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COMPETITION AMONG DIFFERENTIATED HEALTH PLANS UNDER ADVERSE SELECTION

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Competition among Differentiated Health Plans under Adverse Selection*

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Abstract

Most of the literature on adverse selection considers extreme cases: perfect competition or monopoly. We consider a model where insurers are horizontally differentiated. This is well justified for pre-paid health-care plans. Interesting features emerge from the interaction of adverse selection and differentiation. Profits derived from a low risk are higher than from a high risk. In equilibrium insurers offer menus of contracts that may entail inter-type cross-subsidization: the profits derived from high risks could even be negative despite average profits are always non-negative. In this case the equilibrium is not second-best. Our model also provides the necessity of a minimum firm complexity: there cannot be equilibria where each and all firms offer a single contract. All this is accomplished using the Nash (instead of Wilson) equilibrium notion.

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1 Introduction

We explore the effect of horizontal differentiation in a model of competition among pre-paid health care plans in the presence of adverse selection, i.e., when consumers possess privileged information about their true health risks. Pre-paid health care plans, or Health Maintenance Organizations (HMOs) in the US, directly offer medical treatment in exchange for a fee (or capitation rate) per affiliated agent, usually paid by a third party. This way of obtaining health care has gained much popularity in the US and it has been extended to public programs as Medicaid and Medicare that provide health care for the poor and elderly respectively.[1] HMOs are also important in Europe. After the Dekker reform, health organizations in Holland receive a premium for providing health care to their insurees. In Spain, civil servants can obtain health insurance from health plans that work very much as HMOs.

We interpret horizontal differentiation as geographical differentiation, which we take as exogenous. Thus, hospitals are located at different points in a straight line. Attending a hospital implies some transportation cost that increases with distance. This assumption is well justified in the HMO market. Travel distance seems to be an important variable when choosing health care provider. Kessler and McClellan (2000) measure competition using travel distances between patients and hospitals. Propper *et al.* (2002) measures competition faced by a particular hospital by the number of hospitals within 30 minutes of travel time. Shortell and Hughes (1988) use a fixed-radius measure of geographic markets to measure competition. Given that, in general, HMO patients will only visit the providers that the HMO has contracted with, then travel distance must be an important variable to choose among HMOs. Given the above evidence, we are quite comfortable with the geographical interpretation of distance. However, as in Ellis (1998), the model does not preclude interpreting distance as a consumer's disutility from seeking a certain style of health care.

Apart from being realistic, differentiation brings about several results that are relevant from both the theoretical and the empirical point of view. The first one deals with the profits that plans obtain from each type of consumer. The literature on risk selection in health insurance points out that insurance companies have incentives to attract low risks while deterring

[1] In 1993 there were 41million US citizens enrolled in HMOs (Winslow and Anders 1993). According to Town and Liu (2002), HMO Medicare penetration has increased to 15.7% in 1999 from 4.8% in 1993.

high risks.[2] While this is intuitive, it is at odds with the standard model with homogenous product, which predicts zero expected profits per consumer no matter his type. Under product differentiation we can prove that in equilibrium insurance companies will obtain more profits from low risks than from high risks. In fact, high risks entail negative profits for some parameter values.

The second result coming from differentiation is that our model yields more precise predictions on the mix of risks across firms than the perfectly competitive model. In the latter model this mix is left undetermined. Since Rothschild and Stiglitz's (1976) seminal paper it is well understood that one should observe two contracts in equilibrium, one intended for the low risks and the other for the high risks. However, this does not mean that each insurance company offers the two contracts, as in a menu.[3] While this is a possibility, a situation where some firms only offer the contract intended for the high risk and the rest only offer the contract for the low risk also constitutes an equilibrium. We refer to this situation as one with "full specialization."[4] In contrast, in reality insurance companies offer menus of contracts. According to Marquis and Long (1999), in 1997 more than half of the establishments in the US offering choice of multiple plans did so through a single insurer.[5] Our model with product differentiation predicts that in equilibrium at least one firm must offer a menu of contracts. In other words,

[2] See for instance Newhouse (1986), Jack (2001), and Frank *et al.* (2000).

[3] Even in this case it is not necessarily true that all firms obtain the same share of both types. Any other distribution of types is also an equilibrium. For instance, a firm offering the equilibrium menu may monopolize all low risk individuals and attract a single high risk individual while another firm obtains the rest of high risks; and so on.

[4] In other words, it turns out that the same equilibrium set of available contracts emerges independently of whether firms are allowed to offer menus of contracts or just single contracts. However, this matters for the existence of equilibrium. When firms are allowed to offer menus of contracts the parameter set for which an equilibrium exist is smaller than if firms can only offer single contracts.

[5] Specific examples of menus of contracts in the Medicare HMO market are: Neighborhood Health Partnership, Inc. offers in Florida three different plans (Senior Health Choice, NHP Plus and NHO Comprehensive Plus). Physicians Healthcare Plans (Florida) offers both CareFree and CarePlus Plan. Vista Healthplan (Florida) offers both Medicare Value and Medicare Choice. In Oregon, Clear Choice Health Plans offers three different plans (Traditional, Traditional plus and Value). Kaiser Foundation (Oregon) offers both Senior Advantage and Senior Advantage II. Providence Health Plan offers both Plan 1 and Plan 2. Regence HMO Oregon offers both plans: standard and Plus. More examples can be obtained using http://www.medicare.gov/mphCompare/home.asp.

it rules out full specialization as a possible equilibrium outcome.

Notice that our first and second results are closely related. The indeterminacy of the case mix across firms in the homogenous product model follows from zero profits per agent of any type. One of the reasons that we are able to derive more precise predictions is that firms get more profits from the low than from the high risks. (The other is that agents sort themselves out among firms according to their location.)

Our third result relates to the redistributive role of insurance in the absence of public intervention. If in equilibrium firms derive positive profits for the low risk and negative profits for the high risk, the market is providing some, though maybe incomplete, ex-ante insurance. Such a situation is referred to as "cross-subsidization." Behind the veil of ignorance, that is, before nature reveals the type for each individual, providing them with some insurance against being high risk may be optimal. We give an example where cross-subsidization is present in equilibrium. In this example, both the transportation costs and the proportion of high risks are small but positive. This is quite intuitive. If the transportation costs were high then in equilibrium firms would make positive profits from each and all types. If the proportion of high risks were large then cross-subsidization would be to onerous for firms.

One may ask why would firms ever offer a contract that makes losses. In other words, why not drop this contract from the menu? Suppose that a firm unilaterally does so. The high risks who are close enough to the deviating firm will still purchase a contract from this firm in order to save the transportation costs. Since the only contract remaining is the one directed to the low risk, this is the contract that these agents will accept. This has extremely bad consequences for the deviating firm. To sum up, no contract is dropped and we have an equilibrium with cross-subsidization.[6]

Our fourth result regards the welfare properties of the equilibrium. They also differ from the perfectly competitive model. In the latter, the equilibrium

[6] In the homogeneous product case, several authors (Miyazaki 1977 and Spence 1978 among others) have used Wilson's (1977) equilibrium notion to sustain cross subsidization in equilibrium. According to it, when one calculates the payoff of a deviation, the deviating firm assumes that other firms will withdraw those insurance contracts that generate losses due to the deviation. Riley (2001) and Crocker and Snow (1985a) criticize these non-Nash behavioral assumptions. We find cross-subsidization while sticking to the standard Nash equilibrium concept.

menu of contracts is always second-best if firms are allowed to offer menus.[7] In contrast, our equilibria are not always second-best. Moreover, we provide a simple test that ensures that the equilibrium is not second best, namely, if the profits derived from the high risk are negative in equilibrium, then the equilibrium is not second best. In this case there is room for public intervention, consisting in reducing the distortions present in the low risk contract. This efficiency gain is used to finance an improvement in both the high and the low risk contracts while keeping average profits constant.

From a more fundamental point of view, our model justifies the interpretation of a firm as a hierarchy of contracts. That is, firms are more complex than single contracts. Two of our results point to this direction: (i) the existence of cross-subsidization (not all contracts yield the same profits) and (ii) if all firms attract some agents of each and every type, at least one firm must offer a menu of contracts in equilibrium. We want to insist that neither (i) nor (ii) are true in the homogeneous product case, even if one allows firms to offer menus of contracts.

To sum up, apart from being justified empirically, introducing product differentiation provides new insights over the perfectly competitive framework. First, it allows for cross-subsidization to appear in equilibrium without resorting to any non-Nash equilibrium notion. Second, it generates a simple test of constrained Pareto-optimality: if the profits derived from high risks are negative the equilibrium is not second-best. Third, the predictions on case mix at each firm are more precise. In particular, we can rule out configurations where each and every firm only attracts a single type of agent. Finally, our model also provides the necessity of a minimum complexity in the firm. There cannot be equilibria where all firms offer a single contract each.

We are able to derive some comparisons between the set of contracts that emerges with and without differentiation. An invariant property of the equilibrium is that the contract intended for the high risks is never distorted. In contrast, under the premise that cross-subsidization is present, the high risks receive a better contract and the contract intended for the low risks is less distorted due to differentiation. If cross-subsidization is absent, then both risks are worse off due to differentiation and the low risk contract distortions are not corrected.

[7] This point is sometimes not well understood. It fails to be true if firms are constrained to offer single contracts.

We also present a numerical example that suggests two additional results. First, differentiation improves the predictive power of the model in another dimension: equilibrium exists for a larger set of parameters than in the perfectly competitive model. Second, cross-subsidization appears when transportation costs are small but positive.

Biglaiser and Ma (forthcoming) also exploit the combination of adverse selection and differentiation. They analyze the market for two services that can be either sold in a bundle or separately. Although their model is similar to ours, their focus is quite different. First, they compare the equilibrium qualities for these services with their first best values across four different frameworks. These four frameworks are defined by whether the services are sold separately or in a bundle, and by whether firms are allowed to offer menus of contracts or not. In contrast, we compare outcomes between the perfectly competitive and the differentiated markets, assuming both that the services are always sold in a bundle and that menus are always allowed. We believe that both assumptions are justified for health care services. Second, we prove that potential Pareto-improvements exists even if market separation was not a policy option. Third, we analyze the issue of cross-subsidization, which is omitted from their analysis. Finally, when they analyze the case where menus are allowed, they restrict attention to equilibria where menus are indeed offered. We contribute to this by proving that most configurations where firms offer a single contract cannot be part of an equilibrium if menus are allowed.[8] For all these reasons, we believe that our analysis is complementary to theirs.[9]

As for the literature of adverse selection in competitive insurance markets, there are many other extensions to the seminal article by Rothschild and Stiglitz (1976), including the application of non-Nash notions of equilibria, multiperiod contracting, risk categorization, multiple risks, competitive pooling, and the simultaneous consideration of moral hazard and adverse

[8]Let us insist on the point that this is false in the homogeneous product framework: equilibria exist where each firm offers a single contract even if menus are allowed.

[9]The models have also different feautures. First, as they study the case of separated markets for each service, firms choose the price of the services and the consumer pays. In our case, the price is fixed and paid by a third party. This is motivated by the way that Medicaid and Medicare works. In this sense, we closely follow Glazer and McGuire (2000). Second, one of their main assumptions is that the costs of providing each service are correlated. We recognise that this might be realistic in our setting also, but we do not think it would add new insights to our problem as we do not study the advantages of separating markets.

selection. (See Dionne *et al.* 2000 for a survey.)

Finally, to the more specific literature on HMO competition, we contribute by simultaneously analyzing two features that were analyzed before separately: adverse selection and horizontal differentiation. Glazer and McGuire (2000) study HMO competition under adverse selection in absence of product differentiation.[10] Ellis (1998) and Encinosa (1999) consider product differentiation but do not consider adverse selection. Frank *et al.* (2000) focus on an empirical measure of the distortions caused by adverse selection in the presence of horizontal differentiation, but they fix the value of resorting to an alternative HMO as an exogenous parameter. That is, they do not aim at characterizing the equilibrium.

