ISSN 1350-6722

University College London

DISCUSSION PAPERS IN ECONOMICS

THE 3-EQUATION NEW KEYNESIAN MODEL — A GRAPHICAL EXPOSITION

by

Wendy Carlin and David Soskice

Discussion Paper 05-03

DEPARTMENT OF ECONOMICS
UNIVERSITY COLLEGE LONDON
GOWER STREET
LONDON WCIE 6BT

The 3-Equation New Keynesian Model — a Graphical Exposition

Wendy Carlin
UCL & CEPR

David Soskice
Duke, WZB & LSE

November 2005

Abstract

We develop a graphical 3-equation New Keynesian model for macroeconomic analysis to replace the traditional IS-LM-AS model. The new graphical IS-PC-MR model is a simple version of the one commonly used by central banks and captures the forward-looking thinking engaged in by the policy maker. Within a common framework, we compare our model to other monetary-rule based models that are used for teaching and policy analysis. We show that the differences between the models centre on whether the central bank optimizes and on the lag structure in the IS and Phillips curve equations. We highlight the analytical and pedagogical advantages of our preferred model. The model can be used to analyze the consequences of a wide range of macroeconomic shocks, to identify the structural determinants of the coefficients of a Taylor type interest rate rule, and to explain the origin and size of the inflation bias.

Much of modern macroeconomics is inaccessible to undergraduates and non-specialists. There is a gulf between the simple models found in principles and intermediate macro textbooks — notably, the IS-LM-AS approach — and the models currently at the heart of the debates in monetary macroeconomics in academic and central bank circles that are taught in graduate courses. Our aim is to show how this divide can be bridged.[1]

[1]We are grateful for the advice and comments of Christopher Allsopp, James Cloyne, John Driffill, Andrew Glyn, Liam Graham, Matthew Harding, Campbell Leith, Colin Mayer, Edward Nelson, Terry O'Shaughnessy, Nicholas Rau, Daniel Rogger and David Vines. The clarity of the argument has benefited greatly from the feedback provided by the editor, David Romer and two anonymous referees.

Modern monetary macroeconomics is based on what is increasingly known as the 3-equation New Keynesian model: IS curve, Phillips curve and a monetary policy rule. This is the basic analytical structure of Michael Woodford's book *Interest and Prices* published in 2003 and, for example, of the widely cited paper 'The New Keynesian Science of Monetary Policy' by Clarida et al. (1999). An earlier influential paper is Goodfriend and King (1997). These authors are concerned to show how the equations can be derived from explicit optimizing behaviour on the part of the individual agents in the economy in the presence of some nominal imperfections. Moreover, "[t]his is in fact the approach already taken in many of the econometric models used for policy simulations within central banks or international institutions" (Woodford, 2003, p. 237).

Our contribution is to develop a version of the 3-equation model that can be taught to undergraduate students and can be deployed to analyze a broad range of policy issues. It can be taught using diagrams and minimal algebra. The IS diagram is placed vertically above the Phillips diagram, with the monetary rule shown in the latter along with the Phillips curves. We believe that our IS-PC-MR graphical analysis is particularly useful for explaining the optimizing behaviour of the central bank. Users can see and remember readily where the key relationships come from and are therefore able to vary the assumptions about the behaviour of the policy-maker or the private sector. In order to use the model, it is necessary to think about the economics behind the processes of adjustment. One of the reasons IS-LM-AS got a bad name is that it too frequently became an exercise in mechanical curve-shifting: students were often unable to explain the economic processes involved in moving from one equilibrium to another. In the framework presented here, in order to work through the adjustment process, the student has to engage in the same forward-looking thinking as the policy-maker.

The model we propose for teaching purposes is New Keynesian in its 3-equation structure and its modelling of a forward-looking optimizing central bank. However it does not incorporate either a forward-looking IS curve or a forward-looking Phillips curve.[2]

Romer (2000) took the initial steps toward answering the question of how modern macroeconomics can be presented to undergraduates. His alternative to the standard IS-LM-AS framework follows earlier work by Taylor (1993) in which instead of the LM curve, there is an interest rate based monetary policy rule.[3] In Section 1, we motivate the paper by providing a common framework within which several models can be compared. The common framework consists of an IS rela-

[2]Both extensions are provided in Chapter 15 of Carlin and Soskice (2006).

[3]Other presentations of 'macroeconomics without the LM' are provided in Allsopp and Vines (2000), Taylor (2000) and Walsh (2002).

tion, a Phillips curve and a monetary rule. We shall see that the differences between the models centre on (a) whether the monetary rule is derived from optimizing behaviour and (b) on the lag structure in the IS curve and in the Phillips curve.

Using the common framework, we highlight the analytical and pedagogical shortcomings of the Romer–Taylor and Walsh (2002) models and indicate how our model (which we call the Carlin–Soskice or C–S model) overcomes them. In the final part of Section 1 we show how the very similar models of Svensson (1997) and Ball (1999) fit into the common framework. The Svensson–Ball model is not designed for teaching purposes but fits the reality of contemporary central banks better.

In Section 2, we show how our preferred 3-equation model (the C–S model) can be taught to undergraduates. This is done both in equations and in diagrams. We begin by describing how a diagram can be used to illustrate the way an IS shock affects the economy and how the central bank responds so as to steer the economy back to its inflation target. We also analyze inflation and supply-side shocks. We discuss how variations in the structural characteristics of the economy both on the demand and the supply side, and in the central bank's preferences are reflected in the behaviour of the economy and of the central bank following a shock. In the final part of Section 2, we show that by adopting the Svensson–Ball lag structure, the central bank's interest rate rule takes the form of the familiar Taylor Rule in which the central bank reacts to contemporaneous deviations of inflation from target and output from equilibrium. In Section 3, we show how the problems of inflation bias and time inconsistency can be analyzed using the 3-equation model.

1 Motivation

Two significant attempts to develop a 3-equation IS-PC-MR model to explain modern macroeconomics diagrammatically to an undergraduate audience are the Romer–Taylor (R–T) and the Walsh models. While both are in different ways attractive, they also suffer from drawbacks—either expositional or as useful models of the real world. We set out these two models within a common 3 equation framework and compare them to the model developed in this paper, in which we believe the drawbacks are avoided. The defining differences between the models lie in the lags from interest rates to aggregate demand and aggregate demand to inflation. We also use the framework to set out the Svensson–Ball model. Although in our view the lag structure of the Ball–Svensson model best fits the real world, it is—even when simplified —significantly harder to explain to an undergraduate audience than our preferred C–S model. The common framework involves a simplification in the way that the central bank's loss function is treated. We propose a short-cut that enables

us to avoid the complexity of minimizing the central bank's full (infinite horizon) loss function whilst retaining the insights that come from incorporating an optimizing forward-looking central bank in the model.

As shown in Section 1.1, all four models share a 3-equation structure — an IS equation, a Phillips relation and a monetary rule equation. The key differences between the models lie in whether there is an optimizing central bank and in two critical lags. The first critical lag is in the IS equation from the interest rate to output, and the second is in the Phillips equation from output to inflation. In fact the four possible combinations of a zero or one year lag in the IS equation and in the Phillips equation more or less define the four models. After explaining the framework, the four models are set out in order: the Romer-Taylor model initially without and then with central bank optimization; the Walsh model; the C–S model; and the simplified Svensson–Ball model.

1.1 Common framework

The IS equation. It is convenient to work with deviations of output from equilibrium, $x_t \equiv y_t - y_e$, where y is output and y_e is equilibrium output. Thus the IS equation of $y_t = A_t - ar_{t-i}$, where A_t is exogenous demand and r_{t-i} is the real interest rate, becomes

$$x_t = (A_t - y_e) - ar_{t-i}, \qquad (IS \text{ equation})$$

where

$$i = 0, 1$$

captures the lag from the real interest rate to output (a period represents a year). Once central bank optimization is introduced, we shall generally replace $A_t - y_e$ by $ar_{S,t}$ where $r_{S,t}$ is the so-called 'stabilizing' or Wicksellian (Woodford) rate of interest such that output is in equilibrium when $r_{S,t} = r_{t-i}$.

The Phillips curve. In the Phillips curve equation we assume throughout, as is common in much of this literature, that the inflation process is inertial so that current inflation is a function of lagged inflation and the output gap, i.e.

$$\pi_t = \pi_{t-1} + \alpha x_{t-j}, \qquad \text{(Phillips curve)}$$

where

$$j = 0, 1$$

is the lag from output to inflation.

The monetary rule. The monetary rule equation can be expressed in two ways. On the one hand, it can be expressed as an interest rate rule indicating how the current real interest rate should be set in response to the current inflation rate (and

sometimes in response to the current output gap as well, as in the famous Taylor rule). We shall call this form of the monetary rule, the interest rate rule or IR equation. Alternatively, the monetary rule can be in the form that shows how output (chosen by the central bank through its interest rate decision) should respond to inflation (or, as will be seen, to forecast inflation). We call this the MR–AD equation and it is shown as a downward sloping line in the Phillips curve diagram. Each form of the monetary rule can be derived from the other. It is usual to derive the MR–AD equation from the minimization by the central bank of a loss function, and then (if desired) to derive the interest rate rule from the MR–AD equation. We shall follow that practice here. It will be assumed that the loss in any period t is written $x_t^2 + \beta\pi_t^2$, so that the output target is equilibrium output and the inflation target π^T is set equal to zero for simplicity. In Section 3, we relax the assumption that the output target is equilibrium output. The central bank in period t (the period t CB for short) has the discretion to choose the current short-term real interest rate r_t, just as the period $t+n$ CB has the discretion to choose r_{t+n}.

For a graduate audience the standard approach to the central bank's problem is to use dynamic optimization techniques to show how the period t CB minimizes the present value loss, L_t,

$$L_t = \left(x_t^2 + \beta\pi_t^2\right) + \delta\left(x_{t+1}^2 + \beta\pi_{t+1}^2\right) + \delta^2\left(x_{t+2}^2 + \beta\pi_{t+2}^2\right) + ...,$$

where δ is the discount factor with $0 < \delta < 1$. Showing this rigorously is beyond the scope of undergraduate courses. We believe, however, that it is important to see the central bank as an optimizing agent who needs to think through the future consequences of its current decisions and hence engage in forecasting. We have found in teaching undergraduates that a useful compromise approach is therefore to assume that the period t CB minimizes those terms in the loss function L_t that it affects directly via its choice of r_t. Thus, neglecting random shocks that it cannot forecast, it affects x_{t+i} and π_{t+i+j} directly through its choice of r_t. Using this simplified procedure, the period t CB minimizes

$$\begin{aligned}
L &= x_t^2 + \beta\pi_t^2 \text{ if } i = j = 0 && \text{(Walsh)}\\
L &= x_t^2 + \delta\beta\pi_{t+1}^2 \text{ if } i = 0,\ j = 1 && \text{(Romer–Taylor)}\\
L &= x_{t+1}^2 + \beta\pi_{t+1}^2 \text{ if } i = 1,\ j = 0 && \text{(Carlin–Soskice)}\\
L &= x_{t+1}^2 + \delta\beta\pi_{t+2}^2 \text{ if } i = j = 1, && \text{(Svensson–Ball)}
\end{aligned}$$

(Central Bank loss functions)

where each loss function is labelled by the name of the model with the associated lag structure, as we shall explain in the subsections to follow. If these loss functions are minimized with respect to x_{t+i} subject to the relevant Phillips curve, the resulting

MR–AD equations are

$$\begin{aligned} x_t &= -\alpha\beta\pi_t \text{ if } i = j = 0 && \text{(Walsh)} \\ x_t &= -\delta\alpha\beta\pi_{t+1} \text{ if } i = 0,\ j = 1 && \text{(Romer–Taylor)} \\ x_{t+1} &= -\alpha\beta\pi_{t+1} \text{ if } i = 1,\ j = 0 && \text{(Carlin–Soskice)} \\ x_{t+1} &= -\delta\alpha\beta\,\pi_{t+2} \text{if } i = j = 1. && \text{(Svensson–Ball)} \\ & && (MR\text{–}AD \text{ equations}) \end{aligned}$$

Note that the MR–AD equations require that the central bank forecasts the relevant inflation rate using the Phillips curve. We show in the subsequent sub-sections how the interest rate rule equations are derived.

