individual holding a full-time job paying wage w is the same as the value of a part-time job paying wage $q_i(w)$. Since the job destruction rate and the arrival rates for both full-time and part-time jobs are assumed independent of current hours of work, it can be shown that in order to determine $q_i(w)$ it sufficient to compare the instantaneous utility flows between part-time and full-time employment. In other words $q_i(w)$ solves:

$$wh_1 - T_i^1(wh_1) - C_i^1 = q_i(w)h_0 - T_i^0(q_i(w)h_0) \tag{3.1}$$

emphasized by the new-tax responsiveness literature (Feldstein, 1995). However, incorporating this into our model is non-trivial and is left as an extension for future research.

Table 3.2: Worker acceptance strategies

	Full-time offer	Part-time offer
Unemployed	$w' \geq \phi_i(b)$	$w' \geq q_i(\phi_i(b))$
Employed, full-time	$w' > w$	$w' > q_i(w)$
Employed, part-time	$w' > q_i^{-1}(w)$	$w' > w$

Notes: Assumes a current wage w and either a full-time or part-time wage offer w'. $q_i(w)$ is as defined in equation 3.1; $\phi_i(b)$ is as defined in equation 3.2.

which has a unique solution provided that conditional on hours of work marginal tax rates are always strictly less than one (an assumption that we maintain throughout), in which case we have $dq_i(w)/dw > 0$. So while employed workers with wage w will accept any wage offer w' from their current hours sector that is (by convention) strictly greater than their current wage, a full-time worker would find a part-time job offer acceptable if and only if $q_i(w') > w$. Similarly, a part-time worker would find such a full-time wage offer acceptable if and only if $q_i^{-1}(w') > w$.

Unemployed workers also follow a reservation wage strategy, that will again vary depending on whether or not a full-time or part-time offer is received. We let $\phi_i(b)$ denote the reservation wage for *full-time* work conditional on observed type i and unobserved leisure cost b. This takes a similar form to the standard reservation wage equation with on-the-job search (see, for example, Mortensen and Neumann, 1988), but is here modified both by the presence of taxes and because workers are now sampling offers from two distributions; in particular, taxes act to discount future earnings by the net-of-tax rate. In Appendix 3.A we show that $\phi_i(b)$ is the solution to:

$$\phi_i(b)h_1 - T_i^1(\phi_i(b)h_1) - C_i^1 = b - T_i^u + h_1 \int_{\phi_i(b)}^{\infty} B_i(w)dw \tag{3.2}$$

where:

$$B_i(w) \equiv \frac{\left(1 - T_i^{1\prime}(wh_1)\right)\left[(\kappa_{ui}^0 - \kappa_{ei}^0)\overline{F}_0(q_i(w)) + (\kappa_{ui}^1 - \kappa_{ei}^1)\overline{F}_1(w)\right]}{1 + \rho_i/\delta_i + \kappa_{ei}^0\overline{F}_0(q_i(w)) + \kappa_{ei}^1\overline{F}_1(w)}$$

and where $\kappa_{ji}^h = \lambda_{ji}^h/\delta_i$, with $j \in \{u, e\}$ and $h \in \{0, 1\}$. From our discussion above, it follows that the lowest acceptable wage offer for a part-time job is simply given by $q_i(\phi_i(b))$. Before we proceed note that in the case where $\kappa_{ui}^h = \kappa_{ei}^h$ for $h \in \{0, 1\}$, we have $B_i(w) = 0$ for all (w, i) so that the optimal strategy of unemployed workers is independent of the equilibrium wage offer distributions. This is the case analysed in Bontemps et al. (1999). For reference, we summarize the job acceptance strategy of all individuals in Table 3.2.

3.3.3 Steady State Flows

This section derives a number of steady state objects by using flow equations. For now, we treat the distributions of wage offers and their arrival rates as being given, but will later show how they emerge as an equilibrium outcome.

Distribution of Reservation Wages

In deriving steady state flows it is necessary to consider the distribution of (full-time) reservation wages for each demographic group (denoted A_i) which is related to the distribution of work opportunity costs according to $A_i(w) = H_i(\phi_i^{-1}(w))$. We denote the distribution of full-time reservation wages amongst the stock of unemployed and employed workers as A_{ui} and A_{ei} respectively, and these are related to A_i according to:

$$A_i(\phi) = u_i A_{ui}(\phi) + (1-u_i)A_{ei}(\phi). \tag{3.3}$$

In order to describe the equilibrium of the labour market, it is necessary to determine the distribution of reservation wages amongst the unemployed A_{ui}, so that we are able to describe the flows from the unemployment pool into employment at a given wage. This also allows us to determine the equilibrium unemployment rate. In steady state we require that the flow of layoffs must exactly equal the flow out of the unemployment pool to either part-time or full-time employment. Since unemployed workers accept any wage that is at least as high as their reservation wage we have,

$$\delta_i(1-u_i)A_{ei}(\phi) = \lambda_{ui}^0 u_i \int_{-\infty}^{\phi} \overline{F}_0(q_i(w))dA_{ui}(w) + \lambda_{ui}^1 u_i \int_{-\infty}^{\phi} \overline{F}_1(w)dA_{ui}(w). \tag{3.4}$$

Notice that the wage offer distributions (F_0 and F_1) are the only quantities in the above that do not vary with observable worker type i. By differentiating equation 3.4 we obtain a relationship between the densities of employed and unemployed worker reservation wages, which when combined with equation 3.3 allows us to derive the following expression for the distribution of ϕ amongst the unemployed:

$$u_i A_{ui}(\phi) = \int_{-\infty}^{\phi} \frac{dA_i(w)}{1+\kappa_{ui}^0 \overline{F}_0(q_i(w)) + \kappa_{ui}^1 \overline{F}_1(w)}. \tag{3.5}$$

If we define $\underline{w}_i \equiv \min\{\underline{w}_1, q_i^{-1}(\underline{w}_0)\}$ and $\overline{w}_i \equiv \max\{\overline{w}_1, q_i^{-1}(\overline{w}_0)\}$, and let $\phi \to \infty$ in equation 3.5 then we obtain the following expression for the steady-state unemployment rate for workers of a given demographic type:

$$u_i = \frac{1}{1+\kappa_{ui}^0+\kappa_{ui}^1} A_i(\underline{w}_i) + \int_{\underline{w}_i}^{\overline{w}_i} \frac{dA_i(w)}{1+\kappa_{ui}^0 \overline{F}_0(q_i(w)) + \kappa_{ui}^1 \overline{F}_1(w)} + (1-A_i(\overline{w})) \tag{3.6}$$

which decomposes the unemployment rate into three parts; those individuals who accept all job offers, those who accept some and reject others, and those who reject all job offers.

Between Jobs

In what follows we need to consider the fraction of workers currently employed in part-time and full-time jobs, and denote these respective quantities as m_{0i} and m_{1i} so that $m_{0i} + m_{1i} = 1 - u_i$. We also let $G_{1i}(w)$ denote the fraction of full-time workers of type i with a wage no greater than w, and similarly define $G_{0i}(w)$. The respective density functions are given by $g_{1i} \equiv G'_{1i}$ and $g_{0i} \equiv G'_{0i}$, and we shall now determine these objects. Since individuals will only accept jobs that offer a value strictly greater than the value associated with their current wage-hours package, the number of individuals who leave a full-time job paying wage w (whether through their job being destroyed, or receiving a higher valued offer from a part-time or full-time firm) must exactly equal the number of individuals who accept such a job. Hence,

$$\begin{aligned} m_{1i}g_{1i}(w)&\left[\delta_i + \lambda^0_{ei}\overline{F}_0(q_i(w)) + \lambda^1_{ei}\overline{F}_1(w)\right] \\ &= f_1(w)\left[\lambda^1_{ui}u_iA_{ui}(w) + \lambda^1_{ei}m_{0i}G_{0i}(q_i(w)) + \lambda^1_{ei}m_{1i}G_{1i}(w)\right]. \end{aligned} \tag{3.7}$$

Similarly, we can derive an analogous expression for part-time jobs paying wage w:

$$\begin{aligned} m_{0i}g_{0i}(w)&\left[\delta_i + \lambda^0_{ei}\overline{F}_0(w) + \lambda^1_{ei}\overline{F}_1(q_i^{-1}(w))\right] \\ &= f_0(w)\left[\lambda^0_{ui}u_iA_{ui}(q_i^{-1}(w)) + \lambda^0_{ei}m_{0i}G_{0i}(w) + \lambda^0_{ei}m_{1i}G_{1i}(q_i^{-1}(w))\right]. \end{aligned} \tag{3.8}$$

Both equations 3.7 and 3.8 will also feature later when we discuss identification and present our semi-non-parametric estimation technique. Integrating equation 3.7 and equation 3.8 (the LHS of both equations are integrated by parts) we obtain the following alternative representations of these flow equations:

$$\begin{aligned} m_{1i}G_{1i}(w)&\left[\delta_i + \lambda^0_{ei}\overline{F}_0(q_i(w)) + \lambda^1_{ei}\overline{F}_1(w)\right] + \lambda^0_{ei}m_{1i}\int_{-\infty}^{w} G_{1i}(x)dF_0(q_i(x)) \\ &= \lambda^1_{ui}u_i\int_{-\infty}^{w} A_{ui}(x)dF_1(x) + \lambda^1_{ei}m_{0i}\int_{-\infty}^{w} G_{0i}(q_i(x))dF_1(x) \end{aligned} \tag{3.9}$$

and:

$$\begin{aligned} m_{0i}G_{0i}(w)&\left[\delta_i + \lambda^0_{ei}\overline{F}_0(w) + \lambda^1_{ei}\overline{F}_1(q_i^{-1}(w))\right] + \lambda^1_{ei}m_{0i}\int_{-\infty}^{w} G_{0i}(x)dF_1(q_i^{-1}(x)) \\ &= \lambda^0_{ui}u_i\int_{-\infty}^{w} A_{ui}(q_i^{-1}(x))dF_0(x) + \lambda^0_{ei}m_{1i}\int_{-\infty}^{w} G_{1i}(q_i^{-1}(x))dF_0(x). \end{aligned} \tag{3.10}$$

While expressions for G_{0i} and G_{1i} from above are both individually complicated, an appropriately weighted average term, that features prominently in our analysis, has a considerably simpler form. To derive this we add equation 3.9 (evaluated at w) and equation 3.10 (evaluated at $q_i(w)$) so that we are able to eliminate the terms where we integrate over the cross-sectional distributions of part-time and full-time wages. This delivers the following average condition:

$$\begin{aligned} (m_{1i}G_{1i}(w) + m_{0i}G_{0i}(q_i(w)))&\left[\delta_i + \lambda^0_{ei}\overline{F}_0(q_i(w)) + \lambda^1_{ei}\overline{F}_1(w)\right] \\ &= \lambda^0_{ui}u_i\int_{-\infty}^{w} A_{ui}(x)dF_0(q_i(x)) + \lambda^1_{ui}u_i\int_{-\infty}^{w} A_{ui}(w)dF_1(x). \end{aligned} \tag{3.11}$$

Using equation 3.4 we note that the RHS of equation 3.11 can be written as:

$$\delta_i(1-u_i)A_{ei}(w) - u_iA_{ui}(w)\left[\lambda^0_{ui}\overline{F}_0(q_i(w)) + \lambda^1_{ui}\overline{F}_1(w)\right]$$
$$= \delta_iA_i(w) - u_iA_{ui}(w)\left[\delta_i + \lambda^0_{ui}\overline{F}_0(q_i(w)) + \lambda^1_{ui}\overline{F}_1(w)\right] \quad (3.12)$$

so by combining equation 3.12 with equation 3.11 and dividing through by δ_i we then obtain:

$$(m_{1i}G_{1i}(w) + m_{0i}G_{0i}(q_i(w)))\left[1 + \kappa^0_{ei}\overline{F}_0(q_i(w)) + \kappa^1_{ei}\overline{F}_1(w)\right]$$
$$= A_i(w) - u_iA_{ui}(w)\left(1 + \kappa^0_{ui}\overline{F}_0(q_i(w)) + \kappa^1_{ui}\overline{F}_1(w)\right) \quad (3.13)$$

which can be substituted into equations 3.7 and 3.8 to eliminate the averaged earnings distributions and yield expressions for the earnings densities g_{0i} and g_{1i}. These may then be integrated to obtain the respective cumulative distribution functions (G_{0i} and G_{1i}) and employment shares (m_{0i} and m_{1i}).

3.3.4 Firm Behaviour

In this section we characterize the optimal behaviour of firms. In addition to choosing a wage policy, we allow firms to exercise a role as job creators. The expansion of recruiting effort (here referred to as “job vacancies”) allows firms of a given productivity level to increase their visibility in the labour market, with changes in the supply of job vacancies affecting the arrival rate of job offers at the macroeconomic level through an aggregate matching function (see Section 3.3.5).

The flow cost of a full-time productivity p firm with v job vacancies is given by $c_1(p,v)$. We assume that this function is strictly convex in v with $c_1(p,0) = 0$. The profit flow of a firm with strategy (w,v) is given by $(p-w)h_1L_1(w,v) - c(p,v)$ where $L_1(w,v) \equiv \sum_i n_il_{1i}(w,v)$ is the steady-state employment of such a firm, and where $l_{1i}(w,v)$ is the steady-state employment of a type i worker. Letting V_1 denote the aggregate stock of full-time job vacancies, $l_{1i}(w,v)$ solves the flow equation:

$$(\delta_i + \lambda^0_{ei}\overline{F}_0(q_i(w)) + \lambda^1_{ei}\overline{F}_1(w))l_{1i}(w,v) = \frac{v}{V_1}\left[\lambda^1_{ui}u_iA_{ui}(w) + \lambda^1_{ei}m_{0i}G_{0i}(q_i(w)) + \lambda^1_{ei}m_{1i}G_{1i}(w)\right]$$

which balances the number of workers who enter and exit employment, and reflects the assumption that vacancies affect the sampling probability of firms. Given that v enters the above expression for $l_{1i}(w,v)$ multiplicatively, it is convenient in what follows to write $l_{1i}(w,v) = \bar{l}_{1i}(w)v/V_1$, and similarly let $\overline{L}_1(w) \equiv \sum_i n_i\bar{l}_{1i}(w)$, with:

$$\bar{l}_{1i}(w) = \frac{\kappa^1_{ei}A_i(w) + \left[\kappa^1_{ui}(1+\kappa^0_{ei}\overline{F}_0(q_i(w))) - \kappa^1_{ei}(1+\kappa^0_{ui}\overline{F}_0(q_i(w)))\right]u_iA_{ui}(w)}{\left(1+\kappa^0_{ei}\overline{F}_0(q_i(w)) + \kappa^1_{ei}\overline{F}_1(w)\right)^2} \quad (3.14)$$

which is obtained by substituting equation 3.13 in the flow equation for $l_{1i}(w,v)$ to eliminate the cross-sectional wage distributions. Each full-time firm chooses its optimal wage policy $K_1(p)$ and optimal recruiting policy $v_1(p)$ to maximize its steady state profit flow,[11] taking the arrival rate of job offers, together with the behaviour of other firms (both part-time and full-time) and workers as given. Hence:

$$(K_1(p), v_1(p)) = \underset{(w,v)}{\arg\max}\ \pi_1(p,w)\frac{v}{V_1} - c_1(p,v)$$

where:

$$\pi_1(p,w) = (p-w)h_1\overline{L}_1(w). \tag{3.15}$$

The optimal vacancy level $v_1(p)$ equates the marginal cost of an additional vacancy to the expected profit flow from such a vacancy. That is:

$$\left.\frac{\partial c_1(p,v)}{\partial v}\right|_{v=v_1(p)} = \frac{\pi_1(p,K_1(p))}{V_1} \tag{3.16}$$

Rather than working directly with the first order conditions for the optimal choice of wages, we use the envelope theorem to write $\pi_1(p) = \pi_1(p, K_1(p))$ as follows:

$$\begin{aligned} \pi_1(p) &= \pi_1^*(\underline{p}_1) + h_1\int_{\underline{p}_1}^{p}\overline{L}_1(K_1(y))dy \\ &= (\underline{p}_1 - \underline{w}_1^*)\overline{L}_1(\underline{w}_1^*) + h_1\int_{\underline{p}_1}^{p}\overline{L}_1(K_1(y))dy \end{aligned}$$

where $\underline{w}_1^*$ maximizes equation 3.15 for the lowest productivity full-time firm ($p = \underline{p}_1$). Setting the above equal to 3.15 (evaluated at $w = K_1(p)$) we obtain:

$$K_1(p) = p - \left[\pi_1^*(\underline{p}_1) + h_1\int_{\underline{p}_1}^{p}\overline{L}_1(K_1(y))dy\right] \times \frac{1}{h_1\overline{L}_1(K_1(p))} \tag{3.17}$$

which is a form that we exploit when we numerically solve for the equilibrium of our model. Of course, directly analogous expressions holds for the wage policy function and recruiting efforts of part-time firms. Note that equation 3.17 (and the corresponding expression for part-time firms) will depend upon the entire distribution of both part-time and full-time wage offers. Once these wage policy functions have been calculated, it is also necessary to verify whether the second-order conditions of firms in both sectors hold, so that the candidate equilibrium is indeed implementable.

3.3.5 Equilibrium

In order to close the model we endogenize the arrival rate of job offers by complementing it with aggregate matching functions as in Mortensen (2000, 2003). Here, the total flow of

[11] This implicitly assumes that firms have a zero rate of time preference.

matches M_h in each sector $h \in \{0,1\}$ depends on the total stock of vacancies V_h:

$$V_0 = \int_{\underline{p}_0}^{\overline{p}_0} v_0(p)d\Gamma_0(p) \quad \text{and} \quad V_1 = \int_{\underline{p}_1}^{\overline{p}_1} v_1(p)d\Gamma_1(p) \tag{3.18}$$

(with $v_h(p)$ given in equation 3.16) and the total intensity adjusted search effort of workers S_h:

$$S_h = \sum_i n_i \left[s^h_{ui} u_i + s^h_{ei}(1-u_i) \right] \tag{3.19}$$

where s^h_{ji} denotes the exogenous search effort of workers for each i, $j \in \{u,e\}$ and $h \in \{0,1\}$. The matching function in hours sector h is then written as:

$$M_h \left(V_h, \sum_i n_i \left[s^h_{ui} u_i + s^h_{ei}(1-u_i) \right] \right) \tag{3.20}$$

where M_h is assumed to be increasing in both its arguments, concave, and linearly homogeneous. The job offer arrival rates for each worker are then related to the flows of matches according to:

$$\lambda^h_{ji} = \frac{s^h_{ji} M_h}{\sum_i n_i (s^h_{ui} u_i + s^h_{ei}(1-u_i))}. \tag{3.21}$$

The market equilibrium of the economy is now defined in the following definition:

Definition 2. *A market equilibrium in the economy is defined by* $\{F_0, F_1, v_0, v_1\}$ *such that simultaneously:*

1. *The arrival rates of job offers* $\{\lambda^0_{ui}, \lambda^0_{ei}, \lambda^1_{ui}, \lambda^1_{ei}\}_{i \in \mathcal{I}}$ *are given by equation 3.21*
2. *The distribution of wage offers in the economy is:*

$$F_0(K_0(p)) = \int_{\underline{p}_0}^{p} \frac{v_0(p)d\Gamma_0(p)}{V_0} \quad \text{and} \quad F_1(K_1(p)) = \int_{\underline{p}_1}^{p} \frac{v_1(p)d\Gamma_1(p)}{V_1}$$

   with V_0 *and* V_1 *as defined in equation 3.18.*

3. *Each worker of type* (b,i) *follows the strategy described in Table 3.2.*
4. *The strategy of each type-p firm is to choose a vacancy level and wage that maximizes profits given the job offer arrival rates, strategies of other firms' and workers':*

$$\begin{aligned} (K_1(p), v_1(p)) &= \arg\max_{(w,v)} \pi_1(p,w)\frac{v}{V_1} - c_1(p,v) \\ (K_0(p), v_0(p)) &= \arg\max_{(w,v)} \pi_0(p,w)\frac{v}{V_0} - c_0(p,v) \end{aligned}$$

   where $\pi_h(p,w)$ *is as defined in equation 3.15.*

That the wage offer distributions (see part 2 of Definition 2) are equal to a vacancy weighted distribution of firm productivity follows from the observation that more productive firms pay higher wages in equilibrium. See for Chapter 2 of this thesis, or Mortensen (2003), for a simple proof.

3.4 Estimation

In this section we discuss the structural estimation of our model using longitudinal survey data. We first derive the log-likelihood function, and then proceed to discuss the identification of our model and the three step estimation procedure that we adopt. We then discuss our application of the UK tax and transfer system and the data used in estimation. Results are presented in Section 3.4.6.

3.4.1 Likelihood Function

We now derive the likelihood contribution for individuals in different labour market positions, and with different initial transitions. The derivation closely follows that of Bontemps et al. (1999), and here we continue to use u and e to index the respective states of unemployment and employment, and 0 and 1 to denote part-time and full-time jobs. Note that we do not use any information beyond the first observed transition. In what follows elapsed and residual durations are given by:

$$\begin{aligned}
t_{ub} &= \text{elapsed unemployment duration} \\
t_{uf} &= \text{residual unemployment duration} \\
d_{ub} &= 1 \text{ if unemployment duration left-censored, otherwise } 0 \\
d_{uf} &= 1 \text{ if unemployment duration right-censored, otherwise } 0 \\
t_{eb} &= \text{elapsed employment duration} \\
t_{ef} &= \text{residual employment duration} \\
d_{eb} &= 1 \text{ if employment duration left-censored, otherwise } 0 \\
d_{ef} &= 1 \text{ if employment duration right-censored, otherwise } 0
\end{aligned}$$

while earned and accepted wages are denoted as follows:

$$\begin{aligned}
w_u &= \text{full-time wage accepted by unemployed individuals} \\
q_i(w_u) &= \text{part-time wage accepted by unemployed individuals} \\
d_u &= 1 \text{ if } w_u \text{ unobserved, otherwise } 0 \\
w_e &= \text{full-time wage of employees at date of first interview} \\
q_i(w_e) &= \text{part-time wage of employees at date of first interview} \\
d_e &= 1 \text{ if } w_e \text{ unobserved, otherwise } 0
\end{aligned}$$

current employment is indexed by:

$$\begin{aligned} h_e^0 &= 1 \text{ if employed work in the part-time sector, otherwise } 0 \\ h_e^1 &= 1 \text{ if employed work in the full-time sector, otherwise } 0 \end{aligned}$$

and initial transitions are indexed by:

$$\begin{aligned} v_u^0 &= 1 \text{ if unemployed accept a part-time job, otherwise } 0 \\ v_u^1 &= 1 \text{ if unemployed accept a full-time job, otherwise } 0 \\ v_e^0 &= 1 \text{ if employed accept a part-time job, otherwise } 0 \\ v_e^1 &= 1 \text{ if employed accept a full-time job, otherwise } 0. \end{aligned}$$

We first derive the likelihood function contribution for currently unemployed workers; following this, we derive the contribution for workers who are employed. In both cases, the initial conditions use the steady state quantities and distributions first derived in Section 3.3.

