University College London

Wireless D&F Relay Channels: Time Allocation Strategies for Cooperation and Optimum Operation

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I, Elsheikh Mohamed Ahmed Elsheikh, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

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Abstract

Transmission over the wireless medium is a challenge compared to its wired counterpart. Scarcity of spectrum, rapid degradation of signal power over distance, interference from neighboring nodes and random behavior of the channel are some of the difficulties with which a wireless system designer has to deal. Moreover, emerging wireless networks assume mobile users with limited or no infrastructure. Since its early application, relaying offered a practical solution to some of these challenges. Recently, interest on the relay channel is revived by the work on user-cooperative communications. Latest studies aim to re-employ the channel to serve modern wireless networks.

In this work, the *decode-and-forward* (D&F) relay channel with half-duplex constraint on the relay is studied. Focus is on producing analytical results for the half-duplex D&F relay channel with more attention given to time allocation. First, an expression for the mutual information for the channel with arbitrary time allocation is developed. Introduction of the concept of conversion point explains some of the channel behavior and help in classifying the channel into suppressed and unsuppressed types. In the case of Rayleigh fading, *cumulative distribution function* (cdf) and *probability density function* (pdf) are evaluated for the mutual information. Consequently, expressions for average mutual information and outage probability are obtained.

Optimal operation of the channel is investigated. Optimal time allocation for maximum mutual information and optimal time allocation for minimum total transmission time are worked out for the case of *channel state information at transmitter* (CSIT). Results revealed important duality between optimization problems.

Results obtained are extended from a two-hop channel to any number of hops. Only sequential transmission is considered.

A cooperative scheme is also developed based on the three-node relay channel. A two-user network is used as a prototype for a multi-user cooperative system. Based on the model assumed, an algorithm for partner selection is developed. Simulation results showed advantages of cooperation for individual users as well as the overall performance of the network.

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Dedication

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Abbreviation

A&F	amplify-and-forward
AWGN	additive white Gaussian noise
BC	broadcast channel
cdf	cumulative distribution function
CDMA	code-division multiple-access
CSIR	channel state information at receiver
CSIT	channel state information at transmitter
D&F	decode-and-forward
DMC	discrete memoryless channel
FDD	frequency-division duplex
FDMA	frequency-division multiple-access
LAN	local area network
LEO	low earth orbit
LTE	long term evolution
MA	multiple-access
MANET	mobile ad hoc network
MIMO	multiple-input multiple-output
pdf	probability density function
pmf	probability mass function
SCI	statistical channel information
SNR	signal-to-noise ratio
TDD	time-division duplex
TDMA	time-division multiple-access
WAN	wide area network
WBP	wireless broadcast property
WLAN	wireless local area network
WPAN	wireless personal area network
WWAN	wireless wide area network

Notation

$\|x\|$	absolute value or amplitude of x
$\sim$	$\ldots$ is distributed as $\ldots$
$\in$	element of
$\triangleq$	defined as equal to
$n!$	factorial of integer n
$\gg$	much greater than
∞	infinity
$\mathcal{C}$	channel capacity
$\exp(x)$	e^x
$\Gamma(.)$	the Gamma function
$\log_x(y)$	the log, base x, of y
$E[.]$	expectation operator
$\mathbf{F}_X$	cdf of random variable X
$\mathbf{f}_X$	pdf of random variable X
$\mathcal{I}(X;Y)$	mutual information between random variables X and Y
$\mathcal{I}_{\mathsf{A},\mathsf{R}_1,\mathsf{R}_2,\ldots\mathsf{B}}$	mutual information for channel $(\mathsf{A},\mathsf{R}_1,\mathsf{R}_2,\ldots\mathsf{B})$
$\max\limits_{g(x)} f(x)$	maximum value of $f(x)$ maximized over all $g(x)$
$\min\limits_{g(x)} f(x)$	minimum value of $f(x)$ maximized over all $g(x)$
$p(x)$	marginal pmf of x
$p(x,y)$	joint pmf for x and y
$p(x\|y)$	probability of x given y
$(\mathsf{A},\mathsf{R}_1,\mathsf{R}_2,\ldots\mathsf{B})$	a relay channel with a source node A, a destination node B and relays $\mathsf{R}_1,\mathsf{R}_2\ldots$

List of Publications

- K. K. Wong and E. Elsheikh, Optimizing time and power allocation for cooperation diversity in a decode-and-forward three-node relay channel, *Journal of Communications, Academy Publisher*, vol. 3, no. 2, pp. 4352, Apr 2008.
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- K. K. Wong and E. Elsheikh, Optimized cooperative diversity for a three-node decode-and-forward relay channel, *IEEE Int. Symp. on Wireless Pervasive Computing*, Feb 2007.
- E. Elsheikh and K. K. Wong, Wireless cooperative networks: Partnership selection and fairness, *IFIP Wireless Days Conference*, Nov 2008.
- E. Elsheikh and K.-K.Wong, Unleashing the Full Potential of Relaying, *London Communications Symposium*, Sep 2007.

Chapter 1

Introduction

Modern communication systems have become an important part of our day-to-day life; the industry has grown in all dimensions. Telecommunications today has claimed strategic as well as social and economic importance. Services offered expanded from simple voice and texts to live TV broadcast and the Internet. Aided by great advancements in hardware, today's communications systems are diverse and complex like never before. In the heart of this revolution is wireless communications; it is by all means the fastest growing segment of the communications industry. Accessing network resources through the wireless medium offered users with indispensable mobility advantage.

The aim of this chapter is to demonstrate the motivation behind the work presented in the thesis. Section 1.1 briefly introduces some of the widespread wireless systems, challenges faced by these systems and how relaying helps combat some of the challenges. In Section 1.2, objectives of this research are listed along with the methodology followed. Section 1.3 lists contributions of the thesis followed by a list of published materials during the course of the research. Finally, an overview of the thesis is given in Section 1.4.

1.1 Motivation

1.1.1 Wireless Communication Systems and Networks

1.1.1.1 Challenges

Designing a wireless communication system proves to be a challenge; transmission over the wireless medium is ever hard. That, in addition to other hardware restrictions, hampers the performance of wireless systems and complicates their design. There are three key features that differentiate the wireless channels and wireless networks from their wired counterparts:-

- *Scarcity of Wireless Resources*— Radio spectrum is very scarce. It has to be allocated fairly and used efficiently. Usually international organizations and governmental bodies control assignment of frequency bands. Recently cost of spectral licensing in some countries has reached astronomical figures. With the number of wireless users rapidly increasing, it has become more appealing to improve spectrum efficiency by advancing transmission techniques.

- *Wireless Broadcast Property (WBP)*— Signal transmitted by a wireless node is received by other nodes located in the vicinity. As a result, if two nodes transmit at the same time and same frequency band, their signals will interfere. To avoid interference, there number of techniques to achieve orthogonality; such as *time-division multiple-access* (TDMA), *frequency-division multiple-access* (FDMA) or *code-division multiple-access* (CDMA). User-cooperative communications take advantage of WBP. Wireless users can help neighbor nodes by retransmitting their signal.

- *Channel behavior*— It is noticed that power of the wireless signal degrades rapidly with distance. In addition, communication over the wireless medium suffers from large scale and small scale fading caused by shadowing and the multi-path phenomena. Multi-path fading is a result of the constructive and destructive addition of multiple components of the received signal [1]. Small scale fading is even harder to deal with. Random changes in channel status due to nodes' mobility and environment dynamics further worsen the situation. Poor performance in wireless systems is largely due to fading. Probability of experiencing a fade (and associated bit errors) on the channel is a limiting factor in the link's performance.

Modern wireless devices are required to be compact. Size restriction means *full duplex* transmission is usually inapplicable. A wireless terminal can either transmit or receive (but not both) at a given time and frequency band. Although a full-duplex device is possible, it is expensive and may also be ineffective, since the dynamic range of incoming and outgoing signals can easily go beyond the supported range. A *half-duplex* wireless device is more realistic. *Time-division duplex* (TDD) and *frequency-division duplex* (FDD) are mechanisms used to ensure separation between up-link and down-link channels. TDD and FDD are based on time-division multiplexing and frequency-division multiplexing techniques, respectively.

Modern wireless devices are also required to support different applications. At the same time they need to be cheap and lightweight. Moreover, hand-held wireless devices must incorporate small batteries, which leads to more restrictions on power consumption. This is a particular issue for networks without fixed infrastructure, where all nodes are required to be equally capable of carrying out all processing and control tasks. In order to conserve energy transmission and signal processing, they need to be optimized for minimum power consumption.

Different applications usually have different requirements. Voice communication, for example, requires low data rate, can tolerate relatively high probability of error and has harsh delay constraints; while data systems need high data rate with small probability of error and relaxed delay constraints. Other applications such as video conferencing, web browsing, sensing, short messaging and distributed control have different sets of requirements. Wired networks can usually accommodate different applications using a single protocol. With lower rate and higher bit error rate (BER), wireless systems are intolerable to design deficiencies. Therefore, it is more complicated to build a wireless system that satisfies the diverse requirements of multiple applications. Most wireless systems are tailored to accommodate only a few applications, which results in a large number of systems and standards. This kind of diversity imposes restrictions on evolution of future wireless systems and integration between current systems.

To reduce deficiencies associated with layered approaches of designing communications systems, design of wireless systems follows a cross-layer approach. That further complicates design procedure. Wireless systems designers must have interdisciplinary expertise in communications, signal processing, and network theory and design.

1.1.1.2 Modern Wireless Systems

Since Marconi's first demonstration of radio transmission in 1895, we witnessed the emergence of a large number of wireless systems. This continuous evolution is a result and part of advancements in other fields such as information and communication theory, electronics, computational systems, control systems and signal processing, which allowed for complicated systems to be built. Until recently, wireless communication systems have achieved limited success due to high cost and low data rate. During the last two decades the situation has dramatically changed; wireless systems are rapidly growing like never before. Every part of the network is being revolutionized, from the end user's equipment to the core network and from the physical layer to the network layer. Here we explore a few of the most popular systems, discuss the main factors that led to their success and demonstrate some of their features and future prospect.

Two key technical advancements contributed significantly to the current leap in wireless communications. The first is the concept of frequency reuse developed by researchers in AT&T Bell laboratories [2]. According to the principle, since power of transmitted signals falls rapidly with distance, two users can transmit on the same frequency band at the same time without causing serious interference. This led directly to the emergence of cellular systems. Cellular architecture has significantly improved efficiency of spectrum and allowed for large numbers of users. Frequency reuse is not limited to cellular systems, however. *Wireless local area networks* (WLAN) and *wireless personal area networks* (WPAN) are allowed to operate in the unlicensed frequency bands. With restricted transmission power, there is a high probability that these networks can coexist and operate without causing serious interference to each other or to other systems operating on the same band.

Then there was the switch to digital communications. The idea of digitizing analogue data can be linked back to the famous theoretical work of Claude Shannon [3] in 1948. By bringing the idea into practice, doors opened wide for new possibilities in telecommunications and the whole architecture of communications systems was revolutionized. Following are a number of the advantages brought by digital systems:-

1. Allowed for the integration of voice and data systems.
2. Allowed for integration of communication systems and computational systems so that more complicated systems could exist.
3. Significantly improved system efficiency.
4. Intelligent codes could be built which allowed error-correction, data compression and secured transmission.
5. Cost has significantly dropped.
6. Digital devices are in general cheaper, faster, smaller and consume less power.
7. More services and applications could be fitted into the system.

Cellular systems are probably the most successful wireless systems. Mobile phones are replacing their fixed counterparts in developed as well as developing countries, making them a critical social and economical tool. More than 2 billion users are served by these systems [2]. Cellular systems very successfully exploit frequency reuse; coverage area of a cellular system is divided into sub-areas or cells, where each cell is assigned a subset of available channels. The same set of channels can then be reused by another cell far away enough. Consequently, the system can serve more users and spectrum efficiency is significantly improved. By shifting to digital technology in the second generation, cellular systems became more efficient and the range of services offered expanded. It also became possible to provide data service. The *3rd Generation Partnership Project* (3GPP) *long term evolution* (LTE) is the latest standard in cellular systems. LTE specifications include high spectral efficiency, very low latency and inter-working with other wireless systems.

History of wireless data networking goes back to 1971 [2]. The first data system based on packet radio, ALOHANET, was developed at the University of Hawaii. Many of ALOHANET's channel access protocols and routing algorithms are still in use. Commonly, wireless data networks are classified, based on coverage area, into three types: *wireless wide area networks* (WWAN), WLANs and WPANs. In addition to having the widest coverage, WWAN differs from the other two types in that it serves both fixed and mobile users. WWANs are cellular-like systems where geographical areas are covered by a number of access stations. Recently, *Worldwide Interoperability for Microwave Access* WiMAX emerged as a wireless broadband access based on the IEEE 802.16 standard. WiMAX aims to provide broadband wireless access for variety of devices, both fixed and mobile, in range of a few kilometers from a base-station [4]. Due to its relatively low cost of deployment, WiMAX is replacing cellular data services and in some areas wired data services.

WLANs offer high speed data services to fixed and slow-moving users within a small region, e.g., a campus or small building. Based on the popular IEEE 802.11 standard, Wi-Fi is a widely used WLAN technology. An access point (also known as hotspot) can connect number of wireless users on a peer-to-peer basis, to another *local area network* (LAN) or to a *wide area network* (WAN) such as the the Internet. Then number of access points can be arranged to cover larger regions. On the other hand, WPANs have the smallest coverage area. WPANs typically interconnect devices within only a few meters. Bluetooth is a popular WPAN technology based on the IEEE 802.15 standard. Owing to their low transmission power, WLANs and WPANs use unlicensed frequency bands, which has a significant effect on cost and increases the popularity of these networks.

Satellite systems are of the earliest wireless systems; they were mainly used to relay transmission between earth stations where direct connection is not possible. Satellite systems are challenged by low data rate, long propagation delay and high power consumption. TV broadcast is a popular application of these systems. In telephone applications, satellites are used as part of the core network, relaying calls between central switches possibly located in different continents. With the flourishing of optical fiber transmission lines and sub-marine cables, satellites lost much of their significance. However, they continue to offer services to remote areas, such as the Arctic and Antarctica, and to some mobile applications where cabling is impossible such as communication to ships and planes. *Low earth orbit* (LEO) satellite constellations were proposed to provide direct access to mobile phones. LEOs cost less and have lesser latency too. Nevertheless, they did not flourish due to the hard competition from cheap and less power-demanding cellular systems.

All the above systems rely, in one way or another, on some kind of wired infrastructure. Usually, only the last link between end users and access points (or base stations) is wireless. Access points not only act as gateways to the network, but they carry out most control tasks and signal processing so that end user's devices save power for transmission. Still, most wireless systems are challenged by the last mile problem due to the unreliability of the wireless channel. Some specifications of existing systems remain impractical in many regions due to the lack of proper coverage.

1.1.1.3 Ad Hoc Networks

Another problem with systems which rely on infrastructure is that they are unreliable in emergency situations such as natural disasters and wars. A report addressing lessons from emergency response to the 7 July 2005 London bombings states *the use of GSM mobile telephones by front-line staff in the emergency services should decrease with the move to new dedicated digital radio systems which allow the emergency services to communicate between each other more easily* [5].

Wireless ad hoc networks are decentralized networks which can be rapidly deployed in areas where no infrastructure is available. The nature of ad hoc networks makes them suitable for emergency applications. Other applications include wireless sensor networks, automated highways, automated factories and telemedicine.

Mobile ad hoc networks (MANET) are a kind of wireless networks where nodes are mobile. MANETs can be formed by vehicles, ships and planes. Wireless mesh ad hoc networks is another type of ad hoc networks. In addition to user nodes, these networks have mesh routers. Mesh routers are multi-interface nodes, and often have more resources compared to user nodes in the network and thus can be exploited to perform more resource intensive functions.

Recently, some centralized systems also allow limited ad hoc operation. For example, both IEEE 802.11 and IEEE 802.16 standards support ad hoc mode of operation. Peer-to-peer connection can be established without help from an access point. Mesh networking is also supported by 3GPP LTE, where self-organized network (SON) operation is allowed. LTE eNBs (similar to base stations in GSM) can connect directly using wireless links, eliminating the need to connect through the core network [6]. TerraNet has recently developed a system which allows mobile users to connect directly when they are out of coverage area of the traditional cellular network. TerraNet aims to provide access to people in rural areas and where investment in infrastructure is not viable. In disaster areas where the traditional communications networks are down, a TerraNet-based telephone system can be set up in a few hours [7].

Ad hoc networks have a dynamic structure as nodes can freely connect or disconnect from the network. A functioning network must be able to cope with this dynamic restructuring, preferably in a way that is timely, efficient, reliable, robust and scalable. Nodes are required to be able to carry all control and necessary signal processing tasks.

Connectivity in an ad hoc network is a particular issue. Transmission is often carried out over multiple wireless hops. Any routing algorithm must consider the limited resources available to nodes and the dynamic nature of the network. When traditional routing algorithms are applied to ad hoc networks, serious issues arise [8]. This is less

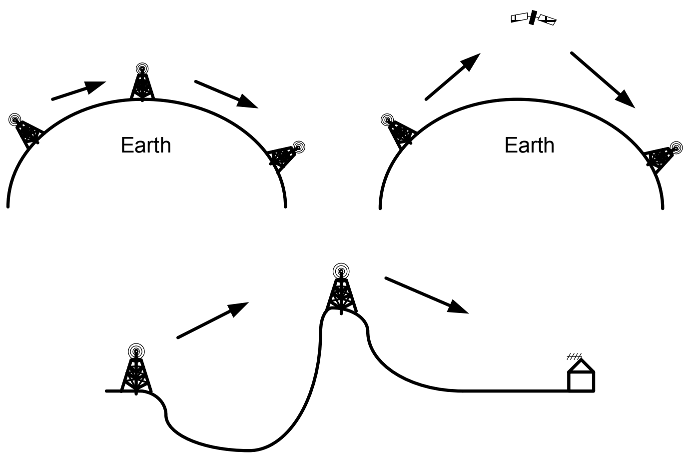


Figure 1.1: Examples of early applications of relays in transmission over long distances.

pronounced in systems which rely on infrastructure.

1.1.2 Wireless Relay Channels

In today's modern society, freedom offered exclusively to wireless users is indispensable. Besides, the Internet and the evolution of applications introduced a need for higher rate and more robust networks. It is thus appealing to improve wireless communications systems even more. Researchers are working on new ideas to solve untackled problems; all parts of the system are targeted. In the thesis, nonetheless, we consider the relay channel as a means to improve wireless connectivity.

Relaying can be important for some systems, such as cellular systems, in order to achieve specified requirements. For ad hoc networks, on the other hand, relaying is essential to ensure connectivity between nodes. Relaying also represents the basis for some of the emerging paradigms such as cognitive radio and user-cooperative communications.

Relaying was in use long before modern telecommunication. Smoke and fire beacons used to send important messages over long distances during ancient times. Use of wireless relaying in telecommunications started in during 1940s in the USA [9]. At the time, it was considered an efficient way to extend the range of transmission between fixed stations. As explained in Figure 1.1, relaying was proposed mainly to tackle problems like earth curvature, path loss and irregular terrains where direct transmission is unattainable. The use of satellites in communications is the best example demonstrating the use of relay channels for range extension. In addition, the relay channel is currently seen as a means to achieve diversity in transmission to combat fading, as will be explained next.

1.1.2.1 Diversity in Communications

The subject of providing diversity in reception to remedy channel impairments has been investigated for decades. Diversity is particularly useful in fading channels where increasing transmission power is ineffective in combating channel impairments. It is known that detection error probability decays exponentially in received *signal-to-noise ratio* (SNR) in a single channel with *additive white Gaussian noise* (AWGN) only, while it decays only inversely with the SNR in fading channels [10]. Diversity is more effective on flat fading channels. [11].

Diversity is achieved by combining multiple copies of the signal transmitted over independent channels. The probability of having all channels in deep fade, which leads to decoding errors, is less than that for a single channel.

In fading channels, as explained above, diversity is used as a means to improve reliability by repeating the same signal over parallel independent channels. Alternatively, by transmitting independent information streams over these independent sub-channels, data rate is increased. Both types of gain can be simultaneously obtained for a given channel, but there is a fundamental trade-off between how much of each any coding scheme can get. This is known as the *diversity-multiplexing trade-off* [12, 13].

Diversity can be obtained over time, frequency, space or any combination of these dimensions. *Temporal diversity* is obtained by retransmitting data packets in time intervals greater than channel *coherence time* (time over which channel changes significantly). It can also be achieved via coding and interleaving, where information bits are dispersed over time in different coherence periods so that different parts of the codeword experience independent fades. Coding and interleaving is implemented successfully in Global System for Mobile Communication (GSM). Analogously, in *frequency selective* channels, diversity is achieved by dividing the channel into a set of orthogonal sub-carriers, each experiencing narrow band *frequency non-selective* (or *flat*) fading. Information bits are then repeated or interleaved across sub-bands.

Spatial diversity, on the other hand, is obtained by transmitting data streams over multiple independent paths. Multiple independent channels are created by attaching multiple antennas to transmitter and receiver. *Multiple-input multiple-output* (MIMO) systems are widely acknowledged as an effective means to improve performance. In cellular systems, for example, multiple antennas spaced sufficiently are employed at base stations (see Figure 1.2). MIMO is also specified in WiMAX, Wi-Fi and LTE (see for example [4, 15]).

Mounting multiple antennas is not always feasible; constraints on power consumption and physical size prevent mobile users from having multiple antennas. In that case, spatial diversity can be achieved through relay channel. A relay node offers an independent alternate transmission path. Diversity obtained by relaying is also referred to

Figure 1.2: Multiple antennas at a base station of a cellular system. Photo obtained from [14].

by *cooperative diversity* [16].

1.1.2.2 Multi-Hop Relaying

Instead of a single relay and two-hop transmission, a source node can transmit its signal over multiple hops using multiple relays. Multi-hopping is an efficient way to expand transmission range for power constraint systems. This is particularly important for ad hoc networks. When users are spread in a wide area, multi-hop is the only means that allows more nodes to remain connected. For example, in sensor systems nodes depend on limited power batteries users need to save power for longer operation life. Multi-hopping is as essential to ad hoc networks as frequency reuse is to cellular systems.

Cognitive networks is another application where multi-hop relaying is useful. Cognitive radio aims to improve spectrum efficiency by allowing secondary users to use licensed frequency bands when that does not cause harm to primary users. The

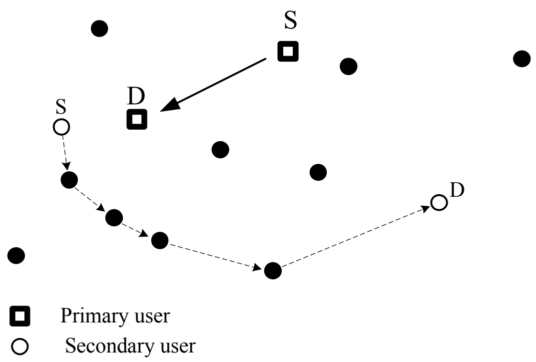


Figure 1.3: Secondary users are allowed to use licensed bands if those bands are unused or if they can make sure that no harmful interference is caused to primary receivers. Multi-hop relaying is helpful in the sense that transmission power can be kept low in order to reduce interference caused to primary users.

spectrum of licensed channels can be categorized into: *white holes*, where the primary network is inactive; *gray holes*, where the primary network is operating with low power and *black holes*, where the primary network is operating with high power [16]. Secondary users have the right to fully exploit white holes while banned from black holes.

Secondary users are allowed to operate in gray holes as well, under the condition that interference caused at primary receivers is tolerable. A typical scenario is shown in Figure 1.3. A secondary source can not exceed a maximum transmission power $P_{\max}$; otherwise connection between primary users is disturbed. On the other hand, a minimum power $P_{\min}$ is needed to establish a connection with the secondary destination. If only direct transmission is allowed, then the secondary source is allowed to transmit only if $P_{\min} \leq P_{\max}$. However, other secondary users can help by establishing a multi-hop route from the secondary user to the secondary destination. A carefully selected route ensures that neither the secondary source nor any of the relay nodes exceeds its maximum allowed transmission power.

1.1.2.3 User-Cooperative Communications

Recently, application of user cooperation techniques in wireless systems has received much attention. User-cooperative communication is a form of communication in which users work together to improve transmission. Cooperative diversity is accomplished by having a cooperating partner acting as a relay to forward the received information from the user.

Cooperative wireless systems exploit the broadcast property of wireless networks. Nearby nodes are able to receive the source's signal, at no extra cost. These neighboring nodes can then act as relays, helping the destination to reveal the transmitted message.

The work presented in the thesis, and most of the recent work in relay channels, is motivated by interest in user-cooperative communications. In multiuser systems, user-cooperative transmission offers a flexible and dynamic alternative to obtain some of the advantages of spatial diversity, especially when hardware restrictions prevent the use of multiple antennas. This is more pronounced in ad hoc networks where relayed transmission by cooperating users can improve connectivity between nodes.

1.2 Objectives and Methodology

Wireless D&F relay channels showed modest performance when considered for cooperative communications. This is mainly due the half-duplex constraint on the relay where time is assumed to be shared equally between the source and each of the relay(s) [17]. Channel performance may improve if time is allocated differently. Information theory, however, does not offer tools necessary to analyze the channel for such cases. The aim of this thesis is to fill this gap by producing formulae necessary to analyze the channel for any time allocation. That can lead to more benefits such as optimizing channel performance by choosing the best time allocation policy and devising more efficient cooperative protocols.

This thesis studies the D&F relay channel with half-duplex nodes and any number of hops. We aim to produce analytical results that are useful for information theoretic analysis of the channel. Mutual information and average mutual information are considered for measuring the throughput while outage probability is assumed for reliability measurement. All results are functions of time allocation. Available time is allocated to the source node and the relay node (or nodes) in order to comply with the half-duplex operation. Similar results can be produced in the case of frequency allocation.

Fundamentally, relaying can be applied to any wireless system; however, the importance and effectiveness of relaying varies for different systems. No particular application is considered; instead, this work considers a general wireless relay channel. Both AWGN and Rayleigh fading channels are considered.

As optimum operation is always a concern for wireless systems, we seek time allocation policies that achieve the optimal performance. Once again, we aim to produce analytical results when possible. In particular, we are interested in the optimal time allocation that maximizes mutual information, maximizes average mutual information or maximizes link reliability. We are also seeking the optimal time allocation that minimizes total time used for transmission while maintaining a minimum required performance. Choosing the optimum route is also considered in the case of multiple relays.

Since this work is partly motivated by user-cooperative communication, we propose a cooperative scheme based on relaying. The proposed scheme is to be practical, reasonably simple and realistic so that it demonstrates some of the important issues related to user-cooperation communications. Particular attention is given to partner

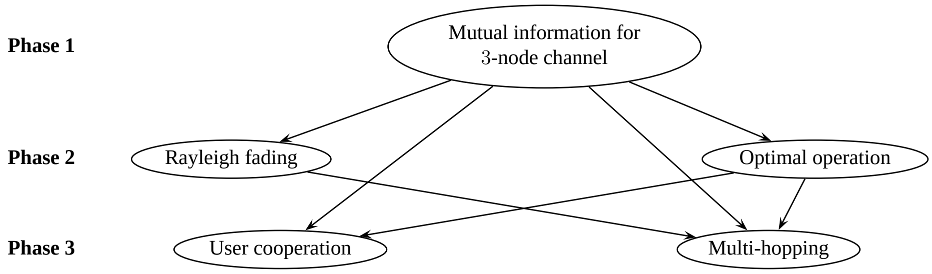


Figure 1.4: Work flow diagram. Arrows indicate order at which topics are dealt with as well as dependencies between topics.

selection, fairness, resource allocation and interaction with upper layers.

Figure 1.4 illustrates the tactic used to deal with the above objectives. Work took place in three stages. We started with the simplest relay channel which is the three-node AWGN channel. Results produced for this channel formed the basis for all results to follow.

In the second stage, results for the three-node AWGN channel are extended in two directions. First, we considered the case of the three-node channel with Rayleigh fading. In addition, solutions to optimization problems are generated for the three-node AWGN channel.

The last stage aimed at two tasks. The first task was to propose a model for user cooperation in a multi-user network. The proposed model is derived from the three-node AWGN channel. Moreover, solutions to optimization problems for that channel were useful when some cooperative issues are addressed.

Finally, we dealt with the multi-hop relay channel. That is done by extending all results produced for the simple two-hop channel to any number of hops. That included mutual information, average mutual information, outage probability and optimum routing (route selection and time allocation). Figure 1.4 shows topics tackled and the connection between them.

1.3 Contributions

The following is a detailed list of contributions:-

1. Mutual information formula for the D&F relay channel with half-duplex constraint on the relay is worked out for the case of AWGN channels with any number of hops and any time allocation policy. Results are generated for the three-node channel and later generalized to any number of relays. Only sequential channels are considered; that is, channels with only one transmitter at a time. The concept of conversion point is introduced. Closely related, channel classification based on channel conditions is also established. The type of the channel

and its conversion point give an indication of its general behavior for different time allocations. This proved useful for relay selection in the case of optimum relaying and partner selection in the proposed user-cooperative communication.

2. In Rayleigh fading channels, cdf and pdf are evaluated for mutual information of the D&F relay channel with half-duplex constraint on the relay and any number of hops. Consequently, it was also possible to work out average mutual information and outage probability. Again, results are first generated for the three-node channel and then extended to any number of hops. Technique used is general and can be utilized to work out the same quantities for other channels with different kind of fading.

3. Optimum relaying strategy is worked out for the channel, which includes the optimum time allocation policy for the three-node channel. In the case of more than one relay, optimum routing involves route selection as well as optimum time allocation. The aim is to maximize throughput, measured in terms of mutual information or to minimize total time while maintaining the same throughput. Solutions revealed an important duality between these optimization problems.

4. Based on the three-node relay channel, a two-user cooperative network setup is proposed. A cooperative network scenario is studied. A framework is proposed to deploy a cooperative prototype to a multi-user network. Several related issues are addressed, which include cross-layer design, fairness, partner selection and time allocation. Simulation results are generated and analyzed, comparing cooperative and non-cooperative performance for multi-user networks. Numerical results suggest a strong connection between cooperation gain and network parameters such as path-loss, node density and average SNR.

1.3.1 Publications

Results presented in this thesis are also published as follows:-

- K. K. Wong and E. Elsheikh, Optimizing time and power allocation for cooperation diversity in a decode-and-forward three-node relay channel, *Journal of Communications, Academy Publisher*, vol. 3, no. 2, pp. 4352, Apr 2008.

- E. Elsheikh, *Wireless D&F Relay Channels: Time Allocation Strategies for Cooperation and Optimum Operation,* Germany, LAP LAMBERT Academic Publishing, to be published.

- E. Elsheikh, K. K. Wong, Y. Zhang, and T. Cui, Chapter 11: User-cooperative Communication, *Cognitive Radio Communications and Networks: Principles* *and Practice,* A. Wyglinski, M. Nekovee and T. Hou (editors). New York, USA: Elsevier Press, Jan 2010.

- K. K. Wong and E. Elsheikh, Optimized cooperative diversity for a three-node decode-and-forward relay channel, *IEEE Int. Symp. on Wireless Pervasive Computing*, Feb 2007.

- E. Elsheikh and K. K. Wong, Wireless cooperative networks: Partnership selection and fairness, *IFIP Wireless Days Conference,* Nov 2008.

- E. Elsheikh and K.-K.Wong, Unleashing the Full Potential of Relaying, *London Communications Symposium,* Sep 2007.

1.4 Thesis Overview

The remainder of the thesis is organized as follows. Chapter 2 gives background information and discusses related work. Topics discussed include wireless communications, relay channels, cooperative communications and some fundamental information theoretical concepts.

Chapter 3 is the most important single chapter in the thesis. All other chapters rely on results presented in that chapter. There, mutual information for the AWGN three-node D&F relay channel with half-duplex constraint on the relay is derived for arbitrary time allocation. Based on realizations of channel behavior for different channel conditions, classification of the relay channel is established. Moreover, the concept of conversion point is introduced. Later in the chapter, a channel with Rayleigh fading is considered. Both average mutual information and outage probability are worked out. In order to do that, it was first necessary to obtain distribution functions for mutual information random function. Average mutual information and outage probability are also presented as functions of time allocation.

Chapter 4 deals with some of the optimization problems of the channel. In particular, optimal time allocation policies are sought to maximize mutual information and to minimize total transmission time. Analytical solutions are obtained for both problems. Furthermore, it is possible to establish an important duality between optimization problems by comparing these solutions.

User-cooperative communication is discussed in Chapter 5. A two-user cooperative model is presented. Later, that model is used as a prototype for cooperation in a multi-user network. Several issues are addressed including time allocation, partner selection, and fairness. Simulation results are presented and analyzed. Connection is established between cooperation gain and network parameter such as node density, path-loss factor and average SNR.

In Chapter 6, results from Chapter 3 and Chapter 4 are extended to multi-hop relay channel. In addition to mutual information, average mutual information and outage probability, this chapter deals with the problem of finding the optimum route and optimum time allocation.

Finally, Chapter 7 concludes and summarizes the thesis and looks at its contributions. This is followed by a discussion of potential future research.

Chapter 2

Background and Related Work

This chapter gives an essential background on some of the topics related to the thesis and explores other researchers' work which can be linked to the problem considered. Giving a comprehensive literature review is impractical; instead, focus is on studies which have a strong connection to the thesis. Other important studies are also referenced. Section 2.1 focusses on wireless channel modeling and examines some of these models and criteria for model selection. The relay channel is reviewed in Section 2.3. In Section 2.4 user-cooperative communications are considered. Finally, a summary of the chapter is given in Section 2.5.

2.1 Modeling The Wireless Channel

Establishing a model is the first step when conducting research. A good model is essential for producing useful results. A model is characterized by its accuracy and practicality. An accurate channel model is the one that successfully imitates all effects of the channel on signals.

When modeling the wireless channel we need to address the following effects of the channel:-

1. Electronic noise produced at the receiver.

2. Interference caused by neighbor nodes transmitting at the same time and in the same frequency band.

3. Path loss, which is the attenuation of the transmitted signal due to dissipation of the transmitted power.

4. Shadowing, which is the attenuation of the transmitted signal due to obstruction caused by obstacles.

5. Multi-path fading caused by constructive and destructive addition of in-phase and out-of-phase multi-path signal components.

In the thesis it is assumed that transmission orthogonality is kept all the time, which means that the channel is always interference-free. Path loss and shadowing are the *large-scale propagation effects* while multipath fading is the *small-scale propagation effect*. This categorization refers to the rate of change in received signal caused by each of these channel impairments. Fading is also classified as *flat* or *frequency-selective*. Flat fading occurs when all of the signal's spectral components are affected in a similar manner. One the other hand, frequency-selective fading occurs when a signal's spectral components are not all affected equally by the channel [1].

The model also has to take into account the time-varying nature of the channel resulting from user mobility and environment dynamics. Fading is classified as *fast* or *slow* based on the rate at which the channel changes.

Researchers have devised a number of models for the wireless model. Selection of a suitable model usually involves a compromise between accuracy and practicality. Ultimate accuracy can be achieved by solving Maxwell's equations with suitable boundary conditions. That requires detailed information of the physical characteristics of surrounding objects. Since it is usually difficult to obtain this information, and sometime impossible, solving Maxwell's equations is impractical.

On the contrary, *free space* model is the most simple model that tells the least about the channel. It assumes an environment which is void of any obstructer. Thus only line-of-sight (LOS) path is considered. According to free space model, the ration of the received signal power to the transmitted signal power is given by,

$$\frac{P_{\mathrm{Rx}}}{P_{\mathrm{Tx}}} = k(f)d^{-\alpha}, \tag{2.1}$$

where α denotes the path loss exponent (typically ranging from 2 to 6 [16]) and k is an appropriate constant, function of frequency f, that accounts for the antenna pattern in the direction of transmission and other hardware losses. k is dependent on carrier frequency.

A practical approximation to solving Maxwell's equations is *ray tracing*. This method considers the number of paths through which the signal propagates. To simplify it further, reflection and refraction are taken into account by ray-tracing, while the more complex scattering is ignored. Ray-tracing requires knowledge of the geometry and dielectric properties of the region through which the signal propagates. The number of paths to be taken into account to achieve reasonable accuracy depends on the complexity of the modeled channel. For example, two-ray model is a good approximation to propagation along highways and rural roads, while more paths must be considered for indoor propagation.

Empirical methods are also used to model the wireless channel. For a particular area, an empirical model is established by measuring the channel at given distances for a given frequency range. A disadvantage of these models is that their accuracy is always questionable when applied to other environments.

There are also several *statistical/probabilistic models* which are widely used to model the channel. These are more appropriate for studying the behavior of a general wireless channel and they fit most of the scenarios where ray-tracing is deemed impractical, that is, when the number the multipath components is very large or dielectric properties of the environment is unknown. Probabilistic models are also suitable for channels which change unpredictably.

Nakagami distribution is a general probability distribution developed to fit a wide variety of empirical measurements. pdf for a Nakagami distributed random variable X is given by [2],

$$\mathbf{f}_X(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)E[X^2]} \exp\left[\frac{-mx^2}{E[X^2]}\right], \quad m \geq 0.5, \tag{2.2}$$

where $E[.]$ is the expected value and $\Gamma(.)$ is the Gamma function. $E[X^2]$ and m determines characteristics of the distribution. They can be adjusted to fit statistical information for a given environment [2]. m, in particular, indicates the severity of the fading, with $m = 0.5$ as the worst case. When $m = \infty$ the received signal has a constant power, that is, an AWGN channel, i.e., without fading.

When $m = 1$ in Nakagami distribution, the channel is Rayleigh distributed. Rayleigh distribution is widely used as a statistical model for the general wireless channel. Rayleigh fading is most applicable when there is no LOS and there are many received signal components through deflection, reflection and scattering. Originally, it was viewed as a reasonable model for tropospheric and ionospheric signal propagations. It is also used to model transmission through heavily built-up areas such as city centers [1].

Rayleigh phenomenon is a direct consequence of the central limit theorem. In simple terms, when many random components are received, the real and imaginary parts of the sum signal tend to be normally distributed, regardless of the distribution of individual components. Consequently, amplitude of the received signal is Rayleigh distributed while the phase has a Uniform distribution. It can also be shown that if the received signal is Rayleigh distributed, then power of the received signal is exponentially distributed.

The thesis assumes a Rayleigh fading channel. Moreover, it is assumed that fading is flat and slow. We also consider the AWGN channel. AWGN channels are the classical model for communication channels where noise is the only channel impairment. Fading channels can be viewed as AWGN channels with randomly changing SNR. That makes

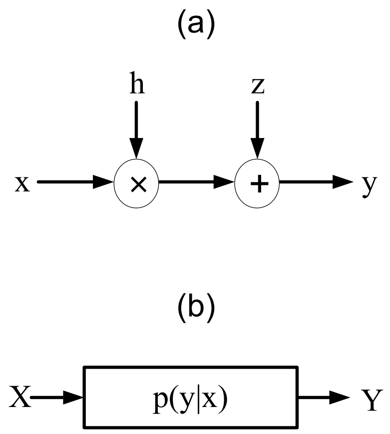


Figure 2.1: Single point-to-point channel, (a) operational (physical) representation and (b) information theoretical (probabilistic) representation.

AWGN channel model a good starting point to study fading scenarios.

2.2 Mutual Information and Channel Capacity

In information theory, a mathematical representation of a point-to-point *discrete memoryless channel* (DMC) communication channel consists of two random variables, X and Y, corresponding to the input and output sequences and a set of conditional *probability mass functions* (pmf), $p(y|x)$, for each $x \in \mathcal{X}$ and $y \in \mathcal{Y}$, $\mathcal{X}$ and $\mathcal{Y}$ finite sets of channel input and output alphabets respectively (see Figure 2.1-b).

Channel capacity is the basic information theoretic performance measure for a communication channel. As first introduced by Claude Shannon in [3], *channel capacity* is the maximal rate at which information can be sent over the channel with arbitrary low probability of error [18]. Mathematically, channel capacity for a single channel equals the maximum mutual information between X and Y, maximized over all possible input distributions, $p(x)$ [2],

$$\mathcal{C} = \max_{p(x)} \; \mathcal{I}(X;Y) \text{ bit/channel use}. \tag{2.3}$$

Mutual information between X and Y, denoted as $\mathcal{I}(X;Y)$, is a quantity that measures their mutual dependence. In other words, it tells us on average how much information we have about X given Y or vice versa [18]. Mathematically that is expressed by,

$$\mathcal{I}(X;Y) = \sum_{x\in\mathcal{X}} \sum_{y\in\mathcal{Y}} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}, \tag{2.4}$$

where $p(x)$, $p(y)$ and $p(x,y)$ are the marginal pmf for X, the marginal pmf for Y and the joint pmf for X and Y, respectively. Unless otherwise mentioned, logarithms are taken to base 2.

