COLLECTIVE LABOUR SUPPLY WITH CHILDREN

Richard Blundell
Pierre-Andre Chiapprori
Costas Meghir

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Richard Blundell[†], Pierre-Andre Chiappori[‡], and Costas Meghir[§]

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Abstract

We extend the collective model of household behavior to allow for the existence of public consumption. We show how this model allows to analyze welfare consequences of policies aimed at changing the distribution of power within the household. In particular, we claim that our setting provides an adequate conceptual framework for addressing issues linked to the 'targetting' of specific benefits or taxes. We also show that the observation of the labor supplies and the household demand for the public good allow to identify individual welfare and the decision process. This requires either a separability assumption, or the presence of a distribution factor.

1. Introduction

The 'targeting' view It is by now widely accepted that intrahousehold distribution of income and decision power matters. Numerous empirical studies have

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[†] University College London, Department of Economics, Gower Street, London, WC1E 6BT and Institute for Fiscal Studies.

[‡]University of Chicago, Department of Economics, 1126 E 59th Street Chicago ILL 60637 USA.

[§] University College London, Department of Economics, Gower Street, London, WC1E 6BT and Institute for Fiscal Studies.

shown that, contrary to an implicit postulate of the standard framework, targeting a benefit to a particular household member (say, the wife) may have important consequences on the ultimate use of the corresponding resources. Thomas (1990) argued early on that male and female non labor incomes have a very different impact on children's health and demographics; similar conclusions have been reached by, among others, Schultz (1990), Phipps and Burton (1998), Bourguignon, Browning, Chiappori and Lechene (1993) and Lundberg, Pollak and Wales (1997). More recently, Thomas *et al.* (1997), using an Indonesian survey, have shown that the distribution of wealth by gender at marriage has a significant impact on children's health in those areas where wealth remains under the contributor's control.[1] Duflo (2000) has derived related conclusions from a careful analysis of a reform of the South African social pension program that extended the benefits to a large, previously not covered black population. Specifically, Duflo finds that the consequences of this windfall gain on child nutrition dramatically depends on the gender of the recipient.[2] Such findings have potentially crucial normative implications on the design of aid policies, social benefits, taxes and other aspects of public policy.

However, while the 'targeting' view has strong empirical support and major policy implications, its theoretical foundations remain somewhat weak. After all, the standard methodological tool for studying household behavior is (or was until recently) the 'unitary' model, which relies on the assumption that the household maximizes a unique utility function. This assumption directly implies that target-

[1] See also Galasso (1999) for a similar investigation.

[2] See also Bertrand *et al.* (1999) for an investigation of labor supply of younger women using the same data base, and Rubalcava and Thomas (2000) for a study of the impact of benefits on female labor supply in the US.

ing *cannot* be effective, since the various sources of income will always be pooled at the household level for the sake of decision making.

An alternative, increasingly popular framework for studying household behavior is the 'collective' approach, whereby individuals with specific (and in general different) preferences make Pareto efficient decisions. This general setting includes, as particular cases, bargaining models (under symmetric information), as well as a number of other settings. The collective model is aimed at formalizing the notion of 'decision powers' within the household, and the idea that changes in respective powers typically generate changes in behavior even when total resources are kept constant - a key insight of the 'targeting' view. In that sense, the collective approach seems to provide the natural theoretical background for issues related to targeting.

The first goal of the present paper is to support this claim. We provide a fully developed theoretical framework within which these issues can be addressed. In this model the household budget constraint is pooled and there are no 'hypothecated' funds. However, complex decisions processes are allowed for, provided they satisfy a basic efficiency requirement; namely, outcomes are assumed to be efficient in the Pareto sense. Different decision processes lead to different locations of the outcomes on the Pareto frontier, and to each location correspond (implicitly or explicitly) a set of Pareto weights for the family members. These weights fully summarize the decision process. A basic goal of the collective approach is precisely to analyze how the decision process, i.e. the individual weights, can be affected by prices, incomes and other exogenous factors, and how such changes influence household decisions. From a collective perspective, targeting may thus matter through its impact on respective weights of household members. Paying a benefit to the wife instead of the husband may twist the balance of powers in favor of the former; everything (i.e., preferences and budget constraints) equal, this may (and in general will) ultimately change the outcome.

We illustrate how the collective approach works by considering a particular problem. Several studies argue that on average, 'mothers care more for children than fathers', in the sense that an increase in the mother's power within the couple results in more expenditures spent on children. We analyze the theoretical underpinnings of this claim within a collective approach. We prove that a shift in the (Pareto) weights favoring a member boosts the demand for a public good if and only if the marginal willingness to pay of this member *is more sensitive to increases in (her) private consumption* than that of the other member. In other words, the key property is not that the mother has a larger willingness to pay for children goods (out of her resources), but that her willingness to pay is more responsive to changes in these resources than the father's.

Mapping theory and observed behavior: identifiability Our first claim, thus, is that the collective model may provide an adequate conceptual toolbox for analyzing policy issues linked to intrahousehold allocation of power. Still, its *empirical* relevance has to be asserted. Two aspects are of particular importance: The empirical *characterization* of household behavior stemming from a collective framework, and the *identifiability* of the structural model from observed behavior.

During the last decade, important progresses have been made on the characterization problem. Chiappori (1988, 1992), Browning and Chiappori (1998) and recently Chiappori and Ekeland (2003) have derived increasingly general necessary conditions for a given function to be the collective demand of a group of given size. However, whenever welfare implications are at stake, characterization is not enough. When theory is used to formulate normative judgments, identifiability becomes a crucial issue. To be useful, a theory should provide ways of recovering the underlying, welfare relevant structure (preferences, decision process) from observed behavior. From this perspective, the collective model exhibits an important weakness: While some identification results have been obtained so far, basically all rely on the assumption that commodities are privately consumed.[3] Such a setting, which excludes public consumption within the household, seems inappropriate for the study of decisions regarding children, since it is natural to assume that both parents derive utility (albeit possibly to a different extent) from children's well-being. In other words, while the collective process seems to provide a very natural conceptual context for analyzing intrahousehold allocation of power, it is fair to say that, in the present state, not enough is known about collective behavior with public goods to actually ground empirical analysis within the collective framework.

The second goal of the present paper is precisely to fill this gap. In a related and complementary paper, Chiappori and Ekeland (2004) analyze identifiability in the collective framework from a very general perspective.[4] In this paper, we concentrate on a version of the collective model that has been extensively used for empirical applications[5], namely Chiappori's (1992) model of collective labor

[3]An important exception is a paper by Fong and Zhang (2001), who consider a model in which leisure can be consumed both privately and publicly. Although the two alternative uses are not independently observed, they can in general be identified under a separability restriction, provided that the consumption of another exclusive good (e.g., clothing) is observed.

[4]For further results on the collective model and identification see also Donni (2000)

[5]See for instance Fortin and Lacroix (1997), Chiappori, Fortin and Lacroix (2002) and Blundell et al. (2003).

supply, and we introduce children in this specific context. We assume that both parents care about children's welfare (or equivalently that expenditures on children are a public good within the household), but possibly to a different extent; identifying from observed behavior how much each parent cares for expenditure on children is precisely one of the goals of this line of research. Using a model that incorporates labour supply is particularly appropriate in the context where the public expenditures are taken to be for children; in a more general setting analyzed in our paper time may be used for market activities, household production and leisure.

We show that this model is fully identifiable; i.e., parents' individual preferences and the decision process (as summarized by the Pareto weights) can generically be recovered from observed behavior.[6] In particular, the observation of changes in expenditures on children in response to changes in wages allows one to recover each parent's willingness to pay for children expenditures. As always identifiability requires restrictions. In our case we show that if leisure and individual consumption are separable from expenditure on children then we can identify individual preferences on private and public goods as well as the Pareto weights, just by observing individual labour supply, aggregate household consumption, the expenditure on the public good and wages. We also show that if separability is not a valid assumption knowledge of a distribution factor, i.e. a variable driving

[6] We use here the standard distinction between identifiability and identification (see f.i. Koopmans 1949). A model is identifiable when an assumed perfect observation of behavior would enable to fully recover the underlying, structural model. For instance, the standard consumer model is identifiable because (as it is well known) preferences can be uniquely recovered from demand functions. Identification, on the other hand, relates to the more general problem of the relationship between data and theory, in which the limits to the observation of behavior are paramount. See Chiappori and Ekeland (2004) for a more precise discussion of this distinction, and Blundell et al (2003) for a discussion of identification in a collective framework.

the Pareto weights but not preferences,[7] allows full identification. Interestingly, the separability assumption may be tested without fully identifying the model, allowing us to ascertain the informational requirements for identification *a priori.*

Finally, the model is extended to include household production. In a way this relaxes further the assumptions of the model because it allows some aspect of time to be public within the household and further reinforces the justification of considering the identification issues within the context of labour supply. We consider a framework in which the public good is used, together with time for each individual, as an input to household production. Again we show that the model is identifiable, under the assumption of productive efficiency, so long as time use data is available, detailing the time inputs that go into household production (as opposed to pure leisure). Of course the measurement problems here can be severe, but this discussion points to both the importance of collecting data on time use and the importance of distinguishing expenditure on private and public goods.

2. The framework

2.1. Commodities, preferences, distribution factors

We consider a static version of the collective model of labor supply for a two-member ($i = 1, 2$) household. There are three commodities: Two individual leisure L^1, L^2 and a Hicksian composite good C; wages (respectively non labor income) are denoted w_1 and w_2 (respectively Y), while the price of the Hicksian good is normalized to one. In contrast with previous versions of the model, we assume that the Hicksian good is used for private expenditures and some public

[7] For instance, of variables affecting each member's threat point in a bargaining framework.

consumption:

$$C = C^1 + C^2 + K \tag{2.1}$$

where K denotes the level of expenditures for the public good. A natural (but not exclusive) interpretation is that K represent the amount spent on children. The identifiability results we will present do *not* require variations in the price of the public good.

