Ardua et Astra: *On the Calculation of the Dates of the Rising and Setting of Stars*

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From our earliest texts onwards, we find classical authors referring to the time of year by means of the rising and setting of various stars and constellations. Hesiod encourages us to begin the harvest at the rising of the Pleiades, and to plough at their setting; Alcaeus encourages us to drink at the rising of Sirius; and Horace assures us that a man content with his lot is not disturbed at the setting of Arcturus or the rising of Haedus.[1] Indeed, Quintilian suggests that a knowledge of the stars is necessary to understand poetry, because poets so often specify the time by means of the rising and setting of stars.[2] However, it is not just poets who rely on the stars to specify the time: we find a similar use in prose authors such as Thucydides or Hippocrates.[3] This is unsurprising, as before the reform of Julius Caesar, the calendars of the Greeks and Romans, with their erratic intercalations, were frequently out of step with the seasons and often gave no reliable indication of the time of year.[4]

It is often helpful to know when these phenomena actually took place; and in some cases, such as Ovid's *Fasti*, it can inform both textual and literary criticism. In the *Fasti* Ovid provides over forty dates for the rising and setting of various stars and constellations, following the tradition of the Greek parapegmata.[5] In the early nineteenth century, the German mathematician and chronologist Ideler published an article comparing Ovid's dates with those he calculated for Ovid's time;[6] he found that Ovid made a large number of errors in his dating, and since then criticism of this aspect of the poem has been common.[7] More recently, however, some scholars have suggested that rather than astronomical incompetence, these 'mistakes' may show evidence of Ovid's deliberate manipulation of his sources for his own literary purposes.[8]

The example of the *Fasti* is instructive, as all analysis of the astronomical passages of the work has been (until very recently) been based on Ideler's nineteenth century article, despite the fact that methods for calculating modern dates have been refined considerably since then. The reluctance of scholars to calculate these dates for themselves is understandably not uncommon: in addition to the recent works on the *Fasti* that rely on the calculations of Ideler, we find for instance that Aujac's 1975 commentary on Geminus relies on the calculations of Hofmann published in 1879.[9]

One of the problems that can arise when one does not make the calculations oneself is a lack of awareness of the many variables that are involved in the process of calculating these dates, and the various uncertainties that arise as a result. For example, although Ideler mentions some of the assumptions he is making, the uncertainties in his calculations have not filtered through into later commentaries, which tend to assume these dates are precise. When Aujac discusses the disagreements between the dates in the parapegma attached to Geminus' *Isagoge* and those in Hofmann's tables, she raises the possibility that the calculations are wrong, but not that they might have a significant error margin.[10] In general it seems that some scholars have the not unreasonable idea that modern computational methods will provide a precise date for these phenomena;[11] and those who do attempt to discover the possible error margins for these calculations are reassured that it is only of the order of plus or minus two days.[12] As we shall see, however, there are good reasons for thinking that the uncertainty surrounding these dates is in many cases considerably larger.

The purpose of this article is to review the various methods currently available to scholars for the calculations of these dates, some of which make it very easy for scholars to perform these calculations themselves; to examine more closely the various assumptions that such calculations require; and also to discuss the various problems involved with actually observing these phenomena.

Some basic astronomy

In order to understand some of the issues involved, it may be helpful to begin with a little basic astronomy. I hope the text below is reasonably clear, but I have made some diagrams and animations illustrating what follows available on the web.[13]

From the earth it seems as if the stars are on a sphere that rotates from east to west around the north celestial pole. So if we were to look vertically upwards at the Earth's north pole, the stars would appear to rotate in horizontal planes parallel to the horizon, never rising or setting. As we move south and our latitude decreases, the north celestial pole appears to move down in the sky, until when we reach the equator it appears to be in the same horizontal plane as the horizon; the stars would appear to be rotating in vertical planes perpendicular to the horizon, and every star would rise and set.

It takes roughly 23 hours, 56 minutes for the celestial sphere to complete a full revolution or one sidereal day.[14] That is to say, any star will reach the same point in the sky roughly four minutes earlier each night: for example, a star which crosses the eastern horizon at about 6pm on January 1st will cross the horizon at about 5.40pm on January 6th. The sun moves slightly slower than the stars – the average time it takes to complete a full revolution being of course 24 hours.

'Rising' and 'setting'

In the range of latitudes of particular interest to the classicist, most stars will rise and set – in the ordinary sense of the words – once every sidereal day. That is to say, in any given 23 hour 56 minute period, all stars apart from the circumpolar ones (which never rise or set) will cross the eastern and western horizon. These risings and settings may take place at any time during the night, when they will be visible, or during the day, when they will be invisible.

However, when Hesiod and others talk of the rising and settings of the stars, they are not using the ordinary sense of the words, but rather refer to the rising and setting of the stars in a particular relation to the sun, namely at or close to sunrise and sunset. In this 'astronomical' sense, there are eight different risings and settings, each of which occurs only once a year: these are tabulated in Table One below.

TABLE ONE - PHENOMENA	OTHER TERMINOLOGY	DESCRIPTION
True Morning Rising (TMR)	(true) cosmical rising	the star crosses the eastern horizon with the sun: for the previous few days it had crossed the eastern horizon after sunrise (and was thus invisible); on the day of the true morning rising, it was below the horizon shortly before sunrise (and thus was invisible), and will remain invisible as it crosses the horizon (as the day has now dawned and the sun's light obscures that of the star).
Apparent Morning Rising (AMR)	heliacal rising 'first visibility'	the star crosses the eastern horizon shortly before the sun, and is thus briefly visible (for the first time). With every morning that follows, the interval between the star's rising and sunrise increases: the star is visible earlier, and for longer. This is the first of many visible risings.
True Morning Setting (TMS)	(true) cosmical setting	the star crosses the western horizon as the sun crosses the eastern horizon: for the previous few days, the sun had risen before the star had set, and so as day dawned the star could be seen some distance above the western horizon.. On the day of the true morning setting, the star was visible before sunrise, but its setting is obscured by the light of the rising sun.
Apparent Morning Setting (AMS)	(visible) cosmical setting	the star crosses the western horizon just before the sunrise, and so can be seen to set (for the first time) in the morning twilight. With every morning that follows, the interval between the star's setting and the sunrise increases: this is the first of many visible settings.
Apparent Evening Rising (AER)	(visible) acronychal rising	The star crosses the eastern horizon just after sunset. On previous days the star had crossed the horizon some time after sunset, and so its rising was easily visible. On subsequent days the interval between sunset and the star's rising diminishes, and the sky is too bright for the star's rising to be seen, and by the time the sky is dark the star is already some distance above the eastern horizon. Thus the apparent evening rising is the last visible rising of the star after sunset.
True Evening Rising (TER)	(true) acronychal rising	the star crosses the eastern horizon as the sun crosses the western horizon, and is thus is invisible. By the time the sky is dark enough for the star to be seen, it will have already risen and be some distance above the eastern horizon.
Apparent Evening Setting (AES)	heliacal setting 'last visibility'	The star crosses the western horizon shortly after sunset. On previous days the star crosses the western horizon some time after sunset, and so it could be seen for some time in the night sky and it setting was easily visible. This is the last visible setting of the star, as on subsequent days the star will have disappeared under the horizon by the time the sky is dark, and so will be invisible.
True Evening Setting (TES)	(true) acronychal setting	The star crosses the western horizon with the sun: by the time the sky is dark enough for stars to be seen, the star is beneath the horizon and so is invisible.