DMC capacity in (2.3) can be extended to the case when X and Y are continuous. For the single channel in Figure 2.1, the received signal at the destination is,

$$y = hx + z, \tag{2.5}$$

where h is a constant accounting for the channel gain. z accounts for the noise and is assumed to be normally distributed with mean equals zero and variance N_0. This AWGN channel has a capacity of,

$$\mathcal{C} = \log\left(1 + \frac{P|h|^2}{N_0}\right) \qquad \text{bit/sec/Hz}, \tag{2.6}$$

where P is the transmission average power constraint given by

$$\tfrac{1}{n}\sum_{i=1}^{n} E[|x_i|^2] \leq P, \tag{2.7}$$

for any codeword $(x_1, x_2 \ldots x_n)$. $\mathcal{C}$ in (2.6) is achievable when X is normally distributed with zero mean and variance P.

In fading channels, h is considered to be a random variable. Goldsmith and Varaiya studied the point-to-point fading channel in [19]. They showed that capacity is achievable by means of adaptive power allocation techniques where transmission power is allocated according to channel state. That required full knowledge of the channel at both transmitter and receiver. The allocated power, $P(h)$ is subject to the long term power constraint

$$\int_0^\infty P(\mathfrak{h}) f_{|h|^2}(\mathfrak{h}) \mathrm{d}\mathfrak{h} \leq \bar{P}, \tag{2.8}$$

where, $|h|^2$ is channel power gain with pdf $f_{|h|^2}$ and $\mathfrak{h}$ is an arbitrary variable that can take any value $|h|^2$ takes. The optimal power allocation that achieves capacity is,

$$P(h) = \begin{cases} WN_0\left(\frac{1}{H_{\text{th}}} - \frac{1}{|h|^2}\right), & |h|^2 \geq H_{\text{th}}, \\ 0, & |h|^2 < H_{\text{th}}, \end{cases} \tag{2.9}$$

for some cut-off value H_{th} which can be obtained by numerically solving

$$\int_{H_{\text{th}}}^\infty \frac{WN_0}{\bar{P}}\left(\frac{1}{H_{\text{th}}} - \frac{1}{\mathfrak{h}}\right) f_{|h|^2}(\mathfrak{h}) \mathrm{d}\mathfrak{h} = 1. \tag{2.10}$$

The corresponding capacity is thus

$$\mathcal{C} = \int_{H_{\text{th}}}^\infty W \log\left(\frac{\mathfrak{h}}{H_{\text{th}}}\right) f_{|h|^2}(\mathfrak{h}) \mathrm{d}\mathfrak{h} \qquad \text{bit/sec}. \tag{2.11}$$

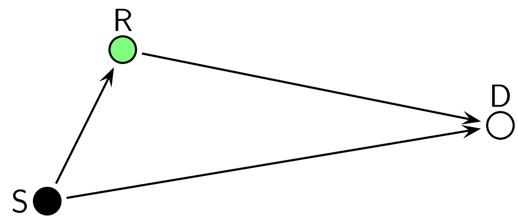


Figure 2.2: A three-node relay channel.

This kind of power allocation is known as *water-filling* in time. Water-filling can also be in frequency for frequency selective channels or in space for MIMO systems.

Channel capacity is in general a quantity that is difficult to find. For most channels, there is no closed-form solution. Capacity of the relay channel has been under investigation for long time. However it remains a challenge, even for the simple three-node case. An upper bound was obtained by Cover and El-Gamal in [20] by an application of the cut-set theorem.

Rather than considering a general relay channel, the thesis studies a specific relaying strategy, that is, D&F relaying. Consequently, focus is on producing mutual information results for the channel. In the case of fading channels, we also try to obtain expressions for average mutual information and outage probability, both derivatives of mutual information.

2.3 Relay Channels

Theoretical study of the relay channel goes back to 1960s. Use of satellite systems in telecommunications in the 1970s motivated extensive work on the channel [9].Van der Meulen was the first to introduce the three-node model consisting of a source node, a destination node and a relay node in [21]. That model was investigated by Cover and El-Gamal in [20]. Cover and El-Gamal considered discrete AWGN relay channels and derived achievable rate based on some random coding techniques [20]. More specifically, they also derived the capacity for the physically degraded class of channels. Later, more discussions on capacity and capacity-achieving codes appeared in [22–26]. Despite all work on the channel, capacity of the general relay channel remained an unsolved problem.

While developing a proper model, deriving capacity and designing coding strategies were the main concern of previous studies, whereas recent work has also taken into consideration other issues and extensions such as multiple relays and multi-hop transmission [27–35], resource allocation [35–40], multiuser relay networks [41], relay selection [42, 43], coding [44–46] and cross layer issues [35, 47].

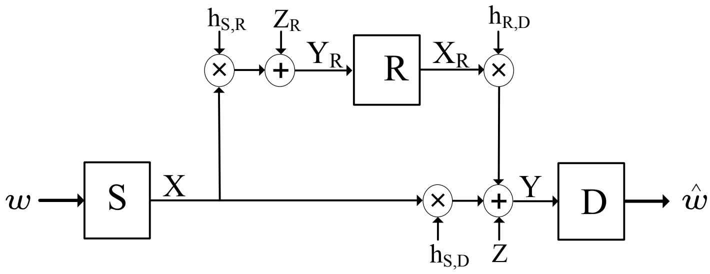


Figure 2.3: Block diagram for the three-node relay channel.

2.3.1 A General Three-Node Channel

A three-node relay channel consists of a source node, a destination node and a relay node. The source intends to send a codeword w to the destination. It first broadcasts w to the destination and the relay nodes. In turn, the relay node sends w_{R} to help the destination decode w. Figure 2.3 shows a block diagram for the general network with a single relay.

There are several ways in which the relay node can make use of the signal it receives from the source node. One approach is *amplify-and-forward* (A&F) relaying, where the relay simply sends a scaled copy of the received noisy signal. Another method is D&F relaying, where the relay transmits a re-encoded copy to the destination after decoding the transmitted message. A&F is easy to implement, but the noise at the relay may be amplified, which makes it unsuitable for multi-hop scenarios. On the other hand, D&F provides a more reliable solution at the cost of increased complexity. Due to the repetition nature, a common disadvantage of A&F and D&F relaying is the inefficient use of the available degree of freedom. This is especially apparent in half-duplex constraint relay systems.

Coded cooperation [48–50], on the other hand, integrates cooperative communication into channel coding. Instead of repeating its partner's data, a cooperating user sends some overhead bits, or just part of the codeword. The receiver makes use of both transmitted parts to generate the correct codeword. Obviously this improves resources' utilization with added coding complexity.

Focus of the thesis is on D&F relaying. D&F relaying is reasonably simple, practical and can be systematically extended to more complex setups. These factors make D&F more suitable for application in existing systems without many modifications. It is also appropriate for ad hoc networks. The efficiency of coded relaying makes it the protocol for future relay channels. More work on coded relaying is still needed.

Three D&F coding strategies are known to achieve the maximum transmission rate possible. Using descriptive names used in [31], these strategies are: *irregular encoding/successive decoding*, *regular encoding/backward decoding* and *regular encoding/sliding window decoding*. All three strategies are based on dividing the message

	Time slot 1	Time slot 2	Time slot 3	Time slot 4
Source	x(w_1)	x(w_2)	x(w_3)	x(1)
Relay	$x_R(1)$	$x_R(w_1)$	$x_R(w_2)$	$x_R(w_3)$

Figure 2.4: Example illustrating regular encoding/sliding window decoding scheme.

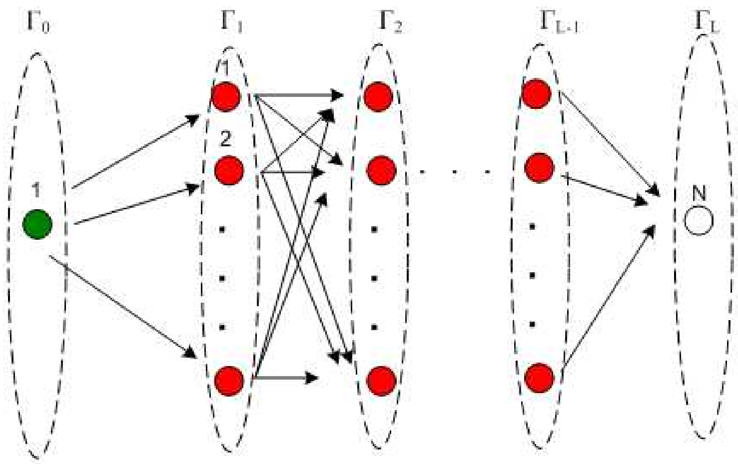


Figure 2.5: Multi-level multiple-relay channel

into blocks before transmission. The first strategy was used by Cover and El-Gamal in [20], the second was introduced by Willems [51], while the last was developed by Carleial [52] (originally for *multiple-access* (MA) channels). Regular encoding/sliding window decoding is practically advantageous over the other two techniques since it is the simplest of the three, expendable to multi-hop relaying and causes limited delay. Figure 2.4 shows a regular encoding/sliding window decoding example in which the message is divided into 3 blocks and sent in 4 time slots. Focus of the thesis is on achievable throughput; it is thus sufficient to assume any of the above coding strategies. As we are not trying to devise a new one, we assume any of these coding techniques is used.

2.3.2 Multi-Hop Relay Channel

Multi-hopping was the focus of a number of studies. In [35] a general multi-level multiple-relay channel was investigated. The channel consisted of a source node, a destination node and number of relay nodes arranged into different levels, as shown in Figure 2.5. Transmission is carried out in multiple hops with relays at the same level using space-time coding to transmit at the same time. The source node is located at level 0, which contains only one node. Similarly level L only contains the destination node.

The efficiency of space-time coding for multiple relay channels was discussed in [17]. Performance analysis of a two-hop multiple-relay channel, shown in Figure

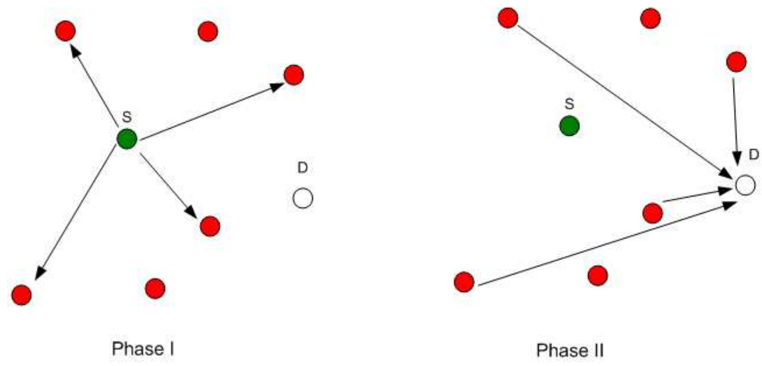


Figure 2.6: Illustration of the two phases of the multiple- relay cooperative diversity algorithm. In the first phase, the source broadcasts to the destination as well as potential relays. In the second phase, involved relays (decoding relays in the case of D&F) either repeat in orthogonal sub-channels or utilize a space-time code to simultaneously transmit to the destination.

2.6, demonstrated superiority over sequential transmission when relays are half-duplex constrained. Several implementation issues arise, however. One problem encountered is that we need a space-time code that works for an unknown number of relays. Another problem is the control and synchronization of such a channel when applied to wireless transmission.

In [35], achievable rate at node m in a multi-level relay channel is shown to be,

$$R_m \leq \max_{P(\mathbf{X}_0,\dots,\mathbf{X}_{L-1})} \min_{1\leq k\leq L} \min_{i:i\in\Gamma_k} I(\mathbf{X}_1,\dots,\mathbf{X}_{i-1};Y_i|\mathbf{X}_i,\dots,\mathbf{X}_m) \tag{2.12}$$

$\mathbf{X}_i$ is the input by nodes in level i. In the same study, useful results are also produced regarding optimum routing and optimum power allocation. An important recursive power-filling procedure is used to allocate power optimally. Authors concluded that sequential transmission is optimum for the channel.

A major difference between multi-hopping as being considered in the thesis and that studied in [35] is the half-duplex constraint on relays. With some considerations, a modified version of the recursive power-filling procedure mentioned above is developed in the thesis to allocate time optimally. Our study of optimal routing is more rigorous compared to [35].

An important scenario is considered in [53]. A useful relationship between route reliability (reliability $=$ $1-$ outage probability), distance between nodes and transmission power is established. Route reliability at node S_m in multi-hop relaying is expressed as,

$$\mathbf{R}_m = \prod_{i=1}^{m} \mathbf{R}_i = \exp\left(-\sum_{i=1}^{m} \frac{d_{i-1,i}^{\alpha}}{\gamma_{i-1,i}}\right) \tag{2.13}$$

where α is the path-loss exponent. $d_{i,j}$ and $\gamma_{i,j}$ are the distance and average SNR

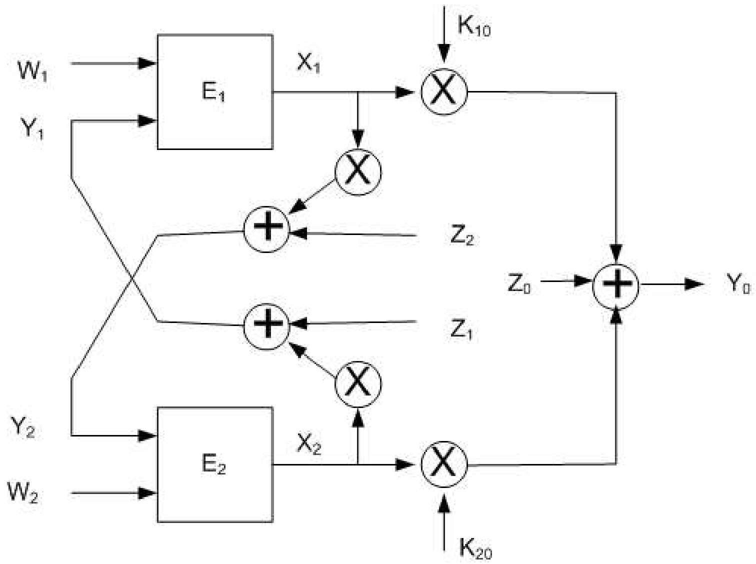


Figure 2.7: Block diagram for two users cooperating to send data to a common destination in a CDMA system. This model is used in [54].

between any two nodes i and j. $\mathbf{R}_i$ the point-to-point reliability.

With regard to (2.13), three optimization problems are investigated:-

1. End-to-end reliability subject to fixed maximum transmission power per link.

2. Minimum total power to achieve a guaranteed end-to-end reliability.

3. Maximum end-to-end reliability subject to fixed maximum total power.

Solutions to the above problems showed that two of them are dual problems. That comes as a consequence of the trade-off between end-to-end reliability and total transmission power. A similar trade-off for the half-duplex channel is investigated in the thesis, that is, a trade-off between mutual information and total transmission time.

2.4 User-Cooperative Communications

User-cooperative communication is a form of communication in which users work together to deliver their data. Users act as relays aiding their partners' transmission. Most of the recent work on relay channels is motivated by user-cooperative communications. In multiuser systems, user-cooperative transmission offers a flexible and dynamic alternative to obtain some of the advantages of spatial diversity, especially when hardware restrictions prevent the use of multiple antennas. This is more pronounced in ad hoc networks.

Studies by King [55], Carleial [56] and Willems *et al.* [51, 57–59] examining MA channels with generalized feedback can be related to the cooperative model [17]. Arguably, work by Sendonaris *et al.* has brought user-cooperative communications to attention and renewed the interest in relay channels. Following this, there has been an extensive amount of work in regard to relay channels and user-cooperative communication, e.g., [38, 39, 49, 50, 60–67]. Due to their mutual relationship, the emergence of user-cooperative communication revitalized researchers' interest in relay channels. In fact, most of the recent work in relay channels is motivated by user-cooperative communication.

In their two-part papers [68, 69], Sendonaris *et al.* presented an extensive set of simulation results demonstrating the great potential of cooperative diversity and discussed some implementation issues. Their model was based on a CDMA cellular system. Proposed cooperative model was a two-user cooperative model and used D&F relaying. Discussions included optimal and sub-optimal receivers.

Work by Laneman *et al.* in [60] is another significant contribution which considered a TDMA system. They developed and analyzed low-complexity user-cooperative diversity protocols, based on A&F and D&F signaling, for delay-constrained wireless channels. Work of Laneman *et al.* is useful for studying the half-duplex relay channel. We are more interested on the fixed D&F relay channel for which a mutual information is shown to be,

$$\mathcal{I}_{\text{D\&F}} = \tfrac{1}{2} \min \left\{ \log(1 + \gamma_{S,R}), \log(1 + \gamma_{S,D} + \gamma_{R,D}) \right\} \tag{2.14}$$

Fixed A&F, on the other hand, is shown to achieve,

$$\mathcal{I}_{\text{A\&F}} = \tfrac{1}{2} \log \left(1 + \gamma_{S,D} + \frac{\gamma_{S,R}\gamma_{R,D}}{1 + \gamma_{S,R} + \gamma_{R,D}} \right) \tag{2.15}$$

Other protocols included D&F selection relaying and D&F incremental relaying. Outage probability expressions are also given. This paper is very useful as it addresses, very clearly, issues related to application of relaying in a wireless scenario. Results produced show the effect of half-duplex constraint on channel performance.

Work of Laneman *et al.* in [60], however, is not well connected to user-cooperative communications. They failed to properly extend their three-node relay channel to a two-user cooperative model. There was the least interaction between cooperating users. Issues like partner selection, fairness, resource allocation and cooperation gain, which naturally arise, were overlooked. Some of these topics are considered in the thesis.

Hunter *et al.* proposed coded cooperation in which cooperation operates through channel coding in the spatial domain [49]. Instead of repeating the received bits (as in A&F and D&F), the cooperating node sends an incremental redundancy for its partner. They also studied outage probability for the coded cooperation in [50]. Although coded cooperation has shown promising results, a lot of work is needed before it can come into practice. In particular, practical coding schemes must be devised for the system.

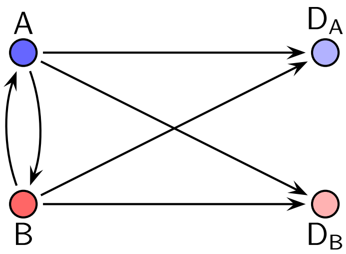


Figure 2.8: Two relay channels formed by two cooperating users, A and B transmitting to two different destinations, D_A and D_B; a typical scenario in ad hoc networks.

Similar to [49, 60, 68, 69], the thesis proposes a two-user cooperative model. Proposed model is based on a D&F three-node relay channel. Like [60], a time-sharing scenario is assumed. However, instead of a half-half fixed time allocation, cooperating users here are allowed to assume any time allocation. Focus is on finding an appropriate time sharing policy to make cooperation useful.

More attention in the thesis is given to partnership selection, a topic that is lightly considered in the literature. Partner selection was considered by Hunter *et al.* in [42] where both it and grouping were investigated without exploiting channel state information nor concerning user fairness. In the thesis a set of analytical results are developed to formalize partner selection which take into consideration network set up, cooperation effectiveness and user fairness. Partner selection is determined by network set up and gain sought from cooperation. Discussions are also concerned with interactions with upper layers.

2.5 Chapter Summary

This chapter introduced the essential components of the thesis. Aspects that distinguish the wireless channel are pointed out as well as channel modeling. Formal definitions of channel capacity and mutual information are included. Introduction also comprised the relay channel and user-cooperative communication.

Chapter 3

Mutual Information and Probability of Outage for The Three-Node D&F Relay Channel with Half-Duplex

This chapter is the first to present the results obtained, which are the cornerstone for the remainder of the thesis. A wireless D&F three-node channel is studied. In Section 3.1, a description of the channel is given. Section 3.2 examines the mutual information for the D&F half-duplex relay channel. A formula is obtained for the mutual information as a function of time allocation. Behaviour of the channel is further inspected and the concept of conversion point is introduced. With regard to Rayleigh fading channels, in Section 3.3 cumulative and probability distribution functions for the channel's mutual information are evaluated. Consequently, that helped to find average mutual information and outage probability.

Subsequent chapters are connected in one way or another to results generated in this Chapter. Chapter 4 investigates optimal time allocation for the channel. Chapter 5 studies user-cooperative communication where users cooperate by forming three-node relay channels. In Chapter 6 results are generalized for multi-hop relay channel.

3.1 Channel Model

A relay channel consists of a single source node, a single destination node and at least one relay node assisting the source on transmitting to the destination. A relay channel with a single relay is the simplest of its kind. In the channel shown in Figure 3.1, S is the source node, R is the relay node and D is the destination node. S transmits only, D receives only, while R switches from one mode to the other.

R is operated on a D&F mode of operation, which results in S transmission rate being restricted so that R is able to reliably decode transmitted codeword. Transmission fails if R is unable to decode S transmitted signal.

R is half-duplex constraint. Therefore, S and R can not transmit at the same time

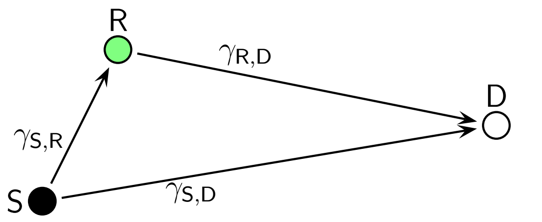


Figure 3.1: A three-node relay channel. Arrows indicate direction of transmission.

and same frequency band. We assume time-division approach to ensure orthogonality between source signal and relay signal. Available time is divided into two time intervals, τ_{S} and τ_{R}. First, S transmits for τ_{S} units of time. Then R repeats S transmission using τ_{R} units of time. This sequential transmission is illustrated in Figure 3.2. We refer to the ordered pair $\boldsymbol{\tau} = (\tau_{\mathsf{S}}, \tau_{\mathsf{R}})$ as the *time allocation vector* or just *time allocation*. Theoretically, a frequency- division approach produces the same results.

The channel could be encountered in a multi-user as well as a single user scenario. To ensure generality, obtained results are normalized by the user's available time. As a result, we have $\tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1$.

Block diagram in Figure 3.3 demonstrates the channel in further detail. The encoder at the transmitter side encodes the message w into a sequence of channel input symbols X. Z and Z_{R} resemble noise produced at the destination and the relay, respectively. Z and Z_{R} are zero-mean normally distributed random variables with variance N_0. On the other hand, $h_{i,j}$ is a complex quantity which captures channel effects such as path loss, shadowing and fading between nodes i and j. In the thesis we consider an AWGN channel where $h_{i,j}$ is constant as well as a Rayleigh fading channel where $h_{i,j}$ is a normally distributed complex random variable. Fading is assumed to be slow and frequency non-selective.

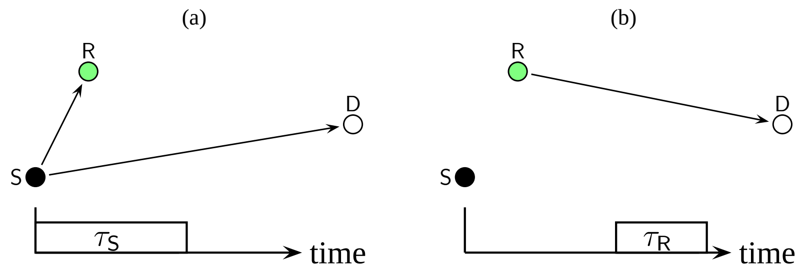


Figure 3.2: A wireless relay channel with half-duplex constraint. Transmission is carried out in two stages: (a) the source node broadcasts the message to both the relay and destination nodes using time τ_{S}, while (b) the relay node forwards the source's signal using time τ_{R}.

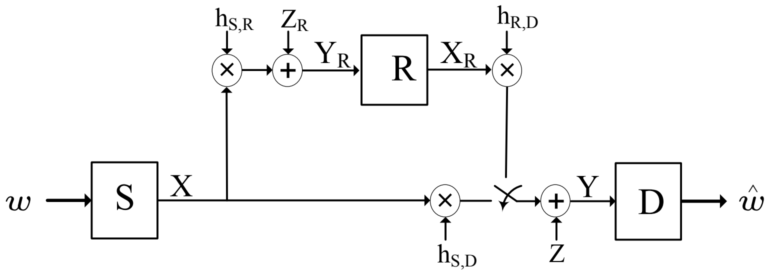


Figure 3.3: A block diagram for the wireless channel with single relay.

Denoting the source's transmitted symbol x, the received signal y_{R} at the relay is given by,

$$y_{\mathsf{R}} = h_{\mathsf{S,R}} x + z_{\mathsf{R}}. \tag{3.1}$$

On the other hand, D has two independent copies of the received signals, one from S and another from R. During direct transmission phase, the received signal at D can be expressed as,

$$y_{\mathsf{D}}^{(1)} = h_{\mathsf{S,D}} x + z_{\mathsf{D}}^{(1)}, \tag{3.2}$$

where the $h_{\mathsf{S,D}}$ and $z_{\mathsf{D}}^{(1)}$ are defined similarly. The received signal at D during the relaying phase is given by,

$$y_{\mathsf{D}}^{(2)} = h_{\mathsf{R,D}} x_{\mathsf{R}} + z_{\mathsf{D}}^{(2)}. \tag{3.3}$$

Detection at D is performed by maximum-likelihood (ML) decoding over the two copies of received signals.

In addition to available degree of freedom, channel performance is parameterized by SNR at receivers. We use $\gamma_{i,j}$ to denote SNR at receiver j when i is transmitting. $\gamma_{i,j}$ is given by,

$$\gamma_{i,j} = \hat{k}(f) \frac{P_i}{N_0} d_{i,j}^{-\alpha} |h_{i,j}|^2 \tag{3.4}$$

$P_i = E[X_i^2]$ is node i's average transmission power. $d_{i,j}$ is the distance between i and j and α is the path-loss coefficient. $|h_{i,j}|^2$ is the power gain between i and j. For Rayleigh fading channels, $|h_{i,j}|^2$ is exponentially distributed. $\hat{k}$ is an appropriate constant, function of frequency f, that accounts for the antenna pattern in the direction of transmission and other hardware losses. In the thesis we assume $\hat{k} = 1$. We also assume *naive* power policy where the same average power is radiated regardless of time allocation [18]. It is thus sufficient to characterize the channel by $\gamma_{i,j}$.

$(\mathsf{A}, \mathsf{R}_1, \mathsf{R}_2, \ldots \mathsf{B})$ is used in the thesis to refer to a relay channel with node A as a source node, node B as a destination node and nodes $\mathsf{R}_1, \mathsf{R}_2 \ldots$ as relays. $\mathcal{I}_{\mathsf{A,B}}$ is used to denote mutual information between two nodes, A and B. When relaying takes place, $\mathcal{I}_{\mathsf{A,R_1,R_2,\ldots B}}$ refers to mutual information between A and B with nodes $\mathsf{R}_1, \mathsf{R}_2 \ldots$ as relays. Independent variables are reserved for time allocation.

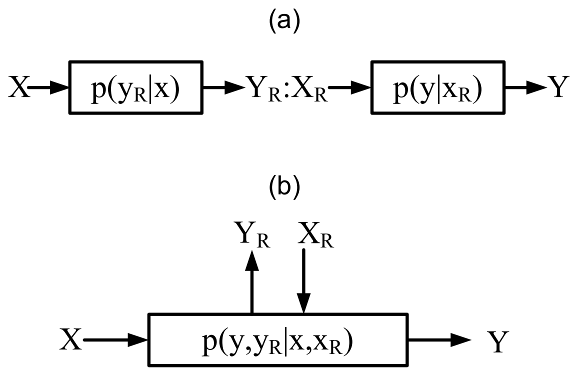


Figure 3.4: Information theoretic representation of a: (a) non-cooperative relay channel and (b) cooperative relay channel.

3.2 Mutual Information for The Relay Channel

3.2.1 Preliminary

In this preliminary section we look into a full-duplex D&F relay channel. Mathematically, a three-node relay channel is modeled as a channel with two random inputs X and X_{R} for the source and the relay, respectively, and two random outputs Y and Y_{R} for the destination and the relay, respectively; and a set of pmf's $p(y, y_{\mathsf{R}}|x, x_{\mathsf{R}})$ for each $(x, x_{\mathsf{R}}, y, y_{\mathsf{R}}) \in \mathcal{X} \times \mathcal{X}_{\mathsf{R}} \times \mathcal{Y} \times \mathcal{Y}_{\mathsf{R}}$.

When there is no direct link between the source and the destination (i.e. when $\gamma_{\mathsf{S,D}} = 0$), the system is not fully connected. In that case the channel is a *non-cooperative* relay channel. A non-cooperative relay channel is represented by two consecutive point-to-point channels as shown in Figure 3.4-a. This channel can achieve any rate as long as it can be supported by both source-relay and relay-destination sub-channels. Achievable rate is thus bounded by the minimum of the two sub-channels, or

$$\mathcal{I}_{\mathsf{S,R,D}} = \min\{\mathcal{I}_{\mathsf{S,R}}, \mathcal{I}_{\mathsf{R,D}}\} \qquad \text{bit/sec/Hz}, \tag{3.5}$$

where, for any channel realization,

$$\mathcal{I}_{\mathsf{S,R}} = \log(1 + \gamma_{\mathsf{S,R}}), \tag{3.6}$$

and

$$\mathcal{I}_{\mathsf{R,D}} = \log(1 + \gamma_{\mathsf{R,D}}). \tag{3.7}$$

Cooperative relaying, on the other hand, is possible only if the network is fully connected. In that case with the aid of the relay, the destination receives two copies of the transmitted signal; from the source and from the relay. To take advantage, the destination must also be able to combine both received signals.

The channel can be viewed as a combination of a *broadcast* (BC) channel (from the sender to the relays and destination) and MA channel (from the relays to the destination), as illustrated by Figure 3.5 [18].

Next derivation borrows substantially from [18]. To work out mutual information for the three-node full-duplex D&F relay channel, we first consider the BC channel from the source to the relay and the destination shown in Figure 3.5-a. As for a general BC channel, the source node may generate three codebooks. Codewords w_{R}, w_{D} and w are chosen from each of the codebooks. w_{R} is sent to R, w_{D} is sent to D while w is sent to both receivers. Let $\mathcal{C}(x) = \log(1+x)$. The total rate at each receiver, $R_{\mathsf{s,R}}$ and $R_{\mathsf{s,D}}$, must lie in the capacity region. The capacity region for this BC channel is,

$$\begin{aligned} R_{\mathsf{S,R}} < \quad \mathcal{I}_{\mathsf{S,R}} = \quad & \mathcal{C}\left(\frac{(1-\beta)(1-\alpha)\gamma_{\mathsf{S,R}} + \beta\gamma_{\mathsf{S,R}}}{(1-\beta)\alpha\gamma_{\mathsf{S,R}} + 1}\right), \\ R_{\mathsf{S,D}} < \quad \mathcal{I}_{\mathsf{S,D}} = \quad & \mathcal{C}\left((1-\beta)\alpha\gamma_{\mathsf{S,D}} + \beta\gamma_{\mathsf{S,D}}\right). \end{aligned} \tag{3.8}$$

where α and β are power sharing coefficients. β of the transmission power is allocated to w. α of the remaining power, that is $\alpha(1-\beta)$, is allocated to w_{D}. What remains, that is $(1-\alpha)(1-\beta)$, is allocated for w_{R}. In our case, the same information is sent to both receivers. That means all power is allocated to w by setting $\beta = 1$. Moreover, only one codebook is needed in order to send w. That means $R_{\mathsf{S,R}} = R_{\mathsf{S,D}} = R^{\mathrm{BC}}$. The capacity region becomes,

$$\begin{aligned} R^{\mathrm{BC}} < \quad \mathcal{I}_{\mathsf{S,R}} = \quad & \mathcal{C}\left(\gamma_{\mathsf{S,R}}\right), \\ R^{\mathrm{BC}} < \quad \mathcal{I}_{\mathsf{S,D}} = \quad & \mathcal{C}\left(\gamma_{\mathsf{S,D}}\right). \end{aligned} \tag{3.9}$$

or

$$R < \min\left\{\mathcal{I}_{\mathsf{S,R}}, \mathcal{I}_{\mathsf{S,D}}\right\} = \min\left\{\mathcal{C}\left(\gamma_{\mathsf{S,R}}\right), \mathcal{C}\left(\gamma_{\mathsf{S,D}}\right)\right\} \tag{3.10}$$

Achievable rate in (3.10) takes into consideration the way the relay operates in a D&F channel. The fact that the relay is required to fully decode the source's signal as well as the destination, resulted in the minimum of the mutual information, $\mathcal{I}_{\mathsf{S,R}}$ and $\mathcal{I}_{\mathsf{S,D}}$. Achievable rate in (3.10), however, did not take into account the fact that the relay node retransmits the message and the destination cooperatively uses both received copies of the transmitted signal to decode the message. Denoted $\mathcal{I}_0$, we expect the achievable rate at the receiver to be greater than $\mathcal{I}_{\mathsf{S,D}}$ shown in (3.10).

To find the cooperative mutual information at the destination we consider the MA channel from the source and the relay to the destination. In general, the source would generate a single codebook with rate $R_{\mathsf{S,D}}$ while the relay generate another codebook

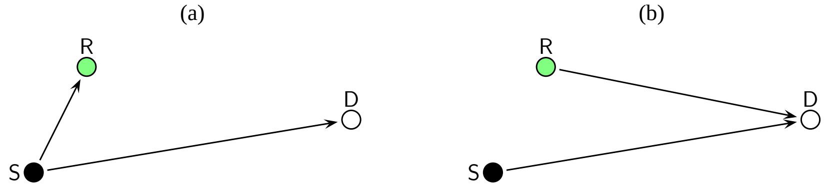


Figure 3.5: (a) From the source's perspective the D&F relay channel is seen as a BC channel, while (b) from the destination's perspective the channel is seen as a MA channel.

with rate $R_{\mathsf{R,D}}$. The capacity region for this MA channel is given by,

$$\begin{aligned} R_{\mathsf{S,D}} <& \quad \mathcal{C}\left(\gamma_{\mathsf{S,D}}\right) \\ R_{\mathsf{R,D}} <& \quad \mathcal{C}\left(\gamma_{\mathsf{R,D}}\right) \\ R_{\mathsf{S,D}} + R_{\mathsf{R,D}} <& \quad \mathcal{C}\left(\gamma_{\mathsf{S,D}} + \gamma_{\mathsf{R,D}}\right). \end{aligned} \tag{3.11}$$

We are interested in the total rate at the destination, $R^{\text{MA}} = R_{\mathsf{S,D}} + R_{\mathsf{R,D}}$. Summing the first two inequalities in (3.11) we get,

$$\begin{aligned} R^{\text{MA}} <& \quad \mathcal{C}\left(\gamma_{\mathsf{S,D}}\right) + \mathcal{C}\left(\gamma_{\mathsf{R,D}}\right) \\ R^{\text{MA}} <& \quad \mathcal{C}\left(\gamma_{\mathsf{S,D}} + \gamma_{\mathsf{R,D}}\right), \end{aligned} \tag{3.12}$$

or,

$$R^{\text{MA}} < \mathcal{I}_0 = \mathcal{C}\left(\gamma_{\mathsf{S,D}} + \gamma_{\mathsf{R,D}}\right). \tag{3.13}$$

Substituting $\mathcal{I}_0$ in (3.10) yields the mutual information for the full-duplex D&F relay channel,

$$\mathcal{I}_{\mathsf{S,R,D}} = \min\left\{\mathcal{I}_{\mathsf{S,R}}, \mathcal{I}_0\right\}. \tag{3.14}$$

3.2.2 Mutual Information for The Relay Channel with Half-Duplex and Arbitrary Time Allocation

Achievable rate as stated by (3.14) corresponds to that of a channel with a full-duplex relay node. In this section we consider a half-duplex constraint relay channel. The relay node can only transmit or receive at one time and in the same band, therefore transmission take places in two stages. First, the source node transmits to both the relay and the destination. Meanwhile, the relay remains silent. In the second stage the source node stays idle while the relay node transmits. Further, we seek mutual information when available time is arbitrarily allocated to the source and the relay, in contrast to equal time allocation when the available time is allocated equally to both nodes.

The following lemma is a step forward.

Lemma 3.1 (Mutual information for the three-node relay channel with equal time allocation)**.** *Consider a three-node D&F relay channel with half-duplex constraint on the relay. Let τ of the available time, $\tau \leq \frac{1}{2}$, be used by the source to transmit the message and the same amount of time be used by the relay to retransmit the message after successful decoding at the relay. Mutual information between the source and the destination is given by,*

$$\mathcal{I}_{\mathsf{S,R,D}}(\tau) = \min\{\mathcal{I}_{\mathsf{S,R}}(\tau), \mathcal{I}_0(\tau)\} \quad \text{bit/sec/Hz}, \tag{3.15}$$

where,

$$\mathcal{I}_{\mathsf{S,R}}(\tau) = \tau \log\left(1 + \gamma_{\mathsf{S,R}}\right), \tag{3.16}$$

and

$$\mathcal{I}_0(\tau) = \tau \log\left(1 + \gamma_{\mathsf{S,D}} + \gamma_{\mathsf{R,D}}\right). \tag{3.17}$$

Proof. In an argument similar to that of the previous section, $\mathcal{I}_{\mathsf{S,R}}(\tau)$ is the maximum rate at which the relay can reliably decode the source message. $\mathcal{I}_0(\tau)$ on the other hand is the maximum rate at which the destination can reliably decode the source message, given repeated transmission from the source and the relay. In a D&F relaying, both the relay and the destination are required to decode the source message. That condition results in taking the minimum of the two mutual information. □

In words, lemma 3.1 tells us that even though the destination is capable of achieving a rate up to $\mathcal{I}_0(\tau)$, it can only do that if the source-relay channel can support that rate, i.e., only if $\mathcal{I}_{\mathsf{S,R}}(\tau) \geq \mathcal{I}_0(\tau)$, $\mathcal{I}_0(\tau)$ is achievable by the channel, otherwise $\mathcal{I}_{\mathsf{S,R,D}}(\tau)$ can not exceed $\mathcal{I}_{\mathsf{S,R}}(\tau)$. Mutual information as stated in Lemma 3.1 is a general form to that discussed in [60] whereas there $\tau = \frac{1}{2}$.

Lemma 3.2 (Equivalent SNR)**.** *With regard to the mutual information $\mathcal{I}$, transmission time τ_1 and received SNR γ_1; there is an equivalent SNR, γ_2, given by,*

$$\gamma_2 = [1 + \gamma_1]^{\frac{\tau_1}{\tau_2}} - 1 \tag{3.18}$$

which has the same effect on $\mathcal{I}$ if τ_2 units of time is used for transmission.

Proof. Figure 3.6 shows allocated time versus SNR for fixed mutual information. For any two arbitrary pairs (γ_1, τ_1) and (γ_2, τ_2) on $\mathcal{I}$, we have,

$$\begin{aligned} \mathcal{I}(\gamma_1, \tau_1) &= \mathcal{I}(\gamma_2, \tau_2) \\ \tau_1 \log\left(1 + \gamma_1\right) &= \tau_2 \log\left(1 + \gamma_2\right) \end{aligned}$$

and (3.18) follows. □

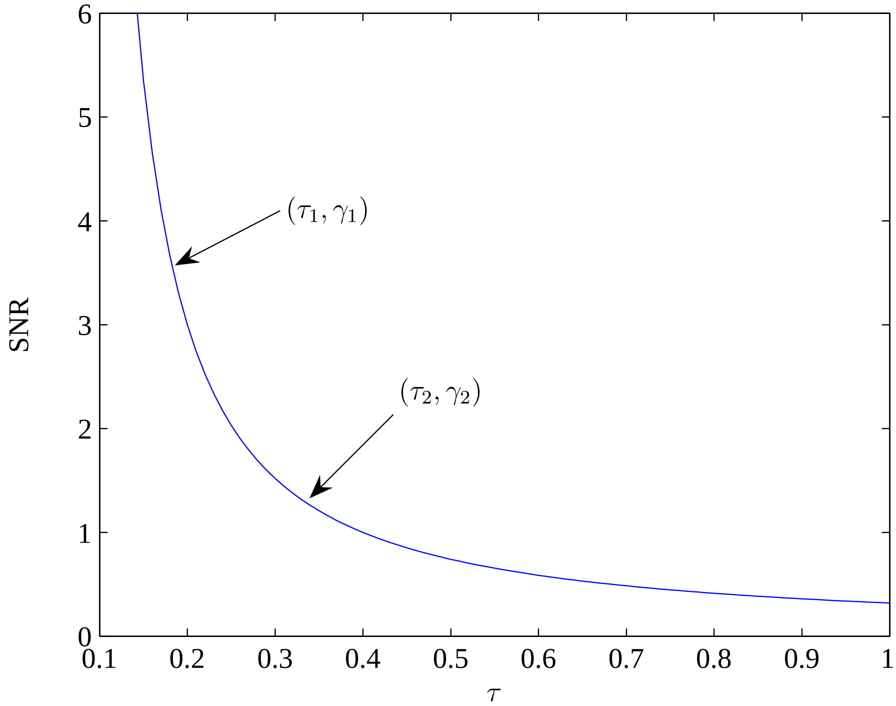


Figure 3.6: Time versus SNR for fixed mutual information. Graph plotted using $\gamma = 2^{\frac{\mathcal{I}}{\tau}} - 1$, where $\mathcal{I}$ is constant. There is an infinite number of SNR and transmission time pairs (τ, γ) that can achieve $\mathcal{I}$. In this figure two pairs are shown, (γ_1, τ_1) and (γ_2, τ_2). These two pairs are therefore exchangeable.

