A Spectrally Efficient Frequency Division Multiplexing Based Communications System

M. R. D. Rodrigues[†] and I. Darwazeh[‡]

[†]Laboratory for Communications Engineering
Dept. Engineering, University of Cambridge
mrdr3@eng.cam.ac.uk

[‡]Telecommunications Research Group
Dept. E&EE, University College London
i.darwazeh@ee.ucl.ac.uk

Abstract-- In this paper we propose a spectrally efficient frequency division multiplexing (FDM) based communications system. This FDM system handles intercarrier frequency separations smaller than those of orthogonal frequency division multiplexing (OFDM) systems. The FDM transmitter consists of a bank of modulators that generate the different information conveying sub-carriers that compose the transmitted FDM signal. The FDM receiver consists of a bank correlators followed by a maximum likelihood (ML) detector. The demodulators extracts a set of sufficient statistics from the received FDM signal and the ML detector uses the sufficient statistics to estimate the transmitted symbols. To alleviate the complexity of ML detection alternative detection methods are examined including linear detectors and genetic algorithms (GA) based detectors. Finally, FDM system fundamental limitations are discussed.

Index Terms—Frequency division multiplexing, Orthogonal frequency division multiplexing, Bandwidth efficiency, Spectral efficiency, Maximum likelihood detection, Linear detection, Genetic algorithm based detection

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) based communications systems have been adopted in/proposed for a number of wireless applications such as digital audio broadcasting (DAB), digital video broadcasting (DVB), indoor wireless networks (e.g. IEEE 802.11a, IEEE 802.11g and HIPERLAN2) and fixed broadband wireless access (e.g. IEEE 802.16). OFDM offers many advantages for wireless applications such as good performance in the presence of frequency selective fading without the need for complex time-domain equalisation techniques, efficient usage of the available bandwidth and efficient digital signal processor based transceiver implementations [1,2].

At an OFDM transmitter, a high rate data stream is partitioned into N lower rate data sub-streams; each sub-stream modulates one of N sub-carriers; and finally, the modulated sub-carriers are added to generate the transmitted OFDM signal. The OFDM receiver consists of a bank of filters matched to the different OFDM sub-carriers followed by a bank of detectors. The intercarrier frequency separation is set to be equal to the inverse of the signalling interval thereby ensuring that the information conveyed by a particular sub-carrier can be recovered independently from the information conveyed by other sub-carriers.

This paper investigates frequency division multiplexing (FDM) systems with intercarrier frequency separations smaller than those of OFDM systems. Note that FDM based systems with intercarrier frequency separations equal to half the inverse of the signalling interval have already been proposed elsewhere [3,4]. However, such proposals only handle BPSK or M-ASK modulation of the FDM sub-carriers whereas this new proposal also handles M-PSK or M-QAM modulation of the FDM sub-carriers.

II. FDM SYSTEM BASICS

The FDM based communications system proposed in this paper works as follows: At the transmitter, a high data rate stream is also partitioned into N lower rate data sub-streams; each sub-stream modulates one of N sub-carriers; and finally, the modulated sub-carriers are added to generate the transmitted FDM signal. However, in this case the intercarrier frequency separation can be smaller than the inverse of the signalling interval and hence the spectral efficiency of an FDM system is higher than that of an equivalent OFDM system.

We write the transmitted FDM signal as follows

$$s(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} S_{k,n} g_n(t - kT) \tag{1}$$

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T}} e^{j2\pi n \Delta f t}, & t \in [0,T] \\ 0, & t \notin [0,T] \end{cases} \tag{2}$$

where $S_{k,n}$ is the symbol transmitted in time slot k and sub-channel n, N is the number of FDM sub-channels, T is the FDM symbol duration and Δf is the intercarrier frequency separation. We consider for simplicity symbols transmitted in different time slots and sub-channels to be independent.

We write the received FDM signal as follows

$$r(t) = s(t) + n(t) \tag{3}$$

where $s(t)$ is the transmitted FDM signal and $n(t)$ is additive white complex Gaussian noise (with power spectral density N_0). We consider for simplicity communication in additive white Gaussian noise channels.

The receiver for FDM signals consists of a bank correlators followed by a detector. The correlators extracts a set of sufficient statistics $R_{k,n}$, n=0,…,N−1, from a received FDM symbol $r(t)$, $kT<t<(k+1)T$, and the detector uses the sufficient statistics $R_{k,n}$, n=0,…,N−1, to estimate the transmitted symbols $S_{k,n}$, n=0,…,N−1. The goal of this process is to minimise the probability of error for the estimate[1].