An important tool to achieve identification is the presence of *distribution factors* (Bourguignon et al., 1995). These are defined as variables that can affect group behavior only through their impact on the decision process. Think, for instance, of the choices as resulting from a bargaining process. Typically, the outcomes will depend on the members' respective bargaining positions, and *any variable in the household's environment that may influence these positions (EEPs in McElroy's (1990) terminology) potentially affects the outcome.* Such effects are of course paramount, and their relevance is not restricted to bargaining in any particular sense. One crucial insight of the 'targeting' literature is precisely that any variable that changes the balance of powers within the household (say, paying a benefit to the wife instead of the husband) may, everything equal, have an impact on observed collective behavior. Throughout the paper, we denote by z a distribution factor, and we assume that z is a continuous variable.[8]

As it is standard, we assume that L^1, L^2, C and K are observed (as functions of w_1, w_2, Y and z), while the distribution of private consumption within

[8]For instance, Chiappori, Fortin and Lacroix (2002) use sex ratio as a continuous distribution factor for the identification of a collective model of labor supply. Other examples include non labor income (Thomas 1990), labor income in a model involving constrained labor supply (Browning et al. 1994), wealth at marriage (Thomas et al 1997), and benefits (Rubacalva and Thomas 2000).

the couple ($C = C^1 + C^2$) is not. We assume that the functions $L^1(w_1, w_2, Y, z)$, $L^2(w_1, w_2, Y, z)$, $C(w_1, w_2, Y, z)$ and $K(w_1, w_2, Y, z)$ are twice continuously differentiable.

Individual i is characterized by differentiable, strictly increasing, strictly convex preferences $\succsim^i$on leisure, private consumption and the level of public expenditures. We assume that the bundle (w_1, w_2, Y, z) varies within a compact subset $\mathcal{K}$ of $\mathbb{R}^3_+ \times R$; then the vector (L^i, C^i, K) varies within some compact set $\mathcal{K}'$ of $\mathbb{R}^3_+$. The preferences are represented by a twice continuously differentiable utility function U^i, and we assume that U^i is such that the Gaussian curvature of any indifference curve is positive at any point of K' (U^i has no critical point on $\mathcal{K}'$). By a well known theorem[9], preferences can then be represented by a differentiable strictly concave function U^i on $\mathcal{K}'$.[10] Throughout the paper, we use such a representation of individual preferences.

In some sections of the paper, we make the assumption that individual consumption and leisure are separable from the public good K.

$$U^i\left(L^i, C^i, K\right) = W^i\left[u^i\left(L^i, C^i\right), K\right] \tag{S}$$

where W^i and u^i are twice continuously differentiable, strictly increasing and strongly concave. The separability assumption is certainly restrictive, since the level of expenditures on children (say, paying for day-care) may be expected to affects the trade-off between consumption and labor supply at the individual level. We shall see below when it can be relaxed. One point has however to be emphasized here. By separability we do *not* imply that individual preferences are

[9]See for instance. Mas Colell 1985, p. 80

[10]A differentiably strictly concave function such that the Gaussian curvature of any indifference curves is positive at any point is sometimes called a strongly concave function.

not affected by the presence of children. Indeed, we do not use or suggest using observations on the behavior of couples without children or on singles to identify preferences. The latter may be different in arbitrary ways for individuals in all these states. The separability concerns *expenditures* on the public good, taken here to be children. In other words, even under the separability assumption, we allow individual preferences (e.g., the marginal rate of substitution between consumption and leisure) to depend on the *presence* on children; we simply assume that when children are present, preferences are not significantly affected by *how much is actually spent* on them.

2.2. The decision process

Pareto efficiency Following the usual strategy of collective models (Chiappori, 1988, 1992, Blundell et all, 2001), we assume that the decisions made by the household are Pareto efficient. This is equivalent to assuming that household allocations are determined as solutions to the problem

$$\begin{gathered} \max_{L^1,L^2,C^1,C^2,K} \lambda U^1 \left(L^1, C^1, K\right) + \left(1-\lambda\right) U^2 \left(L^2, C^2, K\right) \\ \text{subject to the overall budget constraint} \\ w_1 L^1 + w_2 L^2 + C^1 + C^2 + K = w_1 + w_2 + Y \end{gathered} \tag{2.2}$$

(where the time endowment is normalized to one); note that, since the U^i are strictly concave, so is the maximand of program (2.2). The Pareto weight $\lambda \geq 0$ reflect the relative weight of member 1 in the household. This can be a function of wages (w_1 and w_2) as well as of non labor income Y and distribution factors. We assume that the Pareto weight is continuously differentiable in w_1, w_2, Y and the distribution factors, z (if any).

In practice, thus, observable behavior (i.e., labor supplies, children expenditures and aggregate private consumption) stem from the maximization of some 'household utility function' H, defined by

$$H\left(L^1, L^2, C, K; \lambda\right) = \max_{C^1} \lambda U^1\left(L^1, C^1, K\right) + (1-\lambda) U^2\left(L^2, C - C^1, K\right) \quad (2.3)$$

H is not a standard utility function because it depends on the Pareto weight λ, which itself varies with prices and income; technically, H is thus price-dependent, which implies for instance that the resulting demand functions will typically not satisfy Slutsky symmetry. However, H exhibits separability properties, in the sense that some marginal rates of substitution do not depend on the Pareto weight - a property that has crucial implications for the model. Indeed, first order conditions imply that

$$\lambda \frac{\partial U^1}{\partial C} = (1-\lambda) \frac{\partial U^2}{\partial C}$$

and from the envelop theorem

$$\frac{\partial H}{\partial L^1} = \lambda \frac{\partial U^1}{\partial L^1}, \qquad \frac{\partial H}{\partial L^2} = (1-\lambda) \frac{\partial U^2}{\partial L^2}, \qquad \frac{\partial H}{\partial C} = (1-\lambda) \frac{\partial U^2}{\partial C}$$

It follows that

$$\frac{\partial H / \partial L^i}{\partial H / \partial C} = \frac{\partial U^i / \partial L^i}{\partial U^i / \partial C} \quad (2.4)$$

In words: At the equilibrium point, the household's marginal rate of substitution between individual i's leisure and private consumption is equal to the corresponding individual's marginal rate of substitution between her labour supply and consumption. Hence in general it only depends on L^i, C^i and K, and not on the spouse's labor supply nor on the Pareto weight.

Conditional sharing rule Just as in the private consumption case, the solution to the household problem (2.2) can be thought of as a two stage process. At stage one, agents agree on public expenditures, as well as on a particular distribution of the residual non labor income between them. At stage two, each of the two members freely choose their level of consumption and labor supply, conditional on the level of public expenditures and the budget constraint stemming from stage one. Technically, let $L^{*i}(w_1, w_2, Y, z), C^{*i}(w_1, w_2, Y, z)$, $i = 1, 2$, and $K^*(w_1, w_2, Y, z)$ be the solution of problem (2.2), and define ρ_i by:

$$\rho_i(w_1, w_2, Y, z) = w_i L^{*i}(w_1, w_2, Y, z) + C^{*i}(w_1, w_2, Y, z) - w_i$$

Here, ρ_1 and ρ_2 define the *sharing rule*. In words, ρ_i is the fraction of residual non labor income allocated to member i; ρ_i is conditional in the sense that the members share what is left for private consumption after purchasing the public good. Hence ρ_i can be positive or negative, and

$$\rho_1(w_1, w_2, Y, z) + \rho_2(w_1, w_2, Y, z) = Y - K^*(w_1, w_2, Y, z)$$

We now have the following result:

Proposition 1. *Assume $0 < \lambda < 1$. For any given (w_1, w_2, Y, z), the pair*

$$\left(L^{*i}(w_1, w_2, Y, z), C^{*i}(w_1, w_2, Y, z)\right), i = 1, 2$$

solves

$$\max_{L^i, C^i} U^i\left(L^i, C^i, K\right) \tag{2.5}$$
$$w_i L^i + C^i = w_i + \rho_i$$

where $K = K^*(w_1, w_2, Y, z)$ and $\rho_i = \rho_i(w_1, w_2, Y, z)$.

Proposition 1 states that each agent chooses his (her) private consumption and labor supply by maximizing utility, under the constraint that he (she) cannot spend more than his (her) share of residual non labor income. The proof is straightforward: If a higher utility could be achieved at the same cost (say, for some (L'^i, C'^i)), then the maximand in (2.2) could be increased by replacing (L^i, C^i) with (L'^i, C'^i), a contradiction.

An important remark is that the existence of a (conditional) sharing rule, while implied by efficiency, is not *equivalent* to efficiency. Any efficient decision process can be associated with a sharing rule; but, conversely, an arbitrary sharing rule is not in general compatible with efficiency for a given level of public expenditures. The intuition for this result goes as follows. For given preferences, there exists a continuum of Pareto efficient allocations. In general[11], different efficient outcomes correspond to different levels of the public good and different distributions of private consumptions and labor supplies. Hence, although conditional on K a consumption/labour supply allocation maybe (constrained) efficient, this does not mean that the particular level K is itself the solution to an efficient household allocation under the prevailing prices; consequently, the corresponding L^i, C^i allocation may (and will in general) be inefficient. This shows the limit of the 'two stage' interpretation of the decision process; while formally convenient (as it will become clear below), it should not hide the fact that the level of public expenditures cannot be chosen independently of the allocation of private resources.

¿From a more technical perspective, for any given sharing rule compatibility with the observed level of public expenditures imposes additional conditions, a crucial point for the identifiability results below. However, before stating these

[11] i.e., except for very particular cases such as quasi-linear preferences

conditions, it is convenient to introduce two concepts of indirect utility.