Lemma 3.2 is important in order to find mutual information between the source and the destination when τ_{S} is assigned to the source while τ_{R} is assigned to the relay such that $\tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1$ and τ_{S} is not necessarily equal to τ_{R}.

Applying Lemma 3.2 to the relay channel with time allocation $\boldsymbol{\tau} = (\tau_{\mathsf{S}}, \tau_{\mathsf{R}})$ we have the following. When the relay transmits for τ_{R} of the available time and the received SNR from the relay at the destination is $\gamma_{\mathsf{R,D}}$, there is an equivalent SNR $\tilde{\gamma}_{\mathsf{R,D}}$ that has the same effect on $\mathcal{I}_0(\boldsymbol{\tau})$ when the relay transmits for τ_{S} instead. $\tilde{\gamma}_{\mathsf{R,D}}$ is given by,

$$\tilde{\gamma}_{\mathsf{R,D}} = [1 + \gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{\tau_{\mathsf{S}}}} - 1 \tag{3.19}$$

This readily leads us to the MA rate at the destination for any arbitrary time allocation $\boldsymbol{\tau}$,

$$\begin{aligned}\mathcal{I}_0(\boldsymbol{\tau}) &= \tau_{\mathsf{S}} \log(1 + \gamma_{\mathsf{S,D}} + \tilde{\gamma}_{\mathsf{R,D}}) \\ &= \tau_{\mathsf{S}} \log\left(\gamma_{\mathsf{S,D}} + [1 + \gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{\tau_{\mathsf{S}}}}\right) \ \text{bit/sec/Hz}.\end{aligned} \tag{3.20}$$

As a result of Lemma 3.1, Lemma 3.2 and (3.20) we have the following theorem.

Theorem 3.1 (Mutual information for the three-node relay channel with arbitrary time

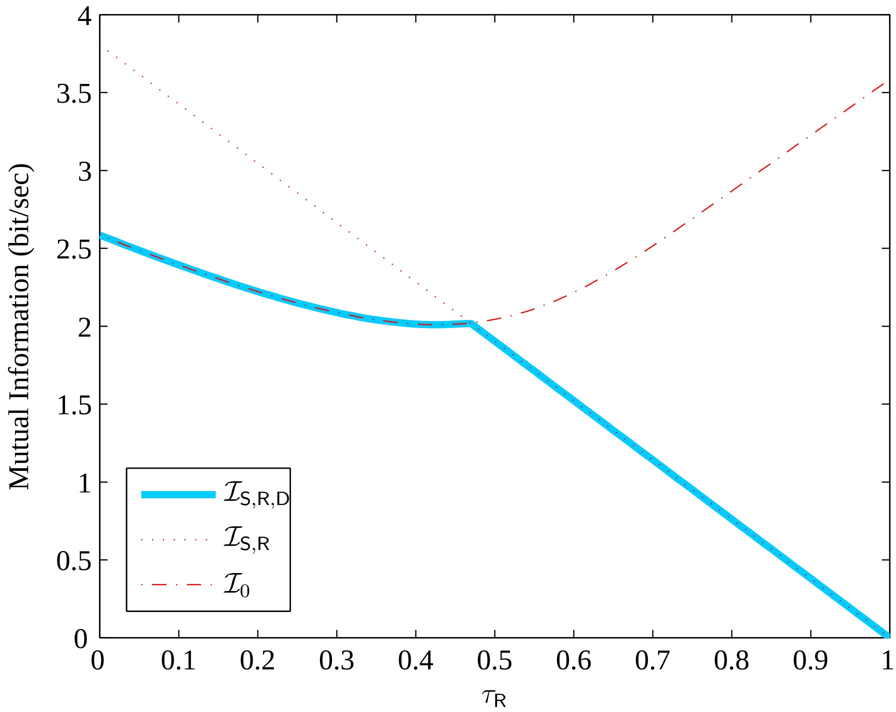


Figure 3.7: Deterministic mutual information for the three-node relay channel with half-duplex constraint on the relay versus relaying time, τ_{R}. Graph produced using $\tau_{\mathsf{S}} + \tau_{\mathsf{R}} = 1$, $\gamma_{\mathsf{S,D}} = 5$, $\gamma_{\mathsf{S,R}} = 13$, and $\gamma_{\mathsf{R,D}} = 11$.

allocation)**.** *Consider a three-node D&F relay channel with half-duplex constraint on the relay. τ_{S} of the available time is used by the source to transmit the message while τ_{R} of the time is used by the relay to retransmit the message after successful decoding at the relay. In general $\tau_{\mathsf{S}} \neq \tau_{\mathsf{R}}$ and $\tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1$. Mutual information between the source and the destination is given by,*

$$\mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right) = \min\{\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right), \mathcal{I}_0\left(\boldsymbol{\tau}\right)\}, \quad \text{bit/sec/Hz} \tag{3.21}$$

where,

$$\mathcal{I}_{\mathsf{S},R}\left(\boldsymbol{\tau}\right) = \tau_{\mathsf{S}} \log\left(1 + \gamma_{\mathsf{S},R}\right), \tag{3.22}$$

and

$$\mathcal{I}_0\left(\boldsymbol{\tau}\right) = \tau_{\mathsf{S}} \log\left(\gamma_{\mathsf{S,D}} + \left[1 + \gamma_{\mathsf{R,D}}\right]^{\frac{\tau_{\mathsf{R}}}{\tau_{\mathsf{S}}}}\right). \tag{3.23}$$

To demonstrate the relationship between all three functions, $\mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right)$, $\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right)$ and $\mathcal{I}_0\left(\boldsymbol{\tau}\right)$ are plotted in Figure 3.7 for some channel realization.

3.2.3 Notes on $\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{S}}, \tau_{\mathsf{R}})$

Following notes on the mutual information for the relay channel, stated in Theorem 3.1, help us to understand the behavior of the D&F relay channel. Along with graphs in Figure 3.8, these notes examine the general behavior of the channel for different scenarios. We have $\boldsymbol{\gamma} = (\gamma_{\mathsf{S,D}}, \gamma_{\mathsf{S,R}}, \gamma_{\mathsf{R,D}})$. We have realized the following:-

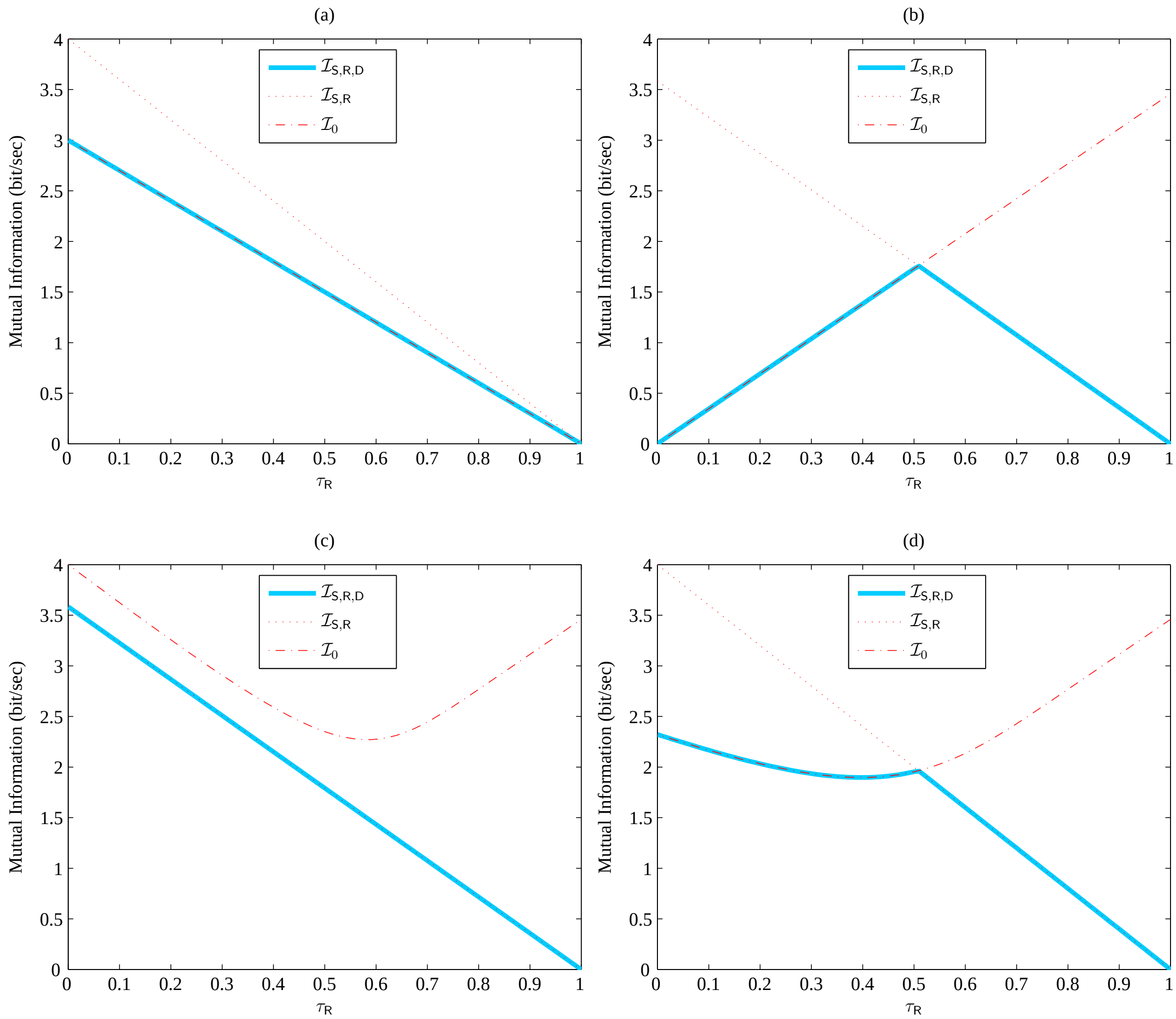


Figure 3.8: Along with notes given in Section 3.2.3, these graphs explain behavior of the relay channel. $\mathcal{I}_{S,R,D}$ is plotted against τ_R with $\tau_S + \tau_R = 1$. (a) $\boldsymbol{\gamma} = (7, 15, 0)$, (b) $\boldsymbol{\gamma} = (0, 11, 10)$, (c) $\boldsymbol{\gamma} = (15, 11, 10)$, (d) $\boldsymbol{\gamma} = (4, 15, 10)$.

1. $\mathcal{I}_{S,R,D}(\boldsymbol{\tau}) = 0$ if $\tau_S = 0$. In other words, transmission can not take place without the source node taking part, no matter how efficient the relay node is. This is trivial and noticeable in all graphs in Figure 3.8. As τ_R approaches 1, τ_S approaches 0 and $\mathcal{I}_{S,R,D}(\boldsymbol{\tau})$ approaches 0.

2. When $\tau_R = 0$, the resulted rate is $\mathcal{I}_{S,R,D}(\boldsymbol{\tau}) = \min\{\mathcal{I}_{S,R}(\boldsymbol{\tau}), \mathcal{I}_{S,D}(\boldsymbol{\tau})\} \leq \mathcal{I}_{S,D}$. $\mathcal{I}_{S,D}$ is direct transmission mutual information. That is, mutual information when the source node completely ignores the relay and uses all available time for transmission. $\mathcal{I}_{S,D}$ is given by,

$$\mathcal{I}_{S,D} = \log(1 + \gamma_{S,D}). \tag{3.24}$$

$\mathcal{I}_{\text{S,D}}(\boldsymbol{\tau})$, on the other hand, is given by,

$$\mathcal{I}_{\text{S,D}}(\boldsymbol{\tau}) = \tau_{\text{S}} \log(1 + \gamma_{\text{S,D}}). \tag{3.25}$$

The relay channel in this case is some kind of BC channel.

3. When $\gamma_{\text{R,D}} = 0$, the channel again turns into a BC channel with a rate as in 2 above. Figure 3.8-a shows a scenario when $\gamma_{\text{S,D}} < \gamma_{\text{S,R}}$ so that $\mathcal{I}_{\text{S,R,D}}(\boldsymbol{\tau}) = \mathcal{I}_{\text{S,D}}(\boldsymbol{\tau})$.

4. When $\gamma_{\text{S,D}} = 0$, $\mathcal{I}_{\text{S,R,D}}(\boldsymbol{\tau}) = \min\{\mathcal{I}_{\text{S,R}}(\boldsymbol{\tau}), \mathcal{I}_{\text{R,D}}(\boldsymbol{\tau})\}$, where,

$$\mathcal{I}_{\text{R,D}}(\boldsymbol{\tau}) = \tau_{\text{R}} \log(1 + \gamma_{\text{R,D}}). \tag{3.26}$$

   This is A non-cooperative half-duplex D&F relay channel. This is a typical model for a channel with no direct link between the source node and the destination. Data can only be transmitted to the destination through the relay. $\mathcal{I}_{\text{S,R,D}}(\boldsymbol{\tau}) = 0$ when $\tau_{\text{R}} = 0$ ($\tau_{\text{S}} = 1$) or when $\tau_{\text{R}} = 1$ ($\tau_{\text{S}} = 0$). This is illustrated in Figure 3.8-b.

5. When $\gamma_{\text{S,D}} \geq \gamma_{\text{S,R}}$, we have $\mathcal{I}_{\text{S,R,D}}(\boldsymbol{\tau}) = \mathcal{I}_{\text{S,R}}(\boldsymbol{\tau})$. This is because, in this case $\mathcal{I}_{\text{S,R}}(\boldsymbol{\tau}) \leq \mathcal{I}_0(\boldsymbol{\tau})$ for all time allocations and all $\gamma_{\text{R,D}}$. Figure 3.8-c explains this behavior.

6. When $\gamma_{\text{S,D}} < \gamma_{\text{S,R}}$, $\mathcal{I}_{\text{S,R,D}}(\boldsymbol{\tau})$ either equals $\mathcal{I}_{\text{S,R}}(\boldsymbol{\tau})$ or $\mathcal{I}_0(\boldsymbol{\tau})$ subject to time allocation. As demonstrated by Figure 3.8-d,

   (a) When more time is allocated to the relay, eventually $\mathcal{I}_{\text{S,R,D}}(\boldsymbol{\tau}) = \mathcal{I}_{\text{S,R}}(\boldsymbol{\tau})$.

   (b) When more time is allocated to the source, eventually $\mathcal{I}_{\text{S,R,D}}(\boldsymbol{\tau}) = \mathcal{I}_0(\boldsymbol{\tau})$.

   (c) Time can be allocated such that $\mathcal{I}_{\text{S,R,D}}(\boldsymbol{\tau}) = \mathcal{I}_{\text{S,R}}(\boldsymbol{\tau}) = \mathcal{I}_0(\boldsymbol{\tau})$. That is where $\mathcal{I}_{\text{S,R}}(\boldsymbol{\tau})$ intersects with $\mathcal{I}_0(\boldsymbol{\tau})$.

Although the above notes are concluded directly from Theorem 3.1, they can also be deduced using an intuitive approach. We conclude from the notes above that the presence of the relay can sometimes become harmful for transmission. This is the case when $\gamma_{\text{S,D}} \geq \gamma_{\text{S,R}}$ as the relay channel matches direct transmission channel at its best. Also when $\gamma_{\text{R,D}} = 0$, mutual information can not exceed that of direct transmission. This is due to the way a D&F relaying is operated. Rate is bounded by $\mathcal{I}_{\text{S,R}}$ to ensure that the relay can always successfully decode the message sent by the source. There are other less obvious situations when the relay channel performs worse than direct channel. In contrast, relaying is essential in some situations. In the absence of a direct

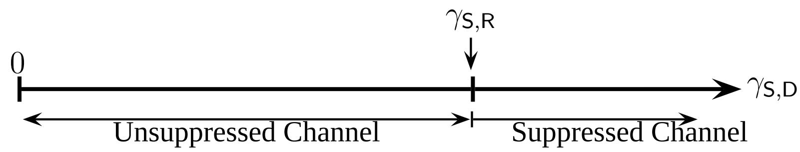


Figure 3.9: Classification of the D&F relay channel based on the SNR between the source and the destination relative to that of the source to the relay.

link between the source and the destination, that is when $\gamma_{S,D} = 0$, the message can only be conveyed through a relay channel.

A less obvious scenario is when $\gamma_{S,D} < \gamma_{S,R}$. Examination of this case would reveal the conditions under which the relay can be useful. Useful relaying is considered in Chapter 5. In a multi-user cooperative network, users select partners based on their mutual usefulness as they relay each other's signal.

One way to tackle drawbacks of D&F relaying is to employ adaptive transmission. Adaptive sources are able to switch to direct transmission when the relay has a negative impact on the performance.

3.2.4 Conversion Point

3.2.4.1 Classification of The Channel

In light of the notes in Section 3.2.3 we may categorize the D&F relay channel, based on channel realizations $\boldsymbol{\gamma}$, into,

1. *Suppressed* channel, when $\gamma_{S,D} \geq \gamma_{S,R}$
2. *Unsuppressed* channel, when $\gamma_{S,D} < \gamma_{S,R}$. Unsuppressed channels are further sub-categorized into,
   (a) *Active* channel, when $\gamma_{R,D} > 0$
   (b) *Passive* channel, when $\gamma_{R,D} = 0$

In fact, as demonstrated before, passive unsuppressed channels represent a kind of BC channel. Therefore, in the thesis, focus is given to suppressed and unsuppressed active channels. The above classification emphasizes the importance of the relationship between $\gamma_{S,D}$ and $\gamma_{S,R}$ in predicting channel performance. Figure 3.9 geometrically demonstrates classification of the D&F relay channel and its relationship to $\gamma_{S,D}$ and $\gamma_{S,R}$.

3.2.4.2 The Concept of Conversion Point

Any time allocation $\boldsymbol{\tau}$ must satisfy two conditions. First, $\tau_S, \tau_R \in [0, 1]$. Second, $\tau_S + \tau_R \leq 1$. Let $\mathbf{C} \subset \mathbb{R}^2$ be the set of all feasible time allocations. In Figure 3.10 $\mathbf{C}$ is

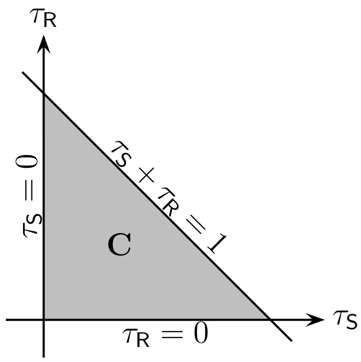


Figure 3.10: Set of all feasible time allocations.

shown as a shaded area in the $\tau_{\mathsf{S}}\tau_{\mathsf{R}}$-plane. It is the area enclosed by,

$$\begin{aligned} \tau_{\mathsf{S}} \geq & \ 0, \\ \tau_{\mathsf{R}} \geq & \ 0, \\ \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq & \ 1. \end{aligned} \tag{3.27}$$

If, however, we know that exactly a proportion c of used time is to be used for transmission, then **C** shrinks to the line $\tau_{\mathsf{S}} + \tau_{\mathsf{R}} = c$. We call it *the operation line*.

Definition 3.1 (Operation Line)**.** *The operation line is the line $\tau_{\mathsf{S}} + \tau_{\mathsf{R}} = c$, where $c \in [0, 1]$ is the total time allocated for transmission.*

We also have the following definition of the conversion point.

Definition 3.2 (Conversion Point, $\boldsymbol{\mu}$)**.** *A relay channel operating on the line $\tau_{\mathsf{S}}+\tau_{\mathsf{R}} = c$, may have a conversion point, $\boldsymbol{\mu}(c) = \big(\mu_{\mathsf{S}}(c), \mu_{\mathsf{R}}(c)\big)$. $\boldsymbol{\mu}(c)$ is the time allocation such that $\mathcal{I}_{\mathsf{S,R,D}}\big(\boldsymbol{\mu}(c)\big) = \mathcal{I}_{\mathsf{S,R}}\big(\boldsymbol{\mu}(c)\big) = \mathcal{I}_0\big(\boldsymbol{\mu}(c)\big)$.*

If it exists, $\boldsymbol{\mu}(c)$ is the point where $\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right)|_{\tau_{\mathsf{S}}+\tau_{\mathsf{R}}=c}$ intersects with $\mathcal{I}_0\left(\boldsymbol{\tau}\right)|_{\tau_{\mathsf{S}}+\tau_{\mathsf{R}}=c}$. In Figures 3.8,-b and 3.8-d, $\boldsymbol{\mu}(c)$ marks time allocation where $\mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right)$ is discontinuous. We adopt the convention of dropping the independent variable from the conversion point when $c = 1$. Therefore, $\boldsymbol{\mu}$, μ_{S} and μ_{R} refer to $\boldsymbol{\mu}(1)$, $\mu_{\mathsf{S}}(1)$ and $\mu_{\mathsf{R}}(1)$, respectively.

Lemma 3.3 (Properties of The Conversion Point)**.** *For a given set of a channel's realization, we have the following properties of the conversion point:-*

1. *Only unsuppressed channels have conversion points.*

2. *There can be no more than one conversion point per operation line.*

3. $\varsigma = \frac{\mu_{\mathsf{R}}(c)}{\mu_{\mathsf{S}}(c)}$ *= constant, for all c. ς is the* conversion ratio *and is given by,*

$$\varsigma = \frac{\log\left(1 + \gamma_{\mathsf{S,R}} - \gamma_{\mathsf{S,D}}\right)}{\log\left(1 + \gamma_{\mathsf{R,D}}\right)}. \tag{3.28}$$

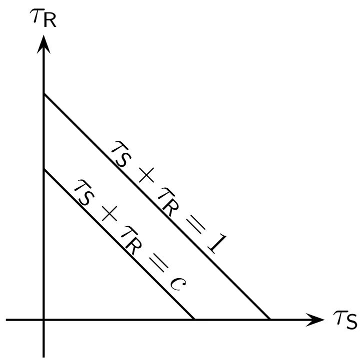


Figure 3.11: The operation line is the set of time allocations for a given total time. Two examples of operation line on the $\tau_{\mathsf{S}}\tau_{\mathsf{R}}$-plane are shown: $\tau_{\mathsf{S}}+\tau_{\mathsf{R}}=1$ and $\tau_{\mathsf{S}}+\tau_{\mathsf{R}}=c$.

Proof. 1. We have,

$$\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\mu}(c)\right)=\mu_{\mathsf{S}}(c)\log\left(1+\gamma_{\mathsf{S,R}}\right), \tag{3.29}$$

and

$$\mathcal{I}_{0}\left(\boldsymbol{\mu}(c)\right)=\mu_{\mathsf{S}}(c)\log\left(\gamma_{\mathsf{S,D}}+\left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\mu_{\mathsf{R}}(c)}{\mu_{\mathsf{S}}(c)}}\right). \tag{3.30}$$

From the definition, at the conversion point: $\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\mu}(c)\right)=\mathcal{I}_{0}\left(\boldsymbol{\mu}(c)\right)$. That means at the conversion point,

$$\mu_{\mathsf{S}}(c)\log\left(1+\gamma_{\mathsf{S,R}}\right)=\mu_{\mathsf{S}}(c)\log\left(\gamma_{\mathsf{S,D}}+\left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\mu_{\mathsf{R}}(c)}{\mu_{\mathsf{S}}(c)}}\right).$$

That can be manipulated to get,

$$\gamma_{\mathsf{S,R}}-\gamma_{\mathsf{S,D}}=\left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\mu_{\mathsf{R}}(c)}{\mu_{\mathsf{S}}(c)}}-1. \tag{3.31}$$

Since $\left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\mu_{\mathsf{R}}(c)}{\mu_{\mathsf{S}}(c)}}\geq 1$, then we must have $\gamma_{\mathsf{S,D}}\leq\gamma_{\mathsf{S,R}}$ for (3.31) to hold. In other words, a channel must be unsuppressed for a conversion point to exist.

2. To prove this property we show that even if $\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right)$ and $\mathcal{I}_{0}\left(\boldsymbol{\tau}\right)$ may have more than one intersection point, only one is an acceptable time allocation. For a given operation line, $\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right)$ and $\mathcal{I}_{0}\left(\boldsymbol{\tau}\right)$ can be written as functions of τ_{R} only. Take operation line $\tau_{\mathsf{S}}+\tau_{\mathsf{R}}=c$. We have,

$$\tau_{\mathsf{S}}=c-\tau_{\mathsf{R}}. \tag{3.32}$$

when substituted in (3.22) and (3.23) gives,

$$\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})=(c-\tau_{\mathsf{R}})\log\left(1+\gamma_{\mathsf{S,R}}\right), \tag{3.33}$$

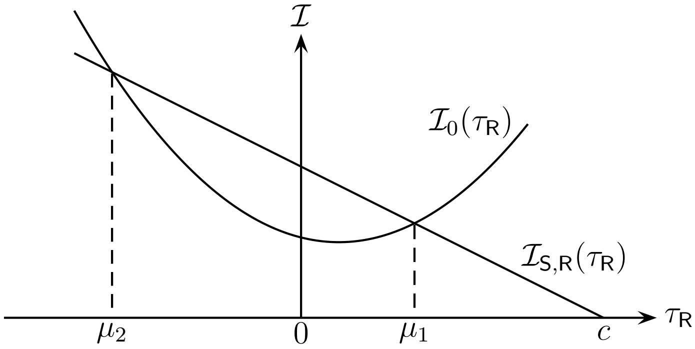


Figure 3.12: An illustration of the proof of Property 2 in Lemma 3.3

and

$$\mathcal{I}_0(\tau_R) = (c - \tau_R)\log\left(\gamma_{S,D} + [1 + \gamma_{R,D}]^{\frac{\tau_R}{c-\tau_R}}\right). \tag{3.34}$$

$\mathcal{I}_{S,R}(\tau_R)$ is a line and $\mathcal{I}_0(\tau_R)$ is a strictly convex function on τ_R (See Lemma 4.1 for proof of convexity of $\mathcal{I}_0(\tau_R)$). Therefore there can be no more than two intersection points between $\mathcal{I}_{S,R}(\tau_R)$ and $\mathcal{I}_0(\tau_R)$. Let $\tau_R = \mu_1$ and $\tau_R = \mu_2$, $\mu_1 > \mu_2$, be two points where $\mathcal{I}_{S,R}(\boldsymbol{\tau})$ intersects with $\mathcal{I}_0(\boldsymbol{\tau})$. We further assume that $\mu_1 > 0$. We will show that in this case μ_2 must be a negative quantity and thus is disqualified from being a conversion point.

Figure 3.12 further demonstrates this proof. From the definition of the convexity we have,

$$\begin{aligned}\mathcal{I}_0(\theta\mu_1 + (1-\theta)\mu_2) &< \theta\mathcal{I}_0(\mu_1) + (1-\theta)\mathcal{I}_0(\mu_2)\\ &= \theta\mathcal{I}_{S,R}(\mu_1) + (1-\theta)\mathcal{I}_{S,R}(\mu_2)\\ &= \mathcal{I}_{S,R}(\theta\mu_1 + (1-\theta)\mu_2) \end{aligned} \tag{3.35}$$

for all $0 < \theta < 1$. The last equality in (3.35) follows from the linearity of $\mathcal{I}_{S,R}(\tau_R)$.

Take θ_0 such that $\theta_0\mu_1 + (1-\theta_0)\mu_2 = 0$, or,

$$\theta_0 = -\frac{\mu_2}{\mu_1 - \mu_2}. \tag{3.36}$$

Substitute θ_0 separately in LHS and RHS of (3.35),

$$\begin{aligned}\mathcal{I}_0(\theta_0\mu_1 + (1-\theta_0)\mu_2) &= \mathcal{I}_0(0)\\ &= \lim_{\tau_R\to 0}(c-\tau_R)\log\left(\gamma_{S,D} + [1+\gamma_{R,D}]^{\frac{\tau_R}{c-\tau_R}}\right)\\ &= c\log(1+\gamma_{S,D}), \end{aligned} \tag{3.37}$$

and

$$\mathcal{I}_{\mathsf{S,R}}\left(\theta_0\mu_1 + (1-\theta_0)\mu_2\right) = \mathcal{I}_{\mathsf{S,R}}\left(0\right) \\ = c\log\left(1+\gamma_{\mathsf{S,R}}\right). \tag{3.38}$$

According to property 1, $\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})$ and $\mathcal{I}_0(\tau_{\mathsf{R}})$ intersect only if,

$$\gamma_{\mathsf{S,D}} < \gamma_{\mathsf{S,R}}. \tag{3.39}$$

From (3.39), (3.37) and (3.38) we see that θ_0 satisfies (3.35), which implies that θ_0 can only be such that $0 < \theta_0 < 1$, or using (3.36),

$$0 < -\frac{\mu_2}{\mu_1 - \mu_2} < 1 \tag{3.40}$$

Inequality (3.40) can only be true if $\mu_2 < 0$ and thus is not an acceptable time allocation.

3. From the definition, at the conversion point we have,

$$\begin{aligned}
\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\mu}\right) &= \mathcal{I}_0\left(\boldsymbol{\mu}\right), \\
\mu_{\mathsf{S}}(c)\log\left(1+\gamma_{\mathsf{S,R}}\right) &= \mu_{\mathsf{S}}(c)\log\left(\gamma_{\mathsf{S,D}} + \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\mu_{\mathsf{R}}(c)}{\mu_{\mathsf{S}}(c)}}\right), \\
1+\gamma_{\mathsf{S,R}} &= \gamma_{\mathsf{S,D}} + \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\mu_{\mathsf{R}}(c)}{\mu_{\mathsf{S}}(c)}}, \\
1+\gamma_{\mathsf{S,R}} - \gamma_{\mathsf{S,D}} &= \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\mu_{\mathsf{R}}(c)}{\mu_{\mathsf{S}}(c)}}, \\
\log\left(1+\gamma_{\mathsf{S,R}} - \gamma_{\mathsf{S,D}}\right) &= \frac{\mu_{\mathsf{R}}(c)}{\mu_{\mathsf{S}}(c)}\log\left(1+\gamma_{\mathsf{R,D}}\right) \\
&= \varsigma\log\left(1+\gamma_{\mathsf{R,D}}\right)
\end{aligned}$$

and 3.28 follows. As it is independent of c, then for a given set of channels realizations ς is constant for all operation lines.

□

According to Property 1 in the above lemma we may redefine suppressed and unsuppressed channel based on the existence of a conversion point. *A suppressed channel is the channel that has no conversion point.* Whereas *an unsuppressed channel is the channel with conversion point.*

From property 2 and property 3, we have,

$$0 < \varsigma \leq \infty. \tag{3.41}$$

On the other hand, Lemma 3.3 also implies that, for a given channel's realizations,

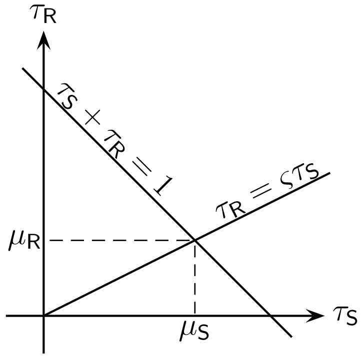


Figure 3.13: As a consequence of Lemma 3.3, $\boldsymbol{\mu}(c)$ must lie on the line $\tau_R = \varsigma\tau_S$.

all conversion points lie on the line $\tau_R = \varsigma\tau_S$; this is demonstrated in Figure 3.13. Hence for a relay channel operating on the line $\tau_S + \tau_R = c$ we may find $\boldsymbol{\mu}(c)$ by solving $\tau_R = \varsigma\tau_S$ and $\tau_S + \tau_R = c$. That gives,

$$\begin{cases} \mu_S(c) & = c\frac{1}{1+\varsigma} = c\,\mu_S, \\ \mu_R(c) & = c\frac{\varsigma}{1+\varsigma} = c\,\mu_R. \end{cases} \tag{3.42}$$

Finally, mutual information for the unsuppressed relay channel can be written as,

$$\mathcal{I}_{S,R,D}(\boldsymbol{\tau})\big|_{\gamma_{S,D}<\gamma_{S,R}} = \begin{cases} \mathcal{I}_0(\boldsymbol{\tau}), & 0 \leq \frac{\tau_R}{\tau_S} < \varsigma, \\ \mathcal{I}_{S,R}(\boldsymbol{\tau}), & \varsigma \leq \frac{\tau_R}{\tau_S} < \infty. \end{cases} \tag{3.43}$$

3.3 Performance in Rayleigh Fading Channels

In fading channels, channel conditions change randomly over time. Therefore, mutual information is a random variable with distribution function related to that of channel state. In this section we derive cdf and pdf for the D&F relay channel with half-duplex as a function of time allocation. Later, we define two quantities, average mutual information and outage probability, used to analyze relay channel performance in fading environments.

3.3.1 Distribution and Density Functions for $\mathcal{I}_{S,R,D}$

In Rayleigh fading channels, channel coefficients are complex Gaussian random variables. Consequently, magnitude and angle of channels' coefficients are Rayleigh distributed (hence the name) and uniformly distributed, respectively. As a result, channel power coefficient for a Rayleigh channel is exponentially distributed. For a random variable $X \sim \text{Exponential}(\lambda_X)$, cdf and pdf are given by,

$$\mathcal{F}_X(x) = \begin{cases} 1 - e^{-\lambda_X x}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases} \tag{3.44}$$

and

$$\begin{aligned} \mathbf{f}_X(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\mathcal{F}_X(x) \\ &= \begin{cases} \lambda_X e^{-\lambda_X x}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases} \end{aligned} \tag{3.45}$$

respectively; where $E[X] = \frac{1}{\lambda_X}$.

$\mathcal{I}_{\mathsf{S,R,D}}$ is function of, exponentially distributed random variables, $\gamma_{\mathsf{S,D}}$, $\gamma_{\mathsf{S,R}}$ and $\gamma_{\mathsf{R,D}}$. To find distribution functions for $\mathcal{I}_{\mathsf{S,R,D}}$ for a given time allocation, we consider $\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau})$ as stated by Theorem 3.1. $\mathcal{I}_{\mathsf{S,R}}$ and $\mathcal{I}_0$ are also random variables. The following Lemma relates the distribution of $\mathcal{I}_{\mathsf{S,R}}$ and $\mathcal{I}_0$ to that of $\mathcal{I}_{\mathsf{S,R,D}}$.

Lemma 3.4 (cdf of the minimum of two independent RVs)**.** *Given two independent RVs X and Y, cdf and pdf of $Z = \min(X, Y)$ are given by,*

$$\mathcal{F}_Z(z) = 1 - [1 - \mathcal{F}_X(z)]\,[1 - \mathcal{F}_Y(z)] \tag{3.46}$$

and

$$\mathbf{f}_Z(z) = \mathbf{f}_Y(z)\,[1 - \mathcal{F}_X(z)] + \mathbf{f}_X(z)\,[1 - \mathcal{F}_Y(z)] \tag{3.47}$$

respectively.

Proof.

$$\begin{aligned} \mathcal{F}_Z(z) &= \Pr\{Z \leq z\} \\ &= \Pr\{X \leq z \cup Y \leq z\} \\ &= 1 - \Pr\left\{\overline{X \leq z \cup Y \leq z}\right\} \\ &= 1 - \Pr\left\{\overline{X \leq z} \cap \overline{Y \leq z}\right\} \\ &= 1 - \Pr\{X > z \cap Y > z\} \\ &= 1 - \Pr\{X > z\} \times \Pr\{Y > z\} \\ &= 1 - \left[1 - \Pr\left\{\overline{X > z}\right\}\right]\left[1 - \Pr\left\{\overline{Y > z}\right\}\right] \\ &= 1 - [1 - \Pr\{X \leq z\}]\,[1 - \Pr\{Y \leq z\}] \\ &= 1 - [1 - \mathcal{F}_X(z)]\,[1 - \mathcal{F}_Y(z)]\,. \end{aligned}$$

where $\{\bar{\epsilon}\}$ denotes the complement of the event ϵ. Differentiating (3.46) gives $\mathbf{f}_Z$ as in

(3.47). See also [70] and [71]. □

Applying Lemma (3.46) to (3.21) we get,

$$\mathcal{F}_{\mathcal{I}_{\mathsf{S,R,D}}}(\boldsymbol{\tau}, r) = 1 - \left[1 - \mathcal{F}_{\mathcal{I}_{\mathsf{S,R}}}(\boldsymbol{\tau}, r)\right]\left[1 - \mathcal{F}_{\mathcal{I}_0}(\boldsymbol{\tau}, r)\right], \tag{3.48}$$

and

$$\mathbf{f}_{\mathcal{I}_{\mathsf{S,R,D}}}(\boldsymbol{\tau}, r) = \mathbf{f}_{\mathcal{I}_0}(\boldsymbol{\tau}, r)\left[1 - \mathcal{F}_{\mathcal{I}_{\mathsf{S,R}}}(\boldsymbol{\tau}, r)\right] + \mathbf{f}_{\mathcal{I}_{\mathsf{S,R}}}(\boldsymbol{\tau}, r)\left[1 - \mathcal{F}_{\mathcal{I}_0}(\boldsymbol{\tau}, r)\right] \tag{3.49}$$

Next we find $\mathcal{F}_{\mathcal{I}_{\mathsf{S,R}}}$, $\mathcal{F}_{\mathcal{I}_0}$, $\mathbf{f}_0$ and $\mathbf{f}_{\mathcal{I}_{\mathsf{S,R}}}$.

3.3.1.1 Distribution and Density Functions for $\mathcal{I}_{\mathsf{S,R}}$

Consider $\mathcal{I}_{\mathsf{S,R}}$ as in (3.22). $\gamma_{\mathsf{S,R}}$ has cdf,

$$\mathcal{F}_{\gamma_{\mathsf{S,R}}}(x) = \begin{cases} 1 - e^{-\frac{x}{\Gamma_{\mathsf{S,R}}}}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases} \tag{3.50}$$

where $\Gamma_{\mathsf{S,R}} = E[\gamma_{\mathsf{S,R}}]$. The following Lemma is useful for evaluating $\mathcal{F}_{\mathcal{I}_{\mathsf{S,R}}}$.

Lemma 3.5 (cdf for $Y = b\log(a + X)$)**.** *If X and Y are two RVs related by,*

$$Y = b\log(a + X), \tag{3.51}$$

cdf of Y is given by,

$$\mathcal{F}_Y(y) = \mathcal{F}_X\left(2^{\frac{y}{b}} - a\right). \tag{3.52}$$

Proof.

$$\begin{aligned} \mathcal{F}_Y(y) &= \Pr\{Y \leq y\} \\ &= \Pr\{b\log(a + X) \leq y\} \\ &= \Pr\left\{X \leq 2^{\frac{y}{b}} - a\right\} \\ &= \mathcal{F}_X\left(2^{\frac{y}{b}} - a\right). \end{aligned} \tag{3.53}$$

□

Application of Lemma 3.5 with $X = \gamma_{\mathsf{S,R}}$, $Y = \mathcal{I}_{\mathsf{S,R}}$, $a = 1$ and $b = \tau_{\mathsf{s}}$ gives,

$$\begin{aligned} \mathcal{F}_{\mathcal{I}_{\mathsf{S,R}}}(\boldsymbol{\tau}, x) &= \mathcal{F}_{\gamma_{\mathsf{S,R}}}\left(2^{\frac{x}{\tau_{\mathsf{s}}}} - 1\right) \\ &= \begin{cases} 1 - e^{-\frac{2^{\frac{x}{\tau_{\mathsf{s}}}} - 1}{\Gamma_{\mathsf{S,R}}}}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases} \end{aligned} \tag{3.54}$$

and by means of differentiation,

$$\mathbf{f}_{\mathcal{I}_{\mathsf{S,R}}}(\boldsymbol{\tau}, x) = \begin{cases} \frac{\ln 2}{\tau_{\mathsf{S}}} \frac{2^{\frac{x}{\tau_{\mathsf{S}}}}}{\Gamma_{\mathsf{S,R}}} e^{-\frac{2^{\frac{x}{\tau_{\mathsf{S}}}}-1}{\Gamma_{\mathsf{S,R}}}}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases} \tag{3.55}$$

Lemma 3.5 is also useful in evaluating distribution functions for $\mathcal{F}_{\mathcal{I}_0}$.

