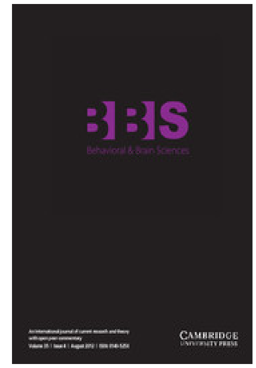

Symmetries and itineracy in nonlinear systems with many degrees of freedom

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on. For example, Kazanovich and Borisyuk (1994; 1999) studied oscillatory networks with a central element and the applications of this network to modelling attention focus formation. The mechanism for controlling the system dynamics is based on synchronisation of neural activity. It has been shown that the regime of partial synchronisation is very promising for the description of neurodynamics. In this regime, some oscillators work synchronously with a central element forming a temporally existing attractor. Makarenko and Llinas (1998) have applied the synchronisation principle to study phase synchronisation of chaotic systems and model the activity of inferior olivary neurons.

Conclusion: The chaotic neurodynamics seems a very intriguing and promising mathematical technique. Further research should be done in mathematics and neuroscience to understand the meaning of chaotic dynamics for modelling of information processing in the brain.

Symmetries and itineracy in nonlinear systems with many degrees of freedom

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Abstract: Tsuda examines the potential contribution of nonlinear dynamical systems, with many degrees of freedom, to understanding brain function. We offer suggestions concerning symmetry and transients to strengthen the physiological motivation and theoretical consistency of this novel research direction: Symmetry plays a fundamental role, theoretically and in relation to real brains. We also highlight a distinction between chaotic "transience" and "itineracy."

Attractor networks and brain-like neural systems: Symmetry is the missing link. Symmetry has been central to the conceptual development of the dynamics of high dimensional nonlinear systems, but is a notable absentee from Tsuda's target article. Basin riddling was first described in the context of dynamical systems with symmetry (Alexander et al. 1992, not Grebogi et al. 1987). Symmetrical systems remain the focus of research into basin riddling (e.g., Heagy et al. 1994) and attractor networks (Buono 2000; Kaneko 1997). Symmetries play the crucial role of enforcing the dynamical invariance of low-dimensional linear manifolds. When these manifolds support chaotic sets, Milnor attractors and basin riddling can arise naturally (unlike the "exceptional" examples that originally motivated Milnor [1985] such as depicted in Fig. 5). Basin riddling occurs when these sets also contain a dense set of periodic orbits which are repelling in the transverse direction. However, typical orbits (of full measure) are transversely attracting. Hence, the "natural" transverse Lyapunov exponent associated with the chaotic set is negative (*not* positive as erroneously stated in para. 6 of sect. 3.4). It is only low-order periodic orbits (that have zero measure) that are associated with positive Lyapunov exponents (and hence "connections" to other attractors). Approaching high dimensional systems from the perspective of symmetry thus permits a clear understanding of the mechanisms of "weak" instability. In addition, it is possible to exploit the different degrees of symmetry exhibited by the attractors to construct a rigorous classification and ordering of the network (e.g., Ashwin et al. 1992). This permits an improvement on the vague notion of attractors arbitrarily distributed throughout phase space, as depicted in Figure 4.

Symmetries arise naturally in systems of coupled nonlinear oscillators (Field et al. 1996). Brain-like neural systems are characterised by networks of coupled nonlinear oscillators – from the scale of the neuron, up to the scale of the macrocolumn. In these systems dense local excitatory and inhibitory interconnections construct individual "nodes," which are coupled into larger ensembles by sparser long-range excitatory connections. Thus, the organisation of the brain motivates a study of coupled nonlinear systems and, hence, symmetry. Moreover, the attractors of symmetrical systems represent synchronous oscillations among clusters of nodes of different sizes (Kaneko 1997) which strengthens this motivation. Attractor networks in symmetrical systems have been used to model normal olfactory perception (Breakspear 2000), visual hallucinations (Bressloff 2001) and animal gaits (Buono et al. 2000). In contrast, systems with skew-product structure (as considered by Tsuda) are not well motivated, because nearly all brain interactions are reciprocal (even the LGN of the thalamus sends many projections to the retina). Symmetry considerations may strengthen the relevance of Tsuda's interesting and original proposals.

Saddles, chaotic transients, and noise: The need for clarity. Tsuda is correct in pointing out that it may be more relevant to study transient or itinerant behaviour rather than attractors in dynamic systems where inputs and parameters change relentlessly (Friston 1997). However, it is important to ensure clarity and consistency in the use of the terms "transience" and "itineracy." Traditionally, the term "chaotic transience" was applied in the following way (Greborgi et al. 1983): A chaotic attractor (A), subject to some parameter perturbation, "collides" with its own basin boundary. Subsequently, orbits on the attractor are mapped into another basin and subsequently onto another attractor. Put another way, A is no longer an invariant of the dynamic. However, a large set of initial conditions will still approach the region of A (now an attractor "ruin") and transiently mimic the behaviour of the former attractor, before collapsing onto the alternative attractor. After this collapse, the transient is not seen again *unless* the system's parameters are tuned back in the opposite direction. If this is the case, attractors may constantly be "ruined" and then "rebuilt." Such relatively rapid changes in attractors may be effected by NMDA-receptor mediated changes in the underlying control parameters (Friston 1997).

On the other hand, the process of chaotic itineracy – which Tsuda exploits – occurs by a different mechanism. A chaotic attractor, A, is subject to a parameter perturbation that weakens its *transverse stability.* At some critical point (the *blowout bifurcation*), the transverse Lyapunov exponent for the attractor (the *natural measure*) becomes positive (Ashwin et al. 1996). A is then a saddle, not an attractor ruin. Note that A is still an invariant of the dynamic, but will attract only a zero measure set of initial conditions. However, if the phase space contains many such saddles, it may be that typical orbits relentlessly shadow these saddles. Hence the evolution of the system is characterised by irregular switching between different types of itinerant chaos corresponding to the shadowing of different saddles. This tuning of the dynamics into a regime of saddle networks may be achieved by enduring monoamine-mediated changes in functional synaptic coupling (Breakspear 2000).

In summary, there are two types of "transient chaos" with potentially distinct neurophysiological mechanisms. "Chaotic transience" induced by dynamically changing control parameters and "Chaotic itineracy" due to an invariant but complex manifold (discussed as engendering type 1 and type 2 complexity in Friston 2000). As brain science calls more upon dynamical systems theory, it is important to keep such distinctions clear.

Summary. The progression from autonomous, low-dimensional strange attractors to systems with noise and many degrees of freedom represents an important advance in the theory of neural systems (Wright 2000). The present paper by Tsuda outlines many potential computational benefits of this progression. Yet, it is critically important that due respect is paid to both neurophysiology and nonlinear theory, before another magic "man in the machine" in cognitive neuroscience research takes shape.