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A CLARIFICATION OF THE GOODWIN MODEL OF THE GROWTH CYCLE

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A Clarification of the Goodwin Model of the Growth Cycle

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Abstract We show that there is a difficulty in the original Goodwin model which is also found in some more recent applications. In it both the labour share and the proportion employed can exceed unity, properties which are untenable. However, we show that the underlying dynamic structure of the model can be reformulated to ensure that these variables cannot exceed unity. An illustrative example extends the original model, and we argue it is both plausible and satisfies the necessary unit box restrictions.

Keywords: Business cycles; Nonlinear dynamics; Goodwin

JEL classification: E32

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1. The Goodwin Model of the Growth Cycle

In “A Growth Cycle” (1967), Goodwin states a simple but elegant model of the struggle between capital and labour for shares in national income, based on the classic Volterra-Lotka predator-prey model for fish populations. (See Minorski (1962)). Since then, the model has been extended in many directions and as such has proved to be a useful framework for combining growth and cycles in a simple non-linear model (For examples see Atkinson (1969), Desai (1973), Wolfstetter (1977) (1979), Pohjola (1979a)(1979b), and for more recent contributions Harvie (2000) and Solow (1990). A comprehensive review is given in Veneziani(2001)).

Despite all these extensions to the Goodwin model, a basic difficulty contained in the original is found in them also, and so far this feature has not received much attention.[3] To explain what this is, we first set out the original mathematical model (the predator-prey model), together with some extensions, before summarising the original Goodwin model. This will be the form of the Goodwin model which we compare with our extension in section 3 below.

The Volterra-Lotka predator-prey model has two variables, x and y, where x is the predator population and y the prey population, and the motion of each is described by the pair of non-linear differential equations

$$\dot{x} = -ax + bxy \qquad (1)$$

$$\dot{y} = cy - dxy \qquad (2)$$

where the dot indicates differentiation with respect to time, all the parameters are positive and explicit mention of time dependence is dropped for convenience.

[3] Previous authors noting it are Desai and Pemberton (1980), Flaschel (1987), Harvie, Kelmanson and Knapp (2000) and Velupillai (1983). Goodwin himself was also clearly aware of this too. (See Goodwin (1967))

There are two singular points for this model, (0,0) and $(c/d, a/b)$ in (*x*,*y*) space. All the trajectories (in the positive orthant) are closed orbits around the upper singular point. The course of the trajectories depends only on the initial values *x*(0), *y*(0). Thus trajectories starting near the origin traverse a larger range of *x* and *y* values than those starting near the upper singular point given by $(c/d, a/b)$.[4]

The Volterra-Lotka (V-L from now on) model was applied by Goodwin to model economic dynamics. Significantly, when he did this, he made one important change. The *x* and *y* variables in his model are defined to lie in the [0,1] interval. Thus his *x* variable (*u* in his notation) is the share of labour in national income and his *y* variable (*v* in his notation) is the proportion of labour force employed. Obviously, neither can exceed unity. To appreciate the problem this can lead to, the model comprising the employment rate and labour share equations needs to be set out in somewhat more detail.

The Goodwin model consists of two dynamic equations, one for the employment rate, *v*, and one for the labour share, *u*. The dynamic equation for the employment rate is

$$\frac{\dot{v}}{v} = \frac{1-u}{\sigma} - (\alpha + \beta) \qquad (3)$$

where α is the exogenous growth in labour productivity, β the exogenous growth in the labour force and σ the capital-output ratio. (For a derivation, see Harvie (2000)). To get the dynamic equation for the labour share, Goodwin used a linear Phillips curve-type relationship in real wages. Thus, where *w*

[4] For a good treatment of the relevant differential equation theory see Birkhoff and Rota (1969) and Hirsch and Smale (1974). The basic form of the Volterra-Lotka model was generalised by Kolmogorov, (see Minorski, op. cit.).