Aside from the teaching benefit, there are two further justifications for our way of simplifying the central bank's problem. First, the MR–AD equations have the same form as in the dynamic optimization case. To take an example, $i = j = 1$ corresponds to the lag structure of the Svensson–Ball model. The MR–AD equation shown above and in the full Svensson–Ball model with dynamic optimization is of the form $x_{t+1} = -\theta\pi_{t+2}$. The difference between the two is of course that the slope of the inflation-output relation here is too steep since our procedure takes no account of the beneficial effect of a lower π_{t+2} in reducing future losses. Thus in comparison to the equation above, $x_{t+1} = -\delta\alpha\beta\ \pi_{t+2}$, the corresponding equation in Svensson (1997) is $x_{t+1} = -\delta\alpha\beta k\ \pi_{t+2}$ (equation B.7, p.1143) where $k \geq 1$ is the marginal value of π^2_{t+2} in the indirect loss function $V(\pi_{t+2})$ (equation B.6).

A second justification for this procedure (again, taking the case of the $i = j = 1$ lag structure) comes from the practice of the Bank of England. As we shall see below, the Bank of England believes the $i = j = 1$ lag structure is a good approximation to reality. And in consequence it uses, at time t, the rate of inflation π_{t+2} as its forecast target. This comes close to the period t CB minimizing the loss function $L = x^2_{t+1} + \delta\beta\pi^2_{t+2}$.

1.2 The simple Romer–Taylor model ($i = 0; j = 1$)

The attraction of the R–T model is its simplicity and ease of diagrammatic explanation. Since $i = 0$ and $j = 1$, the IS and PC equations are

$$\begin{aligned} x_t &= -a(r_t - r_{S,t}) && (IS \text{ equation}) \\ \pi_t &= \pi_{t-1} + \alpha x_{t-1}. && (PC \text{ equation}) \end{aligned}$$

Instead of assuming that the central bank optimizes in choosing its interest rate rule, the R–T model assumes an interest rate rule of the form

$$r_t = \gamma\pi_t. \qquad (IR \text{ equation})$$

The interest rate form of the monetary rule may be easily changed into a relationship between x_t and π_t by substituting the IR equation into the IS equation to get

$$x_t = ar_{S,t} - a\gamma\pi_t, \qquad (AD \text{ equation})$$

which is a downward sloping line in the Phillips curve diagram. With $ar_{S,t} = A_t - y_e = 0$, the AD equation goes through ($x = 0$, $\pi = \pi^T = 0$). A permanent positive aggregate demand shock implies that $A_t > y_e$ and so $r_{S,t} > 0$, which shifts the curve up permanently.

Equilibrium. The equilibrium of the model is easily derived. Stable inflation requires output at equilibrium, i.e. $x = 0$. This implies from the AD equation that the inflation rate at equilibrium, π_e, is as follows:

$$\pi_e = r_{S,t}/\gamma = (A_t - y_e)/a\gamma.$$

Thus in this model inflation at the constant inflation equilibrium differs from the inflation target whenever $A_t \neq y_e$.

Adjustment to equilibrium. The lag structure of the model makes it particularly simple to follow the consequences of a demand shock. The demand shock in t raises output in t without initially affecting inflation since inflation responds to last period's output. Since the interest rate only responds to inflation, the period t CB does not respond to the demand shock in period t. In period $t+1$, inflation increases as a lagged response to the increased output in t; the period $t+1$ CB increases r_{t+1} and hence reduces x_{t+1}. This then reduces inflation in $t+2$, and so on.

Drawbacks. Appealing though the simplicity of the model is, it has three drawbacks:

1. The interest rate rule is not chosen through optimizing behaviour by the central bank.

2. Since $\pi_e = (A_t - y_e)/a\gamma$, then if $A_t - y_e > 0$, inflation in equilibrium is above the target, i.e. $\pi_e > \pi^T$ and conversely for $A - y_e < 0$, inflation in equilibrium is below the target, $\pi_e < \pi^T$.

3. If the slope of the Phillips curve α is less than the absolute value of the slope of the AD curve, $1/a\gamma$, and if the economy starts from $\pi_0 = 0$, then for the initial demand-induced increase in inflation, π_1, $\pi_1 < \pi_e$. Thus the tighter monetary policy will push inflation *up* from π_1 to π_e. This is because $\pi_1 = \alpha(A - y_e)$ while $\pi_e = (A - y_e)/a\gamma$.

Problems (2) and (3) are illustrated in the Fig. 1. Problem (2) arises because π_e is determined by the intersection of the AD curve with the long-run vertical

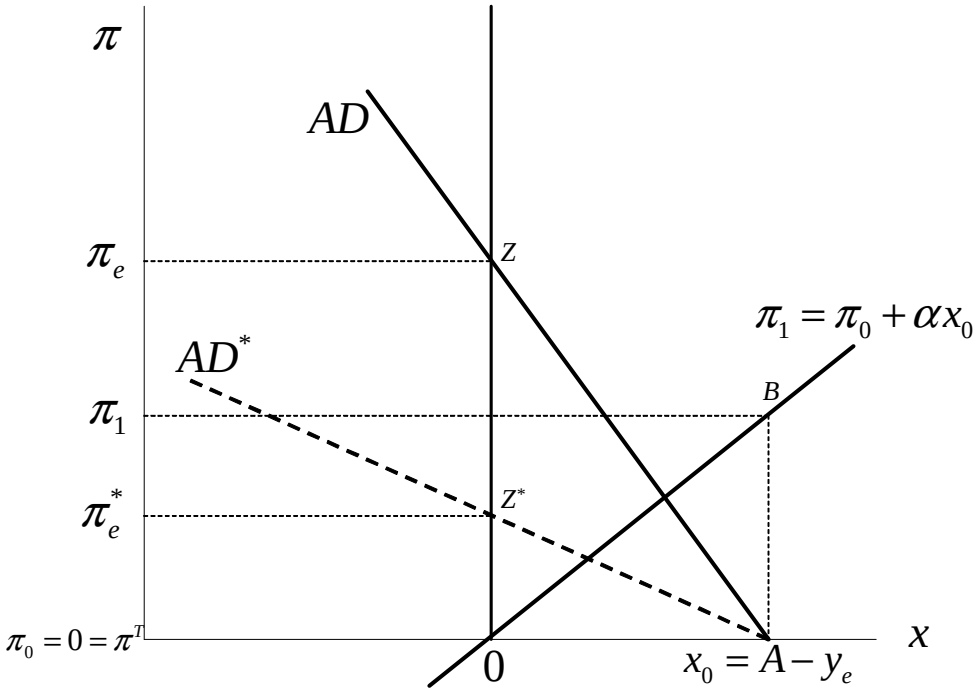


Figure 1: Adjustment to a permanent aggregate demand shock: R–T model

Phillips curve; so a shift in AD resulting from a permanent demand shift, $A - y_e$, shifts π_e above 0 to point Z in the case of the solid AD and to point Z^* in the case of the dashed AD^*. Problem (3) is straightforward to see in the diagram: the *initial* demand induced inflation π_1 is equal to $A - y_e$ multiplied by the Phillips curve coefficient, α (point B in Fig. 1). The *equilibrium* rate of inflation as a result of the demand shock is given by $A - y_e$ multiplied by the coefficient from the AD curve, $1/a\gamma$. Hence, in the case of the AD curve shown by the solid line in Fig. 1, $\alpha < 1/a\gamma$, so that $\pi_e > \pi_1$ and the economy adjusts with rising inflation to tighter monetary policy. With the dashed AD curve $\alpha > 1/a\gamma$, which implies that $\pi_e^* < \pi_1$.[4]

1.3 The R–T model with central bank optimization

If the central bank chooses the monetary rule optimally, all three problems disappear. Remember that in the R–T model with the lag structure $i = 0$, $j = 1$, the central bank minimizes the loss function, $L = x_t^2 + \delta\beta{\pi_{t+1}}^2$, subject to the Phillips curve, $\pi_{t+1} = \pi_t + \alpha x_t$, which implies the MR–AD equation derived in Section 1.1:

$$x_t = -\delta\alpha\beta\pi_{t+1}. \qquad (MR\text{–}AD \text{ equation})$$

The interest rate rule is derived from this as follows. Since $\pi_{t+1} = \pi_t + \alpha x_t$, the MR–AD equation can be rewritten $x_t = -\frac{\delta\alpha\beta}{1+\delta\alpha^2\beta}\pi_t$. Using the IS equation, the optimized interest rule is then

$$r_t - r_{S,t} = \frac{\delta\alpha\beta}{a\left(1+\delta\alpha^2\beta\right)}\pi_t = \gamma\pi_t, \qquad (IR \text{ equation})$$

where $\gamma \equiv \frac{\delta\alpha\beta}{a(1+\delta\alpha^2\beta)}$.

Once central bank optimization is introduced, the MR–AD curve replaces the AD curve and exogenous demand does not shift it. In equilibrium therefore $\pi_e = \pi^T$, so that drawback (2) above disappears. So too does the third drawback (3), since $\pi_{t+1} = \pi^T + \alpha a r_{S,t} > \pi_e = \pi^T$.

Now, however, a new problem arises. This is that a permanent demand shock has no effect on output or inflation. This is because as can be seen from the IR equation, r_t rises by exactly the increase in $r_{S,t}$. A rational central bank will raise the interest rate by the full amount of any increase in the stabilising rate of interest

[4]One possible way of circumventing problem (3) would be to assume $\alpha > 1/a\gamma$. But this carries another drawback from the point of view of realistic analysis, namely that adjustment to the equilibrium cycles. The relevant difference equation is $(\pi_{t+1} - \pi_e) = (1 - \alpha a\gamma)(\pi_t - \pi_e)$. Hence stable non-cyclical adjustment requires $1 > (1 - \alpha a\gamma) > 0$ or $1/a\gamma > \alpha$, which implies that stable non-cyclical adjustment to equilibrium is only consistent with $\pi_1 < \pi_e$.