Indirect utilities Let $V^i(w_i, \rho_i, K)$ denote the value of program (2.5); intuitively, V^i is a conditional (on K) indirect utility function for i. We call it the *individual indirect utility* of agent i because it only depends on i's preferences; i.e., it does not vary with the particular decision process at stake (although its argument ρ_i certainly does). As usual, there is a one-to-one correspondence between direct and indirect utilities; i.e., U^i can be deduced from V^i by:

$$U^i\left(L^i, C^i, K\right) = \min_{w_i, \rho_i} V^i(w_i, \rho_i, K) \tag{2.6}$$

$$w_i L^i + C^i = w_i + \rho_i$$

Note also that since preferences are strictly increasing, $\partial V^i / \partial \rho_i > 0$ at each point.

Secondly, for any particular function $\rho_i(w_1, w_2, Y, z)$, we can express V^i directly as a function of wages, non labor income and public expenditures. This requires a slightly technical construction, however, since we must formally translate the fact that K is kept constant. Specifically, let $\mathcal{O}$ be some open subset of K such that $\partial K^* / \partial z$ does not vanish on $\mathcal{O}$. By the implicit function theorem, the condition $K^*(w_1, w_2, Y, z) = K$ allows one to express z as some function ζ of (w_1, w_2, Y, K). Then we can define, over $\mathcal{O}$, the function $\tilde{V}^i$ by:

$$\tilde{V}^i(w_1, w_2, Y, K) = V^i\left[w_i, \rho_i(w_1, w_2, Y, \zeta(w_1, w_2, Y, K)), K\right]$$

In words, V^i describes i's indirect utility when facing a private allocation ρ_i and a level of children expenditures K, while, for any given function ρ_i, $\tilde{V}_i$ describes i's indirect utility when faced with a wage income bundle w_1, w_2, Y and a distribution factor z such that public expenditures are exactly K. The distribution factor plays a key role here because it provides an additional dimension, thus allowing w_1, w_2, Y to vary freely (i.e., in a three dimensional set) while K is kept constant.

We propose to call $\tilde{V}^i$ the *collective indirect utility* of agent i, to reflect the fact that the definition of $\tilde{V}^i$ implicitly includes the sharing function ρ_i, hence an outcome of the collective decision process. In particular, in contrast with the individual indirect utility V^i, the collective indirect utility $\tilde{V}^i$ can only be defined in reference to a particular decision process. Note that whenever normative judgments are at stake, the collective indirect utility is the relevant concept, since it measures the level of utility that will ultimately be reached by each agent, taking into account the redistribution that will take place within the household.

Determination of public expenditures We can now characterize the efficiency conditions for public good expenditures. These take the standard, Bowen-Lindahl-Samuelson form. Namely, assuming an interior solution, the first order conditions for problem (2.2) give:

$$\frac{\partial U^1/\partial K}{\partial U^1/\partial C} + \frac{\partial U^2/\partial K}{\partial U^2/\partial C} = 1 \tag{2.7}$$

Equivalently, one can use the two-stage representation and express the conditions in terms of individual indirect utilities. The optimal choice of (ρ_1, ρ_2, K) solves:

$$\max_{\rho_1,\rho_2,K} \lambda V^1 (w_1, \rho_1, K) + (1-\lambda) V^2 (w_2, \rho_2, K)$$

$$\rho_1 + \rho_2 + K = Y$$

which gives

$$\lambda \frac{\partial V^1}{\partial \rho} = (1-\lambda) \frac{\partial V^2}{\partial \rho} = \lambda \frac{\partial V^1}{\partial K} + (1-\lambda) \frac{\partial V^2}{\partial K}$$

hence

$$\frac{\partial V^1/\partial K}{\partial V^1/\partial \rho}+\frac{\partial V^2/\partial K}{\partial V^2/\partial \rho}=1 \tag{2.8}$$

The ratio $\frac{\partial V^i/\partial K}{\partial V^i/\partial \rho}$ is i's marginal willingness to pay for the public good; condition (2.8) states that the individuals' marginal willingness to pay must add up to the price of the public good.

Finally, the same condition can be expressed in terms of collective indirect utilities. After simple calculations, one gets

$$\alpha\frac{\partial \tilde{V}^1/\partial K}{\partial \tilde{V}^1/\partial Y}+(1-\alpha)\,\frac{\partial \tilde{V}^2/\partial K}{\partial \tilde{V}^2/\partial Y}=1 \tag{2.9}$$

where

$$\alpha=\frac{\partial \rho_1}{\partial Y}+\frac{\partial \rho_1}{\partial z}\frac{\partial \zeta}{\partial Y}$$

In words: If non labor income is changed by one dollar, z being adjusted so as to keep K constant, α is the fraction of this change born by member 1 (obviously,$1-\alpha$ is born by member 2). Again, (2.9) expresses that agents are indifferent to one marginal dollar being spent on the public good, the cost being divided between the agents according to the proportions thus defined.

Separability For general preferences the level of public consumption influences the optimal choice of consumption and labor supply through two channels: An income effect (i.e., more public expenditures means less total private consumption, hence (presumably) a tighter private budget for both members) *and* the direct impact of public expenditures on the consumption-leisure trade-off. In the separable case (S), however, the second effect disappears. Technically, the second stage problem (2.5) becomes:

$$\max_{L^i, C^i} u^i \left(L^i, C^i\right) \tag{2.10}$$

$$w_i L^i + C^i = w_i + \rho_i$$

These separability properties at the individual level are preserved in the optimal value function of the household, although in a somewhat specific way.[12] If individual utilities satisfy the separability property (S), then from equation (2.4) above, the household's marginal rate of substitution between individual i's leisure and total private consumption C satisfy the equation

$$\frac{\partial H/\partial L^i}{\partial H/\partial C} = \frac{\partial u^i/\partial L^i}{\partial u^i/\partial C} \tag{2.11}$$

where the right hand side only depends on L^i and C^i. This means that, if we control for *individual* consumptions, the household MRS does not depend on public good expenditures K. The property must however be handled with care, since it does not lead to the standard separability tests of consumer theory. The right hand side expression in (2.11) is the MRS of individual i, which is taken at (L^i, C^i). Since C^i is not observed, one cannot *directly* test this property in the usual way.

One can also express separability in terms of indirect utilities. The individual indirect utility V^i is such that the individual wage and sharing rule are separable from the public expenditure K. Specifically, let $v^i (w_i, \rho_i)$ denote the value of program (2.10); note that v^i does not depend on K directly. Then the individual's indirect utility function V^i is defined by:

$$V^i (w_i, \rho_i, K) = W^i \left[v^i (w_i, \rho_i), K\right]$$

[12] We are indebted to the editor, Fernando Alvarez, for pointing out this point.

More interestingly, the definition of collective indirect utility $\tilde{V}^i$ no longer requires the presence of a distribution factor. To see why, assume away for a moment the distribution factor (so that all functions only depend on (w_1, w_2, Y)), and let y denote the portion of non labor income not devoted to the purchase of the public good:

$$y = Y - K(w_1, w_2, Y)$$

Clearly, y can be positive or negative. Again, consider some open subset $\mathcal{O}'$ of $\mathcal{K}$ such that $\partial K / \partial Y \neq 1$ on $\mathcal{O}'$; i.e., an additional dollar of non labor income would not be entirely spent on the public good, a natural requirement. By the implicit function theorem, the equation above allows one to express Y as some function Υ of (w_1, w_2, y). Then we can define, over $\mathcal{O}'$, the function $\tilde{V}^i$ by:

$$\tilde{V}^i(w_1, w_2, y) = V^i[w_i, \rho_i(w_1, w_2, \Upsilon(w_1, w_2, y))]$$

Intuitively, since the subutility u^i can be defined independently of K, the additional dimension provided by the distribution factor is no longer needed.

2.3. Collective analysis of welfare: An illustration

This section illustrates the way collective models allow one to study 'targeting', and more generally issues related to intrahousehold distribution of power and its impact on behavior. Our claim is that an explicit formalization of individual preferences over private and public consumption is crucial for the purposes of analyzing the welfare implications of policy reforms and for understanding issues such as child poverty. The implications are far reaching and they are relevant for policy both in the context of developing countries as well as for industrialized ones.

The question we consider here is the following: How does a change in the distribution of power within the household - i.e., here, a change in the Pareto weight λ - affect the expenditure on the public good? In particular, when is it the case that an improvement of the mother's position (say, because a benefit is now targeted to her) increases children's expenditures? We shall see that an answer can readily be given in the theoretical context just developed.