The 'true' risings and settings take place when the star crosses the horizon at the same time as the sun: as the sky is still bright at this time, these phenomena cannot be observed and the dates can only be reached by calculation. The 'apparent' risings and settings take place just before sunrise or just after sunset, when the sky is just dark enough for the star to be visible. Since the 'apparent' phenomena are the only ones which can actually be seen, it seems clear that they are the phenomena referred to in the majority of literary texts and parapegmata; references to the 'true' phenomena, which cannot be seen and whose dates cannot be ascertained without mathematical or mechanical assistance, are obviously of less practical use and tend to be confined to ancient handbooks on astronomy.[15]

In what follows, I use the terminology in the first column of Table One: it has the significant advantage of being clear and practical, and also corresponds to the terms used by the Greeks themselves in the earliest extant handbooks.[16] The terms 'heliacal', 'acronychal' and 'cosmical' are by contrast somewhat opaque, and have been used by different scholars to refer to different phenomena.[17] The terms 'first visibility' and 'last visibility', while a little more helpful, can also be somewhat misleading, as there are some stars such as Arcturus that are always visible; and they also only apply to two of the eight phenomena.

It is always the case that the apparent morning phenomena follow the true morning phenomena, and that the apparent evening phenomena precede the true evening phenomena. However, the order in which the apparent phenomena appear in relationship to one another, and the order in which the true phenomena appear in relationship to one another varies with a star's position relative to the ecliptic (the sun's apparent path through the sky);[18] and the order of a star's true phenomena is not necessarily the same as that of its apparent phenomena (see Table Two).[19]

TABLE TWO: ORDER OF THE PHENOMENA (DATES CALCULATED FOR ROME, 44 B.C.E.)							
NORTH OF ECLIPTIC				**ON ECLIPTIC**		**SOUTH OF ECLIPTIC**	
Capella (α Aurigae)		Denebola (β Leonis)		Regulus (α Leonis)		o Leonis	
TMR Mar 10	AMR Apr 7	TMR Aug 13	AMR Aug 31	TMR Jul 28	AMR Aug 14	TMR Jul 25	AMR Aug 17
TES Jun 6	AES May 23	TES Sep 15	AER Jan 27	TER Jan 22	AER Jan 11	TMS Jan 12	AER Jan 8
TER Sep 12	AER Aug 21	TER Feb 7	AMS Apr 7	TMS Jan 24	AMS Feb 11	TER Jan 20	AMS Jan 29
TMS Dec 6	AMS Dec 19	TMS Mar 15	AES Aug 7	TES Jul 27	AES Jun 30	TES Jul 15	AES Jun 12

As we can see, both stars located north of the ecliptic show the same order in their true phenomena, but a different order in their apparent phenomena; and the stars on the ecliptic and south of the ecliptic show different orders again in their true phenomena. Table Two also illustrates how the interval between true and apparent phenomena can vary: in this instance we see intervals ranging from 11 days between the apparent and true evening risings of Regulus, to 39 days between the apparent and true evening settings of Denebola.[20]

Problems of calculation

Whatever method of calculation one uses, the first thing to decide upon is the star whose rising or setting we wish to discover. This is by no means a trivial decision. While in some cases classical authors do specify a particular star (e.g. Arcturus) or an easily identifiable star cluster (e.g. the Pleiades), in some cases they do not, talking instead of constellations rather than individual stars. In the case of a large constellation such as Pegasus, this can be extremely confusing: when we read for example in the Geminus parapegma that "On the 17th day of Leo, for Euctemon … the Horse rises",[21] to which star does this refer? Is it the first star of the constellation to rise (e.g. κ Pegasi)? Or the first bright star to rise (possibly ε, η, or β Pegasi)? Or the most constantly bright star (α Pegasi)? Our choice makes a substantial difference to the date, as there is a gap of nearly a month between the apparent morning rising of κ Pegasi and α Pegasi, and a similar gap between the true risings, as illustrated in Table Three.[22] Similarly, when we read of Orion rising, we have an even larger number of stars to choose from.[23]

TABLE THREE: DATES FOR STARS IN PEGASUS RISING				
DATES CALCULATED FOR ATHENS, 432 B.C.E.				
Phenomenon	κ Peg	ε Peg	β Peg	α Peg
AMR	Jan 19	Jan 24	Feb 3	Feb 16
TMR	Dec 22	Dec 30	Jan 8	Jan 18

In some cases, the sources seem to be giving us some help in identifying the star: e.g. again from the Geminus parapegma, "For Eudoxus, Orion begins to set in the evening" or "For Callippus, Sagittarius ceases rising",[24] which might suggest that we should look at the first star to set and last star to rise respectively (though even this is not certain).[25] However, even then there is confusion. Which stars are we to take as the first stars of Orion? To answer that question, we would need to know exactly how Eudoxus constructed the constellation. While we may get some help in this regard from Aratus, who used Eudoxus as a source, what are we to do for Euctemon? Even when we think we have a good idea of the stars which make up a constellation, there are still some surprises. For example, according to Aratus, the star in the head of Andromeda (α Andromedae) is also the star in the belly of Pegasus; this is presumably the 'bright star in the navel' to which Eratosthenes refers.[26] Hipparchus is aware of this star (3.4.5), but when he describes the last star to set, he refers not to this star (which should be the last star of Pegasus to set), but rather to γ Pegasi (see Diagram One).[27] Table Four illustrates the different dates calculated for the apparent evening setting of various stars of Pegasus.[28]

TABLE FOUR: DATES FOR STARS IN PEGASUS SETTING					
DATES CALCULATED FOR ATHENS, 432 B.C.E.					
Phenomena	ε Peg	α Peg	β Peg	γ Peg	α And
AES	Jan 17	Feb 4	Feb 13	Feb 19	Feb 27

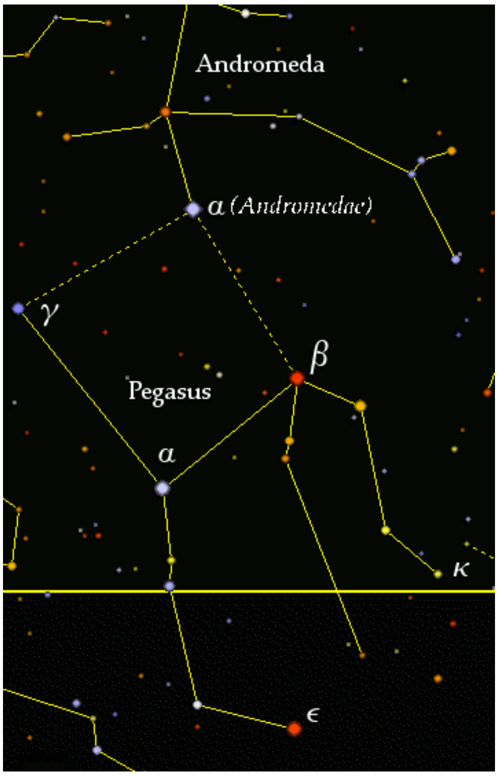

Diagram One: Pegasus Setting

There are two more key variables necessary for all methods of calculation: the geographical latitude of the observer, and the year of the observation. Specifying these variables for an author such as Ovid is not too problematic,[29] but uncertainty can arise in the case of the observations of some of the early Greek astronomers, who are said to have observed in a number of different locations.[30] It is often noted that both variables have an impact on the results of the calculation: this is true, though as we shall see, there is only a slight change with time, and the change with latitude is irregular – it is often, though not always, greater than the change with time.

