ISSN 1350-6722

University College London

DISCUSSION PAPERS IN ECONOMICS

PERSISTENT INEQUALITY WHEN LEARNING REQIRES A MINIMAL STANDARD OF LIVING

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Discussion Paper 03-09

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Persistent inequality when learning requires a minimal standard of living

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February 2003

Abstract

This paper studies the persistence of wealth and utility inequality in a dynamic model of skill acquisition with complete credit markets and rational, perfectly altruistic, dynastic utility-maximization, when efficient learning requires a minimal standard of living. The main result is that, if the minimal standard of living is not trivially small, at any stationary equilibrium without intergenerational mobility there are 'poor', unskilled and 'rich', skilled dynasties. Members of rich dynasties inherit more from their parents than members of poor dynasties. The former in general acquire skill, while the latter remain unskilled, and - most importantly - members of rich families also enjoy strictly higher utility than members of poor dynasties.

Keywords: Inequality, Skill Acquisition, Standard of Living, Bequests.

JEL Classification: D33, D91, J24, J31, J62.

1 Introduction

This paper studies the persistence of wealth and utility inequality in a dynamic model of skill acquisition with complete credit markets and rational, perfectly altruistic, dynastic utility-maximization, when efficient learning requires a minimal standard of living.

There are two occupations or tasks, both essential for the production of commodities. The first requires no skill. The second task can only be performed by sufficiently skilled individuals. Each individual lives for two periods. A person who chooses not to attend school works for both periods at unskilled wages. The acquisition of skill requires a period of exclusive education. All individuals are born uneducated and, if they decide to attend school, they are equally gifted students. Apart from skilled and unskilled labor, physical capital is the third factor of production. Every individual inherits a stock of capital from

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its perfectly altruistic parent and may leave a bequest to its offspring. Furthermore, all individuals can borrow or lend at identical interest rates.

The central assumption of the present paper is that learning is only effective if the standard of living enjoyed during the period of learning is sufficiently high. To attain skill it may therefore be necessary to deviate from the first-best consumption path, i.e. the path that would be chosen in the absence of any minimal standard of living constraint. This is more likely to be the case for poor individuals that have inherited little wealth, and therefore earn a smaller capital income. Hence, the loss in lifetime utility due to the restriction imposed by the minimal standard of living while attending school (which add to the opportunity cost of foregone earnings), are higher for the poor than for the rich.

The main result of this paper is that, if the minimal standard of living is not trivially small, at any stationary equilibrium without intergenerational mobility there are 'poor', unskilled and 'rich', skilled dynasties. Moreover, members of rich families also enjoy strictly higher lifetime utility than members of poor dynasties. Thus, at stationary equilibrium with both types of labor being active the economy inevitably exhibits persistent inequality in lifetime utility.

In the remainder of this introduction we review some findings on the possibility and the necessity of persistent inequality at stationary equilibrium in the related literature and briefly link these to the present paper.

The standard Ramsey-Cass-Koopmans-model The perfect altruism assumed in the present paper makes the Ramsey-Cass-Koopmans framework an adequate benchmark for understanding what happens when there is only one type of labor. Even in the most standard model with complete capital markets and without endogenous skill acquisition persistent inequality of wealth and utility may prevail. If the stationary equilibrium capital stock of this model is distributed in an arbitrary way among otherwise identical agents, the new wealth distribution remains a stationary equilibrium distribution. The personal wealth distribution is thus completely indeterminate in the Ramsey-Cass-Koopmans model.[1] On the one hand, this shows that the *possibility* of persistent inequality neither hinges on non-convexities nor on incomplete credit markets. On the other hand, it makes clear, that something has to be added if inequality is to be explained as the only equilibrium outcome.

[1] Chatterjee (1994) and Caselli and Ventura (2000) study the evolution of the distribution of wealth in the standard neoclassical growth model. Among others, Caselli and Ventura (2000) show that the distribution of bequeathed assets may converge or diverge when the aggregate economy is converging to a steady state, depending on how the initial (aggregate) capital intensity compares to the steady state capital intensity and depending on the elasticity of substitution between capital and labor.

Endogenous skill acquisition and perfect capital markets To explain income inequality, the literature on human capital investments attributes observed differences in individual wages to individual differences in training and/or experience (see, e.g. Neal and Rosen 2000, Freeman 1996). In a framework similar to the one of the present paper, Mincer (1958) already shows that such economies experience earnings inequality at equilibrium, even if all agents are inherently identical. If some young individuals decide to attend school before entering the labor market as skilled workers, while others decide not to do so (in our framework this must be the case at equilibrium because both occupations are essential for production), then skilled workers must earn more than unskilled workers. Otherwise nobody would be prepared to bear the (opportunity) cost of schooling. Nevertheless, in Mincer's model (without a minimal standard of living) lifetime wage incomes of all types of labor employed at stationary equilibrium are the same. The level of attained education thus neither influences wealth nor utility. As in the Ramsey-Cass-Koopmans framework, utility inequality hinges on unexplained initial wealth inequality. Mincer (1958) does not contain a full-fledged equilibrium analysis, however his results are confirmed at stationary equilibrium in our model, if we set the minimal standard of living to zero (our Proposition 10).

Endogenous skill acquisition and imperfect capital markets In contrast to Mincer (1958) or to the present paper, most articles working with endogenous skill acquisition also assume incomplete credit markets. When parents care for their children, sufficient wealth of the parents may be necessary for their children to acquire skill, if credit markets are incomplete. For example, in Loury (1981) credit markets are completely missing. In this case, the cost of deferring consumption is smaller for a rich dynasty than for a poor one, since marginal utility of consumption is smaller and thus expensive set up costs for training are rather borne by the rich (see also the excellent survey by Mookherjee and Ray (2003)). On the other hand, when interest rates differ for borrowing and lending, training costs are higher for borrowers than for lenders (see, e.g., Galor and Zeira (1993) or Aghion and Bolton (1997)). Hence, the poor will rather perform tasks requiring a small amount of training. We arrive at the same conclusion, although in our framework the role of the interest differential between borrowing and lending is now played by the minimal standard of living for efficient learning, which is more restrictive for the poor than the rich.

At this point, the question arises whether the absence of credit markets makes equilibrium inequality inevitable. In a framework in which human capital is the only asset that can be inherited, Mookherjee and Ray (2003) show that this is indeed the case. In their model skilled and unskilled labor are essential for the production of all commodities and the acquisition of skill is costly (as in the present paper). Moreover, there are no non-human capital assets and no credit market (both in contrast to the present paper). They find that all skill levels are active at any stationary equilibrium, that the children of skilled individuals acquire skill while those of the unskilled do not (no intergenerational mobility), that skilled wages exceed unskilled wages, and that skilled individuals enjoy higher (per-period) utility than unskilled. As in our framework for a sufficiently high minimal standard of living, income and utility inequality is inevitable.

However, if alternative forms of assets, such as physical capital, are introduced, this strong conclusion no longer holds (see also Mookherjee and Ray 2002). As in the standard Solow-type models with or without endogenous savings, utility inequality is possible but not inevitable. Moreover, if perfectly altruistic parents can leave non-human capital assets to their children, neither the absence of perfect capital markets nor the existence of financial set up costs for skill acquisition need jeopardize the existence of stationary equilibria in which all 'newborn' individuals (with identical capabilities and time preference) are treated equally.

Non-concave indirect lifetime utility In our framework stationary equilibria with perfect equality in bequests still exist for sufficiently low standard of living requirements. If, however, this standard of living is not sustainable for *all* individuals, no stationary equilibrium with perfect equality exists. Instead, in stationary equilibrium there are a positive number of poor and unskilled individuals and a positive number of rich and skilled individuals. Moreover, sufficiently high standard of living requirements typically generate a non-concavity in the indirect lifetime utility of an individual as a function of net transfers it receives from other generations. This non-concavity implies that in stationary equilibrium the society is partitioned into a class of poor unskilled dynasties and a class of rich skilled dynasties - despite perfect credit markets and despite the presence of non-human capital assets.

Minimal standard of living for efficient working With respect to its central assumption, the present paper is related to Baland and Ray (1991), Bliss and Stern (1978), Dasgupta and Ray (1986), Dasgupta and Ray (1987), Ray and Streufert (1993). These articles assume a minimal nutritional requirement for the effectiveness of physical labor and show how this may lead to efficiency-wage type of unemployment. In the dynamic model of Ray and Streufert (1993) a worker is only productive if he consumes a sufficient amount of calories for a sufficient length of time to attain a certain body weight. They show that depending on the initial distribution of wealth (i.e. land), inequality (i.e. unemployment) can prevail at stationary equilibrium. The assumption of a minimal nutritional requirement for the productivity of *working* is similar to our assumption of a minimal standard of living for the 'productivity' of *learning*. However, in our framework a person that once has acquired skill remains skilled thereafter, while a person that once has achieved sufficient body weight in Ray and Streufert (1993) needs an even higher calorie intake to maintain his weight when working. More importantly, while in our model all individuals are born unskilled, newborns in Ray and Streufert (1993) inherit their parents' body weight! Furthermore, while in our framework altruistic parents leave an intentional bequest, in Ray and Streufert (1993) non-altruistic parents leave an unintentional random bequest, not because they care for their children, but rather because they do not know the time of their own decease.

The remainder of the present paper is organized as follows. Section 2 presents the model. Section 3 solves the individual decision problem in several steps: first the decision problems of an agent *given* his decision whether to attend school or not are examined for a given heritage and a given bequest (Section 3.1 and Section 3.2). Section 3.3 then deals with the schooling decision, again for given heritage and given bequest. In particular, it is shown that the indirect lifetime utility function may be non-concave. Finally, Section 3.4 solves the complete individual decision problem by also incorporating the decision of how much bequest to leave. Section 4 deals with stationary equilibria without intergenerational mobility. First we show that stationary equilibria always exist (Proposition 9, Section 4.1). In fact, we show there is a continuum of two-group equilibria, which differ with respect to the distribution of wealth between skilled and unskilled dynasties. Then, in Section 4.2 we derive the main result: stationary equilibrium implies wealth and utility inequality if the minimal standard of living required for schooling is not trivially small (Theorem 11). In contrast, for low standard of living requirements, e.g. in the absence of a minimum standard of living, the minimal degree of inequality is zero. Section 4.3 deals with equilibria at which students do not consume more than managers and nobody leaves negative bequests.

2 The model

2.1 Dynasties

An individual lives for two periods. When it enters the second period, one offspring enters the economy. An infinite sequence of offsprings constitutes a dynasty. There is a mass L of dynasties such that in every period a mass of $2L$ individuals populates the economy.