Unemployed Workers

Using the above notation, we first derive the likelihood contribution for unemployed workers of demographic type i. The exact form that this will take will depend upon whether an accepted wage is observed, and whether or not the residual and elapsed unemployment durations are subject to any censoring. If an unemployed worker is observed to exit unemployment to either a full-time job paying wage w_u, or a part-time job paying wage $q_i(w_u)$, then we have $d_u = 0$ and $d_{uf} = 0$. The respective likelihood contribution is therefore given by:

$$\begin{aligned} &(\lambda_{ui}^0 + \lambda_{ui}^1)^{2-d_{ub}} \exp\left[-\left(\lambda_{ui}^0 + \lambda_{ui}^1\right)(t_{ub} + t_{uf})\right] \frac{A_i(\underline{w}_i)}{1 + \kappa_{ui}^0 + \kappa_{ui}^1} \\ &\quad \times \frac{(\lambda_{ui}^0 f_0(q_i(w_u)))^{v_u^0} (\lambda_{ui}^1 f_1(w_u))^{v_u^1}}{\lambda_{ui}^0 + \lambda_{ui}^1} + \left\{ \int_{\underline{w}_i}^{w_u} (\lambda_{ui}^0 \overline{F}_0(q_i(b)) + \lambda_{ui}^1 \overline{F}_1(b))^{2-d_{ub}} \right. \\ &\qquad \times \exp\left[-\left(\lambda_{ui}^0 \overline{F}_0(q_i(b)) + \lambda_{ui}^1 \overline{F}_1(b)\right)(t_{ub} + t_{uf})\right] \\ &\qquad\quad \left. \times \frac{(\lambda_{ui}^0 f_0(q_i(w_u)))^{v_u^0} (\lambda_{ui}^1 f_1(w_u))^{v_u^1}}{\lambda_{ui}^0 \overline{F}_0(q_i(b)) + \lambda_{ui}^1 \overline{F}_1(b)} \frac{dA_i(b)}{1 + \kappa_{ui}^0 \overline{F}_0(q_i(b)) + \kappa_{ui}^1 \overline{F}_1(b)} \right\} \end{aligned}$$

where we have integrated over the range of possible reservations wages for unemployed workers using equation 3.5.

If instead we do not observe a wage accepted by the unemployed worker (so that $d_u = 1$), but we nonetheless have $d_{ub} + d_{uf} < 2$, then the worker can not be permanently unemployed. Hence, it still must be the case that the full-time reservation wage of such an unemployed worker is no greater than $\overline{w}_i$, and so it is only necessary to integrate over the distribution of reservation wages no greater than this amount. It therefore follows that their likelihood contribution is:

$$(\lambda^0_{ui}+\lambda^1_{ui})^{2-d_{ub}-d_{uf}} \exp\left[-(\lambda^0_{ui}+\lambda^1_{ui})(t_{ub}+t_{uf})\right] \frac{A_i(\underline{w}_i)}{1+\kappa^0_{ui}+\kappa^1_{ui}}$$
$$\times \left[\frac{(\lambda^0_{ui})^{v^0_u}(\lambda^1_{ui})^{v^1_u}}{\lambda^0_{ui}+\lambda^1_{ui}}\right]^{1-d_{uf}} + \left\{\int_{\underline{w}_i}^{\overline{w}_i} (\lambda^0_{ui}\overline{F}_0(q_i(b))+\lambda^1_{ui}\overline{F}_1(b))^{2-d_{ub}-d_{uf}}\right.$$
$$\times \exp\left[-\left(\lambda^0_{ui}\overline{F}_0(q_i(b))+\lambda^1_{ui}\overline{F}_1(b)\right)(t_{ub}+t_{uf})\right]$$
$$\left.\times \left[\frac{(\lambda^0_{ui}\overline{F}_0(q_i(b)))^{v^0_u}(\lambda^1_{ui}\overline{F}_1(b))^{v^1_u}}{\lambda^0_{u}\overline{F}_0(q_i(b))+\lambda^1_{ui}\overline{F}_1(b)}\right]^{1-d_{uf}} \frac{dA_i(b)}{1+\kappa^0_{ui}\overline{F}_0(q_i(b))+\kappa^1_{ui}\overline{F}_1(b)}\right\}.$$

Finally, if we have both $d_u = 1$ and $d_{ub}+d_{uf} = 2$, then individuals are never observed in the employment state so we must also consider the probability that such individuals have a full-time reservation wage that is greater than $\overline{w}_i$. The likelihood contribution then becomes:

$$\exp\left[-(\lambda^0_{ui}+\lambda^1_{ui})(t_{ub}+t_{uf})\right] \frac{A_i(\underline{w}_i)}{1+\kappa^0_{ui}+\kappa^1_{ui}}$$
$$+\left\{\int_{\underline{w}_i}^{\overline{w}_i} \exp\left[-\left(\lambda^0_{ui}\overline{F}_0(q_i(b))+\lambda^1_{ui}\overline{F}_1(b)\right)(t_{ub}+t_{uf})\right]\right.$$
$$\left.\times \frac{dA_i(b)}{1+\kappa^0_{ui}\overline{F}_0(q_i(b))+\kappa^1_{ui}\overline{F}_1(b)} + (1-A_i(\overline{w}_i))\right\}.$$

Employed Workers

The likelihood contribution of a type i individual working full-time (part-time) at wage w_e ($q_i(w_e)$) is similarly given by:

$$\{m_{0i}g_{0i}(q_i(w_e))\}^{h^0_e}\{m_{1i}g_{1i}(w_e)\}^{h^1_e}\left[\delta_i+\lambda^0_{ei}\overline{F}_0(q_i(w_e))+\lambda^1_{ei}\overline{F}_1(w_e)\right]^{2-d_{eb}-d_{ef}}$$
$$\times \exp\left[-\left(\delta_i+\lambda^0_{ei}\overline{F}_0(q_i(w_e))+\lambda^1_{ei}\overline{F}_1(w_e)\right)(t_{eb}+t_{ef})\right]$$
$$\times \left[\frac{\delta_i^{1-v^0_e-v^1_e}(\lambda^0_{ei}\overline{F}_0(q_i(w_e)))^{v^0_e}(\lambda^1_{ei}\overline{F}_1(w_e))^{v^1_e}}{\delta_i+\lambda^0_{ei}\overline{F}_0(q_i(w_e))+\lambda^1_{ei}\overline{F}_1(w_e)}\right]^{1-d_{ef}}.$$

Note that in the above we do not use any information on the wage accepted following a job-to-job transition. The reason for adopting such a limited information approach is that the model we have developed does not permit transitions associated with lower job values.[12] Finally, if the wage of an employed worker were missing ($d_e = 1$), then the likelihood contribution is simply given by m_{1i} if they are a full-time worker, m_{0i} if working part-time, or $1-u_i$ if hours of work is also unobserved.[13]

[12]The model can be extended to allow for job-to-job transitions associated with lower values by introducing a *reallocation shock* as in Jolivet et al. (2006). These shocks are draws from the wage offer distributions for which the only alternative to acceptance is to become unemployed. The presence of reservation wage heterogeneity would mean that some individuals may wish to exercise the unemployment option upon receiving such a shock.

[13]By construction of the data, if the hourly wage is known then so too is the hours of work.

3.4.2 Identification

The logic behind the non-parametric identification of the model is the same as was discussed in Chapter 2 of this thesis, and for completeness we repeat the main argument here.

We therefore first consider a special case of our model: we abstract from the presence of a tax system, we suppose that the distribution of opportunity costs collapses to a degenerate distribution (i.e. workers are homogeneous), and we assume that there is only a single sector in the economy. This is the model analysed in Bontemps et al. (2000). Conditional on transitional parameters, identification of the wage offer distribution follows directly from a steady state relationship between the wage offer and earnings distributions (a simpler form of equation 3.9 presented here) that permits a simple inversion. Moreover, in such a setting all job offers will be accepted by all unemployed workers so that the accepted wage distribution will coincide with the wage offer distribution. This special case of our more general model is therefore over identified.

Regardless of its source, once we allow for heterogeneity in the reservation wage of unemployed workers the distribution of accepted wages will no longer equal the wage offer distribution. This is because workers are selective in the wages that they are willing to accept, so that the distribution of accepted wages will stochastically dominate the wage offer distribution. We are still able to establish non-parametric identification in this case because we observe as many distributions (starting wages and cross-sectional earnings) as distributions that we wish to recover. If we observe further distributions, such as the distribution of wages that the employed receive in their next job, then we once again will have over identification.[14] These ideas are presented more formally in Appendix 3.B, and are closely related to the estimation procedure that we now present.

3.4.3 Three Step Estimation Procedure

We estimate our model using a three step procedure similar to that proposed by Bontemps et al. (1999, 2000) and extended in Chapter 2 of this thesis. While we no longer have a simple inversion between the observed earnings distributions and the unobserved wage offer distributions, it nonetheless remains possible to perform an inversion by iterating on the relevant flow equations. More specifically:

1. We estimate $\{\underline{w}_0, \overline{w}^0\}$ as the sample minimum and maximum values of w_e amongst part-time jobs ($h = h_0$) and $\{\underline{w}_1, \overline{w}^1\}$ as the sample minimum and maximum values of w_e amongst full-time jobs ($h = h_1$). None of these estimates condition upon worker type. We then calculate estimates of the unconditional earnings densities in each sector using non-parametric (kernel) techniques. We denote these estimated densities as $\hat{g}_0$ and $\hat{g}_1$.

2. We assume a parametric form for the distribution of opportunity costs $H_i(b)$ with a finite

[14] This is related to the approach taken by Barlevy (2008) and Barlevy and Nagaraja (2006) who using record-value theory demonstrate identification of the wage offer distribution by tracking the wage growth of workers as a function of past mobility.

parameter vector $\theta = \{\theta_i\}_{i \in \mathcal{I}}$. Since workers are assumed to sample wage offers from the same distributions F_0 and F_1 regardless of their demographic type i, we weight equations 3.7 and 3.8 by n_i and sum across types to obtain appropriately averaged equations of the form:

$$f_1(w) = \frac{\sum_i n_i m_{1i} g_{1i}(w)}{\sum_i n_i \bar{l}_{1i}(w)} \tag{3.22}$$

and:

$$f_0(w) = \frac{\sum_i n_i m_{0i} g_{0i}(w)}{\sum_i n_i \bar{l}_{0i}(w)} \tag{3.23}$$

with $\bar{l}_{1i}(w)$ (defined in equation 3.14) and the similarly defined $\bar{l}_{0i}(w)$ both depending upon the set of transitional parameters, the reservation wage distribution, and the distribution of full-time and part-time wage offers. We then replace the numerators of equations 3.22 and 3.23 by $m_1 \hat{g}_1(w)$ and $m_0 \hat{g}_0(w)$ respectively, where here we have $m_1 = \sum_i n_i m_{1i}$ and $m_0 = \sum_i n_i m_{0i}$. To recover the part-time and full-time offer distributions that induce our estimates of the unconditional empirical earnings distributions, we provide an initial guess of f_0 and f_1 and then repeatedly (and simultaneously) iterate on these two equations, exploiting the conditional linearity seen above. At each iteration step we scale the densities by a normalization factor to ensure that we have proper distribution functions, and then verify that these normalization factors converge to 1. Conditional on the set of transitional parameters and distribution of opportunity costs, we then obtain consistent estimates of the offer distributions which we denote $\hat{F}_0$ and $\hat{F}_1$. These estimates (together with the corresponding density functions, $\hat{f}_0$ and $\hat{f}_1$) are then substituted into the likelihood function, and are also used to calculate the conditional employment shares and earnings densities: $u_i(\hat{F}_0, \hat{F}_1)$, $m_{hi}(\hat{F}_0, \hat{F}_1)$, and $g_{hi}(\hat{F}_0, \hat{F}_1)$ for all $i \in \mathcal{I}$ and $h \in \{0, 1\}$. Note that we are able to assess the fit of our model by its ability to explain differences in the part-time and full-time earnings distributions across demographic types through variation in the tax schedule, transitional parameters and opportunity cost distribution (see the discussion in Section 3.4.6).

3. Given a parametric form for the matching functions M_h and the vacancy cost functions $c_h(p, v)$, we obtain the implied distribution of firm productivity and the supply of job vacancies by first rewriting the first order conditions from the firms maximization problem as:

$$p_1 = K_1^{-1}(w_1) = w_1 + \overline{L}_1(w_1)/\overline{L}_1'(w_1)$$

$$p_0 = K_0^{-1}(w_0) = w_0 + \overline{L}_0(w_0)/\overline{L}_0'(w_0)$$

and then using equations 3.16, 3.20, and 3.21, together with the two relationships $F_1(K_1(p)) = \int_{\underline{p}_1}^{p} v_1(p)/V_1 d\Gamma_1(p)$ and $F_0(K_0(p)) = \int_{\underline{p}_0}^{p} v_0(p)/V_0 d\Gamma_0(p)$ from Definition 2. If the discount rate ρ_i is assumed known, then the distribution of opportunity costs H_i can then be recovered using equation 3.2.

We construct confidence intervals by bootstrapping the entire three stage estimation procedure. The advantages of this three step procedure versus a completely parametric approach, are essentially threefold.[15] Firstly, it is considerably easier to perform this numerical inversion than it is to solve the full model at every evaluation of the likelihood function. Secondly, it permits greater flexibility than simple parametric forms for the productivity distribution. Thirdly, the estimates of the transitional parameters and wage offer distributions will be valid under a range of models. Conversely, the main disadvantage of this approach compared to a completely parametric specification, is that it does not guarantee a monotonically increasing relationship between wages and productivity (in which case the empirical distribution of wages can not be an equilibrium outcome from our model), and in general it may not be possible to constrain the structural parameters of the model to achieve such monotonicity.

3.4.4 Applying the UK Tax and Transfer System

Our empirical application seeks to accurately represent the main features of the UK tax and transfer system so that we may consider the impact of the WFTC reform.[16] We allow for both income tax and National Insurance (payroll tax), council tax and council tax benefit (a local property based taxation and the corresponding benefit received by lower income families), Income Support and income-based Job-seekers Allowance, FC/WFTC, child benefit (a cash benefit available to all families with children irrespective of income), free school meals, and additional tax allowances given to couples and families with children. The underlying tax and transfer schedules are calculated prior to estimation using FORTAX (see Appendix A.1 in this thesis), and reflect the complex interactions between the tax and transfer system, varying accurately with earnings, hours of work and demographic characteristics.[17]

To economize on the number of groups that we need to consider in our analysis (and potentially structural parameters to estimate), we make a number of further assumptions regarding the set of demographic types $\mathcal{I}$. Specifically, we do not allow taxes and transfers to vary by the age of the claimant or by the age of any children. Taxes and transfers are calculated as if the claimant were at least 25 years old, and as if any children are aged 10 years. Families with more than two children are treated as there were only two children. Since some benefits have asset tests, we also assume that no families in our sample are affected by them. All families are assigned average band C council tax.[18]

The model developed in Section 3.3 assumed the presence of a single economic decision maker. This presents difficulties for our empirical application because benefits and in-work tax

[15]See Chapter 2 of this thesis for more details on parametric estimation of wage posting models with two-sided heterogeneity, and a comparison of results obtained using parametric and semi-parametric approaches.

[16]Recent surveys of the UK tax and transfer system are provided by Adam and Browne (2009) and O'Dea et al. (2007).

[17]A potentially important benefit that we do not consider is housing benefit. While this is modelled by FORTAX, the Labour Force Survey data that we use in our empirical application (see Section 3.4.5) does not contain any information on rents. Since tax credit income results in housing benefit entitlement being withdrawn, families in receipt of housing benefit would gain less from the tax credit reform than otherwise equivalent families not in receipt of housing benefit.

[18]The Labour Force Survey data does not contain information on the council tax band that households are subject to; band C is the most common band.

credits are assessed on family income in the UK. A complete treatment of couples is beyond the scope of this paper (see Guler et al., 2009 for a characterization of the reservation wage strategy of couples with income pooling). Rather than providing a detailed characterisation of the household decision making process, we take an admittedly limited approach, by conditioning upon the current employment status and (discretized) earnings of the individuals' partner. We then subsume partner earnings in the tax-schedule, but allow this tax schedule to accurately vary with the earnings of both individuals. In our empirical application we discretize the empirical distribution of partner earnings (conditional on gender and the presence and number of children) into ten groups, including non-employment (zero earnings); actual partner earnings are then replaced with those observed at either the 10$^{\text{th}}$, 20$^{\text{th}}$, ..., or 90$^{\text{th}}$ percentile point of the relevant empirical distribution.

The above categorization requires that we consider 64 different worker types in our empirical analysis. Conditional on hours of work, the resultant tax schedules for each of these groups i as a function of the wage rate will be a piecewise linear function, with possible discontinuities. We first remove these (small) discontinuities by appropriately modifying parameters of the tax and transfer system.[19] The modified marginal tax rate schedule for fixed hours is then approximated by a differentiable function using the method proposed by MaCurdy et al. (1990). The smoothed tax schedule is then obtained by integration.[20]

3.4.5 Data

We estimate our model using a sub-sample of the UK Labour Force Survey (LFS). The LFS is a quarterly survey of around 60,000 households in Great Britain, with these households followed for five successive quarters or "waves". When individuals first enter the survey they are in wave one, so that in any given quarter, there are roughly equal proportions of individuals in each interview wave. This rolling panel structure means that there is approximately an 80% overlap in the samples for successive quarters. The short panel dimension of the LFS is of some concern, as relatively few transitions and accepted wages are observed. While alternative panel data sets, such as the British Household Panel Survey (BHPS), provide us with a much more extensive panel, the resultant sample sizes are unfortunately too small, especially if we wish to capture demographic heterogeneity in our model.

The LFS provides us with very rich information concerning the respondents labour market status. Crucially, we observe employment status and spell durations, together with hours and earnings information (in the first and fifth waves since 1997) for workers. Our pre-reform

[19] Given it is not possible for working families to receive Income Support with our choice of discrete hours (see Section 3.4.5) this essentially involves setting the minimum payment for all benefits to zero, and starting employee National Insurance payments at slightly lower earnings to remove the entry fee discontinuity.

[20] With K tax brackets, the marginal tax rate approximation at hours h and earnings wh for a type i individual is given by $MTR_i^h(wh) = \sum_{k=1}^{K}[\Phi_{i(k)}^h(wh) - \Phi_{i(k+1)}^h(wh)]\tau_{i(k)}^h(wh)$, where $\tau_{i(k)}^h$ is the marginal tax rate at the k^{th} bracket and $\Phi_{i(k)}^h$ is the normal cumulative distribution function with a mean equal to the value of the k^{th} tax bracket and with variance σ_{ki}^2. The value of σ_{ki} determines how quickly the marginal rates change in the neighbourhood of the break points, with a small value fitting the underlying step function more closely. We set $\sigma_{ki} = 20$ which produces a relatively smooth tax schedule, but our results are not sensitive to this choice.

estimation is performed using data shortly before WFTC was introduced. We follow individuals who are observed in the first quarter of 1997 until (at the latest) the first quarter of 1998. We calculate incomes and construct the likelihood function (see Section 3.4.1) as if individuals always faced the April 1997 system during this period so that the environment is stationary. While we may observe long elapsed spell durations, we nonetheless impose left censoring for durations greater than 24 months as it would be difficult to justify the assumption that they were generated from the same steady state.

We classify individuals as being employed if they have a job, and non-employed if they do not. Since we do not distinguish between the states of unemployment and non-participation, our definition of non-employment is therefore much broader than the standard ILO definition of unemployment. Amongst the employed, women who report working for less than 30 hours per week are classified as being part-time workers, while those working at least 30 hours per week classified as full-time workers. In our model, all part-time workers are treated as if they worked for 20 hours per week, whereas all full-time workers are treated as if they worked 40 hours per week. These hours points correspond well to the respective conditional averages. Empirically, very few men are observed to work part-time, so we treat all male workers as working 40 hours per week regardless of their reported hours of work.[21] In both cases, we calculate gross wages using the reported hours of work, but then proceed to calculate incomes as if they were working at the relevant discrete hours point.

Individuals who are aged below 21 or above 55 are excluded from our sample, as are individuals who are in receipt of disability related benefits, or are either self-employed or in full-time education. Given the assumption that workers are equally productive at any given firm, we additionally restrict our sample to those individuals whose highest qualification is O-level (or equivalent) or below, and assume that any higher educated individuals operate in a separate labour market. After sample selection, we have roughly 23,000 observations. Table 3.3 presents some summary statistics.

While the tax and transfer schedules potentially vary with each observable type $i \in \mathcal{I}$, we only allow the structural parameters of the model to vary with a subset of demographic types. For couples we do not allow the parameters to vary with the earnings and labour market status of their partner; for parents we do not allow them to vary with the number of their children. The distribution of work opportunity costs H_i is assumed to be Normally distributed, with mean μ_i and variance σ_i^2. This gives us 47 parameters to estimate.

3.4.6 Estimation Results and Model Fit

Given our maximum likelihood parameter estimates (Table 3.4), the implied wage policy functions $K_0(p)$ and $K_1(p)$ that are obtained from the first order conditions to the firms'

[21]The derivation of worker behaviour and the flow equations are less complicated when there is a single sector. Nonetheless, the relevant steady-state quantities and distributions can be obtained from our earlier exposition when the job arrival rates in the part-time sector approaches zero. Note also that with a single hours sector it is not possible to identify C_i^1 so we normalize it to be zero for these groups.