3.3.1.2 Distribution and Density Functions for $\mathcal{I}_0$

$\mathcal{I}_0$ distribution and density functions are more complicated than those of $\mathcal{I}_{\mathsf{S,R}}$. We divide the task of evaluating $\mathcal{F}_{\mathcal{I}_0}$ into smaller sub-tasks. We first find distribution and density functions for the random variable $\Xi = (1 + \gamma_{\mathsf{R,D}})^{\frac{\tau_{\mathsf{r}}}{\tau_{\mathsf{s}}}}$. Distribution and density functions are then evaluated for the random variable $\Omega = \gamma_{\mathsf{S,D}} + (1 + \gamma_{\mathsf{R,D}})^{\frac{\tau_{\mathsf{R}}}{\tau_{\mathsf{S}}}}$. Finally, Lemma 3.5 is applied to find $\mathcal{F}_{\mathcal{I}_0}$ and $\mathbf{f}_{\mathcal{I}_0}$.

We start by considering the random variable $\Xi \triangleq (1 + \gamma_{\mathsf{R,D}})^{\frac{\tau_{\mathsf{R}}}{\tau_{\mathsf{S}}}}$. The following lemma illustrates the way to find $\mathcal{F}_{\Xi}$.

Lemma 3.6 (pdf of $Y = (a + X)^b$)**.** *If Y is a RV given by,*

$$Y = (a + X)^b,$$

where, X is an exponentially distributed RV, and a and b are constants; Y is a Weibull distributed random variable with cdf,

$$\mathcal{F}_Y(y) = \begin{cases} 1 - e^{-\frac{y^{\frac{1}{b}} - a}{E[x]}}, & \textit{if } y \geq a^b, \\ 0, & \textit{if } y < a^b, \end{cases} \tag{3.56}$$

and pdf,

$$\mathbf{f}_Y(y) = \begin{cases} \frac{y^{\frac{1}{b}-1}}{bE[X]} e^{-\frac{y^{\frac{1}{b}} - a}{E[X]}} & \textit{if } y \geq a^b, \\ 0 & \textit{if } y < a^b. \end{cases} \tag{3.57}$$

Proof. $\mathcal{F}_X$ is given in (3.44). With regard to $\mathcal{F}_Y$,

$$\begin{aligned} \mathcal{F}_Y(y) &= \Pr\{Y \leq y\} \\ &= \Pr\left\{(a + X)^b \leq y\right\} \\ &= \Pr\left\{X \leq y^{\frac{1}{b}} - a\right\} \\ &= \mathcal{F}_X\left(y^{\frac{1}{b}} - a\right) \\ &= \begin{cases} 1 - e^{-\frac{y^{\frac{1}{b}} - a}{E[x]}}, & \text{if } y \geq a^b, \\ 0, & \text{if } y < a^b \end{cases}. \end{aligned} \tag{3.58}$$

differentiating,

$$\begin{aligned}\mathbf{f}_Y(y) &= \frac{\mathrm{d}}{\mathrm{d}y}\mathcal{F}_Y(y)\\ &= \begin{cases}\frac{y^{\frac{1}{b}-1}}{bE[X]}e^{-\frac{y^{\frac{1}{b}}-a}{E[X]}}, & \text{if } y \geq a^b,\\ 0, & \text{if } y < a^b.\end{cases}\end{aligned} \tag{3.59}$$

□

Applying Lemma 3.6 with $Y = \Xi$, $X = \gamma_{\mathsf{R,D}}$, $a = 1$ and $b = \frac{\tau_{\mathsf{R}}}{\tau_{\mathsf{S}}}$ we have,

$$\mathcal{F}_{\Xi}(\boldsymbol{\tau}, x) = 1 - e^{-\frac{x^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}}-1}{\Gamma_{\mathsf{R,D}}}}, \tag{3.60}$$

and

$$\mathbf{f}_{\Xi}(\boldsymbol{\tau}, x) = \frac{1}{\Gamma_{\mathsf{R,D}}}\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}x^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}-1}e^{-\frac{x^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}}-1}{\Gamma_{\mathsf{R,D}}}}, \tag{3.61}$$

supported over $[1, \infty)$.

Next consider a random variable $\Omega \triangleq \gamma_{\mathsf{S,D}} + \Xi_{\mathsf{R,D}}$.

Lemma 3.7 (cdf of $Z = X + Y$). *If Z is a RV given by,*

$$Z = X + Y, \tag{3.62}$$

wher X and Y are two independent RVs, cdf of Z is given by,

$$\mathcal{F}_Z(z) = \int_{-\infty}^{\infty}\int_{-\infty}^{z-y}\mathbf{f}_X(x)\mathbf{f}_Y(y)\mathrm{d}x\mathrm{d}y \tag{3.63}$$

Proof. Proof is available in several texts on probability theory. See for example [70] and [72]. □

Application of Lemma 3.7 unveils $\mathcal{F}_{\Omega}$,

$$\mathcal{F}_{\Omega}(\boldsymbol{\tau}, z) = 1 - \exp\left(-\frac{z-1}{\Gamma_{\mathsf{S,D}}}\right) - \frac{1}{\Gamma_{\mathsf{S,D}}}\int_0^{z-1}\exp\left(-\frac{(z-y)^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}}-1}{\Gamma_{\mathsf{R,D}}} - \frac{y}{\Gamma_{\mathsf{S,D}}}\right)\mathrm{d}y. \tag{3.64}$$

and $\mathbf{f}_{\Omega}$,

$$\mathbf{f}_{\Omega}(\boldsymbol{\tau}, z) = \frac{1}{\Gamma_{\mathsf{S,D}}\Gamma_{\mathsf{R,D}}}\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}\int_0^{z-1}(z-y)^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}-1}\exp\left(-\frac{(z-y)^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}}-1}{\Gamma_{\mathsf{R,D}}} - \frac{y}{\Gamma_{\mathsf{S,D}}}\right)\mathrm{d}y \tag{3.65}$$

Distribution functions for Ω are supported over $[0, \infty)$. Appendix C explains how Lemma 3.7 is applied to obtain $\mathcal{F}_{\Omega}$ and $\mathbf{f}_{\Omega}$.

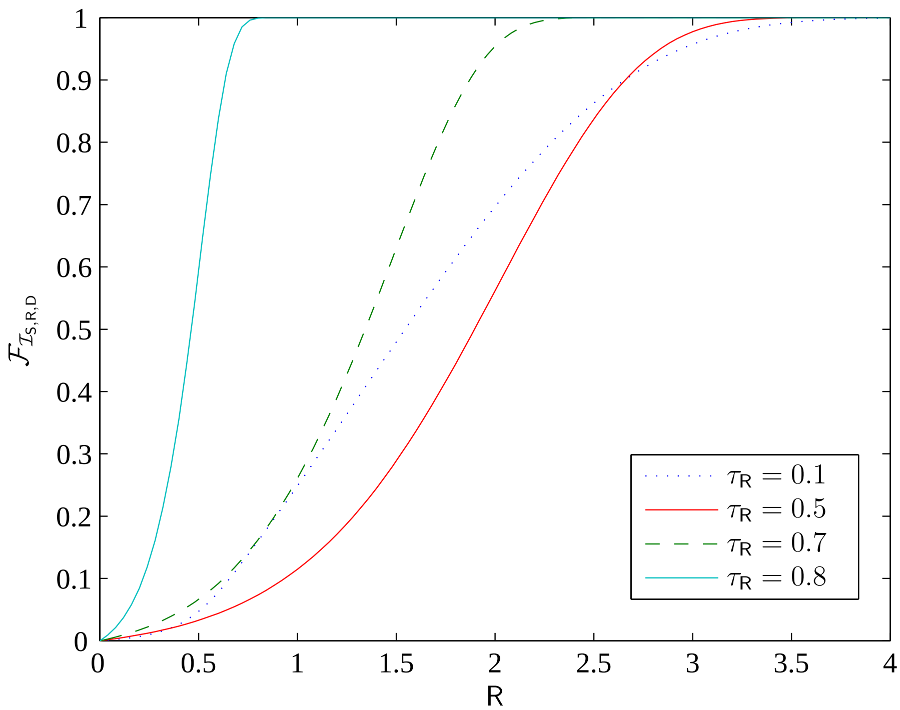


Figure 3.14: cdf for mutual information of a relay channel with $\boldsymbol{\gamma} = (3, 35, 30)$. We have $\tau_S + \tau_R = 1$. Each curve represents a different time allocation.

Eventually, we notice that $\mathcal{I}_0 = \tau_s \log \Omega$. We may thus apply Lemma 3.5 with $X = \Omega$, $Y = \mathcal{I}_0$, $a = 0$ and $b = \tau_s$, to obtain $\mathcal{F}_{\mathcal{I}_0}$ and $\mathrm{f}_{\mathcal{I}_0}$,

$$\begin{aligned}\mathcal{F}_{\mathcal{I}_0}(\boldsymbol{\tau}, z) &= \mathcal{F}_\Omega\left(2^{\frac{z}{\tau_s}}\right)\\ &= 1 - \exp\left(-\frac{2^{\frac{z}{\tau_s}} - 1}{\Gamma_{S,D}}\right)\\ &\qquad - \frac{1}{\Gamma_{S,D}} \int_0^{2^{\frac{z}{\tau_s}}-1} \exp\left(-\frac{\left(2^{\frac{z}{\tau_s}} - y\right)^{\frac{\tau_S}{\tau_R}} - 1}{\Gamma_{R,D}} - \frac{y}{\Gamma_{S,D}}\right) \mathrm{d}y. \qquad (3.66)\end{aligned}$$

and

$$\mathrm{f}_{\mathcal{I}_0}(\boldsymbol{\tau}, z) = \frac{1}{\Gamma_{S,D}\Gamma_{R,D}} \frac{\tau_S}{\tau_R} \int_0^{2^{\frac{z}{\tau_s}}-1} (2^{\frac{z}{\tau_s}} - y)^{\frac{\tau_S}{\tau_R}-1} \exp\left(-\frac{\left(2^{\frac{z}{\tau_s}} - y\right)^{\frac{\tau_S}{\tau_R}} - 1}{\Gamma_{R,D}} - \frac{y}{\Gamma_{S,D}}\right) \mathrm{d}y \qquad (3.67)$$

respectively, for $z \geq 0$.

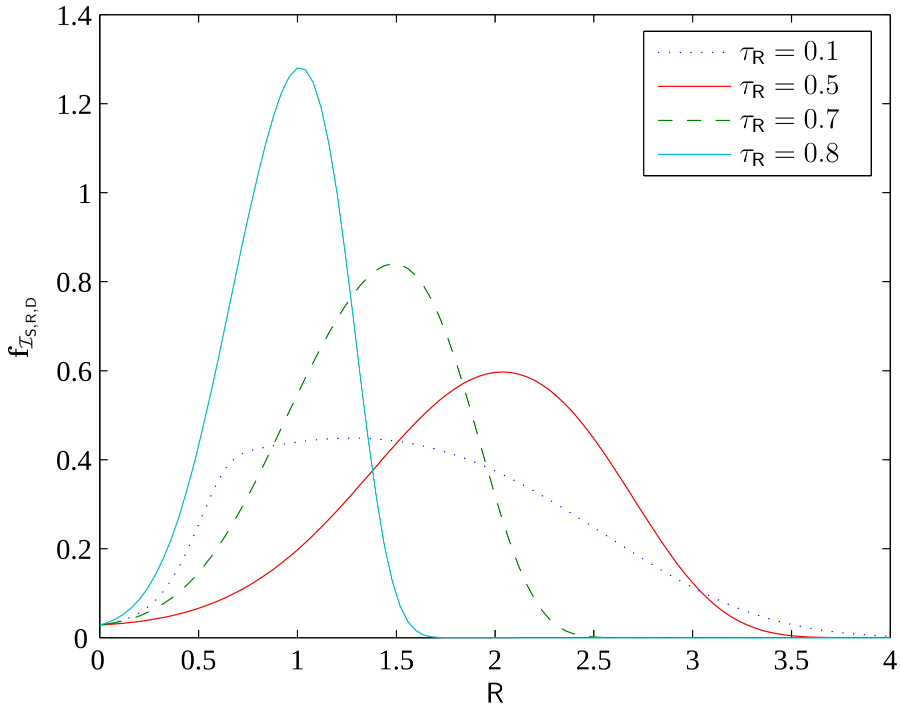


Figure 3.15: pdf for mutual information of a relay channel with $\boldsymbol{\gamma} = (3, 35, 30)$. We have $\tau_S + \tau_R = 1$. Each curve represents a different time allocation.

Finally, we substitute (3.66) and (3.67) in (3.48) and (3.49) to get,

$$\mathcal{F}_{\mathcal{I}_{S,R,D}}(\boldsymbol{\tau}, r) = 1 - \exp\left(-\left(\tfrac{1}{\Gamma_{S,D}} + \tfrac{1}{\Gamma_{S,R}}\right)\left(2^{\frac{r}{\tau_S}} - 1\right)\right) - \frac{1}{\Gamma_{S,D}} e^{-\frac{2^{\frac{r}{\tau_S}}-1}{\Gamma_{S,R}}} \int_0^{2^{\frac{r}{\tau_S}}-1} \exp\left(-\frac{\left(2^{\frac{r}{\tau_S}} - y\right)^{\frac{\tau_S}{\tau_R}} - 1}{\Gamma_{R,D}} - \frac{y}{\Gamma_{S,D}}\right) dy, \tag{3.68}$$

and

$$\mathbf{f}_{\mathcal{I}_{S,R,D}}(\boldsymbol{\tau}, r) = e^{-\frac{2^{\frac{r}{\tau_S}}-1}{\Gamma_{S,R}}} \left[\frac{\ln 2}{\tau_S} \frac{2^{\frac{x}{\tau_S}}}{\Gamma_{S,R}} \exp\left(-\frac{2^{\frac{r}{\tau_S}} - 1}{\Gamma_{S,D}}\right) + \frac{\ln 2}{\tau_S} \frac{2^{\frac{x}{\tau_S}}}{\Gamma_{S,D}\Gamma_{S,R}} \int_0^{2^{\frac{r}{\tau_S}}-1} \exp\left(-\frac{\left(2^{\frac{r}{\tau_S}} - y\right)^{\frac{\tau_S}{\tau_R}} - 1}{\Gamma_{R,D}} - \frac{y}{\Gamma_{S,D}}\right) dy + \frac{1}{\Gamma_{S,D}\Gamma_{R,D}} \frac{\tau_S}{\tau_R} \int_0^{2^{\frac{r}{\tau_S}}-1} \left(2^{\frac{r}{\tau_S}} - y\right)^{\frac{\tau_S}{\tau_R}-1} \exp\left(-\frac{\left(2^{\frac{r}{\tau_S}} - y\right)^{\frac{\tau_S}{\tau_R}} - 1}{\Gamma_{R,D}} - \frac{y}{\Gamma_{S,D}}\right) dy \right]. \tag{3.69}$$

Unfortunately, there is no closed-form for (3.68) and (3.69) and thus it can only be dealt with numerically. Figures 3.14 and 3.15 show plots of $\mathcal{F}_{\mathcal{I}_{S,R,D}}$ and $\mathbf{f}_{\mathcal{I}_{S,R,D}}$, respec-

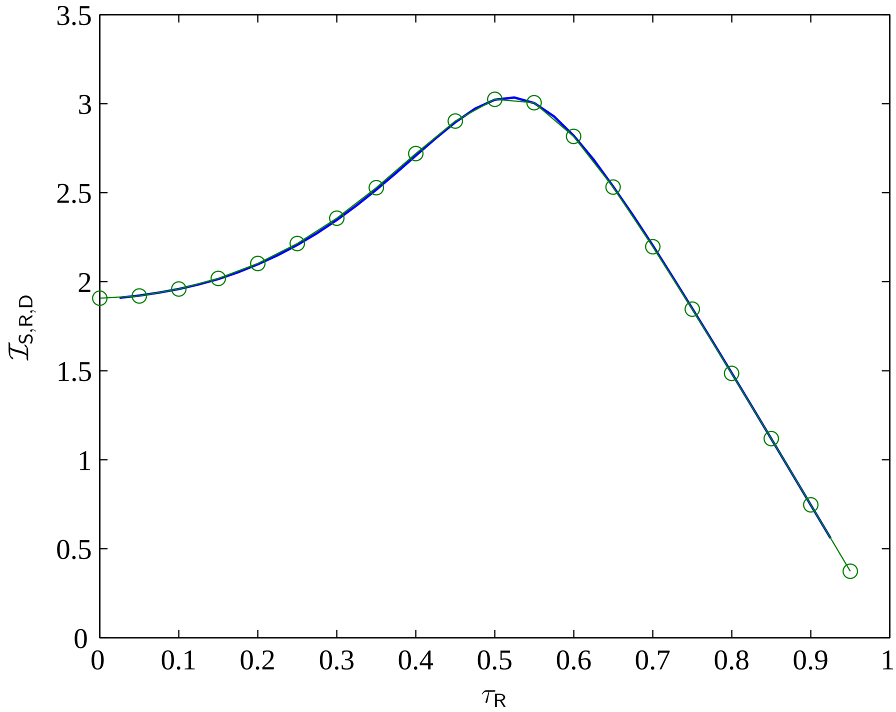


Figure 3.16: Comparing results generated by two different methods for $\bar{\mathcal{I}}_{\mathsf{S,R,D}}$. $\bar{\mathcal{I}}_{\mathsf{S,R,D}}(\boldsymbol{\tau})$ generated by averaging over many channel realizations (green curve with circle marker) is almost identical to that generated using (3.70) (blue curve with no marker). The expected value for the channels was 6, 25, 22 for $\gamma_{\mathsf{S,D}}$, $\gamma_{\mathsf{S,R}}$ and $\gamma_{\mathsf{R,D}}$, respectively. Averaging is carried out over 1000 channel realizations.

tively, for a given relay channel. For comparison, graphs are made for different time allocations. By looking at these graphs we may infer some of the properties of this relay channel. For instance in Figure 3.15, the position and height of the peak of each graph gives an indication of the average rate achievable and its variation for the corresponding time allocation. Comparing $\tau_{\mathsf{R}} = 0.5$ and $\tau_{\mathsf{R}} = 0.8$, the latter indicates that a range of $0 - 1.5$ bit/sec/Hz is likely to occur with average around 1.0 bit/sec/Hz, while in the former the average mutual information is about 2.0 bit/sec/Hz but dispersed over a larger range. Ideally, we seek a time allocation that has a high peak and is far enough from zero.

3.3.2 Average Mutual Information

Probability distribution function for $\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau})$ is useful in finding the average mutual information for the relay channel, $\bar{\mathcal{I}}_{\mathsf{S,R,D}}(\boldsymbol{\tau})$. To get $\bar{\mathcal{I}}_{\mathsf{S,R,D}}(\boldsymbol{\tau})$, the probabilistic average for the random variable $\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau})$ is carried out,

$$\bar{\mathcal{I}}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) \triangleq \int_0^\infty x \, \mathbf{f}_{\mathcal{I}_{\mathsf{S,R,D}}}(\boldsymbol{\tau}, x) \mathrm{d}x \tag{3.70}$$

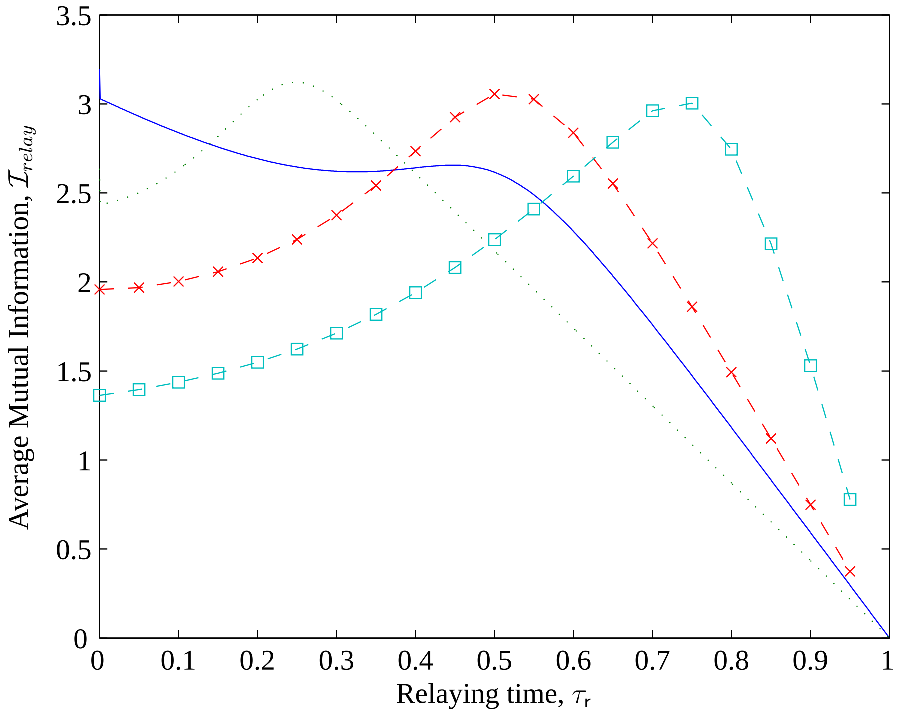


Figure 3.17: Average mutual information for different relay channels.

$\bar{\mathcal{I}}_{\text{S,R,D}}(\boldsymbol{\tau})$ matches the ergodic (Shannon) capacity of the single fading channel. To verify (3.70), average mutual information results are generated first using the above formula and then by means of averaging the mutual information over many channel realizations for the same $\boldsymbol{\gamma}$. Figure 3.16 shows that both graphs are almost identical, which proves that $\bar{\mathcal{I}}_{\text{S,R,D}}$, as well as $\mathcal{F}_{\mathcal{I}_{\text{S,R,D}}}$ and $\mathbf{f}_{\mathcal{I}_{\text{S,R,D}}}$, are correct.

In practice, to achieve $\bar{\mathcal{I}}_{\text{S,R,D}}(\boldsymbol{\tau})$ in (3.70), channel conditions must be accurately known to all transmitters and all receivers. Transmitters use that knowledge to generate new codebooks each time γ changes. In an opportunistic fashion, data is sent in high rate when channel conditions are good and in lower rates when channel conditions deteriorate. In a fading environment, this kind of adaptive transmission is crucial for performance. Performance of non-adaptive systems could be far less than adaptive ones [2, 10]. It is even more beneficial when transmitters are able to optimally allocate resources to achieve the maximum possible performance. Figure 3.17 shows average mutual information graphs for different relay channels in a fading environment. These examples show that time allocation is important for the performance of the channel. A judicious choice of $\boldsymbol{\tau}$ can greatly boost the average mutual information. In Chapter 4 we study optimal time allocation for the relay channel and the effect on performance.

According to Jensen's inequality $E[f(X)] \leq f(E[X])$ for any random variable X. $\mathcal{I}_{\text{S,R,D}}$ is no exception. We have $E[\mathcal{I}_{\text{S,R,D}}(\boldsymbol{\gamma})] \leq \mathcal{I}_{\text{S,R,D}}(E[\boldsymbol{\gamma}])$. In other words, average mutual information for the relay channel is always less than or equal to that of the AWGN relay channel with the same average SNR and same time allocation. This

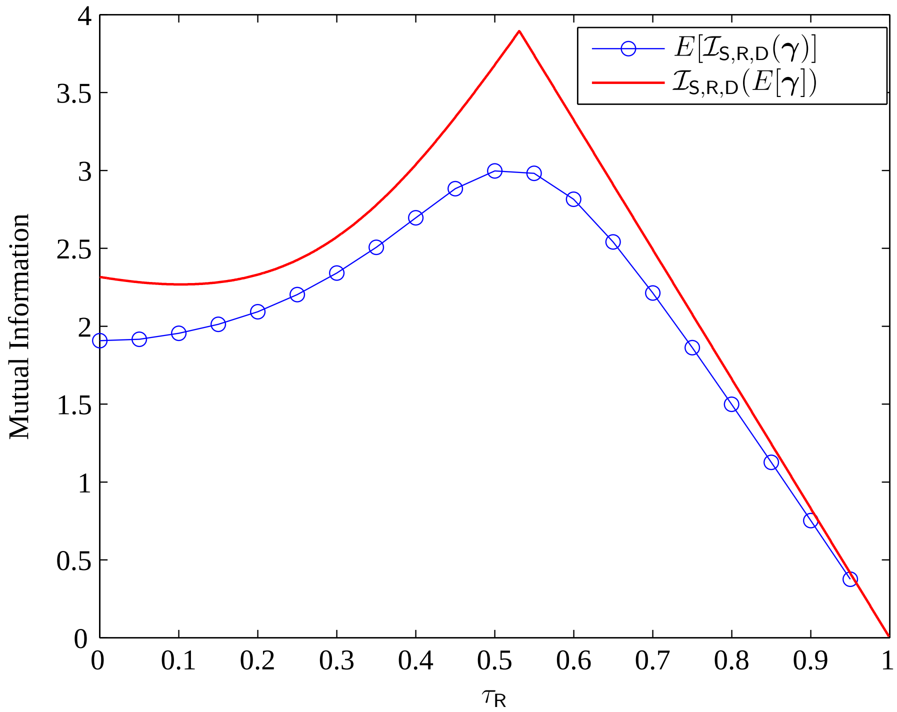


Figure 3.18: Mutual information for the fading relay channel is always less than or equal to that of an AWGN channel with the same average SNR.

is illustrated in Figure 3.18.

Flowchart in Figure 3.19 demonstrates how simulation results for the average mutual information are generated by means of averaging over many time realizations. This is typically the procedure used to generate the results for Figures 3.16, 3.17 and 3.18. The average SNR for the channels, initially in dBs, must first be converted to linear units. Random channel realizations are generated based on these values and according to an Exponential distribution. $\mathcal{I}_{\text{S,R,D}}$ is calculated based on channel realizations. The process of generating random channels and calculating $\mathcal{I}_{\text{S,R,D}}$ is repeated m times, $m \gg 1$, and results are added up. Finally, average is calculated by dividing the accumulated $\mathcal{I}_{\text{S,R,D}}$ by m.

3.3.3 Outage Probability

Network entities are not able to practise adaptive transmission if there is a lack of accurate channel information or due to hardware restrictions such as complexity constraints. In that case transmitters will adopt a single codebook and hence a fixed data rate. Whenever mutual information falls below that rate, the system declares outage and probability of error approaches 1 [2]. Formally, we define *outage probability*, $\mathcal{P}$, as *the probability that mutual information between the source and the destination falls below a minimum required rate* R, where generally R is determined by the application. Mathematically,

$$\mathcal{P}(R) \triangleq \Pr\left\{\mathcal{I} < R\right\}. \tag{3.71}$$

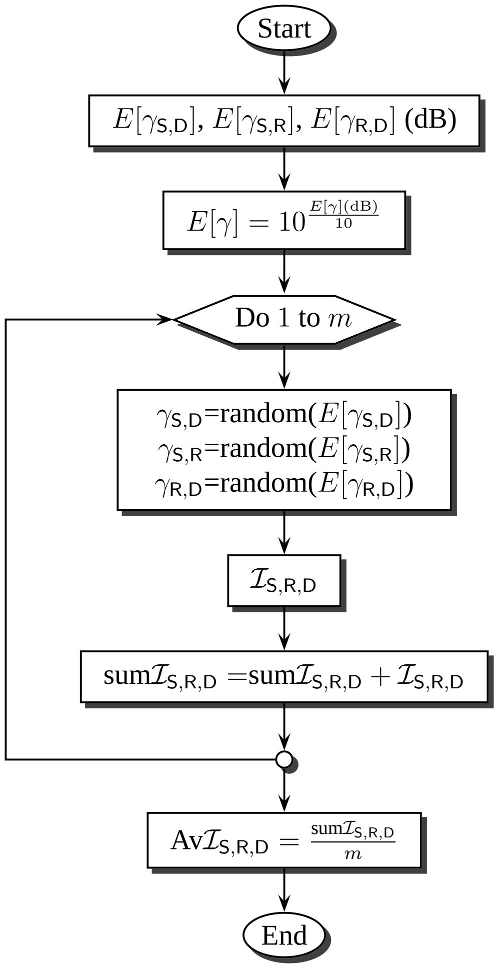


Figure 3.19: Flowchart demonstrating generation of $\bar{\mathcal{I}}_{\mathsf{S,R,D}}$ by means of averaging over many channel realizations.

where $\Pr\{\epsilon\}$ denotes the probability for the event ϵ. As a function of time allocation, outage probability for the relay channel is,

$$\begin{aligned} \mathcal{P}(\boldsymbol{\tau}, R) &= \Pr\{\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) < R\} \\ &= \Pr\{\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) \leq R\} - \Pr\{\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) = R\} \\ &= \mathcal{F}_{\mathcal{I}_{\mathsf{S,R,D}}}(\boldsymbol{\tau}, R) \end{aligned} \tag{3.72}$$

The last equality follows as $\Pr\{X = x\} \to 0$ for every continuous RV X [70]. As we showed previously, in Figure 3.20, outage probability results generated using (3.72) are compared with those generated by means of averaging over many channel realizations. The matching of the two graphs supports the analytical results for $\mathcal{P}$ as well as $\mathcal{F}_{\mathcal{I}_{\mathsf{S,R,D}}}$.

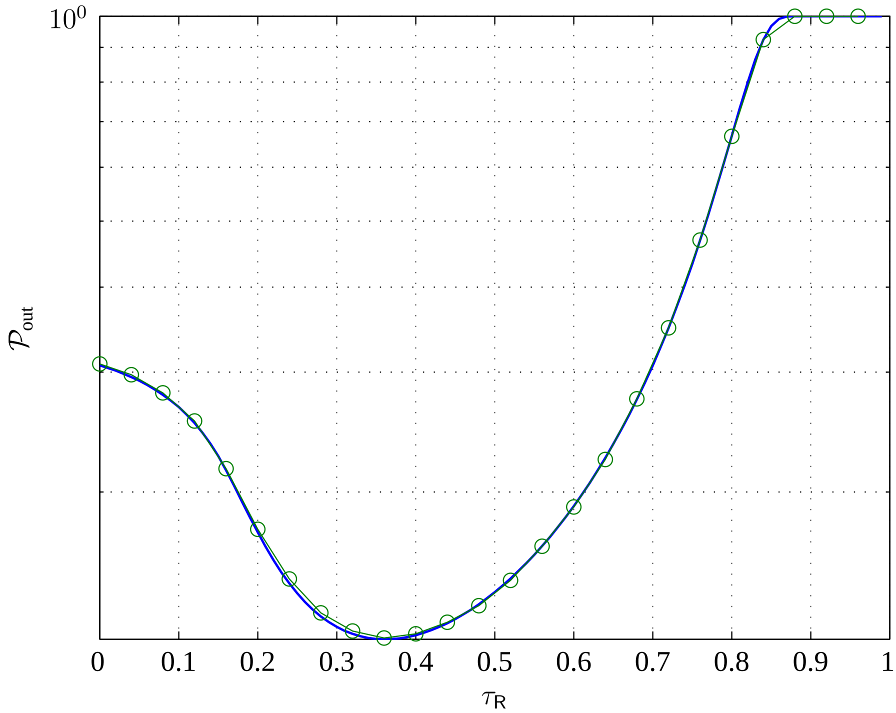


Figure 3.20: Comparing results generated by two different methods for $\mathcal{P}_{out}$. Green curve with circle marker represents $\mathcal{P}_{out}$ generated by averaging over many channel realizations while the blue curve is also for $\mathcal{P}_{out}$, however, produced using (3.72). Both curves are identical. The expected value for the channels was 3, 30, 20 for $\gamma_{S,D}$, $\gamma_{S,R}$ and $\gamma_{R,D}$. Average SNR Averaging is carried out over 100000 channel realizations.

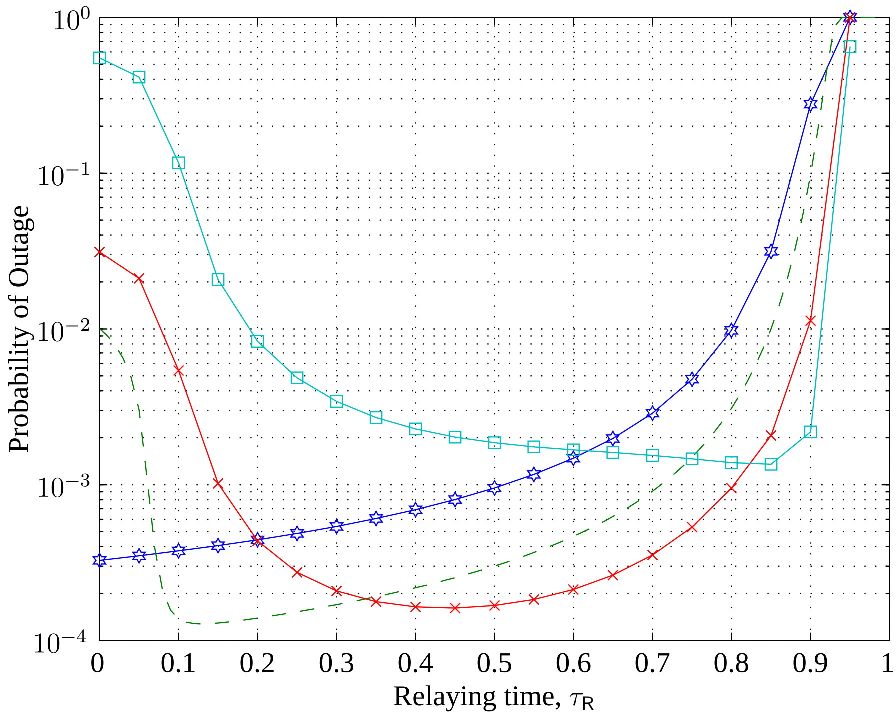


Figure 3.21: Outage probability for different relay channels.

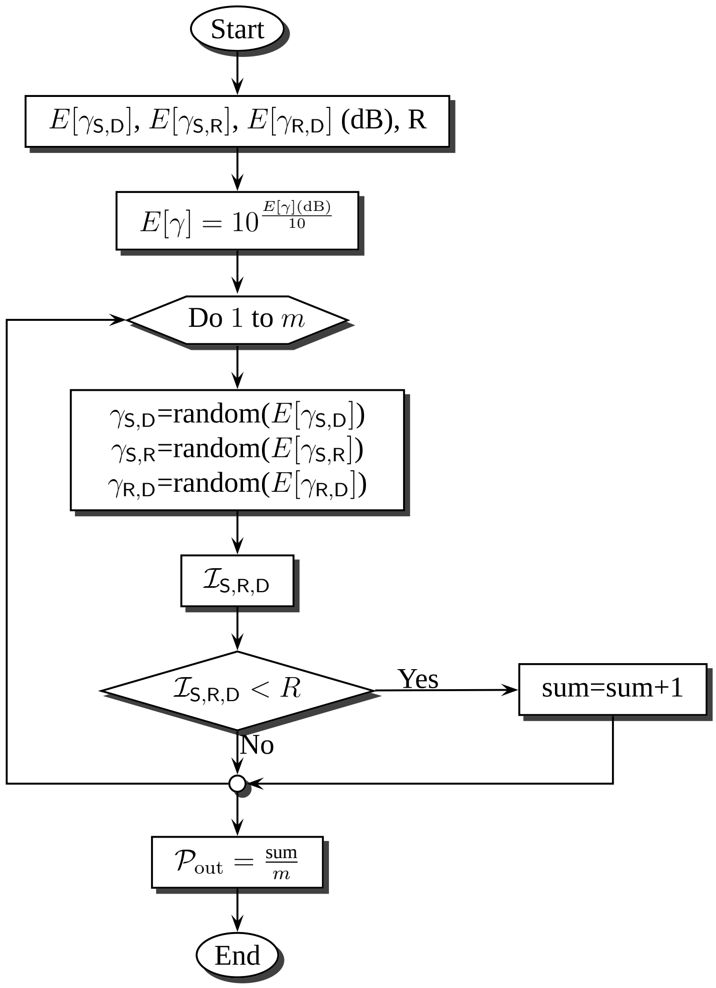


Figure 3.22: Flowchart demonstrating generation of $\mathcal{P}$ by calculating the proportion of the time the outage event occurred.

Flowchart in Figure 3.22 demonstrates the way to generate outage probability results by means of averaging the number outage events over many random channel realizations.

Figure 3.21 shows outage probability graphs for different fading relay channels. Again we see that time allocation is important for the performance of the channel. Outage probability can be kept within an acceptable range by by wisely choosing $\boldsymbol{\tau}$.

$\mathcal{P}$ is more useful for fixed rate transmission scenarios such as voice applications. In other situations, the system is constrained by the maximum allowed outage probability. Instead of a minimum required transmission rate, transmitters can transmit with a maximum rate without outage probability exceeding a predetermined value. In that case outage capacity is more useful in analyzing channel performance. Given a maximum allowed outage probability ϵ, *Outage capacity*, $\mathcal{C}(\epsilon)$, *is the maximum rate at which* *information can be transmitted over the channel such that* $\mathcal{P} < \epsilon$*,*

$$\mathcal{C}(\epsilon) \triangleq \max_{p(x)} \arg \mathcal{P}(R), \quad \text{s.t. } \mathcal{P}(R) < \epsilon. \tag{3.73}$$

3.4 Chapter Summary

In this chapter mutual information for a simple three-node relay channel is derived for arbitrary time allocation and an AWGN channel. Then probability distribution functions for the mutual information are found for the case of Rayleigh fading. This leads to the channel average mutual information and outage probability in a fading environment. The results obtained are useful for the rest of the thesis. In the next chapter we see how to optimize channel performance. In Chapter 4, a cooperative model is proposed based on the three-node relay channel studied here. Chapter 6 extends results to multi-hop relay channels.

Chapter 4

Time Allocation for the D&F Relay Channel

In this chapter we look into optimal operation of the three-node D&F relay channel. Two problems are considered: maximizing mutual information and minimizing transmission time. Solutions to these problems not only help tune for best performance, but also give insight into the channel in order to make further extensions and applications. In Section 4.1 we introduce adaptive relaying and look into how adaptability affects mutual information formula expressed in the previous chapter. Section 4.2 considers mutual maximization problem, whereas, total time minimization problem is investigated in Section 4.3. Section 4.4 establishes an important duality between optimization problems. Finally Section 4.6 summarizes this chapter.

Results obtained in this chapter are useful for the cooperative model proposed in Chapter 5. In particular they help to set useful conditions for partner selection. In multi-hop relaying, studied in Chapter 6, optimum route selection is a generalization for optimization problems considered here.

4.1 Introduction: Adaptive Relaying

In adaptive systems, transmitters change signaling strategy according to channel conditions. It is now well known that employing channel adaptive signaling in wireless communication systems can yield large improvements in almost any performance metric [73]. Adaptive communication requires knowledge of the channel. Channel state information is necessary at receiver as well as transmitter. Channel state information at receiver (CSIR) is obtainable by sending a training sequence prior to message transmission. CSIT is, nevertheless, challenging. Many kinds of channel adaptive techniques have been deemed impractical in the past because of the problem of obtaining channel knowledge at the transmitter. *Statistical channel information* (SCIT) at transmitter can be used instead. Generally, however, SCIT adaptation comes with a performance loss that is not negligible compared with adaptation techniques that use instantaneous channel information [73].

Recently, innovations in the field made instantaneous channel information at transmitter possible. In two-way systems employing TDD, this is achieved by having the transmitter measuring the channel while receiving from what to be a receiver later. That requires the channel to be changing slowly so that obtained information about the channel will not be out of date before or during the transmission period. This technique also assumes that the channel is reciprocal, that is, the up-link and the down-link channels are the same.

FDD systems, forward and reverse are generally highly uncorrelated due to separation in frequency. In this case CSIT can only be obtained by means of limited feedback [10]. A low rate data stream on the reverse side of the link is used to provide information to the transmitter of the forward side of the link [73].

In this chapter we assume that transmitters are able to allocate available time to achieve optimum performance. We also assume that CSIT is available to all transmitters. Finally, we allow the channel to combat the broadcast behavior of D&F signaling. As stated in the next proposition, the source node can ignore the presence of the relay when $\tau_{\mathsf{R}} = 0$, and thus acquire maximum performance through direct transmission.