There are other works that also provide valuable contributions to the literature of HMO competition under adverse selection, but none of them considers horizontal differentiation. Newhouse (1996) takes into account the influence of positive contracting costs. Neudeck and Podczeck (1996) considers market regulations under the Grossman equilibrium concept, which prevents HMOs from offering menus of contracts. Encinosa (2001a and 2001b) also considers market regulation but does so under the Nash Equilibrium concept. Jack (2002) studies risk selection when there is both risk and income heterogeneity but insurance firms are not allowed to offer menus of contracts.

Encinosa and Sappington (1997) is closest in spirit to us. They also simultaneously consider adverse selection and a market imperfection, but the imperfection there is caused by economies of scale. Contrarily to us, they find that a pooling equilibrium may exist. Hence, equilibrium results are very sensitive to the nature and source of market power. This becomes an extremely important issue in the HMO market, where market power seems to be a pervasive phenomenon.[11]

[10] They investigate how to determine the optimal capitation rates set by the health administration as a function of imperfect signals on the patients' true health status. Our results are useful to understand the behavior of HMOs for each given signal realization. Therefore, our analysis can be taken as a first step in a study of optimal risk adjustment under product differentiation. Moreover, our setting is also useful to understand the behavior of some capitation plans in Europe that are not subject to risk adjustment. This is indeed the case in the MUFACE system in Spain (Pellisé, 1994).

[11] Christianson (1997) reports 107 mergers in the US between 1985 and 1995, affecting 143 distinct geographical markets. According to Feldman *et al.* (1999), the US 10 firm concentration ratio for HMOs increased from 58% in 1994 to 67% in 1997. For the Medicare+Choice program, Town and Liu (2002) find that an increase in payments by one

The paper is organized as follows. In Section 2 we present the model and analyze the homogeneous product case as a benchmark. In Section 3 we analyze the game under product differentiation and show that we can rule out most configurations where firms offer a single contract each. In Section 4 we characterize the symmetric separating equilibrium. In subsection 4.1 we conduct the second best analysis of this equilibrium. In subsection 4.2 we compare it with the one in the homogeneous product case. In subsection 4.3 we address the issues of existence and cross-subsidization. In Section 5 we summarize the main findings and discuss some extensions. All proofs are in Appendix A.

2 The model

The players are two health plans with a single hospital each and two continua of agents. Hospitals are located at the two extremes of a straight line of length 1. We refer to the health plan with a hospital located at zero as Firm 0, and similarly for Firm 1. There are two types of agents, low risks and high risks. Both types are located uniformly along the straight line. The type of a particular agent is this agent's private information. There is a mass of γ low risk agents and a mass $1-\gamma$ of high risk agents. We assume that $0<\gamma<1$. Firms are compensated by some third party payer at a rate r per affiliated agent, regardless of his type.

There are two types of treatments. Treatment M is needed by both types of agents with probability one, and is aimed at treating a chronic illness. Treatment N is aimed at treating an acute illness that low risk agents suffer with probability p_L and high risk agents suffer with probability p_H. We assume that $0<p_L<p_H<1$.

Firms choose how much to spend treating each agent. These expenditures translate into treatment qualities, which are observable by agents. Let m be a firm's per patient expenditure on treatment M and let n be the firm's per patient expenditure on treatment N. These expenditures are only realized if agents are treated. Agents' Von-Neumann-Morgenstern utility function v over expenditure is defined directly on these expenditures. We assume that v is twice-differentiable, concave, and increasing. For $J=H,L$; the *gross*

dollar to the HMOs by the administration results in ninety cents increase in profits and only six cents increase in consumer surplus.

expected utility of an agent of type J is given by

$$U^J(m,n) = v(m) + p_J v(n). \quad (1)$$

If an agent resides at point x in the straight line, his transportation costs are tx if he attends Firm 0 and $(1-x)t$ if he attends Firm 1. The *net* expected utility of a type J agent residing at x attending Firm 0 is $v(m)+p_J v(n)-tx$. It is $v(m)+p_J v(n)-t(1-x)$ if he attends the other firm.

2.1 Firms' contract choices

We assume that either firms cannot observe an agents' location, or, equivalently, that if locations are observed, the law prohibits to discriminate on this basis because it is unethical. That is, a firm cannot make its expenditures (m,n) directly depend on an agent's location. We believe this assumption to be in accordance with reality.[12]

Therefore, firm i, $i=0,1$; either chooses a single contract (m_i,n_i) or a menu of *two* contracts $[(m_{iL},n_{iL}),(m_{iH},n_{iH})]$. To ease notation, let $w_{iJ} = (m_{iJ},n_{iJ})$ for all $i=0,1$ and $J=L,H$. We also use $W_i=(w_{iL},w_{iH})$ and $\mathcal{W}=(W_0,W_1)$.

2.2 Agents' decisions

Suppose that an agent confronts two menus of contracts, one offered by each firm. He will of course choose the best contract of the four, taking into account his utility net of transportation costs. This is equivalent to the following sequential decision process. He first compares the two contracts offered by Firm 0 (step 1), then he compares the two contracts in Firm 1 (step 2), and finally he chooses which firm (hospital) to attend and under what contract. Notice that the result of the first two steps depends on type but not on distance since distance enters linearly in the agents utility function. As for the choice of which firm to attend, this does of course depend on distance. Since agents of both types are distributed uniformly, Firm 0 faces

[12] For instance, Ellis (1998) and Biglaiser and Ma (forthcoming) conduct their analyses under the same assumption. In any case, if locations were verifiable and contracts could discriminate according to location, we would return to the standard (homogeneous product) adverse selection model for each location. This would not add new insights to the regular model, except that each of the two firms would offer an extremely complex set of contracts: one for each location.

a demand of type L (γD_{0L} henceforth) that is directly given by γ times the location $D_{0L} \in [0,1]$ of the agent of type L who is indifferent between the two firms. Similarly, Firm 0 faces a demand of type H ($(1-\gamma)D_{0H}$ henceforth) that is directly given by $(1-\gamma)$ times the location $D_{0H} \in [0,1]$ of the agent of type H who is indifferent between the two firms. For $J = L, H$; let $U_0^{J*} = Max\{U^J(w_{0L}), U^J(w_{0H})\}$ and $U_1^{J*} = Max\{U^J(w_{1L}), U^J(w_{1H})\}$. Then D_{0J} is given by

$$D_{0J}(\mathcal{W}) = \begin{cases} 0 & \text{if } U_0^{J*} < U_1^{J*} - t, \\ 1 & \text{if } U_0^{J*} > U_1^{J*} + t, \\ \frac{U_0^{J*} - U_1^{J*}}{2t} + \frac{1}{2} & \text{otherwise.} \end{cases} \tag{2}$$

To ease notation, we omit $(\mathcal{W})$ from the notation in demand. It is clear that each firm demands depend on all the outstanding contracts in the economy. The demands for Firm 1 of types L and H are given by γD_{1L} and $(1-\gamma)D_{1H}$ respectively, where $D_{1L} = 1 - D_{0L}$ and $D_{1H} = 1 - D_{0H}$. Since agents receive treatment for free, we do not need to impose any liquidity or participation constraints.[13]

2.3 Firms' payoffs

Both firms are risk neutral and have access to the same technology of producing quality out of expenditures. For $J = L, H$; the per capita profit obtained by any firm that has attracted an agent of type J with contract $w = (m, n)$ is given by

$$\Pi_J(w) = r - m - p_J n. \tag{3}$$

If Firm 0 attracts some low risks with contract $w_{0L} = (m_{0L}, n_{0L})$ and some high risks with contract $w_{0H} = (m_{0H}, n_{0H})$, then its *average* payoff is given by

$$\gamma D_{0L}\Pi_L(w_{0L}) + (1-\gamma)D_{0H}\Pi_H(w_{0H}),$$

where D_{0L} and D_{0H} are given in (2). The profits for Firm 1 are defined in an analogous way. Since firms are risk neutral while v is concave and the

[13]If transportations costs were very large, patients might decide not to take any treatment. It is enough to assume that $v(0) = -\infty$ to avoid this possibility.

same for the two treatments, a condition for productive efficiency is that expenditures be the same for both illnesses. Formally,

Definition 1 *For $i = 0, 1$; $J = L, H$; if a contract w_{iJ} satisfies $m_{iJ} = n_{iJ}$, we say that it is efficient.*

2.4 The timing of the game

The first mover is Nature, which chooses each agent's type according to γ. The second movers are the firms, who simultaneously offer a contract or a menu of contracts without any capacity constraints. Once a contract is offered, it cannot be withdrawn. Neither can it be denied to an agent demanding it. That is, for all $i = 0, 1$; firm i commits to serve any agent willing to accept a contract w at firm i. The last movers are the agents, who decide which contract to accept.

2.5 Equilibrium

We define equilibrium next, applying the notion of subgame perfection on the timing described above. The equilibrium of the subgame played by agents yields the demand functions given in (2). Given these demands, firms simultaneously choose contracts. We restrict attention to pure-strategy equilibria.

We define and classify equilibria next. In particular, we say that an equilibrium is pooling if an observer cannot infer the type of any agent from his actions. We use quite a stringent notion of pooling equilibrium by assuming that the observer knows the location of every agent. Consequently, there are two ways to infer the type of an agent from his actions. The first way is by his contract choice at a given firm. Namely, all agents attending a firm that offers a menu of incentive compatible contracts will be sorted out. This is the standard way. The other way is by an agent's firm choice. Suppose that demands differ across types, say $D_{0L} < D_{0H}$. Then all agents located in the interval $[D_{0L}, D_{0H}]$ who attend Firm 0 are known to be high risks, while all agents located in the same interval attending Firm 1 are known to be low risks. In this case, at least part of the population is also sorted out. Formally,

Definition 2
1. A vector[14] *of two menus of contracts* $\{[w_{0L}, w_{0H}], [w_{1L}, w_{1H}]\} \in \Re^2 \times \Re^2 \times \Re^2$ *with associated demands defined in (2) is said to be an* equilibrium *if neither firm gains additional profits by offering an alternative menu of contracts* $[\overline{w}_L, \overline{w}_H]$*, given the menu of contracts offered by the other firm.*
2. The equilibrium is said to be pooling *if the following conditions simultaneously hold:*
(i) All firms who attract agents do so with a single contract.
(ii) If both firms attract agents, then $D_{0L} = D_{0H}$ *(so* $D_{1L} = D_{1H}$*).*
3. The equilibrium is said to be separating *if it is not pooling.*
4. The equilibrium is said to be symmetric *if* $[w_{0L}, w_{0H}] = [w_{1L}, w_{1H}]$ *(in which case* $D_{0j} = D_{1j} = 1/2$ *for all* $j = L, H$*).*

Notice that condition (i) ensures that agents are not sorted out by contract choice at any given firm. Condition (ii) ensures that agents are not sorted out by their firm choice. Notice also that our definition of pooling equilibrium includes the pooling equilibrium in Rothschild and Stiglitz (1976), that is, the situation where both firms offer the same single contract, so that $D_{0L} = D_{0H} = 1/2$. Another pooling candidate is that Firm 0 offers a single contract (m_0, n_0) and Firm 1 also offers a single contract (m_1, n_1), with $m_0 > m_1$ but $n_0 = n_1$. It is easy to check that in this case $D_{0L} = D_{0H}$ (although $D_{0L} > 1/2$). No information about the type of any agent is revealed.

2.6 The equilibrium in the homogeneous product case

As a benchmark, we first characterize the (unique) equilibrium set of contracts $\{w_L^0, w_H^0\}$ when there is no differentiation ($t = 0$). As shown by Rotschild and Stiglitz (1976), this set of contracts solves the following system of four equations with four unknowns $(m_H^0, n_H^0, m_L^0, n_L^0)$:

(i) Zero profits from the high risks, that is $\Pi_H(w_H^0) = r - m_H^0 - p_H n_H^0 = 0$,
(ii) Efficiency for the high risk contract, or $m_H^0 = n_H^0$,
(iii) Zero profits from the low risks, or $\Pi_L(w_L^0) = r - m_L^0 - p_L n_L^0 = 0$, and
(iv) High risks' binding incentive compatibility constraint, or
$U^H(w_H^0) = v(m_H^0) + p_H v(n_H^0) = v(m_L^0) + p_H v(n_L^0) = U^H(w_L^0)$.

[14] Each contract is a vector in $\Re^2$. Each firm offers two contracts. There are two firms. Hence the equilibrium can be thought of as a vector in $\Re^8$.

