Competitive Nonlinear Pricing and Bundling*

Mark Armstrong
Department of Economics
University College London

John Vickers
Department of Economics
University of Oxford

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Abstract

We examine the impact of multiproduct nonlinear pricing on profit, consumer surplus and welfare in a duopoly. When consumers buy all their products from one firm (the one-stop shopping model), nonlinear pricing leads to higher profit and welfare, but usually lower consumer surplus, than linear pricing. By contrast, in a unit-demand model where consumers may buy one product from one firm and another product from another firm, bundling generally acts to reduce profit and welfare and to boost consumer surplus. In a more general model where consumers may buy from more than one firm and where consumers have elastic demands for each product, nonlinear pricing has ambiguous effects. Compared with linear pricing, nonlinear pricing tends to raise profit but harm consumer surplus when: (i) demand is elastic, (ii) there is substantial heterogeneity in consumer demand, (iii) consumers face substantial shopping costs when visiting more than one firm, and (iv) a consumer's brand preference for one product is strongly correlated with her brand preference for another product. Nonlinear pricing is more likely to lead to welfare gains when (i), (iii) and (iv) hold, but (ii) does not.

1 Introduction

Most economic analysis of imperfect competition is based on the assumption of linear pricing, where the price of a combination of purchases from a firm, whether of one or more products, is equal to the sum of the prices of the component parts. While many markets operate on that basis, an increasing number feature nonlinear pricing—for example, discounts for purchases of larger volumes or of more products. In the absence of easy arbitrage, which

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would undermine it, such pricing can be observed in more or less competitive markets as well as in some with market power.

Examples include energy markets, which traditionally were monopolies but are now open to varying degrees of competition. Consumers are often able to buy their gas and electricity from a single supplier or from two suppliers. In each case, consumers typically face nonlinear tariffs, and they will often enjoy an additional discount if they purchase both services from a single supplier. Similarly, consumers nowadays can often source telecommunications, cable television and internet services from a single supplier or from several, and nonlinear pricing and bundling are commonplace. Nonlinear pricing and (inter-temporal) bundling is also to be seen in areas such as air travel and supermarkets, where loyalty schemes have become more prevalent with the advent of electronic point-of-sale information. The economic importance of nonlinear pricing goes wider still. Thus labour contracts may include extra pay for longer tenure, so that a two-period worker gets a better deal than two one-period workers.[1] While the provision of incentives is doubtless a prime reason for such nonlinear wages, the nature of multi-period competition by firms for labour may also be relevant, and there may be parallels with multi-product competition among firms for consumers.

There is an extensive literature on nonlinear pricing and bundling.[2] In the context of monopoly supply, analyses include Adams and Yellen (1976), McAfee, McMillan, and Whinston (1989), Armstrong (1996), and Rochet and Choné (1998). In particular, the latter two papers demonstrate the complexity of optimal multiproduct nonlinear pricing for a monopolist, and the profit-maximizing tariff can be derived only in a few isolated examples. There is also a growing literature on (more or less) competitive nonlinear pricing. For example Spulber (1979), Stole (1995), Armstrong and Vickers (2001), Rochet and Stole (2002) and Yin (2004) examine competitive nonlinear pricing in situations in which each consumer purchases all products from a single supplier. The latter three papers suggest that marginal-cost pricing often emerges as a nonlinear pricing equilibrium, in which case welfare is boosted when firms offer such tariffs compared with linear pricing.[3] More generally, a theme in Armstrong and Vickers (2001) is that when consumers are one-stop shoppers, firms choose socially desirable tariffs if given freedom to do so.

But how reasonable is this one-stop shopping framework? It seems implausible in various settings, including dynamic ones where we expect to see some switching between firms by consumers. The competitive bundling literature accordingly investigates nonlinear pricing when some consumers wish to buy products from more than one supplier. Much of the literature on bundling has focussed on anti-competitive behaviour by a multiproduct incumbent facing a potential single-product entrant.[4] More relevant for our purposes, however, is

[1] See, for example, Stevens (2004).

[2] Armstrong (2006) and Stole (2006) are recent surveys.

[3] There is little systematic comparison between nonlinear and linear pricing in terms of profit and consumer surplus in those papers, although Armstrong and Vickers (2001) and Yin (2004) show when consumers have homogeneous demands that profit rises and consumer surplus falls when two-part tariffs are employed.

[4] See Whinston (1990) and Nalebuff (2004), for instance.

the small literature which assumes a more symmetric market structure, including the contributions by Matutes and Regibeau (1992), Anderson and Leruth (1993), Reisinger (2006) and Thanassoulis (2006). This work tends to proceed by way of numerical examples and assumes that consumers wish to buy only one unit of a product. Moreover, the models make the polar opposite assumption from the one-stop shopping models, that consumers face no intrinsic extra "shopping costs" if they buy from more than one firm. The analysis of such models suggests, quite contrary to that of one-stop shopping models, that bundling tends to harm profit and welfare but to be pro-consumer. So different strands of the literature on competitive nonlinear pricing and bundling have yielded starkly conflicting results about the pros and cons of nonlinear pricing, which our analysis seeks to reconcile.

The rest of the paper, and the main results, can be summarized as follows. Our point of departure in section 2 is a Hotelling model of product differentiation with one-stop shopping and consumers with heterogeneous and elastic demands.[5] We show that with nonlinear pricing and all consumers served, the unique symmetric equilibrium of the model has, in effect, two-part tariffs with marginal prices equal to marginal costs. Welfare is maximized with such nonlinear pricing, whereas with linear pricing there is the problem of *excessive marginal prices*. Profit is also higher with nonlinear pricing for two reinforcing reasons, which relate to elasticity of demand and the extent of consumer heterogeneity. Consumer surplus is however typically higher with linear pricing.

A limitation of this model is the assumption that each consumer patronizes just one firm. Section 3 examines the possibility of two-stop shopping—where consumers choose on the basis of prices and brand preferences whether to buy from both firms or just one—in a two-dimensional model of product differentiation with inelastic demand.[6] Discounts for joint purchase—mixed bundling—are a general feature of the nonlinear pricing equilibrium, and have a simple characterization: the elasticity of "demand for two-stop shopping" is equal to 2 in equilibrium. These discounts, however, give rise to the problem of *excessive loyalty*, in that there is too much one-stop shopping. Therefore, in contrast to the one-stop shopping model, welfare is lower when nonlinear pricing is used. Remarkably, the profit and consumer surplus comparisons between linear and nonlinear pricing are also exactly the opposite of those from the one-stop shopping model.

A unifying model—which allows for consumers to have elastic and heterogeneous demands and to purchase from more than one supplier—is analyzed in section 4. This potentially complex model has a remarkably simple equilibrium when nonlinear tariffs are used: firms offer efficient two-part tariffs—i.e., with marginal prices equal to marginal costs—while the fixed elements of the tariffs are precisely the same equilibrium prices, with the same discount for one-stop shopping, as in the model of section 3 with inelastic demand. Therefore, the nonlinear pricing equilibrium, unlike that with linear pricing, is free from the excessive marginal price problem, but it does suffer from the excessive loyalty problem. The models in

[5]This develops the models of Armstrong and Vickers (2001, section 4) and Rochet and Stole (2002).

[6]This develops the model of Matutes and Regibeau (1992).

sections 2 and 3 each had just one of these effects; hence their contrasting results. Perhaps surprisingly, the analysis of competitive nonlinear pricing in our model turns out to be considerably simpler than either monopoly nonlinear pricing or competitive linear pricing.

Section 5 pursues the comparison between linear and nonlinear pricing in terms of five underlying economic influences: (i) demand elasticity, (ii) consumer heterogeneity, (iii) shopping costs of buying from more than one supplier, (iv) correlation in brand preferences, and (v) product differentiation. The impact of the last of these influences is not straightforward, but that of the first four is relatively clear-cut, and Table 1 indicates whether an increase in each of them tends to add to or subtract from the merits of nonlinear pricing relative to linear pricing for welfare, profit and consumer surplus.

	Welfare	Profit	Consumers
(i) demand elasticity	?	+	−
(ii) consumer heterogeneity	−	+	−
(iii) shopping costs	+	+	−
(iv) brand preference correlation	+	+	−

Table 1: Effects on the relative merits of nonlinear pricing

The excessive marginal prices effect becomes more important relative to the excessive loyalty effect—which favours nonlinear pricing in welfare terms—as demand elasticity, consumer homogeneity, shopping costs and brand preference correlation increase. For example, the excessive marginal prices effect and the excessive loyalty effect respectively increase in importance with more elastic demand (because deadweight loss 'triangles' open up) and as shopping costs fall (because more consumers two-stop shop). However, the impact of elasticity on welfare is ambiguous because elastic demand intensifies competition under linear pricing, and this may end up reducing the equilibrium dead-weight losses associated with linear pricing. The impact of product differentiation (v) is generally mixed, as we illustrate.

The direction of effects on profit tends to be the opposite of that on consumer surplus. In respect of (iii) and (iv), the profit effect has the same sign as the welfare effect. But with greater consumer heterogeneity, the relative merits of nonlinear pricing for welfare and consumers tend to be lower. That is because heterogeneity sharpens linear price competition, which is good for consumers and welfare (as the excessive marginal price effect diminishes) but is bad for profit.

Section 6 concludes by suggesting directions for future research on competitive nonlinear pricing and bundling.

2 Nonlinear Pricing and One-Stop Shopping

Two firms, denoted A and B, each supply the same set of $n \geq 1$ products. For exogenous reasons, suppose consumers buy all their suppliers from one firm or the other, i.e., consumers are one-stop shoppers. Consumers differ in their brand preference for the two firms,

x, and their preference for the n products, θ. The (scalar) parameter x is uniformly distributed between 0 (where firm A is located) and 1 (where B is located), while the (possibly multi-dimensional) parameter θ is independently distributed according to some distribution function $F(\theta)$.[7] Gross utility (excluding transport costs and payments to firms) is $u(\theta, q)$ if a type-θ consumer buys quantities q. Lump-sum transport cost is t per unit of distance. In sum, a type-(x, θ) consumer's net utility if she buys quantities q^A from firm A in return for payment T^A is

$$u(\theta, q^A) - tx - T^A \ ,$$

and if she buys quantities q^B from B with payment T^B her utility is

$$u(\theta, q^B) - t(1-x) - T^B \ .$$

Finally, let

$$v(\theta, p) = \max_q : u(\theta, q) - pq \tag{1}$$

denote type-θ consumer surplus with linear prices $p = (p_1, ..., p_n)$.

In the following analysis, we make the important assumption, which is satisfied subject to conditions on willingness-to-pay relative to costs, that all consumers choose to participate in the market with the relevant range of tariffs. Finally, suppose each firm has constant unit cost c_i for supplying product i.

Our first result derives the equilibrium nonlinear tariff, which we will use to compare with linear pricing.[8,9]

Proposition 1 *Suppose that over the relevant range of nonlinear tariffs all consumers are served. Then the unique symmetric equilibrium outcome with nonlinear pricing involves efficient consumption, and a consumer who buys quantities q makes payment*

$$T(q) = t + \sum_i c_i q_i \ . \tag{2}$$

Equilibrium industry profit is $\pi_{NL} = t$.

Proof. That the cost-based two-part tariff in (2) is an equilibrium was established in Armstrong and Vickers (2001, Proposition 5). That this tariff induces the *unique* symmetric

[7] The assumption that x is uniformly distributed is made purely for expositional convenience, and none of the following results depend on this assumption. If x were instead distributed on $[0, 1]$ with density $f(x)$ which is symmetric about $x = \frac{1}{2}$, then subject to regularity conditions on $f(\cdot)$ which ensure a symmetric equilibrium exists, the following results remain valid if 't' is replaced by '$t/f(\frac{1}{2})$'.

[8] Note that this result need not hold if (i) θ and x are correlated, (ii) firms have different costs, or (iii) if some consumers do not buy from either firm. See Rochet and Stole (2002) for further details.

