# A HIGH ORDER FEEDBACK NET (HOFNET) WITH VARIABLE NON-LINEARITY

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## INTRODUCTION

In practical applications of image recognition it is often the case that the input unknown pattern is a noisy version of one known by the recognition system. Therefore, it is important to design the recognition system taking into account the different types of noise which are present in both the input image and in the system itself. These can be divided into two types:

(i) Random time dependent noise, such as that caused by transmission of an image through a turbulent atmosphere or by the passage of a signal through an amplifier in the pattern recognition system

(ii) Fixed time independent noise, such as that caused by transmission of an image past some fixed opaque obstacles

Most neural networks proposed for pattern recognition sample the incoming image at one instant and then analyse it. This means that the data to be analysed is limited to that containing the noise present at one instant. Time independent noise is therefore, captured but only one sample of time dependent noise is included in the analysis. If however, the incoming image is sampled at several instants, or continuously, then in the subsequent analysis the time dependent noise can be averaged out. This, of course, assumes that sufficient samples can be taken before the object being imaged, has moved an appreciable distance in the field of view. High speed sampling requires parallel image input and is most conveniently carried out by optoelectronic neural network image analysis systems. One problem of such systems is the high background levels of system noise which degrades the accuracy of the calculation. The noise is generally caused by spurious scattering of light from defects and impurities in lenses and holograms and depends on the optical signal being processed. So in an iterating optical neural network where the signal is continually being modified until it converges the noise will generally be time dependent during the calculating phase and time independent at the end. Optical technology is particularly good at performing certain operations, such as Fourier Transforms, correlations and convolutions while others such as subtraction are difficult. So for an optical net it best to choose an architecture based on convenient operations such as the high order neural networks.

## HIGH ORDER NEURAL NETWORKS

The probability distribution function of the input data can be modelled by using a polynomial fit which can be achieved using a high order net. Lee [1] has extended the Hopfield [2] equations to higher order as follows:

$$v_i(t+1) = F\left[\sum_{j=1}^{N}\sum_{k=1}^{N}\cdots\sum_{p=1}^{N} T_{ijk\ldots p}\, v_j(t) v_k(t) \ldots v_p(t)\right] \qquad (1)$$

$$T_{ijk\ldots p} = \sum_{m=1}^{M} v_i^{(m)} v_j^{(m)} v_k^{(m)} \ldots v_p^{(m)} \qquad (2)$$

The components or pixels of the stored images are assumed to be binary and bipolar (+1,-1). Note that equations (1) and (2) have been chosen so that when the order, n, is limited to two it reduces to the Hopfield net with non-zero diagonal elements. Equation (2) gives large values for the elements of T when a certain selection of pixels are on in many of the training images, so classifying them. It gives elements of T~N when a certain selection of pixels are on in only one training pattern and are generally random in the other training patterns, so distinguishing the particular features of each pattern. If in a certain selection of pixels the pixels are randomly on or off in each pattern the element of T~0. These equations have been shown to reduce to

$$v_i(t+1) = F\left[\sum_{m=1}^{M} \left(C^{(m)}\right)^n v_i^{(m)}(t)\right] \qquad (3)$$

$$C^{(m)} = \sum_{i=1}^{N} v_i^{(m)} v_i(t) \qquad (4)$$

which expresses the algorithm in terms of correlations (i.e. inner products) between the input pattern and each training pattern. Since correlations are particularly convenient to implement in optics this is an especially helpful arrangement of the algorithm. Figure 1 shows a model of this algorithm based on matched filters [3]. The first and last array in the diagram are arrays of matched filters or correlators storing the training patterns for correlation. The matched filter formalism for modelling neural networks has been found to be very useful for optical neural networks.

## MODIFICATION TO REDUCE TIME DEPENDENT NOISE

The high order net may be implemented in an iterative architecture in which it is convenient to sample the input noisy image at numerous small intervals. By modifying the standard high order equations we can make the net tolerant to time dependent noise. The first modification is that instead of calculating the correlation once and raising it to the power of n we calculate the correlation of the input data sampled at several times and multiply the values obtained

$$v_i(t+1) = F\left[\sum_{m=1}^{M} \left\{\prod_{i=1}^{n} C^{(m)}(\tau_i)\right\} v_i^{(m)}(t)\right] \qquad (5)$$

This effectively calculates the geometric average of the time dependent noise. Each calculation of the correlation will be slightly inaccurate but the accuracy will increase as more correlations are calculated and multiplied.