The equilibrium is obtained graphically in Figure 1. The per capita isoprofit lines are straight and their slope is $-1/p_J$, $J = H, L$. The indifference curves have slope $-v'(m)/p_J v'(n)$, $J = H, L$. Efficient contracts lie in the 45^o line, where $m = n$ and the slopes are equated. Equations (i) and (ii) yield w_H^0. Then use (iii) and (iv) to obtain w_L^0. Notice that the horizontal intercept of the zero-isoprofits for both types is r, since $r - m_J - p_H n_J = 0$ and $n_J = 0$ imply $m_j = r$, for all $J = H, L$.

[FIGURE 1 ABOUT HERE]]

It is important to point out that there are many equilibrium configurations. As explained in the introduction, one is that each of the two firms offers one of the contracts in the set $\{w_L^0, w_H^0\}$, that is, full specialization. This is despite the fact that we are allowing firms to offer menus of contracts. Another is that each firm offers both contracts, as in a menu. In this latter case, any risk mix across firms is consistent with equilibrium. One important result is that under differentiation we can rule out most of these configurations

3 Differentiation: Firms offering single contracts

We start analyzing under which conditions a single contract per firm can be observed in equilibrium when firms are allowed to offer menus of contracts. We are able to prove the following.

Proposition 1 *Suppose that, in equilibrium, both firms attract agents with a single contract each. Then this equilibrium cannot be interior. In other words, either D_{0L} is zero or one, or D_{0H} is zero or one.*

We explain this proposition in several steps. The first step is to understand that if a firm attracts a positive quantity of both types in equilibrium with a single contract, this contract must be efficient ($m = n$). Suppose it is not. This same firm can increase its profits by deviating to a separating menu of contracts. This separating menu offers the low risk the same inefficient contract while it offers the high risk the contract that is efficient and gives him the same utility as the original contract. It turns out that such a menu of contracts is incentive compatible. The deviating firm makes additional per capita profits on the high risk while his demand of both types remains unaltered. In other words, efficiency is a necessary condition for a contract to be robust to menu deviations *by the same firm* (internal stability).

The second step is to understand that efficiency in one firm does not imply that the *rival*'s best response is also an efficient contract. Notice that efficiency puts constraints on the contract. Once a firm offers a single efficient contract, the other finds a profitable deviation through a more flexible non-efficient contract (external stability fails). Therefore, a pair of efficient contracts (one per firm) attracting a positive quantity of both types is not an equilibrium. But if one of the contracts is not efficient then the firm offering it can do better by deviating to a menu (by step 1). This explains Proposition 1.

Let us now address the possibility of pooling equilibria. Notice that Proposition 1 also rules out *interior* pooling candidates, since a necessary condition for pooling is that firms offer a single contract each. Therefore, only non-interior pooling candidates should be considered. This means that at least one of the firms is not attracting some type. However, a pooling equilibrium must satisfy equal demands for both types. Therefore, this firm does not attract the other type either, i.e., it shuts down. It is quite intuitive that such a situation cannot be an equilibrium. Therefore we have the following corollary:

Corollary 1 *No pooling equilibrium exists.*

We can discard still another configuration, namely, the one where each firm offers a single contract and where each firm specializes in one type of agent (full specialization). Notice that this is ruled out neither by Proposition 1 nor by Corollary 1, since full specialization is neither interior nor pooling.

Proposition 2 *There does not exist an equilibrium where firms are fully specialized, that is, where both firms offer a (different) single contract each such that each firm attracts exclusively one of the types.*

Intuitively, suppose that w_0 is offered by Firm 0 and attracts all low risks and w_1 is offered by Firm 1 and attracts all high risks. This means that high risks close to Firm 0 prefer w_1 to w_0, despite transportation costs. Symmetrically, low risks close to Firm 1 prefer w_0 to w_1 despite transportation costs.

Hence, the pair of contracts (w_0, w_1) satisfies all types' incentive compatibility constraints with some slack. Suppose that Firm 0 deviates by offering the menu (w_0, w_1). Then Firm 0 still captures all low risks and steals half of the high risks from Firm 1. Moreover, Firm 0 is able to do this without violating incentive compatibility. Its profits increase by $(1/2)(1-\gamma)\Pi_H(w_1)$. Of course, it could be that $\Pi_H(w_1)$ is zero, but this would mean that Firm 1 was making zero profits in equilibrium to start with. If that was the case, Firm 1 would deviate by lowering both components of w_1 by a sufficiently small $\varepsilon > 0$. Demand would still be close to 1 (or 1) whereas per capita profits would become positive.

Having ruled out most single-contract-per-firm configurations, we turn now to the symmetric separating equilibrium menu of contracts.

4 Differentiation: The symmetric separating equilibrium

We are going to restrict our analysis to symmetric equilibrium candidates, that is, those that verify $D_{0L} = D_{0H} = D_{1L} = D_{1H} = 1/2$; and where both firms separate types by offering incentive compatible menus. To find a separating equilibrium candidate, we must find Firm 0's reaction function. Using the notation established in (1), (2), and (3), Firm 0's best reaction function is found by solving

$$\underset{(w_{0L},w_{0H})\in\Re^2\times\Re^2}{Max} \gamma\Pi_L(w_{0L})D_{0L} + (1-\gamma)\Pi_H(w_{0H})D_{0H} \tag{4}$$

subject to

$$U^L(w_{0L}) \geq U^L(w_{0H}), \tag{5}$$

$$U^H(w_{0H}) \geq U^H(w_{0L}). \tag{6}$$

Conditions (5) and (6) are the incentive compatibility constraints (ICC henceforth). The next proposition characterizes the symmetric separating equilibrium candidate.

Proposition 3 *The candidate for a symmetric separating equilibrium satisfies*

(i) The high risk ICC is binding and the low risk ICC is not.
(ii) The contract intended for the high risk is efficient while the other is distorted. More specifically, $n_{0L} < m_{0H} = n_{0H} < m_{0L}$.
(iii) The per capita profit derived from a low risk is positive and larger than that derived from a high risk. That is, $0 < \Pi_L > \Pi_H$.

Intuitively, firms offer an efficient contract to the type that would have had incentives to lie had the first best menu of contacts been offered. In order to preserve separation, firms offer the low risk a contract that is distorted. This distortion takes the form of an overprovision of treatment quality for the sure (chronic) illness M and an underprovision of quality for the other treatment. Since the probability of needing this other treatment is low for low risks, they are the only ones that are willing to bear lower quality.

This is illustrated in Figure 2. The separating contracts are depicted as points w_H and w_L, corresponding to the high risk and the low risk, respectively. The corresponding isoprofits can be compared by looking at their horizontal intercept.[15] The proposition tells us that $\Pi_H < \Pi_L > 0$, so that $r - \Pi_L$ is to the left of $r - \Pi_H$.[16]

[FIGURE 2 ABOUT HERE]

Our contribution is two-fold. First, in equilibrium firms obtain more profits from each low risk than from each high risk. This is despite the fact that the contract offered to a low risk is distorted and it still has to be good enough to attract low risks. As we show below, the sign of the profits derived from the high risk depends on the parameters γ and t. More importantly, we have an indication that cross-subsidization may exist in equilibrium. This point is shown and further discussed in subsection 4.3.

[15] To see this, let $r - m - p_i n = \overline{\pi}$. Then $m = r - \overline{\pi}$ if $n = 0$.

[16] As a technical aside, it is impossible in our framework to derive the properties of the equilibrium graphically. One cannot determine graphically whether in equilibrium the horizontal intercept $r - \Pi_L$ is to the left (as depicted) or to the right of $r - \Pi_H$ (which would occur if point w_L was lower down the high risk ICC), since the two straight lines depicted need not be zero-isoprofits. This is in contrast to the perfectly competitive situation, where the zero profit condition for each type, together with efficiency at the bottom and the binding high risk ICC constitute a system of four curves that *uniquely* determine the two equilibrium menus (a vector in $\Re^2 \times \Re^2$).

Second, in equilibrium each firm offers a menu of contracts, as in Miyazaki (1977), but without the need to use Wilson's equilibrium notion. The implications of this were already discussed in the introduction.

4.1 Second-Best Analysis

We now ask ourselves whether the symmetric separating equilibrium candidate fails to be second-best (equivalently, fails to be constrained Pareto-optimal). In other words, starting from this candidate, can a planner as informed as the firms improve all players welfare simultaneously (and strictly so for some player)? A partial answer is given in the next proposition.

Proposition 4 *Suppose that the menu* $(w_L^*, w_H^*) = ((m_L^*, n_L^*), (m_H^*, n_H^*))$ *constitutes a symmetric separating equilibrium.*

a) If (w_L^*, w_H^*) *is second-best optimal then the profits derived from a high risk cannot be negative. That is, an equilibrium exhibiting cross-subsidization is not second-best optimal.*

b) If the profits derived from a high risk are *negative then an incentive compatible menu of contracts with the following characteristics exists:*

(i) It improves agents' welfare and leaves firms' average profits constant.

(ii) The contract intended for the low risks is less distorted than in equilibrium (n_L *is higher and* m_L *is lower) and* Π_L *is higher than in equilibrium.*

(iii) The contract intended for the high risks is still efficient, but Π_H *is lower than in equilibrium.*

Part (a) says that a necessary condition for the equilibrium to be second-best is that profits obtained from any high risk agent be non-negative. The Pareto-improvement that we are proposing in part (b) of the proposition is depicted in Figure 3. First, we increase both components of the high risk contract by the same amount, to preserve "efficiency at the bottom". This reduces Π_H. Second, we reduce the distortion of the low risk contract in order to raise Π_L and to maintain average profits. However, this must be done without violating the high risk's ICC. In other words, we raise n_L and lower m_L in such a way that high risks are still indifferent between their improved contract and the new low risk contract. Finally, if these changes do not decrease low risks' utility and preserve each firm's profits, we have found a Pareto-improvement.

[FIGURE 3 ABOUT HERE]

Why is it possible to reach this improvement when we start from an equilibrium with $\Pi_H < 0$? In other words, why didn't firms themselves deviate in the same direction as the proposed improvement? If Π_H is negative, a firm trying to *unilaterally* improve welfare will attract high risk agents. This makes such welfare improvement unattractive for any given firm. Hence in equilibrium welfare is low. Suppose a planner comes in: she will improve welfare at all firms, so any given firm does not attract any additional high risks. This explains part (a) of the proposition.

Notice that Proposition 4 does not contradict the results obtained in a perfectly competitive market. If there was no differentiation ($t = 0$), equilibrium would be characterized by zero profits across types. Therefore the proposition simply does not apply.

One could argue that welfare properties should be conditioned on γ rather than on the sign of $\Pi_H(w_H^*)$, because γ is an exogenous parameter whereas $\Pi_H(w_H^*)$ is endogenous. However, if one wants to prescribe some governmental intervention using the previous proposition, it is sufficient to observe per capita profits. In contrast, it could very well be that γ is only observable to firms. If the planner does not observe γ then the equilibrium would remain intact but the planner would not be able to take a decision on whether to intervene or not. Of course, by fully solving the model one can obtain the relationship between γ and per capita profits and therefore between γ and welfare.

Finally, a comment on the notion of second-best. Most analysis of constrained optimality use Harris and Towsend (1981) notion, which is weaker than the usual concept, since Pareto-improvements are restricted to preserve the *same* profits as the status quo. In other words, to say that an allocation is *not* second-best one must find a change that strictly improves at least one of the agent's utility while keeping the principal's profits constant. Since we have found such a change, we are not only providing necessary conditions for this weaker notion of optimality, but also for any stronger notions.

4.2 Comparisons with the homogeneous product case

In subsection 2.6 we obtained the equilibrium set of contracts under homogeneous product, $\{w_L^0, w_H^0\}$. We now compare this set of contracts with the symmetric separating set of contracts $\{w_L, w_H\}$, characterized by Proposition 3 for the differentiation case. Notice that this proposition does not provide a full characterization. The reason is that profits per agent of any type need not be zero. Therefore, there is no analogous graphical procedure to obtain the equilibrium set of contracts. We therefore must address the aforementioned comparison on a case-by-case basis. There are two cases. Suppose first that the equilibrium presents cross-subsidization, so $\Pi_H < 0 < \Pi_L$. As it will be shown in the next section, this occurs when t is small (but positive). In the second case there is no cross-subsidization, so $0 \leq \Pi_H < \Pi_L$. Once we know the signs of Π_H and Π_L, we are able to derive some comparisons between the equilibria with and without differentiation.