Accordingly, in the first step of reception the received FDM symbol is projected onto a complete set of orthonormal functions that span the FDM symbol space given by

$$b_n(t) = \frac{1}{\sqrt{\xi_n}}\left[g_n(t) - \sum_{i=0}^{n-1}\int_{-\infty}^{\infty} g_n(t) b_i^*(t) dt\, b_i(t)\right] \tag{4}$$

where the value of ξ_n is chosen so that the energy of $b_n(t)$ is equal to one. Note that $b_n(t)$, n=0,…,N−1, are determined from $g_n(t)$, n=0,…,N−1, using the Gram-Schmidt orthonormalisation procedure [5]. The sufficient statistics are then given by

$$\begin{aligned} R_{k,n} &= \int_{kT}^{(k+1)T} r(t) b_n^*(t-kT) \\ &= \int_{kT}^{(k+1)T} s(t) b_n^*(t-kT) dt + \int_{kT}^{(k+1)T} n(t) b_n^*(t-kT) dt \\ &= S'_{k,n} + N_{k,n} \end{aligned} \tag{5}$$

Here $N_{k,n}$ is a circularly symmetric complex Gaussian random variable with mean zero and variance N_0. Note that by virtue of orthonormality of the basis functions $N_{k,n}$, n=0,…,N−1 are independent.

In the second step of reception the sufficient statistics $R_{k,n}$, n=0,…,N−1, are used to estimate the transmitted symbols $S_{k,n}$, n=0,…,N−1, so as to minimise the probability of error for the estimate expressed as

$$P(e) = 1 - \sum_{i=0}^{M^N-1} p_i \int_{\text{Region } i} p\left(\mathbf{R}\middle|\mathbf{S} = \mathbf{S}^{(i)}\right) d\mathbf{R} \tag{6}$$

where $\mathbf{R}=[R_{k,0}\ R_{k,1}\ \cdots\ R_{k,N-1}]$, $\mathbf{S}=[S_{k,0}\ S_{k,1}\ \cdots\ S_{k,N-1}]$, p_i, i=0,…,M^N−1, denotes the probability of each possible transmitted vector $\mathbf{S}^{(i)}$, i=0,…,M^N−1, and Region i denotes a partition of the multi-dimensional vector space for which the transmitted vector $\mathbf{S}^{(i)}$ is assumed to have been transmitted. Note that N denotes the number of FDM sub-channels and M denotes the size of the M-PSK or the M-QAM constellation used in each FDM sub-channel.

The probability density function $p(\mathbf{R}|\mathbf{S}=\mathbf{S}^{(i)})$ is expressed as

$$p\left(\mathbf{R}\middle|\mathbf{S} = \mathbf{S}^{(i)}\right) = \frac{1}{(2\pi N_0)^{N-1}} e^{-\frac{\left(\mathbf{R}-\mathbf{S}'^{(i)}\right)\left(\mathbf{R}-\mathbf{S}'^{(i)}\right)^{*T}}{2N_0}} \tag{7}$$

where $\boldsymbol{S'}=[S'_{k,0}\ S'_{k,1}\ \cdots\ S'_{k,N-1}]$.

The probability of error for $\mathbf{S}$ is minimised if and only if we choose as an estimate for the transmitted vector $\hat{\mathbf{S}}=\mathbf{S}^{(j)}$ such that

$$p_j\, p\left(\mathbf{R}\middle|\mathbf{S} = \mathbf{S}^{(j)}\right) = \max_i p_i \left(\mathbf{R}\middle|\mathbf{S} = \mathbf{S}^{(i)}\right) \tag{8}$$

or, in the case of equally likely transmitted vectors ($p_i=1/M^N$, i=0,…,M^N−1), we choose as an estimate for the transmitted vector $\hat{\mathbf{S}}=\mathbf{S}^{(j)}$ such that

$$p\left(\mathbf{R}\middle|\mathbf{S} = \mathbf{S}^{(j)}\right) = \max_i \left(\mathbf{R}\middle|\mathbf{S} = \mathbf{S}^{(i)}\right) \tag{9}$$

The decision rule in (8) is the maximum a posteriori (MAP) criterion and the decision rule in (9) is the maximum likelihood (ML) criterion. Note that for equally likely transmitted vectors minimisation of the probability of error for $\mathbf{S}$ will also result in minimisation of the probability of symbol error or, in the binary case, probability of bit error. In this paper, we shall only consider equally likely transmitted symbols and consequently ML detectors.