Table 3.3: Descriptive statistics

	Unemployed				Employed						
	$\#N_u$	$u \to h_0$	$u \to h_1$	$\#w_u$	$\#N_e^0$	$\#N_e^1$	$e \to h_0$	$e \to h_1$	$e \to u$	$\#w_e^0$	$\#w_e^1$
single men	1481	–	135	72	–	2560	–	136	112	–	1573
married men, no kids	441	–	43	32	–	1931	–	67	49	–	1391
married men, kids	888	–	85	54	–	3077	–	131	108	–	2163
single women	1031	28	37	36	372	1828	11	72	65	254	1237
lone mothers	1793	85	16	73	676	408	31	14	62	488	306
married women, no kids	579	21	16	25	578	1215	15	47	62	442	916
married women, kids	1713	100	25	78	1444	808	59	35	115	1083	608

Notes: $\#N_u$ refers to the number of unemployed observations in a given category; $\#N_e^0$ and $\#N_e^1$ respectively refer to the number of part-time and full-time employment observations. $\#w_u$ refers to the number of observed accepted wages from unemployment; $\#w_e^0$ and $\#w_e^1$ refer to the number of cross-sectional wage observations in part-time and full-time employment. $i \to j$ refers to the numbers of observed transitions from state i to state j, with states u, e, h_0 and h_1, denoting unemployment, overall employment, part-time employment, and full-time employment respectively.

Table 3.3: (continued)

	Part-time wages							Full-time wages						
	P_{10}	P_{25}	P_{50}	P_{75}	P_{90}	mean	SD	P_{10}	P_{25}	P_{50}	P_{75}	P_{90}	mean	SD
single men	–	–	–	–	–	–	–	3.43	4.16	5.39	7.03	9.28	5.94	2.55
married men, no kids	–	–	–	–	–	–	–	4.02	4.93	6.32	8.29	10.74	6.97	2.90
married men, kids	–	–	–	–	–	–	–	3.74	4.90	6.34	8.46	11.12	6.97	2.99
single women	2.72	3.23	3.85	4.90	6.43	4.27	1.69	3.34	4.04	5.29	6.98	9.26	5.84	2.49
lone mothers	2.70	3.18	3.72	4.66	6.12	4.13	1.62	3.47	4.13	5.28	6.79	8.44	5.68	2.20
married women, no kids	2.81	3.37	3.95	4.96	6.45	4.35	1.66	3.40	4.14	5.09	6.55	8.48	5.63	2.24
married women, kids	2.87	3.37	4.00	5.20	6.77	4.46	1.69	3.31	4.00	5.03	6.45	8.06	5.45	2.03

Notes: All wages are hourly and are expressed in April 1997 prices. P_{10}, P_{25}, P_{50}, P_{75}, and P_{90} respectively refer to the 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} percentiles of the cross-sectional hourly wage distribution; SD refers to the standard deviation.

Table 3.4: Maximum likelihood estimation results

	$1/\delta_i$	$1/\lambda_{ui}^0$	$1/\lambda_{ui}^1$	$1/\lambda_{ei}^0$	$1/\lambda_{ei}^1$	μ_i	σ_i	C_i^1
single men	94.5	–	19.7	–	32.6	18.7	87.8	–
	[88.4,102.3]		[15.6,24.2]		[25.9,38.6]	[-12.7,35.5]	[57.9,135.2]	
married men, no kids	195.4	–	14.5	–	23.9	49.6	66.7	–
	[176.4,217.8]		[10.8,18.7]		[19.8,29.0]	[37.2,60.3]	[53.8,83.2]	
married men, kids	177.3	–	21.1	–	19.3	37.6	48.1	–
	[163.7,190.7]		[17.5,24.8]		[15.5,23.5]	[24.3,51.4]	[31.5,65.2]	
single women	141.9	42.5	38.8	117.2	54.2	-39.8	126.3	24.0
	[128.0,157.0]	[27.5,60.5]	[25.9,56.8]	[62.6,375.8]	[43.1,68.2]	[-156.7,-13.0]	[84.1,248.6]	[13.0,33.1]
lone mothers	66.1	54.0	337.7	118.4	55.2	41.7	28.9	37.0
	[60.1,72.6]	[43.0,81.5]	[188.1,664.7]	[74.1,230.9]	[41.3,72.5]	[36.3,45.7]	[12.9,41.7]	[33.2,42.1]
married women, no kids	171.8	23.4	68.0	147.8	74.7	4.7	71.6	36.7
	[154.2,192.2]	[16.5,32.9]	[39.1,133.1]	[100.4,250.4]	[60.3,92.2]	[-10.1,17.8]	[60.3,85.6]	[25.5,48.0]
married women, kids	99.4	29.2	280.5	37.8	115.6	36.8	36.8	27.7
	[92.1,106.4]	[23.4,35.9]	[174.9,416.5]	[31.0,46.0]	[93.6,135.9]	[33.1,39.9]	[31.9,41.5]	[23.6,34.2]

Notes: All durations are monthly. Incomes are measured in pounds per week in April 1997 prices. The distribution of work opportunity costs H_i is assumed to be Normal, with mean μ_i and variance σ_i^2. The 5^{th} and 95^{th} percentiles of the bootstrap distribution of parameter estimates are presented in brackets, and are calculated using 500 replications.

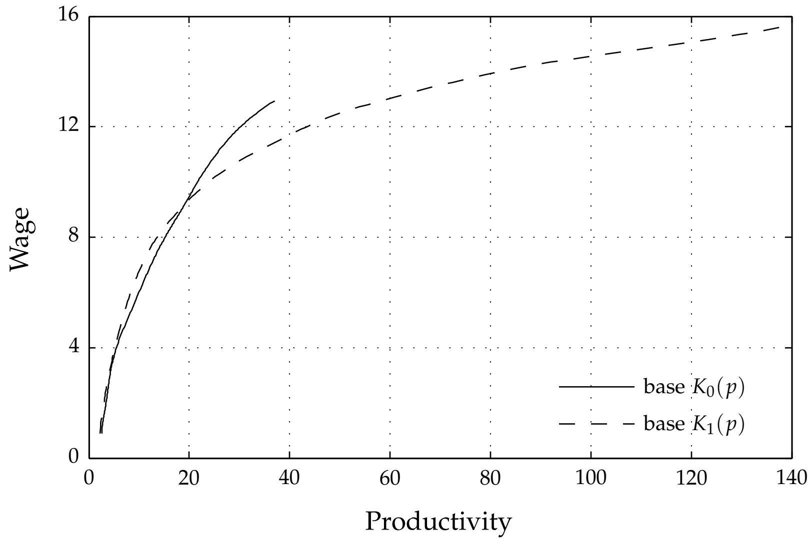


Figure 3.2: Wage policy function (pre-reform). Figure shows how optimal wage policy $K_h(p)$ varies with firm productivity and hours sector under April 1997 tax and transfer system. Figure is truncated at firm productivity greater than $p = 140 \approx K_1^{-1}(G_1^{-1}(0.99))$.

profit maximization problem are found to be monotonically increasing so that the estimated empirical distribution of wages can be an equilibrium outcome from our model. That is, the theoretical model is not rejected by the data.[22] These wage policy functions are presented in Figure 3.2. The first notable feature that is evident in this figure is that the wage policy function for full-time firms becomes very flat as the productivity of firms increases. This implies that high productivity firms have a very high degree of monopsony power. Second, the extent of monopsony power is much lower for part-time firms at high wages. When wages are high, the dis-utility of work C_i^1 becomes small relative to earnings so that part-time firms must offer much higher wages if they are to attract workers from full-time jobs. This additional layer of competition is clearly important for the optimal wage policy of firms and will become apparent in our simulation exercises. The underlying distribution of firm productivity is shown in Figure 3.3, while the wage offer distributions are shown in Figure 3.4; the latter figure shows that there is a larger concentration of part-time firms offering relatively low wages, while the distribution amongst full-time firms is more dispersed and with a longer tail.

The structural parameter estimates suggest that there is considerable heterogeneity across the different demographic groups. The job destruction rate is highest for lone mothers ($\hat{\delta}_i = 0.015$) with this estimate implying that jobs are exogenously destroyed on average every 66 months ($= 1/0.015$). The destruction rates are lowest for married men and married women without dependent children, where they are estimated to be around three times as small. The

[22]Monotonicity is violated for a small proportion of the bootstrap samples. In order to construct bootstrap confidence intervals for the policy responses we therefore first apply a rearrangement procedure (see Chernozhukov et al., 2007). These violations are not a large concern as they typically occur for very high productivity full-time firms where the productivity density is very low.

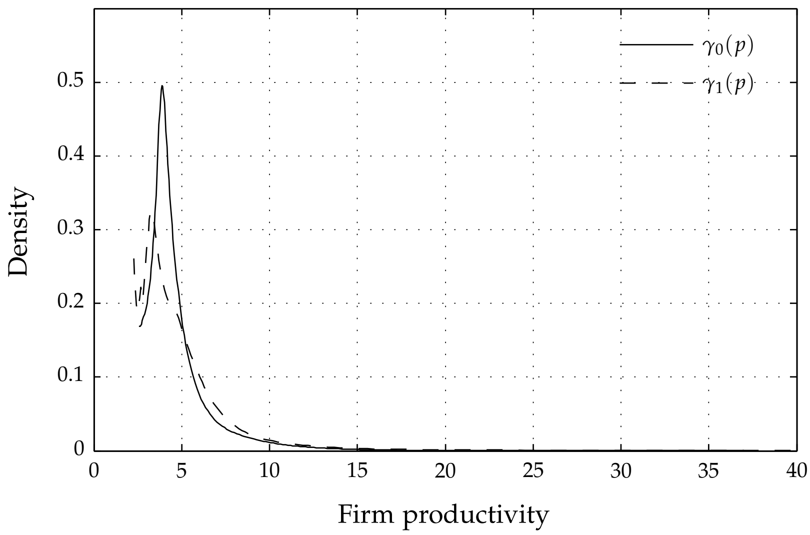


Figure 3.3: Productivity distribution. Distribution of firm productivity is obtained from pre-reform estimation by setting $v_h(p) = 1$ for all (h, p). Figure is truncated at firm productivity greater than $p = 40 \approx K_1^{-1}(G_1^{-1}(0.95))$.

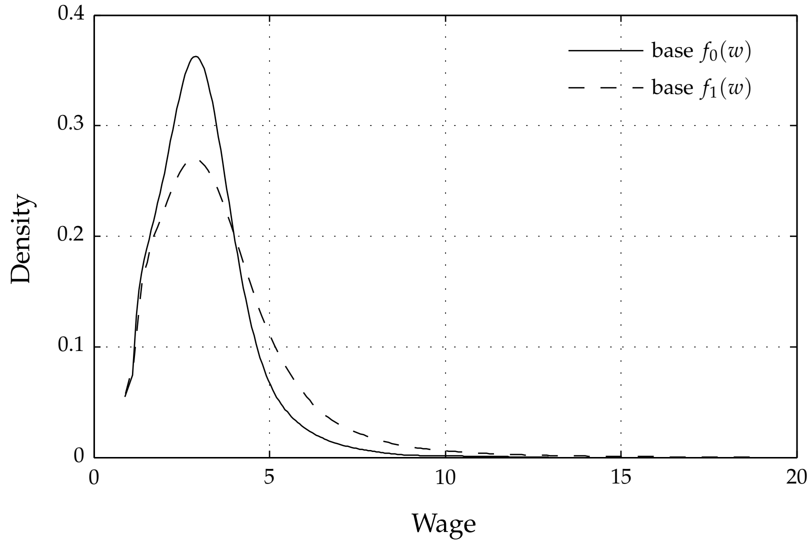


Figure 3.4: Wage offer distribution (pre-reform). Figure shows distribution of wage offers for part-time and full-time firms under April 1997 tax and transfer system.

arrival rates of job offers also varies considerably across the different demographic groups. Job offers arrive most frequently for men: for unemployed married men without children we obtain $\hat{\lambda}^1_{ui} = 0.069$ which implies that offers arrive on average every 14 months ($= 1/0.069$); the arrival rates for unemployed single men and unemployed married men without children are estimated to be slightly lower (0.051 and 0.047 respectively). Of course, the presence of reservation wage heterogeneity means that not all of these job offers will be acceptable to all workers. The estimated total job offer arrival rates $\hat{\lambda}^0_{ui} + \hat{\lambda}^1_{ui}$ for unemployed childless women is similar to the values of $\hat{\lambda}^1_{ui}$ for men. However, for lone mothers this total rate is estimated to be around three times as small; for married women with children it is around one and a half times as small. Note that the arrival rate of full-time offers for unemployed mothers is especially low with $\hat{\lambda}^1_{ui} \approx 0.003$. While our model could potentially explain the proportion of mothers working part-time by a high value of C^1_i (discussed below), this would also require that the accepted wages of full-time working mothers be much higher than those accepted in part-time jobs. We do not observe this in our data.

For a number of groups, the estimated job offer arrival rate when employed ($\hat{\lambda}^h_{ei}$) is similar to that then unemployed ($\hat{\lambda}^h_{ui}$) and in some cases we can not formally reject the null hypothesis that they are the same. While this is similar to the finding of van den Berg and Ridder (1998), it contrasts with Bontemps et al. (2000) which found (using French Labour Force Survey data) that job offers typically arrive ten times as frequently for the unemployed compared to the employed. In our estimation we find that $\hat{\lambda}^1_{ui}$ is around 1.6 times higher than $\hat{\lambda}^1_{ei}$ for childless men, but we can not reject the hypothesis that $\lambda^1_{ui} = \lambda^1_{ei}$ for married men with children. Amongst women, we estimate that λ^1_{ui} is around 1.4 times larger than λ^1_{ei} for single women, very similar for married women without children (no significant difference), but $\hat{\lambda}^1_{ei}$ is much larger than $\hat{\lambda}^1_{ui}$ for both lone mothers (six times larger) and married women with children (more than twice as large). For all groups of women we estimate $\hat{\lambda}^0_{ei} < \hat{\lambda}^0_{ui}$, but we can not reject the hypothesis that they are equal for married mothers.

The monetary dis-utility of full-time work C^1_i is estimated to be equal to around £24 per week for single women, and is somewhat higher for lone mothers and married women (up to around £37 per week), but none of the differences across groups are especially large. We obtain considerable dispersion in the unobserved leisure flow for all groups, and this translates into dispersion in reservation wages. The distribution of (full-time) reservation wages is shown in Table 3.5. The table shows the proportion of workers of each demographic type whose reservation wage is below given percentiles of the (full-time) wage offer distribution and reflects uncertainty in all distributions and structural parameters. For all worker types i we obtain $\hat{A}_i(\underline{\hat{w}}_1) < 1$, so that unemployed workers are indeed selective in the wage offers that they are willing to accept. This feature also implies a negative duration dependence in the exit rate out of unemployment. Furthermore, the value of $\hat{A}_i(\hat{\overline{w}}_1)$ is very close to one for all groups so that essentially all individuals would be willing to accept the highest full-time

Table 3.5: Reservation wage distribution

	Percentile of full-time offer distribution $\hat{F}_1(w)$					
	0	20	40	60	80	100
single men	0.243	0.404	0.523	0.644	0.788	1.000
	[0.086,0.441]	[0.183,0.572]	[0.323,0.654]	[0.496,0.735]	[0.711,0.856]	[1.000,1.000]
married men, no kids	0.145	0.340	0.503	0.651	0.811	1.000
	[0.077,0.237]	[0.200,0.443]	[0.350,0.597]	[0.515,0.723]	[0.713,0.857]	[1.000,1.000]
married men, kids	0.465	0.621	0.702	0.768	0.847	1.000
	[0.398,0.540]	[0.563,0.672]	[0.656,0.750]	[0.729,0.813]	[0.817,0.892]	[1.000,1.000]
single women	0.438	0.571	0.651	0.723	0.806	1.000
	[0.175,0.660]	[0.318,0.720]	[0.464,0.755]	[0.603,0.790]	[0.741,0.859]	[0.995,1.000]
lone mothers	0.232	0.473	0.584	0.659	0.754	1.000
	[0.063,0.413]	[0.245,0.632]	[0.427,0.734]	[0.529,0.830]	[0.655,0.932]	[1.000,1.000]
married women, no kids	0.183	0.479	0.668	0.784	0.882	1.000
	[0.086,0.331]	[0.265,0.627]	[0.472,0.764]	[0.657,0.844]	[0.813,0.919]	[1.000,1.000]
married women, kids	0.270	0.548	0.699	0.793	0.870	1.000
	[0.193,0.393]	[0.380,0.658]	[0.548,0.776]	[0.689,0.844]	[0.812,0.901]	[1.000,1.000]

Notes: Table shows the fraction of individuals whose full-time reservation wage is below various percentiles p of the full-time wage offer distribution, $A_i(\hat{F}_1^{-1}(p))$, and is calculated using the maximum likelihood estimates from Table 3.4. The 5^{th} and 95^{th} percentiles of the bootstrap distribution are presented in brackets, and are calculated using 500 replications.

wage offer.

Since the wage offer distributions are common to workers of all types i, any difference in employment states and earnings distributions must be explained by variation in the transitional parameters, opportunity cost distribution, and the tax and transfer system. Overall, we obtain a good fit to the data. The difference in the empirical and predicted states for the main demographic groups is small and never exceeds more than around 2 percentage points (see Table 3.6). Similarly, we do well in replicating the observed distribution of wages (see Figure 3.5); for most groups the fit is very good, but the model does appear to have some difficulty fitting the full-time earnings distribution for married women with children (Figure 3.5(g)). Finally, we note that the fit is less satisfactory if we compare the empirical and predicted employment states of individuals in couples conditional on the earnings of their partner. More specifically, our model tends to under-predict the non-employment rates for individuals with a non-working partner. In other words, our model is not able to fully explain employment patterns within couples of a given demographic type solely by variation in the tax and transfer system.

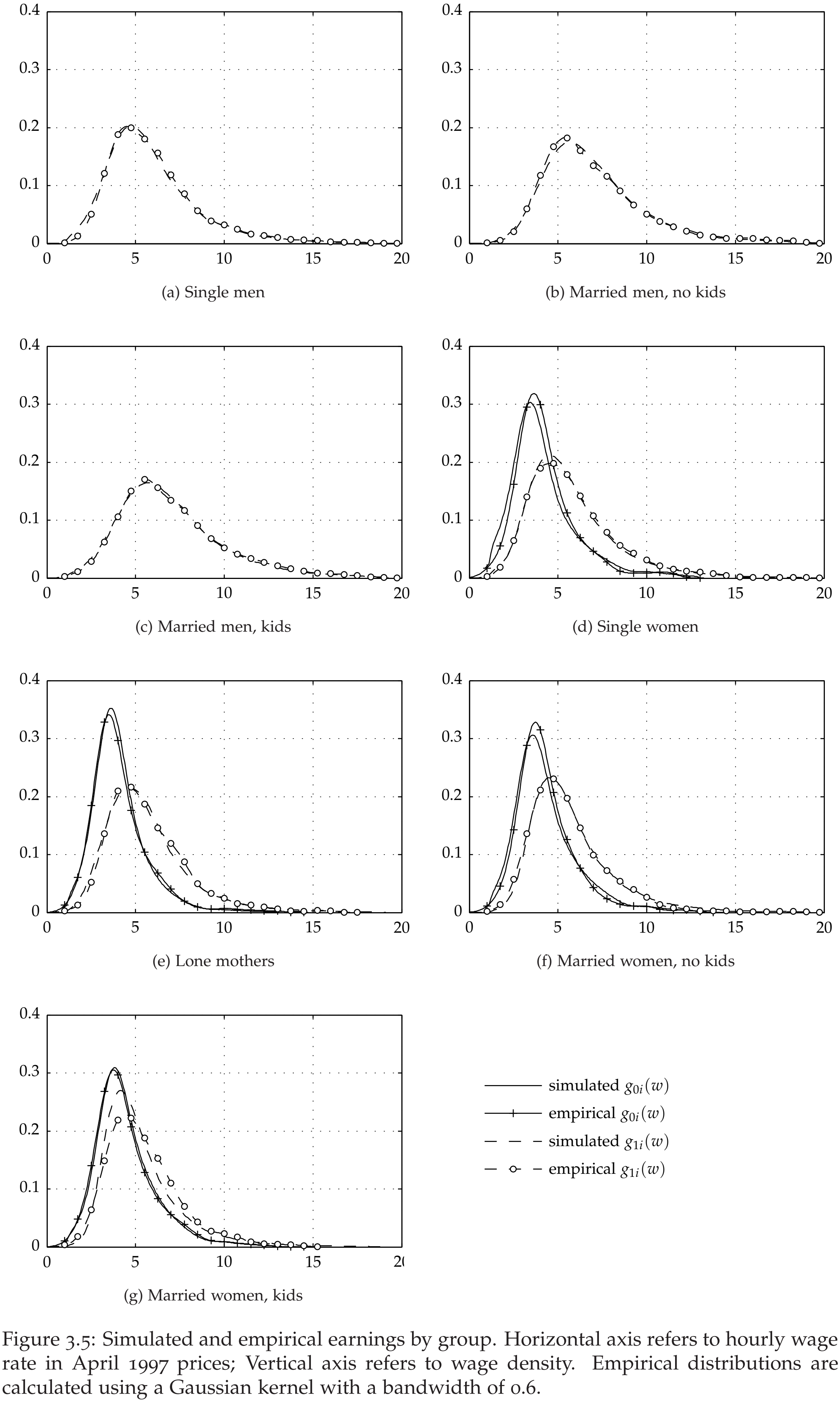
3.5 Simulating WFTC and Contemporaneous Reforms

In this section we simulate the real impact of all changes to the tax and transfer system between April 1997 (the system operating in our pre-reform sample period) and April 2002. As discussed in Section 3.2, this captures the introduction of WFTC together with other changes to the tax and transfer system, including increases in the generosity of out-of-work support for families with children. We do this by using the estimated structural parameters of our model and examining how the equilibrium changes when we impose a different tax and transfer system.

Table 3.6: Empirical and predicted employment states

	Empirical			Predicted		
	u_i	m_{0i}	m_{1i}	u_i	m_{0i}	m_{1i}
single men	0.366	–	0.634	0.346	–	0.654
	[0.354,0.379]		[0.621,0.646]	[0.334,0.356]		[0.644,0.666]
married men, no kids	0.186	–	0.814	0.178	–	0.822
	[0.175,0.200]		[0.800,0.825]	[0.167,0.191]		[0.809,0.833]
married men, kids	0.224	–	0.776	0.211	–	0.789
	[0.214,0.236]		[0.764,0.786]	[0.202,0.220]		[0.780,0.798]
single women	0.319	0.115	0.566	0.307	0.119	0.574
	[0.305,0.332]	[0.106,0.125]	[0.552,0.580]	[0.294,0.320]	[0.111,0.128]	[0.561,0.587]
lone mothers	0.623	0.235	0.142	0.610	0.244	0.146
	[0.609,0.638]	[0.222,0.248]	[0.131,0.153]	[0.597,0.623]	[0.232,0.256]	[0.135,0.158]
married women, no kids	0.244	0.244	0.512	0.241	0.254	0.505
	[0.230,0.259]	[0.228,0.258]	[0.495,0.530]	[0.227,0.255]	[0.239,0.268]	[0.488,0.523]
married women, kids	0.432	0.364	0.204	0.419	0.370	0.211
	[0.419,0.445]	[0.351,0.377]	[0.193,0.214]	[0.408,0.432]	[0.358,0.383]	[0.199,0.221]

Notes: Predicted states are calculated using the maximum likelihood estimates from Table 3.4. Employment states may not sum to one due to rounding. The 5^{th} and 95^{th} percentiles of the bootstrap distribution of employment states are presented in brackets, and are calculated using 500 replications.