For notational simplicity, we define $\rho \equiv \rho_1$; then $\rho_2 = Y - K - \rho = y - \rho$. Condition (2.8) above can be written as

$$\lambda \frac{\partial V^1 (w_1, \rho, K)}{\partial \rho_1} = (1 - \lambda) \frac{\partial V^2 (w_2, Y - \rho - K, K)}{\partial \rho_2}$$

and

$$MWP^1 (w_1, \rho, K) + MWP^2 (w_2, Y - \rho - K, K) = 1$$

where MWP^i denotes i's marginal willingness to pay for the public good. Using the implicit function theorem on these first order conditions, we get

$$\begin{aligned} \frac{\partial \rho}{\partial \lambda} &= \frac{1}{D} \left[\frac{\partial MWP^1}{\partial K} + \frac{\partial MWP^2}{\partial K} - \frac{\partial MWP^2}{\partial \rho_2} \right] \left(\frac{\partial V^1}{\partial \rho_1} + \frac{\partial V^2}{\partial \rho_2} \right) \\ \frac{\partial K}{\partial \lambda} &= -\frac{1}{D} \left[\frac{\partial MWP^1}{\partial \rho_1} - \frac{\partial MWP^2}{\partial \rho_2} \right] \left(\frac{\partial V^1}{\partial \rho_1} + \frac{\partial V^2}{\partial \rho_2} \right) \end{aligned} \tag{2.12}$$

where D is given by

$$\begin{aligned} D = \; & \left[\lambda \frac{\partial^2 V^1}{\partial \rho_1 \partial K} - (1 - \lambda) \left(\frac{\partial^2 V^2}{\partial \rho_2 \partial K} - \frac{\partial^2 V^2}{\partial \rho_2 \partial \rho_1} \right) \right] \left[\frac{\partial MWP^1}{\partial \rho_1} - \frac{\partial MWP^2}{\partial \rho_2} \right] \\ & - \left[\frac{\partial MWP^1}{\partial K} + \frac{\partial MWP^2}{\partial K} - \frac{\partial MWP^2}{\partial \rho_2} \right] \left[\lambda \frac{\partial^2 V^1}{(\partial \rho_1)^2} + (1 - \lambda) \frac{\partial^2 V^2}{(\partial \rho_2)^2} \right] \end{aligned} \tag{2.13}$$

We assume that preferences are such that the 'goods' ρ and K are normal, i.e. an increase in non labor income boosts both private and public consumptions. Then MWP^i is increasing in ρ_i, decreasing in K, and the expression in the square brackets in $\partial\rho/\partial\lambda$ is negative. Also, $\partial V^1/\partial\rho_1$ and $\partial V^2/\partial\rho_2$ are both positive. Finally, we may, without loss of generality, assume that the difference $DMWP \equiv \frac{\partial MWP^1}{\partial\rho_1} - \frac{\partial MWP^2}{\partial\rho_2}$ is positive. Then $\frac{\partial K}{\partial\lambda}$ has the same sign as $\frac{\partial\rho}{\partial\lambda}$. Increasing λ can thus either increase K and ρ or decrease both. But the second case is impossible, because an increase in 1's weight would then reduce the utility of 1, a contradiction. We conclude that $\frac{\partial K}{\partial\lambda} \geq 0, \frac{\partial\rho}{\partial\lambda} \geq 0$.

We can summarize our findings in the following Proposition:

Proposition 2. *Assume that preferences are such that each member i's marginal willingness to pay for the public good is decreasing in the level of public good and increasing in the member's share ρ_i. Then a marginal change in a member's Pareto weight increases the household's expenditures on the public good if and only if the marginal willingness to pay of this member is more sensitive to changes in his/her share than that of the other member*

2.4. Example: Cobb-Douglas preferences

Assume individual preferences are Cobb-Douglas:

$$U^i(c^i, L^i, K) = \alpha_i \log L^i + (1-\alpha_i)\log C^i + \delta_i \log K \tag{2.14}$$

so that indirect utilities are given by:

$$V^i(w_i, \rho_i, K) = \alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(w_i + \rho_i) - \alpha_i \log w_i + \delta_i \log K$$

Thus the MWP of member i is

$$MWP^i = \frac{\partial V^i/\partial K}{\partial V^i/\partial\rho_i} = \delta_i \frac{w_i + \rho_i}{K}$$

and we get that

$$\frac{\partial MWP^i}{\partial \rho} = \frac{\delta_i}{K}$$

Now note that

$$\frac{\partial MWP^1}{\partial \rho} > \frac{\partial MWP^2}{\partial \rho} \Leftrightarrow \delta_1 > \delta_2 \tag{2.15}$$

Assume furthermore that the Pareto weight of member 1 is

$$\lambda = \frac{lw_1}{lw_1 + w_2}$$

where l is a parameter; note that λ is increasing in l. Straightforward computations lead to the following demand functions:

$$\begin{aligned}
L^1 &= \frac{\alpha_1 l}{(1+\delta_1)\, lw_1 + (1+\delta_2)\, w_2} (w_1 + w_2 + y) && (2.16)\\
C^1 &= \frac{(1-\alpha_1)\, lw_1}{(1+\delta_1)\, lw_1 + (1+\delta_2)\, w_2} (w_1 + w_2 + y) && (2.17)\\
L^2 &= \frac{\alpha_2}{(1+\delta_1)\, lw_1 + (1+\delta_2)\, w_2} (w_1 + w_2 + y) && (2.18)\\
C^2 &= \frac{(1-\alpha_2)\, w_2}{(1+\delta_1)\, lw_1 + (1+\delta_2)\, w_2} (w_1 + w_2 + y) && (2.19)\\
K &= \frac{\delta_1 lw_1 + \delta_2 w_2}{(1+\delta_1)\, lw_1 + (1+\delta_2)\, w_2} (w_1 + w_2 + y) && (2.20)
\end{aligned}$$

We are interested in the impact of l on public good expenditure. From the last equation:

$$\frac{\partial K}{\partial l} = (\delta_1 - \delta_2) \frac{w_1 w_2 (w_1 + w_2 + y)}{(lw_1 + \delta_1 lw_1 + w_2 + \delta_2 w_2)^2}$$

which is positive if and only if $\delta_1 > \delta_2$, hence from (2.15) if and only if $\frac{\partial MWP^1}{\partial \rho} > \frac{\partial MWP^2}{\partial \rho}$. Note, however, that even with $\delta_1 > \delta_2$ it may be the case that $MWP^1 = \delta_1 \frac{w_1+\rho_1}{K} < MWP^2 = \delta_2 \frac{w_2+\rho_2}{K}$ (particularly if w_2 is large with respect to w_1). This remark illustrates the fact that increasing member 1's weight may increase children expenditures even though member 2's marginal willingness to pay for children is higher.

3. Identifiability

The identifiability question relates to our ability to recover individual preferences and the Pareto weight from the sole observation of labor supplies and the expenditures on the public good, as functions of wages and non labor income. As argues in the Introduction, identifiability is a key requirement for guaranteeing the empirical relevance of the normative approach described above: Despite all the conceptual insights it helps formalize, the collective approach would be of little help if the concepts at stake could not be recovered from observed behavior, because the analysis would then have limited empirical content.

Technically, the setting is fully determined by the 3-uple (U^1, U^2, λ) where the U^i and λ are functions mapping $\mathbb{R}^3$ to $\mathbb{R}$. Two 3-uples (U^1, U^2, λ) and $\left(\tilde{U}^1, \tilde{U}^2, \tilde{\lambda}\right)$ are *equivalent* if (i) $U^i = f^i\left(\tilde{U}^i\right)$ for some increasing f^i and (ii) for all (w_1, w_2, Y, z) in $\mathcal{K}$, the solution to the household problem (2.2) is the same for (U^1, U^2, λ) and $\left(\tilde{U}^1, \tilde{U}^2, \tilde{\lambda}\right)$. Condition (i) implies that the Pareto set for (U^1, U^2) and $\left(\tilde{U}^1, \tilde{U}^2\right)$ are always identical, while (ii) imposes in addition that the location of the decision on this frontier is always the same for the two 3-uples. We define a *structure* as a set of equivalent 3-uples.

To any given structure, one can associate labor supply and children expenditures functions $L^1(w_1, w_2, Y, z)$, $L^2(w_1, w_2, Y, z)$ and $K(w_1, w_2, Y, z)$, defined as the solution to the household problem (2.2). The structure is *identifiable* if this mapping from structures to behavior functions is one to one, i.e. if two different structures cannot generate the same labor supply and children expenditures functions. Equivalently, the structure is identifiable if to any given labor supply and children expenditure functions corresponds (at most) one structure.

3.1. Identifiability: Separability and Distribution Factors

In general preferences and the Pareto weights are not identifiable: An observed reduced form, which relate each person's labour supply and children expenditure to wages (w_1 and w_2) and non-labor income (Y), can be generated by a continuum of different structural models. However we show below that under two separate conditions identifiability obtains in the sense that if a structural model is compatible with the 'reduced form' functions, all of which are observable, then this structural model is unique.

Most straightforwardly if we know of a *distribution factor*, namely a variable affecting the Pareto weights but not preferences of either individual, then we show below that at most one structural model corresponds to the observed reduced form.[13] If no distribution factor is available then we show that the uniqueness result is preserved but only within the class of separable utility functions. This means that if a reduced form is compatible with separable preferences those preferences will be unique. However, there will be a continuum of non-separable preferences generating the same reduced form.

It turns out that separability has implications for the 'reduced form' conditional labour supply functions. These implications can provide the basis for a statistical test in an empirical model. If these conditions are not valid preferences

[13] Lack of a solution would imply that the reduced forms do not correspond to the solution of a Collective optimisation problem.

are not separable and thus only identifiable based on a distribution factor. If they are valid, we can identify a separable preference structure that generates the reduced form. Nevertheless the true preferences might still be non-separable but there is no information to establish this, unless there is a distribution factor.

3.2. Identifiability with distribution factors

We now proceed to show that the knowledge of L^1, L^2 and K (as functions of w_1, w_2, Y and z) is sufficient for identifiability of the underlying structure. The general strategy goes as follows: We first consider the information embodied in the labor supply function. We show that the basic intuition of the private consumption case can readily be extended; i.e., it is still possible to identify individual consumptions up to an additive constant (which may depend on the level of public good expenditures). This, in turn, allows us to recover individual indirect utilities *up to an increasing function of the public good.* We then show that by using the public expenditure function we are able to identify the structure.