Fig. 1 shows the structure of an FDM based communication system impaired by noise. Note that the FDM receiver structure reduces to the OFDM receiver structure when the intercarrier frequency separation is set to be equal to the inverse of the signalling interval.

[1] Under the assumption that the information conveyed by different OFDM symbols is independent then it suffices to carry out decisions on an OFDM symbol by OFDM symbol basis.

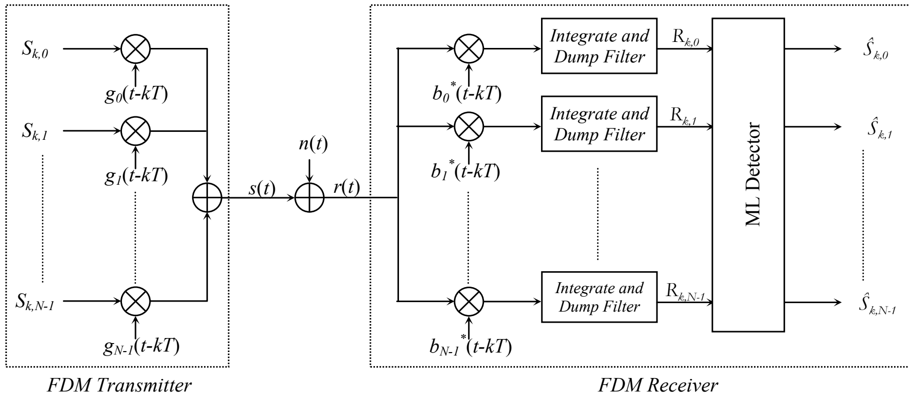

Figure 1: Structure of an FDM based communications system impaired by noise.

III. FDM System Performance

Here we examine the error probability performance of the FDM based communications system in the presence of additive white Gaussian noise. We consider FDM signals with N=2,4,8 and 16, ΔfT=0.1,…,1.0 and BPSK and QPSK modulation of the sub-carriers.

Figs. 1 and 2 show the simulated BER versus normalised intercarrier frequency spacing for various numbers of sub-channels for the FDM/BPSK system and the FDM/QPSK system, respectively. In these simulations E_b/N_0 was set to be equal to 5 dB. Observation of these figures reveals that an FDM system can exhibit a higher spectral efficiency than an equivalent OFDM system without any penalty in the BER. However, the increased receiver complexity associated with ML detection has to be addressed for practical purposes.

IV. Complexity Considerations

The FDM based communications system proposed in this paper employs ML detection. Hence, its complexity grows exponentially with the number of sub-carriers and the cardinality of the constellation. Subsequently, we examine two possibilities to reduce detection complexity.

A. Linear Detection

Here we exploit linear techniques to detect the transmitted symbols in an FDM signal.

The sufficient statistics are related to the transmitted symbols in an FDM based communications system as follows

$$\mathbf{R} = \mathbf{MS} + \mathbf{N} \tag{10}$$

where $\mathbf{R}=[R_{k,0}\ R_{k,1}\ \cdots\ R_{k,N-1}]^T$, $\mathbf{S}=[S_{k,0}\ S_{k,1}\ \cdots\ S_{k,N-1}]^T$, $\mathbf{N}=[N_{k,0}\ N_{k,1}\ \cdots\ N_{k,N-1}]$ and the matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,N} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ m_{N,1} & m_{N,2} & \cdots & m_{N,N} \end{bmatrix} \tag{11}$$

where

$$m_{i,j} = \int_{kT}^{(k+1)T} g_{i-1}(t-kT)\, b_{j-1}^{*}(t-kT)\, dt \tag{12}$$

The matrix $\mathbf{M}$ exhibits the following properties: (i) $m_{i,i} \leq 1$ and $m_{i,i} > m_{j,j}$ for $i>j$ and (ii) $m_{i,j}=0$ for $i>j$ (the matrix is upper triangular). Accordingly, we have two detection options. The first option is based on the inversion of the matrix $\mathbf{M}$ so that an estimate $\hat{\mathbf{S}}=[\hat{S}_{k,0}\ \hat{S}_{k,1}\ \cdots\ \hat{S}_{k,N-1}]^T$ for the transmitted symbols $\mathbf{S}=[S_{k,0}\ S_{k,1}\ \cdots\ S_{k,N-1}]^T$ is given by

$$\hat{\mathbf{S}} = \mathrm{HD}\left[\mathbf{M}^{-1}\mathbf{R}\right] \tag{13}$$

The second option is based on the upper triangular properties of the matrix $\mathbf{M}$ so that an estimate $\hat{S}_{k,n}$ for the transmitted symbol $S_{k,n}$ is given by

$$\hat{S}_{k,n} = HD\left[\frac{1}{m_{n+1,n+1}}\left(R_{k,n} - \sum_{i=n+1}^{N-1} m_{n+1,i}\hat{S}_{k,i}\right)\right] \tag{14}$$

This is an iterative method where first $S_{k,N-1}$ is estimated, second $S_{k,N-2}$ is estimated using $\hat{S}_{k,N-1}$,…, and finally $S_{k,0}$ is estimated using $\hat{S}_{k,n}$, n=0,…,N−2. Note that the operator HD[·] in (13) and (14) performs hard detection.