Let us begin with the first case. We again depict the equilibrium set of contracts in Figure 4. As explained after Proposition 3, $\Pi_H < 0 < \Pi_L$ implies that the horizontal intercept of the high risk isoprofit (i') is to the right of r, whereas that of the low risk isoprofit (iii') is to the left of r. These two isoprofits, plus efficiency at the high risk contract (ii) and the high risks' ICC (iv') determine the equilibrium set of contracts (w_L, w_H) under differentiation, in the same manner as for the homogeneous product case. This directly shows, firstly, that $U^H(w_H) > U^H(w_H^0)$ in terms of gross utility. Therefore, high risks who are close enough to the extremes are better-off with differentiation. Secondly, the contract intended for the low risks is less distorted with differentiation. In other words, low risks enjoy a higher quality of treatment for their acute illness but the opposite is true for their chronic illness. Unfortunately, whether their gross utility is larger with differentiation or not will depend on the other parameters. Their utility net of transportation costs will of course decrease if t is large enough, but this condition may be inconsistent with cross-subsidization. To sum up, although the distortions present in the low-risk's contracts are slightly corrected under differentiation, this may or may not benefit the low risks.

[FIGURE 4 ABOUT HERE]

Consider now the case when there is no cross-subsidization (i.e., when t is large). Market power will allow firms to obtain rents from both types. One can prove that both risks will be worse-off with differentiation[17] and that the distortions in the contract for the low risk are not corrected.[18]

4.3 Existence and cross-subsidization

As it has been recognized in the literature (Rothschild and Stiglitz, 1976), competitive adverse selection models fail to have an equilibrium for some parameter values. Through numerical examples, we will see that in our model this can also be the case. However, we will also see that, at least in our examples, existence is more likely with $t > 0$ than with $t = 0$.[19] We will also prove by example the possibility of cross-subsidization from low risks to high risks in equilibrium.

The maintained assumption in our simulations are the following. We set $v(z) = \ln(z)$,[20] $r = 10$, $p_L = 0.2$, and $p_H = 0.8$. Existence is addressed by means of Table 1. We report there, for different values of t, the critical value for γ such that equilibrium fails to exist if the proportion of low risks is above it. Notice that they are all above 0.74, which is the critical γ for $t = 0$ (the homogeneous product case studied by Rotschild and Stiglitz 1976).[21]

[17] This is true even without taking transportation costs into account. The high risks' contract lies at a lower point in the 45^o-line. As for the low risks, suppose, by contradiction, that their gross utility has increased. Due to the single-crossing condition and also to the fact that the low risk indifference curve crosses the $\Pi^L = 0$ line from below, contract w_L must now lie above the $\Pi^L = 0$ line to preserve incentive compatibility. This contradicts $\Pi^L > 0$.

[18] We have to prove that w_L will never move inside the lens formed by the $\Pi^L = 0$ line and the low risk indifference curve going through w_L^0. This lens is contained inside the lens formed by the $\Pi^L = 0$ line and the *high* risk indifference curve going through w_L^0, due to the single crossing condition. To see that the contract w_L will not move inside this larger lens, notice that both the high risk incentive compatibility constraint and the low risk isoprofit lines shift to the left.

[19] We tried to obtain this result analytically. However, this was not possible because in the cases where a deviation breaks the equilibrium candidate, this deviation is not local. This prevented us from obtaining an analytical result even for specific functional forms for the utility function. In Appendix B we explain the procedure to obtain the reported simulations.

[20] We have repeated a similar exercise using $v(z) = \sqrt{z}$. The qualitative conclusions remain valid.

[21] The reported critical values should be interpreted as follows. If $t = 0.005$, we know that our symmetric candidate is an equilibrium if $\gamma \leq 0'76$ and we know that it is not an equilibrium if $\gamma > 0'77$. We have computed the critical value for γ when $t = 0$ with more accuracy: it lies between 0.74039 and 0.74041.

Notice that for values of t above 0.08, the critical γ is above 0.99. Hence, for relatively small values of t, equilibrium exists for a very wide range of γ. A comforting result is that, as it should be expected, the critical γ is very close to the one obtained for $t = 0$ for t small enough. Namely, for $t = 0.005$, the critical gamma is 0.76.

Table 1. Critical γ for existence of equilibrium

t	0	0.005	0.500	0.060	0.070	0.080	0.090	0.100
Critical γ	0.74	0.76	0.88	0.90	0.92	0.99	0.99	0.99

Here is the intuition for why a larger γ is necessary to break the equilibrium candidate when $t > 0$ than when $t = 0$. In the symmetric candidate, independently of the value of t, each firm has half of the demand of each type, hence the mix of high and low risks is γ, not only in the whole economy but also within each firm. A deviating firm offers a menu of contracts that improves both types. When $t = 0$ there are no frictions and hence an improvement in both types' contracts will attract all the consumers in the market. Consequently the deviating firm will have the same mix of low and high risks as it did when offering the candidate. In contrast, if $t > 0$ an improvement in both the low and the high risk contracts that preserves incentive compatibility will attract a larger number of high risks than low risks. This is so because the improvement in the high risk's contract maintains efficiency, so it involves an increase in both components of the contract, while the improvement in the low risk's contract trades-off a higher n for a lower m. These changes in the menu of contracts raise high risks' utility more than low risks'. Consequently, more high risks than low risks will be willing to pay the travel cost and take the contract offered by the deviating firm. Hence the deviating firm may end up with a less favorable risk mix than in the candidate. Hence, one needs a higher proportion of low risks in the economy for the deviation to be profitable.

By Proposition 3, we know that $\Pi_L > \Pi_H$. We want to go further and show that parameter values exist for which equilibrium entails $\Pi_H < 0$, i.e., that there is cross-subsidization for these parameter values.[22] This is depicted in Figure 5. This figure depicts per capita profits per type for different values

[22] Encinosa (2001a) restores cross-subsidization neither resorting to Wilson's notion nor introducing differentiation. Suppose firms offer a *fixed* quantity of contracts w_H and a

of $t \geq 0.06$. The maintained hypothesis is $\gamma = 0.9$. This ensures, by Table 1, that the allocations in Figure 5 are an equilibrium, since 0.9 is below the critical γ for t equal or larger than 0.06. In the introduction we provided a simple intuitive argument for why firms do not drop loss-making contracts.

[FIGURE 5 ABOUT HERE]

Notice that negative profits are obtained from the high risks for t small enough. On the other hand, when t is very large, market power becomes important and profits obtained from either type are positive. In this case cross-subsidization disappears.

Last but not least, we have used a somewhat restrictive notion of cross-subsidization. We could have defined cross-subsidization as any situation where $\Pi_L > \Pi_H$. With this definition, Proposition 3 tells us that the symmetric separating equilibrium, if it exists, always entails cross-subsidization.

5 Conclusions

In this paper we have investigated the consequences of introducing a realistic component in the competition among health plans: geographical differentiation. Hence, we analyze the coexistence of two simultaneous market imperfections: adverse selection and differentiation. This has proven to be quite fruitful but at the same time it has introduced some technical difficulties. The most important difficulty is that one cannot use the zero-profit condition. As a consequence, one cannot rely on graphical arguments in order to characterize the equilibrium. Despite these difficulties, we have been able to (i) characterize the symmetric equilibrium candidate analytically, (ii) obtain a necessary condition for the equilibrium to be second-best optimal, and (iii) rule out many potential equilibrium configurations.

As for (i), we regard the fact that profits derived from a low risk are higher than from a high risk as one of the important empirical implications of our

fixed quantity of contracts w_L. Suppose that a firm deviates by offering only w_L contracts. There is then an excess demand for the standing contracts in the economy as a whole. Using the proportional rationing rule, demanders are allocated randomly. If $(1-\gamma)$ is large enough, the amount of high risks who return to the deviant firm to accept w_L (the only one available there) is large enough to render the deviation unprofitable.

paper. We have proven by example that there exist equilibria with cross-subsidization, i.e., where the profits derived from high risks are negative.

With respect to (ii), our policy implication is clear: the planner has a role to play for sure precisely when the equilibrium presents cross-subsidization. This contrasts to the homogenous product case for which any equilibrium is second-best efficient if firms are allowed to offer menus of contracts. In our framework, the planner can improve both types' welfare without hurting profits by forcing both firms to correct the distortion imposed on the low risks. This can be implemented through exactly the same tax system introduced by Crocker and Snow (1985a). The fact that the correction is carried out by both firms simultaneously is crucial. A unilateral correction would attract too many high risks. This explains why equilibria that are not second best subsist.

Finally, (iii) has allowed us to give more precise predictions on the equilibrium mix of agents that each health plan attracts and on the minimal firm complexity even if transportation costs are arbitrarily small but positive. There cannot be equilibria where all firms offer a single contract. This is of interest given the proliferation of HMOs that offer menus of contracts (Marquis and Long, 1999).

Unfortunately, the aforementioned technical difficulties have not allowed us to analytically prove several potential results that the numerical simulations suggest. First, as it is standard in competitive adverse selection models that use the Nash equilibrium concept, there are parameter values for which our model does not have a symmetric separating equilibrium. However, we have shown that there exist parameter values for which equilibrium exists even for relatively small transportation costs and for values of γ above the perfectly competitive critical γ. Second, cross-subsidization is more likely in equilibrium if transportation costs are small.

Several extensions to our model can be discussed. If there were more than two firms in the market, to preserve symmetry we would change the model of differentiation by using the circular city model introduced by Salop (1979). We believe that the main insights would remain valid. An increase in the number of firms would be equivalent to a decrease in transportation costs, since only the two adjacent firms are directly competing with any given firm. If free entry is allowed, one would of course have to introduce a fixed entry cost in order to avoid convergence to the perfectly competitive market. We should however be cautious here since entry may lower transportation costs below the threshold below which our equilibrium fails to exist.

The work of Biglaiser and Ma (forthcoming), although in a different setting, suggests that if the costs of providing quality for the acute and the chronic illness were correlated[23] then the Health administration could increase welfare by making it compulsory that firms sell their chronic and acute treatments separately. The idea is that firms would have less of an incentive to dump the high risks.

Allowing firms to locate in other points in the line other than the extremes would introduce two possible extensions. First, a simple one where locations are exogenous but not the extremes. In this case firms would gain market power with respect to the agents located around their closest extreme, but loose market power with respect to the agents located around the center of the line. Since our model does not allow for discrimination among differently located agents, we believe that this should be similar to a change in transportation costs. The main insight of the model, that is, that changes in demand are smooth, remains. Second, one could also assume that firms choose their location simultaneously in a stage previous to competition. By changing the traveling costs from a linear to a quadratic function of distance, we believe we would obtain the maximum-differentiation result and our results would not be altered.

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[23] That is, if also the chronic illness was suffered by agents with a probability q_J that also depends on type, with $q_H > q_L$.

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7 APPENDIX A. Proofs

Proof of Proposition 1

Proof. *Step 1. If a Firm i offers a single contract* (m_i, n_i) *(or two contracts of which one is never chosen) and if* $0 < D_{iL} = D_{iH} < 1$*, then this firm's contract must be efficient. That is,* $m_i = n_i$*.*

Assume first, by contradiction, that a firm offers the contract $w' = (m', n')$ with $m' > n'$. We will find an alternative strategy that increases the profits of this firm, whatever the other firm does. Consider the menu of contracts $[w', w'']$, with $m' > m'' = n'' > n'$, and $U^H(w'') = U^H(w')$. Hence H has no interest in camouflaging when asked to choose w''.We prove next that L has no interest in camouflaging when asked to choose w'. Notice first that $U^H(w'') = U^H(w')$ implies that $v(n') = \frac{v(m'')-v(m')}{p_H} + v(n'')$. Hence $U^L(w') - U^L(w'') = v(m') + p_L v(n') - v(m'') - p_L v(n'') = v(m') + p_L \left[\frac{v(m'')-v(m')}{p_H} + v(n'')\right] - v(m'') - p_L v(n'') = (1 - \frac{p_L}{p_H}) [v(m') - v(m'')] > 0$. Notice that both types are indifferent between the previous single contract and the new menu of contracts. Therefore demands are unaltered whatever the type. Since $D_{0H} > 0$, it suffices to show that profits derived from one H-agent are larger. (The profits derived from the other type are unaltered.) This is straightforward: contracts w' and w'' yield the same expected utility to agents while w'' is efficient while w' is not.