[9] We do not claim that the tariff in (2) is the unique symmetric *tariff*. For instance, if all consumers purchased quantities above some positive lower bound, how the tariff is defined for quantities below this lower bound cannot be uniquely determined.

equilibrium outcome can be argued by contradiction. Suppose that in an alternative symmetric equilibrium, industry profit is Π. Suppose further that the equilibrium tariff $T(\cdot)$ does not always implement efficient consumption. Suppose firm A deviates from the candidate equilibrium, and instead sets the efficient two-part tariff $\hat{T}(q) = \Pi + \sum_i c_i q_i$. A consumer located at x will buy from firm A if $v(\theta, c) - \Pi - tx \geq V(\theta) - t(1-x)$, where

$$V(\theta) = \max_q : u(\theta, q) - T(q) \ .$$

Let

$$x(\theta) = \frac{1}{2} + \frac{v(\theta, c) - \Pi - V(\theta)}{2t}$$

be the marginal x consumer in the θ-segment. Then A's deviation profit is $E_\theta[x(\theta)] \times \Pi$, and so the deviation is profitable if $E_\theta[x(\theta)] > \frac{1}{2}$, i.e., if

$$E_\theta[v(\theta, c)] > \Pi + E_\theta[V(\theta)] \ .$$

(Here and elsewhere, $E_\theta[\cdot]$ refers to taking expectations with respect to θ using $F(\theta)$.) However, the left-hand side of the above is total welfare with marginal-cost pricing, and the right-hand side is total welfare with the candidate tariff T (which by assumption does not always involve marginal-cost pricing). Therefore, this inequality is indeed satisfied, and the unique symmetric equilibrium outcome involves efficient consumption, which is implemented by marginal-cost pricing. ■

Consider next the outcome when linear prices are used. Clearly, the marginal-cost pricing that is implemented by nonlinear pricing is unambiguously beneficial for welfare, as measured by the sum of consumer surplus and profit. (With linear prices, firm will set positive price-cost margins since that is the only way they can generate profit.) A more subtle issue is the impact of nonlinear pricing on profit and on consumer surplus. Suppose that the type-θ consumer has demand functions $q_i(\theta, p)$, where p is a vector of linear prices. (These demands are the quantities q which solve (1).) Let $\pi(\theta, p)$ be type-θ profit, i.e., $\pi(\theta, p) \equiv \sum_{i=1}^n q_i(\theta, p)(p_i - c_i)$. It is convenient to introduce one further piece of notation: for a candidate symmetric equilibrium price vector p, define

$$s(\theta, r) \equiv v(\theta, c + r(p - c)) \ ,$$

where r is a scalar. This allows us to conduct the analysis in terms of the scalar r rather than the vector p. Then $s(\theta, 0) = v(\theta, c)$ and $s(\theta, 1) = v(\theta, p)$. Since v is convex in p, s is convex in r and so $s_{rr}(\theta, r) \geq 0$. Note that

$$\pi(\theta, c + r(p - c)) = -r s_r(\theta, r) \ .$$

In particular $\pi(\theta, p) = -s_r(\theta, 1)$. For p to be a symmetric equilibrium price vector, it is necessary that $r = 1$ maximizes a firm's profit, which is

$$E_\theta\left[\left(\frac{1}{2} + \frac{s(\theta, r) - s(\theta, 1)}{2t}\right)(-r s_r(\theta, r))\right] \ .$$

This has the first-order condition[10]

$$\pi_L = E_\theta[-s_r(\theta,1)] = E_\theta\left[s_{rr}(\theta,1) + \frac{(s_r(\theta,1))^2}{t}\right] \ , \tag{3}$$

where π_L is equilibrium industry profit with linear prices.

Expression (3) implies that

$$\pi_L = E_\theta\left[s_{rr}(\theta,1) + \frac{(s_r(\theta,1))^2}{t}\right] \geq \frac{1}{t}E_\theta\left[(s_r(\theta,1))^2\right] \geq \frac{1}{t}(E_\theta[s_r(\theta,1])^2 = \frac{1}{t}\pi_L^2 \ . \tag{4}$$

The first inequality follows since $s_{rr} \geq 0$, and it is strict if consumer demand is elastic. We refer to this as the "elasticity" effect. The second inequality follows from Jensen's inequality. It is strict if there is some consumer heterogeneity (i.e., if the variance of $s_r(\theta,1)$ is positive); this is the "heterogeneity" effect. Expression (4) shows that $\pi_L \leq t = \pi_{NL}$. Hence profits are lower when linear pricing is employed than with nonlinear pricing.[11] The reason for this is a combination of the elasticity and heterogeneity effects, both of which act in the same direction. (These two effects are discussed in more detail in sections 5.1 and 5.2.)

Turn next to the impact on consumer surplus. Suppose that total welfare is concave in linear prices. This implies that welfare expressed as a function of r, which is

$$w(r) \equiv E_\theta[s(\theta,r) - rs_r(\theta,r)] \ ,$$

is concave in r. In particular, $w(r)$ lies below its tangent at $r=1$. Since welfare with linear pricing is $w(1)$ and welfare with nonlinear pricing (which involves marginal-cost pricing) is $w(0)$, it follows that the welfare difference $\Delta w = w(0) - w(1)$ satisfies

$$\Delta w \leq -w'(1) = E_\theta[s_{rr}(\theta,1)] \ .$$

From (4) we see

$$\Delta w \leq \pi_L - \frac{1}{t}E_\theta\left[(s_r(\theta,1))^2\right] \leq \pi_L - \frac{1}{t}\left(E_\theta\left[s_r(\theta,1)\right]\right)^2 = \pi_L - \frac{1}{t}\pi_L^2 = \frac{\pi_L}{t}(t-\pi_L) \leq t - \pi_L \ ,$$

where the final inequality follows from our previous result that $\pi_L \leq t$. Therefore, the welfare gain is less than the gain in industry profit, and so consumers in aggregate are worse off when nonlinear tariffs are used.

We can summarise our analysis of the one-stop shopping model as:

[10] The second-order condition on r is satisfied provided $E_\theta[s_{rrr}(\theta,1)]$ is not strongly negative, which we assume henceforth.

[11] Note that this profit comparison seems to be valid in an alternative framework in which consumers incur transport costs on a per-unit rather than a lump-sum basis (at least when transport costs are small), although the analysis is considerably more complicated. For instance, see Proposition 5 in Yin (2004) for analysis with linear demand. One major difference is that the equilibrium two-part tariffs are not efficient, and marginal price is above marginal cost.

Proposition 2 *Suppose that over the relevant range of tariffs all consumers are served. Then compared to the outcome with linear pricing, industry profit and total welfare are higher with nonlinear pricing. In addition, if welfare is concave in linear prices, consumer surplus is lower with nonlinear pricing.*

In the unique symmetric equilibrium with nonlinear pricing, firms use cost-reflective two-part tariffs. The optimal fixed fee in such a tariff balances (i) the firm's loss of profit on existing consumers against (ii) its gain in profitable consumers from the other firm. Demand per consumer does not change as the fixed fee varies: it is as if consumers had identical inelastic demands. By contrast, if firms are restricted to linear prices, a firm choosing its price(s) will balance (i) against not only (ii) but also (iii) its gain in average profit per consumer. Profit per consumer can change for two reasons. First, with elastic demands, lower prices expand demand from each type of consumer. Second, and less obviously, if consumers are heterogeneous, lowering prices can alter the *mix* of consumer types coming to a firm—in particular by winning proportionately more high demand (hence high profit) consumers than low demand consumers from the other firm. This second reason why average profit per consumer may rise as linear prices are reduced occurs even if consumers have inelastic demands. These two reasons are reflected in the two inequalities in (4). They are both reasons why there is more incentive to lower prices—hence why consumers do better—with linear pricing. Yet welfare is higher with nonlinear pricing because there is no marginal price inefficiency. It follows that profits must be higher then too.

3 Bundling and Two-Stop Shopping

In this section we analyze a model of multi-product competition where consumers can "mix-and-match" products from two firms. We assume here that consumers have unit demands. This case is of interest in its own right; it will also provide the key to the more general elastic demand analysis in sections 4 and 5.

3.1 The Framework

Consider a two-dimensional Hotelling model. Two firms, A and B, each offer their own brand of two products, 1 and 2. The location (or brand preference) of a consumer is denoted $(x_1, x_2) \in [0,1]^2$. Let x_1 represent a consumer's distance from firm A's brand of product 1 and x_2 represent a consumer's distance from the same firm's brand of product 2. The density of (x_1, x_2) is $f(x_1, x_2)$ and firms are symmetrically placed in terms of consumers:

$$f(x_1, x_2) \equiv f(1-x_1, 1-x_2) \ . \tag{5}$$

The transport cost (or product differentiation) parameter is t_1 for product 1 and t_2 for product 2. In addition to these transport costs, consumers face an exogenous "shopping cost" $z \geq 0$ when they source supplies from two firms. This shopping cost might represent the time or cost involved in visiting two shops rather than one, or it might measure a consumer's perceived cost of dealing with two firms (paying two bills rather than one, for instance). In order to have some two-stop shoppers in equilibrium, it is necessary to constrain the shopping cost not to be too large, and we assume that

$$z < \min\{t_1, t_2\} \ .$$

If this inequality does not hold, all consumers will be one-stop shoppers, and with unit demands the distinction between bundling and linear pricing vanishes.

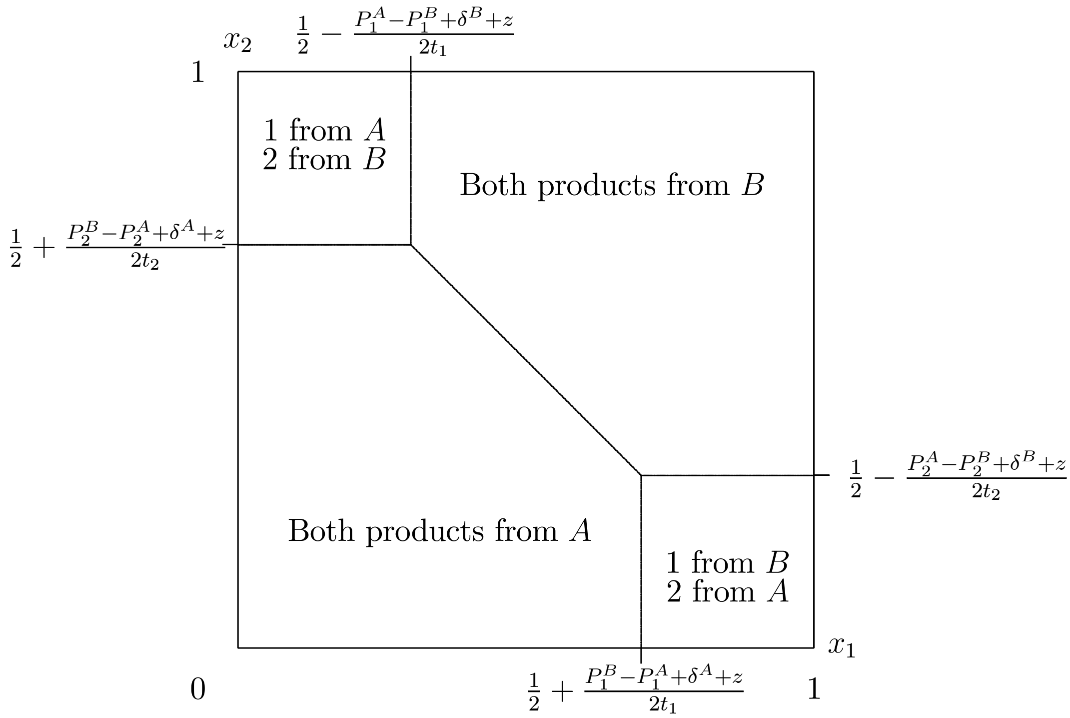

Figure 1: The Pattern of Consumer Demand

Since consumers have unit demand for each product, a firm's tariff consists of three prices. Let P_1^i denote firm i's stand-alone price for its product 1, let P_2^i be its stand-alone price for its product 2, and let δ^i be its discount if a consumer buys both products, so the total charge for buying both products from firm i is $P_1^i + P_2^i - \delta^i$. For simplicity, suppose that conditions in the market are such that all consumers buy both products. The type-(x_1, x_2) consumer's total outlay if she buys both products from firm A is $P_1^A + P_2^A - \delta^A + t_1 x_1 + t_2 x_2$, her total outlay if she buys both products from B is $P_1^B + P_2^B - \delta^B + t_1(1-x_1) + t_2(1-x_2)$, and her total outlay if she buys product i from A and product $j \neq i$ from B is $P_i^A + P_j^B + t_i x_i + t_j(1-x_j) + z$.

The consumer located at (x_1, x_2) will choose the option from among the four possibilities which involves the smallest outlay. The resulting pattern of demand is shown in Figure 1.

For simplicity of notation, suppose that production is costless.[12] In this case, when firms offer the same tariff with stand-alone prices P_1 and P_2 and discount (if applicable) δ, industry profit is

$$(P_1 + P_2) - \delta \times \{\text{proportion of one-stop shoppers}\} \ . \tag{6}$$

In the following analysis, it is useful to introduce some further notation.

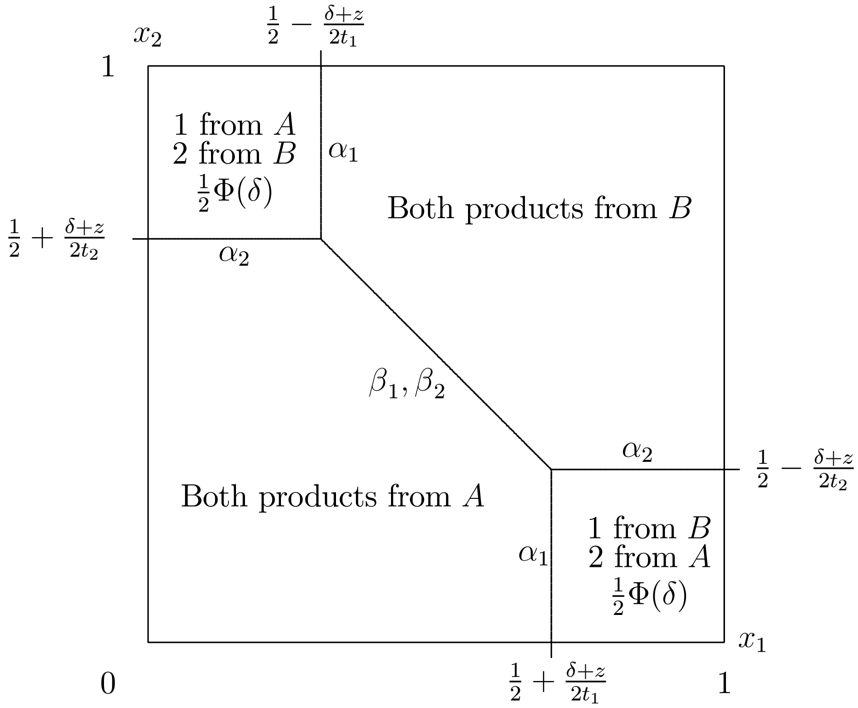

Figure 2: Notation Used in the Analysis

Define the function

$$\Phi(\delta) \equiv 2 \int_0^{\frac{1}{2}-\frac{\delta+z}{2t_1}} \int_{\frac{1}{2}+\frac{\delta+z}{2t_2}}^{1} f(x_1, x_2) \ dx_2 dx_1$$

to be the proportion of consumers who are two-stop shoppers when firms set the same tariff which involves discount δ. The discount δ can be viewed as the "price" for two-stop versus

[12] This is without loss of generality if marginal costs are constant, and the prices P_i derived below can be considered to be prices net of marginal costs. However, it does rule out cases where it is cheaper for a firm to supply the two products together than to supply them separately.

one-stop shopping, and $\Phi(\delta)$ is downward-sloping demand for two-stop shopping.[13] This demand function summarizes many of the economically-relavant features of this market, and it is depicted on Figure 2. (The symmetry assumption (5) implies that the two rectangles of two-stop shoppers each contain the same proportion $\Phi/2$ of consumers.)