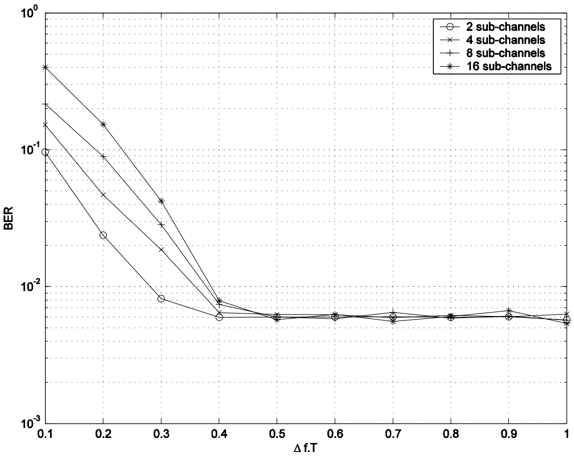

Figure 2: BER versus normalised intercarrier frequency separation for various numbers of sub-channels for the FDM/BPSK system.

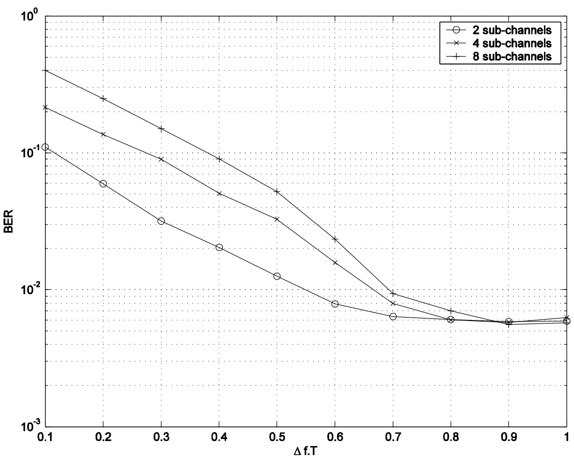

Figure 3: BER versus normalised intercarrier frequency separation for various numbers of sub-channels for the FDM/QPSK system.

Unfortunately, linear detection yields a very high BER as shown in Table I. Linear detection also yields different error rates in different sub-channels.

Finally, we note that the properties of the matrix **M** are such that linear detection may not always be feasible. The matrix determinant rapidly approaches zero as the number of sub-channels is increased or the intercarrier frequency spacing is decreased which prevents linear detection in the first case for the matrix cannot be inverted. Moreover, $m_{N,N}$ rapidly approaches zero as the number of sub-channels is increased or the intercarrier frequency separation is decreased which prevents linear detection in the second case for a high error propagation would occur otherwise. Accordingly, alternative detection procedures have to be sought.

Table I: BER for various linear detection methods for an FDM/BPSK system with various numbers of sub-channels (E_b/N_o=5 dB).

Linear Method	Number of Sub-Channels: 2	4	8	16	32
Inversion Detection	7.7×10^{-3}	2.1×10^{-2}	1.5×10^{-1}	4.1×10^{-1}	4.9×10^{-1}
Iterative Detection	8.4×10^{-3}	1.1×10^{-2}	3.3×10^{-2}	1.6×10^{-1}	3.2×10^{-1}

B. GA Detection

Here we exploit genetic algorithms (GA) based techniques to detect the transmitted symbols in an FDM signal. A GA is a guided random technique which mimics the process of natural evolution. GAs have been frequently used to alleviate the complexity of combinatorial optimisation problems in communications systems [6-8]. A simple GA contains the following steps:

1. Generate an initial population $P(0)$.
2. Evaluate the fitness values of the different elements of the initial population $P(0)$.
3. Create a new population $P(r)$ using crossover and mutation.
4. Evaluate the fitness values of the different elements of the new population $P(r)$.
5. If the termination criterion is not satisfied set r=r+1 and go to step 3 otherwise output solution.