A similar argument can be used to show that a pooling contract $w' = (m', n')$ with $m' < n'$, cannot be part of a pooling equilibrium. (Use a menu of contracts where L receives an efficient contract without changing his utility and which leaves the original pooling unaltered. The high risk will stay put and the low risk will move to the new contract. Profits derived from the low risk increase.)

Step 2 That $0 < D_{0L} < 1$ *and* $0 < D_{0H} < 1$ *(so* $0 < D_{1L} < 1, 0 < D_{1H} < 1$ *as well) cannot be part of an equilibrium.*

Suppose, by contradiction, that $0 < D_{0L} < 1$ *and* $0 < D_{0H} < 1$. By step 1, this implies that $m_0 = n_0$ and $m_1 = n_1$. Since this constitutes an equilibrium then $[m_0, m_0]$ is Firm 0's best response to $[m_1, m_1]$. In general, the contract $w' = (m', n')$ that is the best response to contract $[m_1, m_1]$ solves

$$\begin{aligned} & \underset{\{m',n'\}}{Max} \quad \gamma\Pi_L(w')D_{0L} + (1-\gamma)\Pi_H(w')D_{0H}, \\ \text{where } \Pi_L(w') &= r - m' - p_L n', \\ \Pi_H(w') &= r - m' - p_H n', \\ D_{0L} &= \frac{v(m') - v(m_1) + p_L(v(n') - v(m_1)) + t}{2t}, \\ D_{0H} &= \frac{v(m') - v(m_1) + p_H(v(n') - v(m_1)) + t}{2t}. \end{aligned}$$

It is very important to notice that $0 < D_{iL} < 1$ and $0 < D_{iH} < 1$ for all $i = 0, 1$ implies that the solution is interior. Therefore, the first order conditions with respect to m', n' are necessary conditions. They are:

$$\begin{aligned} \gamma\Pi_L(w')\frac{v'(m')}{2t} - \gamma D_{0L} - (1-\gamma)D_{0H} + (1-\gamma)\Pi_H(w')\frac{v'(m')}{2t} &= 0, \\ \gamma\Pi_L(w')p_L\frac{v'(n')}{2t} - \gamma p_L D_{0L} - p_H(1-\gamma)D_{0H} + (1-\gamma)p_H\Pi_H(w')\frac{v'(n')}{2t} &= 0. \end{aligned}$$

Hence, $n' = m' = m_0$ must satisfy these first order conditions. Letting $L = \gamma\left[\Pi_L(w')\frac{v'(m_0)}{2t} - D_{0L}\right]$ and $H = (1-\gamma)\left[\Pi_H(w')\frac{v'(m_0)}{2t} - D_{0H}\right]$, these first order conditions imply that

$$\begin{aligned} L + H &= 0, \\ p_L L + p_H H &= 0. \end{aligned}$$

Since $p_L \neq p_H$, this implies that $L = H = 0$, or

$$\begin{aligned} \Pi_L(w')\frac{v'(m_0)}{2t} &= D_{0L}, \\ \Pi_H(w')\frac{v'(m_0)}{2t} &= D_{0H}. \end{aligned}$$

It must also be true that $w_1 = [m_1, m_1]$ is Firm 1's best response to $[m_0, m_0]$. Hence, using a symmetric argument, contract w_1 must solve

$$\begin{aligned} \Pi_L(w_1)\frac{v'(m_1)}{2t} &= D_{1L}, \\ \Pi_H(w_1)\frac{v'(m_1)}{2t} &= D_{1H}. \end{aligned}$$

Use $D_{0L} + D_{1L} = 1$ and $D_{0H} + D_{1H} = 1$ to obtain two expressions. Substract them to get

$$[\Pi_L(w') - \Pi_H(w')]\,\frac{v'(m_0)}{2t} + [\Pi_L(w_1) - \Pi_H(w_1)]\,\frac{v'(m_1)}{2t} = 0.$$

However, $\Pi_L(w') = r - m_0(1+p_L) > r - m_0(1+p_H) = \Pi_H(w')$ and $\Pi_L(w_1) = r - m_1(1+p_L) > r - m_1(1+p_H) = \Pi_H(w_1)$, contradiction. This concludes the proof. ■

Proof of Corollary 1

Proof. Since interior equilibria with single contracts per firm have been ruled out in Proposition 1, and since our definition of pooling requires that $D_{0L} = D_{0H}$, it only remains to prove that it is not an equilibrium that one of the firms shuts down.

Without loss of generality we prove that if $W = \{(m_0, n_0), (m_1, n_1)\}$ is such that $D_{0L} = D_{0H} = 1$ (and therefore $D_{1L} = D_{1H} = 0$), then W is not an equilibrium.

Since $D_{0L}, D_{0H} > 0$, we have $m_0 = n_0$, by Step 1 in the proof of Proposition 1. Consider the following deviation by Firm 0. Let $m_0' = m_0 - \varepsilon = n_0 - \varepsilon$. This firm's payoff becomes

$$\begin{gathered} E_0(\varepsilon) = \\ \gamma\,[r - (m_0-\varepsilon)(1+p_L)]\,[\frac{1}{2} + \frac{v(m_0-\varepsilon)(1+p_L) - v(m_1) - p_L v(n_1)}{2t}] + \\ (1-\gamma)\,[r - (m_0-\varepsilon)(1+p_H)]\,[\frac{1}{2} + \frac{v(m_0-\varepsilon)(1+p_H) - v(m_1) - p_H v(n_1)}{2t}]. \end{gathered}$$

Differentiate $E_0(\varepsilon)$ with respect to ε to get

$$\begin{aligned}
E_0'(\varepsilon) &= \gamma(1+p_L)[\frac{1}{2}+\frac{v(m_0-\varepsilon)(1+p_L)-v(m_1)-p_L v(n_1)}{2t}]+ \\
&+\gamma\left[r-(m_0-\varepsilon)(1+p_L)\right]\frac{-v'(m_0-\varepsilon)(1+p_L)}{2t}+ \\
&(1-\gamma)(1+p_H)[\frac{1}{2}+\frac{v(m_0-\varepsilon)(1+p_H)-v(m_1)-p_H v(n_1)}{2t}]+ \\
&+(1-\gamma)\left[r-(m_0-\varepsilon)(1+p_H)\right]\frac{-v'(m_0-\varepsilon)(1+p_H)}{2t}.
\end{aligned}$$

Evaluate this derivative at $\varepsilon = 0$ (where $D_{0L} = D_{0H} = 1$). Let $E[\cdot]$ be the expectation operator using the proportions of low and high risks. Then the aforementioned derivative is positive at $\varepsilon = 0$ if

$$E[1+p] >$$

$$\frac{v'(m_0)}{2t}\left[(1-\gamma)\left[r-m_0(1+p_H)\right](1+p_H)+\gamma\left[r-m_0(1+p_L)\right](1+p_L)\right].$$

The second factor in the right hand side can be rewritten as $rE[1+p] - m_0E[(1+p)^2]$.

Therefore, the expression $E_0'(0) > 0$ can be rewritten as

$$E[1+p] > \frac{v'(m_0)}{2t}\left(rE[1+p]-m_0E[(1+p)^2]\right).$$

Suppose now that the original Firm 0 profits were zero, that is, $E_0(0) = 0$. Then $(1-\gamma)\left[r-m_0(1+p_H)\right]+\gamma\left[r-m_0(1+p_L)\right] = 0$ (recall demands are 1) and therefore $m_0 = \frac{r}{E(1+p)}$. Substitute into last expression to get

$$1 > \frac{v'(m_0)}{2t}r\left(1-\frac{E[(1+p)^2]}{\left(E(1+p)\right)^2}\right).$$

Therefore, a sufficient condition ensuring $E_0'(0) > 0$ is $1 - \frac{E[(1+p)^2]}{(E(1+p))^2} \leq 0$ or $\left(E(1+p)\right)^2 \leq E[(1+p)^2]$, which is true by Jensen's Inequality, since the function $L(x) = x^2$ is convex.

Suppose now that the original Firm 0's profits were positive, that is, $E_0(0) > 0$. Since $D_{1L} = D_{1H} = 0$, Firm 1's profits are zero. Consider the following deviation, this time by Firm 1. Let Firm 1 imitate Firm 0's

contracts, thus capturing half of the market for each type. Doing this Firm 1's profits become
$E_1 = \frac{(1-\gamma)}{2}\left[r - m_0(1+p_H)\right] + \frac{\gamma}{2}\left[r - m_0(1+p_L)\right] =$
$\frac{1}{2}\left[(1-\gamma)\left[r - m_0(1+p_H)\right] + \gamma\left[r - m_0(1+p_L)\right]\right] = \frac{1}{2}E_0(0) > 0$. Thus, Firm 1's profits become positive, and we are done. ■

Proof of Proposition 2

Proof. Without loss of generality, take any candidate (w_0, w_1) such that $D_{0L}(w_0, w_1) = 1, D_{0H}(w_0, w_1) = 0$ (and $D_{1L} = 0, D_{1H} = 1$).

Suppose first that $\Pi_H(w_1) = 0$. Hence $E_1 = (1-\gamma)D_{1H}\Pi_H = 0$. Then let $w_1' = (m_1 - \varepsilon, n_1 - \varepsilon)$. This implies that $\Pi_H(w_1') > 0$, whereas D_{1H} either stays the same at one or decreases slightly below one, by continuity. Hence $D_{1H}' > 0$. Therefore $(1-\gamma)D_{1H}'\Pi_H(w_1') > 0$. Moreover, D_{1L} stays the same at zero (since we have cut both components of the contract). Therefore $E_1' = (1-\gamma)D_{1H}'\Pi_H(w_1) > 0$. We have found a profitable deviation.

Suppose now that $\Pi_H(w_1) > 0$. Then let Firm 0 offer both contracts, w_1 and w_0 simultaneously. Since $D_{0L} = 1$ while $D_{0H} = 0$, it must the case that $U^L(w_0) \geq U^L(w_1) + t$ and $U^H(w_0) \leq U^H(w_1) - t$. Hence $U^L(w_0) > U^L(w_1)$ and $U^H(w_0) < U^H(w_1)$. This implies that (w_0, w_1) is an incentive compatible menu. Therefore all L who decide to go to 0 still prefer w_0. (I.e., to a L, it is as if Firm 0 only offered one contract, w_0.) Hence, D_{0L} and Π_L remain the same. On the other hand, all H prefer w_1. However, since there are now two firms offering the contract w_1, the demand is equally shared among the two firms. In other words, $E_0' = E_0 + \frac{(1-\gamma)}{2}\Pi_H(w_1) > E_0$. We have again found a profitable deviation. This concludes the proof. ■

Proof of Proposition 3

For the purposes of the simulation procedure used in subsection 4.3, we prove an extended version of Proposition 3. Namely,

Proposition 3a
(a) In any interior equilibrium with $w_{0L} \neq w_{0H}$ at least one ICC is binding.
(b) In a separating equilibrium candidate,
(i) If the low risk ICC (5) is binding, then w_{0L} is efficient while w_{0H} overinvests in n and underinvests in m as compared to w_{0L}. That is, $m_{0H} < m_{0L} = n_{0L} < n_{0H}$.

(ii) If the high risk ICC (6) is binding, then w_{0H} is efficient while w_{0L} overinvests in m and underinvests in n as compared to w_{0H}. That is, $n_{0L} < m_{0H} = n_{0H} < m_{0L}$.
(iii) If the candidate is symmetric, the high risk ICC is binding and the low risk ICC is not, so (ii) holds. Moreover, the per capita profit derived from a low risk is positive and larger than that derived from a low type (i.e., $0 < \Pi_L(w_L) > \Pi_H(w_H)$).