(assuming that $\pi_t = \pi^T$ initially). Hence it at once eliminates any effect of the demand shock on output and thus subsequently on inflation. In our view this makes the model unrealistic for teaching undergraduates. This critique also applies to supply shocks (shocks to y_e), although it does not apply to inflation shocks.

1.4 The Walsh model ($i = j = 0$)

The Walsh model assumes $i = j = 0$, although it also assumes that the central bank only learns of demand shocks, u_t, in the next period. Because of this, the Walsh model is not directly comparable with the other models purely in terms of a different lag structure in the IS and PC equations. The Walsh equations are

$$\begin{aligned} x_t &= -a(r_t - r_{S,t}) + u_t && (IS \text{ equation}) \\ \pi_t &= \pi_{t-1} + \alpha x_t. && (PC \text{ equation}) \end{aligned}$$

If the central bank believes that $u_{t+n} = 0$ for $n \geqslant 0$, it will minimize the loss function $L = x_t^2 + \beta(\pi_t - \pi^T)^2$ subject to the Phillips curve $\pi_t = \pi_{t-1} + \alpha x_t$, which implies as shown in Section 1.1, the MR–AD equation:

$$x_t = -\alpha\beta\pi_t. \qquad (MR\text{–}AD \text{ equation})$$

Using a similar argument to that in Section 1.3, the interest rate rule is

$$r_t - r_{S,t} = \frac{\alpha\beta}{a}\pi_t. \qquad (IR \text{ equation})$$

From a teaching perspective, the Walsh model produces a simple diagrammatic apparatus, primarily in the Phillips diagram, and on similar lines to the R–T model, with upward sloping Phillips curves and a downward sloping schedule relating output to the rate of inflation. To derive the downward sloping curve, Walsh substitutes the IR equation into the IS curve. This generates the following equation,

$$x_t = -\alpha\beta\pi_t + u_t, \qquad (MR\text{–}AD(W) \text{ equation})$$

which shows output to be determined jointly by monetary policy, $-\alpha\beta\pi_t$, and exogenous demand shocks, u_t. To prevent confusion, we label this the MR–$AD(W)$ equation.

If the demand shock is permanent, the MR–$AD(W)$ curve only shifts right by u_t in period t. In subsequent periods, $t+n$, $n > 0$, and assuming no further demand shocks, the MR–$AD(W)$ equation is simply $x_{t+n} = -\alpha\beta\pi_{t+n}$. This is because the stabilising rate of interest used by the central bank in period t does not take u_t into account, since u_t is unknown to the central bank in period t. Once u_t becomes

known in period $t+1$, r_S rises to its correct level and u_t disappears. Because of this, Problem (2) in the simple R–T model is absent: inflation in equilibrium is at the target rate.

Drawbacks.

1. The first drawback is that from a teaching point of view the economy is not on the MR–$AD(W)$ curve in period t when the shock occurs (contrary to the way it is set out in Walsh). The reason is as follows: assume the economy was in equilibrium in period $t-1$. Then the central bank will not change the interest rate, r, in period t, since the central bank believes that nothing has changed to disturb the equilibrium. The MR–$AD(W)$ equation should really be written as $x_t = -\alpha\beta\pi_t^{CB} + u_t$, since the monetary policy component of the equation is equal to the level of output corresponding to the inflation rate the central bank believes it is imposing, namely here $\pi_t^{CB} = \pi^T = 0$, so that the x which the central bank believes it is producing is $x_t^{CB} = 0$. Hence output will increase by exactly u_t, so that $x_t = x_t^{CB} + u_t = u_t$. Inflation in period t, π_t, is now determined by the intersection of the vertical line $x_t = u_t$ and the Phillips curve. In Fig. 2 this is shown as point A. Point B, the intersection of the MR–$AD(W)$ curve in period t and the Phillips curve in period t is never reached. Note that B could only be reached if the central bank could reset r_t. But if the central bank could reset r_t, it would now be able to work out the value of u_t so that instead of moving to B, it would adjust $r_{S,t}$ up to take account of u_t and hence go straight to Z, the equilibrium point.

2. The second problem is not an analytic one but a pedagogical one and relates to the forward-looking way in which central banks function. We see this as reflecting the quite long time lags in the transmission of monetary policy. This is difficult to capture in Walsh's model since there are no lags: $i = j = 0$. In our model, which we set out in the next sub-section, the rational central bank is engaged in forecasting the future. How one teaches undergraduates depends a lot on levels and background, but we have found it motivating for them to put themselves in the position of a central bank working out the future impact of its current actions.

1.5 The C–S model ($i = 1, j = 0$)

The model we set out in detail in Section 2 of the paper is characterized by the lag structure of $i = 1$ and $j = 0$, which implies that the the IS, PC and MR–AD

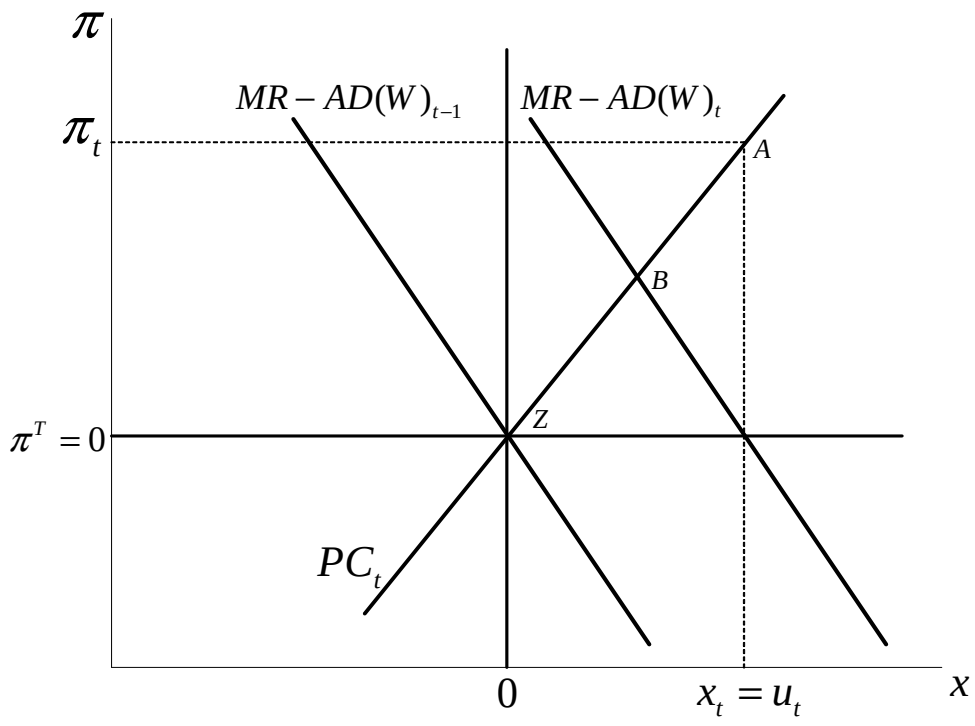


Figure 2: The Walsh Model

equations are as follows:

$$\begin{aligned} x_t &= -a(r_{t-1} - r_{S,t}) && (IS \text{ equation}) \\ \pi_t &= \pi_{t-1} + \alpha x_t && (PC \text{ equation}) \\ x_{t+1} &= -\alpha\beta\pi_{t+1}. && (MR\text{–}AD \text{ equation}) \end{aligned}$$

The IR equation is derived from the MR–AD equation using the Phillips curve equation to substitute for π_{t+1} and the IS equation to substitute for x_{t+1}:

$$\begin{aligned} \pi_t + \alpha x_{t+1} &= -\frac{1}{\alpha\beta} x_{t+1} \\ \pi_t &= -\frac{(1+\alpha^2\beta)}{\alpha\beta} x_{t+1} \\ (r_t - r_{S,t}) &= \frac{\alpha\beta}{a\,(1+\alpha^2\beta)} \pi_t. && (IR \text{ equation}) \end{aligned}$$

We shall show that this model does not suffer the drawbacks of the R–T and Walsh models. Irrespective of the kind of shock, inflation at the constant inflation equilibrium is equal to target inflation: we show in Section 3 how inflation bias arises if the central bank's output target is above the equilibrium, y_e. Moreover, aggregate demand and supply shocks affect output and inflation and cannot be immediately offset by the central bank. The C–S model incorporates central bank optimization, and enables students to see that when the period t CB sets r_t it is having to forecast how to achieve its desired values of x_{t+1} and π_{t+1}. In Svensson's language it is setting r_t in response to current shocks to meet 'forecast targets'. Moreover as we shall see in Section 2 there is a simple diagrammatic apparatus that students can use to explore how a wide variety of shocks and structural characteristics of the economy affect central bank decision-making.

1.6 The Svensson–Ball model ($i = 1, j = 1$)

The Svensson–Ball model is the most realistic one since its lag structure corresponds most closely to the views of central banks. For example, the Bank of England reports:

> The empirical evidence is that on average it takes up to about one year in this and other industrial economies for the response to a monetary policy change to have its peak effect on demand and production, and that it takes up to a further year for these activity changes to have their fullest impact on the inflation rate.[5]

[5] Bank of England (1999), p. 9.

Adopting our simplified treatment of the loss function (Section 1.1), the IS, PC and MR–AD equations in the Svensson–Ball model are:

$$\begin{aligned} x_t &= -a(r_{t-1} - r_{S,t}) && (IS \text{ equation}) \\ \pi_t &= \pi_{t-1} + \alpha x_{t-1} && (PC \text{ equation}) \\ x_{t+1} &= -\delta\alpha\beta\pi_{t+2}. && (MR\text{–}AD \text{ equation}) \end{aligned}$$

As we have seen, it is possible to derive an interest rate rule that expresses how the central bank should react to current data. However, none of the lag structures examined so far delivers an interest rate equation that takes the form of Taylor's empirical rule in which the central bank sets the interest rate in response to deviations in both output and inflation from target. The lag structure in the Svensson–Ball model produces an interest rate in the Taylor Rule form:

$$\begin{aligned} x_{t+1} &= -\delta\alpha\beta\pi_{t+2} \\ -a(r_t - r_{S,t}) &= -\delta\alpha\beta\pi_{t+1} - \delta\alpha^2\beta x_{t+1} \\ &= -\delta\alpha\beta\pi_t - \delta\alpha^2\beta x_t + a\delta\alpha^2\beta(r_t - r_{S,t}) \\ r_t - r_{S,t} &= \frac{\delta\alpha\beta}{a\left(1 + \delta\alpha^2\beta\right)}\left(\pi_t + \alpha x_t\right). && (IR \text{ equation}) \end{aligned}$$

Hence the interest rate responds to current shocks to both output and inflation. In Taylor's empirical rule, the weights on both x_t and π_t are equal to 0.5: that will be the case here if $\delta = \alpha = \beta = a = 1$.