Labor supply and the sharing rule We first concentrate on private expenditures, and fix public expenditures to some arbitrary level $\bar{K}$. Thus technically we consider as above some open subset $\mathcal{O}$ of K such that $\partial K^*/\partial z$ does not vanish on $\mathcal{O}$, and we impose the condition $K^*(w_1, w_2, Y, z) = \bar{K}$, which by the implicit function theorem is equivalent to $z = \zeta\left(w_1, w_2, Y, \bar{K}\right)$. As above, we use the notation $\rho \equiv \rho_1,\ \rho_2 = Y - K - \rho = y - \rho$. Then L^1, L^2 and ρ are functions of $\left(w_1, w_2, Y, \zeta\left(w_1, w_2, Y, \bar{K}\right)\right)$, hence of (w_1, w_2, Y) since $\bar{K}$ is fixed. Using for now the change in variable $y = Y - \bar{K}$, we can express L^1, L^2 and ρ as functions of (w_1, w_2, y); for notational simplicity, we still denote these functions L^1, L^2 and ρ, since no confusion is to be feared.

Now, consider the two programs in (2.5):

$$\max_{L^1,C^1} U^1\left(L^1, C^1, \bar{K}\right) \tag{3.1}$$

$$w_1 L^1 + C^1 = w_1 + \rho$$

and

$$\max_{L^2,C^2} U^2\left(L^2, C^2, \bar{K}\right) \tag{3.2}$$

$$w_2 L^2 + C^2 = w_2 + y - \rho$$

¿From a theorem in Chiappori (1992), the knowledge of the two labor supply functions allows one to recover the sharing rule and the individual utilities up to an increasing constant; moreover, the constant is welfare irrelevant, in the sense that it does not affect the indirect utility. A formal statement is the following:

Lemma 1. *For any given $\bar{K}$, assume that two 3-uples (U^1, U^2, ρ) and $\left(\hat{U}^1, \hat{U}^2, \hat{\rho}\right)$ generate for all (w_1, w_2, y) the same labor supplies in programs (3.1) and (3.2). Then generically on the 3-uple (U^1, U^2, ρ), there exists a constant $A\left(\bar{K}\right)$ such that, for all (w_1, w_2, y),*

$$\begin{aligned}
\hat{\rho}\left(w_1, w_2, y\right) &= \rho\left(w_1, w_2, y\right) + A\left(\bar{K}\right) \\
\hat{U}^1\left(L^1, C^1, \bar{K}\right) &= f^1\left[U^1\left(L^1, C^1 - A\left(\bar{K}\right), \bar{K}\right), \bar{K}\right] \\
\hat{U}^2\left(L^2, C^2, \bar{K}\right) &= f^2\left[U^2\left(L^2, C^2 + A\left(\bar{K}\right), \bar{K}\right), \bar{K}\right]
\end{aligned}$$

where f^1, f^2 are twice continuously differentiable mappings, increasing in their first argument. Moreover, the individual indirect utilities are such that, for all

(w_1, w_2, y) *(with obvious notations):*

$$\begin{aligned} \hat{V}^1\left(w_1, \hat{\rho}, \bar{K}\right) &= f^1\left[V^1\left(w_1, \rho, \bar{K}\right), \bar{K}\right] \\ \hat{V}^2\left(w_2, y-\hat{\rho}, \bar{K}\right) &= f^2\left[V^2\left(w_2, y-\rho, \bar{K}\right), \bar{K}\right] \end{aligned} \tag{3.3}$$

In particular, the collective indirect utilities *corresponding to the two solutions coincide, again up to an increasing function of* $\bar{K}$*.*

Proof. See Chiappori (1992), Proposition 4. The result is 'generic' is the sense it requires a 'regularity' assumption on labour supplies (condition R in Chiappori 1992); specifically, the set of labour supply functions for which the result does *not* hold is characterized by a partial differential equation. The only adjustment with respect to Chiappori (1992) is that in our context, both the additive constant and the increasing mappings f^i are indexed by the level of public expenditures. That is, the indirect utilities are such that, for any (w_1, w_2, R, K)

$$\begin{aligned} \hat{V}^1\left(w_1, R, K\right) &= f^1\left[V^1\left(w_1, R-A\left(K\right), K\right), K\right] \\ \hat{V}^2\left(w_2, R, K\right) &= f^2\left[V^2\left(w_2, R+A\left(K\right), K\right), K\right] \end{aligned}$$

Thus the functions $\hat{V}^i$ and V^i are different; but the value taken by $\hat{V}^1$ (resp. $\hat{V}^2$) for $R = \hat{\rho}$ (resp. $R = y - \hat{\rho}$) and the value taken by V^1 (resp. V^2) for $R = \rho$ (resp. $R = y - \rho$) coincide up to an increasing function of K. ■

A consequence of the Lemma is that the derivatives can readily be computed:

$$\frac{\partial \hat{V}^1\left(w_1, \hat{\rho}, \bar{K}\right)}{\partial \hat{\rho}} = \frac{\partial f^1\left(V^1, \bar{K}\right)}{\partial V^1} \frac{\partial V^1\left(w_1, \rho, \bar{K}\right)}{\partial \rho_1} \tag{3.4}$$

$$\frac{\partial \hat{V}^1\left(w_1, \hat{\rho}, \bar{K}\right)}{\partial \bar{K}} = \frac{\partial f^1\left(V^1, \bar{K}\right)}{\partial V^1} \frac{\partial V^1\left(w_1, \rho, \bar{K}\right)}{\partial \bar{K}} \tag{3.5}$$

$$
\begin{aligned}
&-\frac{\partial f^1\left(V^1,\bar{K}\right)}{\partial V^1}\frac{\partial V^1\left(w_1,\rho,\bar{K}\right)}{\partial \rho_1}A'\left(\bar{K}\right)\\
&+\frac{\partial f^1\left(V^1,\bar{K}\right)}{\partial \bar{K}}
\end{aligned}
$$

$$
\frac{\partial \hat{V}^2\left(w_2,y-\hat{\rho},\bar{K}\right)}{\partial \hat{\rho}} = \frac{\partial f^2\left(V^2,\bar{K}\right)}{\partial V^2}\frac{\partial V^2\left(w_2,y-\rho,\bar{K}\right)}{\partial \rho_2} \tag{3.6}
$$

$$
\begin{aligned}
\frac{\partial \hat{V}^2\left(w_2,y-\hat{\rho},\bar{K}\right)}{\partial \bar{K}} = {} & \frac{\partial f^2\left(V^2,\bar{K}\right)}{\partial V^2}\frac{\partial V^2\left(w_2,y-\rho,\bar{K}\right)}{\partial \bar{K}}\\
&+\frac{\partial f^2\left(V^2,\bar{K}\right)}{\partial V^2}\frac{\partial V^2\left(w_2,y-\rho,\bar{K}\right)}{\partial \rho_2}A'\left(\bar{K}\right)\\
&+\frac{\partial f^2\left(V^2,\bar{K}\right)}{\partial \bar{K}}
\end{aligned} \tag{3.7}
$$

Lemma 1 states that individual preferences and the sharing rule are identifiable up to some additive constant, which may clearly depend on $\bar{K}$. However, this constant does not affect the value taken by the individual indirect utilities V^i and their derivatives with respect to private shares ρ_i.[14] As always, (indirect) utilities can only be identified up to an increasing monotonic transformation; again, this transformation may depend on $\bar{K}$. In other words, one can identify a 3-uple (V^1, V^2, ρ) such that any other solution $\left(\hat{V}^1, \hat{V}^2, \hat{\rho}\right)$ must satisfy conditions (3.3) to (3.7). From now on, thus, (V^1, V^2, ρ) will be *known* functions; what remains to be identified are the functions f^i.

Finally, note that additional, over-identifiability restrictions are generated; an example will be provided below in the Cobb-Douglas case.

[14] The *functions* V^1 and V^2 are identified only up to the same additive constant as the sharing rule. However, the value taken by the functions (and their derivatives) at this sharing rule is the same for all solutions.

Preferences for public consumption We now consider the demand for public goods. Any solution $\left(\hat{V}^1, \hat{V}^2, \hat{\rho}\right)$, assuming an interior solution, must satisfy the following first order conditions:

$$\frac{\partial \hat{V}^1/\partial K}{\partial \hat{V}^1/\partial \rho_1} + \frac{\partial \hat{V}^2/\partial K}{\partial \hat{V}^2/\partial \rho_2} = 1$$

or, given the previous expression:

$$\frac{1}{\partial V^1/\partial \rho_1} \frac{\partial f^1/\partial K}{\partial f^1/\partial V} + \frac{1}{\partial V^2/\partial \rho_2} \frac{\partial f^2/\partial K}{\partial f^2/\partial V} = 1 - \left(\frac{\partial V^1/\partial K}{\partial V^1/\partial \rho_1} + \frac{\partial V^2/\partial K}{\partial V^2/\partial \rho_1} \right) \quad (3.8)$$

where the V^i are known and the f^i are unknown. Clearly, only the ratio $\frac{\partial f^i/\partial K}{\partial f^i/\partial V}$ can (at best) be identifiable, reflecting the fact that f^i is (at best) identifiable up to some increasing transform only.