It will be useful to know the derivative of demand Φ. Write

$$\alpha_1(\delta) = \int_{\frac{1}{2}+\frac{\delta+z}{2t_2}}^{1} f(\tfrac{1}{2} - \frac{\delta+z}{2t_1}, x_2)\, dx_2 \ ; \ \alpha_2(\delta) = \int_0^{\frac{1}{2}-\frac{\delta+z}{2t_1}} f(x_1, \tfrac{1}{2} + \frac{\delta+z}{2t_2})\, dx_1 \tag{7}$$

for the line integrals depicted on Figure 2. (From (5), the two integrals marked α_1 have the same value, as do those marked α_2.) Then

$$\Phi'(\delta) = -\left(\frac{\alpha_1(\delta)}{t_1} + \frac{\alpha_2(\delta)}{t_2}\right) . \tag{8}$$

Finally, write

$$\beta_1(\delta) = \int_{\frac{1}{2}-\frac{\delta+z}{2t_2}}^{\frac{1}{2}+\frac{\delta+z}{2t_2}} f(\tfrac{1}{2} + \frac{t_2}{t_1}(\tfrac{1}{2} - x_2), x_2)\, dx_2 \ ; \ \beta_2(\delta) = \int_{\frac{1}{2}-\frac{\delta+z}{2t_1}}^{\frac{1}{2}+\frac{\delta+z}{2t_1}} f(x_1, \tfrac{1}{2} + \frac{t_1}{t_2}(\tfrac{1}{2} - x_1))\, dx_1 \tag{9}$$

for the line integrals along the diagonal segment depicted on Figure 2. Here, β_1 is the integral in the vertical direction and β_2 is the integral in the horizontal direction, and they are related by $t_2\beta_1 = t_1\beta_2$.

3.2 Linear Pricing

Consider first the case where firms compete with linear prices—that is to say with no bundling discounts in the present context—so $\delta^A = \delta^B = 0$. For simplicity, write $\alpha_i^0 = \alpha_i(0)$ and $\beta_i^0 = \beta_i(0)$. Suppose the two firms initially set the same pair of linear prices P_1 and P_2. Consider firm A's incentive to reduce its price P_1 by ε, keeping its price for product 2 unchanged. At a symmetric equilibrium, half the consumers buy product 1 from firm A, and the firm loses revenue ε from each of these infra-marginal consumers. The price reduction shifts the boundary of the set of consumers who buy product 1 from the firm uniformly to the right by $\varepsilon/(2t_1)$, as depicted on Figure 3.

The profit from these marginal consumers is not constant along this boundary. Those consumers on the two vertical boundaries α_1^0 on the figure each generate profit P_1 to the firm, while the one-stop shoppers on the diagonal boundary β_1^0 generate "double" profit (P_1+P_2). The total profit of these marginal consumers is therefore

$$\frac{\varepsilon}{2t_1} \times \left\{2\alpha_1^0 P_1 + \beta_1^0(P_1 + P_2)\right\} .$$

[13] The "price" of two-stop shopping δ is additional to the exogenous shopping cost z, so the overall cost to the consumer of buying from two firms is $\delta + z$.

Since the profit gained from the marginal consumers must equal the profit lost from the infra-marginal consumers, it follows that in equilibrium

$$2\alpha_1^0 P_1 + \beta_1^0(P_1 + P_2) = t_1 \ . \tag{10}$$

Similarly, the first-order condition for the linear price P_2 to be part of an equilibrium is

$$2\alpha_2^0 P_2 + \beta_2^0(P_1 + P_2) = t_2 \ . \tag{11}$$

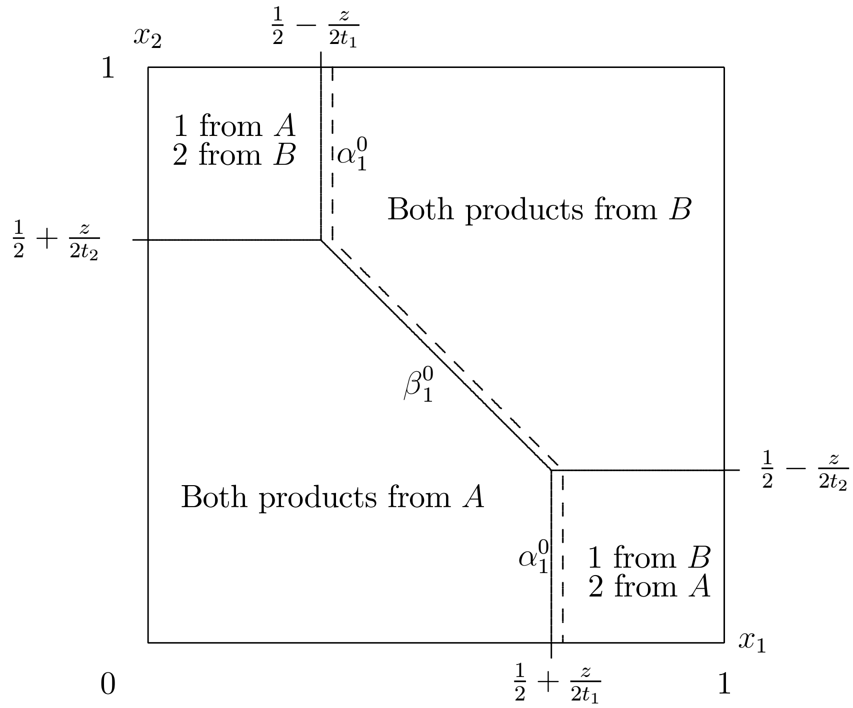

Figure 3: Incentive to Reduce Linear Price for Product 1

Solving the linear equations (10)–(11) yields explicit formulae for equilibrium prices:

$$P_1 = \frac{\alpha_2^0 t_1}{\alpha_1^0\beta_2^0 + \beta_1^0\alpha_2^0 + 2\alpha_1^0\alpha_2^0} \ ; \ P_2 = \frac{\alpha_1^0 t_2}{\alpha_1^0\beta_2^0 + \beta_1^0\alpha_2^0 + 2\alpha_1^0\alpha_2^0} \ . \tag{12}$$

In most cases, prices and profit fall as the shopping cost z becomes greater.[14] The reason is that, when z is large the number of marginal one-stop shopping consumers (measured

[14]For instance, in the case where preferences for the products are symmetric, so that $t_1 = t_2$, $\alpha_1^0 = \alpha_2^0 = \alpha^0$ and $\beta_1^0 = \beta_2^0 = \beta^0$, expression (12) shows that the equilibrium linear price falls with z whenever $\alpha^0 + \beta^0$ rises with z, which is true for a wide range of distributions for (x_1, x_2).

by β_i^0) increases. These marginal consumers are "doubly profitable" with their two-unit demands. So firms compete hard to attract these consumers, with the result that prices decrease with z. We will see in the next section that when firms offer bundling discounts, this acts in a manner similar to exogenous shopping costs to intensify competition.

It is useful to illustrate these, and subsequent, results by means of a simple example.

Uniform Example: $t_1 = t_2 = t > z$ and $f(x_1, x_2) \equiv 1$.

Here, $\alpha_1^0 = \alpha_2^0 = \frac{1}{2} - \frac{z}{2t}$ and $\beta_1^0 = \beta_2^0 = \frac{z}{t}$. From (12) the equilibrium linear price for each product is

$$P_1 = P_2 = \frac{t^2}{t+z} \ , \tag{13}$$

which is decreasing in z.[15]

3.3 Bundling

Suppose next that firms can offer discounts for joint consumption. It turns out that a firm always has a unilateral incentive to do so.

Proposition 3 *Suppose the two firms initially offer the equilibrium linear prices (12). Then a firm's profit increases if it unilaterally introduces a small discount $\delta > 0$ for joint purchase.*

Proof. Suppose both firms initially offer the linear prices in (12), and consider the effect on firm A's profit when it introduces a small joint-purchase discount $\delta > 0$. Let $N < \frac{1}{2}$ denote the proportion of consumers who choose to buy both items from firm A when symmetric linear prices are offered. Then the firm loses revenue δ from the N consumers who previously purchased both products from it in any case, but the discount induces some two-stop shoppers to buy both products from A and also some some one-stop shoppers who previously purchased exclusively from B to buy from exclusively from A.

Specifically, from Figure 4 one sees that $\delta\alpha_1^0/(2t_1)$ consumers switch from buying product 1 from B and product 2 from A to buying both from A, and each of these consumers brings in additional profit $P_1 - \delta$. Similarly, $\delta\alpha_2^0/(2t_2)$ consumers switch from buying product 1 from A and product 2 from B to buying both from A, and each of these consumers brings additional profit $P_2 - \delta$. Finally, $\delta\beta_1^0/(2t_1)$ consumers switch from buying both items from B to buying both from A, and these "doubly profitable" consumers bring profit $P_1 + P_2 - \delta$. In sum, ignoring terms in δ^2, the effect on firm A's profit of introducing the small discount δ is approximately

$$\frac{\delta\alpha_1^0}{2t_1}P_1 + \frac{\delta\alpha_2^0}{2t_2}P_2 + \frac{\delta\beta_1^0}{2t_1}(P_1 + P_2) - \delta N = \delta(\tfrac{1}{2} - N) > 0 \ .$$

[15] In this example one can also show that equilibrium consumer surplus rises with z, despite the extra cost faced by the two-stop shoppers.

Here, the equality follows from expressions (10)–(11), which proves the result. ■

This argument is in the same spirit as that used in the monopoly context by McAfee, McMillan, and Whinston (1989).[16] However, in our duopoly framework a firm *always* has an incentive to introduce a positive discount, whereas in the monopoly context it was not always the case that a discount, as opposed to a premium, was profitable.

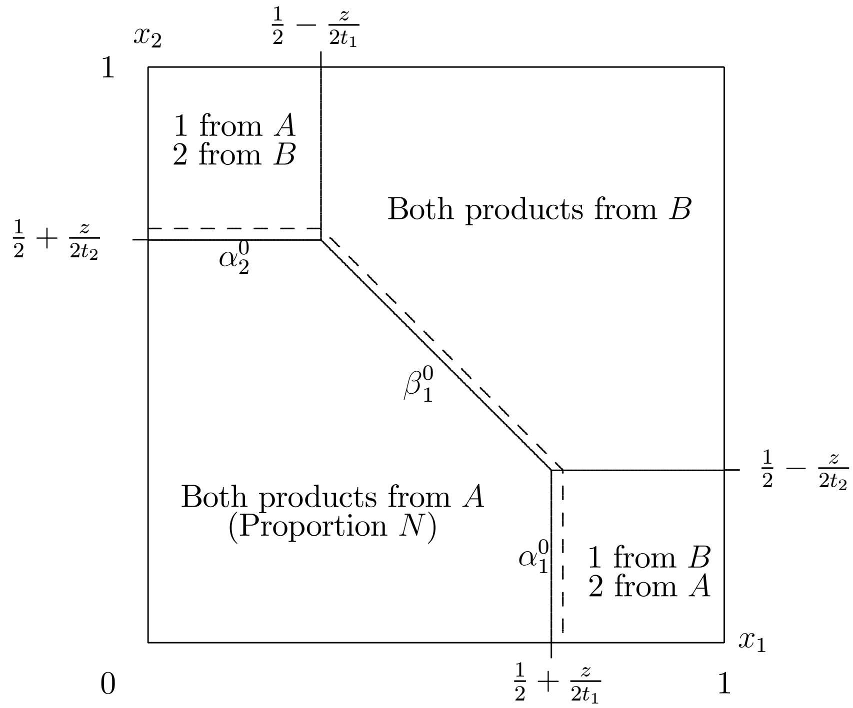

Figure 4: Incentive to Introduce Bundling Discount

An intuition for this result goes as follows. Starting from the symmetric equilibrium with linear pricing, there would be no first-order effect on the profits of a firm that reduced each of its standalone prices by $\frac{1}{2}\delta$ for small δ because those prices are optimal for the firm (i.e., the envelope theorem applies). Suppose instead that the firm does not change its stand-alone prices but introduces a discount δ for joint purchase. From (5), this brings the same gain in custom for each of the two products as the stand-alone price cuts. But it involves strictly less foregone profit on intra-marginal custom because the discount is restricted to the firm's one-stop shoppers rather than paid to all its customers. The firm, having been roughly indifferent about the stand-alone price cuts, will therefore strictly gain from the bundling discount. In sum, offering a bundling discount is a more cost-effective way to boost market share than cuts in linear prices.