In spite of the advantage of greater realism, introducing the second lag ($j = 1$) to the C–S structure makes the diagrammatic analysis significantly harder because the Phillips curve has to be forecast a further period ahead. This does not provide corresponding gains for students in terms of the basic insights of central bank behaviour.[6]

2 The C–S 3-equation model

In this section, we set out the C–S model to show how it can be taught to undergraduates. We present the model in a format useful for teaching, i.e. with the periods numbered zero and one and we work with output, y, rather than directly in terms of the output gap, x. The key lags in the system that the central bank must take into account are shown in Fig. 3. In the IS curve, the choice of interest rate in period zero will only affect output next period ($i = 1$) as it takes time for interest rate changes to feed through to expenditure decisions. In the Phillips curve, this period's inflation

[6]The diagrams are set out in Carlin and Soskice (2006), Chapter 5.

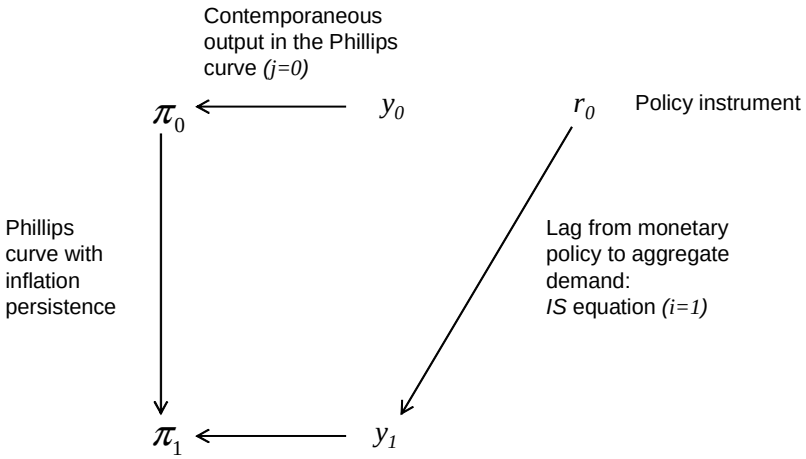


Figure 3: The lag structure in the C–S 3-equation model

is affected by the current output gap ($j = 0$) and by last period's inflation. The latter assumption of inflation persistence can be justified in terms of lags in wage- and or price-setting or by reference to backward-looking expectations: this assumption is common to all the models considered. The lag structure of the model explains why it is π_1 and y_1 that feature in the central bank's loss function: by choosing r_0, the central bank determines y_1, and y_1 in turn determines π_1. This is illustrated in Fig. 3.

2.1 Equations

The three equations of the 3-equation model, the IS equation (#1), the Phillips curve equation (#2) and the MR–AD equation (#3) are set out in this section before being shown in a diagram. The central bank's problem-solving can be discussed intuitively and then depending on the audience, illustrated first either using the diagram or the algebra. The algebra is useful for pinning down exactly how the problem is set up and solved whereas the diagrammatic approach is well-suited to discussing different shocks and the path of adjustment to the new equilibrium.

The central bank minimizes a loss function, where the government requires it to keep next period's inflation close to the target whilst explicitly or implicitly requir-

ing it to avoid large output fluctuations:

$$L = (y_1 - y_e)^2 + \beta(\pi_1 - \pi^T)^2. \qquad \text{(Central Bank loss function)}$$

The critical parameter is β: $\beta > 1$ will characterize a central bank that places less weight on output fluctuations than on deviations in inflation, and vice versa. A more inflation-averse central bank is characterized by a higher β.

The central bank optimizes by minimizing its loss function subject to the Phillips curve ($j = 0$):

$$\pi_1 = \pi_0 + \alpha.(y_1 - y_e). \qquad \text{(Inertial Phillips curve: } PC \text{ equation, \#2)}$$

By substituting the Phillips curve equation into the loss function and differentiating with respect to y_1 (which, as we have seen in Fig. 3, the central bank can choose by setting r_0), we have:

$$\frac{\partial L}{\partial y_1} = (y_1 - y_e) + \alpha\beta.(\pi_0 + \alpha.(y_1 - y_e) - \pi^T) = 0.$$

Substituting the Phillips curve back into this equation gives:

$$(y_1 - y_e) = -\alpha\beta.(\pi_1 - \pi^T). \qquad \text{(Monetary rule: } MR\text{–}AD \text{ equation, \#3)}$$

This equation is the equilibrium relationship between the inflation rate chosen indirectly and the level of output chosen directly by the central bank to maximize its utility given its preferences and the constraints it faces.

To find out the interest rate that the central bank should set in the current period, we need to introduce the IS equation. The central bank can set the nominal short-term interest rate directly, and since implicitly at least the expected rate of inflation is given in the short run, the central bank is assumed to be able to control the real interest rate indirectly. The IS equation incorporates the lagged effect of the interest rate on output ($i = 1$):

$$y_1 = A - ar_0 \qquad (IS \text{ equation, \#1})$$

and in output gap form is:

$$y_1 - y_e = -a(r_0 - r_S). \qquad (IS \text{ equation, output gap form})$$

If we substitute for π_1 using the Phillips curve in the MR–AD equation, we get

$$\begin{aligned} \pi_0 + \alpha(y_1 - y_e) - \pi^T &= -\frac{1}{\alpha\beta}(y_1 - y_e) \\ \pi_0 - \pi^T &= -\left(\alpha + \frac{1}{\alpha\beta}\right)(y_1 - y_e) \end{aligned}$$

and if we now substitute for $(y_1 - y_e)$ using the IS equation, we get

$$(r_0 - r_S) = \frac{1}{a\left(\alpha + \frac{1}{\alpha\beta}\right)}\left(\pi_0 - \pi^T\right). \qquad \text{(Interest-rate rule, } IR \text{ equation)}$$

If $a = \alpha = \beta = 1$,

$$(r_0 - r_S) = 0.5\left(\pi_0 - \pi^T\right).$$

This tells the central bank how to adjust the interest rate (relative to the stabilizing interest rate) in response to a deviation of inflation from its target.

By setting out the central bank's problem in this way, we have identified the key role of forecasting: the central bank must forecast the Phillips curve and the IS curve it will face next period. Although the central bank observes the shock in period zero and calculates its impact on current output and next period's inflation, it cannot offset the shock in the current period because of the lagged effect of the interest rate on aggregate demand and output. This overcomes one of main drawbacks of the optimizing version of the R–T model. We therefore have a 3-equation model with an optimizing central bank in which IS shocks affect output.

2.2 Diagrams: the example of an IS shock

We shall now explain how the 3-equation model can be set out in a diagram. A graphical approach is useful because it allows students to work through the forecasting exercise of the central bank and to follow the adjustment process as the optimal monetary policy is implemented and the economy moves to the new equilibrium.

The first step is to present two of the equations of the 3-equation model. In the lower part of the diagram, which we call the Phillips diagram, the vertical long-run Phillips curve at the equilibrium output level, y_e, is shown. We think of labour and product markets as being imperfectly competitive so that the equilibrium output level is where both wage- and price-setters make no attempt to change the prevailing real wage or relative prices. Each Phillips curve is indexed by the pre-existing or inertial rate of inflation, $\pi^I = \pi_{-1}$.

As shown in Fig. 4, the economy is in a constant inflation equilibrium at the output level of y_e; inflation is constant at the target rate of π^T and the real interest rate required to ensure that aggregate demand is consistent with this level of output is the stabilizing rate, r_S. Fig. 4 shows the IS equation in the upper panel: the stabilizing interest rate will produce a level of aggregate demand equal to equilibrium output, y_e. We now need to combine the three elements: the IS curve, the Phillips curve and the central bank's forecasting exercise to show how it formulates monetary policy.

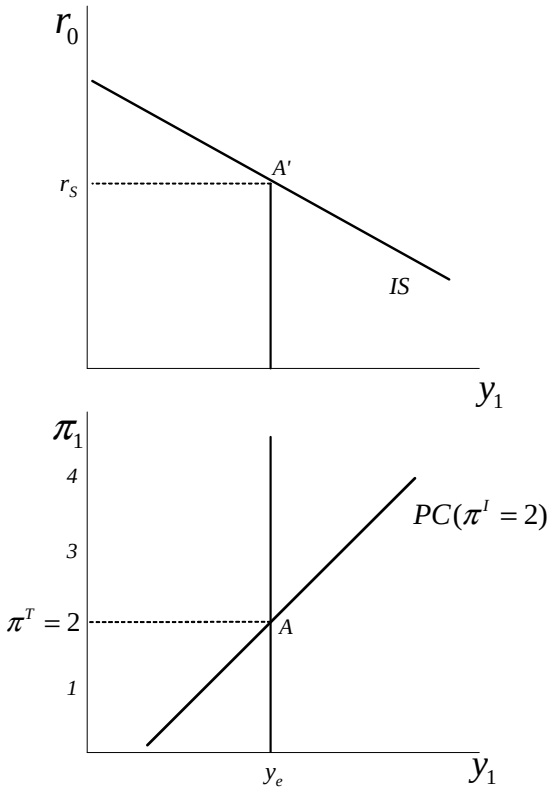


Figure 4: IS and PC curves

In Fig. 5, we assume that as a consequence of an IS shock the economy is initially at point A in the Phillips diagram with output above equilibrium and inflation of 4% above the 2% target. The central bank's job is to set the interest rate, r_0, in response to this new information about economic conditions. In order to do this, it must first make a forecast of the Phillips curve next period, since this shows the menu of output-inflation pairs that it can choose from by setting the interest rate now: remember that changing the interest rate now only affects output next period. Given that inflation is inertial, the central bank's forecast of the Phillips curve in period one will be $PC(\pi^I = 4\%)$ as shown by the dashed line in the Phillips diagram. It is useful to note that the only points on this Phillips curve with inflation below 4% entail lower output. This implies that disinflation will be costly in the sense that output must be pushed *below equilibrium* in order to achieve disinflation.

