A mixed integer quadratic programming formulation for the economic dispatch of generators with prohibited operating zones

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Abstract

In this paper, an optimization-based approach is proposed using a mixed integer quadratic programming model for the economic dispatch of electrical power generators with prohibited zones of operation. The main advantage of the proposed approach is its capability to solve case studies from the literature to global optimality quickly and without any targeting of solution procedures.

Keywords: economic dispatch, power generation, mixed integer quadratic programming, optimisation.

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1. Introduction

The economic dispatch of generators is a key element in the optimal operation of power generation systems. The main goal is the generation of a given amount of electricity at the lowest cost possible. Although the basic objective is straightforward, the problem is typically extended in a number of ways. The main extensions or variations are described by Jayabarathi et al. [1]. One specific case is the consideration of generators which have prohibited zones of operation within their overall domain of operation [2]. Prohibited zones arise from physical limitations of individual power plant components. Lee & Breipohl [2] give the example of the amplification of vibrations in a shaft bearing at certain operating regimes. These physical limitations may lead to instabilities in operation for certain loads. To avoid these instabilities, the concept of prohibited has been developed. The presence of prohibited zones for individual generators leads to a solution space with disjoint, therefore non-convex, feasible regions.

The non-convexity of the feasible space has led researchers in this area to concentrate on the development of direct search and stochastic optimization methods. For a good overview of the types of methods used, see Pereira-Neto et al. [3]. These authors have developed an evolutionary programming method, similar to a genetic algorithm, and have compared it to a number of other stochastic, direct search and artificial intelligence methods. Other examples of methods for solving the dispatch problem are presented in the literature [4-10], the majority concentrating on the use of evolutionary programming methods.

The emphasis on direct search and stochastic methods is due to the observation that mathematical programming approaches are often not suitable for tackling such problems due to the non-convexity of the search space. The advantages of evolutionary programming methods, for instance, include the ability to tackle problems with complex objective functions and constraints, including discontinuities and non-convexities, and the ease of implementation in many cases. The drawbacks of these methods include the lack of guarantee of convergence in finite time and space and the large number of often arbitrary or problem-specific parameters required.

Mathematical programming approaches, however, are able to provide guarantees on convergence and typically have no problem specific parameters to specify. However, they can fail to solve problems adequately in the presence of discontinuities and non-convexities. Although the electricity dispatch problem with prohibited zones has a feasible space which is disjoint, this is not a sufficient complexity to preclude the use of mathematical programming.

This article describes a mathematical programming approach for this problem and presents results for the solution of a number of case studies. Specifically, the next section outlines the economic dispatch problem in detail, including the objective function and the constraints. The following section presents a mixed integer quadratic programming (MIQP) model. The results obtained for the case studies are then presented and the paper concludes with some observations on the applicability of the proposed approach.

2. Problem Statement

The economic dispatch (ED) problem of generators with prohibited operating zones aims at determining the optimal generation levels of all on-line units so as to minimise the total fuel cost subject to a number of constraints. The overall problem can be stated mathematically as follows.

2.1. Objective function

The fuel cost function of each generating unit, i, is usually described by a quadratic function of the power output, P_i *(MW)*, as:

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2$$

where a_i, b_i and c_i are cost coefficients for unit i (here, positive values of c_i are assumed).

The total fuel cost, F, will be the objective function to be minimised:

$$\min F = \sum_{i \in \Omega} F_i(P_i) \tag{1}$$

where Ω is the set of all on-line units.

2.2. Power balance constraint

$$\sum_{i \in \Omega} P_i = P_D \tag{2}$$

where P_D is the total network demand. In this work, it is assumed that there are no network losses.

2.3. Spinning reserve constraints

$$\sum_{i \in \Omega} S_i \geq S_R \tag{3}$$

$$S_i = \min\left\{(P_i^{\max} - P_i), S_i^{\max}\right\} \qquad \forall i \in (\Omega - \omega) \tag{4}$$

$$S_\mathrm{i} = 0 \qquad \forall i \in \omega \tag{5}$$

where S_i is the spinning reserve contribution of unit i, S_R is system spinning reserve requirement, $P_i^{\max}$ is the maximum generation limit of unit i, $S_\mathrm{i}^{\max}$ is the maximum spinning reserve contribution of unit i, and ω is the set of on-line units with prohibited operating zones.

2.4. Power output constraints

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad \forall i \tag{6}$$

where $P_i^{\min}$ is the minimum generation limit of unit i.