In the sequel we index individuals of a given dynasty by the period of their entrance

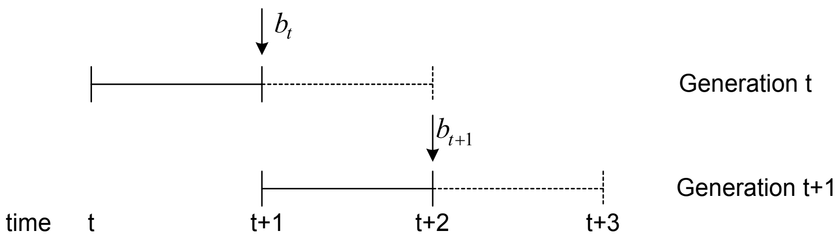

Figure 1:

into the economy. When entering the economy, generation t owns zero assets. In period $t+1$ it inherits a bequest of b_t and in period $t+2$ it leaves a bequest to generation $t+1$ of b_{t+1} (see Figure 1). The capital market is perfect such that all individuals are free to borrow or lend capital at the market interest rate r_t. There is no restriction on the sign of b_t and b_{t+1}, but lenders do not give credit to a household whose descendants will never be able to pay off their dynasty's debt (no-Ponzi condition).

Lifetime utility of generation t depends on the level of consumption in both periods t and $t+1$, i.e. of c_{0t} and c_{1t} respectively. Specifically, we assume that the utility generation t of the respective dynasty derives from its own consumption is

$$V_t = \ln c_{0t} + \overline{\delta} \ln c_{1t} \tag{1}$$

where future consumption c_{1t} is discounted by the factor $\overline{\delta} \in (0,1)$.[2] Every individual cares about the well-being of its offspring and expects its descendants to share this preference. In particular, it seeks to maximize its dynastic utility function

$$J_t = \sum_{j=t}^{\infty} \underline{\delta}^{(j-t)} V_j, \tag{2}$$

where $\underline{\delta} \in (0,1)$ denotes the intergenerational discount factor. Every individual is perfectly altruistic in the sense that its descendants' consumption is valued as is own consumption, i.e. $\underline{\delta} = \overline{\delta}$. We therefore denote both discount factors simply as δ.

When young, every individual has to decide whether to join the unskilled labor force immediately or to attend school in the first period and to work as a skilled worker (manager) in the second. School is free, but no labor income is earned while the young individual is attending school. Every worker supplies inelastically one unit of labor. All newborns share an equal ability to finish school. Schooling is however only effective if a minimum amount of consumption, $\widetilde{c}$, is maintained during the schooling period.

[2] Our results do not depend on this specific functional form. What we need to derive our results is that the indirect lifetime utility functions are concave in lifetime consumption expenditures and satisfy certain conditions such that the transversality condition (20) is indeed necessary for an optimal sequence of bequests.

2.2 Production

There are three factors of production: skilled labor S, unskilled labor U and physical capital K. U_t denotes the mass of unskilled workers and S_t the mass of skilled workers at period t. Denote the wages paid for both types of labor as $w_{U,t}$ and $w_{S,t}$. The production function $F(S,U,K)$ of the economy is Cobb-Douglas:[3]

$$F(S,U,K) = S^{\alpha}U^{\beta}K^{1-\alpha-\beta},\ \alpha,\beta > 0,\ \alpha+\beta < 1. \tag{3}$$

Define $u := U/K$ and $s := S/K$. The economy is in short-run equilibrium in every period, i.e. all markets clear and $\frac{\partial F}{\partial S}(s_t,u_t,1) = w_{S,t}$, $\frac{\partial F}{\partial U}(s_t,u_t,1) = w_{U,t}$, and $\frac{\partial F}{\partial K}(s_t,u_t,1) = r_t$ for all periods t.

2.3 Stationary equilibrium

The present paper studies the existence and the properties of stationary equilibria with rational expectations and zero intergenerational mobility in educational attainment. A stationary equilibrium is defined as a sequence of short-run equilibria with constant (u_t, s_t) at which all generations of all dynasties have perfect foresight and choose the same profession as their ancestors did. In stationary equilibrium, therefore, all factor prices are constant and are denoted by w_S, w_U, and r. In particular, we obtain

$$w_S = \alpha s^{\alpha-1}u^{\beta} \tag{4}$$

$$w_U = \beta s^{\alpha}u^{\beta-1} \tag{5}$$

Let $\overline{w}_S$ and $\overline{w}_U$ denote lifetime labor incomes earned by skilled and unskilled labor, respectively, i.e. $\overline{w}_S := w_S$ and $\overline{w}_U := (2+r)\,w_U$. Since education imposes an additional restriction on lifetime consumption, no individual attends school if $\overline{w}_S < \overline{w}_U$. Further, if the wage gap $\overline{w}_S - \overline{w}_U$ is sufficiently large such that the minimal consumption $\widetilde{c}$ during the schooling period can be financed by this difference in lifetime labor earnings, i.e. if $\overline{w}_S - \overline{w}_U \geq (1+r)\,\widetilde{c}$, it is profitable for *all* individuals—independent of their bequeathed wealth—to attend school. Neither of the inequalities can hold in stationary equilibrium since both types of labor are necessary to produce the final output. For later reference we summarize these first results as a lemma:

Lemma 1 *In stationary equilibrium* $(u,s) \gg 0$ *and* $0 \leq \overline{w}_S - \overline{w}_U < (1+r)\,\widetilde{c}$.

[3] We use the Cobb-Douglas case to guarantee that both skilled and unskilled labor are employed all the time. Note that we could also derive most results under more general Inada-like conditions.

3 Solving the individual problem

We solve the individual problem in two steps. First, for a given heritage b_t and a *given* bequest b_{t+1}, the optimal consumption path is determined both in the case that an individual spends his life working as an unskilled worker (unskilled worker's problem, Section 3.1) and, separately, in the case that an individual decides to attend school (student-manager problem, Section 3.2). We derive the maximal lifetime utility of the respective individual for a given bequest from the solution to both problems. In a second step we characterize the sequence of bequests $\langle b_{t+1}, b_{t+2}, ...\rangle$ that maximizes dynastic utility J_t.

3.1 The unskilled worker's problem

Suppose generation t of a given dynasty decided to work as an unskilled worker in both periods of its life. Then it earns a labor income of w_U in both periods. In the first period it consumes c_{0t}^U and in the second period individual t receives a bequest of b_t (see Figure 1). Thus its second period assets a_t^U amount to

$$a_t^U = b_t + w_U - c_{0t}^U. \tag{6}$$

Depending on its second-period consumption c_{1t}^U, individual t makes a bequest b_{t+1} of

$$b_{t+1} = (1+r)\, a_t^U + w_U - c_{1t}^U. \tag{7}$$

Insertion of (6) into (7) yields the intertemporal budget constraint of individual t:

$$c_{1t}^U + (1+r)\, c_{0t}^U = (1+r)\, b_t - b_{t+1} + \overline{w}_U \tag{8}$$

The discounted value of consumption in both periods, $c_{1t}^U + (1+r)\, c_{0t}^U$, must equal the sum of net transfers, $(1+r)\, b_t - b_{t+1}$, and discounted total labor earnings $\overline{w}_U$. Since consumption in both periods is restricted to be non-negative, net transfers must not be too small such as to render this sum negative. In turn, whenever $(1+r)\, b_t - b_{t+1} + \overline{w}_U \geq 0$ the problem of choosing a consumption profile $\left(c_{0t}^U, c_{1t}^U\right)$ so as to maximize lifetime utility V_t subject to the budget constraint (8) has the following unique solution:

$$c_{0t}^U = \frac{1}{(1+\delta)(1+r)}\left((1+r)\, b_t - b_{t+1} + \overline{w}_U\right) \tag{9a}$$

$$c_{1t}^U = \frac{\delta}{1+\delta}\left((1+r)\, b_t - b_{t+1} + \overline{w}_U\right). \tag{9b}$$

3.2 The student-manager problem

Now let us assume that generation t of a given dynasty decided to attend school in the first period. Then its assets in the second period amount only to

$$a_t^S = b_t - c_{0t}^S \tag{10}$$

since no labor income is earned while attending school. Hence, if in the first period generation t attends school and consumes c_{0t}^S and in the second period it consumes c_{1t}^S, the intertemporal budget constraint it has to respect is

$$c_{1t}^S + (1+r)\, c_{0t}^S = (1+r)\, b_t - b_{t+1} + \overline{w}_S. \tag{11}$$

Moreover, working as a manager in the second period is feasible only if first-period consumption c_{0t}^S is not smaller than the consumption constraint $\widetilde{c}$. Thus, lifetime expenditures $(1+r)\, b_t - b_{t+1} + \overline{w}_S$ must not be smaller than $(1+r)\,\widetilde{c}$ in order to ensure that there is a solution to the student-manager problem

$$\begin{cases} \max_{\left(c_{0t}^S, c_{1t}^S\right)} \ln c_{0t}^S + \delta \ln c_{1t}^S \\ c_{1t}^S + (1+r)\, c_{0t}^S = (1+r)\, b_t - b_{t+1} + \overline{w}_S \\ c_{1t}^S \geq 0,\ c_{0t}^S \geq \widetilde{c}. \end{cases} \tag{12}$$

For sufficiently large lifetime expenditures the consumption constraint $c_{0t}^S \geq \widetilde{c}$ does not bind. Then the solution to the above student-manager problem closely resembles the solution to the unskilled worker's problem: one only has to substitute $\overline{w}_S$ for $\overline{w}_U$ in the expressions (9a) and (9b) to obtain the explicit solution for the optimal level of consumption in both periods.

Whenever $\widetilde{c} > \frac{1}{(1+\delta)(1+r)} \left((1+r)\, b_t - b_{t+1} + \overline{w}_S\right)$, however, the consumption constraint binds. In this case generation t consumes $c_{0t}^S = \widetilde{c}$ in the first and

$$c_{1t}^S = (1+r)\, b_t - b_{t+1} + \overline{w}_S - (1+r)\,\widetilde{c} \tag{13}$$

in the second period such as to be able to make a bequest of b_{t+1} to its offspring.

3.3 Lifetime utility for a given bequest

We want to determine whether, for given bequests b_t, b_{t+1} and a given vector of factor prices (w_S, w_U, r), it is profitable for generation t of a given dynasty to attend school or to work as an unskilled worker. Let T_t denote the net transfers generation t receives if it inherits a bequest b_t and leaves a bequest b_{t+1}:

$$T_t = (1+r)\, b_t - b_{t+1}. \tag{14}$$

Then $T_t + \overline{w}_S$ is the discounted value of consumption in periods t and $t+1$ (measured in goods of period $t+1$) when studying in the first period and working as a manager in the second. Similarly, $T_t + \overline{w}_U$ is the discounted value of total consumption in both periods when working as an unskilled worker.