(a) Single men

(b) Married men, no kids

(c) Married men, kids

(d) Single women

(e) Lone mothers

(f) Married women, no kids

(g) Married women, kids

Figure 3.5: Simulated and empirical earnings by group. Horizontal axis refers to hourly wage rate in April 1997 prices; Vertical axis refers to wage density. Empirical distributions are calculated using a Gaussian kernel with a bandwidth of 0.6.

In the simulations we present in this section, we assume a vacancy cost function for each sector $h \in \{0,1\}$ of the form $c_h(v,p)$=$c_h(p)v^2/2$, together with a Cobb-Douglas matching function $M_h(V_h, S_h) = V_h^{\theta_h} S_h^{1-\theta_h}$, where $S_h \equiv \sum n_i(s^h_{ui}u_i + s^h_{ei}(1-u_i))$ is the total search intensity in sector h. Our main results assume $\theta_0 = \theta_1 = 1/2$ but we do discuss sensitivity with respect to this parameter. Before we proceed we note that with a fixed distribution of firms' productivity and a vacancy cost function that is quadratic in v, our simulation exercises are invariant to the parametrization of $c_h(p)$ provided that $c_h(p) > 0$.[23] Without loss of generality, we therefore assume that $v_h(p) = 1$ for all p and $h \in \{0,1\}$ in the pre-reform period and recover the values of $c_h(p)$ that are consistent with this being an equilibrium. This also implies that $\Gamma_h(p) = F_h(K_h(p))$ under the base system.

To highlight the relative importance that this set of reforms has on job acceptance behaviour and the behaviour of firms, we present our results in two stages. Firstly, we consider the impact of the reform holding the distribution of job offers and their arrival rate constant; secondly we additionally allow firms to respond optimally by changing their wage policy and recruiting effort. We refer to the first channel as the *direct impact* of the reform, and the second channel as the *equilibrium impact* of the reform.

3.5.1 Direct Impact

We present both the direct and equilibrium impact of the reform on employment states in Table 3.7, and we first discuss the direct effect. The table shows that the (non- tax credit) reforms had a small positive effect (around 1 percentage point) on the employment of both singles and couples without children. This increase is mainly due to small reductions in the real value of IS/income-based JSA for families without children, together with reductions in income-tax (the introduction of a new lower starting rate, and a penny reduction is the basic rate – see Section 3.2) which act to raise the value of holding low wage jobs and so lower reservation wages. Since these changes are only small, there is little impact on durations.

Perhaps unsurprisingly, the largest predicted impact of these reforms is on the employment rate of lone mothers, where we predict an increase of 5.6 percentage points. Despite both full-time and part-time reservation wages falling,[24] this employment increase is almost entirely due to a movement into full-time work. This is partly because the lower withdrawal rate of WFTC compared to FC results in full-time incomes increasing by more than part-time incomes over a large range of wages for this group.[25] As such the indifference condition

[23]This is because the marginal cost of a job-vacancy becomes linear in $c_h(p)v$. If we assume an alternative $c_h(p)$ we will identify a new $v_h(p)$ such that $c_h(p)vV_h$ is unchanged (see equation 3.16), and similarly identify a new value of $\gamma_h(p)$ such that distribution of wage offers is preserved, and the search intensities s^h_{ji} such that the arrival rate of offers is maintained. The equilibrium effect of tax reforms is invariant to the choice of $c_h(p)$ as any effect of the reform on $v_h(p)$ is also scaled by $\gamma_h(p)$.

[24]Any reform that lowers reservation wages necessarily relies on parametric identification, as the distribution of reservation wages is only non-parametrically identified on the support of pre-reform wages.

[25]At very low wages the reforms makes part-time work relatively more desirable as tax credit income is counted as income when determining eligibility to other benefits. Consequently, at these low wages other benefits may be withdrawn at full-time hours following the reform, but not with the lower earnings associated with part-time hours. At moderate wages (discussed above) the reduction in the taper rate dominates so that full-time work becomes more desirable. At high wages (where individuals become eligible for tax credits at full-time hours following the reform),

$q_i(w)$ changes so that more part-time workers accept full-time jobs through on-the-job search, while the acceptance of part-time jobs by full-time workers also declines. Note also that the much higher arrival rate of full-time offers relative to part-time offers amongst the employed ($\lambda^1_{ei} \gg \lambda^0_{ei}$) is important for the quantitative impact.

In Figure 3.6a we show the impact of the reforms on the (monthly) unemployment exit rate: $d_{ui}(b) = \lambda^0_{ui}\overline{F}_0(q_i(\phi_i(b))) + \lambda^1_{ui}\overline{F}_1(\phi_i(b))$; the figure also shows the distribution of b in the stock of the unemployed under the base system. The figure illustrates that the reforms increase the exit rate by a considerable amount relative to the pre-reform level, with corresponding large reductions in unemployment durations.[26] Similarly, Figure 3.6b shows the impact on the separation rates of employed lone mothers in part-time and full-time jobs (respectively, $d^0_{ei}(w) = \delta_i + \lambda^0_{ei}\overline{F}_0(w) + \lambda^1_{ei}\overline{F}_1(q_i^{-1}(w))$ and $d^1_{ei}(w) = \delta_i + \lambda^0_{ei}\overline{F}_0(q_i(w)) + \lambda^1_{ei}\overline{F}_1(w)$).[27] Apart from at high wage rates, the separation rate in part-time jobs increases, while it either falls or remains effectively unchanged in full-time jobs over the entire support of wages. Due to the increased flows to the full-time sector, the average duration of a part-time job falls by around 5 months, while there is also a half-month reduction for full-time jobs. This latter reduction reflects a compositional change following the inflow of workers into low paying jobs where the job separation rate is higher.

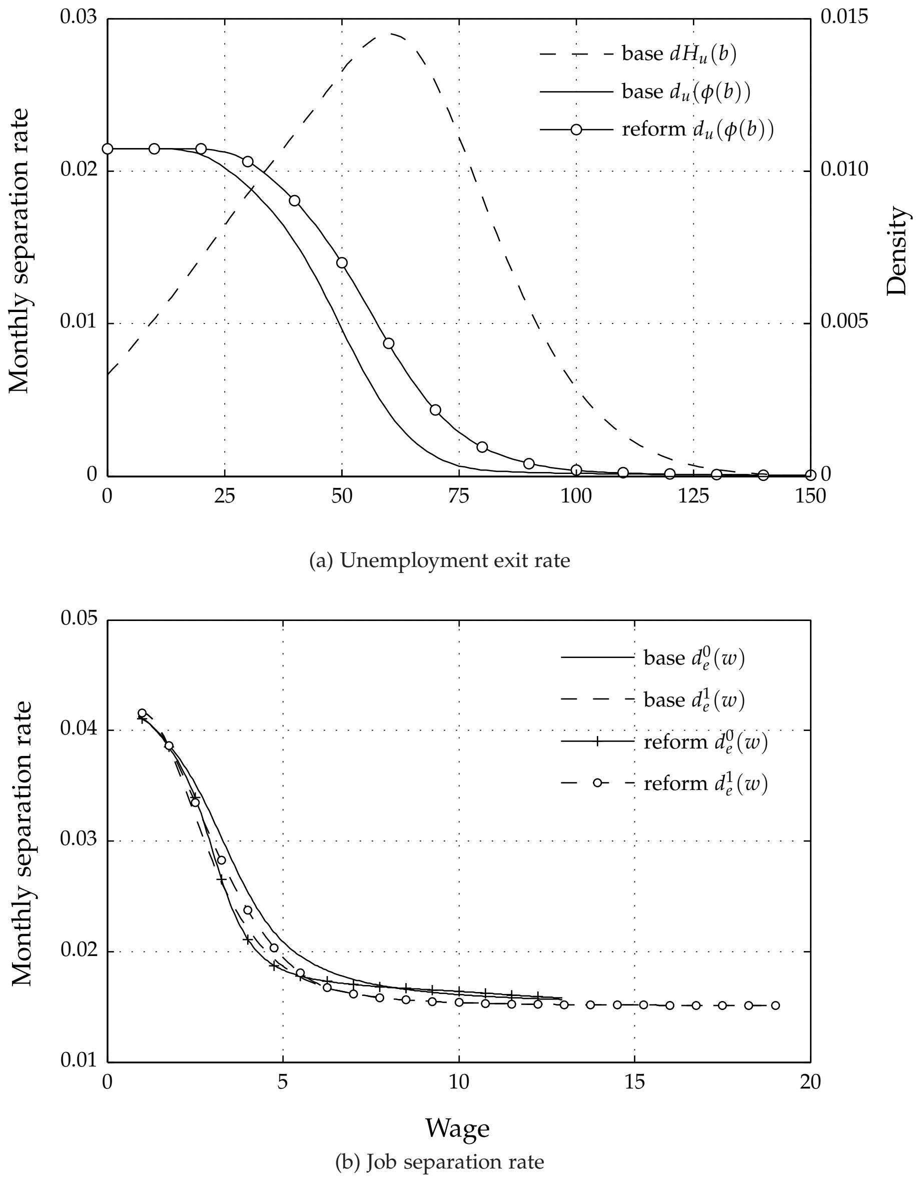
For couples with children the impact of these reforms is more complicated: individuals in couples with a high earning partner are effectively unaffected by the reform as entitlement for tax credits depends upon family income; those with a non-working or very low earning partner partner respond positively with increases in their unemployment exit rate, much like lone mothers; in intermediate cases, movement into work can taper away tax credit awards which may induce negative labour supply responses (particularly among the newly eligible families where there are large relative reductions in the unemployment exit rate). On balance, these factors lead to a small decrease in the labour supply of married women with children (a 1.3 percentage point decrease), but increase the employment rate of married men with children by a little under 3 percentage points. Among married women, the decrease in labour supply comes primarily through a reduction in those working part-time. Since men have no choice of part-time hours, there is no change in the job separation function for married fathers. For married mothers, while there are only small changes in these functions (and job durations) on average, there are much more pronounced changes once we condition on partner earnings.

Note that the simulations performed for couples hold constant the distribution of partner earnings. To understand the importance of this, we use our structural parameter estimates to perform dynamic simulations whereby we allow both individuals in a couple to sample wage

part-time incomes increase by more than full-time incomes. At very high wages individuals are not-affected by the tax credit reform, so there are only small changes in the indifference condition due to the other smaller changes to the tax and transfer system.

[26] The expected unemployment duration conditional on b is given by the $1/d_{ui}(b)$.

[27] The distribution of lone mothers' earnings is shown in Figure 3.5e earlier.



(a) Unemployment exit rate

(b) Job separation rate

Figure 3.6: Lone mother separation rates. Figure shows monthly separation rates for lone mothers under the April 1997 (base) and April 2002 (reform) tax and transfer systems; reform simulations refer to the direct impact only. Panel (a) shows the exit rates from unemployment and is truncated at weekly leisure flows less (greater) than 0 (150); panel (b) shows the job separation rate from part-time and full-time jobs. See text for definitions of d_u, d_e^0 and d_e^1.

offers, but otherwise maintain the same individualistic behaviour described in Section 3.3.[28] That is, we allow each individual to receive wage offers, but then sequentially condition on the current wage and employment state of their partner (subsumed in the tax schedule as before) when job acceptance decisions are made. In contrast to the optimal joint search behaviour analysed in Guler et al. (2009), no voluntary quits are permitted. From this simulation exercise we obtain direct employment impacts which are essentially the same as those presented in Table 3.7 and discussed above. Given this finding, the remainder of our analysis will continue to present results which condition on partner earnings in the base system.[29]

Before we discuss the equilibrium effect of the reforms, we briefly discuss the impact on wages. Note that selection effects alone imply that earnings will change even though the distribution of wage offers is held fixed. This highlights the fact that attempting to estimate the incidence of earned income tax credit programmes by comparing changes in observed wages amongst eligible and non-eligible groups is potentially misleading without carefully controlling for these selection effects. Indeed, selection alone implies some large reductions in full-time average wages. Our simulations imply that lone mothers experience a 7% reduction in average full-time wages and a 1.5% increase in part-time wages, while married men with children see their earnings fall by 1%; the changes for other groups is negligible. Since it is only lone mothers who experience sizeable changes in their earnings, the unconditional (across worker type *i*) distributions change very little due to these selection effects.

3.5.2 Equilibrium Impact

In Table 3.7 we also present the equilibrium impact of the reform. The first immediate thing to note is that the impacts are extremely similar to those obtained from the direct impact. That is, equilibrium considerations do not appear to be very important for *this particular set of reforms*. Looking more closely we can see that equilibrium considerations tend to increase employment in full-time jobs, and decrease employment in part-time jobs. To understand these subtle changes to employment it is useful to consider how the optimal strategy of firms' has changed. Figure 3.7 shows how $v_h(p)$ changes following the reform. Full-time firms are predicted to increase their recruiting effort over much of the distribution, with the increase most pronounced in the middle; only among very high (where the density of firms is particularly low – see Figure 3.3 earlier) and low productivity firms is a decrease predicted. In contrast, the largest increases among part-time firms is for those with the lowest productivity, while the upper half of the distribution tend to decrease recruiting effort. Overall these changes imply that V_0 is effectively unchanged, whereas V_1 increases by around three percent. Given these changes, together with the changes in the total search intensities S_0 and S_1, the flow of part-time matches M_0 is reduced by a negligible amount, while the flow of full-time matches

[28]There is a slight inconsistency here as our sample selection was performed at the level of the individual and not of the family.

[29]Our dynamic simulations were conducted using a population of 100,000 families of each type $i \in \mathcal{I}$, with behaviour simulated over a period of 1,000 years with a monthly time unit.

Table 3.7: Employment impact of reforms

	Direct Impact			Equilibrium Impact		
	Δu_i	Δm_{0i}	Δm_{1i}	Δu_i	Δm_{0i}	Δm_{1i}
single men	-0.010	–	0.010	-0.012	–	0.012
	[-0.013,-0.007]		[0.007,0.013]	[-0.015,-0.009]		[0.009,0.015]
married men, no kids	-0.008	–	0.008	-0.009	–	0.009
	[-0.009,-0.007]		[0.007,0.009]	[-0.010,-0.008]		[0.008,0.010]
married men, kids	-0.029	–	0.029	-0.030	–	0.030
	[-0.042,-0.021]		[0.021,0.042]	[-0.043,-0.022]		[0.022,0.043]
single women	-0.008	-0.000	0.008	-0.008	-0.003	0.011
	[-0.010,-0.004]	[-0.001,0.001]	[0.005,0.011]	[-0.011,-0.005]	[-0.005,-0.002]	[0.008,0.014]
lone mothers	-0.056	-0.005	0.061	-0.053	-0.011	0.064
	[-0.068,-0.043]	[-0.016,0.002]	[0.049,0.075]	[-0.066,-0.041]	[-0.022,-0.002]	[0.052,0.079]
married women, no kids	-0.009	-0.002	0.012	-0.009	-0.006	0.015
	[-0.011,-0.008]	[-0.005,0.001]	[0.009,0.014]	[-0.010,-0.008]	[-0.011,-0.002]	[0.012,0.020]
married women, kids	0.013	-0.012	-0.001	0.015	-0.015	-0.000
	[0.009,0.016]	[-0.014,-0.009]	[-0.003,0.001]	[0.010,0.020]	[-0.019,-0.011]	[-0.002,0.003]

Notes: All employment responses are expressed in percentage points. Changes may not sum to zero due to rounding. The direct impact considers all changes to the tax and transfer system between April 1997 and April 2002, holding the wage offer distributions and arrival rates at their pre-reform levels. The equilibrium impact allows the wage offer distribution and arrival rates to change.

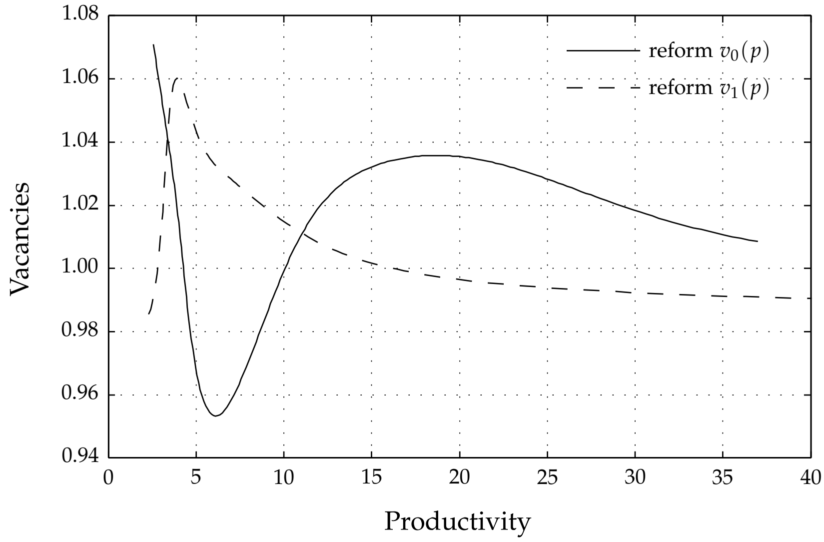


Figure 3.7: Change in recruiting policy function. Figure shows the level of vacancies $v_h(p)$ under with April 2002 tax and transfer system. In pre-reform period we set $v_h(p) = 1$ for all (h, p), so values greater (less) than one correspond to increases (decreases) in recruiting effort. Figure is truncated at firm productivity greater than $p = 40 \approx K_1^{-1}(G_1^{-1}(0.95))$ under base system.

M_1 increases by just 1.6 percent. This increase in full-time matches will benefit all workers, regardless of whether or not they were directly affected by the introduction of WFTC. These additional changes only have a very small impact on the separation rates for all types of workers.

The effect that the reforms have on the optimal wage policy of firms is difficult to predict *a priori* due to the changing competition both within and between sectors. Fixing the optimal strategies of part-time firms at their pre-reform levels, the direct effect on full-time firms acts to increase employment $L_1(w)$ across the distribution of firm productivity. Despite this, changes in the indifference condition $q_i(w)$ mean that some relatively low productivity full-time firms react by increasing the wages that they offer in order to attract workers from low-wage part-time firms, while higher productivity firms decrease wages. Meanwhile, despite the negligible reductions in overall part-time employment, the direct effect of the reforms still increases $L_0(w)$ for a number of firms and these firms respond to this by lowering their wage offers. On balance these changes mean that the equilibrium effects increase full-time employment further, whilst decreasing part-time employment. The full equilibrium effect on the distribution of wage offers is shown in Figure 3.8. The figure shows that there is a small, but noticeable, general shift in the distribution of part-time wage offers towards lower wages. In the full-time sector most changes can be seen to occur in the lower half of the distribution, with a greater proportion of firms posting wages that are close to the median wage offer. The overall effect of the reform on the distribution of part-time and full-time earnings is shown in Figure 3.9. While the direct impact of the reforms on the overall distribution of earnings is relatively minor given the small change in part-time employment, equilibrium effects do

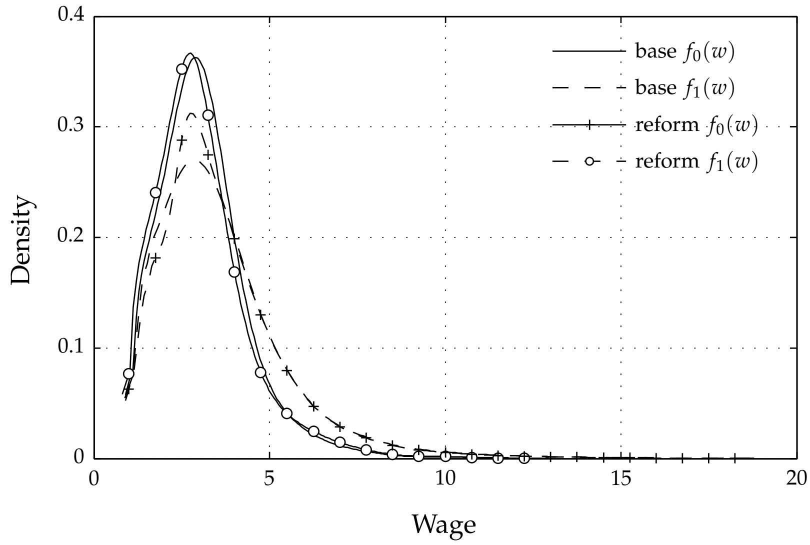


Figure 3.8: Change in wage offer distribution. Figure shows distribution of wage offers for part-time and full-time firms under the April 1997 (base) and April 2002 (reform) tax and transfer systems.

appear to have a noticeable impact on the shape of the part-time earnings distribution as the figure illustrates. To understand these changes further we note that the reason why the changes in the distribution of full-time offers does not have a larger impact on the full-time earnings distribution is because we estimate somewhat higher job offer arrival rates for full-time jobs amongst the employed. This means that workers gravitate to the higher paying full-time jobs much more quickly than in the part-time sector.

We now comment upon the sensitivity of our results to our calibration of the matching functions. Given that the equilibrium effect of the tax reforms is dominated by the direct labour supply effect, it is perhaps unsurprising that our results are not especially sensitive to the choice of θ_h. Higher values of θ_h makes the flow of matches more sensitive to changes in $v_h(p)$ which then acts to increase slightly employment in full-time firms, decrease employment in part-time firms, and raise employment overall. Similarly, lower values of θ_h have the opposite effect. The quantitative importance of these changes is relatively modest: for example, relative to our baseline results with $\theta_0 = \theta_1 = 0.5$, increasing θ_h to 0.9 for $h \in \{0, 1\}$ only increases the employment rate of lone-mothers by a further 0.4 percentage points, while decreasing θ_h to 0.1 just decreases it by a further 0.1 percentage points; other groups experience similar changes.