In addition, ramp-rate limits (up-rate limit, UR_i, down-rate limit, DR_i, and initial power output, P_i^0) further restrict the operating region of all on-line units. These limits are enforced through the following constraints:

$$\max(P_i^{\min}, P_i^0 - DR_i) \le P_i \le \min(P_i^{\max}, P_i^0 + UR_i) \qquad \forall i \tag{7}$$

2.5. Prohibited operating zone constraints

Each generator with *K-1* prohibited zones is characterised by *K* disjoint operating sub-regions $(\hat{P}_{\mathrm{ik}}^{\mathrm{L}}, \hat{P}_{\mathrm{ik}}^{\mathrm{U}})$.

$$\hat{P}_{ik}^{L} \le P_i \le \hat{P}_{ik}^{U} \qquad \forall i \in \omega, k = 1..K \tag{8}$$

Note that $\hat{P}_{i1}^{L} = P_i^{\min}$ and $\hat{P}_{iK}^{U} = P_i^{\max}$ and that only one of the above mutually exclusive constraints should be satisfied.

The optimisation problem described in this section involves a discontinuous objective function and as a result most of the traditional economic dispatch methods, which require continuous cost functions, cannot tackle the above problem. A number of evolutionary programming approaches have recently been proposed in the literature mainly based on genetic and simulated annealing algorithms (see section 1). These latter approaches are efficient but their main drawbacks are lack of guarantee of global optimality in finite time and need of problem-specific parameters. In the next section, the MIQP model is presented for the ED problem. The model can be solved using mathematical programming tools, requiring no problem-specific parameters, and provides a guarantee on global optimality for the particular problem investigated.

3. Mathematical programming approach

The mathematical model described in the previous section exhibits discontinuity in the feasible space defined by the continuous variables. In this section, we reformulate this model using mixed integer optimization techniques to achieve a continuous feasible space, in terms of the continuous variables, through the use of integer variables. First, the following variables are introduced to capture the disjoint operating sub-regions.

Y_{ik} 1 if unit i operates in power output range k; 0 otherwise

Θ_{ik} power output of unit i if operating in range k (*i.e.* if $Y_{ik}=1$ then $\Theta_{ik}=P_i$); 0 otherwise.

Each unit i with prohibited operating zones can operate within only one of the allowed set of ranges.

$$\sum_{k=1}^{K} Y_{ik} = 1 \qquad \forall i \in \omega \tag{9}$$

If unit i operates in range k, then the corresponding Θ_{ik} variable should be equal to P_i otherwise it is forced to the value of zero. This can be achieved by the following two constraints:

$$P_i = \sum_{k=1}^{K} \Theta_{ik} \qquad \forall i \in \omega \tag{10}$$

and

$$\hat{P}_{ik}^{L}\, Y_{ik} \le \Theta_{ik} \le \hat{P}_{ik}^{U}\, Y_{ik} \qquad \forall i \in \omega, k = 1..K \tag{11}$$

It should be noted that constraints (9-11) take the place of the prohibited operating zone constraints (8) from the mathematical model.

The *min* operator involved in the spinning reserve constraint (4) can be expressed mathematically as:

$$S_i \leq P_i^{\max} - P_i \qquad \forall i \in (\Omega - \omega) \tag{12}$$

and

$$S_i \leq S_i^{\max} \qquad \forall i \in (\Omega - \omega) \tag{13}$$

The min and max operators appearing in the ramp constraints (7) can be enforced by combining constraints (6) and the following:

$$P_i^0 - DR_i \leq P_i \leq P_i^0 + UR_i \qquad \forall i \tag{14}$$

Overall, the proposed optimisation formulation for the ED problem with prohibited operating zones can be summarised as follows:

Minimise Objective function (1)

subject to

Power balance constraint (2)

Spinning reserve constraints (3, 5, 12 and 13)

Power output constraints (6, 14)

Prohibited operating zone constraints (9-11)

$Y_{ik} \in \{0,1\}; P_i, S_i, \Theta_{ik} \geq 0$

The resulting mathematical formulation corresponds to an MIQP model, which can be solved to global optimality, due to its convexity, using standard solution techniques such as, for example, branch-and-bound procedures. The applicability of the proposed approach is demonstrated through a number of examples presented in the next section.