The second modification is that we normalise the correlations after each correlation so that the largest is normalised to N. Thereafter we threshold the correlations setting any less than N/2 to zero. This decreases the number of spurious signals in the net giving a much clearer output at times before final convergence. It also has the advantage that the limited dynamic range of the optical system components is not exceeded and so saturation does not occur. Figure 2 shows a schematic diagram of the architecture of this High Order Feedback NET (HOFNET). Repeated iteration by means of the feedback shown calculates the correlations to increasing the exponent by unity each time. So the order of the non-linearity in the net is variable and depends on the number of iterations before the output is observed. In this way extremely high orders can be achieved in a noisy system with limited dynamic range.

## COMPUTER SIMULATIONS

The architecture was simulated to assess the tolerance to both types of noise. The inputs consisted of patterns formed on a grid of 8x8=64 pixels. We defined the characters 0 to 9 and the letters A to Z on this grid. For differing numbers of patterns stored, M=5,10,20,30, we input noisy patterns and found the probability of convergence. For example, we entered the pattern for a letter A but having 3 pixels reversed (i.e. +1 replaced by -1 or vice versa). The positions of these pixels were chosen randomly numerous times and the probability of correct recall was calculated. This was repeated for other memorised characters. These results describe the behaviour for time independent noise. In addition we input patterns in which, using our earlier example, the 3 pixels chosen changed in time so that a different random choice of pixels was made at the next time step. This was used to assess the averaging ability of the net design and to check that time dependent noise was not a problem.

The results are shown in figure 3. These show clearly that as the input noise was increased, in terms of the number of reversed pixels, that the probability of correct recall dropped quite suddenly after a certain noise threshold. The net could tolerate more time dependent noise than time independent noise as a result of the modifications made. If no special modifications had been made we might expect identical curves for both types of noise of equal magnitude. Figure 4 plots the amount of input noise against the number of stored patterns assuming an 70% probability of correct recall. This shows that the tolerance to input noise drops with the number of stored patterns but that more time dependent noise can be tolerated particularly for memory capacities less than 0.5N. This improved performance with respect to time dependent noise is particularly important for optical systems with high levels of internal system noise.

## OPTOELECTRONIC HOFNET EXPERIMENTAL RESULTS

An optical system of the HOFNET described above was constructed [4] and experimental results for pattern recognition were obtained [5]. In the experiment 14 patterns were stored each having 64 pixels to provide some comparison with the simulations. Correct recall was demonstrated in 3 or 4 iterations when 12.5% of the input pattern was obscured corresponding to time independent noise. Time dependent noise was not simulated since the input already had noise due to convection currents in the air, instabilities in the laser, temperature fluctuations, vibration, scattering from dust on lenses, etc. These same sources of noise also introduced noise into the optical system itself. So in this practical experiment there was present both time independent noise and time dependent noise.

## CONCLUSION

The experimental noise was analogue in nature and not digitised so it is difficult to perform a direct comparison with the simulations of binary pixelated noise. Nevertheless, the fact that the optical system was able to operate with 3 or 4 iterations correctly (i.e. third or fourth order non-linearity) proves the viability of the HOFNET design since this is the first time an optical network with an order higher than two has been demonstrated successfully [6-11]. Earlier optical systems have been limited by the high levels of background noise present in the system and by the limited dynamic ranges.

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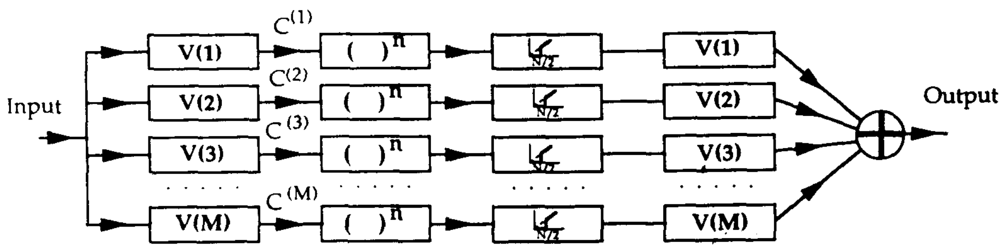