Plugging the definition of net transfers T_t into the intertemporal budget constraint of a student-manager, (11), we see that generation t can afford the minimal lifetime consumption expenditure $(1+r)\,\widetilde{c}$ only if T_t is not smaller than

$$T^{S,\min} = (1+r)\,\widetilde{c} - \overline{w}_S. \tag{15}$$

Equivalently, there is a solution to the unskilled worker's problem only if T_t is not smaller than

$$T^{U,\min} = -\overline{w}_U. \tag{16}$$

It follows from Lemma 1 that only if $T^{U,\min} < T^{S,\min}$ some people may choose to attend school while others work as unskilled workers. Further, there is a minimal T_t, say $\widetilde{T}$, that just suffices for an unconstraint consumption profile of a student-manager. Insertion of (14) into the expression for the first-best level of consumption in the student period,

$$c_{0t}^S = \frac{T_t + \overline{w}_S}{(1+\delta)\,(1+r)},$$

and using $c_{0t}^S = \widetilde{c}$ yields

$$\widetilde{T} = (1+\delta)\,(1+r)\,\widetilde{c} - \overline{w}_S > T^{S,\min}. \tag{17}$$

If and only if $T_t \geq \widetilde{T}$ the consumption profile of a student-manager is unaffected by the consumption constraint $\widetilde{c}$.

For every net transfer T_t we want to determine whether the maximal lifetime utility is greater for an unskilled worker or for a student-manager. To this end, with a slight abuse of notation, let $V^U(T)$ denote the indirect lifetime utility function when working as an unskilled worker in both periods. Since there is a solution to the unskilled worker's problem only if $T_t \geq T^{U,\min}$, we will speak of $T \geq T^{U,\min}$ as the relevant domain of $V^U(T)$ and, for convenience, define $V^U(T) = -\infty$ for all $T < T^{U,\min}$. Analogously, $V^S(T_t)$ denotes the indirect lifetime utility function of a skilled worker and we define $V^S(T) = -\infty$ for all $T < T^{S,\min}$. The following lemma, which is proved in Appendix 1, summarizes important properties of $V^U(\cdot)$ and $V^S(\cdot)$ (see also Figure 2):

Lemma 2 *The indirect lifetime utility function of an unskilled (skilled) worker, $V^U(T)$ (respectively $V^S(T)$), is continuous, increasing and strictly concave with respect to net transfers T in its relevant domain $T \geq T^{U,\min}$ (respectively $T \geq T^{S,\min}$).*

Next, define $V(\cdot)$ as the indirect lifetime utility function with respect to net transfers (see Figure 2):

$$V(T) = \max\left\{V^U(T), V^S(T)\right\} \text{ for every } T \in \mathbb{R}.$$

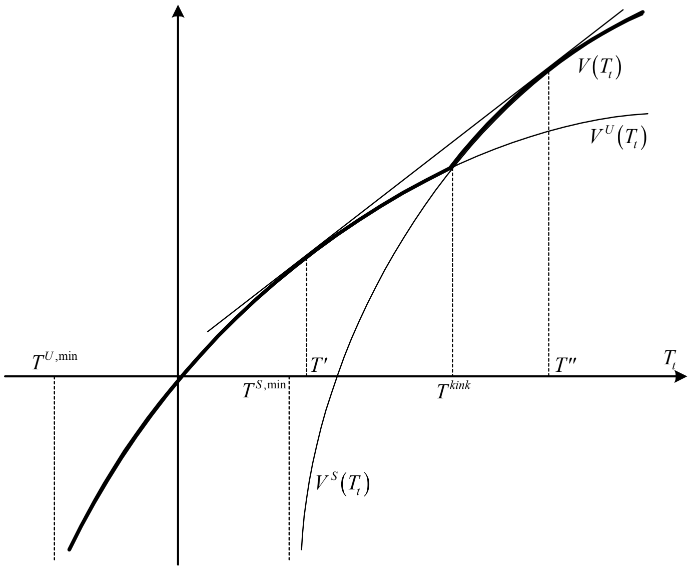

Figure 2:

The following two lemmas 3 and 4 summarize how indirect utility $V(T)$ depends on the wage gap $\overline{w}_S - \overline{w}_U$ (which determines whether $T^{S,\min}$ is greater, smaller, or equal to $T^{U,\min}$) and on net transfers T. Only the two cases considered in Lemma 4, $\overline{w}_S - \overline{w}_U = 0$ and $\overline{w}_S - \overline{w}_U \in (0, (1+r)\,\widetilde{c})$, can occur at stationary equilibrium, as we will see below. For completeness, Lemma 3 also describes the two cases, $\overline{w}_S - \overline{w}_U \geq (1+r)\,\widetilde{c}$ and $\overline{w}_S - \overline{w}_U < 0$, which cannot occur at stationary equilibrium (see Lemma 1). We say the function $V^S(\cdot)$ dominates $V^U(\cdot)$ (and vice versa) if $V^S(T) > V^U(T)$ for every T with $V^U(T) > -\infty$. In stationary equilibrium both types of labor are employed. Hence, neither $V^S(\cdot)$ nor $V^U(\cdot)$ can be dominant.

Lemma 3

- ($\overline{w}_S - \overline{w}_U \geq (1+r)\,\widetilde{c}$) *Suppose the wage gap, $\overline{w}_S - \overline{w}_U$, is not smaller than the expenditures necessary to consume $\widetilde{c}$ in the first period of life. Then $T^{S,\min} \leq T^{U,\min}$ and $V^S(\cdot)$ dominates $V^U(\cdot)$.*
- ($\overline{w}_S - \overline{w}_U < 0$) *If the wage gap is negative, then $T^{U,\min} < T^{S,\min}$ and $V^U(\cdot)$ dominates $V^S(\cdot)$.*

Proof. See Appendix 2. ■

We now turn to the two cases that may occur at stationary equilibrium. If the wage gap is zero, for net transfers not smaller than $\widetilde{T}$ the minimal standard of living requirement does not bind and therefore for every $T_t \geq \widetilde{T}$ working as a skilled worker leaves generation

t indifferent to working as an unskilled worker. For all feasible net transfers smaller than $\widetilde{T}$ the consumption constraint binds and hence generation t does not attend school if $T_t < \widetilde{T}$:

$$V(T) = \begin{cases} V^U(T) \\ V^U(T) = V^S(T) \end{cases} \text{if} \begin{cases} T \leq \widetilde{T} \\ T \geq \widetilde{T} \end{cases}.$$

On its relevant domain, $T \geq T^{U,\min}$, the indirect utility function $V(T)$ is strictly concave in T because $V^U(T)$ is.

Now consider the remaining case that the wage gap is strictly positive but still within the bounds stated in Lemma 1. Then lifetime utility when attending school is strictly greater than lifetime utility when working as an unskilled worker, if the consumption constraint does not bind. This is the case for all net transfers not smaller than $\widetilde{T}$. Furthermore, if the minimal standard of living is attainable only at the expense of extremely low consumption during the manager period, then working as an unskilled worker in both periods is preferred to being a student-manager, since working as an unskilled worker allows for a perfectly smooth consumption path. That is, $V^U(T) > V^S(T)$ for $T > T^{S,\min}$ sufficiently close to $T^{S,\min}$. Hence, for a positive but not too large wage gap both indirect lifetime utility functions $V^S(\cdot)$ and $V^U(\cdot)$ cross at least once. However, the information on $V^S(\cdot)$ and $V^U(\cdot)$ provided by Lemma 2 is not sufficient to guarantee that both functions do not cross more than once. As shown in the proof of the following Lemma 4, the decision to attend school is indeed monotonous in net transfers. Thus, for $0 < \overline{w}_S - \overline{w}_U < (1+r)\widetilde{c}$ the indirect lifetime utility function $V(\cdot)$ is

$$V(T) = \begin{cases} V^U(T) \\ V^S(T) \end{cases} \text{if} \begin{cases} T \leq T^{\text{kink}} \\ T \geq T^{\text{kink}} \end{cases}.$$

Figure 2 depicts this case. In particular, it shows that $V(T)$ is non-concave with respect to net transfers on its relevant domain. The following lemma summarizes how $V(\cdot)$ depends on the wage gap $\overline{w}_S - \overline{w}_U$ if this wage gap respects the bounds stated in Lemma 1.

Lemma 4

- *($\overline{w}_S - \overline{w}_U = 0$) Suppose the wage gap is zero. Then, $T^{U,\min} < T^{S,\min}$, $V(T) = V^U(T) > V^S(T)$ for all $T^{U,\min} < T \leq \widetilde{T}$, and $V(T) = V^U(T) = V^S(T)$ for all $T \geq \widetilde{T}$.*

- *($\overline{w}_S - \overline{w}_U \in (0, (1+r)\widetilde{c})$) Suppose the wage gap is positive and within the bounds of Lemma 1. Then there is a unique T^{kink} such that working as a skilled worker leaves an individual t indifferent to working as an unskilled worker, if $T_t = T^{kink}$. For all $T_t \in [T^{U,\min}, T^{kink})$ individual t does not attend school but instead works as an unskilled worker. For all $T_t > T^{kink}$ it does attend school.*

Proof. See Appendix 3. ■

3.4 The optimal bequest

The optimal bequest of generation $t = 0$ is obtained by determining a feasible sequence of bequests, and hence of net transfers T, that maximizes J_0. As shown above, at stationary equilibrium the bound on net transfers $T^{U,\min}$ must be smaller than the bound $T^{S,\min}$. Thus, we are looking for a solution for

$$\begin{cases} \sup_{\langle T_t\rangle_{t=0}^{\infty}} \Sigma_{t=0}^{\infty}\delta^t V(T_t) \\ \text{s.t. } b_{t+1} + T_t = (1+r)\, b_t \\ b_0 \text{ given, } b_t \geq b^{\text{no-Ponzi}},\ T_t \geq T^{U,\min}\ \forall t \geq 0. \end{cases} \tag{18}$$

Since we have assumed that a no-Ponzi condition holds, i.e. that the debt of no generation exceeds $b^{\text{no-Ponzi}} := -\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^t \overline{w}_U = -\overline{w}_U/r$,[4] the set of feasible bequests is compact for every generation. An optimal path $\langle T_0, T_1, ...\rangle$ exists and must satisfy both the Euler equation

$$V'(T_t) = \delta(1+r)\, V'(T_{t+1}) \tag{19}$$

for all $t \geq 0$ and a transversality condition

$$\overline{\lim_{t\to\infty}}\, \delta^t V'(T_t)\, b_{t+1} \leq 0 \tag{20}$$

as Ekeland and Scheinkman (1986) have shown.

If $1/(1+r)$ is constant δ, it follows from (19) that the sequence $\langle V'(T_t)\rangle_{t\geq 0}$ is stationary. If the factor used to discount future assets, $1/(1+r)$, is not equal to the factor used to discount future utility, δ,[5] this sequence increases at a constant rate $\delta(1+r) - 1$. This in turn implies the following lemma:

Lemma 5 *If $1/(1+r)$ is constant δ, the sequence $\langle V'(T_t)\rangle_{t\geq 0}$ is stationary. Assume $0 < \overline{w}_S - \overline{w}_U < (1+r)\,\widetilde{c}$. Then all members of the dynasty work as unskilled workers at sufficiently large t if $1/(1+r) < \delta$ and as skilled workers if $1/(1+r) > \delta$.*

As one may already expect, in stationary equilibrium $1/(1+r)$ will be equal to δ and therefore the sequence $\langle V'(T_t)\rangle_t$ will be stationary for every dynasty. This does not necessarily imply that, for $\delta(1+r) = 1$, a stationary sequence of net transfers, i.e.