4. Computational results

The MIQP model described in the previous section has been implemented in the GAMS system [11] and solved using the SBB and CONOPT solvers either directly or using the NEOS server [12]. Three case studies from the literature are presented to demonstrate the efficiency of the proposed approach and, more importantly, the consistent quality of the solutions obtained. The results obtained are compared with the results presented in the literature for the first two cases studies.

4.1 Example 1: 4 Generators

The first case study is example 3 from Lee & Breipohl [2], involving four on-line units, with the following characteristics:

$$F_i(P_i) = 500 + 10P_i \ + 0.001P_i^2 \ \ \$/Hr$$
$$100MW \le P_i \le 500MW$$
$$S_i^{\max} = 50MW$$

It should be added that units 3 and 4 can operate using the entire power output range while units 1 and 2 have prohibited zones as described in Table 1.

<< Insert Table 1>>

The system of the four units needs to satisfy a total demand of *1375 MW* and the total spinning reserve requirement is *100 MW*.

Our optimal solution has objective function value 16,223.2125 with 0 optimality gap (*i.e.* the solution obtained is the global optimum). The solution is obtained in approximately 0.1s (PC, 2GHz, 1GB RAM). Table 2 compares the results we obtain (indicated as MIQP in the table) with those obtained by Lee & Breipohl [2] and

Jayabarathi *et al*. [5]. In this table, and in the corresponding table for the second case study, we present not only the best solution obtained in each case but also the average solution obtained. Previous work in this area has concentrated on the use of stochastic methods. Stochastic methods will typically identify a different solution each time they are applied. Therefore, a proper characterization of the performance of such a method should include a statistical analysis of the results including, at a minimum, the average solution obtained. Unfortunately, Jayabarathi et al. [5] did not provide this information so a true comparison is difficult. In any case, the solution we obtain, identified as the MIQP method, is globally optimal and agrees with the best solution obtained by Jayabarathi et al. [5] and improves on that obtained by Lee & Breipohl [2].

<< Insert Table 2>>

4.2. Example 2: 15 generators

The second case study is a larger example, introduced by Lee & Breipohl [2]. This case study is meant to represent a more realistic case study, one which should provide a test of the capabilities of optimisation procedures. The system comprises 15 on-line units that satisfy a demand of 2650 MW and a system spinning reserve requirement of 200 MW.

<<Insert Table 3>>

<<Insert Table 4>>

Tables 3 and 4 show the parameters used by Lee & Breipohl [2] for this case study. Table 5 shows the results we obtain for this problem, again with results from the literature quoted for comparison, noting that we have included statistical information where this is appropriate and available. The result we obtain, in less than 0.25s, is globally optimal and agrees with the best solution obtained by Somasundaram et al. [7].

The mathematical programming approach always yields the same result, as expected. This is not the case for the stochastic procedures. Although Somasundaram et al. [7] do not present any statistical analysis relating to the frequency with which the global optimum is obtained, Su & Chiang [8] do report such statistics. The table shows that the proposed MIQP method improves or matches the results in the literature, both in the best case scenario and in the average case.

<<Insert Table 5>>

Interestingly, a few years after the work by Lee & Breipohl [2], Orero & Irving [4] presented a genetic algorithm for a slightly modified version of this problem. The modified version of this problem is essentially the same as that presented by Lee & Breipohl [2] except for changes to three parameters. Specifically, the changes are two of the cost coefficients, b_8 = 11.21 instead of 11.50 and b_{11} = 10.21 instead of 11.21, and one of the bounds on the power generated, the lower bound on the 5th generator being 105 MW instead of 150 MW. Subsequently, this slightly modified example has been tackled by a number of researchers and we have solved it as well. The results are shown in Table 6 where again we compare not just the best solution obtained by each but also the average solution, where this has been reported.

From Table 6, we can see that the proposed mathematical programming approach finds a solution better than any previously reported in the literature. This solution is globally optimal and, due to the deterministic nature of mathematical programming, is found every time. The solution is again obtained in approximately 0.25s.

<<Insert Table 6>>

4.3. Example 3: 40-unit example

This example is based on the second example of Naresh et al., (2004) without transmission loses in order to demonstrate the efficiency and the scalability of the proposed approach. This case study comprises 40 units, 25 of which exhibit prohibited zones (up to three distinct prohibited zones per unit). The total load demand used is 7000 MW. The complete dataset required for this case study is presented in tables 4 and 7 from Naresh et al. (2004). The problem has been solved to global optimality using the proposed mathematical programming approach in 0.186 CPU seconds. The optimal value of the objective function is $100767.6872. Table 7 presents the optimal power outputs for all 40 units.