is the real wage and v the proportion of labour force employed, then the real wage behaves according to the equation

$$\frac{\dot{w}}{w} = -\gamma + \rho v \qquad (4)$$

Defining the share of labour in national income u as the ratio of the real wage to average product per worker (w/a) and taking the rate of growth of labour productivity $(\dot{a}/a)$ as a Harrod-neutral constant α, gives the differential equation for the share as [5]

$$\frac{\dot{u}}{u} = -(\gamma + \alpha) + \rho v \qquad (5)$$

Equations (3) and (5) then give the model. It is a V-L system with u playing the role of x and v the role of y. As we discuss in section 3 below, the wage equation underpinning (5) has the property that real wage growth is bounded even at full employment. In turn, the labour share equation (3) assumes that all profits are invested.[6] We will show later that both of these features can be relaxed and, in addition, the dynamics of the resulting model are more acceptable. By this second point we mean that while the original V-L system (1), (2) is restricted only to the non-negative orthant, Goodwin's model needs to satisfy the further restriction that all the closed trajectories (and hence also the upper singular point) lie within the $[0,1]\times[0,1]$

[5] This notation is unhelpful, but we retain that used by Goodwin in what follows to facilitate comparisons with the original.
[6] Note that u may exceptionally be greater than unity if there is disinvestment. (See Gandolfo (1997) p461. We are grateful to a referee for pointing this out.

part of the non-negative orthant. In other words, they should lie within the unit box in the non-negative orthant.

Now unless equations (3) and (5) are modified, they will not satisfy this requirement. To see this, note that the upper singular point is at (u^*, v^*) where $u^* = 1 - \sigma(\alpha + \beta)$ and $v^* = (\gamma + \alpha)/\rho$. Thus, when $\alpha + \beta$ is positive, u^* is automatically less than one but it is certainly technically possible for $\alpha + \beta$ to be negative in which case $u^* > 1$. Similarly, it is possible that $v^* > 1$. The upshot of all this is that the singular point may lie outside the unit box and, in this case, trajectories may lie entirely outside the unit box. However, even if the singular point is inside the unit box, trajectories may still lie partly outside the unit box.

But, it is clear that having values above unity for the employment rate and labour share makes no sense. It is, however, a simple task to modify the Goodwin model so that all its trajectories stay within the unit box. So in what follows, we first state the general mathematical conditions which have to be satisfied by the two equations in the model to avoid the variables exceeding the unit constraints just identified. After that, we comment further on the economic implications of the modified system, aiming to show that, with the proposed modification, the model is in certain respects actually improved.

2. A generalisation of the Volterra-Lotka model

We first rewrite (1) and (2) only slightly more generally, since this is sufficient for our purpose. Thus,

$$\dot{x} = -ax + xf(y) = F(x, y) \qquad (1a)$$
$$\dot{y} = cy - yg(x) = G(x, y) \qquad (2a)$$

So $X(x, y) = -a + f(y),\ Y(x, y) = c - g(x)$. Note that the V-L system assumes $f(y) = by$, $g(x) = dx$. We require f and g to be continuously differentiable on (0,1) and a, c to be positive as before. We further require

(i) $f' > 0$ on (0,1)

$g' > 0$ on (0,1)

(ii) $f(0) < a$

$g(0) < c$

(iii) $\lim_{y \to 1-} f(y) = \lim_{x \to 1-} g(x) = \infty$

We will show that these conditions are sufficient for all trajectories in the non-negative orthant to be closed orbits lying entirely in the unit box, $[0,1] \times [0,1]$. We start by observing that there is one singular point in the unit box at $z^* = \begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} g^{-1}(c) \\ f^{-1}(a) \end{bmatrix}$.

Expanding the right hand sides in Taylor series about z^* and remembering $F(x^*, y^*) = G(x^*, y^*) = 0$, we get

$$F(x, y) \approx \frac{\partial F}{\partial x}(x - x^*) + \frac{\partial F}{\partial y}(y - y^*)$$

$$G(x, y) \approx \frac{\partial G}{\partial x}(x - x^*) + \frac{\partial G}{\partial y}(y - y^*)$$

where the derivatives are evaluated at z^*.

The system can be written as

$$\dot{z} \approx \begin{bmatrix} 0 & x^* f'(y^*) \\ -y^* g'(x^*) & 0 \end{bmatrix}(z - z^*) = \mathbf{A}(z - z^*) \text{ say,}$$

where the eigenvalues of **A** are given by

$$\lambda^2 + x^* y^* f'(y^*)g'(x^*) = 0$$

By condition (i),

$$x^* y^* f'(y^*)g'(x^*) > 0$$

and so the eigenvalues are purely imaginary.

It follows that z^* is a centre for the linearised system with all its trajectories being closed orbits. Hence, for the original generalised system ((1a) and (2a)), z^* is either a centre or a focus with the trajectories spiralling inwards towards z^* or outwards away from it.