[16] See Thanassoulis (2006) for a related result.

Notice, however, that if *both* firms offer the same (small) joint purchase discount δ, each firm's profit falls (by approximately $N\delta$) compared to the profit generated with linear pricing. This illustrates the prisoner's dilemma nature of competitive bundling.

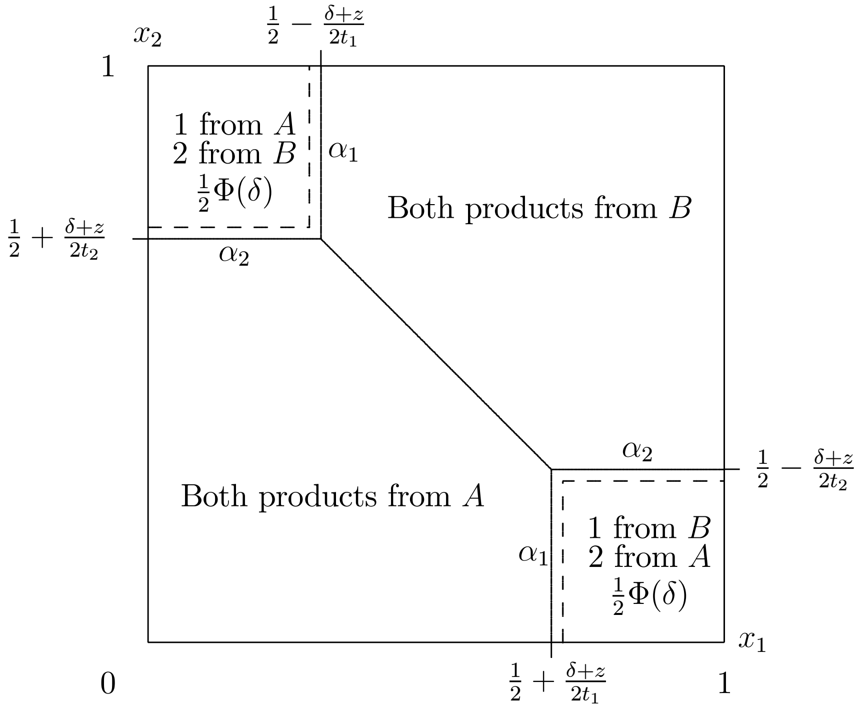

Figure 5: Effect of Changing the Bundle Discount

Having established that linear pricing cannot be an equilibrium when firms can offer bundling discounts, we turn next to analysis of the equilibrium nonlinear tariffs. It turns out that there is a simple and general formula for the equilibrium discount δ:

Proposition 4 *In a symmetric equilibrium the discount δ satisfies*

$$\Phi(\delta) + \tfrac{1}{2}\Phi'(\delta)\delta = 0 \ . \tag{14}$$

Proof. Suppose that the symmetric equilibrium nonlinear tariff involves the stand-alone price P_1 for product 1, the stand-alone price P_2 for product 2, and the bundling discount δ. Consider the effect on firm A's profit if it increases its discount by a small amount 2ε and simultaneously increases each of its stand-alone prices by ε. The result is that its total charge to one-stop shoppers is unchanged, but the total charge for each two-stop shopping option rises by ε. The net effect on the firm's demand is depicted in Figure 5, where the two-stop shopping regions shrink to the dashed regions: the deviation moves the upper boundary marked α_1 to the left by $\varepsilon/(2t_1)$ and lower boundary α_1 to the right by the same amount;

the upper boundary α_2 is moved up by $\varepsilon/(2t_2)$ and the lower boundary α_2 down by the same amount. The net effect on firm A's profit is then approximately

$$(\varepsilon\Phi(\delta)) + \frac{\varepsilon}{2t_1}\alpha_1(P_1-\delta) - \frac{\varepsilon}{2t_1}\alpha_1 P_1 + \frac{\varepsilon}{2t_2}\alpha_2(P_2-\delta) - \frac{\varepsilon}{2t_2}\alpha_2 P_2 = \varepsilon\left(\Phi(\delta) + \tfrac{1}{2}\Phi'(\delta)\delta\right) ,$$

where the equality follows from (8). For this deviation to be unprofitable, expression (14) must hold in equilibrium. ■

Thus, viewing $\Phi(\delta)$ as the demand for two-stop shopping as a function of the price δ, this result shows that in equilibrium the elasticity of this demand $-\delta\Phi'/\Phi$ is equal to 2.

There is a unique solution (with $z < \min\{t_1, t_2\}$) to expression (14) if $\Phi(\delta)$ is log-concave.[17] In this case, a simple corollary to Proposition 4 is that the equilibrium discount is decreasing in z. (Increasing z causes the elasticity $-\delta\Phi'/\Phi$ to rise for given δ.) Indeed, the discount falls to zero as the exogenous shopping cost z approaches $\min\{t_1, t_2\}$. That is to say, if almost all consumers are anyway one-stop shoppers, there is little benefit to firms in inducing still more consumers to become one-stop shoppers by means of a bundle discount. In this sense, the exogenous shopping cost reduces the incentive to bundle. For instance, in the *Uniform Example* we have $\Phi(\delta) = 2(\frac{1}{2} - \frac{\delta+z}{2t})^2$, and so expression (14) implies that $\delta = \frac{1}{2}(t-z)$ in symmetric equilibrium.

An important point to note is that welfare is reduced when firms offer discounts for joint purchase: there is *excessive loyalty*, as more consumers than is efficient buy both products from the same firm. The efficient pattern of consumption requires there to be no bundling discount, so that the pattern of demand is as depicted in Figure 3.

In more detail, by examining Figure 5, one sees that when the discount δ is increased by ε, the number of extra consumers who buy product 1 from the less preferred firm is equal to $\frac{\alpha_1}{t_1}\varepsilon$, and each of these consumers incurs the extra travel cost $(\delta + z)$ compared to buying the product from the closer firm, although these consumers now save the shopping cost z. Thus, their net disutility is δ. Similarly, $\frac{\alpha_2}{t_2}\varepsilon$ extra consumers buy product 2 from the less preferred firm, and these each incur a net disutility δ.

In sum, the extra welfare loss caused by increasing δ by ε is

$$\varepsilon\delta \times \left(\frac{\alpha_1}{t_1} + \frac{\alpha_2}{t_2}\right) = -\varepsilon\delta\Phi'(\delta) .$$

Denote by $w(\delta)$ the level of welfare corresponding to the bundling discount δ relative to the first-best welfare level (i.e., the welfare level corresponding to the case of linear pricing when

[17] There is always another, less interesting, *pure bundling* equilibrium. Suppose one firm offers a tariff involving extremely high stand-alone prices. Then no consumer will ever be a two-stop shopper, and so the rival might as well offer a similar tariff. In the *Uniform Example* one can show that this pure bundling equilibrium involves setting a price for the bundle equal to t (and setting the stand-alone prices prohibitively high). However, there are good reasons to believe that this second equilibrium is non-robust. For instance, it involves firms playing weakly dominated strategies. Moreover, Thanassoulis (2006) shows that when there are some consumers who wish to buy just one item, pure bundling ceases to be an equilibrium.

$\delta = 0$). Thus $-w(\delta)$ is welfare loss relative to the first best. It follows that

$$w'(\delta) = \delta \Phi'(\delta) \ . \tag{15}$$

This condition and $w(0) = 0$ yield $w(\delta)$. For instance, in the *Uniform Example* we have $\Phi(\delta) = 2(\frac{1}{2} - \frac{\delta+z}{2t})^2$ and $\delta = \frac{1}{2}(t-z)$, in which case (15) implies that the equilibrium welfare loss relative to linear pricing is

$$\frac{(t-z)^3}{12t^2} \ . \tag{16}$$

As expected, when the shopping cost is large, this welfare loss falls to zero since the bundling discount also falls to zero.

Having derived the equilibrium discount in (14), we can now derive the equilibrium stand-alone prices P_1 and P_2 in the same way in which the linear prices were derived in section 3.2. For simplicity, given the equilibrium discount δ, write $\alpha_i = \alpha_i(\delta)$ and $\beta_i = \beta_i(\delta)$. The pattern of demand is as shown on Figure 6.

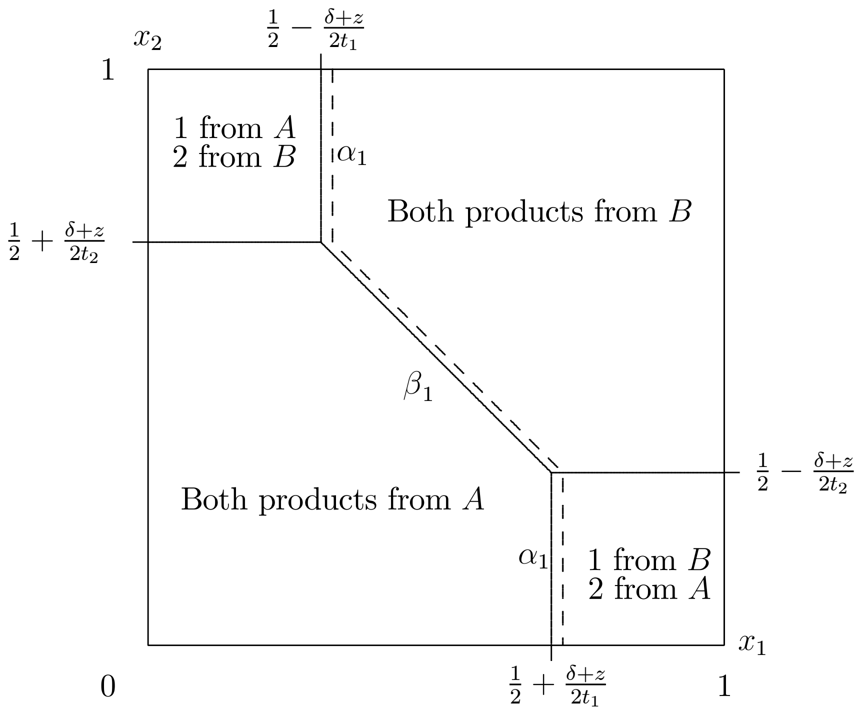

Figure 6: Incentive to Reduce Stand-Alone Price for Product 1

Consider firm A's incentive to reduce its stand-alone price P_1 by ε, keeping its discount unchanged at δ and its stand-alone price for product 2 unchanged at P_2. At a symmetric equilibrium, half the consumers buy product 1 from firm A, and the firm loses revenue ε from each of these infra-marginal consumers. The price reduction shifts the boundary of the set of consumers who buy product 1 from the firm uniformly to the right by $\varepsilon/(2t_1)$. Those consumers on the upper boundary α_1 on the figure generate profit P_1 to the firm, those on the diagonal boundary β_1 generate profit $(P_1 + P_2 - \delta)$, and those on the lower boundary α_1 generate incremental profit $(P_1 - \delta)$.

Since the profit gained from the marginal consumers must equal the profit lost from the infra-marginal consumers, it follows that in equilibrium

$$\alpha_1 P_1 + \beta_1 (P_1 + P_2 - \delta) + \alpha_1 (P_1 - \delta) = t_1 \ .$$

Similarly, the first-order condition for the stand-alone price P_2 is

$$\alpha_2 P_2 + \beta_2 (P_1 + P_2 - \delta) + \alpha_2 (P_2 - \delta) = t_2 \ .$$

Solving these linear simultaneous equations in (P_1, P_2) yields explicit formulae for the stand-alone prices, as reported in the next result.[18]

Proposition 5 *At a symmetric equilibrium, the discount δ satisfies (14) and the stand-alone prices are*

$$P_1 = \frac{\delta}{2} + \frac{\alpha_2 t_1}{\alpha_1 \beta_2 + \beta_1 \alpha_2 + 2\alpha_1 \alpha_2} \ ; \ P_2 = \frac{\delta}{2} + \frac{\alpha_1 t_2}{\alpha_1 \beta_2 + \beta_1 \alpha_2 + 2\alpha_1 \alpha_2} \ . \tag{17}$$

Notice that the stand-alone prices in (17) are uniquely determined given the discount δ. Then, provided there is a unique solution to the first-order condition for the discount in (14), which is the case if Φ is log-concave, there is then only one possible nonlinear pricing equilibrium with some two-stop shoppers.[19]

In the *Uniform Example*, with the equilibrium discount $\delta = \frac{1}{2}(t - z)$ it follows that $\alpha_1 = \alpha_2 = \frac{1}{4} - \frac{z}{4t}$ and $\beta_1 = \beta_2 = \frac{1}{2} + \frac{z}{2t}$. From (17) the equilibrium tariff is then[20]

$$P_1 = P_2 = \tfrac{1}{4}(t - z) + \frac{2t^2}{3t + z} \ ; \ \delta = \tfrac{1}{2}(t - z) \ . \tag{18}$$

Therefore, as in the linear pricing regime, the shopping cost z acts to lower all prices. The reason is the same: the shopping cost expands the margin of "doubly profitable" one-stop shoppers, which intensifies competition.