<<Insert Table 7>>

5. Conclusions

A mathematical programming approach for the economic dispatch problem with prohibited zones has been developed. Results obtained with this approach for a number of case studies from the literature have been presented. A comparison with other approaches shows that the mathematical programming approach always obtains a solution at least as good as any reported. In the last case study, a solution is obtained that is better than any previously reported for the case study.

The advantage of a mathematical programming approach is the consistency of results and the guarantee on solution quality obtained. Although stochastic methods appeal due to their typical ease of implementation, the existence of well established mathematical programming systems, such as GAMS with the assorted solvers which can be accessed (for example, through the NEOS server), means that using mathematical programming is just as easy. Nevertheless, stochastic methods may be appropriate for case studies where mathematical programming demand considerable computational effort.

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List of Tables

Table 1: Prohibited zones for example 1

Unit	Zone 1(MW)	Zone 2 (MW)
1	[200-250]	[300-350]
2	[210-260]	[310-360]

Table 2: Comparison of results for example 1

Method	Best	Average	Reference
λ - δ	16224.175	16224.175	Lee & Breipohl [2]
EP	16223.213		Jayabarathi et al. [5]
MIQP	16223.21256	16223.213	

Table 3: Unit characteristics for example 2

Unit	a_i (\$/Hr)	b_i (\$/MWHr)	c_i (\$/MWHr^2)	P_i^{min} (MW)	P_i^{max} (MW)	S_i^{max} (MW)
1	671.03	10.07	0.000299	150	455	50
2	574.54	10.22	0.000183	150	455	0
3	374.59	8.8	0.001126	20	130	30
4	374.59	8.8	0.001126	20	130	30
5	461.37	10.4	0.000205	150	470	0
6	630.14	10.1	0.000301	135	460	0
7	548.2	9.87	0.000364	135	465	50
8	227.09	11.5	0.000338	60	300	50
9	173.72	11.21	0.000807	25	162	30
10	175.95	10.72	0.001203	20	160	30
11	186.86	11.21	0.003586	20	80	20
12	230.27	9.9	0.005513	20	80	0
13	225.28	13.12	0.000371	25	85	20
14	309.03	12.12	0.001929	15	55	40
15	323.79	12.41	0.004447	15	55	40

Table 4: Prohibited zones for example 2

Unit	Zone 1(MW)	Zone 2 (MW)	Zone 3 (MW)
2	[185-225]	[305-335]	[420-450]
5	[180-200]	[260-335]	[390-420]
6	[230-255]	[365-395]	[430-455]
12	[30-55]	[65-75]	-

Table 5: Results for example 2: 15-unit problem from Lee & Breipohl [2]

Method	Best	Average	Reference
λ - δ	32549.8	32549.8	Lee & Breipohl [2]
IGANUM		32544.99	Su & Chang [8]
FCEPA	32544.97		Somasundaram et al. [7]
MIQP	32544.97	32544.97	

Table 6: Results for example 2: 15-unit problem as presented by Orero & Irving [4]

Method	Best	Average	Reference
SGA-MR	32523	32537.1	Orero & Irving [4]
SGA-SR	32517	32559.6	Orero & Irving [4]
DCGA	32514	32535.1	Orero & Irving [4]
ETQ	32507.5		Lin et al. [6]
FEP	32507.55		Jayabarathi et al. [1]
CEP	32507.55		Jayabarathi et al. [1]
IFEP	32507.46		Jayabarathi et al. [1]
ESO	32506.6		Pereira-Neto et al. [3]
PSO	32506.3		Jeyakumar et al. [13]
MIQP	32506.14	32506.14	

Table 7: Optimal solution for example 3: 40-unit problem (all levels in MW)

Unit	Level	Unit	Level	Unit	Level	Unit	Level
1	40.544	11	205.000	21	456.665	31	20.000
2	60.000	12	205.000	22	460.000	32	20.000
3	140.453	13	125.000	23	460.000	33	20.000
4	24.000	14	132.089	24	460.000	34	20.000
5	26.000	15	125.000	25	460.000	35	18.000
6	115.000	16	125.000	26	460.000	36	18.000
7	110.000	17	125.000	27	460.000	37	20.000
8	217.000	18	456.665	28	10.000	38	25.000
9	265.000	19	458.918	29	10.000	39	25.000
10	130.000	20	456.665	30	10.000	40	25.000