Note also that

$$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{cy - yg(x)}{-ax + xf(y)}$$

Separating the variables

$$\int \left(-\frac{a}{y} + \frac{f(y)}{y}\right) dy = \int \left(\frac{c}{x} - \frac{g(x)}{x}\right) dx$$

and so the equation of the trajectory passing through (x_0, y_0) is

$$-a(\ln y - \ln y_0) + \int_{y_0}^{y} \frac{f(s)}{s} ds = c(\ln x - \ln x_0) - \int_{x_0}^{x} \frac{g(s)}{s} ds$$

Any line of the form $x = p$, where $0<p<1$, meets this trajectory when

$$-a(\ln y - \ln y_0) + \int_{y_0}^{y} \frac{f(s)}{s} ds = c(\ln p - \ln x_0) - \int_{x_0}^{p} \frac{g(s)}{s} ds = P, \text{say.}$$

Denoting the left-hand side by $F(y)$ then

$$F'(y) = -\frac{a}{y} + \frac{f(y)}{y}$$

which, under conditions (i)-(iii), has exactly one zero in (0,1). Hence the line $x = p$ cuts the trajectory in at most two points and so the trajectory cannot be a spiral.

Therefore, the trajectory must be a closed orbit. Furthermore, any trajectory starting inside the unit box will stay inside it because of condition (iii). Otherwise, the trajectory would have to cross one of the lines x=1 or y=1 along which the differential equation system is undefined.

Thus, with what is only a minor generalisation of the V-L model, we have shown that the model satisfies the requirement that all the trajectories are closed orbits and lie entirely within the unit box.

3. A modified Goodwin model

Next, we turn to a reformulation of the Goodwin model which we argue ensures that the problems we have noted in the original model are overcome. The two extensions we consider are first, to specify the real wage equation in a non-linear form (as did Phillips originally for the nominal wage equation), and second to amend the Goodwin assumption that all profits are always invested.

3.1. Amending the Model

Reverting to Phillips' original nonlinear specification, albeit for real rather than money wages, would give an equation of the form,[7]

$$\frac{\dot{w}}{w} = -\gamma' + \rho'(1-v)^{-\delta} \tag{4a}$$

where $\delta > 0$.

In this case, equation (4) above would become

$$\frac{\dot{u}}{u} = -(\gamma' + \alpha) + \rho'(1-v)^{-\delta} \tag{5a}$$

From (4a), as v approaches 1 i.e. full employment, then $\dot{w}/w \to \infty$. This indeed

[7] We use γ' and ρ' as the modified specification results in different parameter values.

makes sense in economic terms and on these grounds is preferable to (4), where even at full employment the rate of growth of real wages is bounded. Of course, it would be even more preferable to respecify the wage bargain in money terms and add a price equation, but we do not discuss these extensions here, since they are not relevant to our main purpose. (But see Desai (1973) for a discussion). Notice also that now (5a) satisfies our conditions (i) and (iii) above and, provided $\rho' < \gamma' + \alpha$, condition (ii) is also satisfied.

The second equation in the Goodwin model assumes that all profits are invested (and all wages are consumed), that the capital-output ratio (σ) is constant and that the labour force grows at a constant rate. Thus if *q* is output and *k* the capital stock

$$\frac{\dot{q}}{q} = \frac{\dot{k}}{k} = \frac{1-u}{\sigma} \qquad (6)$$

$$\frac{\dot{v}}{v} = \frac{\dot{q}}{q} - (\alpha + \beta) \qquad (7)$$

Putting (6) and (7) together, we get

$$\frac{\dot{v}}{v} = [\frac{1}{\sigma} - (\alpha + \beta)] - \frac{u}{\sigma} \qquad (8)$$

Equation (8) is then an analogous equation to equation (3). But in Goodwin's model $\frac{1}{\sigma}(1-u)$ is the profit rate, and hence (6) says that growth of the capital stock always equals the real profit rate. An investment function like (6) which assumes capitalists invest all profits independently of profitability is far fetched.

A plausible alternative specification is that the rate of investment is a function of the gap between the actual rate of profit μ and its reservation rate, $\bar{\mu}$. Taking σ to be constant we can translate the profit terms μ and $\bar{\mu}$ into labour share terms u and $\bar{u}$, where now $\bar{u}$ is the maximum share of labour the capitalists would tolerate. It follows that as the profit rate μ falls towards $\bar{\mu}$, u increases towards $\bar{u}$, and as $u \to \bar{u}, \dot{k} \to -\infty$.