Proof. We focus on the firm located at zero. The Lagrangian of the firm located at 0 is

$$\begin{aligned} \pounds_0 &= \gamma\Pi_L(w_{0L})D_{0L} + (1-\gamma)\Pi_H(w_{0H})D_{0H} + \\ &\quad +\mu_H(U^H(w_{0H}) - U^H(w_{0L})) + \mu_L(U^L(w_{0L}) - U^L(w_{0H})). \end{aligned}$$

According to the necessary Kuhn-Tucker conditions, if $(m_{0H}, n_{0H}, m_{0L}, n_{0L})$ solves this firm's problem then there exist $\mu_L \geq 0$ and $\mu_H \geq 0$ such that

$$\gamma\left[\frac{\Pi_L(w_{0L})v'(m_{0L})}{2t} - D_{0L}\right] - (\mu_H - \mu_L)\, v'(m_{0L}) = 0, \tag{7}$$

$$\gamma p_L\left[\frac{\Pi_L(w_{0L})v'(n_{0L})}{2t} - D_{0L}\right] - (\mu_H p_H - \mu_L p_L)\, v'(n_{0L}) = 0, \tag{8}$$

$$(1-\gamma)\left[\frac{\Pi_H(w_{0H})v'(m_{0H})}{2t} - D_{0H}\right] + (\mu_H - \mu_L)\, v'(m_{0H}) = 0, \tag{9}$$

$$(1-\gamma)p_H\left[\frac{\Pi_H(w_{0H})v'(n_{0H})}{2t} - D_{0H}\right] + (\mu_H p_H - \mu_L p_L)\, v'(n_{0H}) = 0. \tag{10}$$

If the low risk ICC is **not** binding, that is, if

$$v(m_{0L}) + p_L v(n_{0L}) > v(m_{0H}) + p_L v(n_{0H}), \tag{11}$$

then $\mu_L = 0$.

If $\mu_L > 0$ then

$$v(m_{0L}) + p_L v(n_{0L}) = v(m_{0H}) + p_L v(n_{0H}), \tag{12}$$

that is, the low risk ICC is binding.

If the high risk ICC is **not** binding, that is, if

$$v(m_{0H}) + p_H v(n_{0H}) > v(m_{0L}) + p_H v(n_{0L}), \tag{13}$$

then $\mu_H = 0$.

If $\mu_H > 0$ then

$$v(m_{0H}) + p_H v(n_{0H}) = v(m_{0L}) + p_H v(n_{0L}), \tag{14}$$

that is, the high risk ICC is binding.

The proof of Part (a) is straightforward. Suppose, on the contrary, that both ICC constraints are not binding. Then (11) and (13) hold so μ_L and μ_H are zero. Substitute into (7) through (10) to get

$$\begin{aligned}
\Pi_L(w_{0L})\frac{v'(m_{0L})}{2t} &= D_{0L}, &(15)\\
\Pi_L(w_{0L})\frac{v'(n_{0L})}{2t} &= D_{0L}, &(16)\\
\Pi_H(w_{0H})\frac{v'(m_{0H})}{2t} &= D_{0H}, &(17)\\
\Pi_H(w_{0H})\frac{v'(n_{0H})}{2t} &= D_{0H}. &(18)
\end{aligned}$$

Since v' is strictly decreasing, equations (15) and (16) imply that $m_{0L} = n_{0L}$. Equations (17) and (18) imply that $m_{0H} = n_{0H}$. Then, the fact that ICCs are not binding (11) and (13) imply $v(m_{0L}) > v(m_{0H})$ and $v(m_{0H}) > v(m_{0L})$, a contradiction. These proves that at least one of the multipliers (μ_L or μ_H) must be different from zero. This implies that the constraint to which it is associated must be binding.

The proof of Part (b) requires several steps.

Step 1: Show that $w_{0L} \neq w_{0H}$ implies that *exactly one* ICC is binding.
We already proved above that at least one ICC must be binding. We prove now that the two ICCs cannot be simultaneously binding. Adding the two constraints with equality ((12) and (14)) directly implies that $n_{0L} = n_{0H}$. Substitute this into (12) to get $m_{0L} = m_{0H}$. Hence, $w_{0L} = w_{0H}$, contradiction. Therefore, One of the two constraints is not binding.

Step 2. Assume that the low risk ICC is binding (12) and prove part (i) of the proposition.

Step 2.1. Since the low risk ICC is binding, by Step 1 the high risk's is not binding. Therefore $\mu_H = 0$.

Substitute this into (7) and (8) to get

$$\begin{aligned}
v'(m_{0L})[\gamma\frac{\Pi_L(w_{0L})}{2t} + \mu_L] &= \gamma D_{0L},\\
v'(n_{0L})[\gamma\frac{\Pi_L(w_{0L})}{2t} + \mu_L] &= \gamma D_{0L}.
\end{aligned}$$

This implies that $m_{0L} = n_{0L}$. In other words, w_{0L} is efficient.

Step 2.2. Since L's ICC is binding and H's is not, and since $m_{0L} = n_{0L}$, we can write

$$v(m_{0L}) + p_L v(m_{0L}) = v(m_{0H}) + p_L v(n_{0H}),$$

$$v(m_{0H}) + p_H v(n_{0H}) > v(m_{0L}) + p_H v(m_{0L}).$$

Add the last two expressions to get $m_{0L} < n_{0H}$, by using $p_L < p_H$.

Step 2.3. Write the binding constraint of the low risk as $v(m_{0L})+p_L v(m_{0L}) = v(m_{0H})+p_L v(n_{0H})$. Since $p_L v(m_{0L}) < p_L v(n_{0H})$ by Step 2.2, it must be true that $m_{0L} > m_{0H}$.

Step 2.4. To conclude the proof of Step 2, use Steps 2.1, 2.2 and 2.3 to write $m_{0H} < m_{0L} = n_{0L} < n_{0H}$.

Step 3. Assume that the high risk ICC is binding (14) and prove part (ii) of the proposition.

Step 3.1. By Step 1, L's ICC cannot be binding and therefore $\mu_L = 0$. Substitute this into (9) and (10) to get

$$\begin{aligned} v'(m_{0H})[(1-\gamma)\frac{\Pi_H(w_{0H})}{2t} - \mu_H] &= (1-\gamma)D_{0H}, \\ v'(n_{0H})[(1-\gamma)\frac{\Pi_H(w_{0H})}{2t}p_H - p_H\mu_H] &= (1-\gamma)D_{0H}p_H. \end{aligned}$$

This implies (cancel p_H in both sides of last expression) that $m_{0H} = n_{0H}$. Hence w_{0H} is efficient.

Step 3.2. H's binding ICC and L's not binding ICC become

$$v(m_{0H}) + p_H v(m_{0H}) = v(m_{0L}) + p_H v(n_{0L}),$$

$$v(m_{0L}) + p_L v(n_{0L}) > v(m_{0H}) + p_L v(m_{0H}).$$

Adding these two expressions yields $m_{0H} > n_{0L}$ (recall again that $p_H > p_L$).

Step 3.3. In the first of the last two expressions, we have $p_H v(m_{0H}) > p_H v(n_{0L})$, so $m_{0H} < m_{0L}$.

Step 3.4. Steps 3.1, 3.2, and 3.3 imply $n_{0L} < m_{0H} = n_{0H} < m_{0L}$.

Step 4. Concentrate on a *symmetric* separating equilibrium and prove that $L's$ ICC cannot be binding.

Step 4.1. Suppose that $L's$ ICC is binding. Then $\mu_L \neq 0$ and $\mu_H = 0$ by Step 1 and Part (a). Imposing symmetry implies that $D_{0L} = D_{0H} = 1/2$. Substitute these facts into conditions (7) through (10). By inspection, (7) and (8) are equivalent because $m_{0L} = n_{0L}$ by Step 2.1. We are left with the following conditions:

$$\begin{aligned} -\gamma\frac{1}{2} + \gamma\Pi_L(w_{0L})\frac{v'(m_{0L})}{2t} + \mu_L v'(m_{0L}) &= 0, \quad (19) \\ -(1-\gamma)\frac{1}{2} + (1-\gamma)\Pi_H(w_{0H})\frac{v'(m_{0H})}{2t} - \mu_L v'(m_{0H}) &= 0, \quad (20) \\ -(1-\gamma)p_H\frac{1}{2} + (1-\gamma)\Pi_H(w_{0H})\frac{v'(n_{0H})}{2t}p_H - \mu_L p_L v'(n_{0H}) &= 0. \quad (21) \end{aligned}$$

Step 4.2. Isolate $v'(m_{0H})$ from (20) and isolate $v'(n_{0H})$ from (21). Divide the resulting expressions to get

$$\frac{v'(m_{0H})}{v'(n_{0H})} = \frac{(1-\gamma)\frac{\Pi_H(w_{0H})}{2t} - \mu_L\frac{p_L}{p_H}}{(1-\gamma)\frac{\Pi_H(w_{0H})}{2t} - \mu_L},$$

which must be larger than one, since $m_{0H} < n_{0H}$ by Step 2.4 and v' decreasing. Therefore $\mu_L > 0$ since $p_L < p_H$.

Step 4.3. Prove that (19) and (21) imply that $\Pi_L(w_{0L}) < \Pi_H(w_{0H})$.
From (19),

$$\mu_L = \frac{\gamma}{2}\left[\frac{1}{v'(m_{0L})} - \frac{\Pi_L(w_{0L})}{t}\right].$$

Since $\mu_L > 0$, we have $\Pi_L(w_{0L}) < \frac{t}{v'(m_{0L})}$.

From (21),

$$\mu_L = \frac{p_H}{p_L}\frac{(1-\gamma)}{2}\left[\frac{\Pi_H(w_{0H})}{t} - \frac{1}{v'(n_{0H})}\right].$$

Since $\mu_L > 0$, we have that $\Pi_H(w_{0H}) > \frac{t}{v'(n_{0H})}$. Now $n_{0H} > m_{0L}$ by Step 2.2, so v' decreasing implies $\frac{t}{v'(n_{0H})} > \frac{t}{v'(m_{0L})}$. This concludes Step 4.3.

Step 4.4. Show that the fact that the low risk ICC is binding and Jensen's Inequality imply $\Pi_L(w_{0L}) > \Pi_H(w_{0H})$, which contradicts the conclusion in Step 4.3.

By (12) and $m_{0L} = n_{0L}$, we can write

$$m_{0L} = v^{-1}\left[\frac{v(m_{0H}) + p_L v(n_{0H})}{1 + p_L}\right]. \tag{22}$$

Now, that $\Pi_L(w_{0L}) > \Pi_H(w_{0H})$ is equivalent to $r - (1+p_L)m_{0L} > r - m_{0H} - p_H n_{0L}$. This equivalent to

$$m_{0L} < \frac{m_{0H} + p_H n_{0H}}{1 + p_L}.$$

Since $p_H > p_L$, it suffices to prove that

$$m_{0L} < \frac{m_{0H} + p_L n_{0H}}{1 + p_L}.$$

Use (22) to rewrite this as

$$v^{-1}\left[\frac{v(m_{0H}) + p_L v(n_{0H})}{1 + p_L}\right] < \frac{m_{0H} + p_L n_{0H}}{1 + p_L},$$

which is equivalent to

$$\frac{v(m_{0H}) + p_L v(n_{0H})}{1 + p_L} < v\left[\frac{m_{0H} + p_L n_{0H}}{1 + p_L}\right].$$

Finally, let $p \equiv \frac{p_L}{1+p_L}$ and $1 - p = \frac{1}{1+p_L}$. Notice that $0 < p < 1$. Substitute in the previous expression to get

$$(1-p)v(m_{0H}) + pv(n_{0H}) < v\left[(1-p)m_{0H} + pn_{0H}\right].$$

Letting $E(\cdot)$ denote the expectation operator associated to probabilities p and $1-p$, the last expression can be we can rewritten as $E(v(\widetilde{w}_H)) < v(E(\widetilde{w}_H))$, which by Jensen's Inequality is true since v is concave. This concludes Step 4.

Step 5. By Step 4 and Step 1, in a sy*mmetric* separating equilibrium H's ICC must be binding and $L's$ is not. Prove that μ_H is positive and that $\Pi_L(w_{0L}) > \Pi_H(w_{0H})$.