3.5.3 Aggregation

The very selected nature of sample (see Section 3.4.5) implies that the labour market responses presented in Table 3.7 can not be applied to the whole population. The education selection criteria in particular, has the largest impact on the representativeness of the sample, although the impact this has on lone mothers is much smaller relative to other demographic groups. Using survey frequency weights, and assuming that the employment rates of individuals excluded

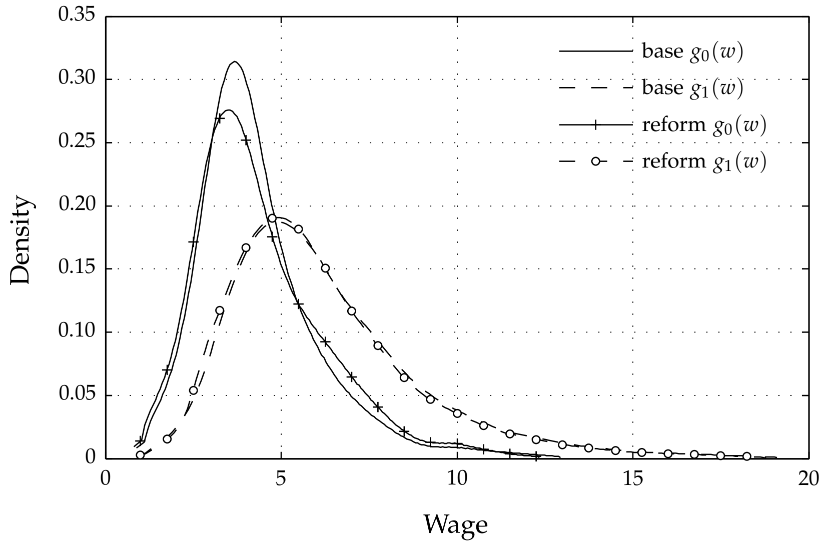


Figure 3.9: Change in earnings distribution. Figure shows distribution of earnings for part-time and full-time workers under the April 1997 (base) and April 2002 (reform) tax and transfer systems.

Table 3.8: Empirical and predicted employment changes

	Empirical			Predicted		
	Δu_i	Δm_{0i}	Δm_{1i}	Δu_i	Δm_{0i}	Δm_{1i}
single men	-0.030	–	0.030	-0.012	–	0.012
married men, no kids	-0.021	–	0.021	-0.009	–	0.009
married men, kids	-0.021	–	0.021	-0.030	–	0.030
single women	0.001	0.003	-0.003	-0.008	-0.003	0.011
lone mothers	-0.052	0.027	0.024	-0.053	-0.011	0.064
married women, no kids	-0.001	-0.013	0.014	-0.009	-0.006	0.015
married women, kids	-0.021	-0.006	0.028	0.015	-0.015	-0.000

Notes: Predicted changes are calculated using the maximum likelihood estimates from Table 3.4 and simulating the equilibrium effect of replacing the April 1997 system with the April 2002 system. Empirical changes refer to the observed changes in our data over this period using the sample selection as described in Section 3.4.5. Changes may not sum to zero due to rounding.

from our sample are unaffected by the reform, our simulations imply that overall employment increases by 135,000. The majority of this increase is though the increased employment of lone mothers (60,000) and married men with children (50,000).

3.5.4 Post-reform Comparison

The simulations that we presented in the previous section allowed us to examine the *ceteris paribus* labour market impact of the set of tax reforms between April 1997 and April 2002. Before comparing these predictions to those which were obtained in previous evaluations of WFTC, we first briefly compare them to the actual changes in the labour market. That is, we ask to what extent did the tax reforms contribute to the observed labour market changes. The simulated and empirical changes are presented in Table 3.8.

Most groups experienced an increase in employment over this period with the exception of women without children where essentially no change was observed. Men without children experienced an increase in employment of between two and three percentage points over this period, and while we did predict a small increase in employment for such individuals, these changes represent a more than doubling of the impact. This therefore suggests that other changes over this period (including robust productivity growth, changes in the distribution of partner earnings, a national minimum wage, and various "New Deal" programmes) may have had a more important effect on employment for this group. The employment increase for married men with children is very similar to that for married men without children, despite very different simulated impacts. To reconcile these findings we therefore require that the other changes in the economy have had an opposite effect on the employment of married men with children.

Lone mothers experienced the largest employment increase by far, with employment increasing by 5.2 percentage points. While the overall impact here line up remarkably well with the simulations, the hours responses differ: the empirical contribution to the increase is approximately evenly split between movements into both part-time and full-time employment; this contrasts with our simulations which suggested that it was exclusively due to a movement into full-time work. Despite predicting small increases in employment for single women and married women without children we effectively see no change for these groups, despite modest employment growth among men. One possible explanation for this is that the labour market is perhaps not as "integrated" as we have modelled, with childless women possibly being more effected by changes in wage offers following the reform than childless men. Of course, to be a credible explanation this would require substantially larger equilibrium effects than we obtained in the previous section (see the discussion in Section 3.5.6). Finally, despite predicting reductions in the employment of married women with children we again observe a modest increase, which again suggests that other changes in the economy boosted employment rates.

3.5.5 Other Evaluations of WFTC

We now compare the labour market responses from our study to those obtained in previous WFTC evaluations. Since our analysis only considers individuals with low levels of educational attainment (see Section 3.4.5), our sample is typically more selective than in the studies cited in this section. As such, the comparisons here are considered more indicative than exact, with the results for lone mothers being most comparable due to lower average education levels for this group. In terms of the employment impact, which is sometime the only outcome considered in these evaluations, the results for lone mothers are broadly similar to those obtained in these other studies. This is perhaps unsurprising given that we did not find evidence of strong equilibrium effects in our analysis.

The most common method that has been used when analysing the impact of WFTC on employment outcomes is difference-in-differences. Such existing evaluations (including Azmat, 2006a; Blundell et al., 2004a; Francesconi and van der Klaauw, 2004;

Gregg and Harkness, 2003; Leigh, 2007) have largely (but not exclusively) focussed upon the impact on lone mothers and essentially involve comparing the changing employment outcomes of lone mothers, with single women without children. As discussed in Section 3.2, the introduction of WFTC was accompanied by a number of other changes to the tax and benefit system. Given this choice of control group, at best these evaluations will be informative about the effect of the set of reforms that only affected parents. Since we predict that the changes to Income Tax and National Insurance acted to increase slightly the labour supply of non-parents, the relevant statistic to compare from our study for lone mothers is an impact of 4.5 percentage points $(= 5.3 - 0.8)$. Before proceeding further, we note that if equilibrium effects were quantitatively important, then the usual stable unit treatment value assumption (or SUTVA) would be violated.

The headline impacts of these estimates on lone mothers (or lone parents) varies somewhat, lying between around 1 and 7 percentage points. The differences across these studies appears to be largely attributable to two factors: (i) the period considered in the estimation; (ii) attempts to control for pre-programme differences in employment trends between treatment and control groups (see Brewer and Shephard, 2004 for a time series). The first factor is important in any comparison because those studies which focus on the period immediately around the introduction of WFTC (such as Leigh, 2007), find considerably smaller impacts. This is unsurprising both because WFTC grew in generosity following its introduction in 1999 (see Table 3.1 earlier), and individuals may require time to obtain an acceptable job. The second issue is more problematic for these studies as it suggests that the usual common trends assumption invoked in difference-in-differences may be violated. Unsurprisingly, studies which assume that this pre-programme differential employment growth would have stopped had WFTC not been introduced typically find larger effects than those which achieve identification through a particular parametric differential time trend specification (using an otherwise similar specification, Blundell et al., 2004a report a 4 percentage point increase, compared to the 2 percentage point impact reported in Azmat, 2006a).

Francesconi and van der Klaauw (2004) offer an interesting interpretation of this pre-reform employment growth, attributing it to an "anticipation effect" so that the pre-programme growth in employment is due to the programme itself. Such effects are possible because the main details of WFTC were announced over a year earlier in the government's March 1998 Budget. Under such an assumption, Francesconi and van der Klaauw estimate that WFTC increased employment by around 7 percentage points, which is somewhat higher than the other evaluations cited here (as well as our simulations for this group). Such effects would be a qualitative implication of a non-stationary labour market search model with anticipation, although other forms of non-stationarity (such as the arrival rate of job offers depending upon calender time) would also be consistent with these trends (see van den Berg, 1990).

An alternative evaluation methodology that has also been adopted involves the estimation and simulation of a static discrete choice labour supply model (Blundell et al., 2000; Blundell and Shephard, 2009; Brewer et al., 2006). These models, which assume away the presence of equilibrium effects, identify preferences by relating changing employment patterns to changing financial work incentives assuming a constant hourly wage and some finite set of work alternatives. Using cross-sectional data from 1997 to 2002, Blundell and Shephard (2009) predicted that the reforms over this period increased employment by around 4 percentage points amongst lone mothers.[30] This study, like other ex-post evaluations, relies upon variation in data caused by the reform itself to obtain this impact.[31] In particular, it explains some of this employment growth by reductions in the "cost" of receiving tax credits.

3.5.6 Why Aren't Equilibrium Effects More Important?

The analysis performed in section 3.5.2 suggests that equilibrium effects may be small. We now explore the extent to which this may be due to the integrated nature of the labour market and the targeted nature of the reforms. As noted previously, lone mothers are the main beneficiaries of the tax credit reforms, and our analysis suggests that labour supply responses are by far the greatest for this group. However, even amongst our sample of workers with low education, they only represent a little over 10% of the sample. While allowing all workers to compete within the same market was a very natural characterisation of the UK labour market, and one which permitted spillover effects, it does severely limit the potential for equilibrium effects following a targeted reform like WFTC if firms are constrained to have a single wage policy.[32]

To understand the importance of our assumptions regarding market segmentation, we re-estimate our model on a sample comprised solely of lone mothers, and perform our simulation exercises as before. Since the model is re-estimated, there are some differences in the direct impact. In particular, the positive employment impact of the reform is now more evenly split between increases in full-time and part-time employment. This is largely due to estimated differences in the wage offer distributions, and because the arrival rates of part-time and full-time wage offers when employed are now estimated to be somewhat more similar (in our previous estimation results, full-time offers arrived more than twice as frequently as part-time offers amongst employed lone mothers – see Table 3.4). An implication of this is that selection effects reduce full-time wages by 5%, and reduce part-time wages by 3%.

Once we allow for equilibrium responses we obtain much larger increases in the flows

[30]Using a similar model, Brewer et al. (2006) reported a similar employment increase for lone mothers, together with a small reduction in the employment of both men and women in couples with children (around half a percentage point).

[31]An ex-ante evaluation using a similar model was provided by Blundell et al. (2000). This predicted a 2 percentage point increase in the employment of lone mothers, together with a small decline for married women with children and essentially no change for men in couples. These results are not comparable to the employment responses that we simulate here as only the "immediate" reform (that is, WFTC in October 1999) was considered.

[32]In a model with worker and firm bargaining, wages essentially become individualistic so that the potential for equilibrium effects is much larger. Lise et al. (2005) used such a model in their analysis of the Canadian Self-Sufficiency Project, and found substantial equilibrium effects.

of matches relative to when the labour market is not segmented: M_0 increases by 5% while M_1 increases by almost 18%. Similarly, we obtain much stronger reductions in full-time wage offers. As a result of these equilibrium responses, full-time (part-time) earnings now fall by 14% (4%) relative to the pre-reform average. Accompanying these changes is a 1 percentage point reduction in full-time employment, with little change in part-time employment. Overall, equilibrium considerations have a non-negligible impact in this experiment, and act to reduce the positive employment impact of the reforms.

3.6 Conclusion

This paper has developed an empirical equilibrium job search model with wage posting, and has used it to analyse the impact of the British Working Families' Tax Credit reform. Our model extends the existing literature in a number of important dimensions: we allow for hours-of-work responses; accurate non-linear tax-schedules; worker and firm heterogeneity (without restrictive arrival rate assumptions); and allow workers of different types to all operate within the same labour market. We close our model by endogenizing the rate of job offer arrivals through aggregate matching functions, and also propose a semi-non-parametric estimation procedure.

We estimate our model using data from before WFTC was introduced, and use our structural parameter estimates to simulate the labour market impact of actual tax reforms. Our analysis suggests that WFTC, together with other reforms to the tax and transfer system between April 1997 and April 2002, had a positive effect on the employment of most groups; only amongst married women with children do we predict a fall in employment. Perhaps unsurprisingly, the increase in employment is found to be strongest for lone mothers, where a 5 percentage point increase is predicted. Our simulations suggest that while equilibrium considerations do play a role, the changes in employment and earnings for most groups are dominated by the direct effect of changing job acceptance behaviour. And while the tax reforms do appear to be able to explain some of the actual changes in employment over the relevant period, for some groups other changes in the economy appear more important.

Even though equilibrium effects may not appear very important for this particular set of reforms, it does not imply that they should always be ignored. Recalling that WFTC is only available to low income families with children, equilibrium effects have the potential to be much more important for tax reforms which are less targeted. We demonstrate that the equilibrium effects of the same reforms may be much larger if we consider a labour market solely comprised of lone mothers, one of the main beneficiaries of WFTC.

We believe that this paper represents an important first step in using empirical equilibrium job search models to evaluate the impact of tax reform policies. Despite performing our empirical analysis on individuals with low education levels, it is likely that differences in worker ability persist within this group. A natural extension could therefore involve incorporating heterogeneity in worker productivity which necessitates a more careful modelling of firm production technologies. Furthermore, given that the tax and transfer systems of many countries (including the UK) depend upon family income to some extent, a more detailed characterisation of the behaviour of couples (building upon the analysis of Guler et al., 2009) would allow us to explore the impact of tax policies on household labour supply allocations. Finally, given the importance of labour supply in our simulation exercises, incorporating micro-level endogenous search intensity (as in Christensen et al., 2005) would create a further dimension along which individuals can respond to changing financial incentives. While each of these represent non-trivial extensions, it does suggest a very exciting agenda of future research.

Chapter 3 Appendix

3.A Worker Strategies

In this appendix we derive the optimal strategies of employed and unemployed workers that were presented in Section 3.3.2.[33] Using the same notation as in the main text, the value of unemployment V_{ui} must satisfy:

$$\begin{aligned}\rho_i V_{ui} = b - T_i^u &+ \lambda_{ui}^0 \mathbb{E}_{w\sim F_0} \max\left\{V_{ei}^0(w) - V_{ui}, 0\right\} \\ &+ \lambda_{ui}^1 \mathbb{E}_{w\sim F_1} \max\left\{V_{ei}^1(w) - V_{ui}, 0\right\}\end{aligned} \tag{3.24}$$

where $V_{ei}^0(w)$ and $V_{ei}^1(w)$ are the values of part-time and full-time employment when receiving wage w. For workers who are employed in a part-time job ($h = h_0$) we have:

$$\begin{aligned}\rho_i V_{ei}^0(w) = wh_0 - T_i^0(wh_0) &+ \lambda_{ei}^0 \mathbb{E}_{x\sim F_0} \max\left\{V_{ei}^0(x) - V_{ei}^0(w), 0\right\} \\ &+ \lambda_{ei}^1 \mathbb{E}_{x\sim F_1} \max\left\{V_{ei}^1(x) - V_{ei}^0(w), 0\right\} + \delta_i(V_{ui} - V_{ei}^0(w))\end{aligned}$$

and for workers employed in a full-time job ($h = h_1$):

$$\begin{aligned}\rho_i V_{ei}^1(w) = wh_1 - T_i^1(wh_1) - C_i^1 &+ \lambda_{ei}^0 \mathbb{E}_{x\sim F_0} \max\left\{V_{ei}^0(x) - V_{ei}^1(w), 0\right\} \\ &+ \lambda_{ei}^1 \mathbb{E}_{x\sim F_1} \max\left\{V_{ei}^1(x) - V_{ei}^0(w), 0\right\} + \delta_i(V_{ui} - V_{ei}^1(w)).\end{aligned}$$

To proceed we define $q_i(w)$ such that $V_{ei}^1(w) = V_{ei}^0(q_i(w))$. This means that a type i worker is indifferent between holding a full-time job with wage w and a part-time job with wage $q_i(w)$. Since marginal tax rates conditional on hours of work are always strictly less than one, this

[33]For notational simplicity, here we do not explicitly write the value functions or the resultant reservation wages as a function of the work opportunity b.

will be a function. It therefore follows that the value of a full-time job may be written as:

$$\rho_i V_{ei}^1(w) = wh_1 - T_i^1(wh_1) - C_i^1 + \lambda_{ei}^0 \int_{q_i(w)}^{\overline{w}_0} (V_{ei}^0(x) - V_{ei}^1(w))dF_0(x) \\ + \lambda_{ei}^1 \int_{w}^{\overline{w}_1} (V_{ei}^1(x) - V_{ei}^1(w))dF_1(x) + \delta_i(V_{ui} - V_{ei}^1(w)).$$

We now wish to obtain the envelope condition $V_{ei}^{1'}(w)$. To do this we first perform integration by parts on the above to obtain:

$$\rho_i V_{ei}^1(w) = wh_1 - T_i^1(wh_1) - C_i^1 + \lambda_{ei}^0 \int_{q_i(w)}^{\overline{w}_0} \overline{F}_0(x)dV_{ei}^0(x) \\ + \lambda_{ei}^1 \int_{w}^{\overline{w}_1} \overline{F}_1(x)dV_{ei}^1(x) + \delta_i(V_{ui} - V_{ei}^1(w)) \tag{3.25}$$

which when differentiated with respect to w yields:

$$\rho_i V_{ei}^{1'}(w) = h_1(1 - T_i^{1'}(wh_1)) - \lambda_{ei}^0 \overline{F}_0(q_i(w))V_{ei}^{0'}(q_i(w))q_i'(w) - \lambda_{ei}^1 \overline{F}_1(w)V_{ei}^{1'}(w) - \delta_i V_{ei}^{1'}(w).$$

Noting that $V_{ei}^{1'}(w) = V_{ei}^{0'}(q_i(w))q_i'(w)$ we may simplify the above equation to obtain:

$$h_1(1 - T_i^{1\prime}(wh_1)) = (\delta_i + \rho_i + \lambda_{ei}^0 \overline{F}_0(q_i(w)) + \lambda_{ei}^1 \overline{F}_1(w))V_{ei}^{1'}(w) \tag{3.26}$$

and performing a similar set of calculations for part-time jobs we arrive at the analogous expression:

$$h_0(1 - T_i^{0\prime}(wh_0)) = (\delta_i + \rho_i + \lambda_{ei}^0 \overline{F}_0(w) + \lambda_{ei}^1 \overline{F}_1(q_i^{-1}(w)))V_{ei}^{0'}(w). \tag{3.27}$$

Note also that equating equation 3.25 (evaluated at wage w) with the analogous expression for part-time employment (evaluated at wage $q_i(w)$) implies that $q_i(w)$ is the solution to:

$$wh_1 - T_i^1(wh_1) - C_i^1 = q_i(w)h_0 - T_i^0(q_i(w)h_0)$$

which is equation 3.1 from the main text. We obtain this simple expression because, conditional on being in employment, the arrival rates for both full-time and part-time jobs are assumed independent of the individuals current hours of work so that it is only necessary to compare the instantaneous utility flows.[34] We can now calculate the reservation wage for unemployed workers. Let us denote ϕ_i as the lowest acceptable wage offer for *full-time* work. Since $V_{ui} = V_{ei}^1(\phi_i) = V_{ei}^0(q_i(\phi_i))$, the lowest acceptable wage offer for part-time work is then

[34] In the more general case, this indifference condition would depend upon the distributions of wage offers.

$q_i(\phi_i)$. We can therefore write equation 3.24 as:

$$\begin{aligned}\rho_i V_{ui} &= b - T_i^u + \lambda_{ui}^0 \int_{q_i(\phi_i)}^{\overline{w}_0} (V_{ei}^0(w) - V_{ui}) dF_0(w) + \lambda_{ui}^1 \int_{\phi_i}^{\overline{w}_1} (V_{ei}^1(w) - V_{ui}) dF_1(w) \\ &= b - T_i^u + \lambda_{ui}^0 \int_{q_i(\phi_i)}^{\overline{w}_0} \overline{F}_0(w) dV_{ei}^0(w) + \lambda_{ui}^1 \int_{\phi_i}^{\overline{w}_1} \overline{F}_1(w) dV_{ei}^1(w).\end{aligned}$$

Substituting our expressions for $V_{ei}^{0\prime}(w)$ and $V_{ei}^{1\prime}(w)$ from equation 3.27 and equation 3.26 in to the above:

$$\begin{aligned}\rho_i V_{ui} = b - T_i^u + \lambda_{ui}^0 \int_{q_i(\phi_i)}^{\overline{w}_0} & \frac{h_0(1 - T_i^{0\prime}(wh_0))\overline{F}_0(w)}{\delta_i + \rho_i + \lambda_{ei}^0 \overline{F}_0(w) + \lambda_{ei}^1 \overline{F}_1(q_i^{-1}(w))} dw \\ + \lambda_{ui}^1 \int_{\phi_i}^{\overline{w}_1} & \frac{h_1(1 - T_i^{1\prime}(wh_1))\overline{F}_1(w)}{\delta_i + \rho_i + \lambda_{ei}^0 \overline{F}_0(q_i(w)) + \lambda_{ei}^1 \overline{F}_1(w)} dw.\end{aligned}$$

By definition of the reservation wage we can set the above equal to $\rho_i V_{ei}^1(\phi_i)$ (from equation 3.25) to obtain the following implicit equation defining ϕ_i in terms of the structural parameters of our model:

$$\begin{aligned}\phi_i h_1 - T_i^1(\phi_i h_1) - C_i^1 = b - T_i^u + (\lambda_{ui}^0 - \lambda_{ei}^0) \int_{q_i(\phi_i)}^{\overline{w}_0} & \frac{h_0(1 - T_i^{0\prime}(wh_0))\overline{F}_0(w)}{\delta_i + \rho_i + \lambda_{ei}^0 \overline{F}_0(w) + \lambda_{ei}^1 \overline{F}_1(q_i^{-1}(w))} dw \\ + (\lambda_{ui}^1 - \lambda_{ei}^1) \int_{\phi_i}^{\overline{w}_1} & \frac{h_1(1 - T_i^{1\prime}(wh_1))\overline{F}_1(w)}{\delta_i + \rho_i + \lambda_{ei}^0 \overline{F}_0(q_i(w)) + \lambda_{ei}^1 \overline{F}_1(w)} dw\end{aligned}$$

dividing both the numerator and denominator of the integral terms by δ_i, and performing a simple change of variable, we then obtain the simplified expression presented in equation 3.2 in the main text.