[4] $b^{\text{no-Ponzi}}$ is the discounted value of all descandants' labor income if they all were to choose $T = T^{U,\min}$. Note the fact that $b^{\text{no-Ponzi}}$ is a function of the wage for unskilled labor only, follows because of $T^{U,\min} < T^{S,\min}$ which must hold in stationary equilibrium.

[5] As becomes obvious from (18) and (19), stationarity of marginal indirect lifetime utility $V'(T)$ requires that $1 + r = 1/\underline{\delta}$. In turn, which interest rate prevails at stationary equilibrium crucially depends on the rate $\underline{\delta}$ which is used to discount lifetime utility of future generations.

$\langle T_0, T_0, T_0, ...\rangle$, and thus a stationary sequence of bequests is the optimal response to *any* initial heritage b_0 since $V(\cdot)$ may be non-concave. If the maximal lifetime utility function $V(\cdot)$ *is* concave on its relevant domain (which is true for zero wage gaps), if $\delta(1+r)=1$ and the no-Ponzi condition is respected, then it is standard to show that a stationary sequence of bequests is indeed the optimal response to *every* heritage $b_0 \geq b^{\text{no-Ponzi}}$.

Next, assume that the wage gap is positive and within the bounds of Lemma 1 and, again, that $\delta(1+r) = 1$. Then, as shown above, the maximal lifetime utility $V(\cdot)$ is not concave on its relevant domain. In this case, studying the evolution of an optimal sequence of bequests for any initial bequest $b_0 \geq b^{\text{no-Ponzi}}$ is in general a difficult task. However, restricting the analyses of initial bequests to bequests within certain subintervals of $\left[b^{\text{no-Ponzi}}, \infty\right)$, we can show that for such initial bequests stationary sequences of bequests indeed maximize dynastic utility.[6] To see this, assume individual $t = 0$ expects to inherit $b_0 \geq b^{\text{no-Ponzi}}$ and intends to make a bequest of $b_1 = b_0$. Let T_0^{stat} denote the corresponding net transfer T_0:

$$T_0^{\text{stat}} = rb_0. \tag{21}$$

The choice of T_0^{stat} is obviously feasible and satisfies the Euler equation, the transversality and the no-Ponzi condition.

Now, let $V^{\text{convex hull}}(\cdot)$ denote the convex hull of the indirect utility function $V(\cdot)$ on its relevant domain.[7] T' and T'' are used to denote those levels of net transfers within the relevant interval of $V(\cdot)$ that bound the interval on which $V(T) \neq V^{\text{convex hull}}(T)$ (see Figure 2).[8] In Appendix 4 we prove that for all T_0^{stat} within the interval $\left[T^{U,\min}, T'\right]$ and all $T_0^{\text{stat}} \geq T''$ dynastic utility is maximized if a sequence of stationary bequests is chosen. The proof uses the fact that for those levels of bequests every point on the secant, connecting two points on $V(\cdot)$, does not lie above $V\left(T_0^{\text{stat}}\right)$. Summarizing:

Lemma 6 *Suppose* $\delta(1+r) = 1$. *For* $0 < \overline{w}_S - \overline{w}_U < (1+r)\widetilde{c}$ *the choice of* $b_1 = b_0$ *is optimal if* $T^{U,\min} \leq T_0^{stat} \leq T'$ *or* $T_0^{stat} \geq T''$. *If* $\overline{w}_S = \overline{w}_U$, *then it is always optimal to choose* $b_1 = b_0$.

Finally, note that, while it is an intricate question whether T_0^{stat} or rather a switching sequence is optimal for a dynasty if T_0^{stat} is in the interval (T', T''),[9] a switching sequence,

[6] For certain parameter restrictions a more complete description of the optimal sequence of bequests is possible (see note 9).

[7] More exactly, $V^{\text{convex hull}}(T)$ is the smallest concave function on the interval $\left(\min\left\{T^{U,\min}, T^{S,\min}\right\}, \infty\right)$ with $V^{\text{convex hull}}(T) \geq V(T)$ for every T within this interval.

[8] $V(T) \neq V^{\text{convex hull}}(T)$ if and only if $T \in (T', T'')$ with $T' < T''$.

[9] From the Euler equation it is obvious that a dynasty may switch only between two distinct values of T, say $\underline{T}$ and $\overline{T}$ (with $\underline{T} < \overline{T}$). A switching equilibrium satisfies the transversality and the no-Ponzi condition

of course, cannot occur at stationary equilibrium. This in turn implies that at stationary equilibrium also individual lifetime utility is stationary.

To summarize the results on the individual problem obtained so far, we want to introduce those levels of stationary bequests that correspond to $T^{S,\min}$, T', T^{kink}, T'', $\hat{T}$, and $\widetilde{T}$ as defined above. Refer to those bequests as $b^{S,\min}$, b', b^{kink}, b'', $\hat{b}$, and $\widetilde{b}$ (for example, $b' = T'/r$ and so forth). Then, we can characterize consumption, schooling, and bequests for the case $(1+r)\,\delta = 1$ (which will be satisfied in any stationary equilibrium) as follows:

Lemma 7 *Suppose $(1+r)\,\delta = 1$ and $0 < \overline{w}_S - \overline{w}_U < (1+r)\,\widetilde{c}$. Then it is optimal for an individual of generation $t = 0$ that inherits a bequest of $b_0 \geq b^{no\text{-}Ponzi}$ to make a bequest of the same amount it inherited itself (i.e. $b_1 = b_0$) if $b_0 \leq b'$ or $b_0 \geq b''$. In this case every generation of the respective dynasty attains the same level of education. If $b_0 \leq b'$ no generation of the respective dynasty attends school and all generations $t \geq 0$ consume*

$$c_{0t}^U = c_{1t}^U = \frac{1}{2+r}\left(rb_0 + \overline{w}_U\right) < \widetilde{c}.$$

If $b'' \leq b_0 < \widetilde{b}$, every generation attends school and, while attending school, consumes $\widetilde{c}$; whereas while working as a manager it consumes

$$c_{1t}^S = rb_0 + \overline{w}_S - (1+r)\,\widetilde{c}.$$

If $b_0 \geq \widetilde{b}$, every generation attends school and consumes in both periods

$$c_{0t}^S = c_{1t}^S = \frac{1}{2+r}\left(rb_0 + \overline{w}_S\right) \geq \widetilde{c}.$$

If $(1+r)\,\delta = 1$ but the wage gap $\overline{w}_S - \overline{w}_U$ is zero, then it is optimal to leave a bequest of $b_1 = b_0$. Further, the individual unambiguously works as an unskilled worker if the heritage b_0 is smaller than $\widetilde{b}$. If $b_0 \geq \widetilde{b}$, the individual is indifferent in his choice to attend school or to work as an unskilled worker.

Proof. See Appendix 5. ■

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only if $\underline{T} < T_0^{\text{stat}} < \overline{T}$. We can show that the lifetime utility of a dynasty switching (not necessarily periodically) between $\underline{T}$ and $\overline{T}$ is $\frac{1}{1-\delta}\left(V\left(\overline{T}\right) - \left(\overline{T} - T_0^{\text{stat}}\right)\frac{V(\overline{T})-V(\underline{T})}{\overline{T}-\underline{T}}\right)$. If for some generation j T_j^{stat} was not in (T', T''), then all generations $i \geq j$ would chose T_j^{stat}. This would violate the Euler equation if $T_j^{\text{stat}} \neq \underline{T}$ and $T_j^{\text{stat}} \neq \overline{T}$. Certainly, if there was a sequence such that $\underline{T} = T'$ and $\overline{T} = T''$ and all the conditions mentioned were satisfied, then lifetime utility is maximized, as is also obvious from Figure 2. We can show that for $\delta > 1/2$, i.e. if the time preference is low and people care a lot for their progeny, such a sequence exists for all $T_0^{\text{stat}} \in (T', T'')$. In case of a strong time preference such sequences still exist but they are not dense in the interval (T', T'').

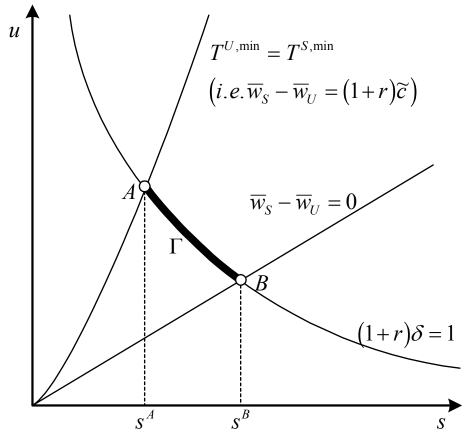

Figure 3:

4 Stationary Equilibrium

In stationary equilibrium the following two conditions are necessarily satisfied: first, since both income and the stock of physical capital are constant and capital does not depreciate, the interest rate r must satisfy $(1+r)\,\delta = 1$. Using Lemma 5, we prove the following Lemma in Appendix 6:

Lemma 8 *In every stationary equilibrium all individuals leave a bequest that equals their own inherited wealth.*

Furthermore, since in equilibrium factor prices satisfy the marginal productivity condition, the following relation between u and s is derived from (3) for $(1+r)\,\delta = 1$ (see also Figure 3):

$$u\,(s) = \left(\frac{1-\delta}{\delta}\frac{1}{(1-\alpha-\beta)\,s^{\alpha}}\right)^{1/\beta}. \tag{22}$$

Second, the difference in lifetime labor income, $\overline{w}_S - \overline{w}_U$, must satisfy the inequalities of Lemma 1 since both types of labor are employed in stationary equilibrium. For any given s (respectively u) exactly one u (respectively s) exists, such that $\overline{w}_S - \overline{w}_U$ equals a given constant. Moreover, the loci on which $\overline{w}_S - \overline{w}_U$ is constant strictly increase in s (respectively u). If (u, s) were below the $\overline{w}_S - \overline{w}_U = 0$ locus nobody would attend school. If (u, s) were on or above the $T^{U,\min} = T^{S,\min}$ locus, i.e. on or above the locus on which $\overline{w}_S - \overline{w}_U = (1+r)\,\widetilde{c}$ holds, everybody would prefer schooling. Let A (respectively B) denote the intersection of the $T^{U,\min} = T^{S,\min}$ (respectively $\overline{w}_S - \overline{w}_U = 0$) locus with the $(1+r)\,\delta = 1$ locus. Define s^A by $A = \left(u\left(s^A\right), s^A\right)$ and, respectively, s^B by $B = \left(u\left(s^B\right), s^B\right)$. In Figure 3 the set Γ, satisfying both $0 < \overline{w}_S - \overline{w}_U < (1+r)\,\widetilde{c}$ and $(1+r)\,\delta = 1$, is emphasized.[10]

[10]Note that both A and B are not within the set Γ.