Tables Five to Eight give the dates of the apparent phenomena for *α* Coronae Borealis (a star north of the ecliptic: Table Five), *α* Leo (a star on the ecliptic: Table Six), *α* Piscis Austrini (a star south of the ecliptic: Table Seven), and Sirius (*α* Canis Maioris, also south of the ecliptic: Table Eight) in Rome over a period of time.[31]

TABLE FIVE: Changes in date over time for α Coronae Borealis – north of the ecliptic; magnitude 2.21					
Phenomena	*300B.C.E.*	*100B.C.E.*	*44B.C.E.*	*401C.E.*	*Over 700 years*
AMR	Oct 6	Oct 8	Oct 8	Oct 11	+5 days
AES	Dec 4	Dec 3	Dec 3	Dec 1	-3 days
AMS	Jul 15	Jul 14	Jul 13	Jun 11	-4 days
AER	Mar 6	Mar 7	Mar 7	Mar 10	+4 days

TABLE SIX: Changes in date over time for α Leonis (Regulus) – on the ecliptic; magnitude 1.4					
Phenomena	*300B.C.E.*	*100B.C.E.*	*44B.C.E.*	*401C.E.*	*Over 700 years*
AMR	Aug 11	Aug 12	Aug 12	Aug 14	+3
AES	Jul 2	Jul 3	Jul 3	Jul 5	+3
AMS	Feb 12	Feb 14	Feb 14	Feb 18	+6
AER	Jan 7	Jan 9	Jan 9	Jan 12	+5

TABLE SEVEN: Changes in date over time for α Piscis Austrini – south of the ecliptic; magnitude 1.23					
Phenomena	*300B.C.E.*	*100B.C.E.*	*44B.C.E.*	*401C.E.*	*Over 700 years*
AMR	Apr 29	Apr 30	Apr 30	May 2	+3
AES	Dec 18	Dec 21	Dec 22	Dec 28	+10
AMS	Jul 23	Jul 25	Jul 26	Jul 31	+8
AER	Sep 4	Sep 5	Sep 5	Sep 8	+4

TABLE EIGHT: Changes in date over time for α Canis Maioris (Sirius) – south of the ecliptic; mag -1.46					
Phenomena	*300B.C.E.*	*100B.C.E.*	*44B.C.E.*	*401C.E.*	*Over 700 years*
AMR	Jul 30	Jul 30	Jul 30	Jul 30	+0
AES	May 2	May 2	May 2	May 3	+1
AMS	Nov 21	Nov 22	Nov 22	Nov 23	+2
AER	Jan 4	Jan 4	Jan 5	Jan 5	+1

It is important to be aware that the precise direction and extent of the change in date depends on the star's position relative to the ecliptic, so for example the AES of α Piscis Austrini (located south of the ecliptic) fell on December 18th in 300B.C.E., but on December 28th in 401C.E., moving forwards by ten days, in contrast to the AES of α Coronae Borealis, which moves backwards by three days over the same period. It should also be noted that as the calendar year gets a little out of sync with the solar year towards the end of our four-year leap-year cycle, a slight change in the date may be noted: so for example, the apparent morning setting of α Leonis fell on February 15th in 41B.C.E., but on February 14th in 40B.C.E.. However, as we can see, in general the rate of change with time remains small.[32]

A variation in latitude, however, causes more substantial changes in the date, though not in a uniform fashion. Tables Nine to Twelve give the dates of the apparent phenomena for a star north of the ecliptic (α Coronae Borealis: Table Nine), a star very close to the ecliptic (Regulus, or α Leonis: Table Ten), and two stars south of the ecliptic

(α Piscis Austrini [Table Eleven] and Sirius, or α Canis Maioris [Table Twelve]) for a variety of latitudes.[33]

TABLE NINE: Changes in date with change in latitude for α Corona Borealis (north of the ecliptic; magnitude 2.21; 44B.C.E.)					
Phenomena	*31 13 (Alexandria)*	*38 (Athens)*	*41 52 (Rome)*	*Athens/Rome*	*Alexandria/Rome*
AMR	Oct 18	Oct 12	Oct 6	-6 days	-12 days
AES	Nov 15	Nov 25	Dec 7	+12 days	+22 days
AMS	Jun 21	Jul 5	Jul 15	+10 days	+24 days
AER	Mar 19	Mar 13	Mar 8	-5 days	-11 days

TABLE TEN: Changes in date with change in latitude for α Leonis (Regulus) (on the ecliptic; magnitude 1.4; 44B.C.E.)					
Phenomena	*31 13 (Alexandria)*	*38 (Athens)*	*41 52 (Rome)*	*Athens/Rome*	*Alexandria/Rome*
AMR	Aug 12	Aug 13	Aug 14	+1	+2
AES	Jul 7	Jul 3	Jul 1	-2	-6
AMS	Feb 6	Feb 9	Feb 11	+2	+5
AER	Jan 12	Jan 11	Jan 11	0	-1

TABLE ELEVEN: Changes in date with change in latitude for α Piscis Austrini (south of the ecliptic; magnitude 1.23; 44B.C.E.)					
Phenomena	*31 13 (Alexandria)*	*38 (Athens)*	*41 52 (Rome)*	*Athens/Rome*	*Alexandria/Rome*
AMR	Mar 23	Apr 12	May 1	+19	+39
AES	Jan 2	Dec 27	Dec 22	-5	-11
AMS	Aug 2	Jul 29	Jul 26	-3	-7
AER	Aug 16	Aug 26	Sep 5	+10	+20

TABLE TWELVE: Changes in date with change in latitude for α Canis Maioris (Sirius) (south of the ecliptic; magnitude -1.46; 44B.C.E.)					
Phenomena	*31 13 (Alexandria)*	*38 (Athens)*	*41 52 (Rome)*	*Athens/Rome*	*Alexandria/Rome*
AMR	Jul 19	Jul 26	Jul 30	+4	+11
AES	May 12	May 6	May 2	-4	-10
AMS	Nov 30	Nov 25	Nov 22	-3	-8
AER	Dec 27	Jan 1	Jan 5	+4	+9

One often reads that a change in latitude has a significant effect on the dates of the phenomena. However, Tables Nine to Twelve show that this statement is misleading: while it is sometimes true, it is not always the case, as the size of the change depends on the position of the star relative to the ecliptic, and also the particular phenomena concerned. The point is, however, that any uncertainty in location can have a substantial impact on the results of our calculations which may not be easy to discern.

These three factors are fundamental to any calculation of the risings and settings. But the different methods of calculation also have other variables that can affect the outcome.

Until recently, the only method used for calculating the dates of the apparent phenomena was based on a method developed by Ptolemy nearly two thousand years ago: the key variable involved here is the *arcus visionis*, the minimum distance necessary between the sun (below the horizon) and the star (assumed by many scholars to be at the horizon) for the star to be visible as it rises or sets (see Diagrams Two to Four).[34] The value of the *arcus visionis* will change with the star's magnitude (as obviously the sun must be further away from a dim star than a bright star for the star to be visible); and some scholars also make an adjustment for the horizontal distance (the difference in azimuth) between the star and the rising (or setting) sun.[35] Diagrams Two to Four illustrate the various ways in which the phrase *arcus visionis* has been interpreted (see note 34).

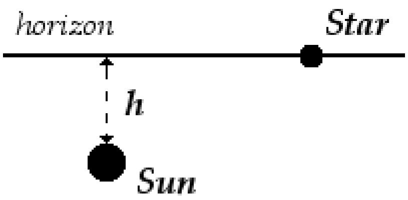

Diagram 1 illustrates the traditional use of the phrase *arcus visionis*: this is the vertical distance *h* between the sun and the star, located on the horizon: this usage assumes that the star is visible as it crosses the horizon.