Hence let us define

$$\phi^i\left(V^i, K\right) = \frac{\partial f^i/\partial K}{\partial f^i/\partial V}$$

then (3.8) can be rewritten as:

$$\frac{1}{\partial V^1/\partial \rho_1} \phi^1\left(V^1, K\right) + \frac{1}{\partial V^2/\partial \rho_2} \phi^2\left(V^2, K\right) = 1 - \left(\frac{\partial V^1/\partial K}{\partial V^1/\partial \rho_1} + \frac{\partial V^2/\partial K}{\partial V^2/\partial \rho_1} \right) \quad (3.9)$$

We now proceed to show that generically (in a sense that will be made precise later), the solution to this equation (if any) is unique. The result is coming from the fact that the unknowns are functions of two variables only, while the equation depends in general on four variables (w_1, w_2, Y, z). To use this feature, let us first note that (3.9) is linear in ϕ^1 and ϕ^2. Thus, if there exist two distinct solutions (ϕ^1, ϕ^2) and (ϕ'^1, ϕ'^2), the differences

$$\psi^i = \phi^i - \phi'^i$$

must satisfy the homogenous equation:

$$\frac{1}{\partial V^1/\partial \rho_1}\psi^1\left(V^1,K\right)+\frac{1}{\partial V^2/\partial \rho_2}\psi^2\left(V^2,K\right)=0 \tag{3.10}$$

At any point such that $\psi^i\left(V^i,K\right)\neq 0$, one must have that $\psi^j\left(V^j,K\right)\neq 0$ for $i\neq j$, and (3.10) can be written as:

$$\log\psi^1\left(V^1,K\right)-\log\psi^2\left(V^2,K\right)=\log\left(\frac{\partial V^1/\partial \rho_1}{\partial V^2/\partial \rho_2}\right)$$

which requires that the right hand side function, $\log\left(\frac{\partial V^1/\partial \rho_1}{\partial V^2/\partial \rho_2}\right)$, be the sum of a function of (V^1,K) and a function of (V^2,K). For generic functions V^1, V^2 and ρ, this property is almost never satisfied, hence it must be the case that

$$\psi^1\left(V^1,K\right)=\psi^2\left(V^2,K\right)=0$$

almost everywhere. A more precise statement can be found in Appendix. We conclude that, when equation (3.9) has a solution, the solution is generically unique.

Lemma 1 states that the labor supply functions allow one to identify the collective indirect utilities up to an increasing function of K. We have showed here that once children expenditures are taken into account, identifiability obtains up to an increasing transform - i.e., the corresponding indirect preferences are exactly identified.

It remains to see whether a solution to equation (3.9) exists at all; this generates additional, over-identifiability restrictions, an example of which is provided below for the Cobb-Douglas example. Finally, once a particular cardinalization has been chosen, one can recover the Pareto weight from the first order conditions:

$$\lambda\frac{\partial V^1}{\partial \rho}=(1-\lambda)\frac{\partial V^2}{\partial \rho}\Leftrightarrow\lambda=\frac{\partial V^2/\partial \rho}{\partial V^1/\partial \rho+\partial V^2/\partial \rho}$$

Our results can be summarized in the following statement:

Proposition 3. *Let L^1, L^2 and K be given functions of (w_1, w_2, Y, z). Generically, the knowledge of these functions identify the corresponding collective indirect utilities up to some increasing mappings. Moreover, for any particular cardinalization, the Pareto weight is exactly identified.*

3.3. Implications of separability for the reduced form labour supply functions in the absence of distribution factors

To provide the basis for test of separability of leisure and consumption from expenditure on children based on observable quantities, we derive the implications of this restriction on the reduced form starting from the equilibrium relationship:

$$\frac{\partial H/\partial L^i}{\partial H/\partial C} = \frac{\partial u^i/\partial L^i}{\partial u^i/\partial C}$$

which only depends on (L^i, C^i) and not on K. However, taking this property to data is a delicate task, because C^i is not observed.

In what follows, we assume that consumption is always a normal good at the individual level, and that aggregate private consumption C is an increasing function of non labor income. We first define $\tilde{L}^i$ as i's *conditional* demand for leisure (i.e., i's demand for leisure as a function of w_1, w_2 and C);[15] note that the

[15] Technically, let $C(w_1, w_2, y)$ denote the aggregate consumption function. From the implicit function theorem, the relationship

$$C = C(w_1, w_2, y)$$

can be inverted into

$$y = \Gamma(w_1, w_2, C)$$

Plugging this into the individual demands for leisure gives

$$\tilde{L}^i(w_1, w_2, C) = L^i(w_1, w_2, \Gamma(w_1, w_2, C)).$$

$\tilde{L}^i$ are known from the data.

Since consumption is a normal good, the individual first order conditions can be inverted and expressed as

$$C^i = \phi^i \left(w_i, L^i \right) \tag{3.11}$$

where ϕ^i is increasing. It follows that the conditional demands for leisure $\tilde{L}^i$ must satisfy a relationship of the form:

$$\phi^1 \left(w_1, \tilde{L}^1 \left(w_1, w_2, C \right) \right) + \phi^2 \left(w_2, \tilde{L}^2 \left(w_1, w_2, C \right) \right) - C = 0 \tag{3.12}$$

for some well chosen ϕ^i. In a lemma presented in the appendix we show that, generically, this is not the case; i.e., such ϕ^i do not exist unless the functions $\tilde{L}^i$ satisfy necessary conditions, which take the form of partial differential equations (PDEs). These can be hard to take to the data but it does show that separability has implications for the observable relationships, even in the absence of a distribution factor.

Moreover, the conditions can readily be tested on a parametric form. Assume, for instance, that conditional labor supply have the quadratic form:

$$\begin{aligned} \tilde{L}^1 \left(w_1, w_2, C \right) &= a_1^1 w_1 + a_2^1 w_2 + b_1^1 \left(w_1 \right)^2 + b_2^1 \left(w_2 \right)^2 + c^1 w_1 w_2 + d^1 C + e^1 C^2 + f^1 \\ \tilde{L}^2 \left(w_1, w_2, C \right) &= a_1^2 w_1 + a_2^2 w_2 + b_1^2 \left(w_1 \right)^2 + b_2^2 \left(w_2 \right)^2 + c^2 w_1 w_2 + d^2 C + e^2 C^2 + f^2 \end{aligned}$$

For generic values of the parameters, whether the PDE are not satisfied can be checked; i.e., one cannot find two functions ϕ^1, ϕ^2 such that (3.12) is satisfied. Moreover, one can derive sufficient conditions for the existence of ϕ^1, ϕ^2. For instance, the conditions are satisfied if $e^1 = e^2 = 0$, or alternatively if $b_j^i = c^i = 0$

for all i, j.[16]

The key point of this result is that it provides the basis for testing for separability, within the context of an empirical analysis, when a distribution factor is unavailable; we exploit both the variation implied by unearned income and the particular structure implied by separability. The conclusion may be that the reduced form is not compatible with any separable preference structure. In the presence of a distribution factor a simpler test can be devised.

3.4. Identification under Separability

The proof of identifiability in the presence of distribution factors applies here noting however that we are now solving for separable preferences. The knowledge of labor supplies and expenditures on children K as functions of wages and non labor income only is sufficient to recover the underlying structure. Indeed, the only role of the distribution factor was to introduce an additional, observable dimension that allows wages and non labor incomes to vary while children expenditures are kept constant. This was needed because changes in expenditures on children would generally modify the individual trade-off between leisure and private consumption, hence hampering the analysis of labor supply. Clearly, this concern does not exist in the separable case. In addition, it is possible that no distribution factor is available, in the sense that any such variable may be considered as affecting preferences. In his case separability is a sufficient condition that allows identifiability.

[16] The computations are particularly tedious, and not reported here. They are available from the authors.

Programs (3.1) and (3.2) become:

$$\max_{L^1,C^1} u^1\left(L^1,C^1\right) \tag{3.13}$$
$$w_1L^1+C^1=w_1+\rho$$

and

$$\max_{L^2,C^2} u^2\left(L^2,C^2\right) \tag{3.14}$$
$$w_2L^2+C^2=w_2+y-\rho$$

While variations in wages and non labor income do change expenditures on children, this effect is irrelevant for the study of labor supply since the only impact is through income effects, which are anyway captured by the sharing rule. In practice, labor supplies can be estimated as functions on wages and residual non labor income $y=Y-K$; changes in K are fully captured through their impact on y. Then the proof proceeds as before. Namely, programs (3.13) and (3.14) identify the value of the individual utilities and of their derivatives; then the first order conditions for public expenditures generically recover the utility functions W^1 and W^2, the argument being exactly the same as above.

3.5. A general example

In the Cobb-Douglas example given above (subsection 2.4), one can readily check that the parameters of the structural model can indeed be recovered from the demand functions, even in the absence of a distribution factor. This fact is an immediate consequence of the results above, since the initial model was separable.

We now consider an example that illustrates the limits of identification in the absence of a distribution factor. Consider the following nonseparable preferences:

$$U^1\left(L^1,C^1,K\right) = \alpha_1\log\left(L^1+\gamma K\right)+(1-\alpha_1)\log C^1+\delta_1\log K$$

$$U^2\left(L^2, C^2, K\right) = \alpha_2 \log L^2 + (1-\alpha_2)\log C^2 + \delta_2 \log K \tag{3.15}$$

Note that U^1 is *not* separable in the sense of (S) unless the coefficient γ is zero. From straightforward computations, the demands implied by this model are:

$$\begin{aligned}
L^1 &= \left(\frac{\alpha_1\lambda}{w_1} - \gamma\frac{\delta_1\lambda + \delta_2(1-\lambda)}{(1-\gamma w_1)}\right)\frac{(w_1+w_2+y)}{(1+\delta_1\lambda+\delta_2(1-\lambda))} \\
L^2 &= \frac{1}{w_2}\frac{\alpha_2(1-\lambda)}{(1+\delta_1\lambda+\delta_2(1-\lambda))}(w_1+w_2+y) \\
C^1 &= \frac{(1-\alpha_1)\lambda}{(1+\delta_1\lambda+\delta_2(1-\lambda))}(w_1+w_2+y) \\
C^2 &= \frac{(1-\alpha_2)(1-\lambda)}{(1+\delta_1\lambda+\delta_2(1-\lambda))}(w_1+w_2+y) \\
K &= \frac{\delta_1\lambda+\delta_2(1-\lambda)}{(1+\delta_1\lambda+\delta_2(1-\lambda))(1-\gamma w_1)}(w_1+w_2+y)
\end{aligned}$$