4.1 Existence

To show that stationary equilibria exist for all parameter specifications, we concentrate on two-group stationary equilibria with stationary bequest distributions. For each level of bequest a bequest distribution determines the proportion of individuals in their second period of life who inherit this bequest. Thus the fraction of these individuals who attend school in their first period of life is stationary at a stationary bequest distribution if within every dynasty the young individuals with identical expected bequests always make identical decisions, as is the case in a stationary equilibrium. In a two-group stationary equilibrium there is one group of identical dynasties with members that inherit a low level of wealth b_U and do not attend school and another group of identical dynasties with members that inherit a high level of wealth b_S and attend school.[11] We will speak of 'poor (or unskilled) dynasties' and of 'rich (or skilled) dynasties' respectively.

We proceed in two steps. In Step 1 we derive a necessary condition for an equilibrium of the asset market. In Step 2 we check whether for a given $(u, s) \in \Gamma$ a bequest distribution exists that clears the asset market and also satisfies the restrictions stated in Lemma 7.

Step 1: Denote the average wealth of poor dynasties by $\overline{a}^U$ and the average wealth of rich dynasties by $\overline{a}^S$. At stationary equilibrium every poor dynasty supplies two units of unskilled labor in every period. Respectively, every rich dynasty supplies only one unit of skilled labor because the young individual of a rich dynasty attends school. Then the clearing of the asset market requires that the $U/2$ dynasties supplying unskilled work and the S households which work as managers or attend school own the stock of physical capital K:

$$K = \overline{a}^U \frac{U}{2} + \overline{a}^S S. \tag{23}$$

Average wealth $\overline{a}^U$ crucially depends on the bequest b_U, while $\overline{a}^S$ depend on b_S. To see exactly how, note that in every period the total stock of assets of a dynasty equals the wealth of the 'old' individual of that dynasty because in the first period of life no individual

[11] As should become clear, the restriction on two-group equilibria is made simply for expositional convenience. In the following, an equilibrium of the asset market imposes certain restrictions on the *average* wealth of both classes and, in case the equilibrium factor intensities (u, s) are within the set Γ, on individual bequests (see condition 25). For two-group equilibria the average wealth of the class of rich and dynasties coincides with the wealth of every single rich dynasty. The same is true for poor dynasties. However, all our results remain valid for bequest distributions with the following two properties: first, *average* bequests of skilled and unskilled dynasties (b_S and b_U respectively) must result in asset market clearing levels of average wealth. Second, all individual bequests must respect the bounds stated in (25) if the wage gap at stationary equilibrium is positive, i.e. if equilibrium factor intensities (u, s) are within the set Γ, or, for a zero wage gap, all individual bequests must not be smaller than $b^{\text{no-Ponzi}}$.

owns any assets. That is, at any period of time t it follows from (6) and (10) that

$$\begin{aligned}\overline{a}^U &= a^U_{t-1} = b_{t-1} + w_U - c^U_{0t-1}\\ \overline{a}^S &= a^S_{t-1} = b_{t-1} - c^S_{0t-1}.\end{aligned}$$

At stationary equilibrium $(1+r)\,\delta = 1$ and for every dynasty the optimal sequence of bequests is stationary. Thus, from (9a) and the fact that individuals from rich dynasties consume $\widetilde{c}$ whenever their bequest b_S is smaller than $\widetilde{b}$ we infer on the following relation between average levels of wealth of rich and poor dynasties and their respective levels of bequests:

$$\overline{a}^U = \frac{2}{2+r} b_U \tag{24a}$$

$$\overline{a}^S \left\{\begin{array}{c}<\\=\end{array}\right\} \frac{1}{2+r}\left(2b_S - \overline{w}_S\right) \text{ if and only if } b_S \left\{\begin{array}{c}<\\=\end{array}\right\} \widetilde{b}. \tag{24b}$$

If the consumption constraint binds, the first-period consumption of an individual from a rich dynasty exceeds its first-best consumption in that period and therefore its wealth in the second period wealth is smaller than $(2b_S - \overline{w}_S)/(2+r)$ if it inherits less than $\widetilde{b}$. Note that the average wealth of any dynasty is smaller than the level of the bequest individuals of this dynasty receive and make themselves.

Step 2: Assume the poor inherit b_U, the rich b_S, and the wage gap satisfies $0 < \overline{w}_S - \overline{w}_U < (1+r)\,\widetilde{c}$. For $b^{\text{no-Ponzi}} \leq b_U \leq b'$ and $b'' \leq b_S$, we infer from Lemma 7 that at stationary equilibrium all poor (rich) individuals work as unskilled (skilled) workers, as their ancestors did, and leave a bequest of b_U (b_S). Using (23), (24), and the fact that by definition $u = U/K$ and $s = S/K$, we see that a stationary equilibrium exists if for some $(u, s) \in \Gamma$ there is a (b_U, b_S) satisfying

$$\left\{\begin{array}{l}\overline{a}^U \frac{u}{2} + \overline{a}^S s = 1\\ b^{\text{no-Ponzi}} \leq b_U \leq b' < b'' \leq b_S\end{array}\right. . \tag{25}$$

To prove the existence proceed as follows: pick any $(u, s) \in \Gamma$. To these factor intensities correspond factor prices (w_S, w_U, r), which in turn determine the critical levels of bequest $b^{\text{no-Ponzi}}$, b', and b''. Now set $b_U = b^{\text{no-Ponzi}} < 0$. We see from (24a) that the average wealth of poor dynasties $\overline{a}^U$ is equal to $2b^{\text{no-Ponzi}}/(2+r)$ and hence also negative. Equilibrium of the asset market requires that $\overline{a}^S = \left(1 - \overline{a}^U u/2\right)/s$ and, hence, that the average wealth of the rich $\overline{a}^S$ is positive. The average wealth $\overline{a}^S$ in turn corresponds to a unique level of bequests b_S which must also be positive since, as mentioned before, $b_S > \overline{a}^S$. If we can show that this b_S is not smaller than b'', the chosen (u, s) are the factor intensities of a stationary equilibrium with a stationary bequests of b_U and b_S. We therefore proceed in the appendix by proving that for all (u, s) in a connected subset of Γ the level of bequest b'' must be negative and thus, since $b_S > 0$, for these (u, s) the bequest of the rich b_S must indeed be greater than b''. As argued in note 9 the restriction on two-group equilibria of the following proposition could be relaxed quite easily, however at the cost of notational convenience.

Proposition 9 *For all parameter specifications of the model and all s in a non-empty interval there is a (unique) $u(s)$ such that $(u(s), s)$ are the factor intensities of infinitely many two-group stationary equilibria. These equilibria differ with respect to the distribution of total wealth between the rich and the poor group.*

Proof. See Appendix 7. ■

4.2 Inequality in stationary equilibrium

To compare our findings with those of the standard model (e.g. Mincer 1958), assume, as a polar case, that there is no minimal standard of living required for learning, i.e. consider $\widetilde{c}$ is zero. Then in stationary equilibrium lifetime labor earnings are the same in both professions and *all* individuals, regardless of the level of inherited wealth, are indifferent in their choice to attend school or to work as an unskilled worker in both periods.[12] Since there are no costs (in addition to foregone earnings in the first period) associated with education that depend on individual wealth, all individuals always compete for both types of labor and thus lifetime wages for both types of occupations must equalize. When lifetime wage incomes are the same for skilled and unskilled labor, wage differentials in any given period solely reflect compensation for the differences in the cost of training for different occupations. Individuals who receive a larger bequest consume more and derive a higher instantaneous and thus lifetime utility, only because they receive larger capital incomes. Moreover, for all bequest distributions that clear the asset market (and respect the no-Ponzi condition) there is a stationary equilibrium with the unique factor intensities $\left(u\left(s^B\right), s^B\right)$. In particular, we have obtained the following proposition:

Proposition 10 *Suppose that there were no minimal standard of living, i.e. suppose that $\widetilde{c}$ equals zero. In this case factor intensities in stationary equilibrium are uniquely determined such as to equate lifetime labor incomes earned in both professions. The class of all stationary equilibrium bequest distributions consists of all bequest distributions that clear the asset market. In particular, in stationary equilibrium all individuals may leave the same bequest and enjoy the same utility.*

[12] Indirect utility functions V^S and V^U are then identical for all net transfers.

As one may expect, the conclusions of the previous Proposition 10 are robust to a small increase of $\widetilde{c}$. However, as Proposition 11 will show, $\widetilde{c}$ should not be too large. Before we turn to this proposition we first want to determine exactly how large $\widetilde{c}$ can be such that a perfect equality equilibrium still exists. To this end, note that skilled labor is performed by the relatively wealthy, that is, by those receiving relatively large capital incomes because the consumption constraint $\widetilde{c}$ is the more restrictive the smaller the inherited wealth is. The fact that in stationary equilibrium $\overline{w}_S \geq \overline{w}_U$ and that both types of labor are employed, implies that for perfect equality to prevail in stationary equilibrium the difference in lifetime labor earnings must be zero: perfect equality can prevail only at point B in Figure 3. Define $\widetilde{c}^{\min}$ as the average per capita production at point B,

$$\widetilde{c}^{\min} = \frac{F(U,S,K)}{2L}\Big|_{(u,s)=B} \; .$$

Since in stationary equilibrium per capita production equals per capita consumption, $\widetilde{c}^{\min}$ denotes the average consumption per capita at point B. $\widetilde{c}^{\min}$ depends on the parameter of the production function and on time preference as measured by the discount factor δ. If $\widetilde{c}^{\min}$ were not smaller than $\widetilde{c}$ and *all* individuals would leave identical bequests, then the consumption constraint would not be binding for anybody. Thus, for $\widetilde{c} \leq \widetilde{c}^{\min}$ there is a stationary equilibrium (at point B) with identical bequests, incomes, lifetime utility, and dynastic utility.

Now assume at point B the consumption constraint $\widetilde{c}$ is larger than the average per capita consumption $\widetilde{c}^{\min}$ and that all individuals would leave identical bequests. Then at B the consumption constraint would bind those individuals attending school, which would leave them in a worse condition than the unskilled workers. Since everybody has the option to work as an unskilled worker, at equilibrium dynastic utility of the skilled workers cannot be smaller than that of the unskilled workers. Hence, at B, for the economy to be in stationary equilibrium there has to be a non-empty class of poor, unfortunate people who do not attend school and a non-empty class of rich, fortunate people who attend school. In particular, the poor leave smaller bequests than the wealthy.[13] This is the essential intuition for Theorem 11.

Finally, as before, let $\widetilde{c}$ be larger than $\widetilde{c}^{\min}$ but assume that the wage gap were positive. Then, any individual who inherits wealth close to b^{kink} could improve its dynastic utility (compared to a stationary choice of profession) by choosing at least once a different profession and leaving a different bequest than one's descendants. This is due to the fact

[13] See also Proposition 12 below for existence of such stationary equilibria. Remember that at B individuals attend school only if their dynasty bequeathes no less than $\widetilde{b}$, that is, only if individuals of such a dynasty are not constrained by the minimal standard of living requirement in their first period of life.

that for net transfers $T^{\text{kink}} = rb^{\text{kink}}$ the marginal maximal lifetime utility is smaller when working as an unskilled worker than the marginal maximal lifetime utility when attending school and working as a manager afterwards. Since at stationary equilibrium the bequest of all dynasties is stationary (Lemma 8), there is a neighborhood of b^{kink} such that at stationary equilibrium no individual makes a bequest which is within this neighborhood.

