

**Rønnow et al. Reply:** In a recent Letter we reported on a comprehensive investigation of the magnetic excitation spectrum of  $\text{Cu}(\text{DCOO})_2 \cdot 4\text{D}_2\text{O}$ , an excellent realization of a 2D quantum ( $S = 1/2$ ) Heisenberg antiferromagnet on a square lattice [1]. We obtained renormalization factors of  $Z_c = 1.21 \pm 0.05$  and  $Z_\chi = 0.51 \pm 0.04$  at low temperature, in good agreement with theory, and discovered a wave-vector dependent quantum renormalization of the excitation energies. By comparing to exact diagonalization and quantum Monte Carlo (QMC) computations, this was shown to be a feature intrinsic to the model. Finally, we studied the temperature dependence of the softening and damping,  $\Gamma(T)$ , of the magnetic excitations. The former was shown to be consistent with higher-order quantum corrections to spin-wave theory, while the latter was in excellent agreement with QMC. We noticed that the damping of the spin waves is in *surprisingly good agreement* with the simple relation  $\Gamma(T) = v_s(T)/\xi(T)$ , where  $v_s(T)$  and  $\xi(T)$  are the spin-wave velocity and correlation length, respectively. In their Comment [2], Kopietz and Spremo address this last observation and propose an alternative functional form for  $\Gamma(T)$  [3].

The magnon damping rate shown in Fig. 1 was extracted by fitting a damped harmonic oscillator line shape to the experimentally measured  $S(\mathbf{k}, \omega)$  (see [4] for details). Within the statistical accuracy of the measurements, no systematic  $\mathbf{k}$  dependence of the damping could be observed. Therefore, to ameliorate statistical quality, we presented an *average* of  $\Gamma_{\mathbf{k}}(T)$  over  $\frac{1}{4a} < |k| < \pi/\sqrt{2}a$ . The validity of this averaging is confirmed by the excellent agreement with the QMC data taken at  $\mathbf{k} = (\frac{\pi}{2a}, \frac{\pi}{2a})$ .

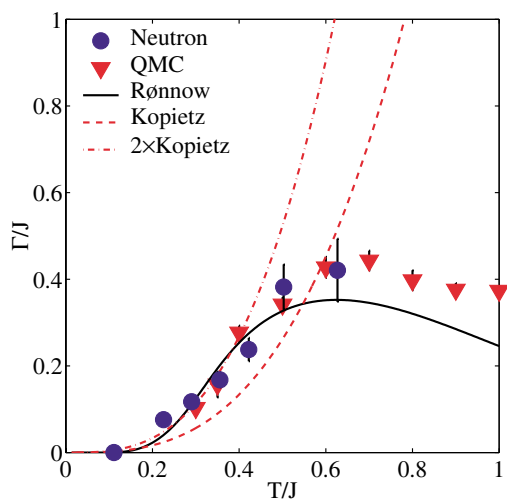


FIG. 1 (color online). Temperature dependence of the damping rate in the 2DQHAFSL. Experimental neutron scattering data (circles) and QMC data (triangles) are in perfect agreement, and are well described by  $\Gamma(T) = v_s(T)/\xi(T)$  (solid line). The expression proposed in [2] (dashed line) can be brought to reasonable agreement up to  $T \approx 0.4J$  if multiplied by a factor of 2 (dot-dashed line).

The functional form  $\Gamma(T) = \frac{2\pi}{3} Z(|v_k|) J(\frac{T}{J})^3$  proposed in [2] is expected to be valid in the large- $S$  and low- $T$  limit for magnons of short wavelength  $|k| \gtrsim \frac{2\pi}{a} [Ta/v_s(0)]^{1/3}$ , where  $v_s(0) = 2^{3/2} Z_c J S a$  and  $Z_c = 1.18$ . Even for the largest wave vector of the experiment,  $|k| = \pi/\sqrt{2}a$ , this leads to the requirement  $T \lesssim \frac{Z_c a}{16}$ , below the experimentally covered temperature range. Still, it is remarkable that allowing a scale factor of  $\sim 2$  the expansion can describe the data up to  $T \approx 0.4J$ . This renormalization factor may arise from fluctuations not included in the large- $S$  expansion. The QMC results demonstrate that the damping saturates at higher temperatures, and this effect is not captured by the low- $T$  expansion.

The behavior over the whole temperature range of the measurements and the QMC calculations could be described by the form  $\Gamma(T) = v_s(T)/\xi(T)$ , for which we have presented a simple phenomenological interpretation: Assume that an excitation belongs to a correlated region of finite spatial extent  $\xi(T)$ . Its lifetime,  $1/\Gamma(T)$ , will be limited to the time it takes for the excitation to propagate across the correlated region,  $\xi(T)/v_s(T)$ . It must be clarified that this argument can be expected to hold for  $|k|$  larger than  $1/\xi(T)$  [5,6]. At wavelengths longer than the correlation length spin waves become unstable and overdamped.

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- [1] H. M. Rønnow, D. F. McMorrow, R. Coldea, A. Harrison, I. D. Youngson, T. G. Perring, G. Aeppli, O. F. Syljuåsen, K. Lefmann, and C. Rischel, *Phys. Rev. Lett.* **87**, 037202 (2001).
- [2] P. Kopietz and I. Spremo, preceding Comment, *Phys. Rev. Lett.* **89**, 079701 (2002).
- [3] P. Kopietz, *Phys. Rev. B* **41**, 9228 (1990).
- [4] H. M. Rønnow, Ph.D. thesis, University of Copenhagen and Risø National Laboratory, 2002 (request copy from hmr@ill.fr).
- [5] P. Kopietz, *Phys. Rev. Lett.* **64**, 2587 (1990).
- [6] D. R. Grempel, in *New Trends in Magnetism*, edited by M. D. Costinho-Filho and S. M. Rezende (World Scientific, Singapore, 1990), p. 133.