3.B Identification

In Section 3.4.2 we discussed the identification of our model, and we now illustrate these ideas more formally. Here we set out to show that conditional on the set of transitional parameters, the observed distributions of part-time and full-time wages, together with the distributions of wages accepted by the unemployed are sufficient to separately identify the wage offer and reservation wage distributions. Once these are known, the structure of the model then permits identification of the opportunity cost and productivity distributions. In what follows, we let $G_{1i}^U(w)$ and $G_{0i}^U(w)$ denote the respective cumulative distribution functions of wages first accepted by type i unemployed workers in full-time and part-time jobs. Since individuals will accept any wage offer that is at least as high as their reservation wage, $G_{1i}^U(w)$ will be given by:

$$\begin{aligned}G_{1i}^U(w) &= \int_{-\infty}^{w} \Pr(W_1 < w | W_1 > x) dA_{ui}(x) = \int_{-\infty}^{w} \frac{F_1(w) - F_1(x)}{\overline{F}_1(x)} dA_{ui}(x) \\ &= A_{ui}(w) - \overline{F}_1(w) \left[\int_{\underline{w}}^{w} \frac{dA_{ui}(x)}{\overline{F}_1(x)} + A_{ui}(\underline{w}) \right]\end{aligned}$$

similarly the fraction of part-time jobs accepted that pay no more than $q_i(w)$ can be shown to be given by:

$$G^U_{0i}(q_i(w)) = A_{ui}(w) - \overline{F}_0(q_i(w)) \left[\int_{\underline{w}}^{w} \frac{dA_{ui}(x)}{\overline{F}_0(q_i(x))} + A_{ui}(\underline{w}) \right].$$

If we combine the above two expressions with the respective density functions of accepted wages, $g^U_{hi}(w) \equiv G^{U\prime}_{hi}(w)$ for $h \in \{0,1\}$, we can write:

$$A_{ui}(w; F_0) = G^U_{0i}(q_i(w)) + \frac{\overline{F}_0(q_i(w))g^U_{0i}(q_i(w))}{f_0(q_i(w))} \tag{3.28}$$

$$A_{ui}(w; F_1) = G^U_{1i}(w) + \frac{\overline{F}_1(w)g^U_{1i}(w)}{f_1(w)} \tag{3.29}$$

which therefore demonstrates that the distribution of reservation wages amongst the unemployed on support $[\underline{w}_i, \overline{w}_i]$ is identified given knowledge of the wage offer functions F_0 and F_1.[35] Furthermore, the requirement that $A_{ui}(w; F_1) = A_{ui}(w; F_0)$ allows us to identify the work cost parameter C^1_i.

Substituting equations 3.28 and 3.29 into equations 3.7 and 3.8 from the main text, we can eliminate the unobserved reservation wage distribution to obtain the following differential equations governing the evolution of the two wage offer distributions:

$$F_1'(w) = \frac{m_{1i}g_{1i}(w)\left(1 + \kappa^0_{ei}\overline{F}_0(q_i(w)) + \kappa^1_{ei}\overline{F}_1(w)\right) - u_i g^U_{1i}(w)\kappa^1_{ui}\overline{F}_1(w)}{\kappa^1_{ui}u_i G^U_{1i}(w) + \kappa^1_{ei}\left(m_{0i}G_{0i}(q_i(w)) + m_{1i}G_{1i}(w)\right)} \tag{3.30}$$

$$F_0'(q_i(w)) = \frac{m_{0i}g_{0i}(q_i(w))\left(1 + \kappa^0_{ei}\overline{F}_0(q_i(w)) + \kappa^1_{ei}\overline{F}_1(w)\right) - u_i g^U_{0i}(q_i(w))\kappa^0_{ui}\overline{F}_0(q_i(w))}{\kappa^0_{ui}u_i G^U_{0i}(q_i(w)) + \kappa^0_{ei}\left(m_{0i}G_{0i}(q_i(w)) + m_{1i}G_{1i}(w)\right)}. \tag{3.31}$$

Equations 3.30 and 3.31 define a system of differential equations, which together with the initial conditions $F_1(\underline{w}_i) = 0$ and $F_0(q_i(\underline{w}_i)) = 0$, establishes non-parametric identification of both wage offer functions conditional on the set of transitional parameters. Identification of the underlying opportunity cost distribution and the productivity distributions then follows as described in Section 3.4.3.

3.C Notation Summary

Indexing

$i \in \mathcal{I}$	individual observed type
$h \in \{0,1\}$	hours of work
$j \in \{u,e\}$	employment state
w	wages
p	firm productivity
v	job vacancies
b	unobserved utility flow for unemployed workers

[35]Recall from Section 3.3.3 that $\underline{w}_i \equiv \min\{\underline{w}_1, q_i^{-1}(\underline{w}_0)\}$ and $\overline{w}_i \equiv \max\{\overline{w}_1, q_i^{-1}(\overline{w}_0)\}$.

Workers

n_i	fraction of type i workers
ρ_i	worker discount rate
C_i^h	additive dis-utility flow of working h hours
s_{ji}^h	exogenous worker search intensity
$\phi_i(b)$	full-time work reservation wage
$q_i(w)$	part-time work indifference wage
$H_i(b)$	cumulative distribution of unobserved utility flow b
$A_i(w)$	cumulative distribution of reservation wages
$A_{ui}(w)$	cumulative distribution of reservation wages amongst unemployed
$A_{ei}(w)$	cumulative distribution of reservation wages amongst employed

Transitional Parameters

λ_{ji}^h	job arrival rates
δ_i	job destruction rate
κ_{ji}^h	defined as λ_{ji}^h/δ_i

Employment States

u_i	unemployment rate
m_{0i}	part-time work rate
m_{1i}	full-time work rate

Taxes

$T_i^h(wh)$	net taxes at hours h and earnings wh
$T_i^{h\prime}(wh)$	marginal tax rate at hours h and earnings wh
T_i^u	net taxes when unemployed

Wage Distributions

$F_h(w)$	distribution of wage offers
$\overline{F}_h(w)$	defined as $1 - F_h(w)$
$G_{hi}(w)$	cumulative distribution of wages amongst employed
$g_{hi}(w)$	density of wages amongst employed
$[\underline{w}_h, \overline{w}_h]$	support of wage distributions
$\underline{w}_i$	defined as $\min\{\underline{w}_1, q_i^{-1}(\underline{w}_0)\}$
$\overline{w}_i$	defined as $\max\{\overline{w}_1, q_i^{-1}(\overline{w}_0)\}$

Firms

$\Gamma_h(p)$	cumulative distribution of firm productivity
$\gamma_h(p)$	density of firm productivity
$[\underline{p}_h, \overline{p}_h]$	support of firm productivity
$K_h(p)$	optimal wage policy
$v_h(p)$	optimal recruiting policy
$c(p, v)$	vacancy flow cost
$l_{hi}(w, v)$	steady state employment of worker i
$L_h(w, v)$	total steady state employment: $\sum_i n_i l_{hi}(w)$
$\bar{l}_{hi}(w)$	defined such that $l_{hi}(w, v) = \bar{l}_{hi}(w)v/V_h$
$\overline{L}_h(w)$	defined as $\sum_i n_i \bar{l}_{hi}(w)$
$\pi(p, v)$	steady state profit flow: $(p - w)h_h\overline{L}_h(w)$

Matching Technology

V_h	aggregate stock of job vacancies
S_h	total search intensity
$M_h(S_h, V_h)$	aggregate matching function
θ_h	matching function elasticity

Further Notation from Appendix

V_{ui}	value of unemployment
$V_{ei}^h(w)$	value of employment
$G_{hi}^U(w)$	cumulative distribution of wages accepted by unemployed
$g_{hi}^U(w)$	density of wages accepted by unemployed

Appendix A

FORTAX

A.1 Introduction to FORTAX

The FORTAX project is centered around the development of the FORTAX library, a micro-simulation tax library programmed in Fortran by Andrew Shephard, with the UK system implementation programmed by Andrew Shephard and Jonathan Shaw. It provides detailed representations of UK tax and transfer systems over time (currently covering the period 1991–2009). The library is efficient and flexible, and is ideally suited to applications where accurate budget sets or components of income need to be calculated repeatedly. The estimation and simulation of labour supply models are therefore natural candidates for the use of the FORTAX library. It has been used extensively in Chapters 1 and 3 of this thesis, and in on-going work by Blundell et al. (2009). Other programs that are part of the FORTAX project, including FORTAX for Stata (which provides easy access to the library from within Stata), the FORTAX Calculator (which provides an intuitive graphical environment for calculating and comparing incomes and budget constraints), and FORTAX Online (an interactive web-based version of the calculator), all use the FORTAX library. FORTAX is freely available and is released under the GNU General Public License version 3 (GPLv3). This document provides a guide to using and developing the FORTAX library, and it should be cited in research that uses FORTAX in any form. It assumes familiarity with the Fortran programming language and the Fortran pre-processor (FPP).

A.2 Overview of the FORTAX Library

The FORTAX library contains a number of modules defined within the following files:

1. `fortax_type.f90` defines the main derived types that describe families, the tax system, and the information returned by the calculation routines.

2. `fortax_calc.f90` is the main calculation module. It calculates various measures and components of income based upon its interpretation of the tax system.

3. `fortax_prices.f90` provides access to various price uprating routines and date utilities.

4. `fortax_read.f90` reads files which describe the tax system into memory.

5. `fortax_write.f90` writes files which describe the tax system in the native FORTAX file format.

6. `fortax_util.f90` provides a number of useful support routines and conversion utilities that are used in various parts of FORTAX.

7. `fortax_kinks.f90` calculates piecewise linear schedules for net-income (or any other component of income), varying either earnings or hours of work.

8. `fortax_extra.f90` provides additional functionality by operating on the tax system. It is specific to the particular system implementation.

A.2.1 Additional Files

The module contained in `fortax_realtype.f90` defines real data types used in FORTAX, and is used in all the other modules. The tax system read/write capabilities of FORTAX in `fortax_read.f90` and `fortax_write.f90` make use of code from the `xml-fortran` project,[1] and we do not document them here. The relevant `xml-fortran` source files required by the FORTAX library are `read_xml_prims.f90`, `write_xml_prims.f90`, and `xmlparse.f90`, together with a small number of included files in the directory `'includes/xml'`. Additionally, the files `xmlfortax_t.f90` and `xmltaxben_t.f90` provide the template that defines the file structure for both TAXBEN and FORTAX system files, and these have been generated using programs from the `xml-fortran` project. Furthermore, a large number of "include" files are located in the subdirectories `'includes'` and `'includes/system'`. These are used in conjunction with the Fortran pre-processor (FPP), and we discuss these in Section A.4.

A.3 FORTAX Source Code

A.3.1 fortax_type.f90

The module `fortax_type` defines the main derived type structures required by FORTAX and provides various initialization routines. There are three derived types that are of primary interest here and are used extensively in other modules:

1. `fam_t` defines the family type structure, containing information on demographic characteristics, earnings, hours of work, and other information. Anything that can affect the taxes and transfer payments of a family is defined in here.

2. `sys_t` defines the tax system structure which families of type `fam_t` face. It describes all the parameters which are interpreted within `fortax_calc.f90`.

[1] Available to download from http://xml-fortran.sourceforge.net/.

3. net_t defines the information returned following calls to the main calculation routines within fortax_calc.f90. It contains measures of net income, together with various tax amounts and other components of income.

Note that none of these types are defined "directly" within this module, but are rather defined through the use of include files together with preprocessor commands. This is discussed further in Section A.4. The type fam_t is implicitly defined by the contents of the files 'includes/fam_t.inc' which is responsible for defining all relevant family characteristics, and 'includes/famad_t.inc' which defines relevant adult level characteristics. Extending the family structure then just requires that appropriate entries are made to fam_t.inc and/or famad_t.inc. Any changes to these files will be recognized and fully reflected in the entire FORTAX library when compiled. The same applies for type net_t which is defined in the files 'includes/nettu_t.inc' and 'includes/netad_t.inc' (respectively reflecting the measures of incomes calculated at the level of the tax unit and individual). Any additional item added to these include files will be accessible whenever any variable of type net_t is.

The definition of the entire tax and transfer system through type sys_t is slightly more complicated than that described above, as there is nested use of the preprocessor. This includes the file 'includes/system/syslist.inc' which tells it about the main components of the tax and transfer system (in the UK context, these would include income support, income tax, national insurance, and others). The parameters within a given part of the system are then defined within the relevant include file (incsup.inc, inctax.inc, natins.inc, etc.) which are references within syslist.inc. This allows additional parameters, or entirely new parts of a tax system, to be introduced easily. Once such a parameter has been defined in the relevant files, it is then only necessary to provide the corresponding code in the main calculation module to interpret these parameters. An implication of this is that it is straightforward to extend FORTAX to implement the tax systems of other countries, or indeed, completely hypothetical systems.

Finally, note that the maximum number of children that information can be stored for is determined by the integer parameter maxkids. By default this is equal to 10, but it can easily be changed at compile time by appropriately defining the macro _maxkids_. We now describe the functions and subroutines of this module.

fam_init will initialize the family variable fam of type fam_t, setting logical variables to .false., integer variables to 0, and real(dp) variables to 0.0_dp.[2] If these initializations are not appropriate, then they should be coded here explicitly. Current exceptions to this default initializations are: ad(1)%age = 25 (single adult, aged 25), tenure = 1 (own property outright), region = 1 (standard region, North East), ctband = 4 (council tax band

[2]Note that dp is defined within fortax_realtype.f90.

D), `banddratio = 1.0_dp` (average band amount), and `intdate = 19900101` (interview date, 1st September 1990).

```
elemental subroutine fam_init(fam)
    type(fam_t), intent(inout) :: fam
end subroutine
```

`fam_gen` will return the variable `fam` of type `fam_t`, setting any characteristics to the values that are specified. It will first call `fam_init` so that any parameters that are not explicitly referenced will be given their default values. If optional `correct` is equal to `.false.` then it will not attempt any consistency checks. Otherwise, it will ensure that any implicit dependencies between the parameters are satisfied (for example, if second adult information is passed it will set `fam%couple=.true.` even if `couple` is not explicitly specified). Note that adult information should be passed by adding a suffix 1 or 2 for the respective adult number, e.g. `fam = fam_gen(age1=25,age2=30)`.

```
pure function fam_gen(...,correct)
    type(fam_t)       :: fam_gen
    logical, optional :: correct
end function
```

`fam_desc` will display the information contained in the family variable `fam` of type `fam_t`. If the optional filename `fname` is specified then it will write this information to the file `fname`, otherwise it will be outputted to the default unit.

```
subroutine fam_desc(fam,fname)
    use fortax_util, only : getunit, inttostr, fortaxerror
    use, intrinsic :: iso_fortran_env
    type(fam_t), intent(in)    :: fam
    character(len=*), optional :: fname
end subroutine
```

`net_init` will initialize the variable `net` of derived type `net_t`. It will initialize logical variables to `.false.`, integer variables to `0`, and real(dp) variables to `0.0_dp`. The structure `net_t` should be suitably defined so that these make sense as default values.

```
elemental subroutine net_init(net)
    type(net_t), intent(inout) :: net
end subroutine
```

`sys_init` will initialize the system variable `sys` of derived type `sys_t`. It will initialize logical variables to `.false.`, integer variables to `0`, and real(dp) variables to `0.0_dp`. The structure `sys_t` should be suitably defined so that these make sense as default values.

```
subroutine sys_init(sys)
    type(sys_t), intent(inout) :: sys
end subroutine
```

`sys_saveF90` will save the system variable `sys` of derived type `sys_t` as Fortran source code with file name `fname`. If `fname` is not specified, then the output will be directed to the default output unit.

```
subroutine sys_saveF90(sys,fname)
  use fortax_util, only : getUnit, fortaxError
  use, intrinsic :: iso_fortran_env
  type(sys_t),  intent(in) :: sys
  character(*), intent(in), optional :: fname
end subroutine
```

This output of this subroutine allows the systems to be "hard-coded", rather than reading in system files. The output file assumes the system name is `sys` and `dp` from `fortax_realtype` must be visible. Sample output is displayed below.

Extract from `sys_saveF90` output

```
! .f90 FORTAX system; generated using sys_saveF90

call sys_init(sys) !deallocates arrays and sets values to default

!inctax
sys%inctax%numbands=2
sys%inctax%pa=63.3653846153846_dp
sys%inctax%mma=33.0769230769231_dp
sys%inctax%ctc=0.000000000000000E+000_dp
...
```

`fam_saveF90` will save the family variable `fam` of derived type `fam_t` as Fortran source code with file name `fname`. If `fname` is not specified, then the output will be directed to the default output unit. The output file assumes the family variable name is `fam` and `dp` from `fortax_realtype` must be visible. Sample output follows the interface.

```
subroutine sys_saveF90(sys,fname)
  use fortax_util, only : getUnit, fortaxError
  use, intrinsic :: iso_fortran_env
  type(sys_t),  intent(in) :: sys
  character(*), intent(in), optional :: fname
end subroutine
```

Extract from `fam_saveF90` output

```
! .f90 FORTAX family; generated using fam_saveF90

call fam_init(fam) !deallocates arrays and sets values to default

!family
fam%couple=.true.
fam%married=.false.
fam%ccexp=40.000000000000000E+000_dp
fam%maint=10.000000000000000E+000_dp
fam%nkids=2
...
```

A.3.2 fortax_prices.f90

The module `fortax_prices` provides date and price uprating capabilities. It is useful for manipulating tax systems, and (when appropriate) determining which tax system was operational for given family types based upon the value of `fam%intdate`. The module defines some variables and a type structure that are private to `fortax_prices`. For purposes of price uprating it stores the arrays `rpidate` and `rpiindex` that define YYYYMMDD dates and the associated price index. For the purposes of accessing systems, it defines the type `sysindex_t` which provides easy look up capabilities for system files.

```
module fortax_prices
    integer,  allocatable :: rpidate(:)
    real(dp), allocatable :: rpiindex(:)
    type sysindex_t
        logical                     :: indexinit = .false.
        integer,        allocatable :: date0(:), date1(:)
        character(255), allocatable :: fname(:)
    end type
contains
end module
```

`loadindex` loads a price index file saved as a comma separated values (CSV) file. If `filename` is not specified it defaults to `'prices/rpi.csv'`. Note this default path is relative to the executable, and not from where the FORTAX library is actually compiled. If no price uprating is to be performed through FORTAX, then it is not necessary for such a file to be present. Otherwise, the first record of this file should contain the number of date entries in this file. It then proceeds in the form *date,index* where *date* is an integer of the form YYYYMMDD and where *index* is a double precision number representing the respective price index level. The list should be sorted by the YYYYMMDD date (in ascending order), and FORTAX does not attempt any consistency checks. An example is provided in the following extract.

Extract from `prices/rpi.csv`

```
261,
19870101,100.0
19870201,100.4
19870301,100.6
19870401,101.8
19870501,101.9
...
```

Subroutine `loadindex` then allocates an integer array `rpidate(:)` and a real(dp) array `rpiindex(:)` which are private to `fortax_prices`, and copies this information to them.

```
subroutine loadindex(filename)
    fortax_util, only : getunit
    character(len=*), intent(in), optional :: filename
end subroutine
```

`getindex` returns the price index associated with the supplied YYYYMMDD date. It accesses data from `rpidate(:)` and `rpiindex(:)` and therefore requires that `loadindex()` is called before.

```
real(dp) elemental function getindex(date)
    integer, intent(in) :: date
end function
```

`upratefactor` uprates prices from `date0` to `date1` prices (both in YYYYMMDD format). It calls the function `getindex` and therefore requires that `loadindex()` has previously been called.

```
real(dp) elemental function upratefactor(date0,date1)
    integer, intent(in) :: date0, date1
end function
```

`upratesys` will uprate the prices in the system file `sys` by the uprating factor `factor`. If present, it will replace the date in `sys%desc%prices` with `newdate`. This makes use of preprocessor commands to automatically perform uprating depending on the original *vartype* description of the tax system elements (see Section A.4). It currently uprates anything declared as being either *amount* or *minamount* through the relevant tax system include files.

```
subroutine upratesys(sys,factor,newdate)
    use fortax_type, only : sys_t
    use fortax_util, only : fortaxwarn
    type(sys_t), intent(inout)        :: sys
    real(dp),    intent(in)           :: factor
    integer,     intent(in), optional :: newdate
end subroutine
```

`checkdate` returns `.true.` or `.false.` depending on whether `date` is a valid YYYYMMDD date.

```
logical pure function checkdate(date)
    integer, intent(in) :: date
end function
```

`freesysindex` deallocates the data structures stored in `sysindex%date0`, `sysindex%date1`, `sysindex%fname`, and sets `sysindex%indexinit` to .false. It may be called if the index files are no longer needed by the program calling the FORTAX library.

```
subroutine freesysindex(sysindex)
    type(sysindex_t), intent(inout) :: sysindex
end subroutine
```

`loadsysindex` will return `sysindex` of type `sysindex_t` using the information contained within `sysindexfile`. If `sysindexfile` is not specified it will default to the file `'systems/sysindex.csv'`. The first line of the CSV file `sysindexfile` should equal the number of records in the file. Subsequent lines should be of the form *date0,date1,sysname*. Both *date0* and *date1* should be of the form YYYYMMDD, and refer to the start (*date0*) and end (*date1*) dates that the system *sysname* operated from. Note that *sysname* should not contain either a file path or file extension, as these will be determined by `systemformat` when `getsysindex` is called. An example is provided below.

Extract from `systems/sysindex.csv`

```
28
19900401,19910331,April90
19910401,19920331,April91
19920401,19930331,April92
19930401,19940331,April93
19940401,19950331,April94
...
```

```
subroutine loadsysindex(sysindex,sysindexfile)
    type(sysindex_t), intent(out)          :: sysindex
    character(len=*), intent(in), optional :: sysindexfile
end subroutine
```

`getsysindex` returns information which allows the user to easily identify which system operated at any given YYYYMMDD date as specified in `sysindex`. `systemformat` refers to the file format of the required system file. As well as its native format (`systemformat='fortax'`), FORTAX can also read the undocumented system files used by the IFS tax and benefit model, TAXBEN (`systemformat='taxben'`), although not all system parameters may be understood if not reflected in the FORTAX code. The use of the native FORTAX format is strongly recommended. It returns the relative file path for this system file in `sysfilepath`, and the sequence number within `sysindex` as `sysnum`. This subroutine calls `checkdate` to verify the consistency of date before searching for the relevant index position.

```
subroutine getsysindex(sysindex,date,systemformat, sysfilepath,sysnum)
    use fortax_util, only : lower
    type(sysindex_t), intent(in)  :: sysindex
    integer,          intent(in)  :: date
    character(len=*), intent(in)  :: systemformat
    character(255),   intent(out) :: sysfilepath
    integer,          intent(out) :: sysnum
end subroutine
```

A.3.3 fortax_read.f90

`readtaxparams` reads tax parameters from `systemfile` into a `sys_t` type structure `sys`. `systemformat` refers to the file format of the system file to be loaded. It can currently be either equal to '`fortax`' for the native FORTAX file format (recommended) or '`taxben`' for the undocumented system files used in the IFS tax and benefit model, TAXBEN. If optional integer `prices` is specified, it will set `sys%desc%prices` to equal this value (which should be specified as YYYYMMDD). If `sysfix` is set to `.true.` then it will call `taxbensysfix` if `systemformat` is '`taxben`', otherwise it is ignored. If the optional YYYYMMDD system date is specified then `taxbensysfix` will also apply further corrections to TAXBEN system files.