Summarizing, we state our fundamental inequality theorem for non-trivial standard of living requirements, relegating the formal proof to Appendix 8:

Theorem 11 *Suppose the minimal standard of living required for schooling, $\widetilde{c}$, is larger than $\widetilde{c}^{\min}$. Then, in stationary equilibrium the average bequest made by all skilled dynasties is strictly greater than the average bequest made by all unskilled dynasties. There is a positive number of poor and unskilled dynasties and a positive number of rich and skilled dynasties. Lifetime and dynastic utility of the rich and skilled individuals is strictly larger than that of the poor and unskilled. Moreover, if the difference in lifetime labor earnings, $\overline{w}_S - \overline{w}_U$, is positive, there is a non-degenerated interval $\left(b^{kink} - \Delta, b^{kink} + \Delta\right)$, $\Delta > 0$, dividing the society into dynasties with poor uneducated members and constant bequests smaller than $b^{kink} - \Delta$ and dynasties with rich educated members with constant bequests larger than $b^{kink} + \Delta$.*

Note that as the minimal standard of living $\widetilde{c}$ approaches $\widetilde{c}^{\min}$ from above, the minimal degree of inequality that must prevail in stationary equilibrium vanishes.

4.3 Equilibria with smooth consumption and no debt

If there is a two-group stationary equilibrium with a bequest distribution which leaves the rich enough wealth to make a bequest of b_S with $b_S \geq \widetilde{b}$, then in this stationary equilibrium managers consume just as much as students do (see Lemma 7). Whether such equilibria exist, depends critically on the level of the consumption constraint $\widetilde{c}$. As argued in Section 4.2, there are always stationary equilibria with identical consumption in both periods of life, if the consumption constraint $\widetilde{c}$ is not greater than $\widetilde{c}^{\min}$. In that section when deriving $\widetilde{c}^{\min}$, we assumed that *all* dynasties leave the large bequest $\widetilde{b}$, while in this section we allow for the poor to leave a smaller bequest. Thus, we can relax the restriction imposed on $\widetilde{c}$ to show the following proposition:

Proposition 12 *If $\widetilde{c} > \widetilde{c}^{\min}$ is sufficiently close to $\widetilde{c}^{\min}$, then there are stationary equilibria at which managers do not consume less than students. At every such stationary equilibrium the average bequest of the class of poor dynasties is strictly smaller than the average bequest of the class of rich dynasties.*

Proof. See Appendix 9. ■

We are interested in stationary equilibria at which skilled workers smooth their consumption perfectly over their lifetime *and* the unskilled do not leave any debt to their progeny. As mentioned, this is trivial for $\widetilde{c} \leq \widetilde{c}^{\min}$. For the non-trivial case, $\widetilde{c} > \widetilde{c}^{\min}$, there is a stationary equilibrium in B with the skilled dynasties leaving a bequest of $\widetilde{b}$ and the unskilled dynasties making non-negative bequests if and only if $\widetilde{c}$ is only slightly larger than $\widetilde{c}^{\min}$. For large $\widetilde{c}$, however, no such stationary equilibria exists in B. The following proposition establishes that for all $\widetilde{c} > \widetilde{c}^{\min}$ such equilibria exist if dynasties are not too patient.

Proposition 13 *For every consumption constraint $\widetilde{c}$ larger than $\widetilde{c}^{\min}$ there are infinitely many stationary equilibria at which nobody makes negative bequests and at which managers do not consume less than students if dynasties are sufficiently patient, i.e. if δ is sufficiently small.*

Proof. See Appendix 10. ■

5 Appendix

Appendix 1 *(Proof of Lemma 2)*

If the optimal consumption profile is unconstraint (which is the case for $T_t > T^{U,\min}$ if individual t works as an unskilled worker in both periods and for $T_t \geq \widetilde{T}$ if individual t pursues a student-manager career), first period consumption is related to second period consumption as follows (see Equations 9a and 9b):

$$c_{0t} = \frac{1}{(1+r)\,\delta} c_{1t}. \tag{26}$$

Taking derivatives of $V^U(\cdot)$ we thus see that

$$\begin{aligned} \frac{dV^U}{dT_t} &= \frac{d}{dT_t}\left(\ln c_{0t}^U + \delta \ln c_{1t}^U\right) \\ &= \frac{d}{dT_t}\left(\ln \frac{1}{(1+r)\,\delta} + (1+\delta)\ln c_{1t}^U\right) \\ &= \frac{(1+\delta)}{c_{1t}^U}\frac{dc_{1t}^U}{dT_t}. \end{aligned} \tag{27}$$

Insertion of (26) and (14) into the budget constraint (8) yields $c_{1t}^U + \frac{1}{\delta}c_{1t}^U = T_t + \overline{w}_U$ and thus

$$\frac{dc_{1t}^U}{dT_t} = \frac{\delta}{1+\delta} > 0. \tag{28}$$

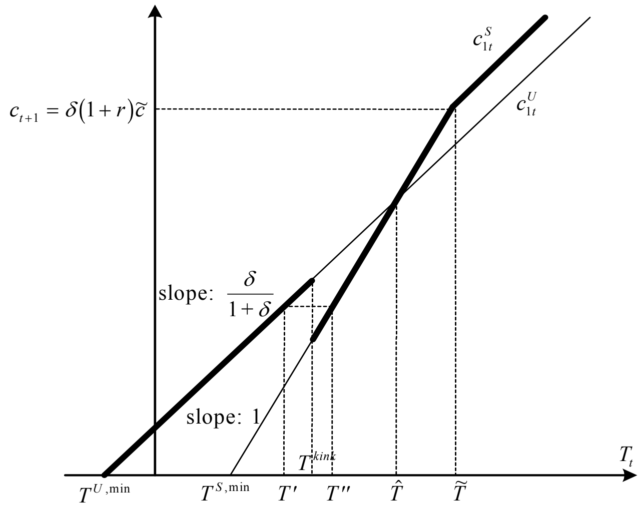

Figure 4:

It follows from (27) and (28) that $V^U(\cdot)$ is indeed strictly concave in net transfers T_t in its relevant domain (see also Figure 4). Similarly, for all $T_t \geq \widetilde{T}$ we obtain $\frac{dV^S}{dT_t} = \frac{(1+\delta)}{c_{1t}^S}\frac{dc_{1t}^S}{dT_t}$ and $\frac{dc_{1t}^S}{dT_t} = \frac{\delta}{1+\delta} > 0$.

If the consumption constraint binds, i.e. for $T^{S,\min} < T_t < \widetilde{T}$, we find that

$$\begin{aligned} \frac{dV^S}{dT_t} &= \frac{d}{dT_t}\left(\ln\tilde{c} + \delta\ln c_{1t}^S\right) \\ &= \frac{\delta}{c_{1t}^S}\frac{dc_{1t}^S}{dT_t} \end{aligned}$$

and from the budget constraint $c_{1t}^S + (1+r)\,\tilde{c} = T_t + \overline{w}_S$ that

$$\frac{dc_{1t}^S}{dT_t} = 1$$

which finishes the proof of the lemma.

Appendix 2 *(Proof of Lemma 3)*

For $\overline{w}_S - \overline{w}_U \geq (1+r)\,\tilde{c}$ we obtain $T^{S,\min} \leq T^{U,\min}$ from (15) and (16). That is, even when the consumption constraint binds, lifetime utility is maximized by attending school in the first period and working as a manager in the second. For $\overline{w}_S - \overline{w}_U < 0$ the choice of attending school is obviously dominated since lifetime earnings are strictly larger for unskilled labor and an unskilled labor does not have to respect the consumption constraint.

Appendix 3 *(Proof of Lemma 4)*

In case of $\overline{w}_S = \overline{w}_U$ we find that $T^{U,\min} < T^{S,\min}$. For all $T_t \geq \widetilde{T}$ the consumption constraint does not bind and therefore attending school leaves individuals t indifferent to working as an unskilled worker. Since the optimal consumption path deviates from the first-best solution for all $T^{S,\min} < T_t < \widetilde{T}$, in this case working as an unskilled worker is preferred.

To show the monotonicity of the schooling decision, assume that, as T_t increases from $T^{S,\min}$ to $\widetilde{T}$, the slope of V^S is greater (smaller) than the slope of V^U for all $T_t > T^{U,\min}$ smaller (greater) than some $\hat{T} \in \left(T^{S,\min}, \widetilde{T}\right)$. Then there is exactly one $T_t > T^{U,\min}$, say T^{kink}, with $V^S\left(T^{\text{kink}}\right) = V^U\left(T^{\text{kink}}\right)$. This unique T^{kink} is in the interval $\left(T^{S,\min}, \hat{T}\right)$.

As should have become clear from Appendix 1, the slope of $V^S(\cdot)$ is equal to the slope of $V^S(\cdot)$ whenever for the same net transfer T_t the consumption of a manager, c^S_{1t}, equals the second period consumption of an unskilled worker, c^U_{1t}. As Figure 4 shows, there is a unique $\hat{T}$ for which this is indeed the case, i.e. this unique $\hat{T}$ satisfies $V^{S\prime}\left(\hat{T}\right) = V^{U\prime}\left(\hat{T}\right)$.

Appendix 4 *(Proof of Lemma 6)*

Assume $0 \leq \overline{w}_S - \overline{w}_U < (1+r)\widetilde{c}$. Remember, a sequence $\langle T_0, T_1, ...\rangle$ is said to be feasible if and only if every T_t, $t = 0, 1, ...$, is not smaller than $T^{U,\min}$ and the corresponding sequence of bequests respects the no-Ponzi condition. Let J_0^{stat} be the dynastic utility corresponding to the stationary sequence $\left\langle T_0^{\text{stat}}, T_0^{\text{stat}}, ...\right\rangle$. We want to show that J_0^{stat} is not smaller than an arbitrary feasible sequence $\langle T_0, T_1, ...\rangle$ if T_0^{stat} satisfies $T^{U,\min} < T_0^{\text{stat}} \leq T'$ or $T_0^{\text{stat}} \geq T''$. From

$$\begin{aligned}
J_0 - J_0^{\text{stat}} &= \lim_{\tau\to\infty} \sum_{t=0}^{\tau} \delta^t \left[V(T_t) - V\left(T_0^{\text{stat}}\right)\right] \\
&\leq \lim_{\tau\to\infty} \sum_{t=0}^{\tau} \delta^t \left(T_t - T_0^{\text{stat}}\right) V'\left(T_0^{\text{stat}}\right) \\
&= V'\left(T_0^{\text{stat}}\right) \lim_{\tau\to\infty} \sum_{t=0}^{\tau} \delta^t \left[\delta^{-1} b_t - b_{t+1} - \left(\delta^{-1} b_0 - b_0\right)\right] \\
&= V'\left(T_0^{\text{stat}}\right) \lim_{\tau\to\infty} \left[\delta^{-1}(b_0 - b_0) + \delta^\tau (b_0 - b_{\tau+1})\right] \\
&= V'\left(T_0^{\text{stat}}\right) \lim_{\tau\to\infty} \delta^\tau (b_0 - b_{\tau+1}) \\
&= -V'\left(T_0^{\text{stat}}\right) \lim_{\tau\to\infty} \delta^\tau b_{\tau+1}
\end{aligned}$$

we infer that J_0 is indeed not greater than J_0^{stat}, since every bequest is confined by the no-Ponzi condition and therefore b_t cannot decrease without bounds. Finally, note that the inequality above is satisfied for every feasible T_0^{stat} if $V(\cdot)$ is concave in its relevant domain. As shown in Lemma 2, this is the case if $\overline{w}_S = \overline{w}_U$, which finishes the proof.