Proposition 4.1 (Mutual information for the relay channel with arbitrary time allocation and adaptive transmission)**.** *Consider a three-node D&F relay channel with half-duplex constraint on the relay. Let* τ_{S} *of the available time,* $\tau_{\mathsf{S}} \leq 1$*, be used by the source to send the message while* τ_{R} *of time is used by the relay for retransmitting the message after successful decoding at the relay. Mutual information between the source and the destination is given by,*

$$\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) = \begin{cases} \mathcal{I}_{\mathsf{S,D}}(\boldsymbol{\tau}), & \text{if } \tau_{\mathsf{R}} = 0, \\ \min\{\mathcal{I}_{\mathsf{S,R}}(\boldsymbol{\tau}), \mathcal{I}_0(\boldsymbol{\tau})\}, & \text{if } 0 < \tau_{\mathsf{R}} < 1. \end{cases} \quad \text{bit/sec/Hz.} \tag{4.1}$$

4.2 Maximizing Mutual Information

The first optimization problem we consider is mutual information maximization problem. We seek the best time allocation policy so that mutual information is maximum. Mathematically, we are looking for a solution to the following problem,

$$\mathbb{P} \mapsto \begin{cases} \max\limits_{\boldsymbol{\tau}} & \mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) \\ \text{s.t.} & \tau_{\mathsf{S}}, \tau_{\mathsf{R}} \geq 0, \\ & c \leq 1. \end{cases} \tag{4.2}$$

Any time allocation τ which satisfies problem constraints is a *feasible* point. The *feasible set*, $\mathbf{C}$, is the set of all feasible time allocations. $\mathbf{C}$ is given by,

$$\mathbf{C} = \{\boldsymbol{\tau} : \tau_{\mathsf{S}} \geq 0, \tau_{\mathsf{R}} \geq 0, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1\}. \tag{4.3}$$

$\mathcal{I}^{\max}_{\mathsf{S,R,D}}$ denotes the optimal value,

$$\mathcal{I}^{\max}_{\mathsf{S,R,D}} = \sup_{\boldsymbol{\tau} \in \mathbf{C}} \mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right). \tag{4.4}$$

As $\mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right)$ is an increasing function of c, the second constraint is active, i.e., satisfied with equality. That reduces the feasibility set to,

$$\mathbf{C} = \{\boldsymbol{\tau} : \tau_{\mathsf{S}} \geq 0, \tau_{\mathsf{R}} \geq 0, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} = 1\}. \tag{4.5}$$

In this case, we have,

$$\tau_{\mathsf{S}} = 1 - \tau_{\mathsf{R}}. \tag{4.6}$$

$\mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right)$ can be written as function of τ_{R} only by substituting (4.6) in (4.1),

$$\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{R}}) = \begin{cases} \mathcal{I}_{\mathsf{S,D}}, & \text{if } \tau_{\mathsf{R}} = 0, \\ \min\{\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}}), \mathcal{I}_0(\tau_{\mathsf{R}})\}, & \text{if } 0 < \tau_{\mathsf{R}} < 1, \end{cases} \quad \text{bit/sec/Hz}, \tag{4.7}$$

where, $\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})$, $\mathcal{I}_0(\tau_{\mathsf{R}})$ and $\mathcal{I}_{\mathsf{S,D}}$ are given by,

$$\begin{aligned} \mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}}) &= (1 - \tau_{\mathsf{R}}) \log\left(1 + \gamma_{\mathsf{S,R}}\right), \\ \mathcal{I}_0(\tau_{\mathsf{R}}) &= (1 - \tau_{\mathsf{R}}) \log\left(\gamma_{\mathsf{S,D}} + [1 + \gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right), \\ \mathcal{I}_{\mathsf{S,D}} &= \log\left(1 + \gamma_{\mathsf{S,D}}\right). \end{aligned} \tag{4.8}$$

We can thus reduce the optimization problem (4.2) to,

$$\mathbb{P}: \max_{0 \leq \tau_{\mathsf{R}} \leq 1} \mathcal{I}_{\mathsf{S,R,D}}\left(\tau_{\mathsf{R}}\right), \tag{4.9}$$

τ_{R} in (4.7) measures the degree of cooperation. $\tau_{\mathsf{R}} = 0$ and $\tau_{\mathsf{R}} = 1$ are the special cases of no-cooperation (or direct transmission) and full cooperation, respectively. Note that full cooperation achieves 0 bit/sec/Hz. $\tau_{\mathsf{R}} = \frac{1}{2}$ is another special case when time is equally allocated for the source and the relay. Conventionally, the half-duplex relay channel is assumed to follow equal time allocation policy.

To solve (4.9) we take advantage of the convexity of $\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})$ and $\mathcal{I}_0(\tau_{\mathsf{R}})$ which is stated in the following lemma.

Lemma 4.1 (Convexity of $\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})$ and $\mathcal{I}_0(\tau_{\mathsf{R}})$**).** *$\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})$ and $\mathcal{I}_0(\tau_{\mathsf{R}})$ are convex over τ_{R} for all $\tau_{\mathsf{R}} \in (0,1)$. Moreover, $\mathcal{I}_0\left(\tau_{\mathsf{R}}\right)$ is strictly convex.*

(a)

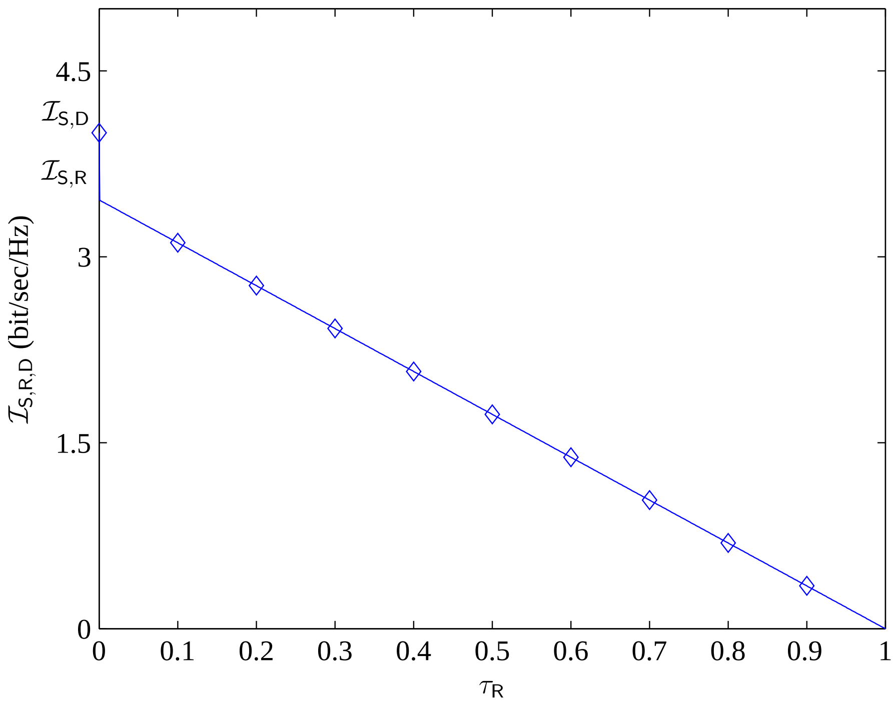


Figure 4.1: Mutual information for the suppressed relay channel vs τ_{R}, $\tau_{\mathsf{S}} = 1 - \tau_{\mathsf{R}}$. Direct transmission is always optimum for the suppressed relay channel. Graph generated using $\gamma_{\mathsf{S,D}} = 15$, $\gamma_{\mathsf{S,R}} = 10$ and $\gamma_{\mathsf{R,D}} = 11$

Proof. As $\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})$ is a line, so it is convex.

Convexity of $\mathcal{I}_0(\tau_{\mathsf{R}})$ is proved by finding the second derivative. Differentiating $\mathcal{I}_0(\tau_{\mathsf{R}})$ twice with respect to τ_{R} we get,

$$\begin{aligned}
\frac{\mathrm{d}^2}{\mathrm{d}\tau_{\mathsf{R}}^2}\mathcal{I}_0(\tau_{\mathsf{R}}) &= \frac{\mathrm{d}^2}{\mathrm{d}\tau_{\mathsf{R}}^2}\left[(1-\tau_{\mathsf{R}})\log\left(\gamma_{\mathsf{S,D}} + [1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)\right] \\
&= \frac{\mathrm{d}}{\mathrm{d}\tau_{\mathsf{R}}}\left[\frac{(1+\gamma_{\mathsf{R,D}})^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\log(1+\gamma_{\mathsf{R,D}})}{(1-\tau_{\mathsf{R}})\left(\gamma_{\mathsf{S,D}} + [1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)} - \log\left(\gamma_{\mathsf{S,D}} + [1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)\right] \\
&= \ln 2 \frac{\gamma_{\mathsf{S,D}}(1+\gamma_{\mathsf{R,D}})^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}(\log(1+\gamma_{\mathsf{R,D}}))^2}{(1-\tau_{\mathsf{R}})^3\left(\gamma_{\mathsf{S,D}} + [1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)^2} \\
&> 0, \qquad \forall \tau_{\mathsf{R}} \in (0,1).
\end{aligned}$$

The above differentiation is found in more details in Appendix A. □

Suppressed and unsuppressed channels are considered separately. Solutions for both cases can then be combined to deduce a general solution to (4.9).

4.2.1 Maximizing $\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{R}})$ When Channel is Suppressed

According to Section 3.2.3, the relay channel is classified as suppressed when $\gamma_{\mathsf{S,D}} \leq \gamma_{\mathsf{S,R}}$. A suppressed channel has got no conversion point. Rewriting (4.7),

$$\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{R}})\,\big|_{\gamma_{\mathsf{S,D}}\geq\gamma_{\mathsf{S,R}}} = \begin{cases} \mathcal{I}_{\mathsf{S,D}}, & \text{if } \tau_{\mathsf{R}} = 0, \\ \mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}}), & \text{if } 0 < \tau_{\mathsf{R}} < 1. \end{cases} \tag{4.10}$$

As demonstrated in the graph of Figure 4.1, $\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})$ is a line with slope $-\log(1+\gamma_{\mathsf{S,R}}) < 0$. Therefore, it is maximized by minimizing τ_{R}. This is,

$$\mathcal{I}^{\max}_{\mathsf{S,R}} = \lim_{\tau_{\mathsf{R}}\to 0} \mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}}) = \log(1+\gamma_{\mathsf{S,R}}). \tag{4.11}$$

But for a suppressed channel, since $\gamma_{\mathsf{S,D}} \geq \gamma_{\mathsf{S,R}}$, we have $\mathcal{I}_{\mathsf{S,D}} = \log(1+\gamma_{\mathsf{S,D}}) \geq \mathcal{I}^{\max}_{\mathsf{S,R}}$. Therefore, in this case direct transmission achieves the maximum mutual information between the source and the destination, or

$$\mathcal{I}^{\max}_{\mathsf{S,R,D}}\big|_{\gamma_{\mathsf{S,D}}\leq\gamma_{\mathsf{S,R}}} = \mathcal{I}_{\mathsf{S,R}}. \tag{4.12}$$

4.2.2 Maximizing $\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{R}})$ When Channel is Unsuppressed

This is the case when $\gamma_{\mathsf{S,D}} < \gamma_{\mathsf{S,R}}$. There is a conversion point given by,

$$\mu_{\mathsf{R}} = \frac{\varsigma}{1+\varsigma} = \frac{\log(1+\gamma_{\mathsf{S,R}}-\gamma_{\mathsf{S,D}})}{\log(1+\gamma_{\mathsf{S,R}}-\gamma_{\mathsf{S,D}})+\log(1+\gamma_{\mathsf{R,D}})} \tag{4.13}$$

obtained using (3.42) with $c = 1$ and ς as in (3.28). As shown in Figure 4.2, $\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{R}})$ is discontinuous at μ_{R}. $\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{R}})$ can be written in the form,

$$\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{R}}) = \begin{cases} \mathcal{I}_0(\tau_{\mathsf{R}}), & \text{if } 0 \leq \tau_{\mathsf{R}} < \mu_{\mathsf{R}}, \\ \mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}}), & \text{if } \mu_{\mathsf{R}} \leq \tau_{\mathsf{R}} \leq 1. \end{cases} \tag{4.14}$$

Due to convexity, $\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})$ and $\mathcal{I}_0(\tau_{\mathsf{R}})$ are maximum at an end point. Then , $\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{R}})$ must be maximum at $\tau_{\mathsf{R}} = 0$, $\tau_{\mathsf{R}} = \mu_{\mathsf{R}}$ or $\tau_{\mathsf{R}} = 1$. Obviously, $\tau_{\mathsf{R}} = 1$ is excluded. As for $\tau_{\mathsf{R}} = 0$ and $\tau_{\mathsf{R}} = \mu_{\mathsf{R}}$, $\mathcal{I}_0(\mu_{\mathsf{R}}) > \mathcal{I}_0(0)$ only if,

$$\mathcal{I}_{\mathsf{S,R}}(\mu_{\mathsf{R}}) > \mathcal{I}_{\mathsf{S,D}},$$
$$(1-\mu_{\mathsf{R}})\log(1+\gamma_{\mathsf{S,R}}) > \log(1+\gamma_{\mathsf{S,D}})$$

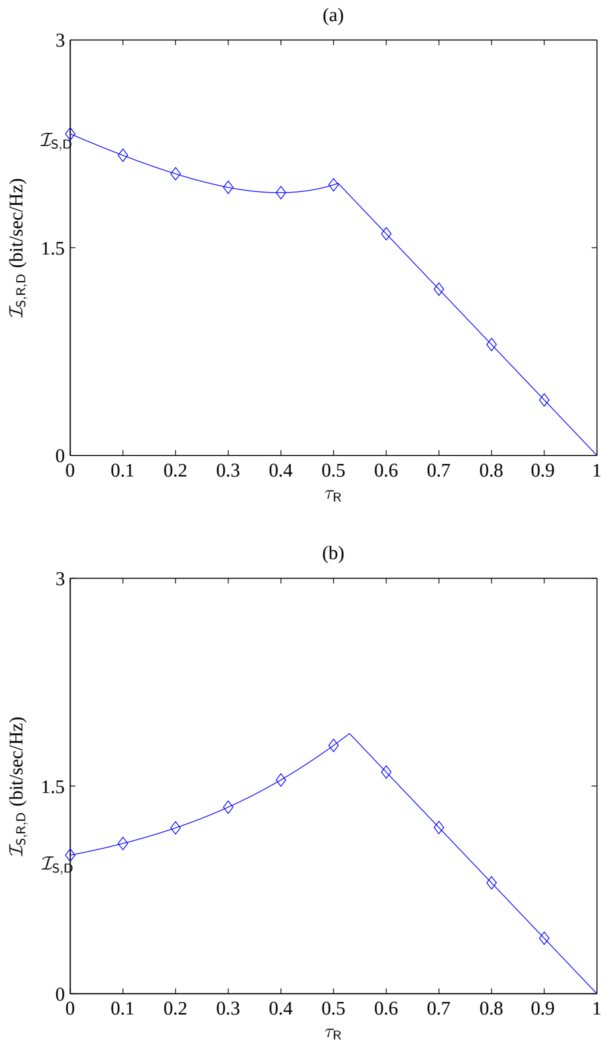


Figure 4.2: Mutual information for the unsuppressed relay channel vs τ_R, $\tau_S = 1-\tau_R$. Two cases are shown, (a) Direct transmission is optimum, graph generated using $\gamma_{S,D} = 4$, $\gamma_{S,R} = 15$ and $\gamma_{R,D} = 10$; and (b) μ_R is optimum, graph generated using $\gamma_{S,D} = 1$, $\gamma_{S,R} = 15$ and $\gamma_{R,D} = 10$.

or when,

$$\begin{aligned} \gamma_{S,D} &< (1+\gamma_{S,R})^{1-\mu_R} - 1 \\ &= (1+\gamma_{S,R})^{\mu_S} - 1 \end{aligned} \tag{4.15}$$

Inequality (4.15) is the condition that verifies optimality of direct transmission for the unsuppressed channel. In words, direct transmission is optimal for the unsuppressed relay channel if (4.15) is not satisfied, otherwise $\mathcal{I}_{S,R,D}(\mu_R)$ is the optimal mutual information.

To sum up, for the unsuppressed relay channel the maximum mutual information

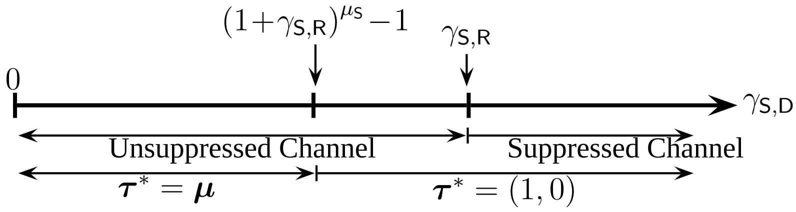


Figure 4.3: Relationship between channels indicates the optimum time allocation for the D&F relay channel. Relaying with time allocated as $\boldsymbol{\mu}$ maximizes mutual information as long as $\gamma_{\mathsf{S,D}} < (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}}$, otherwise direct transmission is optimum.

is given by,

$$\mathcal{I}^{\max}_{\mathsf{S,R,D}}\Big|_{\gamma_{\mathsf{S,D}}<\gamma_{\mathsf{S,R}}} = \begin{cases} \mathcal{I}_{\mathsf{S,R}}(\boldsymbol{\mu}) & \text{if } \gamma_{\mathsf{S,D}} < (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1 \\ \mathcal{I}_{\mathsf{S,D}} & \text{if } \gamma_{\mathsf{S,D}} \geq (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1 \end{cases}. \tag{4.16}$$

achievable with time allocation

$$\boldsymbol{\tau} = \begin{cases} \boldsymbol{\mu} & \text{if } \gamma_{\mathsf{S,D}} < (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1 \\ (1,0) & \text{if } \gamma_{\mathsf{S,D}} \geq (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1 \end{cases}. \tag{4.17}$$

4.2.3 Maximum $\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau})$

To combine results from Section 4.2.2 and Section 4.2.1, it worth noting that $\gamma_{\mathsf{S,R}} \leq (\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1$, which is also illustrated in Figure 4.3. The solution to the maximization problem is stated in the following theorem.

Theorem 4.1 (Maximizing Mutual Information)**.** *For the three-node D&F relay channel, if ς exists and if channels conditions satisfy,*

$$\gamma_{\mathsf{S,D}} < (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1. \tag{4.18}$$

relaying can achieve maximum mutual information of

$$\mathcal{I}^{\max}_{\mathsf{S,R,D}} = \mathcal{I}_{\mathsf{S,R}}(\boldsymbol{\mu}), \tag{4.19}$$

otherwise direct transmission is optimum.

4.3 Minimizing Transmission Time

In the previous section we sought the optimal time allocation so that mutual information between the source and the destination is maximum. In this section time is to be allocated to achieve a predetermined rate while minimizing the total transmission time.

Typically, this can be useful for cognitive networks. Secondary users cooperate to efficiently utilize spectrum not used by primary users. That allows more secondary users to benefit from these *spectrum holes*. Primary users may also cooperate with secondary users to create more spectrum holes.

We seek the best time allocation so that total transmission time, $c(\boldsymbol{\tau})$, is minimum while mutual information achieved is at least equal to predetermined rate R. Mathematically, that is,

$$\mathbb{P} \mapsto \begin{cases} \min\limits_{\boldsymbol{\tau}} & c(\boldsymbol{\tau}) = \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \\ \text{s.t.} & \mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) \geq R \\ & \tau_{\mathsf{S}}, \tau_{\mathsf{R}} \geq 0, \\ & c \leq 1. \end{cases} \tag{4.20}$$

The feasible set $\mathbf{C}$ contains all time allocations $\boldsymbol{\tau}$ which satisfy problem constraints. $\mathbf{C}$ is given by,

$$\mathbf{C} = \{\boldsymbol{\tau} : \mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) \geq R, \tau_{\mathsf{S}} \geq 0, \tau_{\mathsf{R}} \geq 0, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1\} \tag{4.21}$$

The problem is feasible only if $\mathbf{C} \neq \emptyset$. We must then have $R \leq \mathcal{I}^{\max}_{\mathsf{S,R,D}}$, since $\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) \leq \mathcal{I}^{\max}_{\mathsf{S,R,D}}$.

$c^{\max}$ denotes the optimal total transmission time,

$$c^{\max} = \inf_{\boldsymbol{\tau} \in \mathbf{C}} c(\boldsymbol{\tau}). \tag{4.22}$$

As we did for the maximization problem, the suppressed and unsuppressed channels are considered separately. Later, obtained solutions for suppressed and unsuppressed relay channels are combined to deduce a general solution.

4.3.1 Minimizing c when The Channel is Suppressed

Here we have $\gamma_{\mathsf{S,D}} \geq \gamma_{\mathsf{S,R}}$. There is no conversion point and mutual information is discontinuous at $\tau_{\mathsf{R}} = 0$. $\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau})$ takes the form,

$$\mathcal{I}_{\mathsf{S,R,D}}(\boldsymbol{\tau})\big|_{\gamma_{\mathsf{S,D}} \geq \gamma_{\mathsf{S,R}}} = \begin{cases} \mathcal{I}_{\mathsf{S,D}}(\boldsymbol{\tau}), & \text{if } \tau_{\mathsf{R}} = 0, \\ \mathcal{I}_{\mathsf{S,R}}(\boldsymbol{\tau}), & \text{if } 0 < \tau_{\mathsf{R}} < 1. \end{cases} \tag{4.23}$$

The feasible set can, therefore, be written as a union of two sets: $\mathbf{C}_1$ and $\mathbf{C}_2$. $\mathbf{C}_1$ is given by,

$$\mathbf{C}_1 = \{\boldsymbol{\tau} : \mathcal{I}_{\mathsf{S,D}}(\boldsymbol{\tau}) \geq R, 0 \leq \tau_{\mathsf{S}} \leq 1, \tau_{\mathsf{R}} = 0\} \tag{4.24}$$

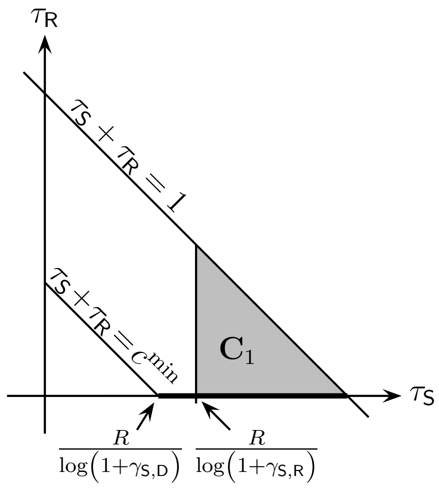


Figure 4.4: In the case of suppressed relay channel, feasible time allocations, $\mathbf{C}$ is the union of $\mathbf{C}_1$ and $\mathbf{C}_2$. In the $\tau_{\mathsf{S}}\tau_{\mathsf{R}}$-plane, $\mathbf{C}_1$ is the shaded area, while $\mathbf{C}_2$ is the the dark line from $(\frac{R}{\log(1+\gamma_{\mathsf{S,D}})}, 0)$ to $(1, 0)$. For the suppressed relay channel, $\boldsymbol{\tau} = (\frac{R}{\log(1+\gamma_{\mathsf{S,D}})}, 0)$ minimizes total time.

which reduces to,

$$\mathbf{C}_1 = \left\{\boldsymbol{\tau} : \frac{R}{\log\left(1+\gamma_{\mathsf{S,D}}\right)} \leq \tau_{\mathsf{S}} \leq 1, \tau_{\mathsf{R}} = 0\right\} \tag{4.25}$$

since,

$$\mathcal{I}_{\mathsf{S,D}}\left(\boldsymbol{\tau}\right) = \tau_{\mathsf{S}} \log\left(1+\gamma_{\mathsf{S,D}}\right). \tag{4.26}$$

On the other hand, $\mathbf{C}_2$ is given by,

$$\mathbf{C}_2 = \{\boldsymbol{\tau} : \mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right) \geq R, \tau_{\mathsf{S}} \geq 0, \tau_{\mathsf{R}} \geq 0, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1\} \tag{4.27}$$

which, in a similar way, reduces to,

$$\mathbf{C}_2 = \left\{\boldsymbol{\tau} : \frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)} \leq \tau_{\mathsf{S}} \leq 1, \tau_{\mathsf{R}} \geq 0, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1\right\} \tag{4.28}$$

because,

$$\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right) = \tau_{\mathsf{S}} \log\left(1+\gamma_{\mathsf{S,R}}\right). \tag{4.29}$$

Both $\mathbf{C}_1$ and $\mathbf{C}_2$ are shown geometrically in Figure 4.4. From Figure 4.4 we can

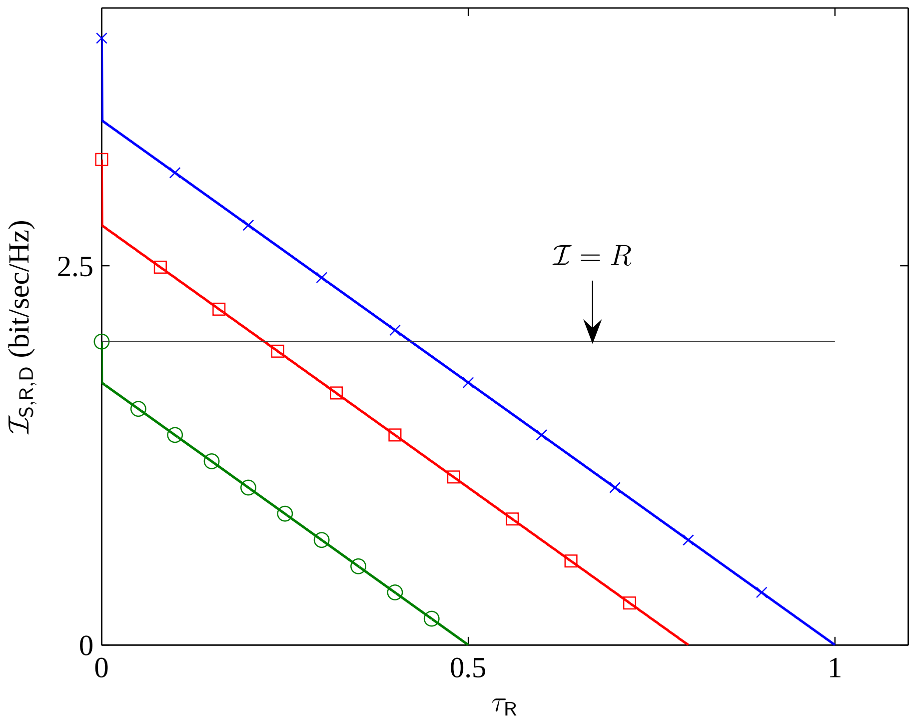


Figure 4.5: Mutual information vs τ_{R} for the suppressed relay channel. For the same channels $\boldsymbol{\gamma} = (10, 15, 11)$, three cases are shown: $c = 1$ (blue), $c = 0.8$ (red) and $c = 0.5$ (green). The minimum transmission time (in this case $c = 0.5$) is achieved by time allocation $\boldsymbol{\tau} = (c, 0)$, where $c = \frac{R}{\log(1+\gamma_{\mathsf{S,D}})}$.

also work out the solution for (4.20) for the suppressed channel as,

$$
\begin{aligned}
c^{\min}\Big|_{\gamma_{\mathsf{S,D}} \geq \gamma_{\mathsf{S,R}}} &= \min_{\boldsymbol{\tau} \in \mathbf{C}} c\,(\boldsymbol{\tau}) \\
&= \min \left[\min_{\boldsymbol{\tau} \in \mathbf{C}_1} \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \,,\, \min_{\boldsymbol{\tau} \in \mathbf{C}_2} \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \right] && (4.30) \\
&= \min \left[\frac{R}{\log(1+\gamma_{\mathsf{S,D}})} \,,\, \frac{R}{\log(1+\gamma_{\mathsf{S,R}})} \right] && (4.31) \\
&= \frac{R}{\log(1+\gamma_{\mathsf{S,D}})}. && (4.32)
\end{aligned}
$$

achievable by time allocation,

$$
\boldsymbol{\tau} = \left(\frac{R}{\log(1+\gamma_{\mathsf{S,D}})}, 0 \right). \tag{4.33}
$$

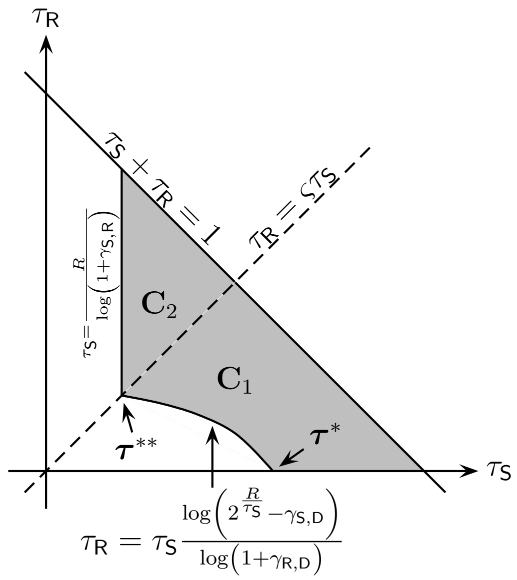


Figure 4.6: In the case of unsuppressed relay channels, feasible time allocations, $\mathbf{C}$ is the union of $\mathbf{C}_1$ and $\mathbf{C}_2$ shown in the $\tau_{\mathsf{S}}\tau_{\mathsf{R}}$-plane. $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ are the candidate time allocations to minimize c.

4.3.2 Minimizing c when Channel is Unsuppressed

Here we have $\gamma_{\mathsf{S,D}} < \gamma_{\mathsf{S,R}}$. For any operation line $\tau_{\mathsf{S}} + \tau_{\mathsf{R}} = c$ there is a conversion point at $\boldsymbol{\mu}(c)$,

$$\begin{cases} \mu_{\mathsf{S}}(c) = c\frac{1}{1+\varsigma} = c\frac{\log\left(1+\gamma_{\mathsf{R,D}}\right)}{\log\left(1+\gamma_{\mathsf{S,R}}-\gamma_{\mathsf{S,D}}\right)+\log\left(1+\gamma_{\mathsf{R,D}}\right)}, \\ \mu_{\mathsf{R}}(c) = c\frac{\varsigma}{1+\varsigma} = c\frac{\log\left(1+\gamma_{\mathsf{S,R}}-\gamma_{\mathsf{S,D}}\right)}{\log\left(1+\gamma_{\mathsf{S,R}}-\gamma_{\mathsf{S,D}}\right)+\log\left(1+\gamma_{\mathsf{R,D}}\right)}. \end{cases} \tag{4.34}$$

$\mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right)$ takes the form,

$$\mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right)\big|_{\gamma_{\mathsf{S,D}}<\gamma_{\mathsf{S,R}}} = \begin{cases} \mathcal{I}_0\left(\boldsymbol{\tau}\right), & \text{if } 0 \le \frac{\tau_{\mathsf{R}}}{\tau_{\mathsf{S}}} < \varsigma, \\ \mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right), & \text{if } \varsigma \le \frac{\tau_{\mathsf{R}}}{\tau_{\mathsf{S}}} \le \infty. \end{cases} \tag{4.35}$$

which illustrates the discontinuity of mutual information at $\boldsymbol{\tau} = \boldsymbol{\mu}$.

The feasible set $\mathbf{C}$ can be written as a union of two sets: $\mathbf{C}_1$ and $\mathbf{C}_2$. Using (4.35) we have,

$$\mathbf{C}_1 = \left\{\boldsymbol{\tau} : \mathcal{I}_0\left(\boldsymbol{\tau}\right) \ge R, \tau_{\mathsf{R}} \ge 0, \tau_{\mathsf{R}} < \varsigma\tau_{\mathsf{S}}, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \le 1\right\}. \tag{4.36}$$

Note that $\mathcal{I}_0\left(\boldsymbol{\tau}\right) \ge R$ means,

$$\tau_{\mathsf{S}} \log\left(\gamma_{\mathsf{S,D}} + \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\tau_{\mathsf{R}}}{\tau_{\mathsf{S}}}}\right) \ge R, \tag{4.37}$$

which yields,

$$\tau_{\mathsf{R}} \ge \tau_{\mathsf{S}} \frac{\log\left(2^{\frac{R}{\tau_{\mathsf{S}}}} - \gamma_{\mathsf{S,D}}\right)}{\log\left(1+\gamma_{\mathsf{R,D}}\right)} \tag{4.38}$$

and that reduces $\mathbf{C}_1$ to,

$$\mathbf{C}_1 = \left\{ \boldsymbol{\tau} : \tau_{\mathsf{R}} \geq \tau_{\mathsf{S}} \frac{\log\left(2^{\frac{R}{\tau_{\mathsf{S}}}} - \gamma_{\mathsf{S,D}}\right)}{\log\left(1+\gamma_{\mathsf{R,D}}\right)}, \tau_{\mathsf{R}} < \varsigma \tau_{\mathsf{S}}, \tau_{\mathsf{R}} \geq 0, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1 \right\}. \tag{4.39}$$

On the other hand, we have,

$$\mathbf{C}_2 = \left\{ \boldsymbol{\tau} : \mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right) \geq R, \tau_{\mathsf{R}} \geq \varsigma \tau_{\mathsf{S}}, \tau_{\mathsf{S}} \geq 0, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1 \right\}. \tag{4.40}$$

Notice that $\mathcal{I}_{\mathsf{S,R}}\left(\boldsymbol{\tau}\right) \geq R$ if,

$$\tau_{\mathsf{S}} \log\left(1 + \gamma_{\mathsf{S,R}}\right) \geq R, \tag{4.41}$$

which yields,

$$\tau_{\mathsf{S}} \geq \frac{R}{\log\left(1 + \gamma_{\mathsf{S,R}}\right)}, \tag{4.42}$$

and that reduces $\mathbf{C}_2$ to,

$$\mathbf{C}_2 = \left\{ \boldsymbol{\tau} : \tau_{\mathsf{S}} \geq \frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}, \tau_{\mathsf{R}} \geq \varsigma \tau_{\mathsf{S}}, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} \leq 1 \right\}. \tag{4.43}$$

$\mathbf{C}_1$ and $\mathbf{C}_2$ are shown in Figure 4.6. Figure 4.6 also shows two candidate time allocations to minimize c. The first time allocation $\boldsymbol{\tau}^*$ is,

$$\begin{cases} \tau_{\mathsf{S}}^* &= \frac{R}{\log\left(1+\gamma_{\mathsf{S,D}}\right)}, \\ \tau_{\mathsf{R}}^* &= 0. \end{cases} \tag{4.44}$$

obtained by solving $\tau_{\mathsf{R}} = \tau_{\mathsf{S}} \frac{\log\left(2^{\frac{R}{\tau_{\mathsf{S}}}} - \gamma_{\mathsf{S,D}}\right)}{\log\left(1+\gamma_{\mathsf{R,D}}\right)}$ and $\tau_{\mathsf{R}} = 0$. $\boldsymbol{\tau}^*$ corresponds to a minimal c,

$$\begin{aligned} c_1^{\min} &= \tau_{\mathsf{S}}^* + \tau_{\mathsf{R}}^* \\ &= \frac{R}{\log\left(1 + \gamma_{\mathsf{S,D}}\right)}. \end{aligned} \tag{4.45}$$

The other time allocation $\boldsymbol{\tau}^{**}$ is such that,

$$\begin{cases} \tau_{\mathsf{S}}^{**} &= \frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}, \\ \tau_{\mathsf{R}}^{**} &= \varsigma \frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}. \end{cases} \tag{4.46}$$

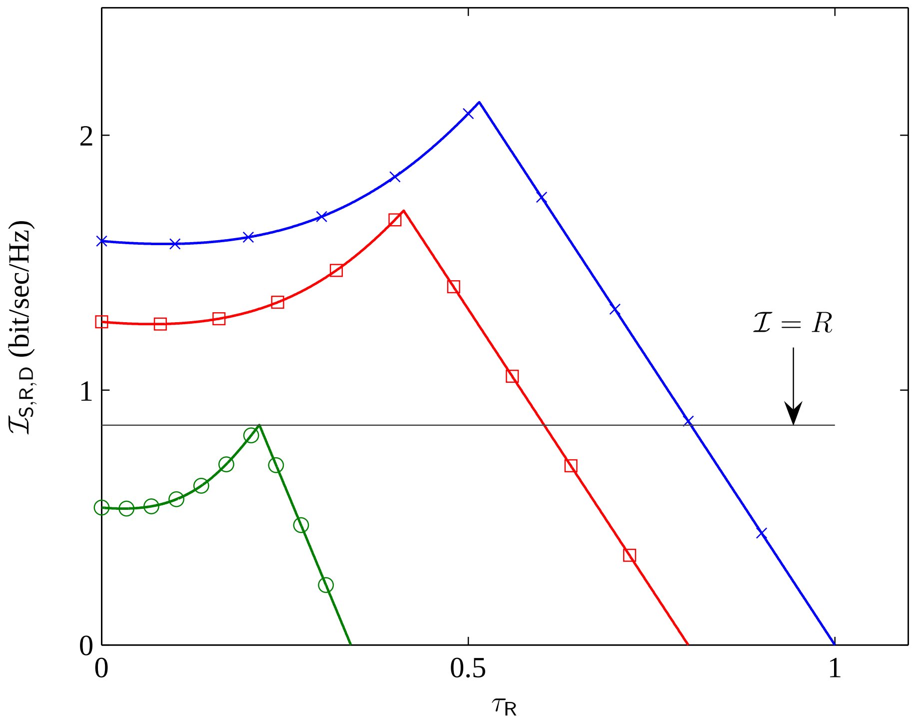


Figure 4.7: Mutual information vs τ_{R} for the suppressed relay channel. For the same channels $\boldsymbol{\gamma} = (20, 2, 15)$, three cases are shown: $c = 1$ (blue), $c = 0.8$ (red) and $c = 0.34$ (green). The minimum transmission time, $c = 0.34$, is achieved for this particular case by time allocation $\boldsymbol{\mu}(0.34)$.

obtained by solving $\tau_{\mathsf{R}} = \varsigma \tau_{\mathsf{S}}$ and $\tau_{\mathsf{S}} = \frac{R}{\log(1+\gamma_{\mathsf{S,R}})}$. $\boldsymbol{\tau}^{**}$ corresponds to a minimal c,

$$\begin{aligned} c_2^{\min} &= \tau_{\mathsf{S}}^{**} + \tau_{\mathsf{R}}^{**} \\ &= \frac{R(1+\varsigma)}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}. \end{aligned} \tag{4.47}$$

$c_2^{\min} < c_1^{\min}$ if,

$$\frac{R(1+\varsigma)}{\log\left(1+\gamma_{\mathsf{S,R}}\right)} < \frac{R}{\log\left(1+\gamma_{\mathsf{S,D}}\right)} \tag{4.48}$$

or

$$\begin{aligned} \gamma_{\mathsf{S,D}} &< \left(1+\gamma_{\mathsf{S,R}}\right)^{\frac{1}{1+\varsigma}} - 1 \\ &= \left(1+\gamma_{\mathsf{S,R}}\right)^{\mu_{\mathsf{S}}} - 1. \end{aligned} \tag{4.49}$$

This condition verifies the optimality of direct transmission for an unsuppressed channel. It is the same as that of the maximization problem. We conclude that for an unsuppressed channel, minimum transmission time is given by,

$$c^{\min}\Big|_{\gamma_{\mathsf{S,D}}<\gamma_{\mathsf{S,R}}} = \begin{cases} \frac{R(1+\varsigma)}{\log\left(1+\gamma_{\mathsf{S,R}}\right)} & \text{if } \gamma_{\mathsf{S,D}} < (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1 \\ \frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)} & \text{if } \gamma_{\mathsf{S,D}} \geq (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1 \text{ or } \varsigma \text{ undefined} \end{cases} \tag{4.50}$$

using optimal time allocation,

$$\boldsymbol{\tau} = \begin{cases} \left(\frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}, 0\right) & \text{if } \gamma_{\mathsf{S,D}} < (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1 \\ \left(\frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}, \varsigma\frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}\right) & \text{if } \gamma_{\mathsf{S,D}} \geq (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1 \text{ or } \varsigma \text{ undefined} \end{cases} \tag{4.51}$$

4.3.3 Minimum c

The following theorem concludes the minimization problem.