How does the central bank make its choice from the combinations of inflation

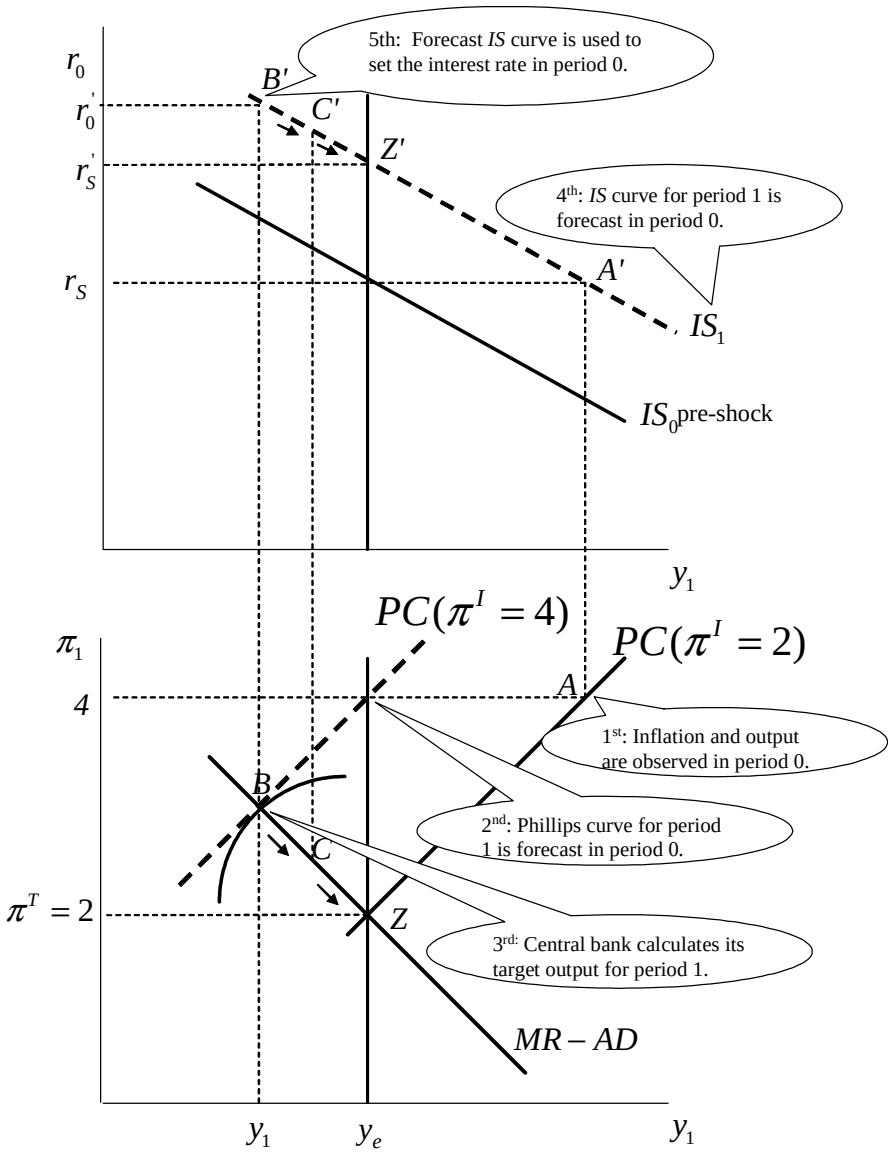


Figure 5: How the central bank decides on the interest rate

and output along the forecast Phillips curve ($PC(\pi^I = 4\%)$)? Its choice will depend on its preferences: the higher is β the more averse it is to inflation and the more it will want to reduce inflation by choosing a larger negative output gap. We show in the appendix how the central bank's loss function can be represented graphically by loss circles or ellipses and we refer to the relevant parts of these circles or ellipses as its indifference curves. In Fig. 5, the central bank will choose point B at the tangency between its indifference curve and the forecast Phillips curve: this implies that its desired output level in period one is y_1. This level of output is the central bank's aggregate demand target for period 1 as implied by the monetary rule. The MR–AD line joins point B and the zero loss point at Z, where inflation is at target and output is at equilibrium. The fourth step is for the central bank to forecast the IS curve for period one. In the example in Fig. 5, the forecast IS curve is shown by the dashed line. With this IS curve, if an interest rate of r'_0 is set now, the level of output in period one will be y_1 as desired. Of course other random shocks may disturb the economy in period 1 but since these are by definition unforecastable by the central bank, they do not enter its decision rule in period zero.

To complete the example, we trace through the adjustment process. Following the increase in the interest rate, output falls to y_1 and inflation falls. The central bank forecasts the new Phillips curve, which goes through point C in the Phillips diagram and it will follow the same steps to adjust the interest rate downwards so as to guide the economy along the IS curve from C' to Z'. Eventually, the objective of inflation at $\pi^T = 2\%$ is achieved and the economy is at equilibrium unemployment, where it will remain until a new shock or policy change arises. The MR–AD line shows the optimal inflation-output choices of the central bank, given the Phillips curve constraint that it faces.

An important pedagogical question is the name to give the monetary rule equation when we show it in the Phillips diagram. What it tells the central bank at $t = 0$ is the output level that it needs to achieve in $t = 1$ if it is to minimize the loss function, given the forecast Phillips curve. Since we are explaining the model from the central bank's viewpoint at $t = 0$, what we want to convey is that the downward-sloping line in the Phillips diagram shows the aggregate demand target at $t = 1$ implied by the monetary rule. We therefore use the label MR–AD.[7]

The MR–AD curve is shown in the Phillips diagram rather than in the IS dia-

[7]It would be misleading to label it AD thus implying that it is the *actual* AD curve in the Phillips diagram because the actual AD curve will include any aggregate demand shock in $t = 1$. If aggregate demand shocks in $t = 1$ are included, the curve ceases to be the curve on which the central bank bases its monetary policy in $t = 0$. On the other hand if an aggregate demand shock in $t = 1$ is excluded — so that the central bank can base monetary policy on the curve — then it is misleading to call it the AD schedule; students would be not unreasonably be surprised if an AD schedule did not shift in response to an AD shock.

gram because the essence of the monetary rule is to identify the central bank's best policy response to any shock. Both the central bank's preferences shown graphically by its indifference curves and the Phillips curve trade-off it faces between output and inflation appear in the Phillips diagram. Once the central bank has calculated its desired output response by using the forecast Phillips curve, it is straightforward to go to the IS diagram and discover what interest rate must be set in order to achieve this level of aggregate demand and output.

2.3 Using the graphical model

We now look at a variety of shocks so as to illustrate the role the following six elements play in their transmission and hence in the deliberations of policy-makers in the central bank:

1. the inflation target, π^T
2. the central bank's preferences, β
3. the slope of the Phillips curve, α
4. the interest sensitivity of aggregate demand, a
5. the equilibrium level of output, y_e
6. the stabilizing interest rate, r_S.

A temporary aggregate demand shock is a one-period shift in the IS curve, whereas a permanent aggregate demand shock shifts the IS curve and hence r_S, the stabilizing interest rate, permanently. An inflation shock is a temporary (one-period) shift in the short-run Phillips curve. This is sometimes referred to as a temporary aggregate supply shock. An aggregate supply shock refers to a permanent shift in the equilibrium level of output, y_e. This shifts the long-run vertical Phillips curve.

2.3.1 IS shock: temporary or permanent?

In Section 2.2 and Fig. 5 we analyzed an IS shock — but was it a temporary or a permanent one? In order for the central bank to make its forecast of the IS curve, it has to decide whether the shock that initially caused output to rise to y_0 is temporary or permanent. In our example, the central bank took the view that the shock would persist for another period, so it was necessary to raise the interest rate to r'_0 above the *new* stabilizing interest rate, r'_S. Had the central bank forecast

that the IS curve would revert to the pre-shock IS curve (i.e. IS_0), then it would have raised the interest rate by less since the stabilizing interest rate would have remained unchanged at r_S. The chosen interest rate would have been as shown in Fig. 5 by IS_0 at y_1.

2.3.2 Supply shock

One of the key tasks of a basic macroeconomic model is to help illuminate how the main variables are correlated following different kinds of shocks. We can appraise the usefulness of the IS-PC-MR model in this respect by looking at a positive aggregate supply shock and comparing the optimal response of the central bank and hence the output and inflation correlations with those associated with an aggregate demand shock. A supply shock results in a change in equilibrium output and therefore a shift in the long-run Phillips curve. It can arise from changes that affect wage- or price-setting behaviour such as a structural change in wage-setting arrangements, a change in taxation or in unemployment benefits or in the strength of product market competition, which alters the mark-up.

Fig. 6 shows the analysis of a positive supply-side shock, which raises equilibrium output from y_e to y'_e. Before analyzing the impact of the shock and the adjustment process as the central bank works out and implements its optimal response, it is useful to identify the characteristics of the new constant-inflation equilibrium. In the new equilibrium, equilibrium output will be at the new higher level, y'_e, and inflation will be at its target of 2%. The long-run Phillips curve will be at y'_e. There will a new MR–AD curve, MR–AD', since it must go through the inflation target and the new equilibrium output level, y'_e: the zero loss point for the central bank following this shock is at point Z. Note also that as a consequence of the supply shock, the stabilizing interest rate has fallen to r'_S.

We now examine the initial effect of the shock. Since the long-run Phillips curve shifts to the right so too does the short-run Phillips curve corresponding to inflation equal to the target (shown by the $PC(\pi^I = 2, y'_e)$). The first consequence of the supply shock is a fall in inflation (from 2% to zero) as the economy goes from A to B: this is observed by the central bank in period zero. To decide how monetary policy should be adjusted to respond to this, we follow the same steps taken in Section 2.2. The central bank forecasts the Phillips curve constraint ($PC(\pi^I = 0, y'_e)$) for period one and chooses its optimal level of output as shown by point C. Next the central bank must forecast the IS curve: since there is no information to suggest any shift in the IS curve, it is assumed fixed. To raise output to the level desired, the central bank must therefore cut the interest rate in period zero to r' as shown in the IS diagram. Note that since the stabilizing interest rate has fallen to r'_S, the central bank reduces the interest rate below this in order to achieve its

desired output level of y'. The economy is then guided along the MR–AD' curve to the new equilibrium at Z.

The positive supply shock is associated initially with a fall in inflation in contrast to the initial rise in both output and inflation in response to the positive aggregate demand shock. In the aggregate demand case, the central bank has to push output below equilibrium during the adjustment process in order to squeeze the higher inflation caused by the demand shock out of the economy. Conversely in the aggregate supply shock case, a period of output above equilibrium is needed in order to bring inflation back up to the target from below. In the new equilibrium, output is higher than its initial level in the supply shock case whereas it returns to its initial level in the case of the aggregate demand shock. In the new equilibrium, inflation is at target in both cases. However, whereas the real interest rate is higher than its initial level in the new equilibrium following a permanent positive aggregate demand shock, it is lower following a positive aggregate supply shock.

2.3.3 IS shock: the role of the interest-sensitivity of aggregate demand

In the next experiment (Fig. 7), we keep the supply side of the economy and the central bank's preferences fixed and examine how the central bank's response to a permanent aggregate demand shock is affected by the sensitivity of aggregate demand to the interest rate. It is assumed that the economy starts off with output at equilibrium and inflation at the target rate of 2%. The equilibrium is disturbed by a positive aggregate demand shock such as improved buoyancy of consumer expectations, which is assumed by the central bank to be permanent. Two post-shock IS curves are shown in the upper panel of Fig. 7: the more-interest sensitive one is the flatter one labelled IS''. To prevent the diagram from getting too cluttered, only one of the pre-shock IS curves is shown. The step-by-step analysis of the impact of the shock is the same as in Section 2.2. The consequence of output above y_e is that inflation rises above target — in this case to 4% (point B). To calculate its desired output level, the central bank forecasts the Phillips curve (i.e. $PC(\pi^I = 4)$) along which it must choose its preferred point for the next period: point C. Since the supply side and the central bank's preferences are assumed to be identical for each economy, the Phillips diagram and hence the PC and MR–AD curves are common to both. However, in the next step, the structural difference between the two economies is relevant. By going vertically up to the IS–diagram, we can see that the central bank must raise the interest rate by less in response to the shock (i.e. to r'' as compare to r') if aggregate demand is more responsive to a change in the interest rate (as illustrated by the flatter IS curve, IS'').