So an alternative to (6) with these properties can be derived as follows:

$$\frac{\dot{k}}{k} = \lambda \ln[(\bar{u} - u)/(1 - \bar{u})]$$

$$= -\lambda \ln(1 - \bar{u}) + \lambda \ln(\bar{u} - u) \quad (9)$$

where $\lambda > 0$, is the speed of adjustment, and the equation embodies the idea that the difference between the actual and desired profit rate, and hence the actual and " tolerable" labour shares, determines investment.

This then gives growth in the employment rate as

$$\frac{\dot{v}}{v} = [-\lambda \ln(1 - \bar{u}) - (\alpha + \beta)] + \lambda \ln(\bar{u} - u) \quad (10)$$

Notice that this satisfies a slightly modified version of condition (iii) since

$$\lim_{u \to \bar{u}-} \ln(\bar{u} - u) = -\infty$$

Condition (i) is also satisfied and so is (ii) provided

$$\alpha + \beta < \lambda \ln[\bar{u}/(1 - \bar{u})]$$

Typically, $\bar{u}/(1 - \bar{u}) > 1$ and hence this inequality will be satisfied for a suitable choice of $\bar{u}$.

Hence, the modified model consists of a differential equation for the share of labour (5a) underpinned by a nonlinear real wage Phillips curve (4a) and the nonlinear investment function (10). These equations satisfy the requirement that all their trajectories stay within the unit box. Indeed, since $\bar{u} < 1$, the trajectories will be confined to a smaller part of the unit box given by $[0,1] \times [0, \bar{u}]$.

3.2. Illustrating the Differences.

The aim of this section is to illustrate our main point: that for some values of the parameters, the original Goodwin model has trajectories that may not be restricted to the unit box. Our reformulation of the model ensures that they are. Hence, for the following exercise we take parameter values to illustrate this claim. It may be argued that these values are in some way "unrealistic". This is to miss the point. We argue that, as a mathematical model, it should yield interpretable dynamic solutions (i.e. solutions within the unit box) for all definable values of the parameters. It is also not an acceptable argument to say that the original model does give acceptable dynamic paths for certain values of parameters. As a mathematical model, it needs to give such solutions in general. Hence, rather than restrict solutions only to a particular set of parameter values, we show that a modest reformulation of the original, in the spirit of the original, ensures that it gives sensible solutions for all parameter values. To illustrate this we need to take values that push trajectories of the original model outside the unit box.

To illustrate the behaviour of the two alternatives, we plot trajectories for both the original Goodwin model and for our modified version of it.[8]

In particular:

[8] These use Matlab (Version 6.1.0.450 Release12.1).

(1) Figure 1 shows trajectories for the Goodwin model, ie equations (3) and (5), with $\alpha = 0.001,\ \beta = 0.001,\ \gamma = 0.95,\ \rho = 1,\ \sigma = 3.$

(2) Figure 2 shows trajectories for our modified version, ie equations (10) and (5a), with $\alpha = 0.001,\ \beta = 0.001,\ \gamma' = 1,\ \delta = 1,\ \lambda = 1,\ \rho' = 0.05,\ \bar{u} = 0.9.$

When considering these parameter values, it should be noted that the role of ρ in (1) is performed by δ and ρ' in (2) and hence that the two sets of parameter values are broadly compatible.

In both Figures 1 and 2, the trajectories are traversed in a clockwise direction. We note further that, in Figure 1, the trajectories go outside the unit box which, we have argued, clearly does not make sense. This does not happen, however, in Figure 2, illustrating that the amended model produces more acceptable economic outcomes as claimed.