Theorem 4.2 (Minimizing Total Transmission Time)**.** *For the three-node D&F relay channel, if the rate to be achieved, R, is less than the maximum achievable mutual information, $\mathcal{I}^{\max}_{\mathsf{S,R,D}}$; ς exists and the channel's conditions satisfy,*

$$\gamma_{\mathsf{S,D}} < (1+\gamma_{\mathsf{S,R}})^{\mu_{\mathsf{S}}} - 1. \tag{4.52}$$

then relaying can achieve R using minimum transmission time,

$$c^{\min} = \frac{R(1+\varsigma)}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}, \tag{4.53}$$

using time allocation $\boldsymbol{\tau}^{**}$ given by,

$$\begin{cases} \tau^{**}_{\mathsf{S}} &= \frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}, \\ \tau^{**}_{\mathsf{R}} &= \varsigma\frac{R}{\log\left(1+\gamma_{\mathsf{S,R}}\right)}. \end{cases} \tag{4.54}$$

If $R < \mathcal{I}^{\max}_{\mathsf{S,R,D}}$; but ς does not exist or condition (4.52) *is not satisfied, then relaying can achieve R using minimum transmission time,*

$$c^{\min} = \frac{R}{\log\left(1+\gamma_{\mathsf{S,D}}\right)}, \tag{4.55}$$

using time allocation $\boldsymbol{\tau}^{*}$ given by,

$$\begin{cases} \tau^{*}_{\mathsf{S}} &= \frac{R}{\log\left(1+\gamma_{\mathsf{S,D}}\right)}, \\ \tau^{*}_{\mathsf{R}} &= 0. \end{cases} \tag{4.56}$$

,

If, however, $R > \mathcal{I}^{\max}_{\mathsf{S,R,D}}$, total transmission time can not be minimized.

4.4 Duality Between Optimization Problems

Optimization problems studied in the previous sections have shown similarities in different aspects. In this section we take a closer look at these similarities to draw some formal conclusions. It will be shown that there is duality between mutual information maximization and total time minimization problems.

4.4.1 General Maximization Problem

To make a connection between optimization problems studied earlier, we consider another problem general to the maximization problem of Section 4.2. We seek the best time allocation policy for a relay channel operating on $\tau_{\mathsf{S}} + \tau_{\mathsf{R}} = c$, for any $c \in [0, 1]$, so that mutual information is maximum. That is,

$$\mathbb{P} \mapsto \begin{cases} \max\limits_{\boldsymbol{\tau}} & \mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right) \\ \text{s.t.} & \tau_{\mathsf{S}}, \tau_{\mathsf{R}} \geq 0, \\ & \tau_{\mathsf{S}} + \tau_{\mathsf{R}} = c. \end{cases} \tag{4.57}$$

Optimization problem studied in Section 4.2 is a special case where $c = 1$. The feasible set is given by,

$$\mathbf{C} = \{\boldsymbol{\tau} : \tau_{\mathsf{S}} \geq 0, \tau_{\mathsf{R}} \geq 0, \tau_{\mathsf{S}} + \tau_{\mathsf{R}} = c\} \tag{4.58}$$

Since the solution is a function of c, we have $\mathcal{I}^{\max}_{\mathsf{S,R,D}}(c)$ to denote the optimal mutual information,

$$\mathcal{I}^{\max}_{\mathsf{S,R,D}}(c) = \sup_{\boldsymbol{\tau} \in \mathbf{C}} \mathcal{I}_{\mathsf{S,R,D}}\left(\boldsymbol{\tau}\right). \tag{4.59}$$

Just as before, we have,

$$\tau_{\mathsf{S}} = c - \tau_{\mathsf{R}}, \tag{4.60}$$

and (4.7) changes to,

$$\mathcal{I}_{\mathsf{S,R,D}}(\tau_{\mathsf{R}}) = \begin{cases} \mathcal{I}_{\mathsf{S,D}}(\tau_{\mathsf{R}}), & \text{if } \tau_{\mathsf{R}} = 0, \\ \min\{\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}}), \mathcal{I}_0(\tau_{\mathsf{R}})\}, & \text{if } 0 < \tau_{\mathsf{R}} < c. \end{cases} \tag{4.61}$$

where, $\mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}})$, $\mathcal{I}_0(\tau_{\mathsf{R}})$ and $\mathcal{I}_{\mathsf{S,D}}(\tau_{\mathsf{R}})$ are given by,

$$\begin{cases} \mathcal{I}_{\mathsf{S,R}}(\tau_{\mathsf{R}}) = (c - \tau_{\mathsf{R}}) \log\left(1 + \gamma_{\mathsf{S,R}}\right), \\ \mathcal{I}_0(\tau_{\mathsf{R}}) = (c - \tau_{\mathsf{R}}) \log\left(\gamma_{\mathsf{S,D}} + [1 + \gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{c - \tau_{\mathsf{R}}}}\right), \\ \mathcal{I}_{\mathsf{S,D}} = (c - \tau_{\mathsf{R}}) \log\left(1 + \gamma_{\mathsf{S,D}}\right). \end{cases} \tag{4.62}$$

	Maximizing mutual information		Minimizing total time	
	$\gamma_{S,D} < \left[1+\gamma_{S,R}\right]^{\mu_S} - 1$	$\gamma_{S,D} \geq \left[1+\gamma_{S,R}\right]^{\mu_S} - 1$	$\gamma_{S,D} < \left[1+\gamma_{S,R}\right]^{\mu_S} - 1$	$\gamma_{S,D} \geq \left[1+\gamma_{S,R}\right]^{\mu_S} - 1$
Optimal time allocation	$\tau_S = c\frac{1}{1+\varsigma}$, $\tau_R = c\frac{\varsigma}{1+\varsigma}$.	$\tau_S = c$, $\tau_R = 0$.	$\tau_S = \frac{R}{\log\left(1+\gamma_{S,R}\right)}$, $\tau_R = \varsigma\frac{R}{\log\left(1+\gamma_{S,R}\right)}$.	$\tau_S = \frac{R}{\log\left(1+\gamma_{S,D}\right)}$, $\tau_R = 0$.
Total time	c	c	$c^{\min}(R) = \frac{R(1+\varsigma)}{\log\left(1+\gamma_{S,R}\right)}$	$c^{\min}(R) = \frac{R}{\log\left(1+\gamma_{S,D}\right)}$
Mutual information	$\mathcal{I}^{\max}_{S,R,D}(c) = c\frac{1}{1+\varsigma}\log\left(1+\gamma_{S,R}\right)$	$\mathcal{I}^{\max}_{S,R,D}(c) = c\log\left(1+\gamma_{S,D}\right)$	R	R

Table 4.1: Comparison between optimization problems.

The problem is thus reduced to

$$\mathbb{P}: \max_{0 \leq \tau_R < c} \mathcal{I}_{S,R,D}\left(\tau_R\right). \tag{4.63}$$

a solution to which is,

$$\mathcal{I}^{\max}_{S,R,D} = \begin{cases} \mathcal{I}_{S,R}\left(\mu_R(c)\right) & \text{if } \gamma_{S,D} < (1+\gamma_{S,R})^{\mu_S} - 1 \\ \mathcal{I}_{S,D}\left(0\right) & \text{if } \gamma_{S,D} \geq (1+\gamma_{S,R})^{\mu_S} - 1 \text{ or } \varsigma \text{ undefined} \end{cases}.$$

obtainable by following steps similar to those used in previous sections. The optimum time allocation is,

$$\boldsymbol{\tau} = \begin{cases} \left(c\frac{1}{1+\varsigma}, c\frac{\varsigma}{1+\varsigma}\right) & \text{if } \gamma_{S,D} < (1+\gamma_{S,R})^{\mu_S} - 1 \\ \left(c, 0\right) & \text{if } \gamma_{S,D} \geq (1+\gamma_{S,R})^{\mu_S} - 1 \text{ or } \varsigma \text{ undefined} \end{cases}.$$

Proposition 4.2 (Maximizing mutual information on a given line of operation)**.** *For the three-node D&F relay channel operating on $\tau_S + \tau_R = c$, if ς exists and if the channel's conditions satisfy,*

$$\gamma_{S,D} < (1+\gamma_{S,R})^{\mu_S} - 1. \tag{4.64}$$

relaying can achieve a maximum mutual information of

$$\mathcal{I}^{\max}_{S,R,D}(c) = c\frac{1}{1+\varsigma}\log\left(1+\gamma_{S,R}\right), \tag{4.65}$$

using time allocation $\boldsymbol{\tau} = (c\frac{1}{1+\varsigma}, c\frac{\varsigma}{1+\varsigma})$.

Otherwise time allocation $(c, 0)$ is optimum achieving maximum mutual information,

$$\mathcal{I}^{\max}_{S,R,D}(c) = c\log\left(1+\gamma_{S,D}\right), \tag{4.66}$$

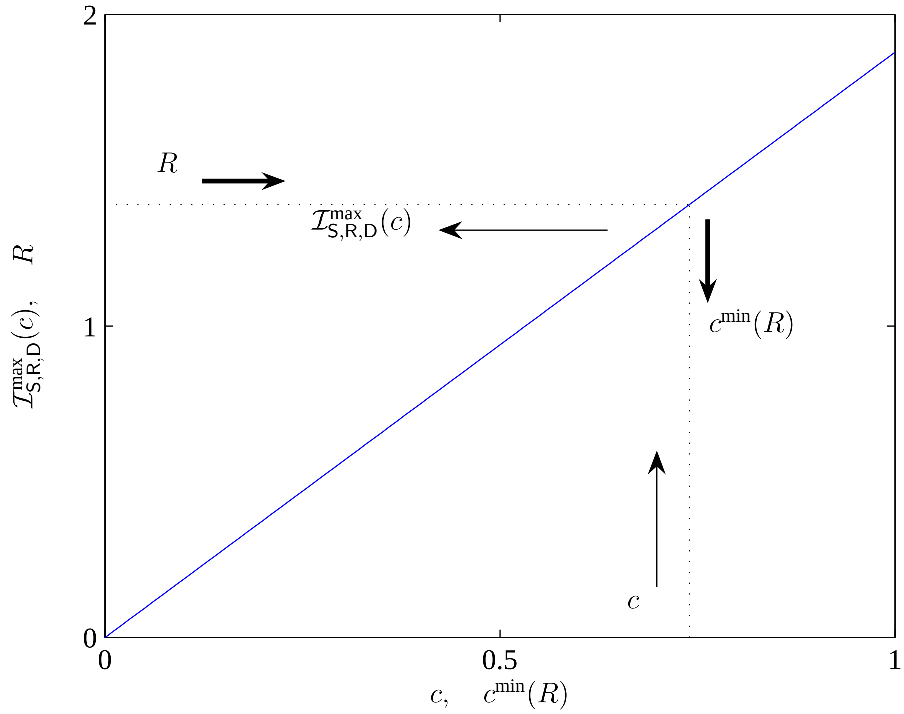


Figure 4.8: $\mathcal{I}^{\max}_{\mathsf{S,R,D}}$ versus c and R versus $c^{\min}$ are identical curves due to the duality between total time minimization problem and mutual information maximization problem for fixed total time. One solution implies the other. c can be used to find $\mathcal{I}^{\max}_{\mathsf{S,R,D}}$ and R can be used to find $c^{\min}$ from the same curve. Plot generated using $\gamma_{\mathsf{S,R}} = 15$, $\gamma_{\mathsf{s,D}} = 1$ and $\gamma_{\mathsf{R,D}} = 10$.

4.4.2 Duality

Table 4.1 makes a comparison between solutions to the general optimization problems. It compares results summarized in Theorem 4.2 with those in Proposition 4.2. Firstly, it shows that there is the same direct link optimality condition, $\gamma_{\mathsf{S,D}} < [1 + \gamma_{\mathsf{S,R}}]^{\mu S}$. Further, a little manipulation proves that optimal time allocation is typical for both problems. Finally, curve specified by $\mathcal{I}^{\max}_{\mathsf{S,R,D}}(c)$ is identical to that specified by $c^{\min}(R)$. This is easy to see by solving the former for c or the latter for R. Both quantities are plotted as a single curve in Figure 4.8.

We conclude that time minimization problem and mutual information maximization problem for fixed total time are dual problems. The solution to either problem is the solution to the other. Figure 4.8 explains how a single curve can be used to find a solution to either problem. An absolute maximum for mutual information is obtained by allocating the maximum possible total time, that is $c = 1$; while an absolute minimum total time is obtained when R is minimum, that is $R = 0$. This duality also formalizes the trade-off between total transmission time and mutual information. A similar trade-off exists between reliability and power as established in [53].

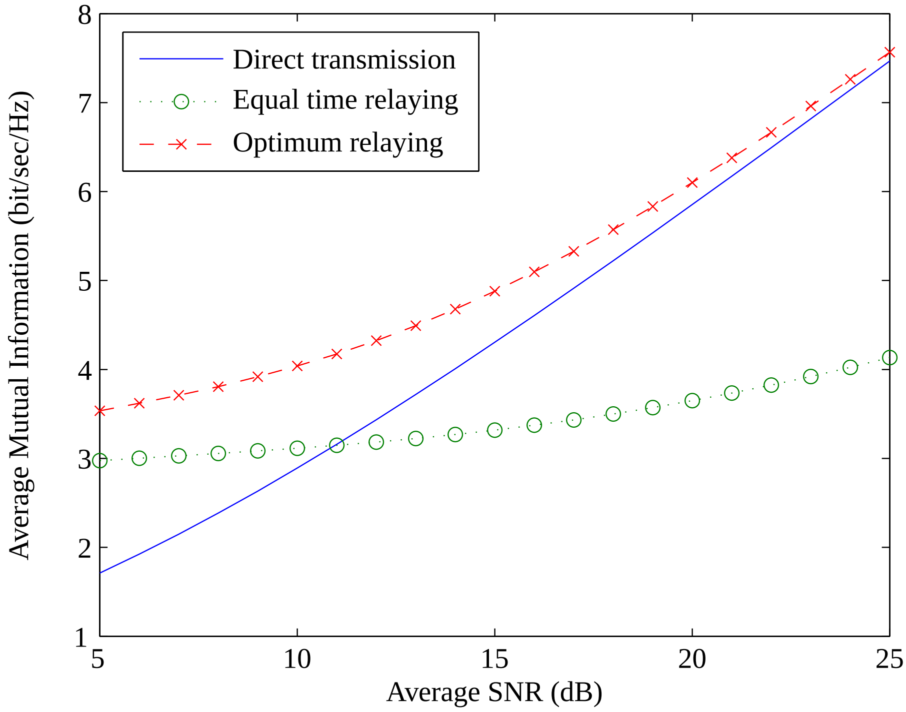


Figure 4.9: Performance comparison and demonstration of possible gain from relaying with optimum time allocation. Comparison is made with direct transmission and equal time relaying. To generate these results it is assumed that all channels are Rayleigh fading channels. Relay transmission power is fixed such that $E[\gamma_{\mathsf{R,D}}] = 20$ dB. Source transmission power is increasing to have $E[\gamma_{\mathsf{S,D}}]$ to vary from 5 dB to 25 dB and $E[\gamma_{\mathsf{S,R}}]$ from 30 dB to 50 dB.

4.5 Maximizing Average Mutual Information and Minimizing Outage Probability in Fading Channels

In fading channels if channel information is available for all nodes, then transmitters can allocate their time optimally each time channels change. The resultant average mutual information is the expected value of the maximum mutual information averaged over all possible channel conditions, i.e.,

$$\bar{\mathcal{I}}^{\max}_{\mathsf{S,R,D}} = \iiint_0^\infty \mathcal{I}^{(\max)}_{\mathsf{S,R,D}}(\boldsymbol{\gamma})\mathbf{f}_{\boldsymbol{\gamma}}(\mathbf{x})\mathrm{d}\mathbf{x} \tag{4.67}$$

where, $\mathbf{f}_{\boldsymbol{\gamma}}$ is the joint distribution for $\gamma_{\mathsf{S,D}}$, $\gamma_{\mathsf{S,R}}$ and $\gamma_{\mathsf{R,D}}$; and $\mathbf{x} = (x_1, x_2, x_3)$.

Figure 4.9 compares the performance of the relay channel with optimum time allocation with that of the direct transmission and relaying with equal time allocation for a given setup. Source transmission power increases such that both $\gamma_{\mathsf{S,D}}$ and $\gamma_{\mathsf{S,R}}$ increase from 5-25 dB and 30-50 dB, respectively. At the lowest source transmission power, both relaying techniques are superior to direct transmission. As the source's transmission power increases, equal time allocation relaying achieves insignificant gain. Not only that, but its performance becomes much worse than direct transmission as $\gamma_{\mathsf{S,D}}$ exceeded 11 dB.

Optimum relaying, on the other hand, keeps its superiority. At high SNR, direct transmission performance almost matches that of the optimum relaying. This indicates that as the source-destination channel improves, more time is allocated to the source. That also explains the poor performance of equal time allocation.

Figure 4.10 explains how simulation results for Figure 4.9 are generated. Program is initiated by assigning values to $E[\gamma_{\mathsf{S,D}}]$, $E[\gamma_{\mathsf{S,R}}]$ and $E[\gamma_{\mathsf{R,D}}]$, which are the expected value for channels' SNRs. These expected values are then used to generate random channel realizations based on Exponential distribution. Next, generated channels' realizations are used to calculate mutual information between the source and the relay first using optimum relaying then equal time allocation relaying and finally direct transmission. These quantities are denoted in Figure 4.10 as $\mathcal{I}^{\max}_{\mathsf{S,R,D}}$, $\mathcal{I}^{\mathrm{EQ}}_{\mathsf{S,R,D}}$ and $\mathcal{I}_{\mathsf{S,D}}$, respectively. They are calculated repeatedly as $\gamma_{\mathsf{S,D}}$ and $\gamma_{\mathsf{S,R}}$ increases by a factor $\Delta\gamma_{\mathsf{S,D}}$ and $\Delta\gamma_{\mathsf{S,R}}$, respectively. This change in $\gamma_{\mathsf{S,D}}$ and $\gamma_{\mathsf{S,R}}$ matches the change in the source's transmission power. $\mathcal{I}^{\max}_{\mathsf{S,R,D}}$, $\mathcal{I}^{\mathrm{EQ}}_{\mathsf{S,R,D}}$ and $\mathcal{I}_{\mathsf{S,D}}$ are array variables of length l each. $\Delta\gamma_{\mathsf{S,D}}$ and $\Delta\gamma_{\mathsf{S,R}}$ are functions of the maximum and minimum transmission power and l. The larger l, the smoother the resulted curve, although at a cost of extra memory and longer processing time. The process of generating random channels' realizations and calculating mutual information for the whole range of the source's transmission power is repeated and summed up m times, $m \gg 1$. The sum is then averaged over m. Finally results are plotted as in Figure 4.9.

Outage probability can similarly be minimized when channel information is available by allocating time so that mutual information is greater than or equal to the required rate, R. In that case, outage occurs only when the maximum mutual information is less than R. That is formulated as,

$$\mathcal{P}^{(\min)}(R) = \Pr\left\{\mathcal{I}^{(\max)}_{\mathsf{S,R,D}}(\boldsymbol{\tau}) < R\right\}$$

When, however, only statistical channels information is available at transmitters, the data transmission rate is fixed as well as time allocation. Outage probability as a function of time allocation is given by (3.72) obtained in Chapter 3. Since no closed-form is obtained, $\mathcal{P}$ can only be dealt with numerically. Matlab code is developed in order to generate numerical results for minimizing $\mathcal{P}$. Results in Figure 4.11 compare outage probability performance for optimum time allocation, equal time allocation and direct transmission for a wireless three-node channel. Three cases are considered. In Figure 4.11-a broadcast channels ($\gamma_{\mathsf{S,D}}$ and $\gamma_{\mathsf{S,R}}$) are the same and the relay-destination channel is relatively weak. The optimum allocation policy achieves superiority over the other two with a small margin. In the second scenario, Figure 4.11-b, the source-

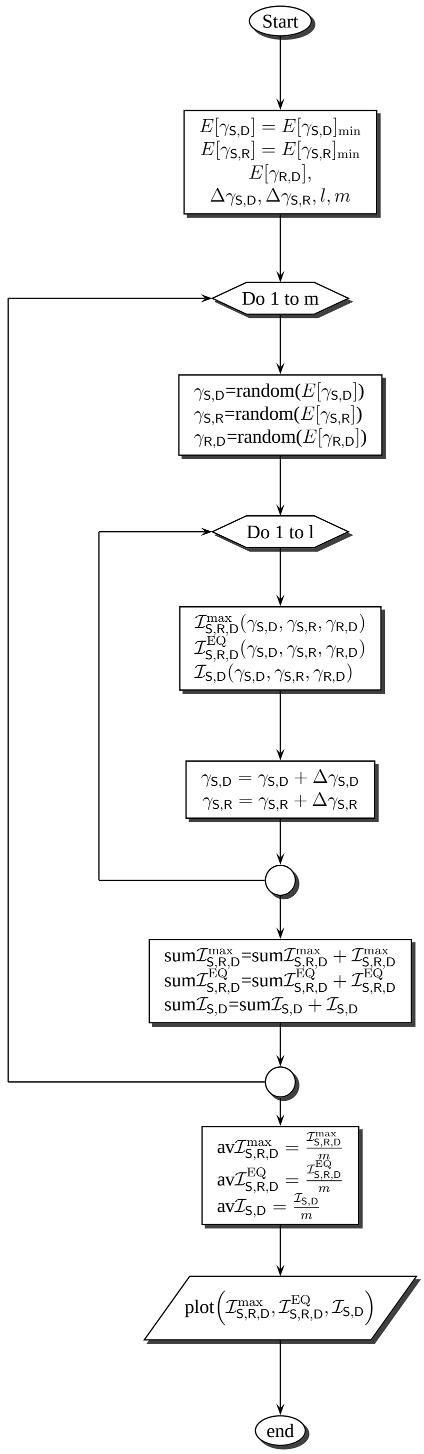


Figure 4.10: Flowchart diagram demonstrating the generation of the simulation results in Figure 4.9.

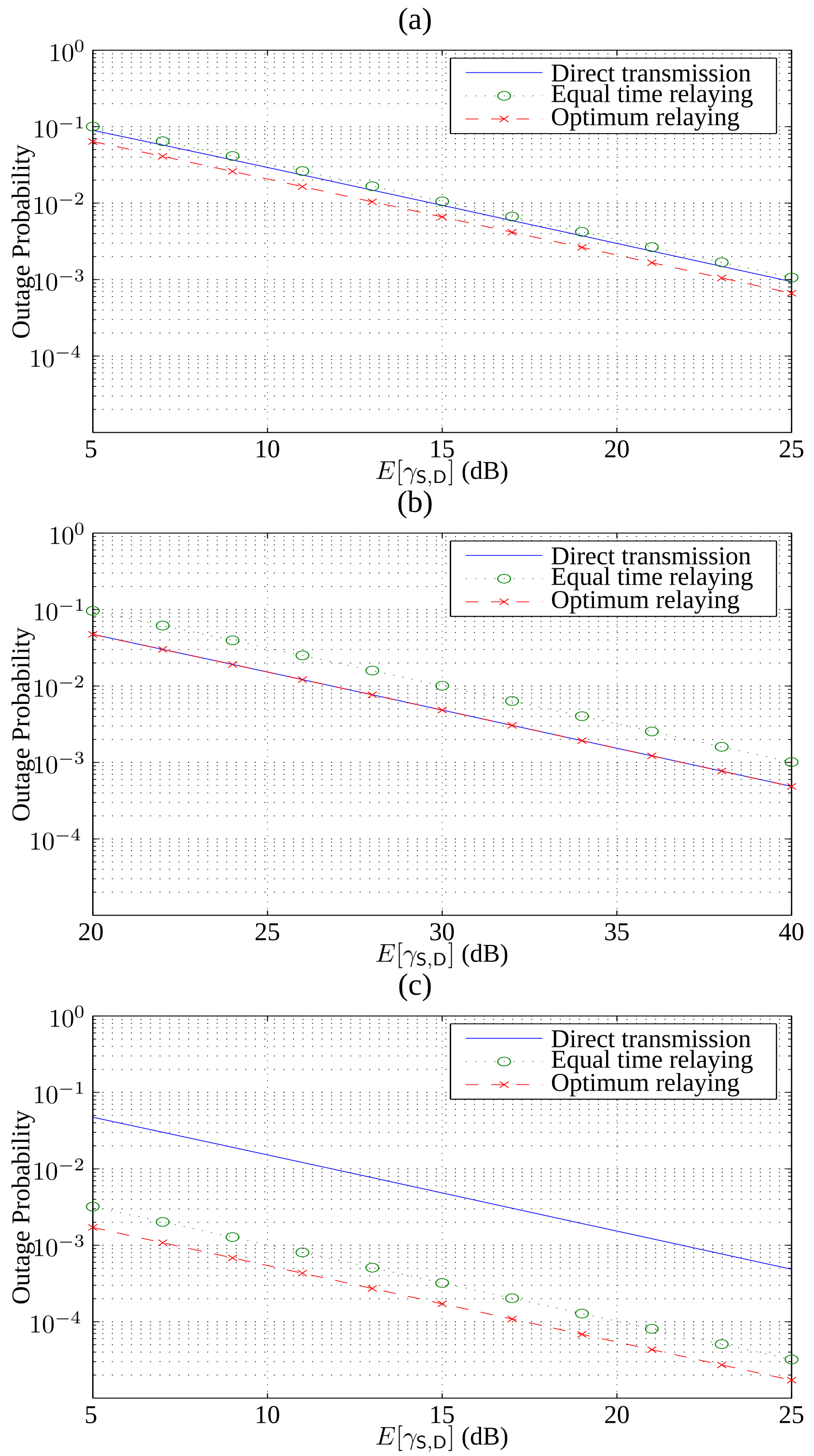


Figure 4.11: Comparing outage probability performance of different transmission techniques. We have $R = 0.2$. The source transmission power is increased while relay transmission power is fixed. (a) $E[\gamma_{\mathsf{S,R}}] = 5 - 25$ dB and $E[\gamma_{\mathsf{R,D}}] = 5$ dB, (b) $E[\gamma_{\mathsf{S,R}}] = 5 - 25$ dB and $E[\gamma_{\mathsf{R,D}}] = 5$ dB, and (c) $E[\gamma_{\mathsf{S,R}}] = 20 - 45$ dB and $E[\gamma_{\mathsf{R,D}}] = 30$ dB.

destination channel is improved by 20 dB while other channels are not changed. As a result, direct transmission becomes optimal. In Figure 4.11-c, the source-destination channel is similar to that in Figure 4.11-a, while the source-relay channel and the relay-destination channel improved by 15 dB and 25 dB, respectively. Consequently, both optimal time allocation and equal time allocation outbeat direct transmission with a significant gain. Optimal time allocation, nevertheless, remains superior over equal time allocation policy.

4.6 Chapter Summary

In this chapter we studied some of the optimization problems for the relay channel. Solutions are worked out for mutual information maximization problem and total transmission time minimization problem. Interestingly, an important duality is established between optimization problems. Results obtained in this chapter are useful for partner selection in cooperative networks addressed in Chapter 5 and multi-hop relay channels studied in Chapter 6.

Chapter 5

User-Cooperative Networks

In this chapter we propose a user-cooperative model based on the three-node D&F relay channel. The aim is to give a practical demonstration on how relay channels form the basis for user cooperation. Cooperative techniques are particularly important for networks with a reduced or no infrastructure. More attention is given to partner selection, as it is crucial for the success of cooperation schemes. Useful user and useful partner are defined and associated conditions based on the model assumed are derived. Simulation results are generated which gives some insight into understanding the performance of a cooperative network and how it is affected by different network parameters. To a lesser extent, this chapter also brings to attention some other issues associated with user-cooperative schemes such as fairness between users, resource allocation, cross-layer issues, network simulation issues and added complexity. To study and understand the cooperative model proposed in this chapter, we rely on the three-node relay channel studied in Chapter 3. Results obtained in Chapter 4 are also useful.

In the next section an introduction is given. In Section 5.2 a two-user cooperative model is proposed. Section 5.3 considers the application of the two-user model to a multi-user ad hoc network. Partner selection occupies most of Section 5.3, followed by network simulation in the same section. Finally, Section 5.4 summarizes the chapter and gives some concluding remarks.

5.1 Introduction

Modern wireless networks aim to maximize users' freedom. There are, however, many technical challenges. Taking the example of ad hoc networks, there could be multiple sources and multiple destinations. Geographical distribution of nodes is random. No centralized control is available, so users have to take decisions. Without infrastructure, connectivity between users depends on innovating efficient routing and user-cooperative protocols.

Cooperative communication is a promising technique to improve the quality of service for ad hoc networks. Users share resources for higher throughput and more re-

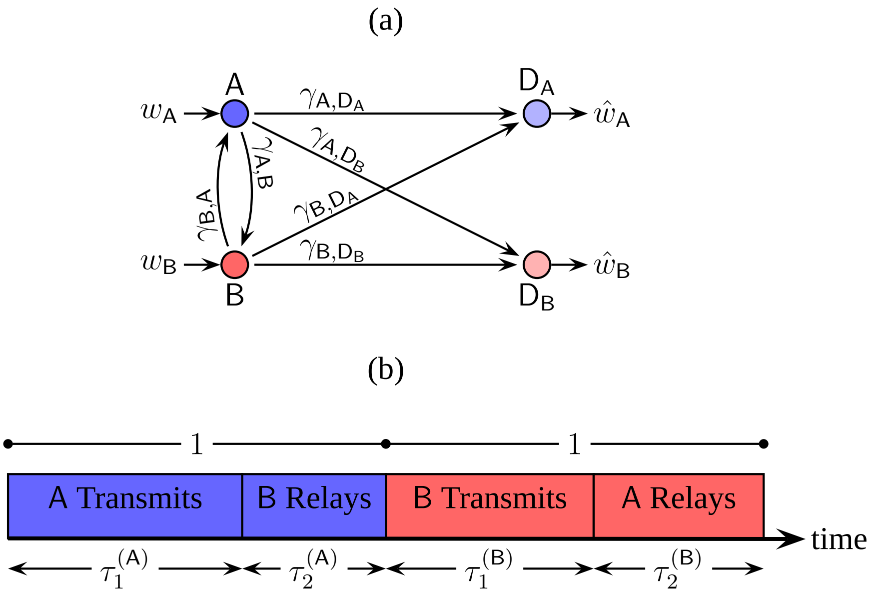


Figure 5.1: (a) A prototype for a two-user cooperative network and (b) the associated time allocation.

liable connectivity. The variety of hardware and applications results in different types of networks. In addition, users can be arranged in clusters of different sizes. An individual user could be a member of more than one cluster, which means an infinite number of cooperative strategies for a single network. These strategies vary in their complexity and the outcome.

A cooperative communication policy could benefit individual users as well as the network. There are many reasons a user may want to cooperate, e.g., to increase throughput, improve reliability or save resources. Cooperative communication can also improve the overall performance of the network, improve efficiency of resources' usage and improve fairness among users.

We learned from Chapter 3 that relaying could be advantageous only when the suitable relay exists and the right time allocation is chosen. As so, in a cooperative network any partnership selection must be made under certain conditions determined by network and individual users. These conditions should reflect the aim from cooperation, constraints on users due to hardware and resources limitations and the nature of the application running on the network.

5.2 A Two-User Cooperative Model

We start by considering a model for user clustering and focus on a type of networks where there is no dedicated relays. Users cooperate by relaying each other's message. Specifically, they form D&F relay channels in which they exchange source and relay roles. In what follows, the word *user* is used to mean a source node. Only clusters of size two are considered. That is, no more than two users are allowed to cooperate. We assume that users can transmit as well as receive. Nevertheless, they are half-duplex constraint. Destinations, on the other hand, can only receive.

In addition to the limitation on cluster size, two other assumptions are made solely for the sake of reducing complexity. First, cooperation has to be reciprocal. Each user is allowed only to help another user who helped him/her. Moreover, only symmetric cooperation is allowed. Cooperating partners allocate their resources (power and degree of freedom) similarly. Relaxation of these could improve the outcome from cooperation with some added complexity.

Figure 5.1 shows a prototype for a four-node cooperative network. There are two users, User A and User B; and two destinations, D_A and D_B. User A wants to send a message w_A to node D_A. Likewise, User B has got a message w_B to be sent to node D_B. User A and User B cooperate to send their messages by forming two relay channels $(\mathsf{A}, \mathsf{B}, \mathsf{D}_\mathsf{A})$ and $(\mathsf{B}, \mathsf{A}, \mathsf{D}_\mathsf{B})$. Subsequently, each user divide its available time into two parts. User A uses the first part of its available time, $\tau_1^{(\mathsf{A})}$, to transmit w_A. The second part, $\tau_2^{(\mathsf{A})}$, is allocated for User B to repeat w_A upon successful decoding at the end the first part. Time available for User B is allocated in the same way.

There are two sets of channels, $\boldsymbol{\gamma}^{(\mathsf{A})} \triangleq (\gamma_{\mathsf{A},\mathsf{D}_\mathsf{A}}, \gamma_{\mathsf{A},\mathsf{B}}, \gamma_{\mathsf{B},\mathsf{D}_\mathsf{A}})$ and $\boldsymbol{\gamma}^{(\mathsf{B})} \triangleq (\gamma_{\mathsf{B},\mathsf{D}_\mathsf{B}}, \gamma_{\mathsf{B},\mathsf{A}}, \gamma_{\mathsf{A},\mathsf{D}_\mathsf{B}})$ associated with $(\mathsf{A}, \mathsf{B}, \mathsf{D}_\mathsf{A})$ and $(\mathsf{B}, \mathsf{A}, \mathsf{D}_\mathsf{A})$, respectively. We assume that CSIT is available for users. Mutual information achievable by each relay channel is exactly the same as that for the adaptive D&F relay channel studied in Chapter 4. Therefore for channel $(\mathsf{A}, \mathsf{B}, \mathsf{D}_\mathsf{A})$,

$$\mathcal{I}_{\mathsf{A},\mathsf{B},\mathsf{D}_\mathsf{A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) = \begin{cases} \mathcal{I}_{\mathsf{A},\mathsf{D}_\mathsf{A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right), & \text{if } \tau_2^{(\mathsf{A})} = 0, \\ \min\{\mathcal{I}_{\mathsf{A},\mathsf{B}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right), \mathcal{I}_0^{(\mathsf{A})}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right)\}, & \text{if } 0 < \tau_2^{(\mathsf{A})} \le 1, \end{cases} \tag{5.1}$$

where

$$\begin{cases} \mathcal{I}_{\mathsf{A},\mathsf{D}_\mathsf{A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) = \tau_1^{(\mathsf{A})} \log\left(1 + \gamma_{\mathsf{A},\mathsf{D}_\mathsf{A}}\right), \\ \mathcal{I}_{\mathsf{A},\mathsf{B}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) = \tau_1^{(\mathsf{A})} \log\left(1 + \gamma_{\mathsf{A},\mathsf{B}}\right), \\ \mathcal{I}_0^{(\mathsf{A})}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) = \tau_1^{(\mathsf{A})} \log\left(\gamma_{\mathsf{A},\mathsf{D}_\mathsf{A}} + [1 + \gamma_{\mathsf{B},\mathsf{D}_\mathsf{A}}]^{\frac{\tau_2^{(\mathsf{A})}}{\tau_1^{(\mathsf{A})}}}\right). \end{cases} \tag{5.2}$$

Similarly for channel $(\mathsf{B}, \mathsf{A}, \mathsf{D}_\mathsf{B})$,

$$\mathcal{I}_{\mathsf{B},\mathsf{A},\mathsf{D}_\mathsf{B}}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right) = \begin{cases} \mathcal{I}_{\mathsf{B},\mathsf{D}_\mathsf{B}}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right), & \text{if } \tau_2^{(\mathsf{B})} = 0, \\ \min\{\mathcal{I}_{\mathsf{B},\mathsf{A}}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right), \mathcal{I}_0^{(\mathsf{B})}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right)\}, & \text{if } 0 < \tau_2^{(\mathsf{B})} \le 1, \end{cases} \tag{5.3}$$

where

$$\begin{cases} \mathcal{I}_{\mathsf{B,D_B}}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right) = \tau_1^{(\mathsf{B})} \log\left(1+\gamma_{\mathsf{B,D_B}}\right), \\ \mathcal{I}_{\mathsf{B,A}}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right) = \tau_1^{(\mathsf{B})} \log\left(1+\gamma_{\mathsf{B,A}}\right), \\ \mathcal{I}_0^{(\mathsf{B})}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right) = \tau_1^{(\mathsf{B})} \log\Big(\gamma_{\mathsf{B,D_B}} + \left[1+\gamma_{\mathsf{A,D_B}}\right]^{\frac{\tau_2^{(\mathsf{B})}}{\tau_1^{(\mathsf{B})}}}\Big). \end{cases} \tag{5.4}$$

Note in (5.1) and (5.3) that rate is normalizes by the number of degrees of freedom available to each user. As before, a naive power policy is adopted. Transmission power remains fixed regardless of time allocation. The sum rate achievable by both users is the average of their individual rates,

$$\mathcal{I}_{\mathsf{A,B;D_A,D_B}}(\boldsymbol{\tau}^{(\mathsf{A})},\boldsymbol{\tau}^{(\mathsf{B})}) = \frac{(\tau_1^{(\mathsf{A})}+\tau_2^{(\mathsf{A})})\mathcal{I}_{\mathsf{A,B,D_A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) + (\tau_1^{(\mathsf{B})}+\tau_2^{(\mathsf{B})})\mathcal{I}_{\mathsf{B,A,D_B}}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right)}{\tau_1^{(\mathsf{A})}+\tau_2^{(\mathsf{A})}+\tau_1^{(\mathsf{B})}+\tau_2^{(\mathsf{B})}}. \tag{5.5}$$

We assume both users are allocated the same amount of degree of freedom, so we have,

$$\mathcal{I}_{\mathsf{A,B;D_A,D_B}}(\boldsymbol{\tau}^{(\mathsf{A})},\boldsymbol{\tau}^{(\mathsf{B})}) = \tfrac{1}{2}\mathcal{I}_{\mathsf{A,B,D_A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) + \tfrac{1}{2}\mathcal{I}_{\mathsf{B,A,D_B}}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right) \text{ bit/sec/Hz}. \tag{5.6}$$

5.2.1 Time Allocation

Time allocation for each relay channel can be carried out separately. Ideally we would allocate time to maximize mutual information (or minimize total time) for each user. Nonetheless, this is not always possible due to constraints imposed by the system such as maximum average power, minimum throughput or maximum complexity. In our case the symmetry requirement prevent optimum time allocation.

A general time allocation problem may take the form,

$$\mathbb{P} \mapsto \left\{ \max_{\boldsymbol{\tau}^{(\mathsf{A})}\boldsymbol{\tau}^{(\mathsf{B})}} \quad f\left(\mathcal{I}_{\mathsf{A,B,D_A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right), \mathcal{I}_{\mathsf{B,A,D_B}}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right)\right) \right. \tag{5.7}$$

for some function $f(.)$. We arbitrarily choose $f(.)$ to maximize the minimum rate of the cooperating partners,

$$\mathbb{P} \mapsto \left\{ \max_{\boldsymbol{\tau}^{(\mathsf{A})}\boldsymbol{\tau}^{(\mathsf{B})}} \quad \min\{\mathcal{I}_{\mathsf{A,B,D_A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right), \mathcal{I}_{\mathsf{B,A,D_B}}\left(\boldsymbol{\tau}^{(\mathsf{B})}\right)\} \right. \tag{5.8}$$

One advantage of this time allocation policy is that it achieves some degree of fairness between cooperating users. It also ensures a win-win cooperation strategy. Other time allocation policies could be assumed to maximize one user's rate, maximize the sum rate or minimize the total time used by users while achieving a predetermined rate.

5.3 A Cooperative Network

We consider an ad hoc wireless network, where users are permitted to work in pairs cooperatively for helping each other. In particular, users are allowed to choose their partners on the condition that cooperation must increase the mutual information for both users. The prototype presented in the previous section is applied by cooperating users.

5.3.1 Partner Selection

An important issue to consider in a cooperative network is partnership selection and formation of clusters. This subsection formalizes the partnership selection for user cooperation based on the constructiveness or mutual usefulness to be defined shortly.

5.3.1.1 Useful User and Useful Set

Consider User A in the network looking for a single partner among other users, labeled $\mathsf{B}, \mathsf{C}, \mathsf{D}, \ldots$. Taking User B as an example, User B is not considered for partnership unless it is classified by User A as a useful user. A definition for the useful user varies depending on the network and partnership selection rules. A general definition of a useful user is *one who helps to achieve cooperation aims without breaching system constraints*. In what follows, a definition for useful is given based on the model assumed.