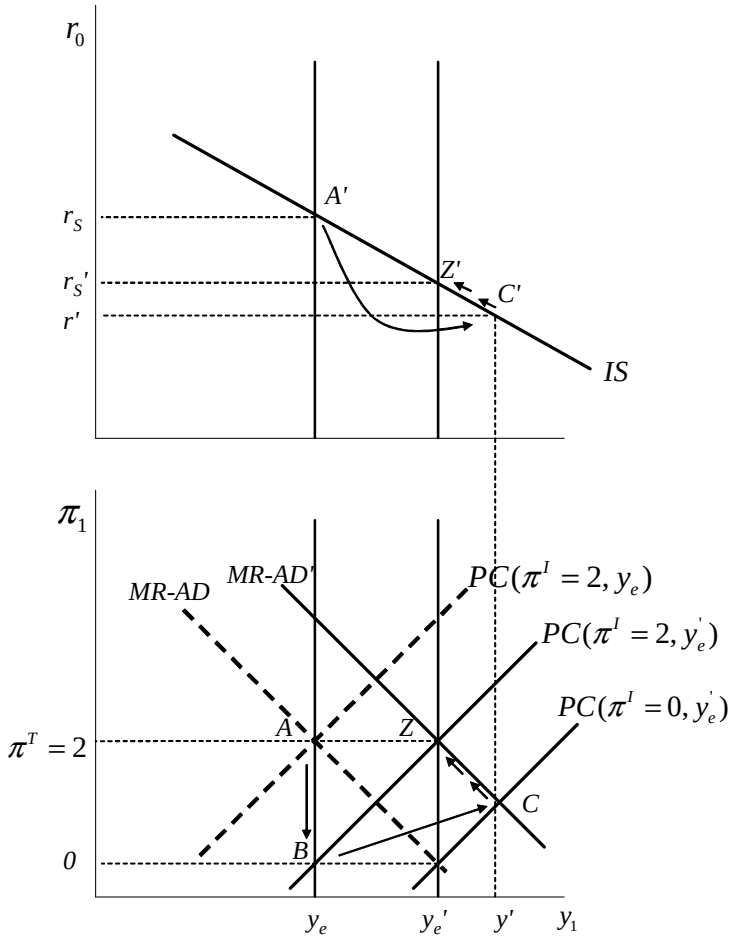


Figure 6: The response of the central bank to a positive supply-side shock, a rise in equilibrium output

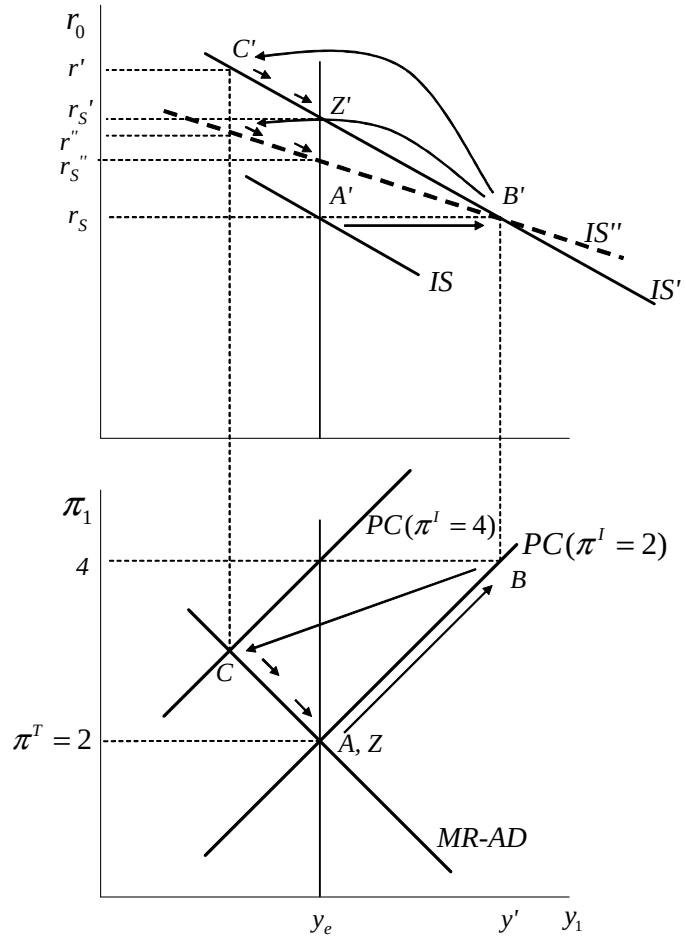


Figure 7: The monetary policy response to a permanent IS shock: the role of the slope of the IS

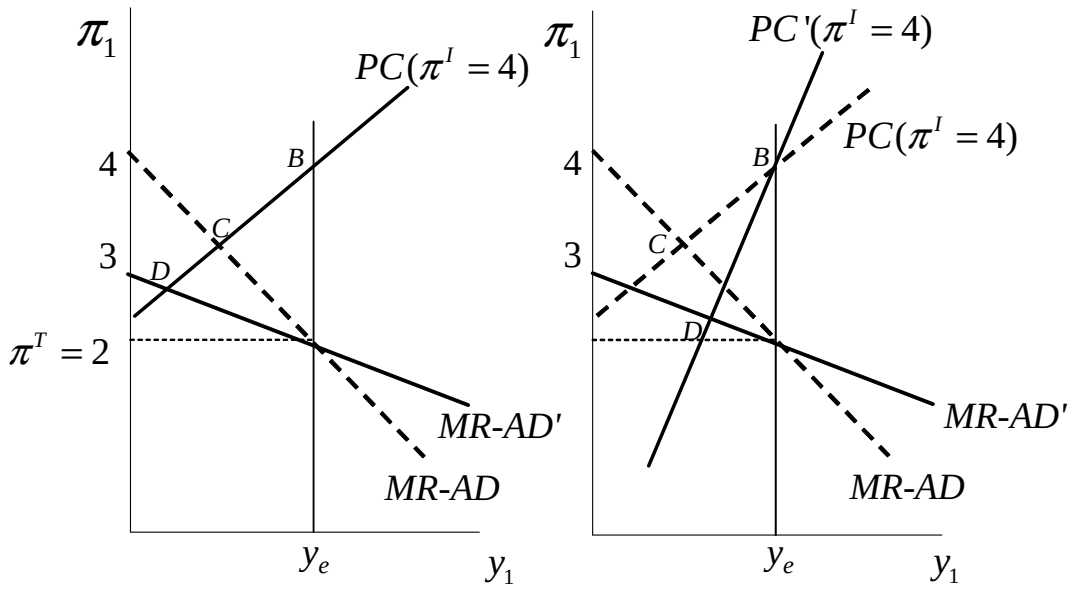
2.3.4 How central bank inflation aversion and the slope of the Phillips curve affect interest rate decisions

To investigate how structural features of the economy such as the degree of inflation aversion of the central bank and the responsiveness of inflation to the output gap impinge on the central bank's interest rate decision, we look at the central bank's response to an inflation shock. A one-period shift in the Phillips curve could occur as a result, for example, of an agricultural disease outbreak that temporarily interrupts supply and pushes inflation above the target level. We assume the economy is initially in equilibrium with inflation at the central bank's target rate of 2% and experiences a sudden rise in inflation to 4%. The short run Phillips curve shifts to $PC(\pi^I = 4\%)$ and the economy moves to point B in Fig. 8.

In our first example in Fig. 8a, we focus attention on the consequences for monetary policy of different degrees of inflation aversion on the part of the central bank (β): the other five structural characteristics listed at the beginning of Section 2.3 are held constant. From the MR–AD equation (i.e. $(y_1 - y_e) = -\alpha\beta.(\pi_1 - \pi^T)$) and from the geometry in Fig. 11 in the Appendix, it is clear that if the indifference curves are circles (i.e. $\beta = 1$) and if the Phillips curve has a gradient of one (i.e. $\alpha = 1$), the MR–AD line is downward sloping with a gradient of minus one. It follows that the MR–AD line will be *flatter* than this if the weight on inflation in the central bank's loss function is greater than one ($\beta > 1$). The more inflation-averse central bank is represented by the solid MR–AD' line in Fig. 8a. In response to the inflation shock, the more inflation-averse central bank wishes to reduce inflation by more and will therefore choose a larger output reduction: point D as compared with point C for the less inflation-averse central bank.

We turn to the second example in Fig. 8b. In this case, we hold the central bank's preferences constant ($\beta = 1$, so in geometric terms, the central bank indifference curves are circles) and look at the implications of the responsiveness of inflation to output as reflected in the slope of the Phillips curve. The economy with the steeper Phillips curve ($\alpha > 1$) shown by the solid line has the flatter MR–AD curve: this is the solid one labelled MR–AD'. As in Fig. 8a, the inflation shock takes the economy to point B on the long-run Phillips curve. When $\alpha = 1$ (i.e. with the dashed Phillips curve and MR–AD curve), the central bank's optimal point is C, whereas we can see that if the Phillips curve is steeper, the central bank cuts aggregate demand by *less* (point D). The intuition behind this result is that a steeper Phillips curve means that, holding central bank preferences constant, it has to 'do less' in response to a given inflation shock since inflation will respond sharply to the fall in output associated with tighter monetary policy.

Using the diagram underlines the fact that although the MR–AD curve is flatter in both of our experiments, i.e. with a more inflation-averse central bank or with



a. Greater inflation-aversion

b. Steeper Phillips curve

Figure 8: Inflation shock: the effect of (a) greater inflation aversion of the central bank and (b) a steeper Phillips curve

greater sensitivity of inflation to output, the central bank's reaction to a given inflation shock is different. In the left hand panel, the flatter MR–AD curve is due to greater inflation-aversion on the part of the central bank. Such a central bank will always wish to cut output by more in response to a given inflation shock (choosing point D) as compared with the neutral case of $\beta = 1$ (where point C will be chosen). By contrast in the right hand panel, a central bank facing a more responsive supply-side (as reflected in steeper Phillips curves) will normally choose to do less in response to an inflation shock (choosing point D) than would a central bank with the same preferences facing a less responsive supply-side (point C).

The examples in Fig. 8b and Fig. 7 highlight that if we hold the central bank's preferences constant, common shocks will require different optimal responses from the central bank if the parameters α (reflected in the slope of the short run Phillips curve) or a (reflected in the slope of the IS curve) differ. This is relevant to the com-

parison of interest rate rules across countries and to the analysis of monetary policy in a common currency area. For example in a monetary union, unless the aggregate supply and demand characteristics that determine the slope of the Phillips curve and the IS curve in each of the member countries are the same, the currency union's interest rate response to a common shock will not be optimal for all members.