The population comprises a set of the possible transmitted symbols (the chromosomes). The fitness value of the transmitted symbols (the chromosomes) are given by the probability density function in (7).

Based on the fitness values parent chromosomes are selected from the population $P(r)$ to reproduce thereby generating offspring chromosomes for the population $P(r+1)$. Here we employ fitness proportionate selection whereby the expected number of times an individual is selected to reproduce is equal to the individual's fitness divided by the average fitness of the population.

Crossover and mutation are two types of genetic operators. The crossover operation combines sub-strings of two parent chromosomes to generate two new offspring chromosomes. Here we employ single-point crossover whereby a single crossover position is chosen at random and the parts of the two parent chromosomes after the crossover position are exchanged to generate the two offspring chromosomes. The mutation operation introduces a random variation into every element of an offspring chromosome with a given probability.

Note that crossover combines parts of highly fit chromosomes to generate possibly even fitter chromosomes whereas mutation avoids the permanent fixation at any particular locally optimum solution.

In this work, we use a population size of 50, a single-point crossover rate of 0.85 and a mutation rate of 0.01. Moreover, we copy 20% of the fittest chromosomes in population $P(r)$ to population $P(r+1)$. This operation is known as elitism.

Fig. 4 shows the average BER vs. number of iterations for FDM/BPSK signals with N=8,16 and 32 and ΔfT=0.75. In these simulations E_b/N_0 was set to be equal to 5 dB. Note that to achieve the ideal ML detector BER 10, 20 and 30 iterations are required for 8, 16 and 32 sub-channels, respectively. Taking the number of times the probability density function in (7) is evaluated as a rudimentar complexity measure it follows that the improvement in complexity that a GA based detector offers over an ML detector is $2^8/(50\times10)\approx5.1\times10^{-1}$, $2^{16}/(50\times20)\approx6.6\times10^1$ and $2^{32}/(50\times30)\approx2.9\times10^6$ for 8, 16 and 32 sub-channels, respectively. For a small number of sub-channels the GA detector offers no improvement in complexity. However, for a large number of sub-channels the GA offers a substantial complexity improvement. Accordingly, a GA based detector constitutes a viable option to detect the transmitted symbols in an FDM signal.

V. Conclusions and Future Work

In this paper we have proposed a spectrally efficient FDM based communications system. This FDM system handles intercarrier frequency separations smaller than those of OFDM systems. The FDM transmitter consists of a bank of modulators that generate the different information conveying sub-carriers that compose the transmitted FDM signal. The FDM receiver consists of a bank correlators followed by a ML detector. The demodulators extracts a set of sufficient statistics from the received FDM signal and the ML detector uses the sufficient statistics to estimate the transmitted symbols.

Linear detection and GA based detection have been proposed to alleviate the complexity associated with ML detection. Linear detectors were shown to yield very poor BER whereas GA detectors were shown to yield very good BER with reasonable complexity.

Future work should concentrate on evaluating the performance of FDM based communications systems in the presence of various degradation factors such as multipath fading channels with delay and Doppler spread and timing and frequency offsets.

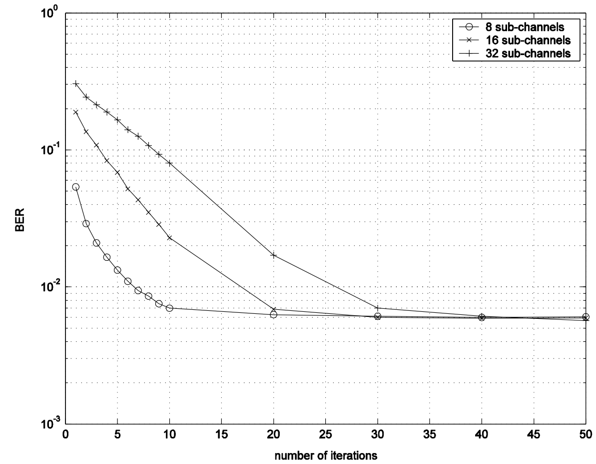

Figure 4: Average BER vs. number of iterations for FDM/BPSK signals with *N*=8,16 and 32 and Δ*fT*=0.75.

Finally, we note that the fact that the determinant of the matrix **M** approaches zero rapidly may in fact constitute a fundamental limitation for proper operation of the system for small intercarrier frequency spacings and large numbers of sub-channels. This topic should be investigated further.

VI. Acknowledgements

M.R.D. Rodrigues gratefully acknowledges the financial support from the Portuguese Foundation for Science and Technology through a postdoctoral fellowship.

VII. References

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