Definition 5.1 (Useful User)**.** *With regard to User* A *and destination* D_A*, User* B *is a useful user if User* A*, with User* B *as a relay, can achieve rate greater than that achievable by direct transmission,* $\mathcal{I}_{\mathsf{A},\mathsf{D}_\mathsf{A}}$*. Otherwise User* B *is a harmful user.*

Lemma 5.1 (Useful User)**.** *User* B *can not be a useful user for user* A *unless,*

$$\gamma_{\mathsf{A},\mathsf{D}_\mathsf{A}} < \gamma_{\mathsf{A},\mathsf{B}} \tag{5.9}$$

Moreover, User B *can only be a useful user for User* A *if,*

$$\gamma_{\mathsf{A},\mathsf{D}_\mathsf{A}} < (1+\gamma_{\mathsf{A},\mathsf{B}})^{\mu_1^{(\mathsf{A})}} - 1, \tag{5.10}$$

where,

$$\mu_1^{(\mathsf{A})} = \frac{\log\left(1+\gamma_{\mathsf{B},\mathsf{D}_\mathsf{A}}\right)}{\log\left(1+\gamma_{\mathsf{A},\mathsf{B}}-\gamma_{\mathsf{A},\mathsf{D}_\mathsf{A}}\right)+\log\left(1+\gamma_{\mathsf{B},\mathsf{D}_\mathsf{A}}\right)} \tag{5.11}$$

Proof. Consider relay channel $(\mathsf{A}, \mathsf{B}, \mathsf{D}_\mathsf{A})$. From Definition 5.1, User B is useful only if there is time allocation $\boldsymbol{\tau}^{(\mathsf{A})}$ such that $\mathcal{I}_{\mathsf{A},\mathsf{D}_\mathsf{A}} < \mathcal{I}_{\mathsf{A},\mathsf{B},\mathsf{D}_\mathsf{A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) \leq \mathcal{I}^{\max}_{\mathsf{A},\mathsf{B},\mathsf{D}_\mathsf{A}}$. From Theorem 4.1, $\mathcal{I}_{\mathsf{A},\mathsf{D}_\mathsf{A}} < \mathcal{I}^{\max}_{\mathsf{A},\mathsf{B},\mathsf{D}_\mathsf{A}} = \mathcal{I}_{\mathsf{A},\mathsf{B},\mathsf{D}_\mathsf{A}}\left(\boldsymbol{\mu}^{(\mathsf{A})}\right)$ only if (5.10) is satisfied. Nevertheless, $\boldsymbol{\mu}^{(\mathsf{A})} \triangleq (\mu_1^{(\mathsf{A})}, \mu_2^{(\mathsf{A})})$ must exist for (5.10) to be tested. $\boldsymbol{\mu}^{(\mathsf{A})}$ exists only if (5.9) is satisfied. □

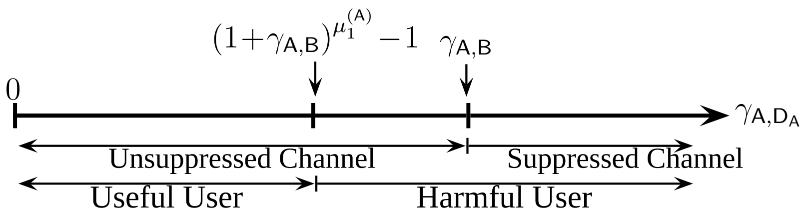


Figure 5.2: $\gamma_{\mathsf{A,D_A}}$ relative to other channels determines if User B is a useful user or not.

Condition (5.9) is a necessary condition, while Condition (5.10) is a sufficient condition for User B to be a useful user. Recalling that $(1+\gamma_{\mathsf{A,B}})^{\mu_1^{(\mathsf{A})}}-1 \leq \gamma_{\mathsf{A,B}}$ we may combine (5.9) and (5.10),

$$\gamma_{\mathsf{A,D_A}} < \ (1+\gamma_{\mathsf{A,B}})^{\mu_1^{(\mathsf{A})}} - 1 \ \leq \gamma_{\mathsf{A,B}} \tag{5.12}$$

Lemma 5.1 only states the conditions for a user to satisfy in order to become useful. It does not say, however, what time allocation policies make $\mathcal{I}_{\mathsf{A,B,D_A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) > \mathcal{I}_{\mathsf{A,D_A}}$. Generally, with user B as a relay, subject to time allocation $\boldsymbol{\tau}^{(\mathsf{A})}$, user A can achieve any rate R, $R < \mathcal{I}^{\max}_{\mathsf{A,B,D_A}}$. In a cooperative scenario, it is appealing to find out all time allocations to make User B a useful user.

Definition 5.2 (Useful Set)**.** *With regard to relay channel* $(\mathsf{A},\mathsf{B},\mathsf{D_A})$*, there is a useful set,* $\Lambda_{\mathsf{A,B,D_B}}$*, that contains all time allocations for the channel to achieve rates greater than direct transmission.*

The following Lemma explains how to find $\Lambda_{\mathsf{A,B,D_B}}$ accordingly.

Lemma 5.2 (Useful Set)**.** *For the relay channel* $(\mathsf{A},\mathsf{B},\mathsf{D_A})$, $\Lambda_{\mathsf{A,B,D_B}}$ *is the convex set,*

$$\begin{aligned}
&\tau_2^{(\mathsf{A})} \leq 1-\tau_1^{(\mathsf{A})},\\
&\tau_1^{(\mathsf{A})} > \frac{\log\left(1+\gamma_{\mathsf{A,D_A}}\right)}{\log\left(1+\gamma_{\mathsf{A,B}}\right)},\\
&\tau_2^{(\mathsf{A})} > \tau_1^{(\mathsf{A})}\frac{\log\left(\left[1+\gamma_{\mathsf{A,D_A}}\right]^{\frac{1}{\tau_1^{(\mathsf{A})}}}-\gamma_{\mathsf{A,D_A}}\right)}{\log\left(1+\gamma_{\mathsf{B,D_A}}\right)}.
\end{aligned} \tag{5.13}$$

Proof. From Definition 5.2, we may write $\Lambda_{\mathsf{A,B,D_B}}$ as,

$$\Lambda_{\mathsf{A,B,D_B}} = \left\{\boldsymbol{\tau}^{(\mathsf{A})} : \mathcal{I}_{\mathsf{A,B,D_A}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) > \mathcal{I}_{\mathsf{A,D_A}}, \tau_1^{(\mathsf{A})} \geq 0, \tau_2^{(\mathsf{A})} \geq 0, \tau_1^{(\mathsf{A})} + \tau_2^{(\mathsf{A})} \leq 1\right\} \tag{5.14}$$

$\Lambda_{\mathsf{A,B,D_B}}$ is similar to $\mathbf{C}$ in Section 4.3.2 and can be determined in an analogous way. R in (4.21) is replaced with $\mathcal{I}_{\mathsf{A,D_A}}$ in (5.14). Alike (4.21), $\Lambda_{\mathsf{A,B,D_B}}$ can be written

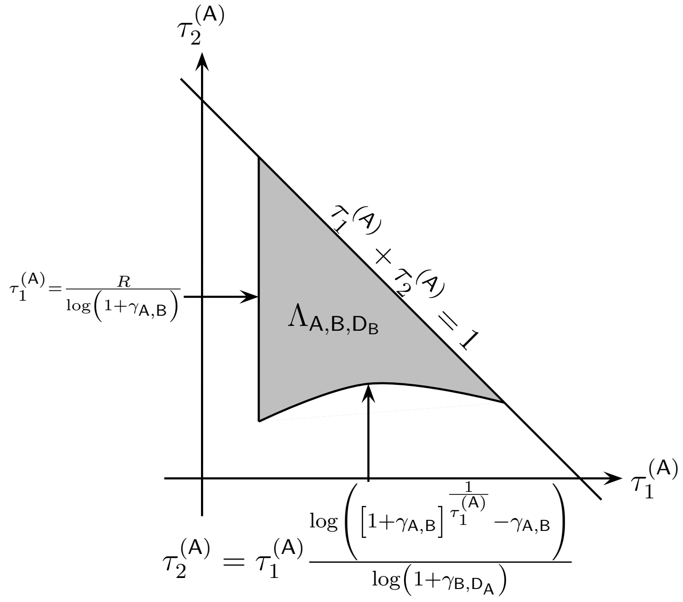


Figure 5.3: Shaded is $\Lambda_{\mathsf{A,B,D_A}}$, the set of time allocations in order for the mutual information for the channel $(\mathsf{A}, \mathsf{B}, \mathsf{D_A})$ to exceed that of direct transmission.

as a union of two sets,

$$\Lambda_{\mathsf{A,B,D_B}} = \left\{\boldsymbol{\tau}^{(\mathsf{A})} : \mathcal{I}_0^{(\mathsf{A})}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) > \mathcal{I}_{\mathsf{A,D_A}}, \tau_2^{(\mathsf{A})} \geq 0, \tau_2^{(\mathsf{A})} < \varsigma\tau_1^{(\mathsf{A})}, \tau_1^{(\mathsf{A})} + \tau_2^{(\mathsf{A})} \leq 1\right\} \cup \left\{\boldsymbol{\tau}^{(\mathsf{A})} : \mathcal{I}_{\mathsf{A,B}}\left(\boldsymbol{\tau}^{(\mathsf{A})}\right) > \mathcal{I}_{\mathsf{A,D_A}}, \tau_2^{(\mathsf{A})} < \varsigma\tau_1^{(\mathsf{A})}, \tau_1^{(\mathsf{A})} + \tau_2^{(\mathsf{A})} \leq 1\right\}$$

which reduces to,

$$\Lambda_{\mathsf{A,B,D_B}} = \left\{\boldsymbol{\tau}^{(\mathsf{A})} : \tau_1^{(\mathsf{A})} + \tau_2^{(\mathsf{A})} \leq 1,\ \tau_1^{(\mathsf{A})} > \frac{\log\left(1+\gamma_{\mathsf{A,D_A}}\right)}{\log\left(1+\gamma_{\mathsf{A,B}}\right)}, \tau_2^{(\mathsf{A})} > \tau_1^{(\mathsf{A})} \frac{\log\left(\left[1+\gamma_{\mathsf{A,D_A}}\right]^{\frac{1}{\tau_1^{(\mathsf{A})}}} - \gamma_{\mathsf{A,D_A}}\right)}{\log\left(1+\gamma_{\mathsf{B,D_A}}\right)}\right\}, \tag{5.15}$$

same as (5.13). $\Lambda_{\mathsf{A,B,D_B}}$ is shown in Figure 5.3. □

5.3.1.2 Useful Partner and Constructive Partnership

Intuitively, if User B is a useful user, the best option for User A is to use time allocation $\boldsymbol{\mu}^{(\mathsf{A})}$ to achieve the maximum mutual information. A cooperative model differs from a relay channel in that the former is a *give-and-take* kind of relationship. It is expected that both users benefit from cooperation. Thus $\boldsymbol{\mu}^{(\mathsf{A})}$ is not always an acceptable time allocation. Especially with the asserted condition that only symmetric cooperation is allowed. In this context, we may recognize three types of partnerships between any two users in the network:-

1. *Constructive Partnership*, when both users are useful to each other.

2. *Destructive Partnership*, when both users are harmful to each other.

3. *Unfair Partnership*, when one user is a useful partner while the other is a harmful one.

Apparently, for all scenarios constructive cooperation is preferred while destructive cooperation should be avoided. Unfair cooperation is prohibited in the system assumed in this chapter but it could be be useful for some situations.

It is necessary then for User A and User B to make constructive partners to have,

$$\begin{cases} \Lambda_{\mathsf{A,B,D_B}} \neq \emptyset, \\ \Lambda_{\mathsf{A,B,D_A}} \neq \emptyset. \end{cases} \tag{5.16}$$

This, however, is not sufficient to make constructive partnership possible. Time allocation must be chosen from the constructive set, $\Lambda_{\mathsf{A}\cap\mathsf{B}}$, defined below

Definition 5.3 (Constructive Set)**.** *The constructive set, $\Lambda_{\mathsf{A}\cap\mathsf{B}}$, is the set of all time allocations to make the cooperation between User A and User B a constructive cooperation.*

Due to symmetric time allocation, in what follows, subscript is dropped from time allocation. That is $\boldsymbol{\tau} \triangleq (\tau_1, \tau_2)$ refers to User B's as well as User A's time allocation.

$\Lambda_{\mathsf{A}\cap\mathsf{B}}$ is given by,

$$\begin{aligned} \Lambda_{\mathsf{A}\cap\mathsf{B}} &= \Lambda_{\mathsf{A,B,D_B}} \cap \Lambda_{\mathsf{A,B,D_A}} \\ &= \Big\{ \boldsymbol{\tau} : \tau_1 + \tau_2 \leq 1,\ \tau_1 > \max\{f_1(\boldsymbol{\gamma}^{(\mathsf{A})}), f_1(\boldsymbol{\gamma}^{(\mathsf{B})})\}, \\ &\qquad \tau_2 > \max\{f_2(\boldsymbol{\gamma}^{(\mathsf{A})}, \tau_1), f_2(\boldsymbol{\gamma}^{(\mathsf{B})}, \tau_1)\} \Big\}, \end{aligned} \tag{5.17}$$

where,

$$f_1(\boldsymbol{\gamma}^{(\mathsf{A})}) = \frac{\log\left(1 + \gamma_{\mathsf{A,D_A}}\right)}{\log\left(1 + \gamma_{\mathsf{A,B}}\right)} \tag{5.18}$$

$$f_1(\boldsymbol{\gamma}^{(\mathsf{B})}) = \frac{\log\left(1 + \gamma_{\mathsf{B,D_B}}\right)}{\log\left(1 + \gamma_{\mathsf{B,A}}\right)} \tag{5.19}$$

$$f_2(\boldsymbol{\gamma}^{(\mathsf{A})}, \tau_1) = \tau_1 \frac{\log\left(\left[1+\gamma_{\mathsf{A,D_A}}\right]^{\frac{1}{\tau_1}} - \gamma_{\mathsf{A,D_A}}\right)}{\log\left(1+\gamma_{\mathsf{B,D_A}}\right)} \tag{5.20}$$

$$f_2(\boldsymbol{\gamma}^{(\mathsf{B})}, \tau_1) = \tau_1 \frac{\log\left(\left[1+\gamma_{\mathsf{B,D_B}}\right]^{\frac{1}{\tau_1}} - \gamma_{\mathsf{B,D_B}}\right)}{\log\left(1+\gamma_{\mathsf{A,D_B}}\right)}. \tag{5.21}$$

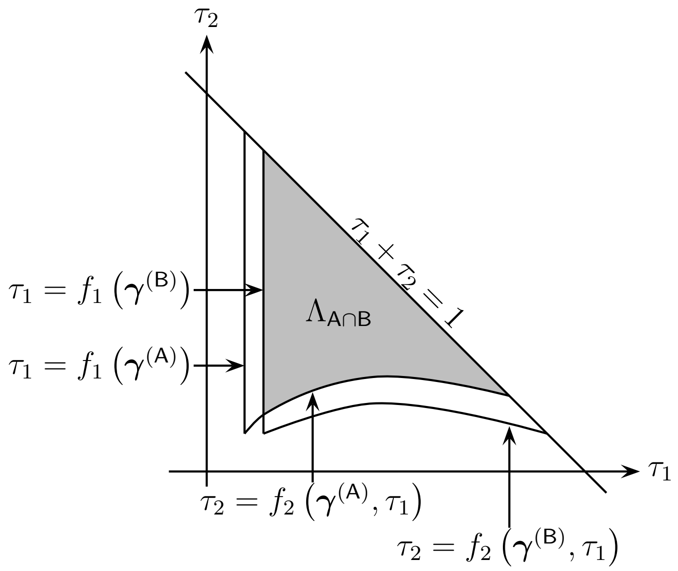


Figure 5.4: Shaded is $\Lambda_{\mathsf{A}\cap\mathsf{B}}$, the set of time allocations in order to make cooperation between User A and User B a constructive cooperation.

User A and User B can cooperate constructively only if,

$$\Lambda_{\mathsf{A}\cap\mathsf{B}} \neq \emptyset, \tag{5.22}$$

The next lemma states conditions to guarantee a nonempty $\Lambda_{\mathsf{A}\cap\mathsf{B}}$.

Theorem 5.1 (Constructive Set). $\Lambda_{\mathsf{A}\cap\mathsf{B}} \neq \emptyset$ *only if both condition* (5.9) *and condition* (5.10) *are satisfied for channels* $(\mathsf{A}, \mathsf{B}, \mathsf{D}_\mathsf{A})$ *and* $(\mathsf{B}, \mathsf{A}, \mathsf{D}_\mathsf{B})$*; and,*

$$\begin{cases} f_2\left(\boldsymbol{\gamma}^{(\mathsf{A})}, f_1(\boldsymbol{\gamma}^{(\mathsf{B})})\right) + f_1(\boldsymbol{\gamma}^{(\mathsf{B})}) < 1, \\ f_2\left(\boldsymbol{\gamma}^{(\mathsf{B})}, f_1(\boldsymbol{\gamma}^{(\mathsf{A})})\right) + f_1(\boldsymbol{\gamma}^{(\mathsf{A})}) < 1, \end{cases} \tag{5.23}$$

Proof. From (5.17), in the $\tau_1\tau_2$-plane, $\Lambda_{\mathsf{A}\cap\mathsf{B}}$ is the area,

$$\tau_1 + \tau_2 \leq 1, \tag{5.24}$$
$$\tau_1 > \max\{f_1(\boldsymbol{\gamma}^{(\mathsf{A})}), f_1(\boldsymbol{\gamma}^{(\mathsf{B})})\}, \tag{5.25}$$
$$\tau_2 > \max\{f_2(\boldsymbol{\gamma}^{(\mathsf{A})}, \tau_1), f_2(\boldsymbol{\gamma}^{(\mathsf{B})}, \tau_1)\}. \tag{5.26}$$

illustrated in Figure 5.4.

From (5.24) and (5.25),

$$\max\{f_1(\boldsymbol{\gamma}^{(\mathsf{A})}), f_1(\boldsymbol{\gamma}^{(\mathsf{B})})\} < \tau_1 \tag{5.27}$$
$$\leq 1 - \tau_2 \tag{5.28}$$
$$\leq 1, \tag{5.29}$$

yielding,

$$f_1(\boldsymbol{\gamma}^{(\mathsf{A})}) = \frac{\log\left(1+\gamma_{\mathsf{A},\mathsf{D}_\mathsf{A}}\right)}{\log\left(1+\gamma_{\mathsf{A},\mathsf{B}}\right)} < 1 \tag{5.30}$$

and

$$f_1(\boldsymbol{\gamma}^{(\mathsf{B})}) = \frac{\log\left(1+\gamma_{\mathsf{B},\mathsf{D}_\mathsf{B}}\right)}{\log\left(1+\gamma_{\mathsf{B},\mathsf{A}}\right)} < 1 \tag{5.31}$$

which is true only if (5.9) is satisfied.

Now consider (5.25) and (5.26), we have,

$$\max\{f_2(\boldsymbol{\gamma}^{(\mathsf{A})}, \tau_1), f_2(\boldsymbol{\gamma}^{(\mathsf{B})}, \tau_1)\} < 1 - \tau_1. \tag{5.32}$$

We seek τ_1 that makes (5.32). Take $\tau_1 = \tau_1^{\min} > \max\{f_1(\boldsymbol{\gamma}^{(\mathsf{A})}), f_1(\boldsymbol{\gamma}^{(\mathsf{B})})\}$.

$$\max\{f_2(\boldsymbol{\gamma}^{(\mathsf{A})}, \tau_1^{\min}), f_2(\boldsymbol{\gamma}^{(\mathsf{B})}, \tau_1^{\min})\} < 1 - \tau_1^{\min}, \tag{5.33}$$

or,

$$\max\{f_2(\boldsymbol{\gamma}^{(\mathsf{A})}, \tau_1^{\min}), f_2(\boldsymbol{\gamma}^{(\mathsf{B})}, \tau_1^{\min})\} + \tau_1^{\min} < 1. \tag{5.34}$$

To make (5.34) correct, it is sufficient to have,

$$f_2\left(\boldsymbol{\gamma}^{(\mathsf{A})}, f_1(\boldsymbol{\gamma}^{(\mathsf{B})})\right) + f_1(\boldsymbol{\gamma}^{(\mathsf{B})}) < 1, \tag{5.35}$$

$$f_2\left(\boldsymbol{\gamma}^{(\mathsf{B})}, f_1(\boldsymbol{\gamma}^{(\mathsf{A})})\right) + f_1(\boldsymbol{\gamma}^{(\mathsf{A})}) < 1, \tag{5.36}$$

and,

$$f_2\left(\boldsymbol{\gamma}^{(\mathsf{A})}, f_1(\boldsymbol{\gamma}^{(\mathsf{A})})\right) + f_1(\boldsymbol{\gamma}^{(\mathsf{A})}) < 1, \tag{5.37}$$

$$f_2\left(\boldsymbol{\gamma}^{(\mathsf{B})}, f_1(\boldsymbol{\gamma}^{(\mathsf{B})})\right) + f_1(\boldsymbol{\gamma}^{(\mathsf{B})}) < 1. \tag{5.38}$$

It is easy to show that (5.37) and (5.38) are true when (5.9) is satisfied for both relay channels, $\Lambda_{\mathsf{A},\mathsf{B},\mathsf{D}_\mathsf{B}}$ and $\Lambda_{\mathsf{A},\mathsf{B},\mathsf{D}_\mathsf{A}}$. □

5.3.1.3 Procedure for Partner Selection

From User A's view, all users in the network can be arranged into three groups, $\mathcal{L}$, $\mathcal{M}$ and $\mathcal{N}$, such that $\mathcal{N} \subseteq \mathcal{M} \subseteq \mathcal{L}$. $\mathcal{L}$ is the set of all users in the network except User A. $\mathcal{M}$ is the set of all useful users, while $\mathcal{N}$ is the set of all useful partners. $\mathcal{L}$, $\mathcal{M}$ and $\mathcal{N}$ differ for each other user in the network. If $\mathcal{N}$ contains more than one user, User A can select one of them as a partner. We make an assumption that partners are selected to maximize the sum rate. User A thus calculates the appropriate time allocation for each useful partner U using (5.8) and chooses the one that satisfies,

$$\mathbb{P} \mapsto \left\{ \max_{\mathsf{U}\in\mathcal{N}} \ \mathcal{I}_{\mathsf{A},\mathsf{U},\mathsf{D}_\mathsf{A}}(\boldsymbol{\tau}) + \mathcal{I}_{\mathsf{U},A,D_U}(\boldsymbol{\tau}) \right. \tag{5.39}$$

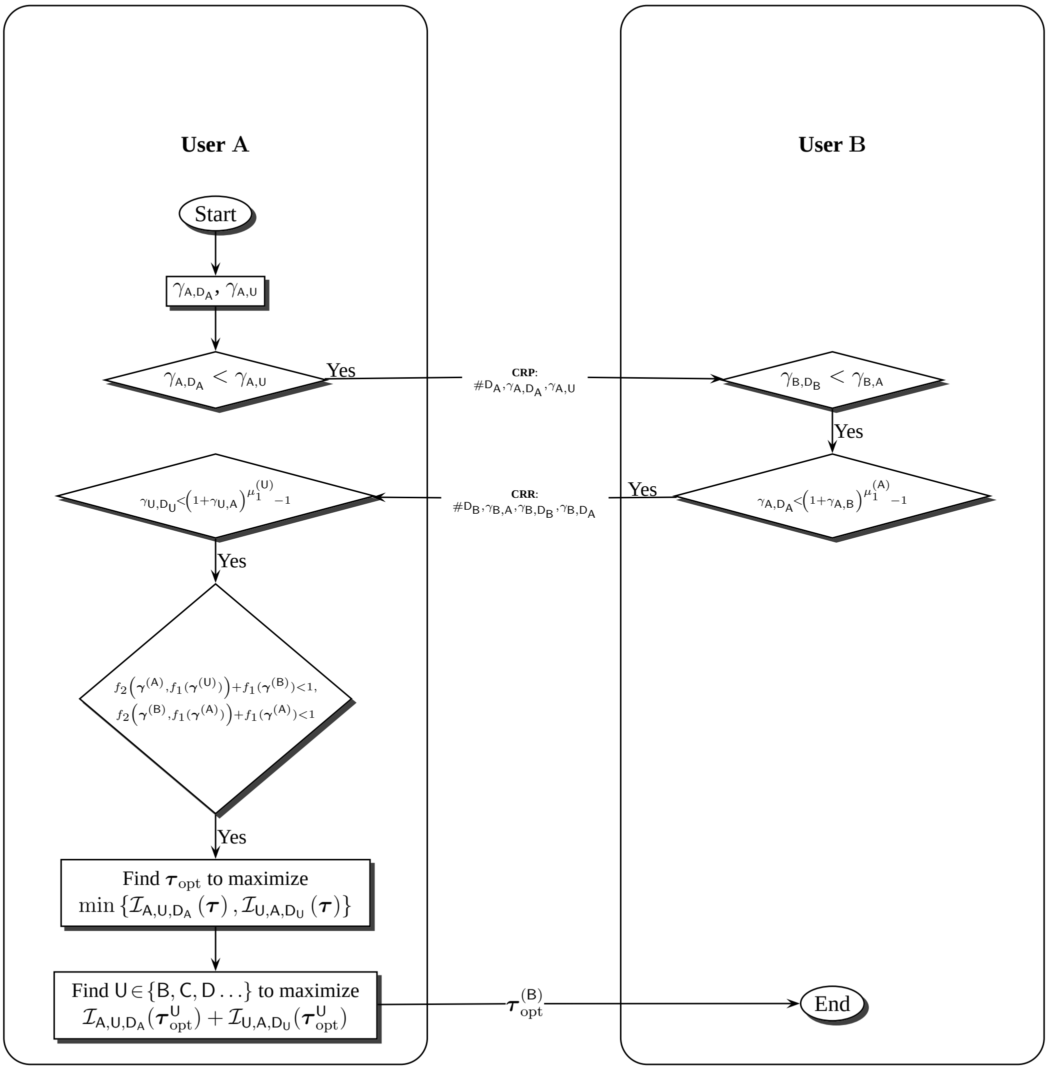


Figure 5.5: Flowchart showing how user B is chosen by user A for cooperation. User U in User A's side refers to any user contacted by User A for cooperation.

This selection is in favor of improving network overall performance.

To conclude this section, an upper layer procedure for partner selection is presented. In what follows we show the steps taken by User A and appropriate users in the network in order for User A to select a partner. It is assumed that each user in the network keeps limited information about the network. This is necessary from the upper layers' view to keep the size of the memory at a minimum. Extra information can indeed be obtained when required. In particular, it is assumed that each user knows about the channel to its destination in addition to channels to all neighboring nodes. A node is considered as a neighbor if it is within the range to the destination. User A, for instance, knows γ_{A,D_A} and all $\gamma_{A,U}$ for every User U with,

$$\gamma_{A,D_A} < \gamma_{A,U}, \tag{5.40}$$

So User A's neighbors are users who satisfy condition (5.9). On the other hand, each of these users knows the channel to its destination, γ_{U,D_U}. User U only knows $\gamma_{U,A}$, however, if,

$$\gamma_{U,D_U} < \gamma_{U,A}, \tag{5.41}$$

i.e., if User A is a neighboring node. The following steps are taken in order for User A to select a partner:-

1. User A multicasts a *Cooperation Request Packet* (CRP) to all neighboring nodes. The CRP contains additional information. It tells neighboring nodes the ID for User A's destination (D_A), γ_{A,D_A} and $\gamma_{A,U}$.

2. Users who receive the CRP check first if User A is a neighboring node by checking condition (5.9). If User A is not a neighboring node, the CRP is ignored.

3. Those who have User A as a neighbor check condition (5.10) using information sent by User A in addition to γ_{U,D_A}. If condition (5.10) tests positive, it means that the user is a useful user to User A. If the test fails, however, the CRP is discarded and no action is taken by the user.

4. If a user, e.g, User U, is declared as a useful user to User A, it replies to the CRP with a *Cooperation Request Reply* (CRR) packet. The CRR also contains the ID for its destination in addition to $\gamma_{U,A}$, γ_{U,D_U} and γ_{U,D_A}.

5. Upon receiving the CRR, User A checks condition (5.10) and (5.23) on all replying users, using information attached to the CRR, in addition to γ_{A,D_U}. User A is a useful user to those who pass condition (5.10). User A can make constructive partnerships with those who pass (5.23) as well.

6. User A calculates the appropriate time allocation for each useful partner using (5.8).

7. User A works out the sum mutual information which would result from cooperating with each of the useful partners using time allocation calculated in the previous step. User A chooses as a partner the user who maximizes the sum rate.

8. Finally, User A sends to the chosen partner the calculated time allocation.

The above procedure is illustrated in the flow chart in Figure 5.5. Although this negotiation may involves several users, in Figure 5.5 only User A and User B are presented. Figure 5.5 also assumes that User B is the successful partner.

The above procedure takes into consideration the number of overhead packets and the amount of information kept by each user. From upper layer perspective in a non-centralized network, there is a trade-off between these two parameters. Keeping more information about the network needs extra hardware and costs more, while less information leads to more overhead packets exchanged which wastes much of the resources. In the procedure proposed, we took advantage of the conditions established in Sections 5.3.1.1 and 5.3.1.2 to keep minimum information stored by a user in order to establish partnership negotiation and at the same time minimize overhead traffic. Efficiency of this protocol needs further investigation in different scenarios and networks using cross-layer analysis.

5.3.2 Network Simulation

A simulation program is developed based on the model and partner selection procedure discussed in the preceding part of this chapter. The aim of the simulation is to show how partner selection is affected by some of the network parameters like average SNR and node density. Results are also generated to show the effect of user-cooperative partnership on the network throughput.

5.3.2.1 Network Model

We consider an ad hoc wireless network, where nodes are randomly positioned. With regard to some reference point, node i is positioned at $(x_\mathsf{i}, y_\mathsf{i})$ where x_i and y_i are randomly generated according to a zero-mean normally distributed random variable with variance σ^2. That makes the distance from nodes to the origin, d_0, a Rayleigh distributed random variable, that is $d_0 \sim \mathrm{Rayleigh}(\sigma)$. The expected value of the distance from a randomly picked node to the origin, $E[d_0]$, is used as a parameter that determines the spread of the nodes. In the simulation we have $E[d_0] = 100$ m. This is used to calculate σ since,

$$\sigma = \sqrt{\frac{2}{\pi}} E[d_0]. \tag{5.42}$$

It can be shown that the distance between any two nodes, d, is also Rayleigh distributed, that is $d \sim \mathrm{Rayleigh}(\sqrt{2}\sigma)$ (see appendix B). The network is parameterized by the average SNR, SNR, given by,

$$\mathsf{SNR} = \frac{P}{N_0} E[d^{-\alpha}]. \tag{5.43}$$

where, P is the fixed transmission power, N_0 is the noise variance and α is the path-loss and shadowing coefficient. For a Rayleigh distributed random variable [74],

$$E[d^\alpha] = (2\sigma^2)^{\frac{\alpha}{2}} \Gamma\left(1 + \frac{\alpha}{2}\right), \tag{5.44}$$

where $\Gamma(.)$ is the Gamma function. Derivation of $E[d^{\alpha}]$ is given in Appendix B.

Each node has a unique identification (ID) number. Half of the nodes are sources and the other half are destinations arranged randomly into source-destination pairs. The available degree of freedom is equally divided between users. All nodes are subject to half-duplex constraint.

Users are allowed to form partnerships in order to increase their rates as well as the total rate achieved by network. The two-user model discussed earlier in this chapter is used as a prototype for cooperation and partnership selection. So a maximum of two users per cluster is allowed; and partnership has to be reciprocal and symmetric. Starting from the user with the least ID number, a partner is sought in a similar way to that described in Section 5.3.1.3. When a user succeed in finding a partner both partners are declared as unavailable for cooperation.

Results are generated to show the percentage of users succeeds in forming a partnership in the network. This is carried out for a range of SNR, α and node densities (function of number of users in the network and $E[d_0]$). Results are also generated to compare network throughput before and after cooperation. The average mutual information in the network without user cooperation is given by,

$$\overline{\mathcal{I}} = \frac{2}{N} \sum_{\mathsf{i} \in \mathcal{T}} \mathcal{I}_{\mathsf{i},\mathsf{D_i}}, \tag{5.45}$$

where, $\mathcal{T}$ is the set of all users, N is the total number of nodes in the network and $\mathcal{I}_{\mathsf{i},\mathsf{D_i}}$ is the mutual information between node i and its destination $\mathsf{D_i}$ (in bit/sec/Hz), normalized by the available degree of freedom for user i. The percentage gain in mutual information is calculated according to,

$$\mathcal{G} = \frac{\overline{\mathcal{I}}_{coop} - \overline{\mathcal{I}}}{\overline{\mathcal{I}}} \times 100\%, \tag{5.46}$$

where $\mathcal{I}_{\text{coop}}$ is the average mutual information in the network after forming partnerships given by,

$$\overline{\mathcal{I}}_{\mathsf{coop}} = \frac{\sum_{\mathsf{i},\mathsf{j(i)} \in \mathcal{C}} \mathcal{I}_{\mathsf{i},\mathsf{j(i)},\mathsf{D_i}}(\tau^{(\mathsf{i})}_{\text{opt}}) + \sum_{\mathsf{i} \in \mathcal{T}-\mathcal{C}} \mathcal{I}_{\mathsf{i},\mathsf{D_i}}}{N/2}, \tag{5.47}$$

where, $\mathcal{C}$ is the set of all users succeeded to find partners, $\mathsf{j(i)}$ is the partner for User i and $\tau^{(\mathsf{i})} = \tau^{(\mathsf{j})}$ is the time allocation for user i (and j).

5.3.2.2 Numerical Results

Flowchart in Figure 5.6 outlines the code used to produce results presented in this section. The developed code so is long and complex that it could be impractical to include all the details in the flowchart. Typically, each of the boxes in Figure 5.6 represents a

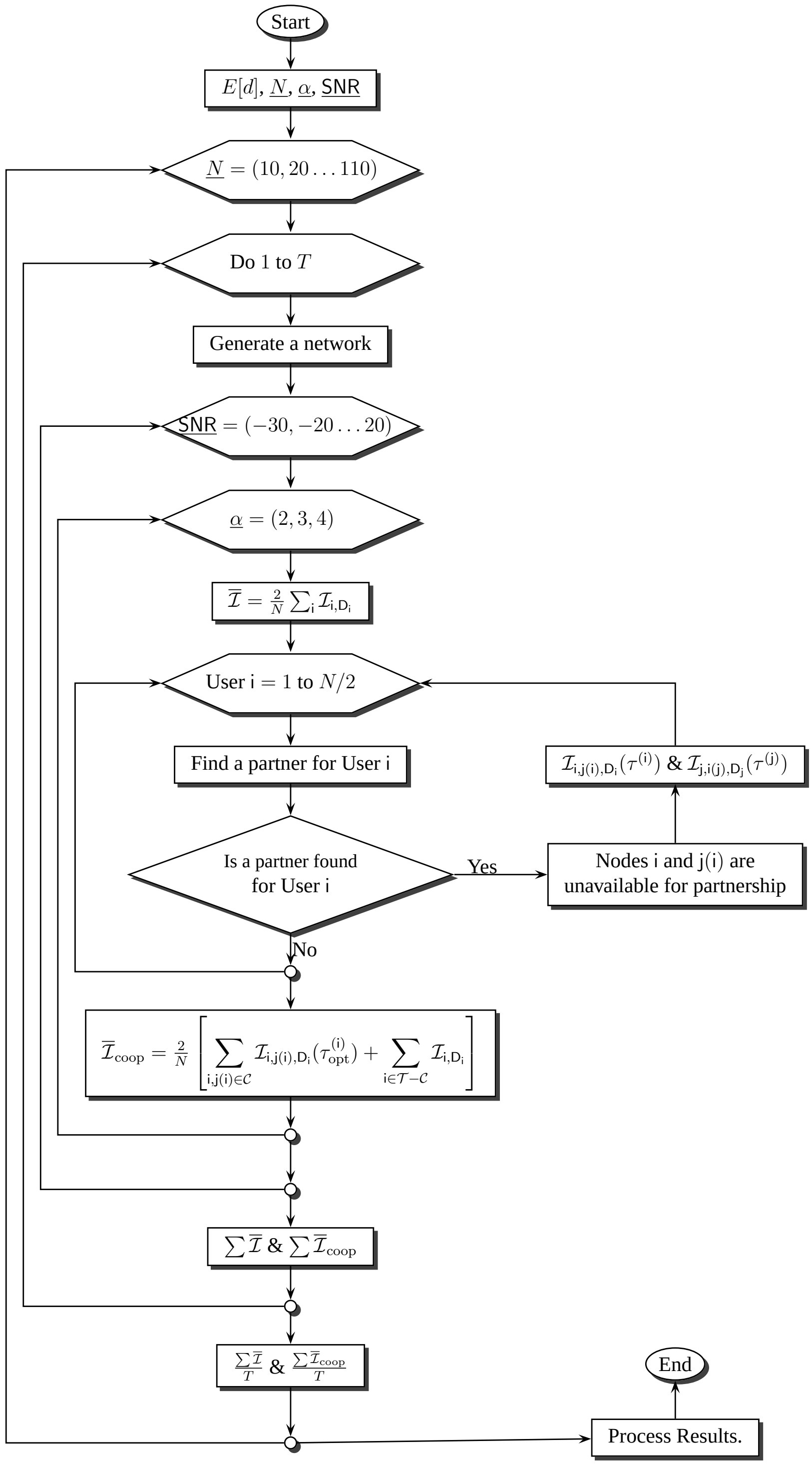


Figure 5.6: Flowchart to outline the code used to produce numerical results for the cooperative network. Most of the variables are multidimensional arrays.

separate program with routines and sub-routines. For instance, box saying '*Find a partner for User* i' in the tenth row is a program that includes several subroutines matching the procedure proposed in Section 5.3.1.3 and the flowchart in Figure 5.5.

Initializing parameters for the program are the $E[d]$ which determines the dispersion of the nodes from the origin, the set of total number of the nodes in the network, the set of path-loss coefficients and the set of SNR. Those parameters are used to generate a random network according to the specifications stated in the previous section. $\overline{\mathcal{I}}$ is then calculated using (5.45). Next, partners are sought according to the conditions stated in Section 5.3.1. If a user succeeds in finding a partner, both users are removed from the list of available partners. Mutual information is recalculated for those who formed cooperative partnerships. $\overline{\mathcal{I}}_{\text{coop}}$ is worked out using (5.47). Calculation of $\mathcal{I}$ and $\mathcal{I}_{\text{coop}}$ is carried out for all N, α and SNR in $\underline{N}$, $\underline{\alpha}$ and $\underline{\text{SNR}}$, respectively. Network generation and all calculations are repeated T times, where $T \gg 1$, and average is taken. Finally, a separate sub-program arranges the accumulated results for presentation.

Figure 5.7 shows the percentage of transmitters who succeed in building constructive partnerships versus N for different SNR. Results are generated for $\alpha = 2, 3$ and 4. Produced graphs suggest a strong connection between the likelihood of a user successfully finding a partner and network parameters. For a given α the percentage of cooperating users increases with the increase in network density and/or decrease in SNR. This behavior is even more apparent for larger α. Nevertheless, the rate of increase of the percentage of cooperating users with SNR is higher at high SNR than at low SNR. That is to say, for a given node density, although the number of users who tend to cooperate in high SNR is small, that number increases significantly with small drops in SNR, while in low SNR that number increases only insignificantly as SNR decreases. For example, the percentage of cooperating users increased by more than 20% at $N = 70$ as SNR drops fron 30 dB to 10 dB, while that percentage increased by less than 10% when SNR drops from -10 dB to -30 dB for the same node density.

We conclude from observing Figure 5.7 that in general finding a partner is easier in dense networks and when channel conditions deteriorate.

Figure 5.8 exhibit another set of graphs in which the percentage gain of the network sum rate versus SNR is plotted for different node densities. Results are generated for $\alpha = 2, 3$ and 4. In general, the network seems to benefit more from cooperation for smaller node densities. Moreover, gain is also bigger for bigger α. The behavior of the gain becomes more interesting as SNR changes. For a given α and given node density, gain in network rate starts from almost zero and increases as SNR drops. Eventually, as SNR exceeds -10, gain starts to decrease. This behavior is typical for all α and all node densities. It differs only on the rate of change in gain as the SNR changes and the maximum gain attained in each case.