2.4 Lags and the Taylor Rule

A Taylor Rule is a policy rule that tells the central bank how to set the current interest rate in response to shocks that result in deviations of inflation from target or output from equilibrium or both. In Section 1.1, we used the expression Interest Rate rule or IR equation to refer to the Taylor-type rules derived from each model. In Taylor's original empirical rule, $(r_0 - r_S)$ responds to $(\pi_0 - \pi^T)$ and $(y_0 - y_e)$ with the coefficients 0.5 and 0.5:

$$r_0 - r_S = 0.5 \cdot (\pi_0 - \pi^T) + 0.5 \cdot (y_0 - y_e). \qquad \text{(Taylor rule)}$$

We derived the Taylor-type rule for the 3-equation C–S model:

$$(r_0 - r_S) = \frac{1}{a\left(\alpha + \frac{1}{\alpha\beta}\right)} \left(\pi_0 - \pi^T\right), \qquad (IR \text{ equation, C–S model})$$

which with $a = \alpha = \beta = 1$, gives $r_0 - r_S = 0.5 \cdot (\pi_0 - \pi^T)$. Two things are immediately apparent: first, only the inflation and not the output deviation is present in the rule and second, as we have seen in the earlier examples, all the parameters of the three equation model matter for the central bank's response to a rise in inflation. If each parameter is equal to one, the weight on the inflation deviation is one half. For a given deviation of inflation from target, and in each case, comparing the situation with that in which $a = \alpha = \beta = 1$, we have

- a more inflation averse central bank ($\beta > 1$) will raise the interest rate by more;
- when the IS is flatter ($a > 1$), the central bank will raise the interest rate by less;
- when the Phillips curve is steeper ($\alpha > 1$), the central bank will raise the interest rate by less.[8]

[8]This is always true for $\beta = 1$ (as in the right hand panel of Fig. 8). In fact, with $\beta \geq 1$, the output cut in response to a given inflation shock is always less when $\alpha > 1$ as compared with $\alpha = 1$. For $\beta < 1$, the output cut is less as long as $\alpha > 1/\beta$.

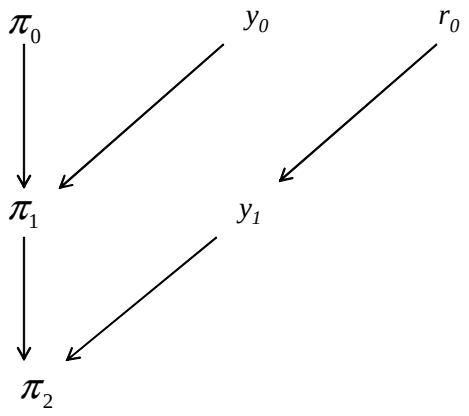


Figure 9: Double lag structure in the 3-equation model ($i = j = 1$)

As shown in the discussion of the Svensson–Ball model in Section 1, in order to derive a Taylor rule in which both inflation and output deviations are present, it is necessary to modify the lag structure of the three equation C–S model. Specifically, it is necessary to introduce an additional lag ($j = 1$), i.e. the output level y_0 affects inflation a period later, π_1. This means that it is y_0 and not y_1 that is in the Phillips curve for π_1.

The double lag structure is shown in Fig. 9 and highlights the fact that a decision taken today by the central bank to react to a shock will only affect the inflation rate two periods later, i.e. π_2. When the economy is disturbed in the current period (period zero), the central bank looks ahead to the implications for inflation and sets the interest rate r_0 so as to determine y_1, which in turn determines the desired value of π_2. As the diagram illustrates, action by the central bank in the current period has no effect on output or inflation in the current period or on inflation in a year's time.

Given the double lag ($i = j = 1$), the central bank's loss function contains y_1 and π_2 since it is these two variables it can choose through its interest rate decision:[9]

$$L = (y_1 - y_e)^2 + \beta(\pi_2 - \pi^T)^2$$

and the three equations are:

$$\begin{aligned} \pi_1 &= \pi_0 + \alpha(y_0 - y_e) && \text{(Phillips curve)} \\ y_1 - y_e &= -a(r_0 - r_S) && (IS\,) \\ \pi_2 - \pi^T &= -\frac{1}{\alpha\beta}(y_1 - y_e). && (MR\text{–}AD) \end{aligned}$$

By repeating the same steps as we used in Section 2.1, we derive the interest rate rule:

$$(r_0 - r_S) = \frac{1}{a\left(\alpha + \frac{1}{\alpha\beta}\right)}\left[\left(\pi_0 - \pi^T\right) + \alpha(y_0 - y_e)\right].$$

(Interest Rate (Taylor) rule in 3-equation (double lag) model)

We note that Taylor's empirical formulation emerges if $a = \alpha = \beta = 1$, i.e.

$$(r_0 - r_S) = 0.5\left(\pi_0 - \pi^T\right) + 0.5(y_0 - y_e).$$

Implicitly the interest rate rule incorporates changes in the interest rate that are required as a result of a change in the stabilizing interest rate (in the case of a permanent shift in the IS or of a supply-side shift): r_S in the rule should be interpreted as the post-shock stabilizing interest rate.

It is often said that the relative weights on output and inflation in a Taylor Rule reflect the central bank's preferences for reducing inflation as compared to output deviations. However, we have already seen in the single lag version of the model that although the central bank cares about both inflation and output deviations, only the inflation deviation appears in the interest rate rule. Although both the output and inflation deviations are present in the IR equation for the double lag model, the *relative* weights on inflation and output depend only on α, the slope of the Phillips curve. The relative weights are used *only* to forecast next period's inflation. The central bank's preferences determine the interest rate response to next period's inflation (as embodied in the slope of the MR curve). Another way to express this result is to say that the output term only appears in the IR equation because of the lag from a change in output to a change in inflation, i.e. because $j = 1$.

[9]For clarity when teaching, it is probably sensible to ignore the discount factor, i.e. we assume $\delta = 1$.

3 Inflation bias and time inconsistency

3.1 Introducing inflation bias

In the 3-equation models analyzed to this point (with the exception of the R–T model without an optimizing central bank), medium-run equilibrium is characterized by inflation equal to the central bank's inflation target and by output at equilibrium. However, since imperfect competition in product and labour markets implies that y_e is less than the competitive full-employment level of output, the government may have a higher target output level. We assume that the government can impose this target on the central bank. How do things change if the central bank's target is full-employment output, or more generally a level of output above y_e? For clarity, we use the C–S model ($i = 1, j = 0$).

A starting point is to look at the central bank's new objective function. It now wants to minimize

$$L = (y_1 - y^T)^2 + \beta(\pi_1 - \pi^T)^2, \tag{1}$$

where $y^T > y_e$. This is subject as before to the Phillips curve,

$$\pi_1 = \pi_0 + \alpha.(y_1 - y_e). \tag{2}$$

In Fig. 10 the central bank's ideal point is now point A (where $y = y^T$ and $\pi = \pi^T$) rather than where $y = y_e$ and $\pi = \pi^T$ (i.e. point C). Since nothing has changed on the supply-side of the economy, the Phillips curves remain unchanged. To work out the central bank's monetary rule, consider the level of output it chooses if $\pi^I = 2\%$. Fig. 10 shows the Phillips curve corresponding to $\pi^I = 2\%$. The tangency of $PC(\pi^I = 2)$ with the central bank's indifference curve shows where the central bank's loss is minimized (point D). Since the central bank's monetary rule must also pass through A, it is the downward sloping line MR–AD in Fig. 10.

We can see immediately that the government's target, point A, does not lie on the Phillips curve for inertial inflation equal to the target rate of $\pi^T = 2\%$: the economy will only be in equilibrium with constant inflation at point B. This is where the monetary rule (MR–AD) intersects the vertical Phillips curve at $y = y_e$. At point B, inflation is above the target and the gap between the target rate of inflation and inflation in the equilibrium is the inflation bias.

We shall now pin down the source of the inflation bias and the determinants of its size. We begin by showing why the equilibrium is at point B. If inflation is initially at its target rate of 2%, the central bank chooses its preferred point on the $PC(\pi^I = 2)$ and the economy is at point D (see Fig. 10). But with output above equilibrium, inflation goes up to 3% and the Phillips curve shifts up (see the dashed Phillips curve in Fig. 10). The process of adjustment continues until

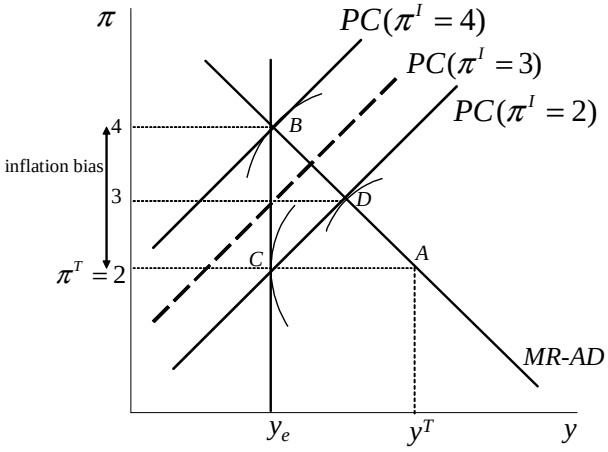


Figure 10: The Inflation Bias

point B is reached: output is at the equilibrium and inflation does not change so the Phillips curve remains fixed. Neither central bank nor price- or wage-setters have any incentive to change their behaviour. The economy is in equilibrium. But neither inflation nor output are at the central bank's target levels (see Fig. 10). The inflation bias arises because target output is above y_e: in equilibrium, the economy must be at y_e and on the MR–AD curve. It is evident from the geometry that a steeper MR line will produce a larger inflation bias.

We can derive the same result using the equations. Minimising the central bank's loss function — equation (1) — subject to the Phillips curve — equation (2) — implies

$$\begin{aligned} y_1 - y^T + \alpha\beta.(\pi_0 + \alpha.(y_1 - y_e) - \pi^T) &= y_1 - y^T + \alpha\beta.(\pi_1 - \pi^T) \\ &= 0. \end{aligned}$$

So the new monetary rule is:

$$y_1 - y^T = -\alpha\beta.(\pi_1 - \pi^T). \qquad (MR\text{–}AD \text{ equation})$$

This equation indeed goes through (π^T, y^T). Since from the Phillips curve, we have

$\pi_0 = \pi_1$ when $y_1 = y_e$, it follows that

$$\begin{aligned} y_e &= y^T - \alpha\beta.(\pi_0 - \pi^T) \\ \Rightarrow \quad \pi &= \pi^T + \underbrace{\frac{(y^T - y_e)}{\alpha\beta}}_{\text{inflation bias}}. \end{aligned} \qquad \text{(Inflation bias)}$$

In equilibrium, inflation will exceed the target by $\frac{(y^T - y_e)}{\alpha\beta}$, the inflation bias.[10] The significance of this result is that $\pi > \pi^T$ whenever $y^T > y_e$. In other words, it is the fact that the central bank's output target is higher than equilibrium output that is at the root of the inflation bias problem. The inflation bias will be greater, the less inflation-averse it is; i.e. the lower is β. A lower α also raises the inflation bias. A lower α implies that inflation is less responsive to changes in output. Therefore, any given reduction in inflation is more expensive in lost output; so in cost-benefit terms for the central bank, it pays to allow a little more inflation and a little less output loss.