Fig. 1 Matched Filter Model of High Order Net

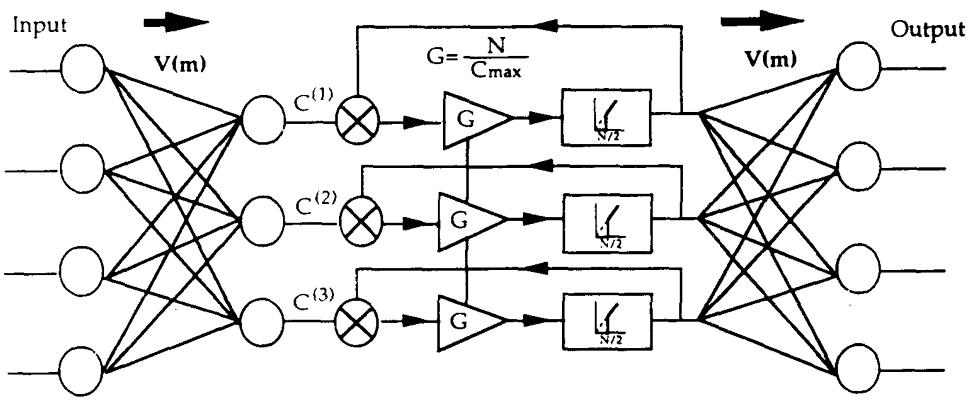


Fig. 2 High Order Net with Geometric Averaging of Time Dependent Noise

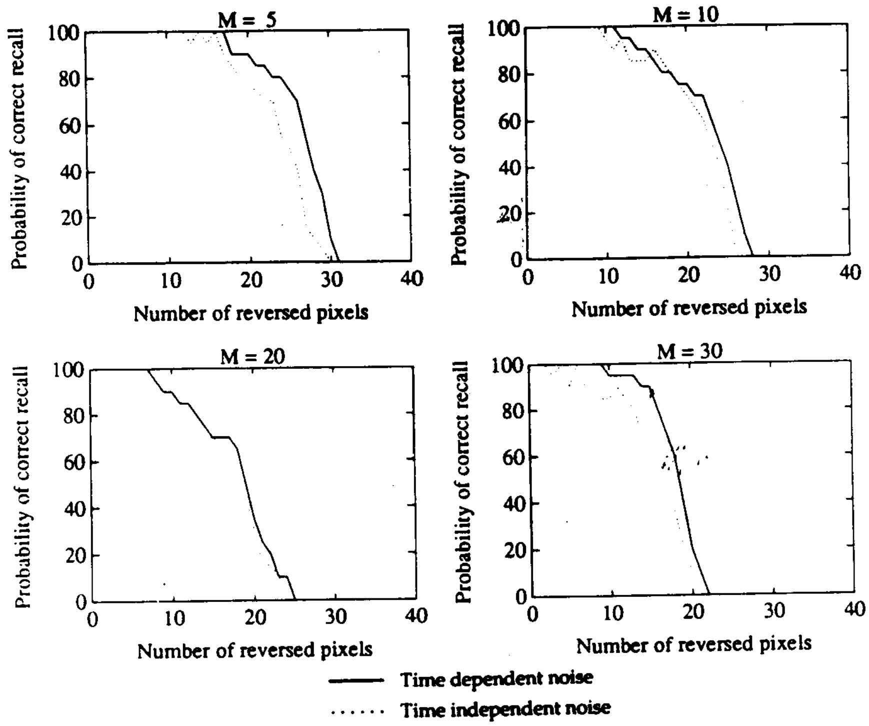


Fig. 3 Probability of Correct Recall as a Function of Input Noise (N=64)

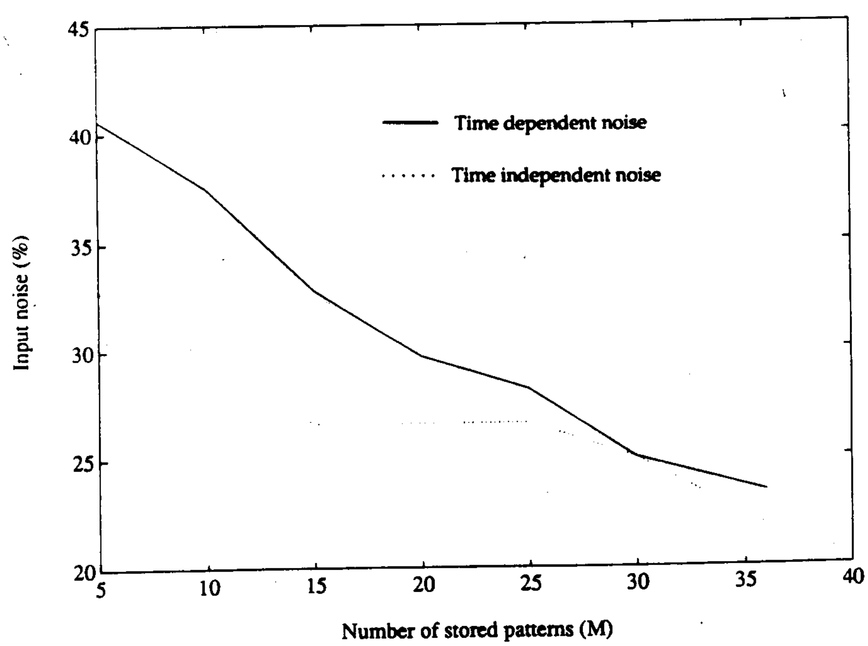


Fig. 4 Maximum Input Noise for Correct Recall as a Function of Memory Capacity (N=64)