[18] Observe that, although our presentation gives the impression that the discount is chosen first and stand-alone prices chosen subsequently, in fact the game is that all three elements of a firm's tariff are chosen simultaneously.

[19] We have not investigated second-order conditions in general for the tariff in Proposition 5, so cannot be sure that a (pure strategy) equilibria exists. However, we have examined the *Uniform Example* and verified here that this tariff is a global best response for one firm if the rival offers the same tariff.

[20] The model of Matutes and Regibeau (1992) corresponds to the case $z = 0$, when $P_i = \frac{11}{12}t$ and $\delta = \frac{1}{2}t$. That example was extended in Armstrong (2006, section 4.2) to situations where $t_1 \neq t_2$ and where $z > 0$. These analyses took a "brute force" approach by simply calculating the areas of the various regions in Figure 1. This method is only practical when the distribution of (x_1, x_2) is uniform, and it cannot be used to derive more general results, such as Propositions 3, 4 and 6 in this paper.

3.4 Comparing the Regimes

The regimes of linear pricing and bundling are easily compared in the *Uniform Example.* First, notice that as z tends to t, so that the proportion of two-stop shoppers vanishes, the tariffs associated with linear pricing (13) and with bundling (18) converge. That implies that profit, consumer surplus and welfare in the two regimes converge for large z.

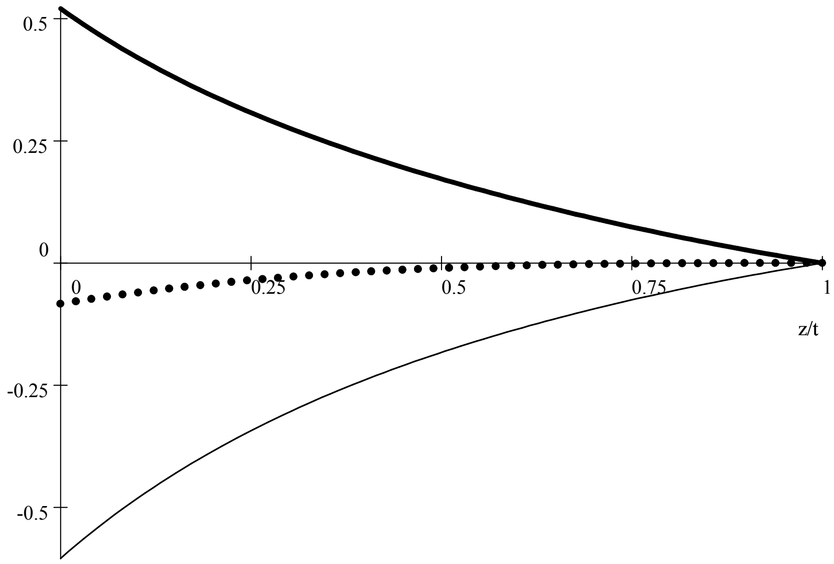

Figure 7: The Effect of z on Relative Profit, Consumer Surplus and Welfare

Figure 7 depicts relative profit, welfare and consumer surplus for all $z \leq t$. The analytic expression for industry profit with bundling is not illuminating, but can be calculated using expression (6). We depict the profit with bundling relative to that with linear pricing (divided by t) as the thin line in the figure. Here, z ranges from zero, where the profit loss with bundling relative to linear pricing is about 30%, to t, where profit is the same in the two regimes. In particular, bundling acts to destroy profit relative to linear pricing. The thick line depicts the difference in aggregate consumer surplus (divided by t) in the two regimes, which is positive but decreases to zero as z approaches t.[21] In particular, consumers in aggregate are always better off in the bundling regime. Finally, the dotted line depicts welfare with bundling (16) relative to linear pricing (divided by t). Of course, welfare falls when bundling is used. For instance, when $z = 0$ only an eighth of consumers are two-stop

[21] Thanassoulis (2006) examines effects of mixed bundling on consumer welfare in a version of this framework where there are some consumers who just want one product. He too finds that with no extra costs of two-stop shopping mixed bundling is better for consumers and worse for firms than stand-alone pricing, but with (prohibitive) shopping costs the opposite can occur as competition for two-product consumers protects one-product consumers if mixed bundling is disallowed.

shoppers with bundling, whereas maximal efficiency requires that half the consumers should use two suppliers (as occurs with linear pricing).

In this example, the stand-alone price with bundling in (18) is lower than the corresponding linear price in expression (13) whenever z is sufficiently small.[22] In such cases, all prices fall when bundling is used, and so all consumers benefit. For larger z, the two-stop shoppers are worse off in the bundling regime, although (as shown on Figure 7) overall consumer surplus is always higher with bundling.

Moving beyond this example, it appears to be widely true that aggregate consumer surplus rises, and profit (and of course welfare) falls, when bundling is used. The next result establishes this in the context of independently distributed brand preferences, while a simple form of correlated brand preferences is analyzed subsequently.

Proposition 6 *Suppose that x_1 and x_2 are independently distributed, with respective density functions $f_1(x_1)$ and $f_2(x_2)$ and respective distribution functions $F_1(x_1)$ and $F_2(x_2)$. (The densities satisfy $f_i(x) \equiv f_i(1-x)$.) Suppose the distributions satisfy the hazard rate conditions*

$$\frac{d}{dx}\frac{F_1(x)}{f_1(x)} \geq \tfrac{1}{4}\ ;\ \frac{d}{dx}\frac{F_2(x)}{f_2(x)} \geq \tfrac{1}{4} \tag{19}$$

for $x \leq \frac{1}{2}$. Then compared to the outcome with linear pricing, industry profit and welfare fall and aggregate consumer surplus rises when bundling is used.

Proof. See appendix. ■

Before discussing this result, it is worth checking its robustness to the presence of correlation in brand preferences. In some situations, it is plausible that if a consumer prefers firm A's product 1 then she is more likely to prefer the same firm's product 2, so that x_1 and x_2 are correlated. That is to say, firm-level brand preference may be an important factor in a consumer's brand preferences over individual products. As is the case with substantial shopping costs, this situation implies that the proportion of two-stop shoppers is smaller than in the uncorrelated case. How does this impact on the incentive to offer bundling discounts?

For instance, consider this variant of the *Uniform Example*.[23] Suppose product differentiation is symmetric ($t_1 = t_2 = t$). Suppose that a fraction ρ of consumers have perfectly correlated preferences, so that $x_1 = x_2$, and this common brand preference is uniform on $[0, 1]$. The remaining $1 - \rho$ consumers have locations (x_1, x_2) uniformly distributed on the unit square. Thus, ρ represents the degree of correlation in brand preferences.

[22] The stand-alone price with bundling is lower than the equilibrium linear price when $z/t < \sqrt{5} - 2 \approx 0.24$.

[23] A similar framework is used in Nalebuff (2004, section III.C). See Reisinger (2006) for further analysis of the effect of correlation in a related framework.

In this example, $\alpha_1^0 = \alpha_2^0 = (1-\rho)(\frac{1}{2} - \frac{z}{2t})$ and $\beta_1^0 = \beta_2^0 = \frac{1}{2} - (1-\rho)(\frac{1}{2} - \frac{z}{t})$.[24] Therefore, from (12), the equilibrium linear price for each product is

$$P_1 = P_2 = \frac{t^2}{t + (1-\rho)z} \; .$$

Thus, correlation acts to relax competition (unless the shopping cost is zero, in which case correlation has no impact on linear prices).

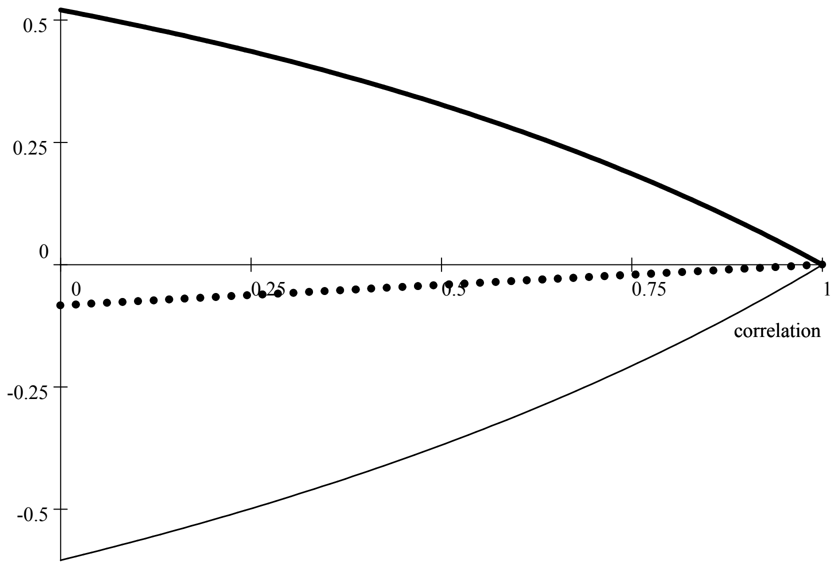

Figure 8: The Effect of ρ on Relative Profit, Consumer Surplus and Welfare

Turning to the case where bundling is employed, $\Phi(\delta)$ is here equal to $2(1-\rho)(\frac{1}{2} - \frac{\delta + z}{2t})^2$, and so expression (14) shows that the equilibrium discount does not depend on ρ and equals $\delta = \frac{1}{2}(t - z)$. From (15), the welfare cost of bundling with this discount is just scaled down by the factor $(1-\rho)$, and so from (16) this welfare loss is $(1-\rho)\frac{(t-z)^3}{12t^2}$. With the discount $\delta = \frac{1}{2}(t-z)$, we have $\alpha_1 = \alpha_2 = \frac{1}{4}(1-\rho)(1 - \frac{z}{t})$ and $\beta_1 = \beta_2 = \frac{1}{2} + \frac{1}{2}(1-\rho)\frac{z}{t}$. Expression (17) then shows that the equilibrium bundling tariff is

$$P_1 = P_2 = \frac{1}{4}(t-z) + \frac{2t^2}{2t + (1-\rho)(t+z)} \; ; \; \delta = \frac{1}{2}(t-z) \; . \tag{20}$$

Thus, as with linear prices, the equilibrium bundling prices increase with correlation. The stand-alone prices with bundling are higher than the equilibrium linear prices whenever

[24] Strictly speaking, $\beta_i(\delta)$ in (9) is not defined in this example, since there is no (two-dimensional) "density" on the line $x_2 = x_1$. However, by examining small changes in P_i as depicted on Figure 3, one can verify that this value for β_i^0 is the appropriate value to use in expression (12).

correlation is sufficiently strong (e.g., when $z = 0$, stand-alone prices rise with bundling whenever $\rho > \frac{1}{3}$). When there is perfect correlation (or if $z = t$), there are no two-stop shoppers in equilibrium, and the outcomes with and without bundling coincide.

Figure 8 represents the relative profit (thin line), welfare (dotted line) and consumer surplus (thick line) associated with bundling versus linear pricing (all divided by t), as correlation ρ varies. (The shopping cost z is set equal to zero in the figure.) Thus, the profit-destroying effect of bundling is mitigated, though never overturned, when there is positive correlation. By comparing Figures 7 and 8 we see that the effects of increased correlation in brand preferences on relative profit, consumer surplus and welfare is qualitively similar to the effects of increasing the shopping cost. However, the impacts of correlation and the shopping cost on *absolute* profit and consumer surplus are quite distinct: higher brand preference correlation tends to relax competition, while higher shopping costs tends to intensify competition.

In this model why do firms do worse, and consumers do better, with nonlinear pricing than with linear pricing? With nonlinear pricing, Propositions 3 and 4 establish that firms will choose to offer discounts to their one-stop shoppers. Such discounts can intensify price competition generally so that stand-alone purchases might become cheaper too (or at least they do not rise by enough to overturn the consumer benefits of the discount). When there are discounts for joint purchase, a wider margin of competition for one-stop shoppers opens up—i.e., consumers for whom the operative choice is to buy both products from firm A or both from B. Such consumers are "doubly profitable"; they bring two profit margins (less the discount for joint purchase) so their existence often intensifies price competition generally. Thus larger bundling discounts, by creating more of these doubly profitable consumers, may strengthen incentives to reduce stand-alone prices as well. (The effect is similar to the impact of the exogenous shopping cost z.) Bundling discounts do not necessarily do so, because the discount itself reduces the profit obtained from the doubly profitable consumers. Nonetheless, even when bundling raises the stand-alone prices, the price rise is not (under the conditions of Proposition 6) sufficient to outweigh the consumer benefits of the discount.

Therefore consumers usually do better with nonlinear pricing than with linear pricing, when these effects are lessened. Yet discounts induce the excessive loyalty inefficiency, so depress welfare. It follows that profit is higher with linear pricing.

It is striking that these are exactly the opposite comparative statics to those obtained in the one-stop shopping model presented in Proposition 2. In that model with elastic demand, giving greater pricing freedom to firms—in particular the freedom to engage in nonlinear pricing—enhanced profit and welfare but was detrimental to consumers. But with consumers able to choose between one- and two-stop shopping, and with inelastic demand, giving firms the freedom to offer discounts for joint purchase lowers profit and welfare but is good for consumers. We will now analyze a model general enough to encompass and reconcile these contrasting findings.