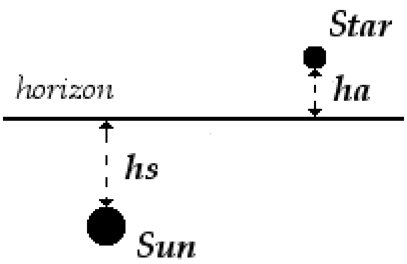

Diagram 2 illustrates an alternative use of the phrase *arcus visionis* (as used in *PSLV*): here the *arcus visionis* refers to the vertical distance *hs* between the sun and the horizon. This usage assumes that the star must be a certain distance above the horizon to be visible. The vertical distance between sun and star (*hs* + *ha*) is therefore greater than the *arcus visionis*.

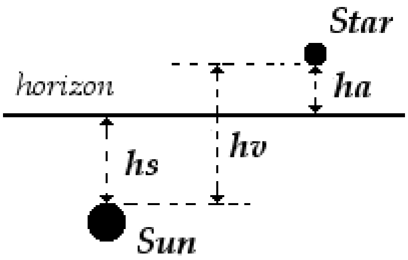

Diagram 3 illustrates another use of the phrase *arcus visionis* (as used by e.g. Schaefer): here the *arcus visionis* (*hv*) refers to the vertical distance between the sun and the star, i.e. the distance between the sun and the horizon (*hs*) added to the distance between the horizon and the star (*ha*).

It is somewhat surprising to discover that for many years the values in use for the *arcus visionis* of stars of differing magnitudes were those deduced by Ideler from dates found in Ptolemy for stars of the first and second magnitude;[36] the *arcus visionis* for less bright stars were then extrapolated from these.[37] However, the precise values of the *arcus visionis* are far from certain: for example the values in Neugebauer's first set of tables,[38] used by West in his Hesiod commentary,[39] were corrected in Neugebauer in later versions,[40] following the work of Schoch, who suggested a revised set of values, derived from the Babylonian observations in the first millennium B.C.E..[41] Other scholars use other sets of values: for example, Aveni uses those of Lockyer, while Bruin compares a variety of values with those he obtained through observation.[42] In some cases we find individual values for the calculation of the individual phenomena.[43] Table Thirteen illustrates the difference that a change in the *arcus visionis* can make for each of the various phenomena.[44]

TABLE THIRTEEN: CHANGES IN DATE CORRESPONDING TO CHANGES IN *ARCUS VISIONIS*							
Phenomena	Regulus (α Leonis) - mag 1.36 – close to the ecliptic. av = *arcus visionis*; ca = critical altitude (zero unless specified otherwise)						
	av = 6	av = 7; ca = 0	av = 6; ca = 1	av = 8	av = 9	av = 10	av = 11
AMR	Aug 1	Aug 2	Aug 2	Aug 4	Aug 5	Aug 6	Aug 7
AES	Jul 15	Jul 13	Jul 13	Jul 12	Jul 10	Jul 8	Jul 6
AER	Jan 12	Jan 11	Jan 14	Jan 10	Jan 9	Jan 8	Jan 7
AMS	Feb 2	Feb 3	Jan 31	Feb 5	Feb 7	Feb 9	Feb 10

As we can see, a change in the *arcus visionis* of one degree can affect the date of the calculation by one to two days. This is somewhat reassuring in that calculations based on different values will be roughly the same, but again it alerts us to the fact that there are uncertainties in the results obtained.

A more serious concern is that many calculations for the dates of the apparent phenomena assume that the star is visible at the moment that it crosses the horizon; however, this is almost never the case, owing to haze, refraction (the bending of light in the atmosphere) and atmospheric extinction (that is, the reduction in brightness of a star as its light passes through the atmosphere, due to dust, pollution, water, movement of air molecules, etc.). A star must reach a certain angular height above the horizon in order to be visible, known as the critical altitude, and the value we assign to this variable can have a significant impact on the results, as we shall see.

Until recently, the calculation of the dates of rising and setting via the Ptolemaic method was an extremely laborious process involving the use of tables such as those of Neugebauer and Baehr.[45] However, there is now available an extremely flexible and accessible tool for this purpose, namely the PC program *Planetary, Stellar and Lunar Visibility*, by Noel Swerdlow and Rainer Lange.[46] The program uses a range of values for the *arcus visionis* based on Schoch,[47] but also allows for an *arcus visionis* that varies with directly with magnitude; it also allows the user to adjust the *arcus visionis* manually. The program also allows the user to take account of the 'critical altitude' of the star, that is, the minimum height above the ground necessary for the star to be visible, as described above. The problem is to decide what value to choose for this variable. The authors raise this problem in the extremely clear documentation that accompanies the program, and admit that there is no simple solution. This is how they conclude the section entitled 'Sources of Computations and Cautions concerning Accuracy' (their italics):

> It appears from trial calculations that changes in the critical altitude produce greater differences in the dates of phenomena than *reasonable* changes in the arcus visionis, so *the critical altitude must be set with great care*. The user is encouraged to experiment with different parameters to find which appear to produce the most accurate, or most reasonable, results, although in the absence of reliable observations for comparison, it is difficult to say what most accurate or most reasonable is.

While this is an honest answer and one that impresses upon the user just how uncertain the business of calculating dates can be, it is not terribly helpful to the scholar whose interest is primarily in, say, Ovid's *Fasti* and just wants a quick answer to the question "when did Sirius have its apparent morning rising?". Such a scholar may be relieved to hear of the existence of a rule of thumb commonly used by archaeoastronomers, according to which the critical altitude is taken to be equal to the magnitude of the star.[48] As we shall see, however, this rule of thumb is not without some serious problems, to which we shall return below. Table Fourteen illustrates the difference a change in critical altitude makes to the calculations for the fairly dim star α Delphini (magnitude 3.76); also included are values for the critical altitude calculated according to Schaefer's method (discussed below), using a limiting magnitude of 6, and visual extinction factor of .2 and .3 (these variables correspond to an observer with good eyesight observing on a clear night and a night of average visibility respectively). The calculations are made for Rome in 44B.C.E., using *PLSV* with variable *arcus visionis* based on the magnitude.

TABLE FOURTEEN: PHENOMENA FOR α DELPHINI FOR ROME IN 44B.C.E.					
Critical Altitude:	*0*	*3.76*	*5.3 [BS 6/0.2]*	*8.9 [BS 6/0.3]*	*Difference 0 - 9*
AMR	Dec 29	Jan 3	Jan 5	Jan 10	+12
AES	Jan 13	Jan 8	Jan 7	Jan 3	-10
AER	May 21	May 25	May 27	May 31	+10
AMS	Aug 22	Aug 18	Aug 16	Aug 13	-9