Assume, as above, that member 1 (respectively 2) has a Pareto weight λ (respectively $1-\lambda$), and the Pareto weights are proportional to wages; i.e.,

$$\lambda = \frac{lw_1}{lw_1+w_2} \tag{3.16}$$

then

$$L^1 = \left(\frac{\alpha_1 l(1-\gamma w_1) - \gamma(\delta_1 lw_1+\delta_2 w_2)}{(1-\gamma w_1)}\right)\frac{w_1+w_2+y}{(1+\delta_1)lw_1+(1+\delta_2)w_2} \tag{3.17}$$

$$\begin{aligned}
C^1 &= \frac{(1-\alpha_1)lw_1}{(1+\delta_1)lw_1+(1+\delta_2)w_2}(w_1+w_2+y) \\
L^2 &= \frac{\alpha_2}{(1+\delta_1)lw_1+(1+\delta_2)w_2}(w_1+w_2+y) \qquad (3.18) \\
C^2 &= \frac{w_2(1-\alpha_2)}{(1+\delta_1)lw_1+(1+\delta_2)w_2}(w_1+w_2+y) \\
K &= \frac{\delta_1 lw_1+\delta_2 w_2}{((1+\delta_1)lw_1+(1+\delta_2)w_2)(1-\gamma w_1)}(w_1+w_2+y)
\end{aligned}$$

hence

$$C = \frac{(1-\alpha_1)lw_1+(1-\alpha_2)w_2}{(1+\delta_1)lw_1+(1+\delta_2)w_2}(w_1+w_2+y)$$

Conditional demands for leisure are:

$$\begin{aligned}
\tilde{L}^1 &= \left(\frac{\alpha_1 l \left(1-\gamma w_1\right) - \gamma \left(\delta_1 l w_1 + \delta_2 w_2\right)}{\left(1-\gamma w_1\right)}\right) \frac{C}{\left(1-\alpha_1\right) l w_1 + \left(1-\alpha_2\right) w_2} \\
\tilde{L}^2 &= \frac{\alpha_2}{\left(1-\alpha_1\right) l w_1 + \left(1-\alpha_2\right) w_2} C
\end{aligned}$$

Now suppose we check whether the conditions characterizing separability are satisfied. Not surprisingly, they are satisfied when $\gamma = 0$ (i.e., when the initial utilities are indeed separable); indeed, one then has that

$$\frac{1-\alpha_1}{\alpha_1} w_1 \tilde{L}^1 + \frac{1-\alpha_2}{\alpha_2} w_2 \tilde{L}^2 - C = 0$$

More surprisingly, however, the conditions are also satisfied when $\gamma \neq 0$. Indeed, define ϕ^1 and ϕ^2 by:

$$\begin{aligned}
\phi_1\left(w_1, L^1\right) &= -\frac{\left(1-\alpha_1\right)}{\gamma\left(\alpha_1+\delta\right)} \left(1-\gamma w_1\right) L^1 \\
\phi_2\left(w_2, L^2\right) &= \left(\left(\frac{1-\alpha_2}{\alpha_2} - \frac{1-\alpha_1}{\left(\alpha_1+\delta_1\right)} \frac{\delta_2}{\alpha_2}\right) w_2 + \frac{1-\alpha_1}{\gamma\left(\alpha_1+\delta_1\right)} \frac{\alpha_1 l}{\alpha_2}\right) L^2
\end{aligned}$$

then one can check that property (3.12) is satisfied.

While this result may seem paradoxical, it is in fact fully compatible with the previous results, and it helps understanding their exact scope. In the absence of a distribution factor, the model is not identifiable; hence the reduced forms (3.18) are consistent with the structural model defined by (3.15) and (3.16), but also with a continuum of different structural models. As it turns out, this particular example is such that one of these alternative structures involves separable preferences. In other words, there exist two separable utilities $\hat{U}^1$ and $\hat{U}^2$, obviously different of the U^1 and U^2 defined by (3.15), that generate the same observable labor supply functions.[17] This stresses a point made earlier. If the reduced forms have been generated by some separable structure, they must satisfy conditions (3.12) above. But the converse is not true. If the reduced forms satisfy conditions (3.12), they are consistent with separable preferences, but this does not exclude the possibility that they have been generated by some non-separable structure. Separability is an identifying assumption, precisely in the sense that (without distribution factors) uniqueness obtains only *within the class* of separable structures.

Finally, assume that a distribution factor is available; i.e., the coefficient l is a function of some observable variable z. Then identification obtains within the general set. The trick is that the separable utilities computed from the functions ϕ_1 and ϕ_2 above can no longer be considered as solutions, because the second utility explicitly depends on l, hence on z - which contradicts the definition of a distribution factor. In other words, the identifying assumption is precisely that the distribution factor z has no *direct* impact on preferences, and matters only through the Pareto weight. Among the numerous structural models that generate the reduced form (3.18), only the initial one (3.15) satisfies this property.

3.6. Application: Household production

Finally, we extend our basic model to include household production.[18] Specifically, we assume that child utility is 'produced' using specific expenditures and parental time. The child's welfare is modeled as

$$u^K\left(K, h^1, h^2\right)$$

[17]This case is peculiar in the sense that, in general, arbitrary reduced form are not consistent with any separable structure, as illustrated by the derivation of the implications of separability provided above.

[18]Ssee also Chiappori (1997) for identification results with marketable household goods and discussion of identification issues with incomplete markets.

where h_i^K denotes the time spent by member i on household production. In particular, the time constraint for member i becomes

$$L^i + l^i + h^i = 1$$

where L^i denotes leisure, l^i market work and h^i household work. Child welfare is a public good, so that individual preferences take the form

$$U^i\left(C^i, L^i, u^K\left(K, h^1, h^2\right)\right)$$

Since the outcome of the production is the child's utility, it is not observable and it is defined only up to an increasing transformation. Namely, if a structure entails the utility functions $\left(U^1, U^2, u^K\right)$ and the Pareto weight λ, for any strictly increasing function g one can define $\left(\hat{U}^1, \hat{U}^2, \hat{u}^K\right)$ by:

$$\begin{aligned} \hat{u}^K\left(K, h^1, h^2\right) &= g\left[u^K\left(K, h^1, h^2\right)\right] \\ \hat{U}^i\left(C^i, L^i, u^K\right) &= U^i\left(C^i, L^i, g^{-1} \circ u^K\right) \end{aligned}$$

and the structure $\left(\hat{U}^1, \hat{U}^2, \hat{u}^K\right)$, λ is observationaly equivalent to the initial one. It follows that only the functions

$$\varphi^1\left(K, h^1, h^2\right) = \frac{\partial u^K / \partial h^1}{\partial u^K / \partial K}, \qquad \varphi^2\left(K, h^1, h^2\right) = \frac{\partial u^K / \partial h^2}{\partial u^K / \partial K}$$

are identifiable (at best).

Clearly, identifiability of such a structure depends on the type of data that are available. Without time use data one cannot identify household production since neither the inputs nor the output is observed. Now suppose time use data is available; i.e., h^1 and h^2 are known functions of (w_1, w_2, Y, z). Then productive efficiency requires:

$$\varphi^1\left(K, h^1, h^2\right) = w_1 \tag{3.19}$$

and

$$\varphi^2\left(K, h^1, h^2\right) = w_2$$

A first result states that, generically, these equations (3.19) are sufficient to identify the φ^i almost everywhere.[19]

Lemma 2. *Take any point $P = (w_1, w_2, Y, z)$ at which the Jacobian determinant of the reduced forms*

$$D_{(w_1,w_2,Y)}\left(h^1, h^2, K\right) = \begin{pmatrix} \frac{\partial h^1}{\partial w_1} & \frac{\partial h^1}{\partial w_2} & \frac{\partial h^1}{\partial Y} \\ \frac{\partial h^2}{\partial w_1} & \frac{\partial h^2}{\partial w_2} & \frac{\partial h^2}{\partial Y} \\ \frac{\partial K}{\partial w_1} & \frac{\partial K}{\partial w_2} & \frac{\partial K}{\partial Y} \end{pmatrix}$$

[19] To see why, note first that, generically, the Jacobian matrix

$$D_{(w_1,w_2,Y)}\left(h^1, h^2, K\right) = \begin{pmatrix} \frac{\partial h^1}{\partial w_1} & \frac{\partial h^1}{\partial w_2} & \frac{\partial h^1}{\partial Y} \\ \frac{\partial h^2}{\partial w_1} & \frac{\partial h^2}{\partial w_2} & \frac{\partial h^2}{\partial Y} \\ \frac{\partial K}{\partial w_1} & \frac{\partial K}{\partial w_2} & \frac{\partial K}{\partial Y} \end{pmatrix}$$

is invertible almost everywhere on $\mathcal{K}$. In a neighbourhood of any point where the matrix is invertible, one can consider the mapping:

$$(w_1, w_2, Y) \rightarrow \left(h^1, h^2, K\right)$$

This mapping can be inverted as:

$$w_1 = \omega_1\left(h^1, h^2, K\right),\ w_2 = \omega_2\left(h^1, h^2, K\right),\ Y = \theta\left(h^1, h^2, K\right)$$

where ω_1, ω_2 and θ are known functions. Then (3.19) simply implies that $\phi^i = \omega_i$, $i = 1, 2$.

is non zero. There exists an open, connected neighborhood $\mathcal{V}$ of P such that the functions φ^1 and φ^2 satisfying (3.19), if they exist, are unique over $\mathcal{V}$.

Proof: On $\mathcal{V}$, consider the change in variables in 3.19 *defined by the following mapping* Γ*:*

$$(w_1, w_2, Y, z) \to \left(h^1, h^2, K, z\right)$$

By the implicit function theorem, this mapping can be inverted as:

$$w_1 = \omega_1\left(h^1, h^2, K, z\right),\ w_2 = \omega_2\left(h^1, h^2, K, z\right),\ Y = \theta\left(h^1, h^2, K, z\right),\ z = z$$

where ω_1, ω_2 and θ are known, continuously differentiable functions, and the functions $\omega_1, \omega_2, \theta$ are unique on $\mathcal{V}$. If one of the $\partial\omega_i/\partial z$ does not vanish at some point of $\mathcal{V}$, then (3.19) does not have a solution. If, alternatively, the ω_i do not depend on z over $\mathcal{V}$, then $\varphi^i = \omega_i$, $i = 1, 2$ are the unique solution to (3.19) on $\mathcal{V}$.