Step 5.1. Substitute $\mu_L = 0$ and $D_{0L} = D_{0H} = 1/2$ into (7) through (10) to check that (10) is redundant. Conditions (7) through (9) become

$$
\begin{aligned}
-\gamma\left[\frac{1}{2} - \frac{\Pi_L(w_{0L})v'(m_{0L})}{2t}\right] - \mu_H v'(m_{0L}) &= 0, & (23)\\
-\gamma p_L\left[\frac{1}{2} - \frac{\Pi_L(w_{0L})v'(n_{0L})}{2t}\right] - \mu_H p_H v'(n_{0L}) &= 0, & (24)\\
-(1-\gamma)\left[\frac{1}{2} - \frac{\Pi_H(w_{0H})v'(m_{0H})}{2t}\right] + \mu_H v'(m_{0H}) &= 0. & (25)
\end{aligned}
$$

Step 5.2. Rewrite (23) and (24) as

$$
\begin{aligned}
v'(m_{0L})[\gamma\frac{\Pi_L(w_{0L})}{2t} - \mu_H] &= \frac{1}{2}\gamma,\\
v'(n_{0L})[\gamma\frac{\Pi_L(w_{0L})}{2t}p_L - \mu_H p_H] &= \frac{1}{2}\gamma p_L.
\end{aligned}
$$

Notice that the expressions in brackets are positive, since both v' and the right hand side is positive. Dividing this two expressions, get

$$
\frac{v'(m_{0L})}{v'(n_{0L})} = \frac{\gamma\frac{\Pi_L(w_{0L})}{2t} - \mu_H\frac{p_H}{p_L}}{\gamma\frac{\Pi_L(w_{0L})}{2t} - \mu_H}. \tag{26}
$$

By Step 3.2, $m_{0L} > n_{0L}$, so $\frac{v'(m_{0L})}{v'(n_{0L})} < 1$ since v' is decreasing. Hence the right hand side of (26) is also smaller than one. This, together with $p_H > p_L$ implies that $\mu_H > 0$.

Step 5.3. Isolate μ_H in (23) to get

$$
\mu_H = \frac{\gamma}{2}\left[\frac{\Pi_L(w_{0L})}{t} - \frac{1}{v'(m_{0L})}\right].
$$

By Step 4.2, $\mu_H > 0$ so $\Pi_L(w_{0L}) > \frac{t}{v'(m_{0L})} > 0$. Isolate μ_H in (25) to get

$$
\mu_H = \frac{(1-\gamma)}{2}\left[\frac{1}{v'(m_{0H})} - \frac{\Pi_H(w_{0H})}{t}\right].
$$

Again, since $\mu_H > 0$, we have $\Pi_H(w_{0H}) < \frac{t}{v'(m_{0H})}$. Finally, since $m_{0L} > m_{0H}$ by Step 3.3, we have $\Pi_L(w_{0L}) > \frac{t}{v'(m_{0L})} > \frac{t}{v'(m_{0H})} > \Pi_H(w_{0H})$. Since $t/v'(\cdot) > 0$, this concludes Step 4 and the proof of the whole proposition. ■

Proof of Proposition 4

Proof. *Step 1.* Consider the following candidate for a Pareto-improvement: Change contracts from (w_L^*, w_H^*) to $W = (w_L, w_H) = ((m_L, n_L), (m_H, n_H))$ so that high risk's utility is increased while the efficiency of his contract is preserved, without violating the ICC and while preserving average profits. Such change must satisfy:

(i) $m_H = n_H$ (efficiency),

(ii) $\frac{\gamma}{2}\Pi_L(w_L) + \frac{1-\gamma}{2}\Pi_H(w_H) = \frac{\gamma}{2}\Pi_L(w_L^*) + \frac{1-\gamma}{2}\Pi_H(w_H^*) \equiv \Pi^*$ (same profits as in equilibrium), and

(iii) $U^H(w_H) = U^H(w_L)$ (incentive compatibility).

This can be rewritten as the following system of equations.

$$\begin{cases} r - \gamma(m_L + p_L n_L) - (1-\gamma)(m_H + p_H n_H) = 2\Pi^* \\ v(m_H)(1 + p_H) = v(m_L) + p_H v(n_L). \end{cases}$$

This implicitly defines the schedules $m_L = m_L(m_H)$ and $n_L = n_L(m_H)$. Differentiate totally the system with respect to m_H and solve for m_L' and n_L' to get

$$n_L' = (1 + p_H)\frac{v'(m_H) + v'(m_L)\frac{1-\gamma}{\gamma}}{p_H v'(n_L) - p_L v'(m_L)}$$

and

$$m_L' = -(1 + p_H)\frac{p_L v'(m_H) + \frac{1-\gamma}{\gamma} p_H v'(n_L)}{p_H v'(n_L) - p_L v'(m_L)}.$$

Notice that $m_L' < 0$ while $n_L' > 0$, as expected, since one has to improve efficiency of the contract w_L in order to finance the increase in m_H. This efficiency improvement implies reducing distortions, so one must rise n_L and lower m_L.

Step 2. Check whether $U^L(m_L(m_H), n_L(m_H))$ does not decrease in m_H. If it does, this means that we have found a Pareto-improvement. This condition can be rewritten as $\frac{dU^L(m_L(m_H), n_L(m_H))}{dm_H} = v'(m_L)m_L' + p_L v'(n_L)n_L' > 0$.

Substitute m_L' and n_L' by the respective expressions to get (after some algebra)

$$p_L v'(n_L)\frac{v'(m_H)+v'(m_L)\frac{1-\gamma}{\gamma}}{p_H v'(n_L)-p_L v'(m_L)} \geqslant v'(m_L)\frac{p_L v'(m_H)+\frac{1-\gamma}{\gamma}p_H v'(n_L)}{p_H v'(n_L)-p_L v'(m_L)},$$

or, since $p_H > p_L$, $m_L > n_L$, and v' decreasing imply $p_H v'(n_L) - p_L v'(m_L) > 0$, the denominators cancel. The previous expression becomes (again after some algebra)

$$v'(m_H) \geqslant \frac{v'(m_L)v'(n_L)}{v'(n_L)-v'(m_L)}\frac{1-\gamma}{\gamma}\frac{p_H-p_L}{p_L}. \tag{27}$$

Since we are constructing a local Pareto-improvement, we must check whether (27) is satisfied at w^*.

Step 3. Use the equations (23), (24), and (25); which characterize the symmetric separating equilibrium.

From (25) get

$$\mu_H = \frac{(1-\gamma)}{2}\left[\frac{1}{v'(m_H^*)} - \frac{\Pi_H(w_H^*)}{t}\right].$$

Substitute into (24) to get (after some algebra)

$$v'(n_L^*) = \frac{\gamma\frac{p_L}{p_H}}{\frac{\Pi_L(w_L^*)}{t}\gamma\frac{p_L}{p_H} + (1-\gamma)\left[\frac{\Pi_H(w_H^*)}{t} - \frac{1}{v'(m_H^*)}\right]}.$$

Letting $A = (1-\gamma)\left[\frac{\Pi_H(w_H^*)}{t} - \frac{1}{v'(m_H^*)}\right]$ this can be rewritten as

$$v'(n_L^*) = \frac{\gamma\frac{p_L}{p_H}}{\frac{\Pi_L(w_L^*)}{t}\gamma\frac{p_L}{p_H} + A}.$$

Repeat for (23) to get (after some algebra)

$$v'(m_L^*) = \frac{\gamma}{A + \gamma\frac{\Pi_L(w_L^*)}{t}}.$$

Substitute $v'(m_L^*)$ and $v'(n_L^*)$ by its respective expressions in (27) to get (after some more algebra and substituting A by its expression)

$$v'(m_H) \geqslant \frac{1}{\frac{1}{v'(m_H^*)} - \frac{\Pi_H(w_H^*)}{t}}. \tag{28}$$

Now, if or $\Pi_H(w_H^*) < 0$, then $\frac{1}{v'(m_H^*)} - \frac{\Pi_H(w_H^*)}{t} > \frac{1}{v'(m_H^*)}$ so $\frac{1}{\frac{1}{v'(m_H^*)} - \frac{\Pi_H(w_H^*)}{t}} < v'(m_H^*)$ and we are done. We have found a local Pareto-improvement. ■

8 APPENDIX B. Procedure to derive the numerical results

In subsection 4.3 we show the results of solving our model for the logarithmic utility function and specific parameter values. Though we cannot extrapolate the conclusions obtained to other specifications, the exercise is useful to show some properties of our model, at least for some parameter values, when the mathematical complexity prevents us from obtaining analytical results. Other authors have found it useful to use numerical methods to solve problems in agency theory. See, for instance, Haubrich (1994).

Figure 5 depicts the equilibrium profits for the logarithmic utility function and specific parameter values. The equilibrium profits were derived from the symmetric equilibrium candidate. This is found by numerically solving the first order conditions, taking into account that ICC is binding for the high risk.

Table 1 gives the critical values of γ for the existence of equilibrium. To do that, we specify a grid of different values of γ. For each value of the grid, we compute the symmetric equilibrium candidate, say W^*. Without loss of generality, we then compute the best deviation of firm 0 if firm 1 offers W^*. This candidate is indeed an equilibrium if the best deviation profits are not higher than under W^*.

Conceptually, finding the best deviation consists in obtaining the menu $((m_L', n_L'), (n_H', n_H'))$ that maximizes:

$$\gamma \left(r - m_L' - p_L n_L'\right) D_{0L} + (1-\gamma)\left(r - m_H' - p_H n_H'\right) D_{0H}, \tag{29}$$

and subject to the ICCs:

$$\begin{aligned} U^L(m'_L, n'_L) &\geq U^L(m'_H, n'_H), \\ U^H(m'_H, n'_H) &\geq U^H(m'_L, n'_L); \end{aligned}$$

where, for $J = L, H$;

$$D_{0J} = \begin{cases} 0 & \text{if } \frac{v(m'_J)-v(m^*_J)+p_J(v(n'_J)-v(n^*_J))+t}{2t} < 0 \\ 1 & \text{if } \frac{v(m'_J)-v(m^*_J)+p_J(v(n'_J)-v(n^*_J))+t}{2t} > 1 \\ \frac{v(m'_J)-v(m^*_J)+p_J(v(n'_J)-v(n^*_J))+t}{2t} & \text{if otherwise.} \end{cases}$$

Although the problem is conceptually simple, the numerical procedure must take into account that the problem is not entirely differentiable. The demand functions present a flat segment whenever $\frac{v(m'_J)-v(m^*_J)+p_J(v(n'_J)-v(n^*_J))+t}{2t}$ is above one or below zero, while they are strictly increasing otherwise. Moreover, the objective function is not necessarily concave.

In order to deal with these problems, we use a robust maximization routine called Simulated Annealing (see for instance Goffe *et al.* 1994 for a discussion of the algorithm) that is designed to escape from local optima, and hence it can deal with functions that are not globally concave. One should take into account that the symmetric equilibrium candidate is always a critical point and may be a local optima of the above problem, as it verifies the interior first order conditions by construction. Consequently, in order to check whether the symmetric candidate is an equilibrium, it is crucial to use a maximization algorithm that is able to escape from local optima. Unlike gradient-based maximization routines, Simulated Annealing works evaluating the function at a selection of points and it does not require to take derivatives, hence it is convenient for non-differentiable maximization problems as ours. In the following, we describe the procedure to find $((m'_L, n'_L), (n'_H, n'_H))$. In order to solve the problem we use the result that, in a best reply, exactly one ICC should be binding and the contract associated to it should be efficient. This is shown as Steps 1 trough 3 in the proof of Proposition 3a in Appendix A.