As Table Fourteen shows, our choice regarding whether or not to take the critical altitude into account, and what value to use, has a significant impact on the results of our calculations. As mentioned above, a rule of thumb in common use has been to use the magnitude of the star for the critical altitude. However, this is one of a number of common assumptions regarding the calculation of apparent phenomena that has been questioned by the astrophysicist Bradley Schaefer, on the basis of his research into the visibility of celestial objects.[49] He argues that this rule of thumb for the critical altitude is erroneous, and only begins to approximate to the correct figure when dealing with bright stars in *exceptionally* clear conditions, such as those found in the Chilean desert:[50] such a rule of thumb will clearly be of little use for Greece or Rome; he also notes that the traditional formula for calculating refraction sometimes underestimates the effect of refraction at the horizon;[51] and he has developed a new method for calculating the dates of the apparent morning rising and apparent evening setting, based not on the *arcus visionis* but rather on estimates of the limiting magnitude of the night sky (that is, the magnitude of the faintest star that can be seen at the zenith in a fully dark sky), and visual extinction.[52] These variables are influenced by various factors, such as the position of the moon, humidity of the sky, air temperature, altitude, the eyesight of the observer, pollution and ambient light.[53] For bright stars in good conditions, the results from Schaefer's method are roughly similar to those reached by the traditional method; but for dim stars in poor conditions, there can be a substantial difference.[54] Of course, Schaefer's method is not without its own uncertainties: for example, how does one establish the air quality, temperature or humidity for Athens on a particular day in the 4th century B.C.E.?[55]

So what does this mean for anyone wishing to calculate the dates of these apparent phenomena? To get some idea of the spread of dates that result from using different methods (or different values within the same method), it will be helpful to compare the dates that we can find either in modern sources or by using modern tools for calculation.[56] In addition to the programs I have already mentioned, included in Tables Fifteen to Eighteen are also calculations from one other source, the web-based utility from the astrophysicist Karine Gadre.[57] All the calculations I have performed are for Rome, using a latitude of 41º 52' 48", for 44B.C.E..

TABLE FIFTEEN: FOR ARCTURUS (α BOOTIS), IN ROME, 44B.C.E.. (MAGNITUDE 0.16, NORTH OF THE ECLIPTIC)

α Bootis	Hofmann	Ideler	Ginzel	PLSV 0.16	Schaefer 6/0.2	Schaefer 6/0.3	Schaefer 5/0.3
MR	Sep 12	-	Sept 19	Sept 20	Sept 20	Sept 23	Sept 23
ES	Nov 9	-	Nov 9	Nov 10	Nov 10	Nov 4	Nov 4
ER	Feb 25	Feb 27	Feb 26	Feb 23	-	-	-
MS	June 16	Jun 10	Jun 12	Jun 17	-	-	-

TABLE SIXTEEN: FOR ALCYONE (ONE OF THE PLEIADES, η TAURI) IN ROME 44B.C.E. (MAGNITUDE 2.84), CLOSE TO THE ECLIPTIC

η Tauri	Hofmann	Ideler	Ginzel	PLSV 2.84	Gadre	Schaefer 6/0.2	Schaefer 6/0.3	Schaefer 5/0.3
MR	27 May	28 May	27 May	30 May	29 May	30 May	Jun 3	Jun 7
ES	11 April	8 April	7 April	6 Apr	-	4 Apr	Apr 1	Mar 31
ER	19 Sep	25 Sep	24 Sept	26 Sep	-	-	-	-
MS	13 Nov	9 Nov	7 Nov	8 Nov	-	-	-	-

TABLE SEVENTEEN: FOR VINDEMIATRIX (ε VIRGINIS) IN ROME, 44B.C.E. (MAGNITUDE 2.82) (NORTH OF THE ECLIPTIC)

ε Virginis	Hofmann	Ideler	PLSV 2.82	Gadre	Schaefer 6/0.2	Schaefer 6/0.3	Schaefer 5/0.3
MR	Oct 6	Sep 18	Sep 20	Sep 21	Sep 21	Sep 23	Sep 26
ES	Nov 22	-	Aug 31	-	Aug 26	Aug 20	Aug 15
ER	Mar 4	Feb 14	Feb 15	-	-	-	-
MS	Jun 18	-	May 15	-	-	-	-

TABLE EIGHTEEN: FOR REGULUS (α LEO) IN ROME, 44B.C.E. (MAGNITUDE 1.4), CLOSE TO THE ECLIPTIC.							
α Leonis	Hofmann	Ideler	PLSV 1.4	Gadre	Schaefer 6/0.2	Schaefer 6/0.3	Schaefer 5/0.3
MR	Aug 12	-	Aug 14	Aug 14	Aug 13	Aug 16	Aug 16
ES	Jul 8	Jul 6	Jul 1	-	Jul 2	Jun 27	Jun 27
ER	Jan 14	-	Jan 11	-	-	-	-
MS	Feb 12	Feb 6	Feb 11	-	-	-	-

The tables show that for the most part scholars are in rough agreement as to when these phenomena take place (with the exception of Hofmann's calculations for Vindemiatrix, where he comes up with a very different set of dates, perhaps owing to a mistake in his calculations). They also illustrate, as expected, that Schaefer's method produces similar results to the Ptolemaic method for bright stars in good conditions; but once conditions becomes less than optimal we begin to notice a substantial difference. The largest difference between any two dates in Tables Fifteen to Eighteen (with the exception of Vindemiatrix) is found between Hofmann's calculations for the apparent evening setting of Regulus (July 8) and those of Schaefer for bad conditions (June 27), a difference of 11 days; but a considerable difference is found between the dates obtained via Schaefer's method for good and bad conditions for Vindemiatrix (6/0.2 ES Aug 26 vs 5/0.3 Aug 15).

As we have seen, the calculation of the dates for the apparent phenomena is based on a number of factors: even if we are confident about the star, latitude and epoch involved in our calculations, there are assumptions to be made about the *arcus visionis*, the critical altitude, the limiting magnitude, or the extinction co-efficient depending on one's choice of method. All these variables can have a significant effect on the outcome of the calculations, but there is no clear agreement as to what the right value for these variables is. As Swerdlow and Lange remark, the 'calculation of visibility phenomena is plagued by uncertainties that will only be resolved by a body of reliable observations that does not yet exist'.[58] However, if we use these dates or make the calculations with an awareness of the assumptions we are making, we can nevertheless hope that they provide a rough estimate of the dates of the actual phenomena, and the above tables show that the various methods are in general agreement.

Observing the actual phenomena

However, having examined the problems of calculation, we need to pay particular attention to the problems of observation. If we have predicted by the best methods that Arcturus will have its apparent morning rising the following week, can we be certain that we will see it on the appropriate day? Things are perhaps not quite that simple: the above calculations assume a flat horizon, whereas in reality areas of the sky may be obscured by aspects of the landscape. Furthermore, as Bruin remarks, "[t]he precise observation of heliacal phases is difficult, because at such a moment the star is only just visible and there is often a haze near the horizon (particularly near the sea…). Even for a trained observer the date may easily be a few days wrong”.[59] Schaefer’s account of his attempt to observe the heliacal rise of κ Gemini is particularly instructive:[60]

> During a recent 20-day trip to CTIO [Cerro Tololo Inter-American Observatory], I tried to spot the heliacal rise of k Gem on every morning. On the 3rd and 4th mornings, the visibility of θ Gem (of equal magnitude but 2° higher than κ Gem) promised the heliacal rising of κ Gem on the 5th morning. However, five cloudy nights occurred. Then on the last moonless clear night, the zodiacal light and occasional cirrus prevented κ Gem from being sighted. Then came three cloudy nights. For the next five days the waning Moon moved closer to the eastern horizon with the effect of keeping κ invisible … the next two nights were cloudy. The final result is that I never did see κ Gem during my trip to CTIO. In this (not untypical) case, the zodiacal lights, clouds, and Moon delayed the heliacal rise date by over two weeks.