The Lemma above is local; it can however be extended to any connected set over which the determinant $D_{(w_1,w_2,Y)}\left(h^1, h^2, K\right)$ does not vanish. Also, the condition $\partial\omega_1/\partial z = \partial\omega_2/\partial z = 0$ generates testable conditions on the functions h_i and K; namely, if the vector t is defined by

$$t = \begin{pmatrix} \frac{\partial h^1}{\partial w_1} & \frac{\partial h^1}{\partial w_2} & \frac{\partial h^1}{\partial Y} \\ \frac{\partial h^2}{\partial w_1} & \frac{\partial h^2}{\partial w_2} & \frac{\partial h^2}{\partial Y} \\ \frac{\partial K}{\partial w_1} & \frac{\partial K}{\partial w_2} & \frac{\partial K}{\partial Y} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial h^1}{\partial z} \\ \frac{\partial h^2}{\partial z} \\ \frac{\partial K}{\partial z} \end{pmatrix}$$

then the first two components of t are identically zero on $\mathcal{V}$.

When the functions φ^i are exactly known, additional testable restrictions must be imposed to reflect the fact that the φ^i are ratios of partials of the same function u^K. Namely, one can readily show that in the neighborhood of any point such that $\frac{\partial u^K}{\partial K} \neq 0$, the φ^i must satisfy

$$\frac{\partial\varphi^1}{\partial h^2} + \varphi^1\frac{\partial\varphi^2}{\partial K} = \frac{\partial\varphi^2}{\partial h^1} + \varphi^2\frac{\partial\varphi^1}{\partial K}$$

If this condition is satisfied, then u^K is known up to an increasing transform. By the remark above, we can without loss of generality choose arbitrarily this transform (i.e., a particular cardinalization of u^K); then u^K is known as a function of (K, h^1, h^2). Since (K, h^1, h^2) are themselves known functions of (w_1, w_2, Y, z), u^K is ultimately known as a function of (w_1, w_2, Y, z). Then Proposition 3 applies (replacing $K(w_1, w_2, Y, z)$ with $u^K(w_1, w_2, Y, z)$), and we conclude that the structure is identifiable.

3.7. Taking the model to the Data

The development of our results has had empirical analysis in mind. Indeed the identifiability results relate to the type of data one may have available such as the UK Family Expenditure Survey or the US Consumer Expenditure Survey. These include information on household composition, labour supply of individual members and household consumption broken down by very detailed categories. We require information on expenditures on children, such as food, clothing, education etc. which are not always available but can clearly be collected on the basis of diaries. For identifying models of household production we would also require a survey in which household members keep a time use diary such as in the American Time Use Survey published by the Bureau of Labor Statistics or other similar surveys listed in the BLS web site.[20] The time use data would need to be combined with expenditure data which is feasible and can be justified in terms of the insights that can be gained for public policies.

However any empirical analysis would require a stochastic specification and of course additional identification results in the presence of such randomness.

[20]See http://www.bls.gov/tus/home.htm#overview

Typically we would allow some of the parameters in the utility functions of the individual to be random, in which case we would end up generally with a model which includes non-separable errors. This is a well know problem in empirical demand and labour economics.

General identification results are not possible here but we can illustrate the issues given our Cobb-Douglas example earlier. In the utility functions 2.14 we could allow the parameters α_1, α_2 and δ_1, δ_2 to be random. Although this seems excessive, note that it is plausible to treat the members of the household in a symmetric way. Moreover, the randomness in the parameters δ_1 and δ_2 ensure that the expenditure on children is not a deterministic function given observables. This specification would imply non-separable errors in the two observable labour supply equations and the equation for expenditures on children, which is common in structural models other than in the simplest of cases. Identification and estimation with exogenous wages and no corners is straightforward given suitable distributional assumptions on the errors. With endogenous wages we would need to specify a joint model of wages, labour supply and expenditures on children and we could then use standard maximum likelihood methods for estimation. The hardest issue one would encounter in an empirical implementation would be the existence of corner solutions in labour supply, since censoring can lead to serious identifiability issues both from the point of view of the theoretical collective model and from the point of view of the econometric model.[21] Finally note that one can construe specifications with much simpler stochastic specifications, in which errors are additive as in Blundell et al. (2003).

[21] See Blundell, Chiappori, Magnac and Meghir (2003)

4. Conclusion

It is now becoming widely accepted that in order to analyze the way that resources are shared within a household we need to model the household as a collective of individuals rather than as one individual unit. This framework can address issues such as targeting of benefits or distribution of income and consumption within the household. In addition to the clear welfare and policy implications of the collective model it may also hold the key to why the restrictions from utility theory are often rejected when the unitary model is applied to multi-member households. Indeed recent evidence suggests that this may well be the case (Browning and Chiappori, 1999 and Blundell et al., 2001).

In this paper we extend the Chiappori (1992) framework to include expenditure on public goods, which we like to think of as expenditure on children. We derive the welfare implications of such a model and show that it offers important insights on the issue of targeting. This is uppermost in the policy agenda both in developed and in developing countries since governments are particularly concerned about delivering benefits to children, such as schooling or nutrition subsidies. We show that a shift in the Pareto weight towards member one, say, will always increase member one's private consumption (under separability). It will also increase the demand of the public good if and only if the marginal willingness to pay of member one is more sensitive to increases in private consumption than member 2. The result emphasizes that basing policy on presumptions about the level of marginal willingness to pay for the public good (e.g. children) is wrong.

The critical parameters for such a policy are an empirical question and that is precisely why our identifiability theorems are of central importance for the empirical analysis of policies that are supposed to affect the distribution of welfare within a household. We prove identifiability of the structure (which consists of the preferences of each adult in the couple over leisure, consumption and expenditure on the public good and the Pareto weights) from data that is typically observable in practice, namely labour supply of individual members, aggregate household expenditure and expenditure on the public good.

Identifiability obtains under two different conditions. First if a variable affecting the Pareto weights but not preferences is available - we call this a distribution factor since it affects the distribution of power in the household. Alternatively if a distribution factor is not available, the structure is still identifiable if preferences for consumption and leisure of each household member are weakly separable from expenditures on public goods. Interestingly, we show that separability can has implications for the reduced form labour supply functions that can be checked; this can provide the basis for an empirical test even without the availability of a distribution factor and allows the validity of the identification strategy to be examined. Even so the conclusion can be that the structure is inconsistent with separable preferences and hence not identifiable without a distribution factor.

Finally we develop identifiability results for the case where the public good is an input to household production together with time for each individual. Again we show that the model is identifiable, under the assumption of productive efficiency, so long as time use data is available, detailing the time inputs that go into household production (as opposed to pure leisure). Of course the measurement problems here can be severe, but this discussion points to the importance of collecting data both on time use and the importance of distinguishing expenditure on private and public goods.

5. Appendix

The Lemma and its proof on which the separability test is based.

Lemma 3. *Let f and g be some arbitrary twice continuously differentiable functions of (x, y, z). Assume that there exist two functions a, b such that*

$$a\left(x, f\left(x, y, z\right)\right) + b\left(y, g\left(x, y, z\right)\right) = z \qquad (R)$$

for all (x, y, z). Then for any point $(\bar{x}, \bar{y}, \bar{z})$ such that $\partial f/\partial z\left(\bar{x}, \bar{y}, \bar{z}\right) \neq 0$, the functions f and g must satisfy, in an open neighborhood of $(\bar{x}, \bar{y}, \bar{z})$, either one or two PDE.

Proof: (R) implies that

$$-\frac{f_y}{f_z} = \frac{b_y + b_g g_y}{1 - b_g g_z}$$

where h_a denotes the partial $\partial h/\partial a$. Hence

$$\begin{aligned} b_y &= -\frac{f_y}{f_z} + b_g\left(\frac{f_y}{f_z} g_z - g_y\right) \\ &= u + b_g v \end{aligned}$$

where the functions u and v are defined by $u = -\frac{f_y}{f_z}$ and $v = \frac{f_y}{f_z} g_z - g_y$; note that these functions only depend on f and g, so that a condition involving these functions is a condition on f and g.

Differentiating with respect to x and z:

$$\begin{aligned} b_{yg} g_x &= u_x + b_{gg} g_x v + b_g v_x \\ b_{yg} g_z &= u_z + b_{gg} g_z v + b_g v_z \end{aligned}$$

hence

$$g_x u_z - g_z u_x = \left(g_z v_x - g_x v_z\right) b_g$$

Two cases can be distinguished:

- In the non generic case where $g_z v_x - g_x v_z = 0$, it must be the case that $g_x u_z - g_z u_x = 0$, which gives a PDE.

- In the alternative, general case in which $g_z v_x - g_x v_z \neq 0$, then

$$b_g = \frac{g_x u_z - g_z u_x}{g_z v_x - g_x v_z}$$

The right hand side expression must be expressed as a function of y and g only, which gives a first PDE, namely

$$g_z \frac{\partial}{\partial x}\left(\frac{g_x u_z - g_z u_x}{g_z v_x - g_x v_z}\right) - g_x \frac{\partial}{\partial z}\left(\frac{g_x u_z - g_z u_x}{g_z v_x - g_x v_z}\right) = 0$$

In addition, we have that

$$b_y = u + \frac{g_x u_z - g_z u_x}{g_z v_x - g_x v_z} v$$

and the two partials of b must satisfy the standard cross derivative restrictions, which provides a second PDE.

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