Appendix 5 *(Proof of Lemma 7)*

Lemma 6 proved that a stationary sequence of bequests is indeed optimal for the given initial bequests. This implies a stationary sequence in educational attainment. The stated levels of consumption follow from (9a), (9b), (13), and the fact that $(1+r)\,\delta = 1$ holds in stationary equilibrium.

Appendix 6 *(Proof of Lemma 8)*

In stationary equilibrium the sequence $\langle V'(T_t)\rangle_t$ is stationary (see Lemma 5). Since there is no switching of professions at stationary equilibrium, this implies that the sequence of net transfers $\langle T_t\rangle_t$ is stationary as well. Let us refer to this (stationary) T_t as T. Then, if $T \neq T_0^{\text{stat}}$ either the no-Ponzi condition would be violated in finite time or the transversality condition would not be satisfied: solving the recurrence equation (14) for a given b_0, we obtain

$$b_t = \frac{1}{r}\left((1+r)^t\left(T_0^{\text{stat}} - T\right) + T\right). \tag{29}$$

In case of $T > T_0^{\text{stat}}$, i.e. if bequest are decreasing from generation to generation, the no-Ponzi condition would be violated in finite time. In case of $T \leq T_0^{\text{stat}}$ the strengthened transversality condition

$$\lim_{t\to\infty} \delta^t V'(T)\, b_{t+1} = 0 \tag{30}$$

must hold: either $T < T_0^{\text{stat}}$ holds and thus bequests would be positive for all large t. Or, if $T = T_0^{\text{stat}}$, then bequests would be stationary. In both case (20) coincides with (30). Inserting (29) and $1+r = 1/\delta$ into (30), we see from

$$\begin{aligned} &\lim_{t\to\infty} \delta^t \left(\frac{\delta^{-t}}{r}\left(T_0^{\text{stat}} - T\right) + \frac{T}{r}\right) \\ = \; &\frac{T_0^{\text{stat}} - T}{r} + \frac{T}{r}\lim_{t\to\infty}\delta^t = \frac{T_0^{\text{stat}} - T}{r} \end{aligned}$$

and the fact that $V'(T)$ is constant that the transversality condition (20) is satisfied only if $T = T_0^{\text{stat}}$.

Appendix 7 *(Proof of Proposition 9)*

We want to show that there is an interval I in $\left(s^A, s^B\right)$ such that for all $s \in I$ with corresponding equilibrium factor intensities $(u(s), s)$ the level of bequest b'' is negative. The fact that the bequest of the poor b_U can be negative and thus that the asset market clearing bequest of the rich is positive then proves the proposition.

Using (24) and the condition of an equilibrium of the asset market (Step 1), $(1 - \overline{a}_U u/2)/s = \overline{a}^S$, we see that

$$\frac{1 - \frac{b_U}{2+r}u}{s} = \overline{a}^S \leq \frac{1}{2+r}\left(2b_S - w_S\right). \tag{31}$$

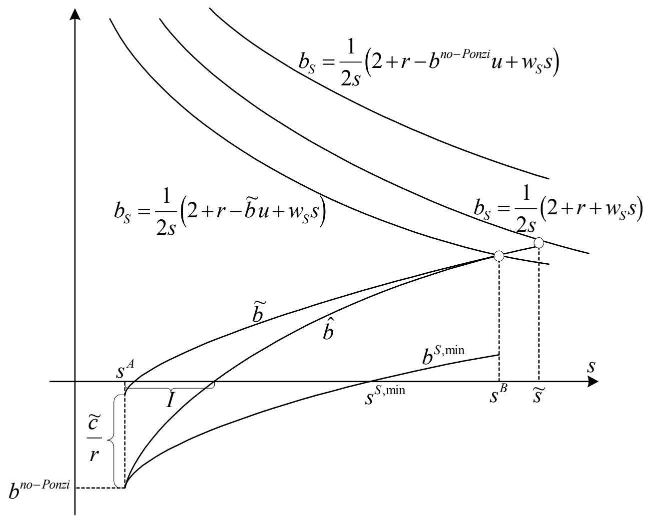

Figure 5: The graph of $b_S(s)$ intersects the curve $\widetilde{b}(s)$ in s^B which is the case if and only if $\widetilde{c} = \widetilde{c}^{\min}$.

For sufficiently small b_U, in particular for $b_U = b^{\text{no-Ponzi}} < 0$, the LHS of this inequality is positive for all (u, s) in Γ and therefore so must be b_S. Solving the above inequality for b_S yields

$$b_S \geq \frac{1}{2s}\left(2 + r - b_U u + w_S s\right). \tag{32}$$

Figure 5 plots for three different levels of b_U (i.e. for $b_U = b^{\text{no-Ponzi}} < 0$, $b_U = 0$, and $b_U = \widetilde{b}$) the graphs of the lower bound of the wealth b_S that must be bequeathed by individuals from rich dynasties such as to guarantee an equilibrium of the asset market. In this figure it is assumed that for any skill intensity $s > 0$ the unskilled intensity u has adapted accordingly such as to guarantee $(1 + r)\delta = 1$, as stated in (22). The b_S-curves clearly fall in s for all $b_U \leq 0$ since $u(s)/s$ falls in s (see Figure 3) as does the wage for skilled labor w_S.

We still have to show that $b'' \leq 0$ in an interval I in $\left(s^A, s^B\right)$. In general, b'' is difficult to determine. However, by resorting to the level of bequest $\hat{b}$ we can check for the sign of b'' since we can infer from the ordering $\widetilde{T} > \hat{T} > T'' > T' > T^{S,\min}$ (see Figure 4) and (21) on the ordering of the corresponding stationary bequests $\widetilde{b} > \hat{b} > b'' > b' > b^{S,\min}$. Remember that $\hat{b}$ is that stationary bequest which leads to $V^{U\prime}\left(\hat{T}\right) = V^{S\prime}\left(\hat{T}\right)$. Since this condition is always satisfied if $c_1^S = c_1^U$ (see Figure 4), using (9b) and (13) we can solve for $\hat{b}$:

$$\hat{b} = \frac{2 + r}{r(1 + r)}\left((1 + r)\widetilde{c} - (w_S - w_U)\right).$$

$\hat{b}$ depends on u and s because the wage rates w_S and w_U depend on both factor intensities. (Again, remember that for every $(u(s), s)$ the interest rate r is constant.) Figure 5 illustrates the shape of $\hat{b}$. Consider a $(u,s) \in \Gamma$ and then move rightwards on the $(1+r)\delta = 1$ locus in Figure 3. This is equivalent to studying an increase of s in Figure 5. In this case the wage for skilled labor decreases and the wage for unskilled labor increases. Hence, on the $(1+r)\delta = 1$ locus $\hat{b}$ increases as s increases. Moreover, in point A of Figure 3 wages of skilled and unskilled labor are related by $w_S = (2+r)\,w_U + (1+r)\,\widetilde{c}$. In A, hence, the bequest $\hat{b}$ is equal to $b^{\text{no-Ponzi}} < 0$. Thus for all s in a non-empty interval I in $\left(s^A, s^B\right)$ we find that $0 \geq \hat{b}$ (see Figure 5). Since $\hat{b} > b''$, this implies that on I, in fact, b'' is negative.

To finish the proof, we want to show that for every $s \in I$ there are infinitely many two-group stationary equilibria with $b_U > b^{\text{no-Ponzi}}$. As noted earlier, for infinitely many small b_U, in particular for all $b_U < 0$, the average wealth $\overline{a}^S$—and thus the (stationary) bequest b_S that corresponds with $\overline{a}^S$—is positive for any given (u,s). Hence, for all $(u(s),s)$ with $s \in I$ there are infinitely many b_U with corresponding asset market clearing bequests b_S satisfying $b_S \geq b''$. Thus all (b_U, b_S) with $b_U < 0$ and $b^{\text{no-Ponzi}} \leq b_U \leq b'$ are equilibrium bequest distribution for any $(u(s),s)$ with $s \in I$. The condition $b_U \leq b'$ is always satisfied when $b_U \leq b^{S,\min}$ because $b^{S,\min} < b'$. From (15) and (21) we can determine $b^{S,\min}$:

$$b^{S,\min} = \frac{(1+r)\,\widetilde{c} - w_S}{r}. \tag{33}$$

As does $\hat{b}$, the level of bequest $b^{S,\min}$ is equal to $b^{\text{no-Ponzi}}$ in point $A = \left(u\left(s^A\right), s^A\right)$ of Figure 3 and increases along the $(1+r)\delta = 1$ locus as s increases. We infer from this finding that there are indeed infinitely many b_U satisfying $b^{\text{no-Ponzi}} < b_U \leq b^{S,\min}$ for every $(u(s),s)$ with $s \in I$, which finishes the proof.

Appendix 8 *(Proof of Theorem 11)*

If the wage gap is zero in stationary equilibrium (point B in Figure 3), only those individuals inheriting no less than $\widetilde{b}$ consider attending school (see Lemma 4). From (17) and (21) we can determine $\widetilde{b}$:

$$\widetilde{b} = \frac{(2+r)\,\widetilde{c} - w_S}{r} = b^{S,\min} + \frac{\widetilde{c}}{r}. \tag{34}$$

(The interest payments for the inherited wealth, $r\widetilde{b}$, plus the labor income, w_S, just suffice to finance a consumption of $\widetilde{c}$ in both periods of life.) Using (24b) and (34) we see that in B for $b_U = b_S \geq \widetilde{b}$ to hold, the asset market is in equilibrium and hence a perfectly equal stationary equilibrium exists if only if $\widetilde{c} \leq \widetilde{c}^{\min}$: the largest $\widetilde{c}$ that satisfies

$$1 = \overline{a}^U \frac{u}{2} + \overline{a}^S s = \frac{1}{2+r}\left(\widetilde{b}\,(u+2s) - w_S s\right)$$

is $\widetilde{c}^{\min}$. In contrast, consider a $\widetilde{c} > \widetilde{c}^{\min}$. Still, bequests made by skilled dynasties cannot be smaller than $\widetilde{b}$ at equilibrium. Since $\widetilde{b}$ increases in $\widetilde{c}$ as obvious from (34), the minimal average wealth of skilled dynasties $\overline{a}^S$ strictly increases in $\widetilde{c}$ as well. Thus the difference between $\overline{a}^S$ and $\overline{a}^U$ strictly increases in $\widetilde{c}$. This implies that the average bequest made by the dynasties that attend school is strictly larger than the average bequest made by the dynasties that work as unskilled workers. Since in stationary equilibrium there are skilled dynasties, this proves the first part of the theorem.