The above example demonstrates the difficulties that can surround an attempt to observe a particular phenomenon on a particular occasion. We might hope that early astronomers such as Euctemon and Eudoxus, when they were compiling their *parapegmata*, or lists of risings and settings, were able to factor out such atmospheric disturbances by compiling their lists from observations made over a number of years and thus obtain a certain degree of accuracy. However, lest we assume that astronomy in the ancient world was somehow more straightforward in a time without street-lights and pollution, let us see what Claudius Ptolemy, the father of astronomy himself, has to say on the subject.

In his *Almagest*, Ptolemy describes a method for calculating the dates of the apparent phenomena of the fixed stars. However, he concludes the passage with a list of the reasons why he will not actually use the method himself: the computation is too complicated, he says, and too time consuming; he also has this to say:

> in respect of the actual observations of the phases it is laborious and uncertain, since [differences between] the observers themselves and the atmosphere in the regions of observation can produce variation in and doubt about the time of the first suspected occurrence, as has become clear, to me at least, from my own experience and from disagreements in this kind of observations.[61]

He concludes: “for the time being we content ourselves with the approximate phases which can be derived either from earlier records or from actual manipulation of the [star-]globe for any particular star”.[62]

Somewhat later he appears to have modified his position, and returned to this mathematical method and made various calculations which are recorded in the work known as the *Phaseis*.[63] However, he only calculated the dates for stars for the first and second magnitude. In defending this decision, he has this to say:

> But one should pardon the fact that we have not incorporated some of the dimmer stars that are named by the more ancient [authorities] either in the treatise on this subject itself or here, e.g. Sagitta, the Pleiades, the Haedi, Vindemiatrix, Delphinus, and any other such [constellation], since the fault is not grave, especially since the last and first appearances of such small stars are absolutely difficult to judge and observe, and one might remark that our predecessors handled them more by guesswork than by observation of the actual phenomena.[64]

This is a very telling comment – not only does he point to the difficulty of observing the risings and settings of stars such as the Pleiades, but he also is very unimpressed by the accuracy of his predecessors' observations, who were presumably no amateurs. From the point of view of observation, we can compare the table in Schaefer in which Schaefer compares the dates he reached by observation with those reached by calculation: there are frequent discrepancies,[65] and for average viewing conditions his findings suggest an error margin of at least plus or minus five or six days for a dim star such as Alcyone.[66]

It seems then that not only the calculation but also the observation of the apparent phenomena of these stars is far from being an exact science.[67] We have seen that when calculating dates for these phenomena we have to make a number of choices, each of which can affect the result of our calculations. Any uncertainty in these choices can correspond to a greater or lesser uncertainty in our results. We have also seen that there are uncertainties when observing apparent phenomena: while a particular star and the sun may be in the same position in the sky at the same time every year, atmospheric conditions do not show the same precise regularity, and one could not be certain that one would actually observe the same phenomenon on the same date every year, so when Hesiod tells us that Arcturus rises sixty days after the solstice (*Op.* 564-7), this has to be at best a rough estimate. Further complications in observation and calculation could be explored: the difference that observational experience and acuteness of vision can make to the dating of a phenomenon; the question of which data to use for the calculations (different modern star catalogues have different standards of accuracy); how to access that data (magnitudes in the Tycho-2 catalogue are not all given in the same system). There may also be further decisions to be made: which phase is the author referring to?[68] Is it true or apparent?[69]

Were we to have all the right data, such as precise figures for the visual extinction factor of the skies in Ancient Greece (for the appropriate season), or exact details of the critical altitude for a particular star, then it may be that these calculations could indeed provide an accuracy of plus or minus two days. It may be that further research will provide this data, and refine these methods further. In the meantime, however, for those whose who just want to know when they should be 'soaking their lungs with wine' in accordance with the instructions of Alcaeus,[70] or beginning to plough their fields, following Hesiod,[71] it is hoped that the above discussion will enable them to use either modern tools such as *Planetary, Lunar and Stellar Visibility* or the calculations of previous scholars with a clearer understanding of the uncertainties involved, and thus of the kind of accuracy that we can expect, namely a rough guide rather than a precise date.

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[1] Cf. Hes. *Op.* 383-4; Alc. fr. 347a; Hor. *Carm.* 3.1.25-8.

[2] Quint. *Inst.* 1.4.4 *nec si rationem siderum ignoret poetas [grammatice] intellegat, qui, ut alia mittam, totiens ortu occasuque signorum in declarandis temporibus utuntur....*

[3] Cf. e.g. Thuc. 2.78; Hippoc. *Aer.* 2.10-18; 11.

[4] Cf. e.g Caes. *B Civ.* 3.6.2 *ii Nonas Ianuarias naves solvit…*; 3.9.8 *iamque hiems adpropinquabat…;* 3.25.1 *multi iam menses erant et hiems praecipitaverat….*

[5] For more information on the tradition of the parapegma, see Rehm 1941, 1949; Lehoux 2000; Hannah 2005, ch. 3.

[6] Cf. Ideler 1822-3, available on the web at http://bibliothek.bbaw.de/bbaw/bibliothek-digital/digitalequellen/schriften/anzeige/index_html?band=07-abh/18221823&seite:int=572.

[7] Cf. e.g. Frazer 1929, vol. 1, p. xx: "I can only hope that, in turning into English the German astronomer's exposure of Ovid's many errors concerning the starry heavens, I have not been guilty of fresh blunders, for my ignorance of astronomy is as profound as that of my author appears to have been'. For lists of more such criticisms, see Robinson 2000, 40; Fox 2004, 92-3. For a defence of Ovid's accuracy, see Robinson 2000, 40-43; Fox 2004; Robinson 2007.

[8] Cf. e.g. Gee 2002. For a discussion of the importance (or otherwise) of accuracy as a basis for literary criticism in the *Fasti*, see Robinson 2007.

[9] An exception is M.L. West, who in his 1978 commentary on Hesiod's *Works and Days* calculated the dates for the various stars mentioned using the tables of Neugebauer 1922. He notes that these dates involve some margin of error: cf. pages 376-82. It is not clear if he made use of the corrections in Neugebauer 1925 and 1929.

[10] Aujac 1975, 158. In her defence, however, it is generally said that these calculations have an accuracy of ± 2-3 days. As I hope to show below, for practical purposes the error margin may be substantially wider.

[11] Cf. e.g. Gee 2000, 205: "With the aid of computer programmes, one might calculate the exact dates of rising and setting of a given star at a given latitude in any particular year…".

[12] Cf. e.g. Neugebauer 1922. Schoch in Langdon 1928 strongly asserts that his figures give an error margin of plus or minus one day.

[13] These can be found at http://www.ucl.ac.uk/GrandLat/permanent/robinson/astronomy.html.

[14] The precise figure for the length of the mean sidereal day is 23 hours, 56 minutes and 4.09 seconds.

[15] Cf. Geminus 13.10 'For this reason the visible risings of the stars are announced in the public decrees. For the true [risings] are unobserved and unobservable, while the visible are both announced and observed.' (trans. Evans and Berggren 2006). Not everyone agrees with this analysis, however: cf. Pritchett and van der Waerdern 1961, who argue that Thucydides divided up year according to some apparent and some true phenomena; cf. also Ideler's analysis of the star-passages in Ovid's *Fasti* (see note 6), which assumes that the sources used by Ovid contained dates for true as well as apparent phenomena. While it is not impossible that such sources, which would have been more academic than practical, existed by the time Ovid was writing in the early first century C.E., it seems highly unlikely (given what we know of Greek mathematical and mechanical expertise) that they would have existed in Thucydides' lifetime.