4 A Unified Model

In this section we extend the unit-demand bundling model to allow consumers to have elastic and heterogenous demands. As well as being of interest for its own sake, the more general model will enable us to explain the sharp contrast between the results from the one-stop shopping model (section 2) and the unit-demand bundling model (section 3).

We make the innocuous assumption that consumers buy all their supplies of a given product from one firm or the other. In general, firm i's tariff then consists of three options: $T_1^i(q_1)$ is the charge for q_1 units of product 1 if the consumer does not buy any product 2 from the firm; $T_2^i(q_2)$ is the corresponding stand-alone tariff for product 2, and $T_{12}^i(q_1, q_2)$ is the tariff if the consumer buys all her supplies from the firm.

Suppose a type-θ consumer has gross utility $u(\theta, q_1, q_2)$ if quantity q_i of product i is consumed. (The parameter θ could be multi-dimensional.) If the type-(θ, x_1, x_2) consumer buys quantities (q_1, q_2) from firm A, her net utility is $u(\theta, q_1, q_2) - T_{12}^A(q_1, q_2) - t_1x_1 - t_2x_2$. If she buys quantities (q_1, q_2) from firm B, her net utility is $u(\theta, q_1, q_2) - T_{12}^B(q_1, q_2) - t_1(1 - x_1) - t_2(1 - x_2)$. And if she buys quantity q_i of product i from A and quantity q_j of product j from B, her utility is $u(\theta, q_1, q_2) - T_i^A(q_i) - T_j^B(q_j) - t_ix_i - t_j(1 - x_j) - z$. The consumer will choose quantities and suppliers to maximize this utility. As in section 2, suppose that θ is distributed independently from the brand preference parameters (x_1, x_2). Suppose that each firm incurs a marginal cost c_i for serving a consumer with a unit of product i. Finally, suppose that parameters are such that all consumers wish to buy some of each product.

In this model two kinds of inefficiency can arise. First, marginal prices may diverge from marginal costs, and as a result a sub-optimal amount of each product will be consumed. This welfare effect was seen with linear pricing in the one-stop shopping analysis of section 2, but not in the basic bundling model of section 3 where the unit demand framework meant that high prices had no welfare impact. This is the excessive marginal price effect. Second, tariffs may encourage excessive one-stop shopping, and consumers may be induced to one-stop shop more often than is socially efficient. This "excessive loyalty" inefficiency was seen in the basic bundling model, but did not arise in the one-stop shopping framework. In the unified model presented in this section, both inefficiencies can be present. We will see that with linear pricing, inefficiency stems from excessive marginal prices but not from excessive loyalty, whereas with nonlinear pricing the reverse is true.

4.1 Linear Pricing

Write $v(\theta, p_1, p_2) = \max_{q_1, q_2} u(\theta, q_1, q_2) - p_1q_1 - p_2q_2$ for the consumer surplus of the type-θ consumer when she pays linear prices p_1 and p_2. If both firms set the same linear prices, the pattern of consumer demand is exactly as depicted on Figure 3. (The θ parameter has no impact on a consumer's choice of firm when firms offer the same tariff.) In particular, in a symmetric equilibrium there will be no inefficiency due to excessive loyalty, although there will be inefficiency due to excessive marginal prices.

If firm A undercuts B's product 1 price by ε, this shifts the boundary of the set of type-θ consumers who buy the product from A uniformly to the right by $\frac{\varepsilon}{2t_1}q_1(\theta, p_1, p_2)$. Each of the marginal consumers on the vertical boundaries α_1^0 brings extra profit $(p_1 - c_1)q_1(\theta, p_1, p_2)$, while each "doubly profitable" consumer on the diagonal boundary β_1^0 brings profit $(p_1 - c_1)q_1(\theta, p_1, p_2) + (p_2 - c_2)q_2(\theta, p_1, p_2)$. Set against this is the impact of the price cut on the profit from infra-marginal consumers. Each of A's existing consumers of product 1 contribute product 1 profit which is changed by $\varepsilon[q_1(\theta, p_1, p_2) + (p_1 - c_1)\frac{\partial}{\partial p_1}q_1(\theta, p_1, p_2)]$, and in a symmetric equilibrium the proportion of such consumers equals a half. Finally, there is the effect of changing p_1 on demand for the firm's product 2. This is only relevant for the firm's one-stop shoppers, who are N in number in equilibrium. Each of these one-stop shoppers generates product 2 profit which is changed by $\varepsilon(p_2 - c_2)\frac{\partial}{\partial p_1}q_2(\theta, p_1, p_2)$.

Putting all this together and taking expectations over θ implies that the first-order condition for p_1 to be the equilibrium price for product 1 is this generalization of (10):

$$E_\theta\left[\left(2\alpha_1^0(p_1 - c_1)q_1 + \beta_1^0((p_1 - c_1)q_1 + (p_2 - c_2)q_2\right)q_1\right]$$
$$= t_1 \times E_\theta\left[q_1 + (p_1 - c_1)\frac{\partial q_1}{\partial p_1} + 2N(p_2 - c_2)\frac{\partial q_2}{\partial p_1}\right] . \qquad (21)$$

(Here, the dependence of demands q_i on p_1, p_2 and θ has been suppressed.) A similar expression holds for product 2.

Formula (21) is complex, and reflects the effects of own and cross-price elasticities, consumer heterogeneity (via the quadratic terms q_1^2 and $q_1 q_2$), the extent of product differentiation, the shopping cost, and correlation in product brand preferences (via the size of N). An extension to the *Uniform Example* illustrates some of these effects.

Linear Uniform Example: $t_i = t > z$; $f(x_1, x_2) \equiv 1$; $q_i(\theta_1, \theta_2, p_1, p_2) = \theta_i(1 - bp_i)$; $c_i = 0$.

Here, demand functions are linear and exhibit no cross-price effects, and consumer heterogeneity is represented by an idiosyncratic multiplicative term θ_i for each product. Suppose that each θ_i has mean 1 and variance σ^2, and let the covariance of θ_1 and θ_2 be $\kappa\sigma^2$ for $-1 \leq \kappa \leq 1$. It is natural to suppose that there is positive correlation in the scale of demands for the two products across consumers (i.e., $\kappa > 0$), since it is likely that a consumer's income will be positively correlated with her demand for each product. The parameter b represents the sensitivity of demand to marginal price. With a linear price p for each product, industry profit is $2p(1 - bp)$ and welfare relative to the first best is $-bp^2$.

As in section 3.2, $\alpha_i^0 = \frac{1}{2} - \frac{z}{2t}$ and $\beta_i^0 = \frac{z}{t}$, and so (21) implies that the equilibrium linear price for a unit of either product, p, satisfies

$$\frac{p(1 - bp)^2}{1 - 2bp} = \frac{t^2}{t + z + \sigma^2(t + \kappa z)} , \qquad (22)$$

which generalizes (13). This equilibrium price increases with t and falls with z, b, σ^2 and κ.[25] The impact of these comparative statics, together with their intuition, will be explored in section 5.

4.2 Nonlinear Pricing

In this section we establish that it is an equilibrium for firms to offer tariffs with marginal prices equal to marginal costs, and with fixed charges corresponding to the bundling prices derived in section 3.[26]

Proposition 7 *Suppose that over the relevant range of tariffs all consumers wish to purchase both products. Then it is an equilibrium for each firm to offer the following tariff:*

$$T_1(q_1) = P_1 + c_1 q_1 \ ; \ T_2(q_2) = P_2 + c_2 q_2 \ ; \ T_{12}(q_1, q_2) = T_1(q_1) + T_2(q_2) - \delta \ , \qquad (23)$$

where P_1, P_2 and δ comprise the mixed bundling tariff described in Proposition 5.

Proof. See appendix. ■

Despite the generality of the demand structure and consumer heterogeneity, this equilibrium is remarkably simple. As with Proposition 1 the shape and heterogeneity of consumer demand functions have no impact on either industry profit or on welfare relative to the first best, so long as all consumers participate. In particular, and perhaps surprisingly, whether or not the two products are complements or substitutes in consumer demand makes no difference to the equilibrium incentive to offer bundling discounts. In this equilibrium there is efficient marginal-cost pricing, but there is inefficiency due to excessive loyalty. (Welfare continues to be determined by expression (15).) In this more general model, the impact of nonlinear pricing on profit, consumer surplus and welfare is ambiguous. In the next section, we discuss how various aspects of consumer preferences determine the net impact of nonlinear pricing.

5 Comparing the Regimes

In this section we use the unified model to compare linear pricing and unrestricted nonlinear tariffs.[27] At the end of section 3 we noted the contrast between the model of one-stop

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[25] The left-hand side of (22) is increasing in p and b over the relevant range $0 \leq p \leq 1/(2b)$. The right-hand side is increasing in t and decreasing in z, σ^2 and κ.

[26] Notice that, unlike Proposition 1, we have not been able to show that this is the *unique* symmetric equilibrium. One reason is the existence of at least one other, pure bundling equilibrium (see footnote 17).

[27] One could also consider an intermediate regime in which firms offer quantity discounts for a particular product, but cannot offer bundling discounts across products. In welfare (and profit) terms, this pricing

shopping (with elastic demand) and the model with consumer choice between one- and two-stop shopping (with inelastic demand) in respect of the consequences for profit, welfare and consumer surplus of allowing firms freedom to depart from linear pricing. The analysis of the unified model of section 4 now allows us to reconcile these findings and to undertake a more general comparison of linear and nonlinear pricing, and to show the importance of five kinds of economic effect: (i) demand elasticity, (ii) consumer heterogeneity, (iii) shopping costs, (iv) correlation in brand preferences, and (v) product differentiation. The impact of the first four of these effects was summarised in Table 1 in the introduction.

5.1 The Effect of Demand Elasticity

When consumer demand is sufficiently inelastic, the unified model is approximated by the unit demand model of section 3, and there is no significant excessive pricing inefficiency. In this case, nonlinear pricing harms profit and welfare, but benefits consumers, relative to the case of linear pricing. What happens when demand is more elastic?

When nonlinear pricing is used, the shape of the demand functions has no effect on profit or welfare (see Proposition 7). With linear pricing, on the other hand, it is plausible that profit is reduced when demand becomes more elastic. For instance, in section 2 we argued that the "elasticity effect" was one reason why linear prices yielded lower profits than nonlinear prices. In the limit as demand becomes very elastic, equilibrium linear prices are close to marginal costs and profit is close to zero. This implies that profit with nonlinear pricing is greater than that with linear pricing whenever demand is sufficiently elastic. Similar arguments apply to consumer surplus. When demand is highly elastic, there is (approximate) marginal-cost pricing in both regimes but consumers also pay fixed charges in the nonlinear pricing regime. Therefore, consumers are worse off with nonlinear pricing when demand is sufficiently elastic.

The impact of elasticity on welfare is not so clear cut, however. Linear pricing has the advantage that there is no excessive loyalty inefficiency, but there is the inefficiency due to excessive marginal prices. The latter problem may be expected to become more prominent as demand becomes more elastic, because for a given price the welfare loss is larger if demand is more elastic. On the other hand, greater demand elasticity lowers the equilibrium linear price, which may mitigate or overturn these increased deadweight losses.

To illustrate these elasticity effects consider the *Linear Uniform Example*, where the linear price is given by expression (22). With nonlinear pricing, Proposition 7 shows that profit does not depend on the elasticity parameter b. However, profit with linear pricing is decreasing in b. When demand is sufficiently sensitive to price, profit with nonlinear pricing

regime delivers the best of both worlds, since the excessive loyalty effect and the excessive marginal price effect are both avoided. However, it would be hard for public policy to enforce this intermediate regime, since in practice it is not always clear what constitutes a distinct product. (For instance, should a season ticket to a concert series count as an inter-product or intra-product discount?) For this reason, we focus on the transparent distinction between linear pricing and unrestricted nonlinear tariffs.

exceeds that with linear pricing. Thus, in the unified model of section 4, the impact of nonlinear pricing on profit is ambiguous.

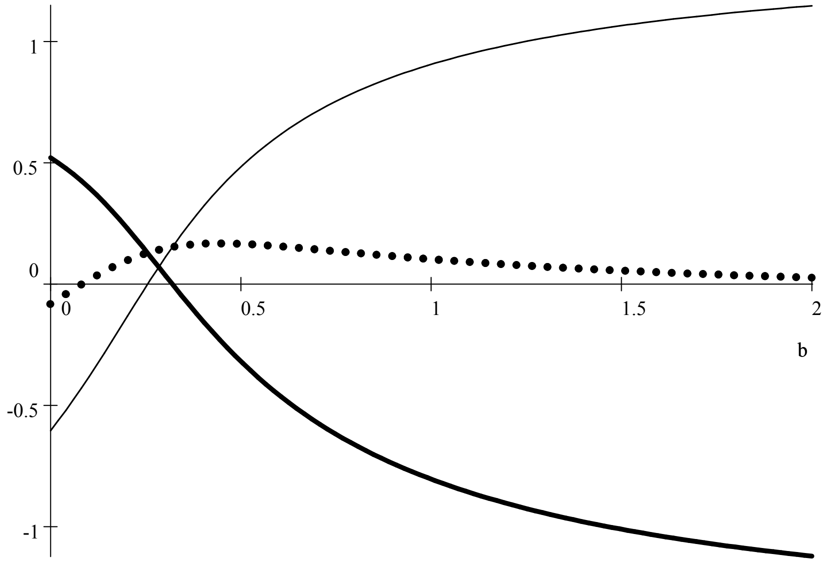

Figure 9: The Effect of Elasticity on Relative Profit, Consumer Surplus and Welfare

Figure 9 plots (as the thin solid line) the difference between the profit with nonlinear pricing and with linear pricing as a function of the elasticity term b (here $t = 1$, $z = 0$ and $\sigma^2 = 0$). The welfare difference between nonlinear and linear pricing is plotted as the dotted line on the figure, while the difference in consumer surplus is the thick solid line. When $b = 0$ we return to the unit demand setting, where profit and welfare are reduced with bundling, while consumers benefit. With sufficiently elastic demand, the reverse holds. In this example, welfare is improved by nonlinear pricing for moderate elasticities, but for very elastic demand (outside the range of Figure 9) welfare is again higher with linear pricing. Therefore, the impact of elasticity on relative welfare is non-monotonic, and this explains the "?" in Table 1.