For non-native formats it is necessary to interpret any parameter value into the equivalent element in the `sys_t` structure. The reading code for the native FORTAX format is completely self-maintaining and does not need to be changed even if further elements (or completely new structures) are added to tax system definition in `sys_t`. This is performed through the use of pre-processing. If an element defined in `sys_t` is not present in the system file, then they will default to `.false.` if a logical variable, `0` if an integer, and `0.0_dp` if real(dp). The FORTAX system files are XML documents (described in Section A.6), and call XML reading routines to provide the relevant reading capabilities. The FORTAX library makes use of code from the `xml-fortran` project (see Section A.2.1).

```
subroutine readtaxparams(sys,systemfile,systemformat, prices,sysfix,sysdate)
    use xml_data_xmltaxben_t, only : read_xml_file_xmltaxben_t, object, &
        namedFields_t, field_t
    use xml_data_xmlfortax_t, only : read_xml_file_xmlfortax_t, system
    use fortax_util,          only : StrToDouble, StrToInt, StrToLogical, lower
    use fortax_type,          only : sys_t, sys_init
    type(sys_t),       intent(out) :: sys
    character(len=*),  intent(in)  :: systemfile
    character(len=*),  intent(in)  :: systemformat
    integer, optional, intent(in)  :: prices
    logical, optional, intent(in)  :: sysfix
    integer, optional, intent(in)  :: sysdate
end subroutine
```

`taxbensysfix` applies a number of "corrections" to the undocumented TAXBEN system files. In particular, it sets values for some necessary parameters that are not contained within the relevant TAXBEN system file, and so are not set when reading. It is called by `readTaxParams` if `sysfix=.true.`.

```
subroutine taxbensysfix(sys,sysdate)
    use fortax_type, only : sys_t
    type(sys_t), intent(inout)        :: sys
    integer,     intent(in), optional :: sysdate
end subroutine
```

Additional subroutines are provided through `fortax_read_assign`, which is used when reading the native FORTAX system files. They are used in the subroutine `readtaxparams` in conjunction with various preprocessor commands, and are private to the module `fortax_read`.

```
interface fortax_read_assign
    module procedure assign_integer
    module procedure assign_logical
    module procedure assign_double
    module procedure assign_integer_array
    module procedure assign_logical_array
    module procedure assign_double_array
end interface fortax_read_assign
```

A.3.4 fortax_write.f90

`fortaxwrite` writes the system file `sys` to disk with file name `fname` in the native FORTAX file format (see the description in Section A.6). This writing code is completely self-maintaining and does not need to be changed even if further elements (or completely new structures) are added to tax and transfer system definition in `sys_t`. This is performed through the use of pre-processing.

```
subroutine fortaxwrite(sys,fname)
    use fortax_type,  only : sys_t
    use xmlparse,     only : xml_parse, xml_open, xml_close
    character(len=*), intent(in) :: fname
    type(sys_t),      intent(in) :: sys
end subroutine
```

`fortaxprint` outputs a summary of the tax system `sys` to the default output unit if `fname` is not specified. Otherwise, this output summary will be written to disk with the file name `fname`. Note that the file it saves is not the FORTAX file format, but is rather outputted in a format that is easy to read. This printing code is completely self-maintaining and does not need to be changed even if further elements (or completely new structures) are added to tax system definition in `sys_t`. This is performed through the use of pre-processing.

```
subroutine fortaxprint(sys,fname)
    use, intrinsic :: iso_fortran_env
    use fortax_type,  only : sys_t
    use fortax_util,  only : upper, getunit
    type(sys_t),      intent(in) :: sys
    character(len=*), intent(in), optional :: fname
end subroutine
```

Additional subroutines are provided through `ftxmlwrite` and `ftprint`, and are used when calling the subroutines `fortaxwrite` and `ftprint` respectively. They are used in conjunction with pre-processor commands and are private to `fortax_write`.

```
interface ftxmlwrite
    module procedure xml_write_finteger
    module procedure xml_write_fdouble
    module procedure xml_write_flogical
    module procedure xml_write_fintegerarray
    module procedure xml_write_fdoublearray
    module procedure xml_write_flogicalarray
end interface

interface ftprint
    module procedure ftprint_finteger
    module procedure ftprint_fdouble
    module procedure ftprint_flogical
    module procedure ftprint_fintegerarray
    module procedure ftprint_fdoublearray
    module procedure ftprint_flogicalarray
end interface
```

A.3.5 fortax_calc.f90

The module `fortax_calc` performs the main tax and transfer calculations. All functions and subroutines in this module should be declared as pure and should not alter either the tax system variable `sys` or the family variable `fam`. The only public subroutine is `calcnetinc`. This returns net-income `net` of type `net_t` given a family `fam` of type `fam_t`, and tax system `sys` of type `sys_t`. It calls a large number of other subroutines which correspond to the various parts of the tax and transfer system. These are discussed in Section A.7 when we describe the implementation of the UK system.

```
pure subroutine CalcNetInc(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)  :: sys
    type(fam_t), intent(in)  :: fam
    type(net_t), intent(out) :: net
end subroutine
```

Since FORTAX may be used in applications where incomes are only required to be calculated for certain family types, it defines a number of macros which allow application specification optimizations to be performed. Currently, it defines the macros _famcouple_, _fammarried_ and _famkids_ which default to fam%couple, fam%married and fam%nkids>0 respectively. If FORTAX is only required to calculate net incomes for lone parents, say, then these can be replaced with .false., .false. and .true. which will remove the need for these expressions to be evaluated at run-time. If the calculation routines are extended (or new ones introduced), then they should use these macro definitions rather than referring to the derived family type directly.

A.3.6 fortax_kinks.f90

The module fortax_kinks provides support programs that have been integrated into the main FORTAX library due to their general applicability and usefulness. These particular support programs produce piecewise linear schedules, varying either earnings (for fixed hours) or hours (for a fixed hourly wage) by repeatedly calling the main calculation routines and using a bisection method to identify the location of any marginal rate changes or discontinuities. This can be performed on any component with the tax system structure of type sys_t. The module defines integer parameter maxkinks which is equal to the maximum number of "kink" points the program will consider (by default this is equal to 200, but it can be set at compile time by appropriately defining the macro _maxkids_), as well as a type structure bcout_t which stores the relevant summary information for the piecewise linear schedule: the actual number of kink points kinks_num, together with the hours kinks_hrs, earnings kinks_earn, tax component amount kinks_net, and marginal rate of this tax component kinks_mtr. Both maxkinks and bcout_t are publicly visible.

```
module fortax_kinks
#   ifndef _maxkinks_
    integer, parameter :: maxkinks = 200
#   else
    integer, parameter :: maxkinks = _maxkinks_
#   endif /* _maxkinks_ */
#   undef _maxkinks_
    type :: bcout_t
        integer :: kinks_num
        real(dp), dimension(maxkinks) :: kinks_hrs, kinks_earn, kinks_net, kinks_mtr
    end type bcout_t
contains
end module
```

kinkshours calculates a piecewise linear schedule under the tax system sys for a family fam of type fam_t by varying hours of work from hours1 to hours2 with a fixed hourly wage wage. In the case of a couple, you may specify which adult this is to be applied to by setting ad=1 or ad=2. The piecewise linear schedule of type budcon_t is stored in bcout. By default, it will output total family net income net%tu%dispinc. It can output different measures by specifying taxlevel (tu,ad1, or ad2 – respectively corresponding to net_t%tu, net_t%ad(1), and net_t%ad(2)) and taxout which determines the income measure in net_t. Note that if either taxout or taxlevel is specified, then both must be specified. taxout is an array, which allows the user to combine income measures which are summed by default. The pre-fix operators + and - can be included in taxout to explictly control any addition or subtraction of income measures. For example, taxout=(/'+a','-b'/) will produce a piecewise linear constraint for the income measure a-b. Optional logical correct performs some rounding to the final schedule, while logical verbose=.true. prints the schedule to the default unit. Note that any positive (negative) discontinuities in the schedule will have marginal rates reported as 9.99999_dp (-9.99999_dp).

```
subroutine kinkshours(sys,fam,ad,wage,hours1,hours2,bcout, &
    taxlevel,taxout,correct,verbose)
    use fortax_type,  only : fam_t, sys_t, net_t
    use fortax_util,  only : lower, inttostr, fortaxerror
    use fortax_calc,  only : calcnetinc
    type(sys_t),      intent(in)  :: sys
    type(fam_t),      intent(in)  :: fam
    integer,          intent(in)  :: ad
    real(dp),         intent(in)  :: wage
    real(dp),         intent(in)  :: hours1, hours2
    type(bcout_t),    intent(out) :: bcout
    character(len=*), intent(in), optional :: taxlevel
    character(len=*), intent(in), optional :: taxout(:)
    logical,          intent(in), optional :: correct
    logical,          intent(in), optional :: verbose
end subroutine
```

kinksearn calculates a piecewise linear schedule under the tax system sys for a family fam of type fam_t by varying earnings from earn1 to earn2 with fixed hours of work hours. The calling syntax is otherwise identical to kinkshours as detailed above.

```
subroutine kinksearn(sys,fam,ad,hours,earn1,earn2,bcout, &
    taxlevel,taxout,correct,verbose)
    use fortax_type,  only : fam_t, sys_t, net_t
    use fortax_util,  only : lower, inttostr
    use fortax_calc,  only : calcnetinc
    type(sys_t),      intent(in)  :: sys
    type(fam_t),      intent(in)  :: fam
    integer,          intent(in)  :: ad
    real(dp),         intent(in)  :: hours
    real(dp),         intent(in)  :: earn1, earn2
    type(bcout_t),    intent(out) :: bcout
    character(len=*), intent(in), optional :: taxlevel
    character(len=*), intent(in), optional :: taxout(:)
    logical,          intent(in), optional :: correct
    logical,          intent(in), optional :: verbose
end subroutine
```

Note: A (double precision) parameter `maxstep` is defined in both of the module subroutines `kinksearn` and `kinkshours`. This controls the initial hours/earnings steps used to identify changes in marginal rates prior to bisection methods being used. Higher values will typically require fewer calls to the main calculation routine (although sufficiently values will likely require more calls), but may result in the respective routines failing to detect certain kink points. The default parameters values appear to work well with UK tax systems, but they can be reduced if the routine fails to identify all the kinks.

`evalKinksHours` uses the piecewise linear budget constraint structure `bcout` as calculated in `kinkshours` to evaluate the respective income measure at hours `hours`. It returns the earnings `earn`, income measure `net` and marginal tax rate `mtr` that are associated with this hours level. If optional `iin` is not specified then it will use bisection to search through `bcout` to identify the relevant linear section. Otherwise, an incremental search will be performed from index `iin`. If optional `iout` is specified, the index of the relevant linear section for hours `hours` will be returned. If `hours` is out-of-range, linear extrapolation will be performed.

```
pure subroutine evalKinksHours(bcout,hours,earn,net,mtr,iin,iout)
    type(bcout_t),  intent(in)  :: bcout
    real(dp),       intent(in)  :: hours
    real(dp),       intent(out) :: earn,net,mtr
    integer,        intent(in),  optional :: iin
    integer,        intent(out), optional :: iout
end subroutine
```

`evalKinksEarn` uses the piecewise linear budget constraint structure `bcout` as calculated in `kinksEarn` to evaluate the respective income measure at earnings `earn`. The calling syntax is as in `evalKinksHours`.

```
pure subroutine evalKinksEarn(bcout,earn,hours,net,mtr,iin,iout)
    type(bcout_t),  intent(in)  :: bcout
    real(dp),       intent(in)  :: earn
    real(dp),       intent(out) :: hours,net,mtr
    integer,        intent(in),  optional :: iin
    integer,        intent(out), optional :: iout
end subroutine
```

A.3.7 fortax_extra.f90

`setminamount` will set any element of the tax system `sys` whose *vartype* is specified as *minamt* (in the relevant include file – see Section A.4) to equal `minamt`. It makes use of the Fortran pre-processor.

```
subroutine setminamount(sys,minamt)
    use fortax_type, only : sys_t
    type(sys_t), intent(inout) :: sys
    real(dp),    intent(in)    :: minamt
end subroutine
```

abolishnifee will modify the tax system sys so that any national insurance entry fee (whereby national insurance is paid on *total* earnings once a threshold is reached) is abolished. This therefore removes a discontinuity in the budget constraint.

```
subroutine abolishnifee(sys)
    use fortax_type, only : sys_t
    use fortax_util, only : fortaxwarn
    type(sys_t), intent(inout) :: sys
end subroutine
```

fsminappamt will modify the tax system sys so that the value of free school meals is included in the *applicable amount* for income support calculations when taper=.true.. This therefore removes a discontinuity in the budget constraint, as entitlement to free school meals is lost when income support is completely tapered away. Default behaviour is obtained when when taper=.false.. An alternative to calling this subroutine is to simply set sys%extra%fsminappamt to be either .true. or .false..

```
subroutine fsminappamt(sys,inappamt)
    use fortax_type, only : sys_t
    type(sys_t), intent(inout) :: sys
    logical,     intent(in)    :: inappamt
end subroutine
```

tapermatgrant will modify the tax system sys so that the value of maternity grant is included in the *applicable amount* for income support calculations when taper=.true.. This therefore removes a discontinuity in the budget constraint, as entitlement to maternity grant is lost when income support is completely tapered away. Furthermore, the value of maternity grant is also tapered away with tax credit entitlement when taper=.true.. Default behaviour is obtained when when taper=.false.. An alternative to calling this subroutine is to set sys%extra%matgrant to be either .true. or .false..

```
subroutine tapermatgrant(sys,taper)
    use fortax_type, only : sys_t
    type(sys_t), intent(inout) :: sys
    logical,     intent(in)    :: taper
end subroutine
```

A.3.8 fortax_util.f90

A number of support utilities are available in fortax_util, as well as generic error handling routines. Function inttostr returns the integer N as a variable length string. It uses intToStrLen to calculate the required length.

```
pure function intToStr(N)
    integer, intent(in)       :: N
    character(intToStrLen(N)) :: intToStr
end function
```

`intToStrLen` returns the length of string representing the integer `N`. It is called by `intToStr`.

```
pure function intToStrLen(N)
    integer, intent(in) :: N
    integer             :: intToStrLen
end function
```

`strToDouble` returns input `string` as a double precision number. It does not perform any user input checks to verify whether `string` contains a valid numeric type.

```
pure function strToDouble(string)
    character(len=*), intent(in) :: string
    real(dp)                     :: strToDouble
end function
```

`strToInt` returns input `string` as an integer number. It does not perform any user input checks to verify whether `string` contains a valid numeric type.

```
pure function strToInt(string)
    character(len=*), intent(in) :: string
    integer                      :: strToInt
end function
```

`strToLogical` returns input `string` as a logical data type. It interprets the string `"0"` to be `.false.`, and anything else to be `.true.`.

```
pure function strToLogical(string)
    character(*), intent(in) :: string
    logical                  :: strToLogical
end function
```

`lower` returns a string of the same length as `str` with all characters converted to lower case. This can be useful for case insensitive string comparisons.

```
pure function lower(str)
    character(len=*), intent(in) :: str
    character(len(str))          :: lower
end function
```

`upper` returns a string of the same length as `str` with all characters converted to upper case. This can be useful for case insensitive string comparisons.

```
pure function upper(str)
    character(len=*), intent(in) :: str
    character(len(str))          :: upper
end function
```

`compact` modifies the original string `str`, converting multiple spaces and tabs to single spaces, deleting control characters and removing any initial spaces.

```
pure subroutine compact(str)
    character(len=*), intent(inout) :: str
end subroutine
```

`trimZero` modifies the original string `str` by trimming any leading zeros. It does not perform any user input checks to verify whether `string` contains a valid numeric type.

```
pure subroutine trimZero(str)
    character(len=*), intent(inout) :: str
end subroutine
```

`getunit` returns a free file unit number `funit` that can be used for file input and output. It searches for free units from one above the standard output unit number to 99.

```
subroutine getunit(funit)
    use, intrinsic :: iso_fortran_env
    integer, intent(out) :: funit
end subroutine
```

`fortaxError` displays the error message `errmsg` and halts execution of the program. If `funit` is specified it will output this error message to file unit `funit`. Otherwise, it will be displayed in the default unit.

```
subroutine fortaxError(errmsg,funit)
    character(len=*),  intent(in) :: errmsg
    integer, optional, intent(in) :: funit
end subroutine
```

`fortaxWarn` displays the warning message `errmsg` and then continues execution of the program. If `funit` is specified it will output this warning message to file unit `funit`. Otherwise, it will be displayed in the default unit.

```
subroutine fortaxWarn(errmsg,funit)
    character(len=*),  intent(in) :: warnmsg
    integer, optional, intent(in) :: funit
end subroutine
```

A.4 Include Files and the Fortran Pre-processor

FORTAX makes extensive use of included source files, saved in the relative path `'includes'`, which are then processed using the Fortran pre-processor (FPP). These allow the user to easily modify parts of FORTAX, by extending and changing the main derived types, `sys_t`, `net_t` and `fam_t`. These are not written in Fortran. The role of the pre-processor is to interpret these files into Fortran source code, and to do this differently depending on the context in which these files are encountered.

All the main parts of the tax and transfer system (income tax, national insurance, child benefit, income support, etc.) are defined within the include file `'includes/system/syslist.inc'`. Whenever FORTAX is performing an operation on the entire tax system it will make use of this include file to cycle through the structures. If a new part is added to the tax system, then it should be reflected in this list. It is structured as follows:

Code extract from `includes/system/syslist.inc`

```
#undef   _$typelist
#define  _$typelist inctax
#include 'includes/system/inctax.inc'
#undef   _$typelist
#define  _$typelist natins
#include 'includes/system/natins.inc'
...
```

The include files which then define the various parts of the tax system (for example, `'includes/system/inctax.inc'`) begin with `_$header` and end with `_$footer`. These allow specific operations to be performed when first entering, and then exiting, a given include file. Parts of the tax system are then defined in the form *storagetype(varname,vartype)* when *storagetype* is either `_$integer`, `_$double` or `_$logical` (corresponding to the Fortran types `integer`, `real(dp)` and `logical`). When they are defined in the form *storagetype(varname,vartype,vardim)*, then *storagetype* is either `_$integerarray`, `_$doublearray` or `_$logicalarray`. In both cases *varname* will refer to the internal variable name, whereas *vartype* tells FORTAX something about what this variable represents in the tax system; for example, it could be specified as a *rate* or *amount*. These specifications allow various operations, such as price uprating which we discuss below, to be performed very efficiently and do not require large data structures to be held in memory. When *vardim* is present it refers to the one dimensional array size, either a valid storage size, or '`:`' for an allocatable array. The main contents of the file `'includes/system/inctax.inc'` is shown below.

Code extract from `includes/system/inctax.inc`

```
...
_$header
_$integer(numbands,range)
_$double(pa,amount)
_$double(mma,amount)
_$double(ctc,amount)
_$double(ctcyng,amount)
_$double(mmarate,rate)
_$double(ctctaper,rate)
_$double(c4rebate,rate)
_$doublearray(bands,amount,:)
_$doublearray(rates,rate,:)
_$footer
...
```

A simple example of the use of preprocessing in FORTAX is illustrated by the subroutine `upratesys` from the module `fortax_prices`. Here, an uprating factor `factor` is passed to the subroutine, and the subroutine defines parameters (which correspond to *vartype* as discussed above) to either `.true.` or `.false.` depending on whether FORTAX should attempt to uprate these tax parameters by scaling by `factor`. In this case, uprating is only performed if *vartype* is equal to `amount` or `minamount`.

Code extract from subroutine `upratesys`

```
subroutine upratesys(sys,factor,newdate)
    use fortax_type, only : sys_t
    use fortax_util, only : fortaxwarn
    type(sys_t), intent(inout)       :: sys
    real(dp),    intent(in)          :: factor
    integer,     intent(in), optional :: newdate

    logical, parameter :: null      = .false.
    logical, parameter :: range     = .false.
    logical, parameter :: scale     = .false.
    logical, parameter :: rate      = .false.
    logical, parameter :: amount    = .true.
    logical, parameter :: minamount = .true.

    if (present(newdate)) sys%desc%prices = newdate

#   include 'includes/fortax_uprate.inc'

end subroutine
```

The actual uprating is then performed by including and processing the file '`includes/fortax_uprate.inc`'. The main body of this code is presented below.