[16] Cf. for example the works of Autolycus (*On Risings and Settings*), and Geminus (*Introduction to Astronomy*), ch. 13.

[17] Cf. for example Lockyer 1894, 120-122 seems to use the term 'heliacal' instead of 'apparent', and refers to the TES as the 'cosmic evening setting'; Bickerman 1980, 143 seems to have misunderstood Ginzel's terminology and claims that 'heliacal' means 'near sunrise' and that 'acronical' (sic) and 'cosmical' imply 'near sunset'.

[18] The order can also influenced by the latitude of the observer: see Evans and Berggren 2006, 63-70.

[19] The results of the true phenomena have been obtained using the algorithms of Meeus 1998, and thus are free of corrections for refraction. The dates for the apparent phenomena have been obtained using the program *Planetary, Lunar and Stellar Visibility* 3.0.1, details of which are given below (see note 22). The calculations have been made for Rome, in 44 B.C.E.. For further details on the relationship between true and apparent risings, see Evans 1998, 190-97; Evans and Berggren 2006, 63-70.

[20] Compare the rule of thumb given by Autolycus in his *On Risings and Settings* (2.1), that a star becomes visible once the sun is half a zodiac sign below the horizon: this is equivalent to fifteen degrees, which is approximately equivalent to fifteen days (see Evans and Berggren 2006, 67-8 for more details). The table illustrates the possibility for error that such a rule of thumb would introduce if it were ever used to compile lists of dates.

[21] Cf. Aujac 1975, 99.

[22] The calculations for the true phenomena were made using the algorithms of Meeus 1998, using data from the Tycho 2 catalogue; those for the apparent phenomena were made using the program *Planetary, Lunar and Stellar Visibility* 3.0.1 (henceforth *PLSV*), by Rainer Lange and Noel Swerdlow, available from http://www.alcyone.de. For the *arcus visionis* I used the program's default variable *arcus*; for the critical altitude I used the magnitude of the star, except for those stars with magnitudes below 0.5, in which case I used 0.5 (for discussion of these values, see below). It should be noted that since writing this article *PLSV* has been updated, and may now produce slightly different results (usually within a day or so of the dates in this article). The table shows that the intervals between various true and apparent risings, while not identical, are similar: in subsequent tables I will give only the dates calculated for apparent risings, using the program and values mentioned above.

[23] Of course, in the case of smaller constellations, which may contain only one very bright star, the choice may be more obvious: but it is important to emphasise that in many cases we do not know for certain what exactly Euctemon or Eudoxus (for example) observed when they decided that they had witnessed the apparent morning rising of the Dolphin; and that as a consequence the choice of star is one of many uncertainties involved in the calculations.

[24] Cf. Geminus, p. 106 Aujac; p. 104 Aujac.

[25] One of the referees of this article suggests that in the case of Euctemon and Eudoxus, a constellation 'began to rise' when its first recognizable star became visible, rather than the first star of the constellation, but that different authorities may have used different criteria.

[26] Cf. Arat. *Phaen.* 205-207; Eratosth. [*Cat.*] 1.18 Olivieri.

[27] Cf. Hipp. 2.6.11 ἔσχατος δὲ ὁ ἐπὶ τῆς ὀσφύος λαμπρός. This is the same description he uses to identify the last star to rise (cf. 2.5.11), which cannot refer to α Pegasi. The picture of the constellation in Diagram One was generated by Chris Marriott's *SkyMap Lite 2005* (available from www.skymap.com).

[28] The star ε Pegasi is the first star to set; α Pegasi is located in the middle of the constellation; β and γ Pegasi follow; with α Andromedae the last star to set. For the program and values used, see note 22.

[29] Though it may be of course that he used sources from a different latitude, or a different time, or both.

[30] Cf. e.g. Ptolemy, *Phaseis* p. 67 in vol. 2 of Heiberg's edition: "[Of the various astronomers] the Egyptians observed here, Dositheus in Cos, Philippus in the Peloponnese and Locris and Phocis, Callipus in the Hellespont, Meton and Euctemon at Athens and the Cyclades and in Macedonia and Thrace, Conon and Metrodorus in Italy and Sicily, Eudoxus in Asia and Sicily and Italy, Caesar in Italy, Hipparchus in Bithynia, Democritus in Macedonia and Thrace."

[31] The dates were calculated using *PLSV*: for more details and values, see note 22. For Sirius I have used a critical altitude of 0.5. The inclusion of Sirius in these tables was the suggestion of one of the referees of this article: as the referee noted, and the table confirms, Sirius is unusual in that its apparent morning rising, which was of great importance in antiquity, keeps the same date for hundreds of years. The changes in dates for the true phenomena of these various stars show a similar pattern.

[32] This means that observations made in one location remain valid for that location (for all practical purposes) for many years.

[33] The calculations for the apparent phenomena were made with the program *Planetary, Lunar and Stellar Visibility*, using the option "calculate arcus visionis from magnitude". The critical altitude used in each case was the magnitude of the star, according to the Tycho 2 catalogue. This is a rule of thumb for the critical altitude that Schaeffer has called into question, but my concern here is not to accurately predict the dates but rather to give a rough idea of the variation in dates.

[34] Different scholars use the term *arcus visionis* in different ways: for Ptolemy, Ideler, Schoch, and *PLSV*, the *arcus visionis* refers to the distance between the sun and the horizon (where in many cases it is assumed that the star is to be found); scholars such as Bruin 1979a; 1979b and Schaefer 1987b use the term to include the distance between the sun beneath the horizon and the star above the horizon.

[35] Cf. e.g. Schoch 1924a; 1924b and in Langdon et al. 1928; followed by Baehr 1955.

[36] Cf. Ideler 1816-17. This can be found on the web, at http://bibliothek.bbaw.de/bibliothek-digital/digitalequellen/schriften/anzeige/index_html?band=07-abh/18161817&seite:int=778. Vogt 1920 performed similar calculations, but with different results.

[37] Cf. Ideler 1822-23, 140.

[38] Cf. Neugebauer 1922.

[39] West 1978.

[40] Cf. Neugebauer 1925; 1929. West does not mention that he uses the revised values for the *arcus visionis*. He is aware that these values are based on assumptions, however, and that other values may be more appropriate – for example, on 383-4 he suggests using an *arcus visionis* of 18 degrees, rather than Ideler's 16.

[41] Cf. Schoch 1924a; 1924b; Langdon et al. 1928.

[42] Cf. Aveni 1972; Lockyer 1894; Bruin 1979a; 1979b.

[43] Cf. e.g. Baehr 1955.

[44] These values were calculated using *PLSV*, for Athens in 432B.C.E..

[45] Cf. Neugebauer 1922; 1929 and Baehr 1955.

[46] Available from http://www.alcyone.de. The program calculates the apparent phenomena directly, but the dates of the true phenomena can be obtained from the list of rising and setting times that the program can create – it should be noted, however, that these calculations involve correction for refraction and thus give times for a visible rather than theoretical rise. The accompanying documentation provides a useful account of the methods for calculating the dates of the various phenomena, and discusses in some detail the question of accuracy.

[47] Cf. Schoch 1928.

[48] As used, for example, by Thom 1967, 15.

[49] Cf. particularly Schaefer 1993b. For a brief summary, see Schaefer 2000. On the general application of his findings to the use of astronomy in historical studies, see Schaefer 1999.