5.2 The Effect of Consumer Heterogeneity

The discussion in section 2 noted the heterogeneity effect, where the presence of consumers with different demands acted to intensify linear price competition and to reduce the profit obtained with linear pricing. In this section we explore this in more detail. The reason why consumer heterogeneity acts to depress the equilibrium linear prices can be seen from expression (21). Speaking loosely, a "mean-preserving spread" of the demand functions has no impact on the right-hand side of (21), but it raises the left-hand side via the quadratic terms. The impact of this is similar to a reduction in the product differentiation parameter

t_1, which will typically cause prices to fall. The economic intuition is that with heterogeneity a price cut attracts proportionally more high demand (hence high profit) consumers from the rival firm, and so improves, at the margin, the *mix* of consumers (an effect absent with homogeneity). Since prices are then closer to marginal costs, this acts to boost the welfare and consumer surplus associated with linear pricing, but to depress profits. We deduce that welfare and consumer surplus are more likely to be higher with linear pricing when there is substantial consumer heterogeneity, while profit is then more likely to be higher with nonlinear pricing. However, the reason is not that heterogeneity boosts the profits from engaging in price discrimination (as one might perhaps have expected), but rather that it harms profit when linear prices are used.

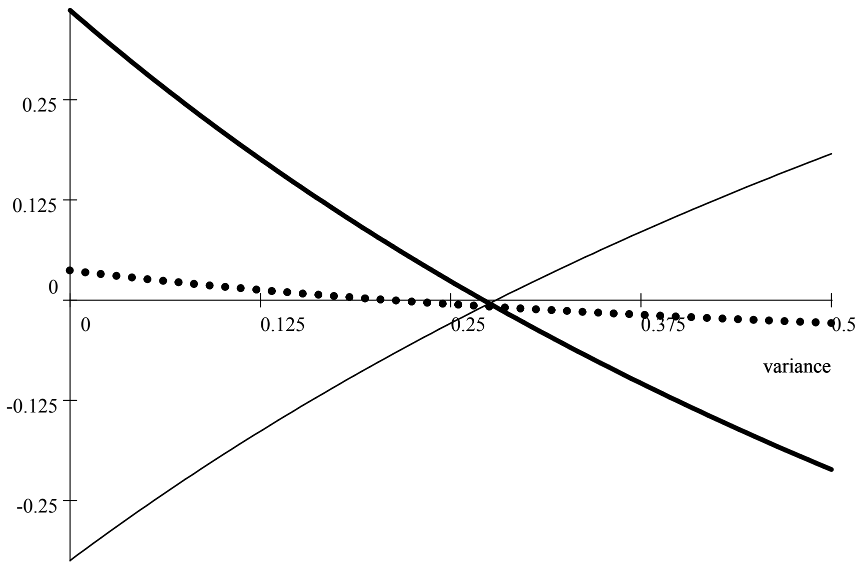

Figure 10: The Effect of Demand Heterogeneity

The effect is illustrated by the *Linear Uniform Example*. Expression (22) shows that the equilibrium linear price for each product is decreasing in σ^2, which confirms the intuition that heterogeneity in consumer demand pushes down linear prices. In addition, (22) shows that the price is decreasing in the correlation in demands for the two products, κ, at least when there is a positive shopping cost z. The intuition for this is as follows. When $z > 0$, there is a set of consumers for whom the relevant margin is whether to buy both products from either firm A or firm B. When there is more correlation in the scale of demand for the two products, the variance of the total profit from both products rises, and this intensifies competition for these one-stop shoppers yet further.

By contrast, when firms use nonlinear tariffs, Proposition 7 shows that profit, welfare and consumer surplus do not depend on the variance of, or correlation between, the scale of consumer demands. Thus, the relative profitability of using nonlinear pricing increases with σ^2 and κ. The reductions in linear prices caused by increased heterogeneity result in relative welfare with nonlinear pricing decreasing with σ^2 and κ. Consumers are better off with nonlinear pricing when σ^2 is relatively small, but when their demands are more varied, consumers (in aggregate) prefer linear pricing. This is illustrated in Figure 10, which shows relative profit (the thin line), consumer surplus (the thick line) and welfare (the dotted line) associated with nonlinear versus linear pricing as σ^2 varies. (Here, $b = \frac{1}{8}$, $t = 1$ and $z = 0$.)

5.3 The Effect of the Shopping Cost

When the shopping cost z is large (i.e., near to $\min\{t_1, t_2\}$), the equilibrium bundling discount δ is small and the excessive loyalty welfare loss is small—see Figure 7 above—and the only relevant factor for welfare is the excessive marginal price problem associated with linear pricing. We can deduce that for large z, the welfare effect of nonlinear pricing is always positive. (Of course, the welfare effect of nonlinear pricing may be positive for small z too, depending on demand elasticities and the other factors we have discussed.) Similarly, for large z the impact of nonlinear pricing on relative profit is surely positive, while the impact on relative consumer surplus is ultimately negative. In sum, as z becomes large, the one-stop shopping analysis in section 2 applies.

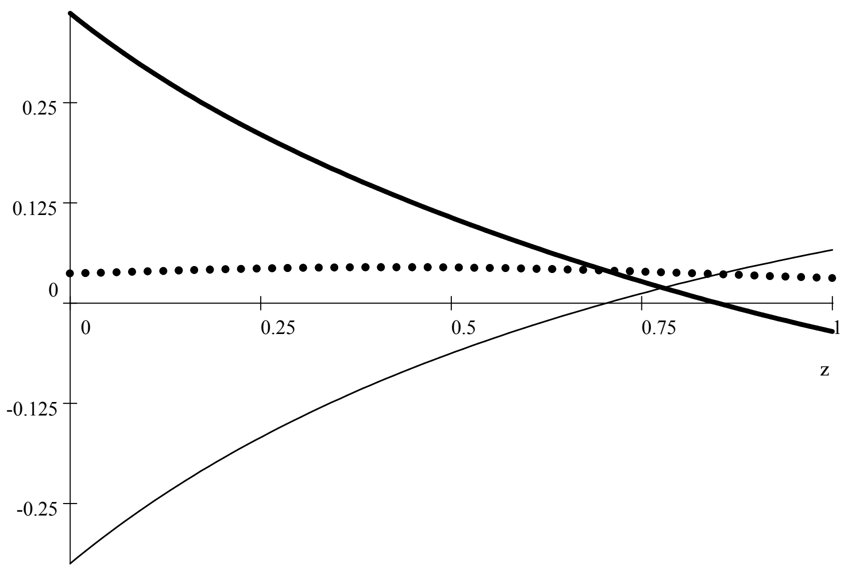

Figure 11: The Effect of Shopping Costs

This discussion can be illustrated with the *Linear Uniform Example*. Figure 11 shows relative profit (the thin line), consumer surplus (the thick line) and welfare (the almost flat dotted line) associated with nonlinear versus linear pricing as z varies. (Here, $b = \frac{1}{8}$, $t = 1$ and $\sigma^2 = 0$.)

5.4 The Effect of Correlation in Brand Preferences

Section 5.2 argued that correlation in the scale of demands for the two products, κ, tends to intensify linear pricing competition (while leaving the outcome with nonlinear pricing unchanged). The impact of this form of correlation is to increase effective consumer heterogeneity, and this decreases welfare and consumer surplus associated with nonlinear pricing relative to linear pricing. In this section we consider an alternative form of correlation—that involving the brand preferences for the two products—which has distinct economic effects.

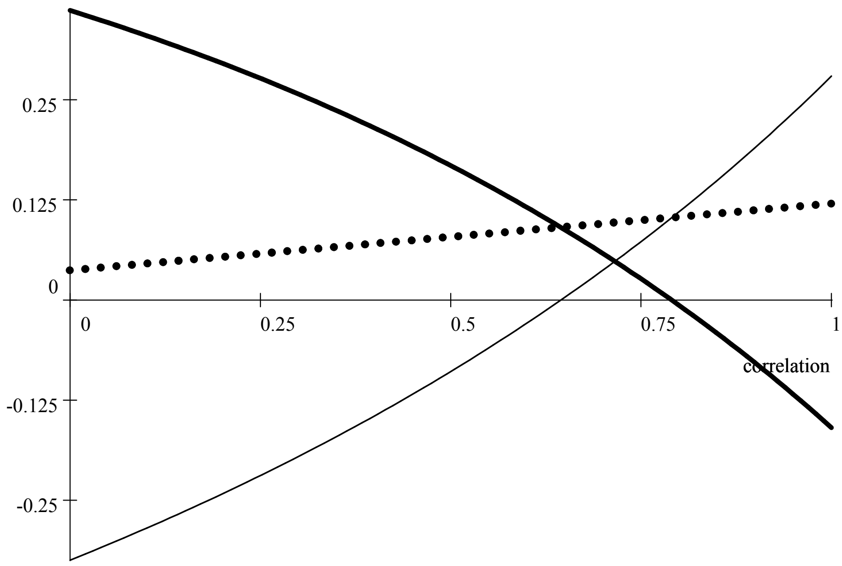

Figure 12: The Effect of Brand Preference Correlation

The impact of strong correlation in brand preferences on relative profit, consumer surplus and welfare is qualitatively similar to that of large shopping costs. When this form of correlation is strong, the fraction of consumers who might be two-stop shoppers is small, and the effect of nonlinear pricing mirrors the analysis of one-stop shopping in section 2: profits and welfare rise, while consumers are harmed. This can be illustrated using the *Linear Uniform Example* modified to allow for a fraction ρ of consumers to have perfectly correlated brand preferences (as suggested in section 3.4). Figure 12 shows relative profit (the thin line), consumer surplus (the thick line) and welfare (the dotted line) associated with nonlinear versus linear pricing as ρ varies. (Here, $b = \frac{1}{8}$, $t = 1$, $z = 0$ and $\sigma^2 = 0$.)

5.5 The Effect of Product Differentiation

In the unified model, the extent of differentiation in the two firms' versions of product i is captured by the parameter t_i. When both parameters (t_1, t_2) are small, so that firms offer closely substitutable products, tariffs converge to marginal costs in both regimes, and linear and nonlinear pricing yield approximately the same profit, welfare and consumer surplus. When at least one parameter is small, so that $z > \min\{t_1, t_2\}$, all consumers are one-stop shoppers regardless of whether nonlinear or linear pricing is used. In this case there is no *extra* welfare cost involved in bundling, and the excessive marginal pricing problem dominates. Then nonlinear pricing is sure to yield higher welfare and profit, while consumers are worse off (as was demonstrated in section 2).

At the other extreme, when (t_1, t_2) both become large, from (21) linear prices converge to the monopoly linear prices (at least when cross-price effects are absent), and profit, consumer surplus and welfare asymptotes to the monopoly outcome. With nonlinear pricing, however, the fixed elements of the two-part tariffs increase without bound, so profit becomes unboundedly large while welfare and consumer surplus become unboundedly negative. As such, when (t_1, t_2) is sufficiently large we expect that relative profit with nonlinear pricing is positive, while the impact on welfare and consumer surplus is eventually negative.

Between these extremes, though, the impact of product differentiation is ambiguous and potentially complex. When there is greater product differentiation, linear prices rise in the linear pricing regime while fixed charges (and the bundling discount) rise in the nonlinear regime. The welfare costs associated with both excessive loyalty and with excessive marginal prices rise. The net impact on profit, consumer surplus and welfare is hard to predict without further assumptions on the model.

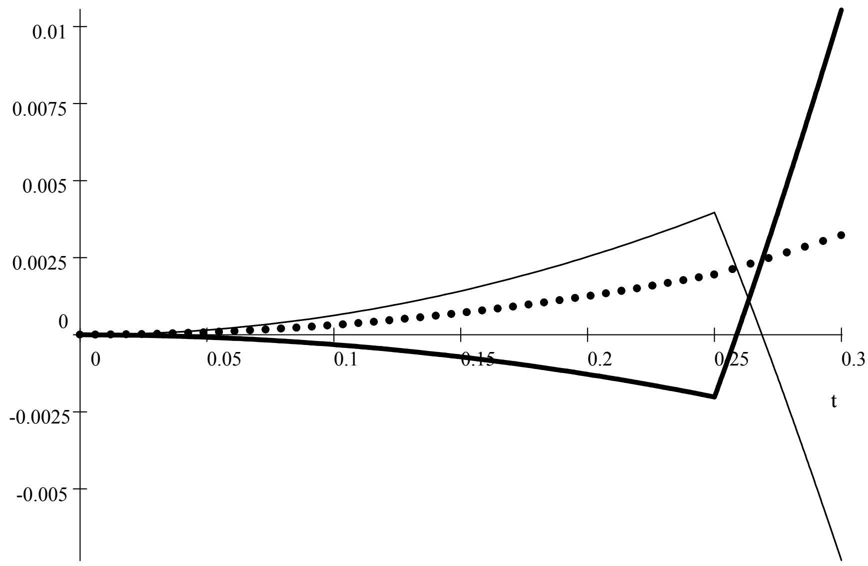

Figure 13: The Effect of Product Differentiation (small t)

To illustrate, consider the *Linear Uniform Example* with $b = \frac{1}{8}$, $z = \frac{1}{4}$ and $\sigma^2 = 0$. Figure 13 first illustrates this example for small t. When t is smaller than the shopping cost $z = \frac{1}{4}$, the rankings corresponding to the one-stop shopping model in Proposition 2 apply. However, when t is slightly above $z = \frac{1}{4}$, the impact of nonlinear pricing on profit and consumer surplus quickly reverses. When t is larger however, the situation with profit and consumer surplus reverses once more: see Figure 14.