To prove the second part of the theorem, assume generation $t = 0$ knows that it is going to inherit a bequest b_0 in its second period of life with b_0 being close to b^{kink}. It is sufficient to show that in this case dynastic utility for a non-stationary sequence $\langle T_t \rangle_{t \geq 0}$ and, hence, for a non-stationary sequence of schooling decisions, is strictly greater than dynastic utility for a stationary sequence of T_t. Without loss of generality suppose b_0 is in the interval $\left[b^{\text{kink}}, b^{\text{kink}} + \Delta\right]$ with $\Delta > 0$. For this dynasty it is feasible to maintain stationary net transfers of $T_0^{\text{stat}} > T^{\text{kink}}$ which implies that every generation would attend school (see Figure 6). Now consider the case that generation $t = 0$ would decide not to attend school but to choose some $\underline{T} < T^{\text{kink}}$, while all its descendants would attend school and receive net transfers of $\overline{T} > T^{\text{kink}}$. (Note, that for our argument $\overline{T}$ does not necessarily have to be the optimal T_t for future generations $t > 0$.) Then there is a positive $\underline{\Delta}$ with $\underline{T} = T_0^{\text{stat}} - \underline{\Delta}$. This implies that generation $t = 0$ makes a bequest of $b_1 = (1 + r)\, b_0 - \underline{T} = b_0 + \underline{\Delta}$ (see Equation 14) and all its descendants make a bequest of $b_1 = b_2 = ...$ Then all descendants choose net transfers of $\overline{T}$ defined by $\overline{T} = rb_1$. Thus, $\overline{T} = r\underline{\Delta} + T_0^{\text{stat}}$. It follows that $\overline{T} - \underline{T} = (1 + r)\, \underline{\Delta}$. Dynastic utility of this sequence $\left\langle \underline{T}, \overline{T}, \overline{T}, ... \right\rangle$ is

$$\begin{aligned} \underline{J} &= V\left(\underline{T}\right) + \sum_{t=1}^{\infty} \delta^t V\left(\overline{T}\right) = \frac{1}{1-\delta}\left(V\left(\underline{T}\right) + \delta\left[V\left(\overline{T}\right) - V\left(\underline{T}\right)\right]\right) \\ &= \frac{1}{1-\delta}\left(V\left(\underline{T}\right) + \underline{\Delta}\frac{V\left(\overline{T}\right) - V\left(\underline{T}\right)}{\overline{T} - \underline{T}}\right) \end{aligned}$$

where the last equality uses the fact that $(1 + r)\, \delta = 1$. If $\underline{T}$ and T_0^{stat} are sufficiently close to T^{kink}, then $\left\langle \underline{T}, \overline{T}, \overline{T}, ... \right\rangle$ dominates the stationary sequence $\left\langle T_0^{\text{stat}}, T_0^{\text{stat}}, ... \right\rangle$, as is obvious from Figure 6. Thus, we have proved that in stationary equilibrium there is a neighborhood of b^{kink} such that no individual inherits wealth in this neighborhood and, therefore, that inherited wealth of the unskilled workers is strictly smaller than the inherited wealth of those attending school. This further implies that lifetime utility and, thus, dynastic utility of those dynasties working as unskilled workers is strictly smaller than the lifetime and the dynastic utility of dynasties working as skilled workers.

Appendix 9 *(Proof of Proposition 12)*

We will show that, if $\widetilde{c}$ is greater than $\widetilde{c}^{\min}$ but still sufficiently small, then for infinitely many $(u, s) \in \Gamma$—remember, the wage gap is positive for all these (u, s)—there is a bequest

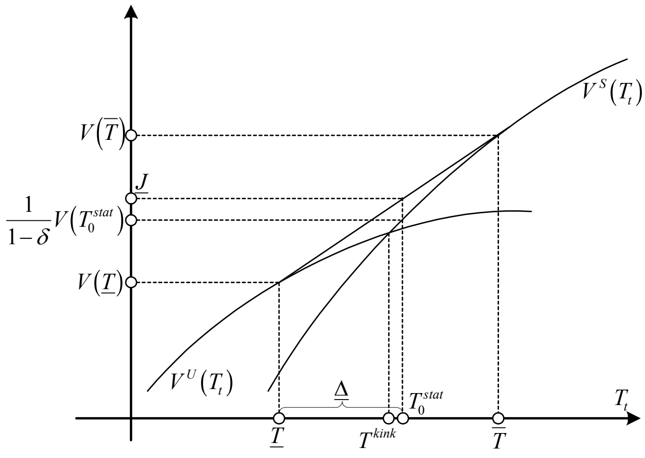

Figure 6:

distribution (b_U, b_S) with $b_U \in \left[b^{\text{no-Ponzi}}, b'\right]$ and $b_S \geq \widetilde{b}$, such that the asset market is in equilibrium. This proof closely resembles the proof in the above Appendix 7; here we just require that $b_S \geq \widetilde{b}$, while in Appendix 7 we demanded that $b_S \geq \hat{b}$.

Assume every generation of a rich dynasty is not constrained by the minimal standard of living. This is the case for $b_S \geq \widetilde{b}$. (b_U, b_S) with $b_S \geq \widetilde{b}$ is an equilibrium bequest distribution if and only if there is a $(u, s) \in \Gamma$ satisfying (32) with the equality holding strictly and $b_U \in \left[b^{\text{no-Ponzi}}, b'\right]$. As in Appendix 7, the b_S-curve (as a function of s with $(u(s), s) \in \Gamma$) shifts downwards as b_U increases. In particular, the b_S-curve for $b_U = \widetilde{b}$ is below the b_S-curve for $b_U = b^{\text{no-Ponzi}}$. Now, if the consumption constraint is $\widetilde{c}^{\min}$, the b_S-curve for $b_U = \widetilde{b}$ intersects the $\widetilde{b}$-curve defined in (34) in s^B (see Figure 5). Thus, for small increases of $\widetilde{c}$ above $\widetilde{c}^{\min}$ there still exist equilibrium bequest distributions for all $(u, s) \in \Gamma \cup \{B\}$. Since the $\widetilde{b}$-curve shifts upwards as $\widetilde{c}$ increases, for sufficiently large consumption constraints no stationary equilibrium exists at point B. However, for $(u, s) \in \Gamma$ with $s < s^B$ such equilibria may still exist because the b_S-curve decreases in s (for b_U sufficiently small) and the $\widetilde{b}$-curve increases in s (see Figure 5). It follows that there exist stationary equilibria whenever the consumption constraint $\widetilde{c}$ is so small that in s^A the $\widetilde{b}$-curve is below the b_S-curve for $b_U = b^{\text{no-Ponzi}}$.

The second part of the proposition follows directly from Theorem 11.

Appendix 10 *(Proof of Proposition 13)*

As argued in Appendix 9, for $\widetilde{c} = \widetilde{c}^{\min}$ the b_S-curve for $b_U = \widetilde{b}$ intersects the $\widetilde{b}$-curve in s^B. By the same argument, there exists a $\widetilde{c}^{\text{bound}} > \widetilde{c}^{\min}$ such that the b_S-curve for $b_U = 0$ and the respective $\widetilde{b}$-curve intersect in s^B. Obviously, there exist stationary equilibria with $b_S \geq \widetilde{b}$ and $b_U \geq 0$ in point B if only if $\widetilde{c} \leq \widetilde{c}^{\text{bound}}$.

For consumption constraints $\widetilde{c}$ larger than $\widetilde{c}^{\text{bound}}$ the factor intensities (u, s) of stationary equilibria must thus be in the set Γ. For all these factor intensities, however, the wage gap is positive and to show that such equilibria exist also in an interval in $\left(s^A, s^B\right)$ for $\widetilde{c}$ being larger than $\widetilde{c}^{\text{bound}}$ we must therefore also verify, in addition to $b_S \geq \widetilde{b}$, that $b_U \leq b^{S,\min} < b'$.

To this end, let $s^{S,\min}$ denote the skill intensity s—with corresponding $u(s)$—for which $b^{S,\min} = 0$ (see Figure 5). For all $s \geq s^{S,\min}$ the condition $b^{S,\min} \geq 0$ is satisfied because $b^{S,\min}$ increases in s (since w_S decreases in s). Moreover, $s^{S,\min}$ always exists in our Cobb-Douglas case because $b^{S,\min}$ approaches $b^{\text{no-Ponzi}} < 0$ as s approaches s^A and $b^{S,\min}$ is positive for sufficiently large s, since the wage for skilled labor approaches zero as the skill intensity increases without bounds. Insertion of (4), (22), and (33) into $b^{S,\min} = 0$ yields

$$s^{S,\min} = \frac{1}{\widetilde{c}} \frac{r}{1+r} \frac{\alpha}{1-\alpha-\beta}. \tag{35}$$

Similarly, $\widetilde{s}$ denotes the skill intensity s—with corresponding $u(s)$—for which the b_S-curve for $b_U = 0$ intersects the $\widetilde{b}$-curve (see Figure 5). Using (4) and (22) we obtain

$$\widetilde{s} = \frac{r}{2\widetilde{c}} \frac{1-\beta}{1-\alpha-\beta}. \tag{36}$$

By construction, $\widetilde{s} = s^B$ if and only if $\widetilde{c} = \widetilde{c}^{\text{bound}}$ and $\widetilde{s} < s^B$ for all $\widetilde{c} > \widetilde{c}^{\text{bound}}$. Now, if $\widetilde{s}$ is not smaller than $s^{S,\min}$ and $\widetilde{c} \geq \widetilde{c}^{\text{bound}}$, then for all $(u(s), s) \in \Gamma$ with $s \in \left[s^{S,\min}, \widetilde{s}\right]$ there are infinitely many two-group stationary equilibria with $b_S \geq \widetilde{b}$ and $b_U \geq 0$.

Next, we show that $\widetilde{s}$ is not smaller than $s^{S,\min}$ if and only if δ is sufficiently small, which finishes the proof. Using (35) and (36) we find that the condition $\widetilde{s} \geq s^{S,\min}$ is equivalent to

$$\delta \leq \frac{1-\beta}{2\alpha}. \tag{37}$$

Noteworthy, whether $\widetilde{s} \geq s^{S,\min}$ is satisfied or not does not depend on the consumption constraint.

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