[50] Schaefer provides BASIC code for a program to calculate the critical altitude in Schaefer 1987a. The underlying equations are presented in more detail in Schaefer 1986. See also Schaefer 1993a; 1993b. The code can be downloaded at http://skyandtelescope.com/resources/software/programs/extinc.bas. The results which support to the rule of thumb correspond to observations made in Cerro Tololo observatory, in the middle of the Chilean desert, which as Schaeffer remarks is 'one of the very best sites in the world' (1986, 39) for astronomical observation.

[51] Cf. Schaefer and Liller 1990.

[52] Cf. Schaefer 1985, which contains BASIC code to perform these calculations, and some typical values for the limiting magnitude and visual extinction in various locations; and Schaefer 1987b, which presents more detailed information about the method and the dataset on which it was based. The code can be downloaded at http://skyandtelescope.com/resources/software/programs/heliac.bas.

[53] Schaefer provides BASIC code for a program to calculate limiting magnitude in Schaefer 1998. The underlying equations are presented in more detail in Schaefer 1993b (along with various extinction co-efficients for various geographical locations) and in the abbreviated version found in Schaefer 1993a. The code can be downloaded at http://skyandtelescope.com/resources/software/programs/vislimit.bas.

[54] Cf. e.g. Schaefer 2000, where his calculations for the dates of the apparent morning rising of Sirius in Ancient Egypt are roughly similar to those reached by traditional methods.
[55] All these values will of course change with the seasons. Schaefer attempts such a reconstruction: for ancient Egypt in Schaefer 2000, and for ancient Greece in Schaefer 2001.
[56] Ginzel's results come from Ginzel 1906-14, 520-23. As regards the results based on Schaefer's code: the code given in Schaefer 1985 is based on a vernal equinox of March 21st. The figures produced here have been adjusted for the date of the vernal equinox in Rome in 44B.C.E. (March 23rd). The variables in tables fifteen to eighteen correspond to an observer with good eyesight, observing on a clear night (limiting magnitude of 6) in good and average conditions (extinction factor of 0.2 and 0.3 respectively), and to an observer observing on a less clear night (limiting magnitude of 5), in average conditions (extinction factor of 0.3).
[57] Cf. http://www.culturediff.org. At the time of writing, a utility on her site calculates the date of the heliacal rising of a star without requiring the user to estimate any variables such as the *arcus visionis*, critical altitude, limiting magnitude or visual extinction. However, it does not calculate the dates for any other of the phases, nor will it at the current time give a date for stars visible all year round, such as Arcturus. Mlle Gadre informs me that various updates are planned for the near future, so by the time of publication the site may be very different.
[58] From the help-file to *PLSV*, under the section 'Sources of Computations and Cautions concerning Accuracy'. The largest collection of observations we have – and as a consequence, the data against which methods of calculations are most often calibrated – consists of ancient observations, the very data which we are hoping to use the modern calculations to verify. It should be noted that Aveni 1972, Bruin 1979b and Schaefer 1987b have tested their figures against a number of their own observations.
[59] Bruin 1979a, 387.
[60] Schaefer 1987b, 27.
[61] Ptol. *Alm.* 8.6 Toomer (his translation) = Heid. Vol 1.2, p. 203. τὸ κατ᾿ αὐτὰς τῶν ἀστέρων φάσεων τηρήσεις ἐργῶδές τε εἶναι καὶ οὐκ εὐκατανόητον καὶ τῶν ὁρῶν των αὐτῶν καὶ τῶν κατὰ τοὺς ὁρωμένους τόπους ἀέρων ἀνόμοιον καὶ ἀβέβαιον τὸν χρόνον τῆς πρώτης ὑποψίας ποιεῖν δυναμένων, ὡς ἔμοιγε ἀπό τε αὐτῆς τῆς πείρας καὶ τῆς ἐν ταῖς τοιαύταις τηρήσεσι δια φορᾶς γέγονεν εὐκατανόητον.
[62] Ptol. *Alm.* p. 204H = Toomer 8.6. Some have argued that the variety of values for the *arcus visionis* that can be derived from the dates in the *Phaseis* suggest that rather than calculating all the various dates, Ptolemy read many of them off a star globe: cf. e.g. Neugebauer 1975, 930-931.
[63] The first part of the work, in which he discussed his method, appears to have been lost.
[64] Ptol. *Phaseis* p. 12 H, vol. 2. τὸ μέντοι τινὰς τῶν παρὰ τοῖς παλαιοτέροις κατωνομασμένων ἀμαυροτέρων ἀστέρων μὴ προσεντετάχθαι παρ᾿ ἡμῖν μήτε ἐν αὐτῇ τῇ τῆς πραγματείας συντάξει μήτε νῦν, οἷον ᾿Οιστόν, Πλειάδας, ᾿Ερίφους, Προτρυγη τῆρα, Δελφῖνα, καὶ εἴ τις τοιοῦτος, συγχωρητέον, εἰ μὴ βαρὺ τὸ αἴτημα, μάλιστα μὲν διὰ τὸ δυσδιακρίτους καὶ δυσκατανοήτους εἶναι παντάπασιν τὰς τῶν οὕτω σμικρῶν ἀστέρων ἐσχάτας καὶ πρώτας φαντασίας, κεχρῆσθαί τε τοὺς πρὸ ἡμῶν αὐταῖς ἀπὸ στοχασμοῦ τινος μᾶλλον ἢ τηρήσεως ἐξ αὐτῶν τῶν φαινομένων ἄν τις κατανοήσειεν. The translation is that of Jones (http://www.chass.utoronto.ca/~ajones/ptolgeog/Phaseis.pdf).
[65] Cf. Schaefer 1987b, 20.
[66] Ancient sources present a number of problems of their own: it is quite possible that some sources preserve dates taken from observations made in a different latitude and in a different epoch (e.g. a first century B.C.E. Roman calendar using a source originating from third-century B.C.E. Alexandria), or even dates taken from a variety of sources (cf. e.g. Pliny's agricultural calendar in his *Natural History*, book 18); in the process of transmission accurate dates may be miscopied, or names of constellations muddled; and as Professor Swerdlow reminds me, there is always the possibility that the dates are in some way conventional or schematic and not dependent on real observations at all.
[67] I have said little about the calculation of true phenomena here. While the dates reached by modern calculations are more secure (though even then they can vary depending on whether or not one includes

correction for refraction, which one probably should not), given that these dates can only be reached by calculation or by use of an instrument such as a star globe, again they can only give us a rough idea of what the ancients thought of as the date of the true phenomena: it would depend on the data and methods used for their calculations, the accuracy of the sky-globes, etc..

68 For example, Horace's contented man need not fear the setting of Arcturus nor the rise of the Kids, though he does not specify in either case whether these phenomena took place in the morning or evening. The morning setting took place in Rome in 44B.C.E. in mid-June (June 16th according to *PLSV*); the evening setting in early November (November 9th, according to *PLSV*). The morning rising of the Kids took place in early to mid May (May 8 for ε Aurigae; May 19 for η Aurigae according to *PLSV*), the evening rising in late August to early September (Aug 30th for ε Aurigae; Sep 10 for η Aurigae according to *PLSV*). It is usually thought that the contented man here does not fear the autumnal / wintry rains and winds associated with these two stars.

69 Again, it seems reasonable to assume that these would usually be apparent phenomena: but see note 15.

70 The morning rising of Sirius took place in late June in Mytilene in 600B.C.E. (June 22nd according to *PLSV*).

71 The morning rising of Alcyone (η Tauri) took place in late May in Ascra in 700B.C.E. (May 22nd according to *PLSV*).