Code extract from `includes/fortax_uprate.inc`

```
...
#define _$logical(x,y) if (y) call fortaxwarn('can''t uprate logical '//#x)
#define _$integer(x,y) if (y) sys%_$typelist%x = factor*sys%_$typelist%x
#define _$double(x,y)  if (y) sys%_$typelist%x = factor*sys%_$typelist%x
#define _$logicalarray(x,y,z) if (y) call fortaxwarn('can''t uprate logicalarray
    '//#x)
#define _$integerarray(x,y,z) if (y) sys%_$typelist%x = factor*sys%_$typelist%x
#define _$doublearray(x,y,z)  if (y) sys%_$typelist%x = factor*sys%_$typelist%x
#define _$header
#define _$footer
#include 'includes/system/syslist.inc'
...
```

As `fortax_uprate.inc` includes '`includes/system/syslist.inc`', it is able to operate on all the individual elements of the tax system defined in type `sys_t`. To understand what this code does, note that before the file `inctax.inc` which appears in `syslist.inc` is included, `_$typelist` is defined as `inctax`. Therefore, the line `_$integer(numbands,range)` from `inctax.inc` will be replaced with the line of Fortran code `if (.false.)  sys%inctax%numbands=factor*sys%inctax%numbands`,

Table A.1: FORTAX file dependencies

File	Depends
xmlparse	
read_xml_prims	xmlparse, <*includes/xml*>
write_xml_prims	xmlparse
xml_taxben_t	xmlparse, read_xml_prims
xml_fortax_t	xmlparse, read_xml_prims, write_xml_prims
fortax_realtype	
fortax_util	fortax_realtype
fortax_type	fortax_realtype, includes/sys_t.inc, includes/sys_init.inc, includes/fam_t.inc, includes/famad_t.inc, includes/nettu_t.inc, includes/netad_t.inc, <*includes/systems*>
fortax_calc	fortax_realtype, fortax_type
fortax_extra	fortax_realtype, fortax_util, fortax_type, includes/fortax_minamt.inc, <*includes/system*>
fortax_prices	fortax_realtype, fortax_util, fortax_type, includes/fortax_uprate.inc, <*includes/system*>
fortax_read	xmltaxben_t, xmlfortax_t, fortax_realtype, fortax_util, fortax_type, fortax_calc, includes/fortax_typeread.inc includes/fortax_read.inc, <*includes/system*>
fortax_write	xmlparse, fortax_realtype, fortax_util, fortax_type, includes/fortax_write.inc, includes/fortax_print.inc, <*includes/system*>
fortax_kinks	fortax_realtype, fortax_util, fortax_type, fortax_calc, includes/nettu_t.inc, includes/netad_t.inc

Notes: <*includes/xml*> and <*includes/system*> refer to the files contained within the respective file directories. File extension is `.f90` unless stated otherwise.

and will therefore do nothing. Moreover, because `.false.` is a known parameter at compile time, an efficient compiler would remove this statement if suitable optimizations are enabled, so that no evaluation would be performed here. Similarly, the line `_$integer(pa,amount)` would be replaced with the line of code, `if (.true.) sys%inctax%pa=factor*sys%inctax%pa`, in which case an operation would be performed. Once again, compiler optimizations may remove the initial logical evaluation as this is known at compile time.

A.5 Compiling FORTAX

A makefile is provided to perform compilation. The provided makefile assumes the use of the Intel Fortran Compiler, although other compilers can be used by modifying the relevant macro definitions. The file dependencies of FORTAX are shown in table A.1 and are reflected in the provided makefile. Microsoft Windows users using Microsoft Visual Studio with Intel Visual Fortran integration may easily include these files in a project. It is generally preferable to compile FORTAX as a library, and then link the library to your particular application.

A.6 FORTAX System File Format

FORTAX saves system files as XML documents. Note that if the tax system is extended in any way, then this will automatically be recognized by FORTAX such that no changes in the read/write routines will be required. An example portion of the system file is shown below. Here *basename* corresponds to the main parts of the tax system as contained in the file `'include/system/syslist.inc'`, while the individual *name* parameters correspond to the description contained in the respective include files (for example, `'includes/system/inctax.inc'`). The *storagetypes* `_$integer`, `_$double`, `_$logical`, `_$integerarray`, `_$doublearray`, and `_$logicalarray`, are respectively referred to as `finteger`, `fdouble`, `flogical`, `fintegerarray`, `fdoublearray`, and `flogicalarray` in the system file.

Extract from example FORTAX system file

```
<?xml version="1.0"?>
<fortax>
<system basename="inctax">
   <finteger name="numbands" value="3"/>
   <fdouble name="pa" value="88.7500000000000"/>
   <fdouble name="mma" value="0.000000000000000E+000"/>
   <fdouble name="ctc" value="10.1730769230769"/>
   <fdouble name="ctcyng" value="10.1730769230769"/>
   <fdouble name="mmarate" value="0.100000000000000"/>
   <fdouble name="ctctaper" value="6.666666666666667E-002"/>
   <fdouble name="c4rebate" value="0.000000000000000E+000"/>
   <fdoublearray name="bands" value="36.9230769230769 575.000000000000
       19230.7692307692"></fdoublearray>
   <fdoublearray name="rates" value="0.100000000000000 0.220000000000000
       0.400000000000000"></fdoublearray>
</system>
<system basename="natins">
   <finteger name="numrates" value="2"/>
    ...
</system>
....
</fortax>
```

A.7 Implementation of the UK Tax System

FORTAX currently models UK tax systems from April 1990 to April 2009. This section describes the implementation of the systems, and in doing so briefly describes some of the main features. It does not attempt to provide a detailed guide to how individual parts of the tax and transfer system are calculated and how they interact. Recent surveys of the the UK tax and transfer system are provided in Adam and Browne (2009) and O'Dea et al. (2007). Before we proceed we note that the current implementation is incomplete. In particular it does not currently model any disability related benefits, or incomes for the non-working-age population. It also ignores non-dependants and anything to do with capital and capital income. Whether or not these are a limitation will depend upon the specific application of FORTAX.

We now describe the implementation. These are discussed in the order in which they are modelled by FORTAX and so reflect the dependencies between parts of the UK tax and transfer system. These routines are all called directly or indirectly from `calcNetInc` within the `fortax_calc` module. Typically these routines and functions require that the tax system `sys` of type `sys_t` and family structure `fam` of type `fam_t` are passed to them and are not allowed to be modified in any way (intent is specified as `in`). The net income structure `net` of type `net_t` is also passed and is modified as calculations are performed (intent is set to `inout`). Note that the subroutines and functions may refer to previously calculated elements of `net_t`.

A.7.1 National Insurance

National insurance contributions (NICs) are taxes paid by employees and employers on earnings. Individuals who have made sufficient contributions are entitled to certain ("contributory") transfer payments. It is necessary to calculate national insurance before income tax because the rebate for Class 4 contributions between April 1985 and April 1995 reduces taxable income. The subroutine `natIns` calculates the NICs of adult `i` in family `fam`. National insurance is defined in `sys%natins` and stores class 1, 2 and 4 contributions in `net%ad(i)%natinsc1`, `net%ad(i)%natinsc2` and `net%ad(i)%natinsc4`. The sum of these NICs components is then stored in `net%ad(i)%natins`. Note that `sys%natins%rates(1)` acts as the "entry fee" (up to April 1999) if earnings exceed `sys%natins%bands(1)`.

```
pure subroutine natIns(sys,fam,net,i)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
    integer,     intent(in)    :: i
end subroutine
```

A.7.2 Income Tax

Income tax operates through a system of allowances and bands of income. Each individual has a personal allowance, which is deducted from total income before tax to give taxable income. Age related personal allowances are ignored in our implementation because we currently only consider working-age individuals. Before the amount of income tax payable is calculated, subroutine `tearn` calculates taxable earnings for the tax unit. It subtracts the income tax personal allowance from individual earnings, and where relevant also deducts a rebate for class 4 NICs and pre-April 1994 deducts married couples allowance and additional person allowance (more below). The relevant system parameters are defined in `sys%inctax`. Individual level taxable earnings are stored in `net%ad(:)%taxable`.

```
pure subroutine tearn(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
end subroutine
```

Income tax for adult `i` is then calculated in `incTax`. The relevant system parameters are defined in `sys%inctax`. Individual level taxable earnings are stored in `net%ad(i)%inctax`.

```
pure subroutine incTax(sys,net,i)
    use fortax_type, only : sys_t, net_t
    type(sys_t), intent(in)    :: sys
    type(net_t), intent(inout) :: net
    integer,     intent(in)    :: i
end subroutine
```

Children's tax credit was only available in two years, 2001/02 and 2002/03, and reduced the tax liability of those with children by a flat-rate amount (tapered away for higher-rate taxpayers). The amount of tax after children's tax credit is calculated by the subroutine `taxAfterCTC`. The system parameters are defined within `sys%inctax` and it modifies the values of `net%ad(:)%inctax`.

```
pure subroutine taxAfterCTC(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
end subroutine
```

Until April 1994 married couple's allowance (MCA) was available to married couples. Up until April 1993, MCA was set against the income of the husband (with any unused allowance transferable to the wife). From April 1993, half or all of the MCA could be transferred to the wife (any unused allowance could still be transferred to the other member of the couple). Since April 1990, the additional personal allowance (APA) – an allowance available to lone parents and unmarried couples with children – has been equal to the MCA. Between April 1994 and April 2000, both MCA and APA reduced tax payable rather than acting as an allowance, while in April 2000 both were abolished for people born after 1935. While the pre-April 1994 allowances are calculated in `tearn`, in later years these are calculated in the subroutine `taxAfterMCA`. In both cases the relevant parameters are defined in `sys%inctax` and subroutine `taxAfterMCA` modifies the value of `net%ad(:)%inctax`.

```
pure subroutine taxAfterMCA(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
end subroutine
```

After calling these subroutines, it sets the values of both `net%ad(:)%posttaxearn` and `net%tu%posttaxearn` which are referenced by subsequent routines.

A.7.3 Child Benefit

Child Benefit (defined in `sys%chben`) is a universal benefit available for families with children. A child is someone aged under 16, or aged 16-18 and in full-time education. The lone parent rate of child benefit (which replaced the former one parent benefit in April 1997) was abolished from 6 July 1998 except for existing claimants. FORTAX ignores the fact that the lone parent rate can still be claimed by existing claimants. It sets `net%tu%chben`.

```
pure subroutine chben(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
end subroutine
```

A.7.4 Family Credit and Working Families' Tax Credit

Family Credit (up to October 1999) and Working Families' Tax Credit (from October 1999 up to April 2003) are means tested in-work benefits/tax-credits payable to families with children. They both have minimum hours conditions, with a further payment for full-time work. They are calculated by the subroutine `famCred`, which calls `maxFCamt` to determine the maximum pre-taper entitlement level (which depends upon the number and age of children, together with childcare expenditure). The subroutine `FCdisreg` calculates the earnings disregard for childcare expenditure. The system parameters are defined in `sys%fc` and it sets the values of both `net%tu%fc` and `net%tu%chcaresub`.

```
pure subroutine famCred(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
end subroutine

pure subroutine maxFCamt(sys,fam,net,MaxFC)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
    real(dp), intent(out)      :: MaxFC
end subroutine

real(dp) pure function FCDisreg(sys,fam)
    use fortax_type, only : sys_t, fam_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
end function
```

A.7.5 New Tax Credits

The new tax credits (working tax credit and child tax credit) were introduced in April 2003. Working Tax Credit provides in-work support for low-income working families both with children (when at least one adult works at least 16 hours per week) and without children (at least one adult aged 25 or above working at least 30 hours per week). It also includes a childcare credit. Child tax credit is payable to families with children, and comprises a family element with additional elements for each child. A joint CTC/WTC means tested is applied and depends on family level income. For both these tax credits FORTAX ignores any disability related payments, and also the working tax credit supplement for the over 50s who are returning to work. Entitlement for the new tax credits is calculated by the subroutine `NTC`. This first calls `maxWTCamt`, `maxCTCfam` and `maxCTCkid` to determine the respective maximum pre-taper entitlement for working tax credit, and both the family and child elements of the child tax credit. The subroutine `NTCtaper` then applies the taper, assuming receipt for the entire year. The entire new tax credits system is defined in `sys%ntc`, `sys%wtc`, and `sys%ctc`. It sets the values of `net%tu%wtc`, `net%tu%ctc`, and `net%tu%chcaresub`.

```
pure subroutine NTC(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
end subroutine

real(dp) pure function maxCTCfam(sys,fam)
    use fortax_type, only : sys_t, fam_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
end function

real(dp) pure function maxCTCkid(sys,fam)
    use fortax_type, only : sys_t, fam_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
end function

pure subroutine maxWTCamt(sys,fam,net,maxWTC)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
    real(dp),    intent(out)   :: maxWTC
end subroutine

pure subroutine NTCtaper(sys,fam,net,maxWTC, maxCTCFam,maxCTCKid)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
    real(dp),    intent(in)    :: maxWTC,maxCTCFam,maxCTCKid
end subroutine
```

A.7.6 Income Support and Income-based JSA

Income support is a means-tested benefit paid to people on low incomes and is not available to those in full-time work. Families are eligible if a measure of net family income (less any disregards) is less than their “applicable amount”. The applicable amount is equal to the sum of a personal allowance, together with lone parent and family premiums and child additions. Income support entitlement is then equal to the amount of income required such that the sum of net family income and income support is equal to the applicable amount. Note that up until October 1996, unemployed workers who satisfied the relevant contributory conditions received unemployment benefit, while those who did not could claim income support. In October 1996, unemployment benefit was renamed contributory job seekers allowance (JSA), and, for those who did not work, income support was renamed income-based JSA. Income support entitlement is calculated in subroutine `incSup`, which calls the functions `ISappamt` and `ISdisreg` to calculate the relevant applicable amount and income disregard. The income support parameters in the following subroutines and functions are defined in `sys%incsup` and it sets the values of `net%tu%incsup`.

```
pure subroutine incSup(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
end subroutine

real(dp) pure function ISappAmt(sys,fam)
    use fortax_type, only : sys_t, fam_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
end function

real(dp) pure function ISdisreg(sys,fam)
    use fortax_type, only : sys_t, fam_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
end function
```

A.7.7 Maternity Grant

Sure start maternity grant (formerly known as the maternity expenses payment), is a one-off payment for families with a new baby. Families can receive it if they receive Income Support, income-based Jobseeker’s Allowance, FC/WFTC (up to April 2003), or more than the family element of Child Tax Credit (from April 2003). Receipt due to receipt of disability related benefits or Pension Credit is not modelled by FORTAX.

```
pure subroutine matGrant(sys,fam,net,calcmax)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)          :: sys
```

```
    type(fam_t), intent(in)           :: fam
    type(net_t), intent(inout)        :: net
    logical,     intent(in), optional :: calcmax
end subroutine
```

A.7.8 Local Taxation

Council tax was introduced in April 1993 and is the current form of local taxation in England, Scotland and Wales (varying with local authority area) and is based on the value of the property occupied according to some council tax "band", with certain discounts and exemptions applied. It does not apply to Northern Ireland where domestic rates still exist. Council tax liabilities are calculated in the subroutine `ctax`, with the relevant parameters defined in `sys%ctax`. It sets the value of `net%tu%ctax`.

```
pure subroutine ctax(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t, &
    ctax_banda, ctax_bandb, ctax_bandc, ctax_bandd, &
    ctax_bande, ctax_bandf, ctax_bandg, ctax_bandh
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
end subroutine ctax
```

Between April 1990 and April 1993, the community charge (poll tax) operated in England, Scotland and Wales, and was a flat rate that also depended on the local authority area. Community charge liabilities are calculated by the subroutines `polltax`, with the relevant parameters defined in `sys%ccben`. It sets the value of `net%tu%polltax`.

```
pure subroutine polltax(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
end subroutine polltax
```

A.7.9 Housing Benefit, Council Tax Benefit, and Community Charge Benefit

Since housing benefit, council tax benefit, and community charge benefit are all calculated in much the same way, FORTAX reduces the number of calculations that need to be performed by first calling the subroutine `prelimCalc`. It returns the applicable amount `appamt` by calling the function `HBappAmt`, the standard earnings disregard `disreg1` by calling the function `stdDisreg`, the sum of the full-time and childcare disregard `disreg2` by calling the functions `FTdisreg` and `chcareDisreg`, and the maintenance disregard by calling `maintDisreg`. These are discussed further below.

```
pure subroutine prelimCalc(sys,fam,net,appamt,disreg1, disreg2,disreg3)
    use fortax_type, only : sys_t, fam_t, net_t
```

```
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
    real(dp),    intent(out)   :: appamt,disreg1,disreg2, disreg3
end subroutine

real(dp) pure function HBappAmt(sys,fam)
    use fortax_type, only : sys_t, fam_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
end function

real(dp) pure function stdDisreg(sys,fam)
    use fortax_type, only : sys_t, fam_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
end function

real(dp) pure function FTdisreg(sys,fam)
    use fortax_type, only : sys_t, fam_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
end function

real(dp) pure function chcareDisreg(sys,fam,net)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
    type(net_t), intent(in) :: net
end function

real(dp) pure function maintDisreg(sys,fam)
    use fortax_type, only : sys_t, fam_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
end function
```

Housing benefit is a means tested benefit payable to families with low incomes who rent their homes (for home owners, income support may provide support for mortgage interest payments). Families who receive income support, income-based jobseeker's allowance or the guarantee credit element of the pension credit (currently not modelled by FORTAX) are automatically entitled to the maximum level of housing benefit. This maximum level is equal to "eligible rent" minus possible deductions. If `sys%rebatesys%docap` is `.false.` then the level of eligible rent is interpreted to be their actual rent. Otherwise, a rent cap is imposed for private renters (as identified by `fam%tenure`), so that eligible rent is equal to the minimum of `fam%rent` and `fam%rentcap`. Families are eligible to receive housing benefit if a measure of net family income (less any disregards as calculated by the subroutine `prelimCalc`) is less than their "applicable amount". As with income support, the applicable amount is equal to the sum of a personal allowance, together with lone parent and family premiums and child additions. Housing benefit entitlement is calculated by the subroutine `HBen` and is equal to the amount of income required such that the sum of net family income and housing benefit is equal to the applicable amount. The parameters of housing benefit are defined in `sys%rebatesys` and it sets the value of housing benefit in `net%tu%hben`.

```
pure subroutine HBen(sys,fam,net,appamt,disreg1, disreg2,disreg3)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
    real(dp),    intent(in)    :: appamt,disreg1,disreg2, disreg3
end subroutine
```

The function `HBfull` returns value `.true.` if the family is entitled to the full amount of their eligible rent, and `.false.` otherwise.

```
logical pure function HBfull(sys,fam,net,appamt,disreg1, disreg2,disreg3)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in) :: sys
    type(fam_t), intent(in) :: fam
    type(net_t), intent(in) :: net
    real(dp),    intent(in) :: appamt,disreg1,disreg2,disreg3
end function
```

Council tax benefit is payable to families with low incomes who are liable to pay council tax on a property in which they are resident. Many of the conditions for claiming are the same as those for housing benefit and the benefit is calculated by the subroutine `ctaxBen`. The relevant parameters are defined in `sys%rebatesys` and `sys%ctax` and it sets the value of `net%tu%ctaxben`.

```
pure subroutine ctaxBen(sys,fam,net,appamt,disreg1, disreg2,disreg3)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
    real(dp),    intent(in)    :: appamt,disreg1,disreg2,disreg3
end subroutine
```

The community charge benefit is calculated by subroutine `polltaxBen`. The relevant parameters are defined in `sys%ccben` and it sets the value of `net%tu%polltaxben`.

```
pure subroutine polltaxBen(sys,fam,net,appamt,disreg1, disreg2,disreg3)
    use fortax_type, only : sys_t, fam_t, net_t
    type(sys_t), intent(in)    :: sys
    type(fam_t), intent(in)    :: fam
    type(net_t), intent(inout) :: net
    real(dp),    intent(in)    :: appamt,disreg1,disreg2, disreg3
end subroutine
```

A.7.10 Net Income

Following the calculation of these income components, FORTAX proceeds to construct a number of summary income measures: `net%tu%tottax` (total tax paid by the tax unit), `net%tu%pretax` (pre-tax income of the tax unit), `net%tu%dispinc` (disposable income of the tax unit). It also calculates the values of `net%ad%pretaxearn` (pre-tax earnings of each adult – same as `net%fam%ad%earn`) and `net%tu%pretaxearn` (pre-tax earnings of the tax unit).

A.8 Example Code

This section illustrates the basic use of FORTAX. It defines a family (in this case, a lone parent with two children aged 2 and 10, rent of £50 per week, childcare expenditure of £20 per week, and council tax band C) by calling `fam_gen`. Any unspecified components will be set to their default values. Note also that it will automatically satisfy the relevant dependencies in `fam`, so setting the appropriate number of children (`fam%nkids=2`), and age of youngest child (`fam%yngkid=2`). It uses `readTaxParams` to load the systems `April98.xml` and `April02.xml` into `sys(1)` and `sys(2)` respectively. We now want to modify `sys(1)` so that it is expressed in the same prices as `sys(2)`. We do this by first loading the price index via `loadindex` (default is `'prices/rpi.csv'`). We then calculate the uprating factor `factor` using `upratefactor` and by referencing the prices information saved in the system files. To demonstrate the use of the system file writing procedure, We call `fortaxwrite` which saves the modified system `sys(1)` as `'April98rpi02.xml'`. We then loop over the range of hours, from `hrs1` to `hrs2` with `nhrs` steps. At each cycle We modify the hours and earnings information in `fam%ad(1)`, using the wage defined earlier in `wage`. The subroutine `calcNetInc` is called twice (once for each system) to calculate incomes under the respective systems, which are then saved in `net(1)` and `net(2)`. The values of hours and net disposable income under `sys(1)` and `sys(2)` are then written to the default unit. The loop then continues.

Example code using FORTAX

```
program fortaxexample

    !load the required modules
    use fortax_realtype
    use fortax_type
    use fortax_read
    use fortax_write
    use fortax_calc
    use fortax_prices

    implicit none

    type(fam_t) :: fam
```

```
type(sys_t) :: sys(2)
type(net_t) :: net(2)

integer  :: i, nhrs
real(dp) :: wage, hrs1, hrs2, hrstep, factor

!generate a family
fam = fam_gen(kidage=(/2,10/),rent=50.0_dp, ccexp=20.0_dp,ctband=3)

!hourly wage rate, and hours range
wage = 5.0_dp
hrs1 = 0.0_dp
hrs2 = 100.0_dp
nhrs = 100

!step size when looping over hours
hrstep = (hrs2-hrs1)/real(nhrs-1,dp)

!load fortax system files
call readTaxParams(sys(1),'systems/fortax/April98.xml', 'fortax')
call readTaxParams(sys(2),'systems/fortax/April02.xml', 'fortax')

!load the price index to perform uprating
call loadindex()

!uprate prices so April 98 system is in April 02 prices
factor = upratefactor(sys(1)%extra%prices, sys(2)%extra%prices)
call upratesys(sys(1),factor,sys(2)%extra%prices)

!write the uprated system. this is not necessary here,
!but allows new system to be used directly in future
call fortaxwrite(sys(1),'systems/fortax/April98rpi02.xml')

!loop over the range of hours, from hrs1 to hrs2
do i = 1, nhrs

    !modify the hours and earnings information in fam
    fam%ad(1)%hrs  = hrs1 + (i-1)*hrstep
    fam%ad(1)%earn = wage*fam%ad(1)%hrs

    !call the main calculation routine
    call calcNetInc(sys(1),fam,net(1))
    call calcNetInc(sys(2),fam,net(2))

    !write hours, and net income from both systems
    write (*,'(3(F12.4,2X))') fam%ad(1)%hrs, net(1)%tu%dispinc, net(2)%tu%dispinc

end do
```

```
end program
```

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