The maximization routine is solved in three stages. The first one obtains the best deviation assuming that the ICC of the high risk is binding. The second one obtains the best deviation assuming that the ICC of the low risk is binding. The final stage chooses the solution that gives more profits between the two previous solutions.[24]

In order to solve the problem under the assumption that high risk ICC is binding, the routine works as follows. For each contract (m'_L, n'_L) that the Simulated Annealing selects, we obtain (m'_H, n'_H) such that high risk ICC is binding and $m'_H = n'_H$. In the following step, the demands for each type are obtained. By construction (m'_H, n'_H) is incentive compatible, hence we set $w_{0H} = (m'_H, n'_H)$ and $w_{1H} = (m^*_H, n^*_H)$ to obtain the demand for the high risk. As it is not guaranteed that the pair (m'_L, n'_L) that is selected by the Simulated Annealing is incentive compatible, the demand for the low risk is computed using $w_{1L} = (m^*_L, n^*_L)$ and either $w_{0L} = (m'_L, n'_L)$ or $w_{0L} = (m'_H, n'_H)$, whichever maximizes the utility of the low risk. The procedure then checks that demands be between 0 and 1.

In order to solve the problem under the assumption that low risk ICC is binding, we proceed in the same fashion, changing H by L and viceversa.

The programs have been written in GAUSS and they are available from the authors upon request. We have benefited from the Simulated Annealing code written by E. G. Tsionas.[25]

[24] In our case, we have always found that the best deviating strategy is the one obtained when the high risk ICC is binding.

[25] Available from http://www.american.edu/academic.depts/cas/econ/gaussres/optimize/optimize .htm.

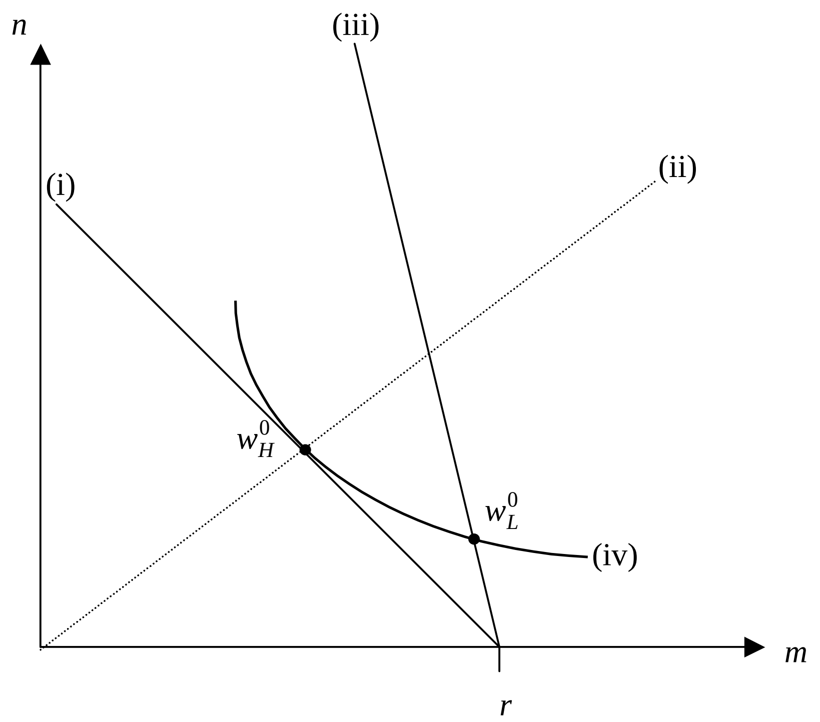

Figure 1. The contracts under homogeneous product.

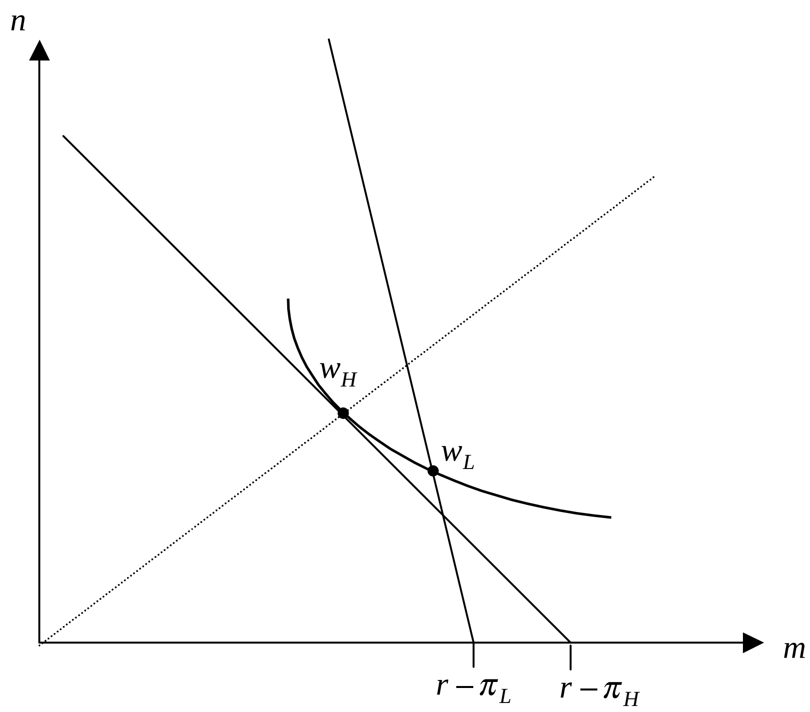

Figure 2. The symmetric separating equilibrium.

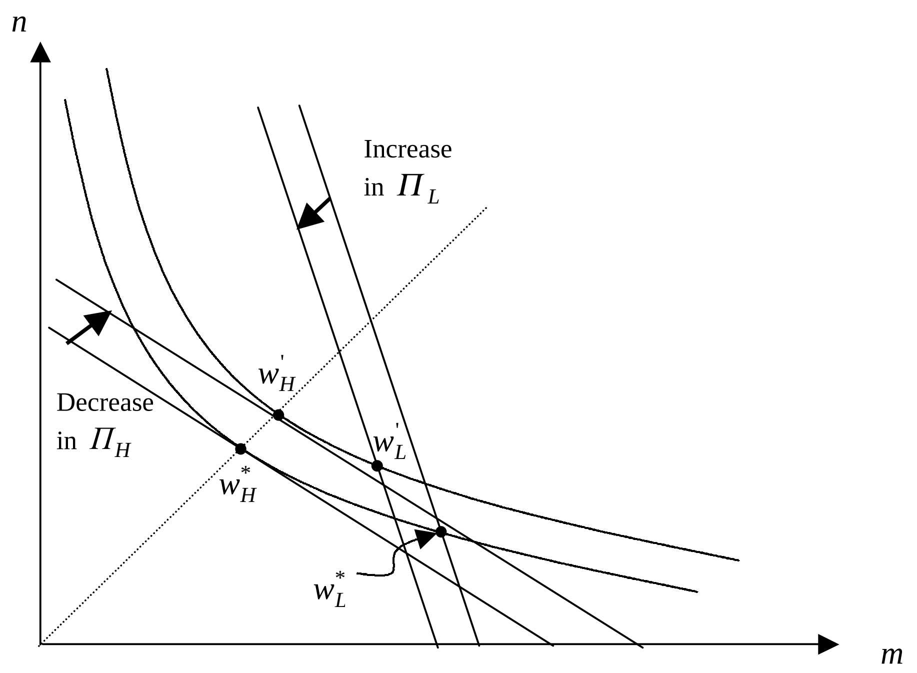

Figure 3. The Pareto-improvement. Average profits remain constant, U^H increases, and U^L increases if $w_L^{'}$ lies on an indifference curve that is above the one going through w_L^{*} (not depicted).

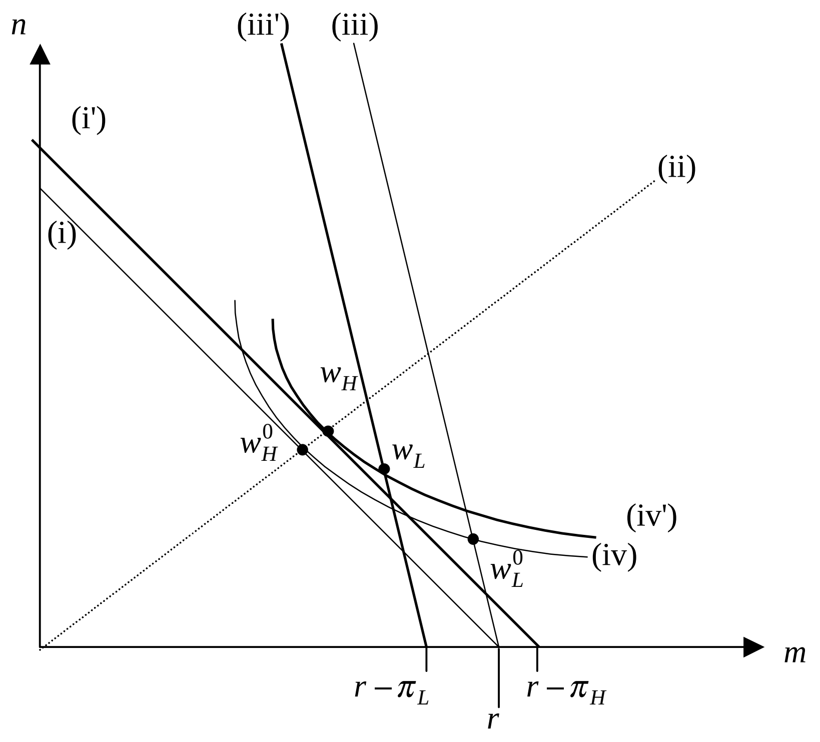

Figure 4. Comparing the equilibrium contracts with and without differentiation under cross subsidization.

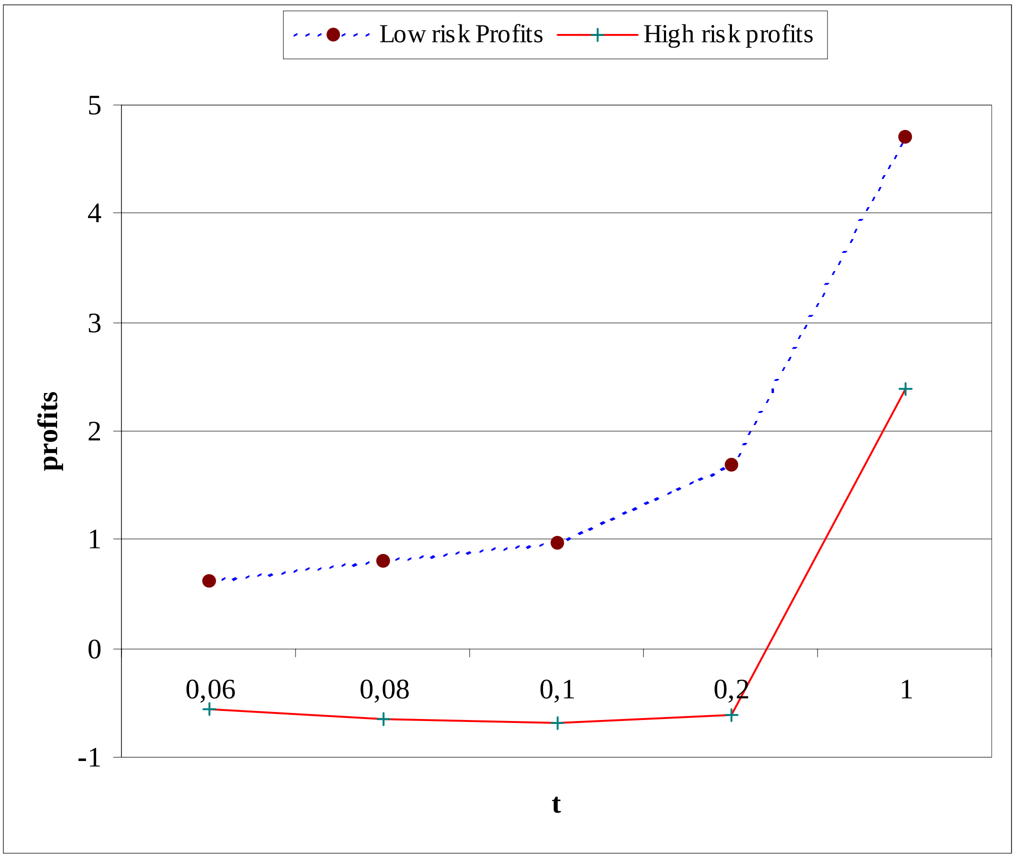

Figure 5. Firm's per-capita profits.

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