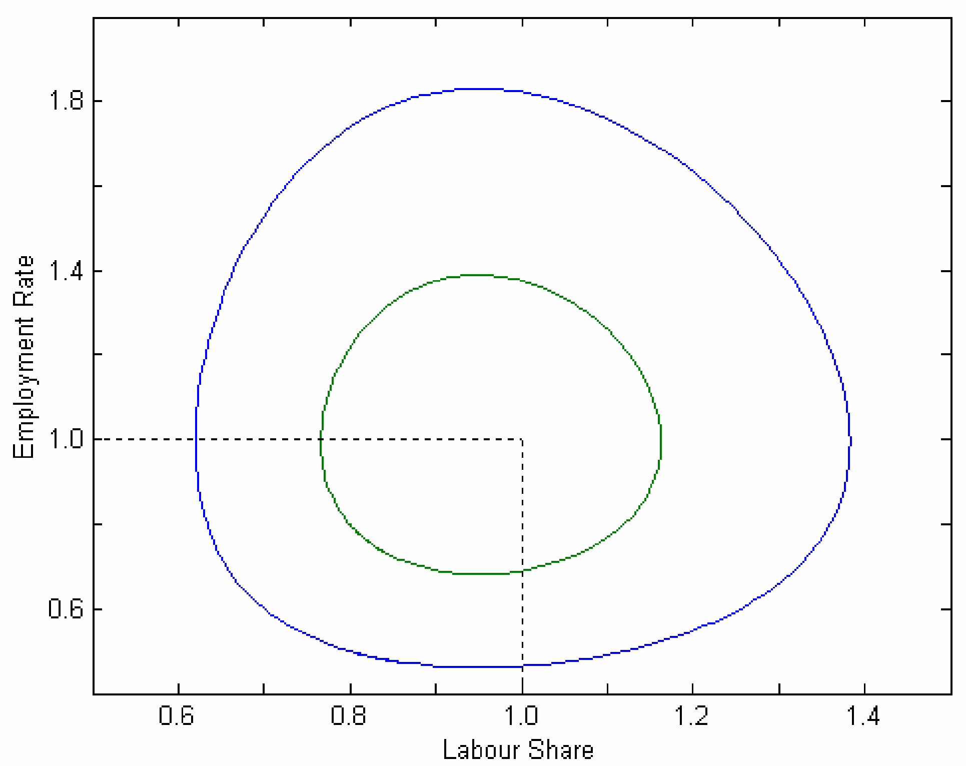

Figure 1

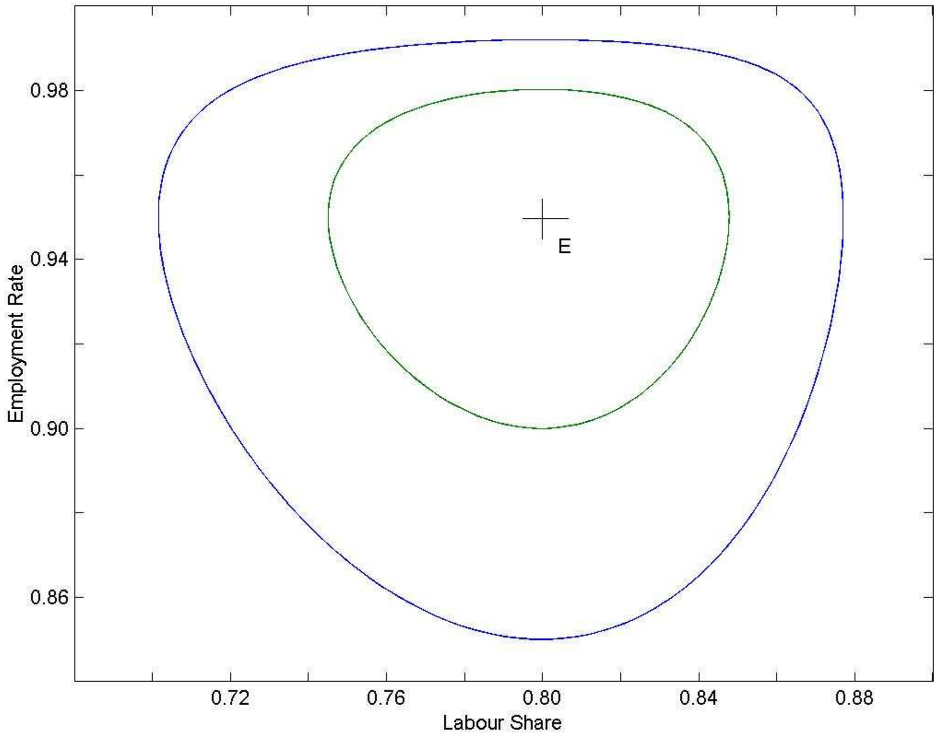

Figure 2

4. Conclusions

The influential Goodwin version of the Volterra-Lotka predator-prey model can fall foul of a basic restriction when adapting it to economic growth cycles. This restriction is simply that the endogenous variables in the model, the labour share and the proportion of the labour force employed, are bounded at unity. We have shown that ensuring this restriction is met in the model is relatively straightforward and, moreover, when it is done, the economic logic of the basic relationships is enhanced.

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