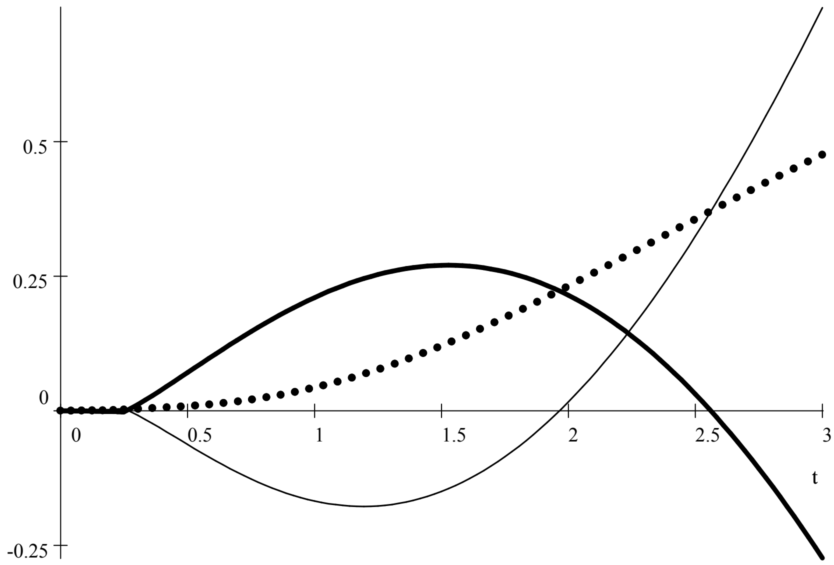

Figure 14: The Effect of Product Differentiation (large t)

Indeed, as t is extended beyond the limit of Figure 14, the dotted welfare line eventually becomes negative. In sum, the dependence of profit, consumer surplus and welfare on the extent of product differentiation—outside extreme parameter values—is potentially non-monotonic and complex.

6 Conclusions

Our analysis of competitive nonlinear pricing and bundling—and its effects on consumer surplus, profit and welfare—has proceeded from the special models of sections 2 and 3 to the considerably broader framework of section 4, which allowed for consumer heterogeneity, elastic demands, consumer choice between one- and two-stop shopping, and shopping costs. Yet the broader model was shown to have a remarkably simple equilibrium when firms can offer nonlinear tariffs—namely, efficient two-part tariffs with fixed charges equal to the equilibrium prices in the model with inelastic demand. The model illuminated the importance of keen competition for one-stop shoppers when bundling is feasible, and the inefficiencies arising from excessive marginal prices (with linear pricing) and excessive loyalty (with nonlinear pricing). We identified five economic influences on these sources of inefficiency—and hence on the pros and cons of nonlinear pricing relative to linear pricing—which were summarised in the introduction.

The economic effects discussed in this paper may operate more widely, along with other influences no doubt. However, though general in some respects, our framework has been confined to static competition between symmetric two-product duopolists in a setting where all consumers buy some of each product. Natural next steps would be to relax these restrictions.

For example, allowing for free entry instead of duopoly would open up the issue of the effect of nonlinear pricing on the equilibrium number of firms.[28] In the unit demand bundling model, nonlinear pricing acts to depress profit and welfare. With free entry, this implies that the equilibrium number of firms will fall, and this could act to mitigate possible "excess entry". Bundling might then have a positive impact on welfare, despite the excessive loyalty problem.

Another extension would be to make the model dynamic, and to allow for "customer poaching". Existing models of customer poaching, where a firm sets its current price on the basis of whether a buyer is a previous or a new customer of the firm, assume unit demands in each period.[29] In the basic version of these models, firms do not commit to their future prices, and so price low to their rival's existing customer base. Like the (static) bundling analysis in this paper, customer poaching tends to benefit consumers and to harm firms and welfare relative to linear pricing. However, the welfare problem is quite different: since firms offer low prices to existing customers of their rival (rather than discounts to their own loyal customers as in the bundling framework), customer poaching models involve *insufficient* rather than excessive loyalty. It would be interesting to see whether the extension of those models to elastic demand means that nonlinear pricing can benefit firms and welfare, just as we have shown it can benefit firms and welfare in static bundling models.

These are but two lines of possible further analysis. There is much more to be understood about the economics of competitive nonlinear pricing and bundling.

[28] See Stole (1995) for early work in this direction in the one-stop shopping context.
[29] See Chen (1997) and Fudenberg and Tirole (2000).

APPENDIX

Proof of Proposition 6: From (17), for any δ the sum of the stand-alone prices is

$$P_1 + P_2 = \delta + \frac{t_1\alpha_2 + t_2\alpha_1}{\alpha_1\beta_2 + \alpha_2\beta_1 + 2\alpha_1\alpha_2} \ ,$$

or

$$P_1 + P_2 = \delta + \frac{t_1}{\hat{\alpha} + \beta_1}$$

where

$$\hat{\alpha} = \frac{2\alpha_1\alpha_2}{\frac{t_2}{t_1}\alpha_1 + \alpha_2} \ .$$

From (6), for a particular δ industry profit is

$$\pi(\delta) = \delta + \frac{t_1}{\hat{\alpha} + \beta_1} - \delta(1 - \Phi(\delta)) \ ,$$

where Φ is defined in (14). Therefore,

$$\pi(\delta) = \delta\Phi(\delta) + \frac{t_1}{\hat{\alpha} + \beta_1} \ . \tag{24}$$

From (24), consumer surplus relative to the case of linear pricing, denoted $v(\delta)$, satisfies

$$v(\delta) = w(\delta) - \left[\delta\Phi(\delta) + \frac{t_1}{\hat{\alpha} + \beta_1}\right]$$

and so

$$v'(\delta) = -\Phi(\delta) - \frac{d}{d\delta}\left[\frac{t_1}{\hat{\alpha} + \beta_1}\right] \ . \tag{25}$$

(Here, $w(\delta)$ is total welfare with discount δ which satisfies expression (15).) Therefore, a sufficient condition for consumer surplus to rise with bundling is that expression (25) be positive. (Consumer surplus with linear pricing corresponds to $\delta = 0$, and we know that firms in equilibrium will choose to offer a positive bundling discount.)

To make progress, specialize to the case of independence, so that $f(x_1, x_2) \equiv f_1(x_1)f_2(x_2)$. Write $F_i(\cdot)$ for the distribution function corresponding to $f_i(\cdot)$ and write

$$\eta_i(\delta) \equiv \frac{f_i(\frac{1}{2} - \frac{\delta+z}{2t_i})}{F_i(\frac{1}{2} - \frac{\delta+z}{2t_i})} \ .$$

Then one can show

$$\frac{d}{d\delta}(\hat{\alpha} + \beta_1) = 2t_1F_1F_2\frac{t_1\eta_2^2\eta_1' + t_2\eta_1^2\eta_2'}{(t_1\eta_2 + t_2\eta_1)^2} = t_1\Phi\phi' \ , \tag{26}$$

where

$$\phi(\delta) \equiv \frac{\eta_1(\delta)\eta_2(\delta)}{t_1\eta_2(\delta) + t_2\eta_1(\delta)} \ .$$

Then (25) implies that

$$v'(\delta) = -\Phi + t_1^2 \Phi \frac{\phi'}{(\hat{\alpha} + \beta_1)^2} \ . \tag{27}$$

Assumption (19) implies that $\eta_i'(\delta) \geq 0$, which in turn implies $\phi'(\delta) \geq 0$. Notice that (26) implies

$$\frac{d}{d\delta}(\hat{\alpha} + \beta_1) \leq \tfrac{1}{2} t_1 \phi' \ .$$

And since $\hat{\alpha} + \beta_1 = \frac{1}{2} t_1 \phi$ when $\delta = -z$, we deduce that

$$\hat{\alpha} + \beta_1 \leq \tfrac{1}{2} t_1 \phi \ .$$

From (27) it follows that $v(\delta)$ is increasing if

$$\frac{\phi'}{\phi^2} = \frac{t_1 \eta_1'}{\eta_1^2} + \frac{t_2 \eta_2'}{\eta_2^2} \geq \tfrac{1}{4} \ .$$

A sufficient condition for this inequality to hold is the (relatively mild) hazard rate condition (19), in which case consumer surplus necessarily rises when bundling is used.

Since welfare $w(\delta)$ is deceasing in δ, it follows immediately that industry profit (which equals $w(\delta) - v(\delta)$) is decreasing in δ under the same conditions. This proves Proposition 6.

Proof of Proposition 7: Suppose that firm B offers the menu of two-part tariffs $T_1(\cdot)$, $T_2(\cdot)$ and $T_{12}(\cdot,\cdot)$ described in (23). We establish that firm A's best response is to use the same tariff by means of the following argument.[30]

Suppose firm A can directly observe a consumer's parameter θ (but not x_1 or x_2). We will calculate firm A's best response to B's tariff, given θ. Suppose firm A's tariff is $\{T_1^A(q_1)$, $T_2^A(q_2)$, $T_{12}^A(q_1, q_2)\}$. Then let

$$\begin{aligned}
V_1 &= \max_{q_1,q_2} : u(\theta, q_1, q_2) - T_1^A(q_1) - T_2(q_2) \\
V_2 &= \max_{q_1,q_2} : u(\theta, q_1, q_2) - T_1(q_1) - T_2^A(q_2) \\
V_{12} &= \max_{q_1,q_2} : u(\theta, q_1, q_2) - T_{12}^A(q_1, q_2)
\end{aligned}$$

be the type-θ consumer's gross utility (i.e., the utility excluding travel and shopping costs) when she buys only product 1 from A, only product 2 from A or buys all supplies from A,

[30] A similar argument was used to prove Armstrong and Vickers (2001, Proposition 5).

respectively. Next, consider firm A's most profitable way to generate the utilities V_1, V_2 and V_{12}. The most profitable way to generate utility V_1 is the solution to the problem

$$\max_{T_1,q_1} : \ T_1 - c_1 q_1 \text{ subject to } V_1 = \max_{q_2} u(\theta, q_1, q_2) - T_1 - T_2(q_2)$$

which is the same problem as

$$\max_{q_1,q_2} : u(\theta, q_1, q_2) - c_1 q_1 - T_2(q_2) - V_1 \ .$$

Clearly, the solution to this problem involves marginal-cost pricing for product 1. That is to say, given θ it is a dominant strategy for firm A to choose a tariff of the form $T_1^A(q_1) = P_1^A(\theta) + c_1 q_1$ for some fixed charge $P_1^A(\theta)$. (We write the fixed charge as a function of θ, since in general it will depend on θ.) A similar argument serves to show that $T_2^A(q_2) = P_2^A(\theta) + c_2 q_2$ and $T_{12}^A(q_1, q_2) = P_1^A(\theta) + P_2^A(\theta) - \delta^A(\theta) + c_1 q_1 + c_2 q_2$ for some choice of $P_2^A(\theta)$ and $\delta^A(\theta)$.

Since both firms are setting marginal prices equal to marginal costs, the net utility of the consumer is

$$v(\theta, c_1, c_2) - [P_1 + P_2 - \delta + t_1(1 - x_1) + t_2(1 - x_2)]$$

if both products are purchased from B,

$$v(\theta, c_1, c_2) - [P_1^A + P_2^A - \delta^A + t_1 x_1 + t_2 x_2]$$

if both products are purchased from A, and

$$v(\theta, c_1, c_2) - [P_i^A + P_j + t_i x_i + t_j(1 - x_j) + z]$$

if product i is purchased from A and product j is purchased from B. In particular, the consumer's decision over where to buy depends only on her total outlay (the terms in square brackets above), and the benefit from consumption, $v(\theta, c_1, c_2)$, does not depend on her choice of suppliers. Therefore, given θ, firm A will choose the fixed charges P_1^A, P_2^A and the bundling discount δ^A in order to maximize its profit, given B's tariff (23). But this is precisely the problem analyzed in section 3, where Proposition 5 established that the optimal response to the bundling tariff (P_1, P_2, δ) in (17) is to use the same bundling tariff.

In sum, we have shown that (i) if firm B sets the tariff (23) and (ii) if firm A can observe a consumer's type θ, then the tariff (23) maximizes A's profit. However, since the tariff (23) does not depend on θ, this tariff must also be the firm's best response in the more constrained problem in which A cannot observe θ, which proves the result.

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