3.2 Time inconsistency and inflation bias

The problem of inflation bias is usually discussed in conjunction with the problem of time inconsistency in which the central bank or the government announces one policy but has an incentive to do otherwise. For this kind of behaviour to arise, it is necessary to introduce forward-looking inflation expectations. The simplest assumption to make is that inflation expectations are formed rationally and that there is no inflation inertia: i.e. $\pi^E = E[\pi]$, so $\pi = \pi^E + \varepsilon_t$, where ε_t is uncorrelated with π^E. We continue to assume that the central bank chooses y (and hence π) *after* private sector agents have chosen π^E. This defines the central bank as acting with *discretion*. Now, in order for firms and workers to have correct inflation expectations, they must choose π^E such that it pays the central bank to choose $y = y_e$. That must be where the central bank's monetary rule cuts the $y = y_e$ vertical line, i.e. at point B in Fig. 10. Note that the positively sloped lines are now interpreted as Lucas supply equations rather than as short-run Phillips curves.[11] This is the so-called

[10] For an early model of inflation bias with backward-looking inflation expectations, see Phelps (1967).

[11] The usual interpretation of the former is that an inflation surprise leads output to deviate from equilibrium whereas in the latter, a shift of output away from the equilibrium leads inflation to deviate from its expected level.

Lucas surprise supply equation:

$$\begin{aligned} y_t - y_e &= \frac{1}{\alpha}\left(\pi_t - \pi_t^E\right) \\ y_t &= y_e + \underbrace{\frac{1}{\alpha}\left(\pi_t - \pi_t^E\right)}_{\text{inflation surprise}}. \end{aligned}$$

For an expectations equilibrium, inflation must be sufficiently high to remove the temptation of the central bank to raise output toward its target. With $\pi = 4\%$ and $y = y_e$, the temptation has been removed because any increase in output from B would put the central bank on a loss circle more distant from its bliss point A: firms and workers therefore rationally expect an inflation surprise of 2% over and above the target inflation rate of 2% (compare point B, which is an expectations equilibrium for the private sector and the central bank, with point C, which is not an equilibrium for the central bank).

The inflation bias presents a problem. As is clear from Fig. 10, the loss to the central bank at B is greater than its loss at C since output is the same but inflation is higher at B. So the central bank would clearly be better off at C. Moreover, firms and workers would be just as happy at C as at B, since output, employment and the real wage are the same in each case. What is to stop the central bank being at C? When private sector agents are forward-looking, the problem is called that of *time inconsistency.* Although the central bank claims to have an inflation target of π^T, if firms and workers act on the basis of this target (2%), when it comes to set the interest rate, the central bank does not choose the output level consistent with its target. In short, at point B there is no incentive for the central bank to cheat; whereas at point C, there is an incentive.

4 Conclusions

The graphical 3-equation (C–S) model with $i = 1$, $j = 0$ is a replacement for the standard IS-LM-AS model. It provides undergraduate students and non-specialists with the tools for analyzing a wide range of macroeconomic disturbances and with access to contemporary debates in the more specialized monetary macroeconomics literature. It has a number of features that distinguish it from other models that replace the LM equation with a monetary policy rule. First, it conforms with the view that monetary policy is conducted by optimizing forward-looking central banks. Second, since aggregate demand responds to interest rate changes with a lag, aggregate demand and aggregate supply shocks cannot be fully offset even by a forward-looking central bank. Third, in response to a shock, the central bank guides the economy back to equilibrium with target inflation.

The graphical approach helps illuminate the role played by structural characteristics of the aggregate supply and demand sides of the economy and by the central bank's preferences in determining the optimal interest rate response to shocks. It is straightforward to demonstrate the determinants of the size of the inflation bias using this model and the origin of the time inconsistency problem.

The model brings to the fore the relationship between the central bank's preferences and the form of the interest rate rule. In the C–S model, the interest rate rule shows the interest rate responding to current deviations of inflation from target. An advantage of the model is that, modified with an additional lag it becomes the Svensson-Ball model. In that case, the response of inflation to output is lagged (i.e. $j = 1$ rather than $j = 0$ as in the C–S model) and the central bank must forecast the Phillips curve a further period ahead, which produces an interest rate rule that takes the familiar Taylor Rule form to include contemporaneous inflation and output shocks. Although we believe the Svensson–Ball model is too complex to use as an undergraduate teaching model, it will be useful for students to see the relationship between the two, and hence the derivation of the standard Taylor rule.

5 References

Allsopp, Christopher and David Vines (2000). 'The Assessment: Macroeconomic Policy'. Oxford Review of Economic Policy 16(4): 1-32.

Ball, Lawrence (1999). 'Efficient Rules for Monetary Policy'. International Finance 2(1): 63-83.

Bank of England (1999). 'The Transmission of Monetary Policy'
http://www.bankofengland.co.uk/publications/other/monetary/montrans.pdf

Carlin, Wendy and David Soskice (2006). *Macroeconomics: Imperfections, Institutions and Policies*. Oxford: University Press.

Clarida, Richard, Jordi Gali and Mark Gertler (1999). 'The Science of Monetary Policy: A New Keynesian Perspective'. Journal of Economic Literature 37(4): 1661-1707.

Goodfriend, Marvin and Robert King (1997). 'The New Neoclassical Synthesis and the Role of Monetary Policy'. NBER Macroeconomics Annual: 231-283.

Phelps E. S. (1967). 'Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time'. Economica New Series 34/135: 254-281.

Romer, David (2000). 'Keynesian Macroeconomics without the LM Curve'. Journal of Economic Perspectives 14(2): 149-169.

Svensson, Lars E.O. (1997). 'Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets'. European Economic Review 41: 1111-1146.

Taylor, John B. (1993). 'Discretion Versus Policy Rules in Practice'. Carnegie-Rochester Conference Series on Public Policy 39(1): 195-214.

Taylor, John B. (2000). 'Teaching Modern Macroeconomics at the Principles Level'. American Economic Review Papers and Proceedings 90 (2), 90-94.

Walsh, Carl E. (2002). 'Teaching Inflation Targeting: An Analysis for Intermediate Macro'. Journal of Economic Education 33(4): 333-347.

Woodford, Michael (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton: University Press.

6 Appendix

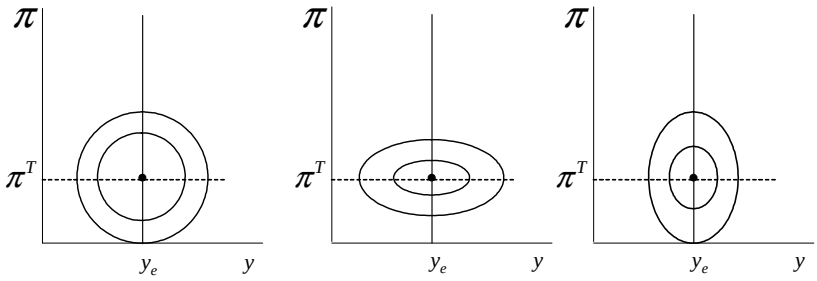
The central bank's loss function: graphical representation

The geometry of the central bank's loss function can be shown in the Phillips diagram. The loss function with $i = 1, j = 0$, i.e.

$$L = (y_1 - y_e)^2 + \beta(\pi_1 - \pi^T)^2,$$

is simple to draw. With $\beta = 1$, each indifference curve is a circle with (y_e, π^T) at its centre (see Fig. 11(a)). The loss declines as the circle gets smaller. When $\pi = \pi^T$ and $y = y_e$, the circle shrinks to a single point (called the 'bliss point') and the loss is zero. With $\beta = 1$, the central bank is indifferent between inflation 1% above (or below) π^T and output 1% below (or above) y_e. They are on the same loss circle.

Only when $\beta = 1$, do we have indifference *circles*. If $\beta > 1$, the central bank is indifferent between (say) inflation 1% above (or below) π^T and output 2% above (or below) y_e. This makes the indifference curves ellipsoid as in Fig. 11(b). A central bank with less aversion to inflation ($\beta < 1$) will have ellipsoid indifference curves with a vertical rather than a horizontal orientation (Fig. 11(c)). In that case, the indifference curves are steep indicating that the central bank is only willing to trade off a given fall in inflation for a smaller fall in output than in the other two cases. Such a central bank is sometimes referred to as unemployment-averse.



(a) Balanced: *ß =1*

(b) Inflation-averse: *ß >1*

(c) Unemployment-averse: *ß <1*

Figure 11: Central bank loss function: varying the degree of inflation aversion

DISCUSSION PAPERS 1999-2002 SERIES
ISSN 1350-6722

No	Author(s)	Titles
2001		
01-2001	T de Garidel	Information Sharing in Dynamic Insurance Markets
02-2001	N Rosati	How has Economic Inequality Evolved over the Past Two Decades? A Look at the Italian Experience
03-2001	N Rosati	A Measurement Error Approach to the Study of Poverty
04-2001	I Preston S F Ross S Szymanski	Seizing the Moment: a Blueprint for Reform of World Cricket
05-2001	I Preston J Thomas	Rain Rules for Limited Overs Cricket and Probabilities of Victory
06-2001	D Sonedda	Employment Effects of Progressive Taxation in a Unionised Economy.
07-2001	B M Koebel	Separabilities and Elasticities
2002		
02-01	S Borghesi D Ulph	Sustainability For All? A North-South-East-West Model
02-02	S Lemos	The effects of the minimum wage on wages and employment in Brazil - A menu of minimum wage variables
02-03	J Banks R Blundell J P Smith	Wealth portfolios in the US and the UK
02-04	P Legros A Newman	Assortative matching in a non-transferable world
02-05	A Calveros M Vera-Hernandez	Quality externalities among hotel establishments: What is the role of tour operators?

02-06	J McClaren A Newman	Globalisation and insecurity
02-07	E Duguet S Monjon	Creative destruction and the innovative core: Is innovation persistent at the firm level?
03-01	W Carlin S Fries M Schaffer Paul Seabright	Competition, Restructuring and Firm Performance: Evidence of an Inverted-U relationship from a Cross-country Survey of Firms in Transition Economies
03-02	S Lemos	A menu of minimum wage variables for evaluating wages and employment effects: evidence from Brazil
03-03	S Lemos	Policy variables as instruments for the minimum wage
03-04	P Olivella M Vera-Hernandez	Competition among differentiated health plans under adverse selection
03-05	R Blundell M Costa Dias C Meghir J Van Reenen	Evaluating the employment impact of a mandatory job search program

2004

04-01	S Lemos	The effect of minimum wages on prices in Brazil
04-02	B Groom P Koundouri E Panopoulou T Pantalidis	Model selection for estimating certainty equivalent discount rates
04-03	P Aghion R Blundell R Griffith P Howitt S Prantl	Entry and productivity growth : evidence from plant level data
04-04	M Desai B Henry A Mosley M Pemberton	A clarification of the Goodwin model of the growth cycle

04-05	GL Albano C Leaver	Transparency, recruitment and retention in the public sector
04-06	P Aghion N Bloom R Blundell R Griffith P Howitt	Competition and innovation : an inverted U relationship ?
04-07	J Heckman D Masterov	Skill policies for Scotland
04-08	T Börgers P Postl	Efficient Compromising
04-09	J Banks R Blundell Z Oldfield JP Smith	House price volatility and housing ownership over the life-cycle
05-01	T Borgers P Norman	A note on budget balance under interim participation constraints: The case of independent types
05-02	F Cunha J Heckman L Lochner D Masterov	Interpreting the evidence on life-cycle skill formation
05-03	W Carlin D Soskice	The 3 equation New Keynesian Model: A graphical exposition