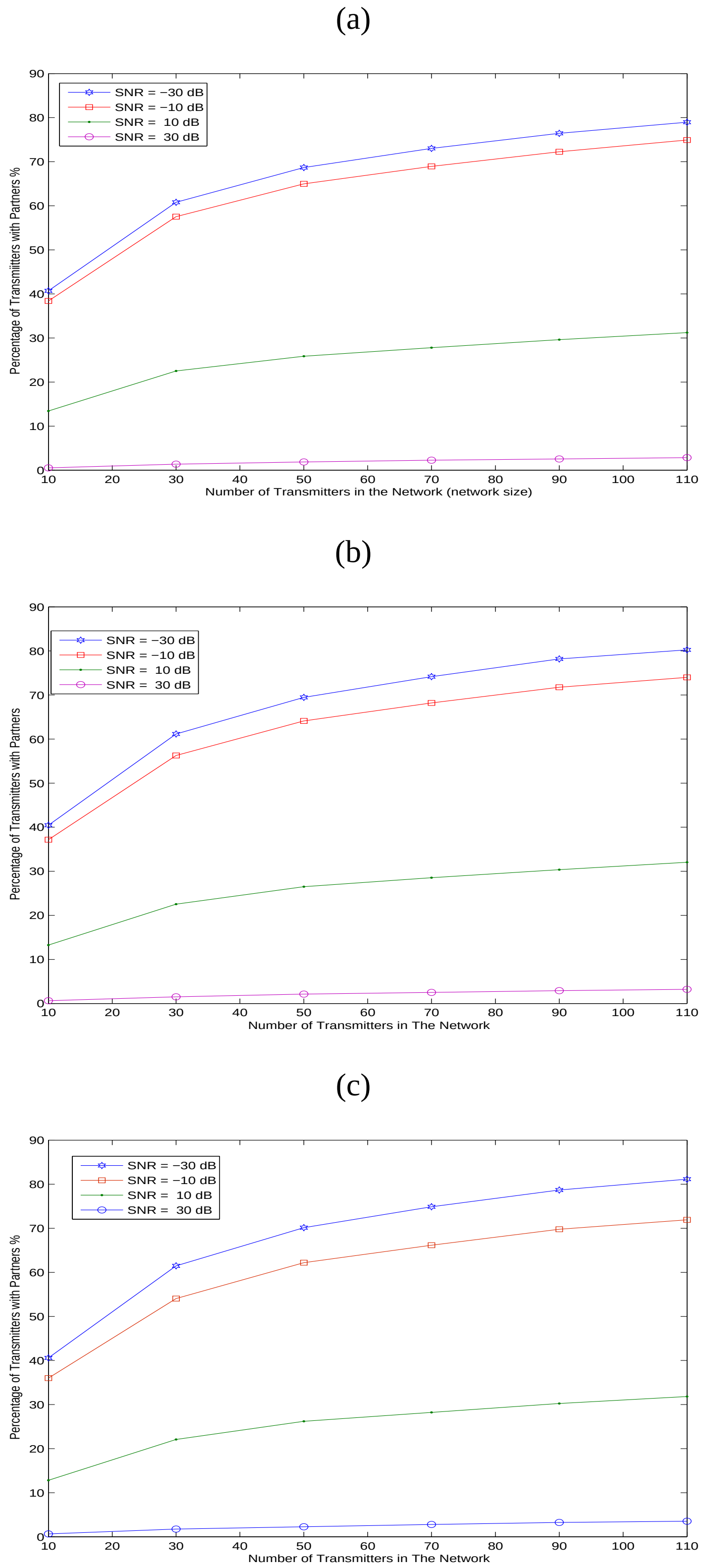
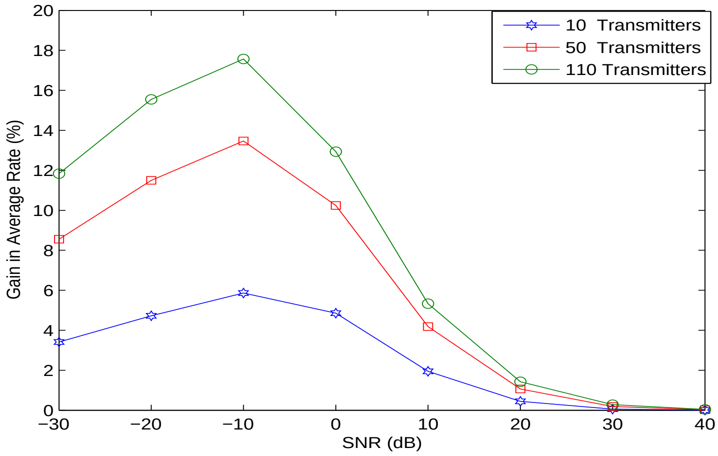
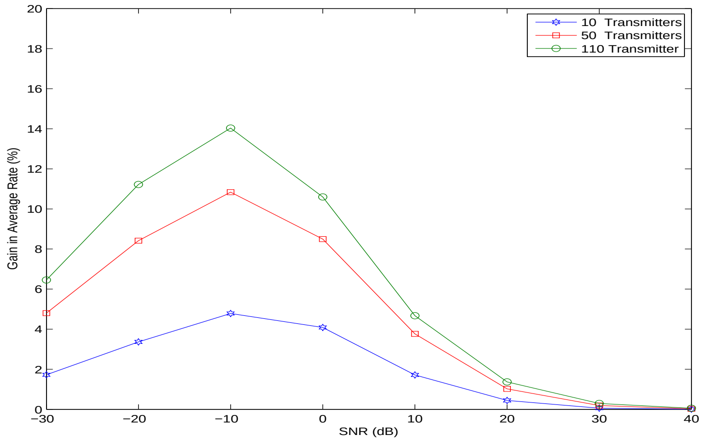


Figure 5.7: The percentage of transmitters with partners is plotted against the total number of transmitters in the network for different SNR. (a) $\alpha = 2$ (b) $\alpha = 3$, and (c) $\alpha = 4$.

(a)



(b)



(c)

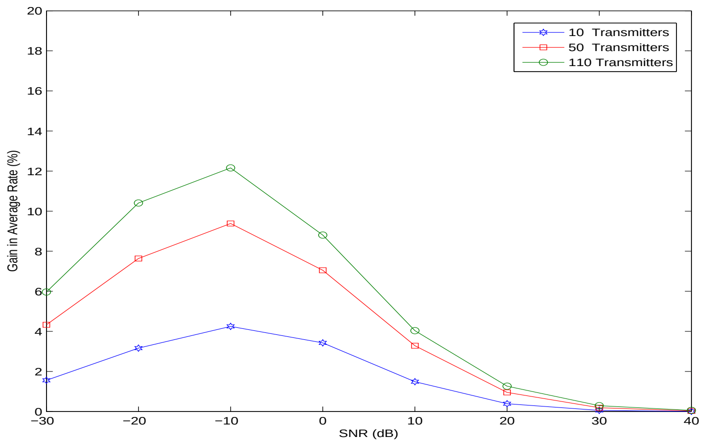


Figure 5.8: $\mathcal{G}$ plotted versus SNR for different node densities, (a) $\alpha = 2$, (a) $\alpha = 3$ and (b) $\alpha = 4$.

To understand the reason behind this behavior, it is useful to look back and connect these results to those in Figure 5.7. There are two factors affecting the gain in network sum rate. One is the number of cooperating users and the other is the gain attained by the individual cooperating user. In turn, both factors are affected by SNR. As suggested by Figure 5.7, as SNR decreases, the percentage of cooperating users increases. Figure 5.8 is justifiable only under an assumption that the gain for individual cooperating users decreases as SNR decreases. To explain, starting at SNR $= 40$ dB, no user wants to form a partnership and thus gain from cooperation is zero. As SNR drops, more users form cooperative partnerships. This is accompanied by a decrease in the rate gain per cooperating user. The increase in the number of cooperating users, however, is significant and dominates the decrease in the gain for individual users for SNR < -10 dB. As a result, the overall gain in the network increases. As SNR falls below -10 dB, the increase in cooperating users is minor and thus is dominated by the decrease in gain per cooperating user. As a result, the overall gain in the network starts to decrease again. This justification agrees with the observations from Figure 5.7.

We conclude from Figure 5.7 and Figure 5.8 that at high SNR, users are unlikely to form cooperative partnerships and thus the network benefits less. In low SNR network's benefit from cooperation is humble too, due to the insignificant gain in rate per cooperative user. There is an optimum SNR where gain from cooperation is maximum, which in this case is approximately -10 dB.

It is important to remember that these results only reflect the physical layer's view. A cross-layer analysis would reveal more reliable results. For example, when considering the moderate gain from cooperation accompanied by the large number of cooperating users at low SNR, it is necessary to consider wasted resources due to the introduced overhead needed to establish partnerships. The actual throughput could be much less than that shown in Figure 5.8. It is also important to consider overhead traffic for rapidly changing channels and rapidly changing networks where partnerships are to be constructed and destructed continuously.

It is also important to remember that results obtained for this section are a result of the model assumed at the first part of the chapters for partnership and network. We should expect that cooperative networks would perform better if some of the constraints are removed. For instance, the limitation in cluster size and the condition on partnership to be symmetric and reciprocal forces users to choose less favored partners and the resultant rate is not the optimum. Procedure for partner selection also involves some policies, which does not allow for the best options. In particular, the order at which users select their partners is not optimal. Giving priority to those who can benefit more from cooperation would result in a better performance for the network. This could also be treated by allowing a selected partner to make its own decisions before committing itself to any partnership offer. This chapter, nonetheless, aimed at developing a framework for dealing with cooperative networks rather than seeking the optimum cooperating strategy.

5.4 Chapter Summary

In this chapter we considered user-cooperative communications in wireless networks. Based on the three-node D&F relay channel, a two-user cooperative model is proposed. More focus is given to partner selection and necessary conditions are generated for partner selection. The prototype is then applied to a multi-user network. Numerical results gave us some insight into the relationship between network performance and network parameters. The proposed two-user model and application of this model to a multi-user network offers a framework to deal with a broader range of networks. Most of the terms defined here and conditions established can be extended to other scenarios.

Chapter 6

Multi-Hop D&F Relay Channel

In this chapter we extend results obtained for the two-hop channel to any number of hops. In section 6.1 we introduce the channel. In Section 6.2 mutual information is worked out. Conversion point is also introduced for the channel. In Section 6.3 distribution functions for mutual information in Rayleigh fading channels is found. Then average mutual information and outage probability formulas are evaluated. Section 6.4 discusses the problem of optimal routing. Some examples are given in Section 6.5. Finally, this chapter is summarized in Section 6.6.

6.1 Introduction

In the classical multi-hop relay channel, information is sent in a consecutive fashion: first from the source to the first relay, then from the first relay to the second relay and so on until the message reaches the destination terminal. This mode of transmission is also called non-cooperative relaying. Apart from the source and the destination, non-cooperative transmission assumes that each node is connected to one node in the downstream and another node in the upstream. In contrast, in cooperative relaying, each node combines all the signals received from all nodes in the upstream. It is thus necessary to have a fully connected network, such as the one in Figure 6.1, in order to carry out cooperative transmission.

A general multi-relay channel consists of a number of relays arranged into different groups or levels, where relays in the same level cooperatively decode and transmit the received information [35]. In this chapter, we are interested in the multi-hop relay channel with a single relay per hop. We further consider the wireless medium where the network is fully connected and relays have half-duplex constraint. We consider that an M-hop relay channel comprises a source node, labeled as node 0, a destination node, labeled as node M and $M-1$ relay nodes labeled accordingly by $\{1, 2 \ldots, M-1\}$.

Due to the half-duplex constraint, only one node can transmit at a time. Available time is thus shared by all transmitting nodes. Node i listens during time periods $\tau_0, \tau_1 \ldots \tau_{i-1}$, transmits during τ_i and remains idle during periods $\tau_{i+1}, \tau_{i+2} \ldots \tau_{M-1}$.

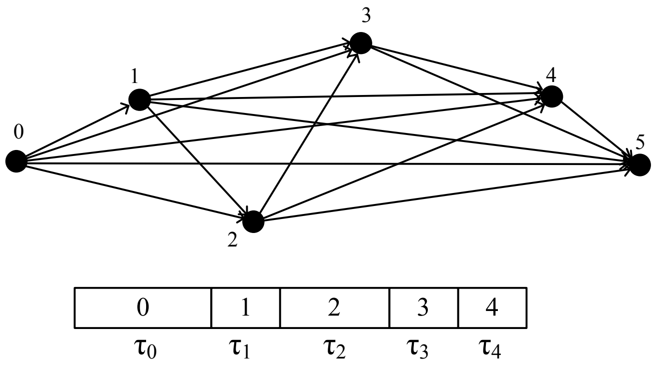


Figure 6.1: Multiple-hop wireless relay channel and time allocation. Here cooperative transmission is possible since the network is fully connected.

An example is given in Figure 6.1 for a 5-hop relay channel. Figure 6.2, further, describes transmission from a source node, 0, to a destination node, 5. The first node 0 broadcasts the message to all nodes $1, 2 \ldots 5$. Node 1 starts decoding the message immediately at the end of phase 1 and retransmits the message in the second phase. Node $2, 3$ and 4 follow. Each node combines all received copies of the signal to decode the message before transmission. For example as shown in Figure 6.3, node 4 combines signals transmitted during time periods $\tau_0, \tau_1 \ldots \tau_3$ and retransmits the message during τ_4. D&F is advantageous over A&F as transmission is extended over many hops.

6.2 Mutual Information

According to the D&F signaling, each relay must be able to correctly decode the transmitted codeword. Mutual information between the source and destination nodes, therefore, is bound by the minimum achievable rate at the destination and each of the relay nodes. For the channel $(0, 1 \ldots M)$, mutual information, denoted $\mathcal{I}_M$, is given by,

$$\mathcal{I}_M(\boldsymbol{\tau}) = \min_{j \in \{1,2,\ldots,M\}} \mathcal{I}_0^{(j)}(\boldsymbol{\tau}), \text{ bit/sec/Hz} \tag{6.1}$$

$\boldsymbol{\tau} = (\tau_0, \tau_1, \ldots \tau_{M-1})$ is the time allocation vector. $\mathcal{I}_0^{(j)}$ denotes decoding rate at node j given repeated transmission from nodes $0, 1 \ldots j-1$. The first relay is particularly important, as we will see in Section 6.4, thus we give it a name; we call it the *primary relay*. All other relays are *secondary relays*. Only one copy of the signal is received at the primary relay, thus we have,

$$\mathcal{I}_0^{(1)}(\boldsymbol{\tau}) = \tau_0 \log(1 + \gamma_{0,1}). \tag{6.2}$$

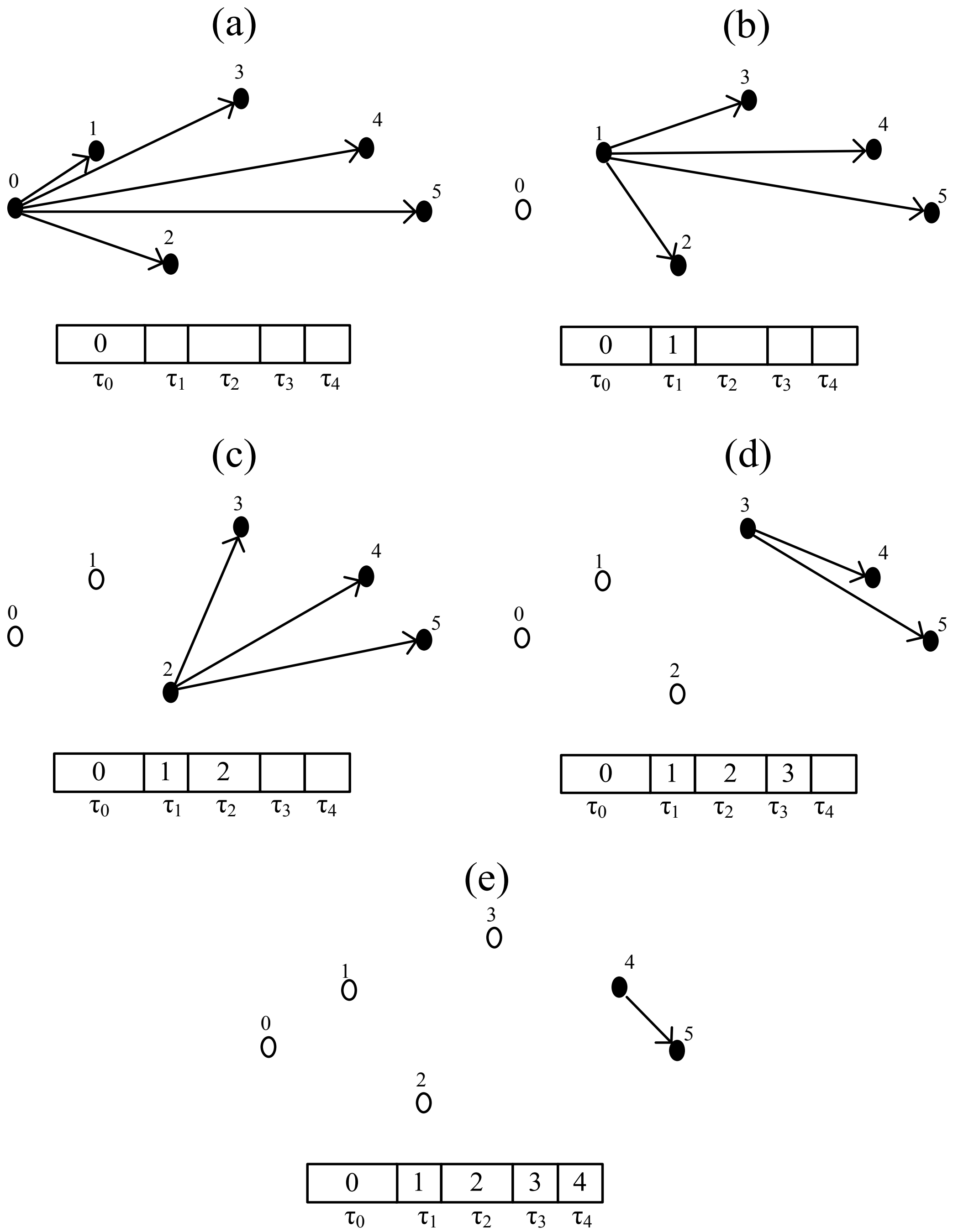


Figure 6.2: Transmission from node 0 (source node) to node 5 (destination node) over 5 hops. (a)-(e) Nodes $0 \ldots 4$ transmit/forward the signal using $\tau_0 \ldots \tau_4$ of the available time in each hop.

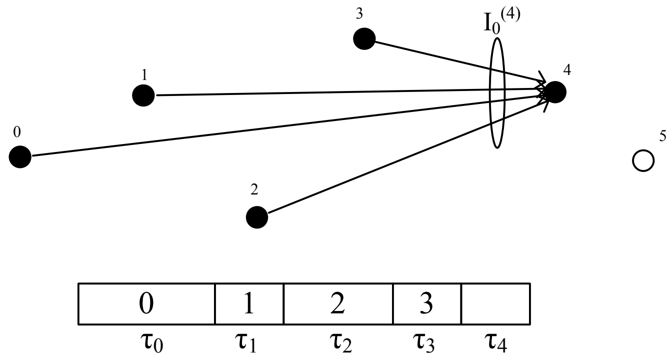


Figure 6.3: The accumulated rate at node 4 after 4 hops, $\mathcal{I}_0^{(4)}$.

$\mathcal{I}_0^{(2)}$ can be be copied from Chapter 3 using the appropriate notation,

$$\mathcal{I}_0^{(2)}(\boldsymbol{\tau}) = \tau_0 \log\left(\gamma_{0,2} + [1+\gamma_{1,2}]^{\frac{\tau_1}{\tau_0}}\right) \ \text{bit/sec/Hz.} \tag{6.3}$$

Similarly for $j = 3$, with help of Lemma 3.2,

$$\mathcal{I}_0^{(3)}(\boldsymbol{\tau}) = \tau_0 \log\left(\gamma_{0,3} + [1+\gamma_{1,3}]^{\frac{\tau_1}{\tau_0}} + [1+\gamma_{2,3}]^{\frac{\tau_2}{\tau_0}} - 1\right) \ \text{bit/sec.} \tag{6.4}$$

This can be generalized to any node j,

$$\mathcal{I}_0^{(j)} = \tau_0 \log\left(\sum_{i=0}^{j-1} (1+\gamma_{i,j})^{\frac{\tau_i}{\tau_0}} - j + 1\right). \tag{6.5}$$

The following proposition formalizes mutual information for the M-hop relay channel.

Proposition 6.1 (Mutual Information for Multi-Hop D&F Relay Channel)**.** *Consider a cooperative multi-hop relay channel where node* 0 *is the source node, node* M *is the destination node and nodes* $1, 2 \ldots M-1$ *are relay nodes with half-duplex constraint.* $\boldsymbol{\tau} = (\tau_0, \tau_1 \ldots \tau_{M-1})$ *is the corresponding time allocation. Mutual information between the source and the destination nodes is given by,*

$$\mathcal{I}_M(\boldsymbol{\tau}) = \min_{j \in \{1,2,\ldots,M\}} \mathcal{I}_0^{(j)}(\boldsymbol{\tau}), \ \textit{bit/sec/Hz} \tag{6.6}$$

where,

$$\mathcal{I}_0^{(j)} = \tau_0 \log_2\left(\sum_{i=0}^{j-1} (1+\gamma_{i,j})^{\frac{\tau_i}{\tau_0}} - j + 1\right). \tag{6.7}$$

6.2.1 Conversion Point for The Multi-Hop Relay Channel

Any time allocation, $\boldsymbol{\tau}$, must be such that $\tau_j \geq 0$ and $\sum_j \tau_j \leq 1$. In the following, concepts of operation line and conversion point are redefined for the multi-hop channel.

Definition 6.1 (Operation Line). *The operation line is the line* $\sum_{i=0}^{j-1} \tau_i = c$*, where* $c \in [0,1]$ *is proportion of the available time used for transmission.*

Unless otherwise specified, it is assumed that all available time is used for transmission. That is, always $c = 1$. Extension to $c < 1$ is straightforward.

Definition 6.2 (Conversion Point for Multi-Hop Channel, $\boldsymbol{\mu}$). *Conversion point for the multiple-hop D&F relay channel is the time allocation* $\boldsymbol{\mu} = (\mu_0, \mu_1 \ldots \mu_{M-1})$ *so that we have* $\mathcal{I}_M(\boldsymbol{\mu}) = \mathcal{I}_0^{(1)}(\boldsymbol{\mu}) = \ldots \mathcal{I}_0^{(M)}(\boldsymbol{\mu})$.

It can be shown that there can be no more than one conversion point per channel.

Lemma 6.1 (There is No More Than One Conversion Point per Channel). *For a given set of a channel's realization and a given operation line, there can be no more than one conversion point.*

Proof. From definition, $\boldsymbol{\mu}$ is the time allocation where $\mathcal{I}_0^{(1)}(\boldsymbol{\mu}), \mathcal{I}_0^{(2)}(\boldsymbol{\mu}), \ldots \mathcal{I}_0^{(M)}(\boldsymbol{\mu})$ intersect. We know from proof of Property 2 in Lemma 3.3 that $\mathcal{I}_0^{(1)}(\boldsymbol{\mu})$ and $\mathcal{I}_0^{(2)}(\boldsymbol{\mu})$ may have a maximum of two intersections, of which only one is an eligible conversion point. Therefore, there can be no more than one intersection points for $\mathcal{I}_0^{(1)}(\boldsymbol{\mu}), \mathcal{I}_0^{(2)}(\boldsymbol{\mu}), \ldots \mathcal{I}_0^{(M)}(\boldsymbol{\mu})$. □

We may classify the channel as suppressed or unsuppressed. A suppressed channel is the one which has no conversion point. We also define the conversion ratio vector associated with each unsuppressed channel.

Definition 6.3 (Conversion Ratio Vector, $\boldsymbol{\varsigma}$). *Conversion ratio vector for a multi-hop D&F relay channel is the vector* $\boldsymbol{\varsigma} = \frac{1}{\mu_0}\boldsymbol{\mu} = (1, \varsigma_1, \varsigma_2 \ldots \varsigma_{M-1})$.

The following lemma explains how to calculate $\boldsymbol{\varsigma}$ and conditions for its existence.

Lemma 6.2 (Properties of The Conversion Ratio Vector). *For an unsuppressed multi-hop relay channel* $(0, 1 \ldots M)$*,*

1. ς_j *is independent of the operation line and is given by,*

$$\varsigma_j = \frac{\log\left(1 + j + \gamma_{0,1} - \sum_{i=0}^{j-1}\left[1 + \gamma_{i,j+1}\right]^{\varsigma_i}\right)}{\log\left(1 + \gamma_{j,j+1}\right)}, \tag{6.8}$$

for $j = 0, 1, \ldots, M-1$.

2. ς_0 *always exists and is equal to* 1.

3. *ς_j, $j = 1, 2 \dots M-1$, exists only if $\varsigma_1, \varsigma_2 \dots \varsigma_{j-1}$ exist and the following inequality is satisfied,*

$$\sum_{k=0}^{i-1} \left[1 + \gamma_{k,i}\right]^{\varsigma_k} - i > \sum_{k=0}^{j-1} \left[1 + \gamma_{k,j+1}\right]^{\varsigma_k} - j \tag{6.9}$$

for all $i = 1, 2 \dots j$.

Proof. 1. From definition, at the conversion point $\mathcal{I}_0^{(1)}(\boldsymbol{\mu}) = \mathcal{I}_0^{(j+1)}(\boldsymbol{\mu})$, or,

$$\begin{aligned}
\mu_0 \log(1 + \gamma_{0,1}) &= \mu_0 \log \left(\sum_{i=0}^{j} \left[1 + \gamma_{i,j+1}\right]^{\frac{\mu_i}{\mu_0}} - j \right) \\
&= \mu_0 \log \left(-j + \left[1 + \gamma_{0,j+1}\right]^{\frac{\mu_0}{\mu_0}} + \left[1 + \gamma_{1,j+1}\right]^{\frac{\mu_1}{\mu_0}} \dots + \left[1 + \gamma_{j,j+1}\right]^{\frac{\mu_j}{\mu_0}} \right) \\
&= \mu_0 \log \left(-j + \left[1 + \gamma_{0,j+1}\right]^{\varsigma_0} + \left[1 + \gamma_{1,j+1}\right]^{\varsigma_1} \dots + \left[1 + \gamma_{j,j+1}\right]^{\varsigma_j} \right)
\end{aligned}$$

Solving for ς_j we get (6.8).

In fact there are several ways in which (6.8) can be formulated. In general ς_j can be expressed as a function of $\varsigma_0, \dots \varsigma_{j-1}$, $\gamma_{0,j+1} \dots \gamma_{j,j+1}$ and $\gamma_{0,i} \dots \gamma_{i-1,i}$; $0 < i \leq j$. By choosing $i = 1$ we get the simplest form in (6.8).

2. This is clear from the definition of conversion ratio vector.

3. This is obviously because ς_j is a function of $\varsigma_0, \varsigma_1 \dots \varsigma_{j-1}$ as stated by (6.8).

Equating $\mathcal{I}_0^{(i)}(\boldsymbol{\mu})$ and $\mathcal{I}_0^{(j+1)}(\boldsymbol{\mu})$, $0 < i \leq j$, we get,

$$\begin{aligned}
&\mu_0 \log \left(1 - i + \left[1 + \gamma_{0,i}\right]^{\varsigma_0} + \left[1 + \gamma_{1,i}\right]^{\varsigma_1} \dots + \left[1 + \gamma_{i-1,i}\right]^{\varsigma_{i-1}} \right) \\
&\qquad = \mu_0 \log \left(-j + \left[1 + \gamma_{0,j+1}\right]^{\varsigma_0} + \left[1 + \gamma_{1,j+1}\right]^{\varsigma_1} \dots + \left[1 + \gamma_{j,j+1}\right]^{\varsigma_j} \right)
\end{aligned}$$

Solving for ς_j we get,

$$\varsigma_j = \frac{\log \left(1 + (j - i) + \sum_{k=0}^{i-1} \left[1 + \gamma_{k,i}\right]^{\varsigma_k} - \sum_{k=0}^{j-1} \left[1 + \gamma_{k,j+1}\right]^{\varsigma_k} \right)}{\log(1 + \gamma_{j,j+1})} \tag{6.10}$$

Since it can be only a positive real number, then (6.9) must be satisfied for ς_j to exist.

□

6.2.1.1 Finding $\boldsymbol{\mu}$

$\boldsymbol{\mu}$ exists only if $\boldsymbol{\varsigma}$ exists. There are several way to find $\boldsymbol{\mu}$. One way to find $\boldsymbol{\mu}$ is by finding $\boldsymbol{\varsigma}$ first. $\boldsymbol{\varsigma}$ can be calculated using (6.8). We can then use,

$$\sum_{i=0}^{M-1} \mu = \mu_0 \sum_{i=0}^{M-1} \varsigma_i = 1. \tag{6.11}$$

to get,

$$\mu_0 = \frac{1}{\sum_{i=0}^{M-1} \varsigma_i} \tag{6.12}$$

Then $\boldsymbol{\mu}$ can simply be calculated using,

$$\boldsymbol{\mu} = \mu_0 \boldsymbol{\varsigma} \tag{6.13}$$

6.3 Average Mutual Information and Outage Probability

When links between nodes experience fading, $\mathcal{I}_M$ becomes a random variable. It is function of random variables $\gamma_{i,j}$, where $i, j \in \{0, 1 \ldots M\}$ and $i < j$. In that case, average mutual information ($\bar{\mathcal{I}}_M$) and outage probability ($\mathcal{P}_M$) are of greater interest. To evaluate $\bar{\mathcal{I}}_M$ and $\mathcal{P}_M$ we need to work out $\mathcal{F}_{\mathcal{I}_M}$, cdf for $\mathcal{I}_M$.

To find $\mathcal{F}_{\mathcal{I}_M}$, we follow techniques similar to those used in Section 3.3.1. We start by finding cdf for the minimum of sequences of random variables. Given random variables $X_1, X_2 \ldots X_M$, $Z = \min(X_1, X_2 \ldots X_M)$ has cdf,

$$\mathcal{F}_Z(z) = 1 - \prod_{i=1}^{M} \Big[1 - \mathcal{F}_{X_i}(z) \Big] \tag{6.14}$$

which implies that

$$\mathcal{F}_{\mathcal{I}_M}(\boldsymbol{\tau}, r) = 1 - \prod_{j=1}^{M} \Big[1 - \mathcal{F}_{\mathcal{I}_0^{(j)}}(\boldsymbol{\tau}, r) \Big]. \tag{6.15}$$

$\mathcal{F}_{\mathcal{I}_0^{(j)}}$ is the cdf for $\mathcal{I}_0^{(j)}$. To derive $\mathcal{F}_{\mathcal{I}_0^{(j)}}$ we find it useful to rewrite (6.5) in the form,

$$\mathcal{I}_0^{(j)} = \tau_0 \log \left(1 + \sum_{i=0}^{j-1} \Xi_{i,j} \right). \tag{6.16}$$

where

$$\Xi_{i,j} \triangleq (1 + \gamma_{i,j})^{\frac{\tau_i}{\tau_0}} - 1. \tag{6.17}$$

$i, j \in \{0, 1 \dots M\}$ and $i < j$

Next we consider $\Xi_{i,j}$. Since $\gamma_{i,j}$ is exponentially distributed with cdf,

$$\mathcal{F}_{\gamma_{i,j}}(x) = \begin{cases} 1 - e^{-\frac{x}{\Gamma_{i,j}}}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases} \tag{6.18}$$

then $\Xi_{i,j}$ is Weibull distributed with cdf and pdf respectively given by,

$$\mathcal{F}_{\Xi_{i,j}}(x) = \begin{cases} 1 - e^{-\frac{\left[(x+1)^{\frac{\tau_0}{\tau_i}} - 1\right]}{\Gamma_{i,j}}}, & \text{for } x \geq 0, \\ 0, & \text{for } x < 0. \end{cases} \tag{6.19}$$

and,

$$\mathbf{f}_{\Xi_{i,j}}(x) = \begin{cases} \frac{1}{\Gamma_{i,j}} \frac{\tau_0}{\tau_i} (x+1)^{\frac{\tau_0 - \tau_i}{\tau_i}} e^{-\frac{\left[(x+1)^{\frac{\tau_0}{\tau_i}} - 1\right]}{\Gamma_{i,j}}}, & \text{for } x \geq 0, \\ 0, & \text{for } x < 0. \end{cases} \tag{6.20}$$

obtainable by applying Lemma 3.6.

Now define $\Omega^{(j)} \triangleq \sum_{i=0}^{j-1} \Xi_{i,j}$. Then for independent channels, we have the following cdf for $\Omega^{(j)}$,

$$\mathcal{F}_{\Omega_0^{(j)}}(z) = \int_0^z \int_0^{z-x_0} \cdots \int_0^{z - \sum_{i=0}^{j-2} x_i} \prod_{i=0}^{j-1} \mathbf{f}_{\Xi_{i,j}}(x_i) \, \mathrm{d}x_{j-1} \cdots \mathrm{d}x_1 \mathrm{d}x_0. \tag{6.21}$$

supported over $z \geq 0$.

Eventually, notice that $\mathcal{I}_0^{(j)} = \tau_0 \log\left(1 + \Omega^{(j)}\right)$. It can be shown that,

$$\mathcal{F}_{\mathcal{I}_0^{(j)}}(z) = \mathcal{F}_{\Omega^{(j)}}\left(2^{\frac{z}{\tau_0}} - 1\right) \tag{6.22}$$

which reveals,

$$\begin{aligned} \mathcal{F}_{\mathcal{I}_0^{(j)}}(\boldsymbol{\tau}, r) = \int_0^{2^{\frac{r}{\tau_0}} - 1} \int_0^{2^{\frac{r}{\tau_0}} - 1 - x_0} \cdots \int_0^{2^{\frac{r}{\tau_0}} - 1 - \sum_{i=0}^{j-2} x_i} \\ \prod_{i=0}^{j-1} \mathbf{f}_{\Xi_{i,j}}(x_i) \, \mathrm{d}x_{j-1} \cdots \mathrm{d}x_1 \mathrm{d}x_0. \end{aligned} \tag{6.23}$$

supported over $r \geq 0$.

Finally, $\mathcal{F}_{\mathcal{I}_M}$ is found by substituting $\mathcal{F}_{\mathcal{I}_0^{(1)}}, \mathcal{F}_{\mathcal{I}_0^{(2)}} \dots \mathcal{F}_{\mathcal{I}_0^{(M)}}$ in (6.15). $\mathbf{f}_{\mathcal{I}_M}$ can be found by differentiating $\mathcal{F}_{\mathcal{I}_M}$.

$\bar{\mathcal{I}}_M$ can be calculated using,

$$\bar{\mathcal{I}}_M(\boldsymbol{\tau}) = \int_0^\infty x \mathbf{f}_{\mathcal{I}_M}(\boldsymbol{\tau}, x)\,\mathrm{d}x. \tag{6.24}$$

On the other hand, $\mathcal{P}_M$ is found using,

$$\mathcal{P}_M(\boldsymbol{\tau}, R) = \mathcal{F}_{\mathcal{I}_M}(\boldsymbol{\tau}, R). \tag{6.25}$$

Both (6.24) and (6.25) have no closed-form and can only be dealt with numerically.

6.4 Optimal Routing

When there is a number of relays available to aid transmission between a source and destination nodes, different channels can be formed by different subsets of the set of available relays. We refer to each of these channels as a route. Two characteristics distinguish one route from another:-

1. The set of relays taking part in transmission.

2. The order of relays taking part in transmission.

A relay is considered as a member of a route only if it is allocated time for transmission greater than 0.

Similar to Chapter 4, we seek the optimal routing strategy to,

1. Maximize mutual information.

2. Minimize total time for a given minimum rate.

From Chapter 4 we know that there is duality between these two problems. Solving one implies the solution for the other. Therefore, we focus on the maximization problem and later generalize to time minimization problem. We aim to:-

1. find the optimal route, that is, to find a set of relays chosen from the set of available relays and arrange them in a particular order for transmission; and

2. find the optimal time allocation for the chosen route,

so that no other route with any time allocation is able to achieve higher mutual information.

6.4.1 Optimum Time Allocation for The Unsuppressed Relay Channel

We start by considering the optimal time allocation for an unsuppressed relay channel. Focus is on unsuppressed routes only, since a suppressed route can never be the optimal route, as it will be shown later. This problem is a generalization to the problem worked out in Chapter 4. Some of the lessons learned there are useful in solving the current one. A mathematical expression for the problem is as follows,

$$\mathbb{P} \mapsto \begin{cases} \max\limits_{\boldsymbol{\tau}} & \mathcal{I}_M(\boldsymbol{\tau}) \\ \text{s.t.} & \tau_0, \ldots \tau_{M-1} > 0, \\ & \sum_{i=0}^{M-1} \tau_i \leq 1. \end{cases} \tag{6.26}$$

$\mathcal{I}_M^{\max}$ denotes the optimal value.

Solution to the maximization problem in Chapter 4 relied on convexity of $\mathcal{I}_0^{(1)}(\boldsymbol{\tau})$ and $\mathcal{I}_0^{(2)}(\boldsymbol{\tau})$. On the contrary, it can be shown that convexity of $\mathcal{I}_0^{(j)}(\boldsymbol{\tau})$ for $j > 2$ is subject to channel conditions. However, a similar solution to that of the three-node channel is obtained as stated in the following Lemma.

Lemma 6.3 (Optimal Time Allocation for an unsuppressed Multi-Hop Relay Channel)**.** *For an unsuppressed multi-hop relay channel, conversion point is the optimal time allocation.*

Proof. To prove this lemma we rely on the duality discussed in Section 4.4. We will try to allocate time optimally to minimize the total time and achieve mutual information equal to some rate R. If R can be chosen so that the minimum total time is equal to the available time, then in that case R is the maximum mutual information.

We follow a recursive approach to find the optimal time allocation to minimize the total time while achieving a small rate R. Aa similar method is used in [35] to optimally allocate power for a sequential multi-hop relay channel. Starting from node 1, the primary relay relies solely on direct transmission from the source node to decode the message. Thus time allocated to the source node must be long enough so that decoding rate at the primary relay is no less than R. That is to find minimum τ_0 so that $\mathcal{I}_0^{(1)}(\boldsymbol{\tau}) = \tau_0 \log(1+\gamma_{0,1}) \geq R$. We thus get,

$$\tau_0 = \frac{R}{\log(1+\gamma_{0,1})} \tag{6.27}$$

Relay node 2, on the other hand, is unable to decode the message at that rate (this is the case in unsuppressed channels). It needs repeated transmission from the primary relay in order to reliably decode the message. We thus allocate τ_1 so that decoding rate at relay node 2 is no less than R, or,

$$\tau_1 = \tau_0 \frac{\log(2^{\frac{R}{\tau_0}} - \gamma_{0,2})}{\log(1+\gamma_{1,2})} \tag{6.28}$$

In a similar manner τ_j is calculated using $\mathcal{I}_0^{(j+1)}(\boldsymbol{\tau}) = R$, $0 \leq j \leq M-1$. Thus time allocated to node j gives,

$$\tau_j = \tau_0 \frac{\log\left(1 + j + 2^{\frac{R}{\tau_0}} - \sum_{i=0}^{j-1}\left[1+\gamma_{i,j+1}\right]^{\frac{\tau_i}{\tau_0}}\right)}{\log\left(1+\gamma_{j,j+1}\right)} \tag{6.29}$$

The partial first derivative of $\mathcal{I}_0^{(j+1)}(\boldsymbol{\tau})$ with respect to τ_j, given by,

$$\frac{\partial}{\partial \tau_j}\mathcal{I}_0^{(j+1)}(\boldsymbol{\tau}) = \frac{(1+\gamma_{j,j+1})^{\frac{\tau_j}{\tau_0}} \log(1+\gamma_{j,j+1})}{\sum_{i=0}^{j}(1+\gamma_{j,j+1})^{\frac{\tau_j}{\tau_0}} - j} \geq 0, \tag{6.30}$$

reveals that $\mathcal{I}_0^{(j+1)}(\boldsymbol{\tau})$ is always an increasing function of τ_j. Hence (6.29) is the minimum time needed by node j to achieve R at the next node, node $j+1$. Consequently, total time in this case is the minimum time to achieve R.

R may be chosen small enough that the total time resulting from the above procedure is less than the available time. That is,

$$\sum_{i=0}^{j-1} \tau_j \leq 1. \tag{6.31}$$

If we gradually increase R and repeat the same allocation procedure we will eventually reach a point where R can not be increased. That is when (6.31) is satisfied with equality. Then we have $R = \mathcal{I}_M^{\max}$. Since we have,

$$\mathcal{I}_M(\boldsymbol{\tau}) = \mathcal{I}_0^{(1)}(\boldsymbol{\tau}) = \ldots \mathcal{I}_0^{(M)}(\boldsymbol{\tau}) = \mathcal{I}_M^{\max}, \tag{6.32}$$

then the conversion point is the optimal time allocation to maximize mutual information. □

For a suppressed route it can be shown that the optimal time allocation strategy involves allocating 0 seconds to some relays. In other words, a new route which is unsuppressed is created, to which time can be allocated as stated in Lemma 6.3.

6.4.2 Optimum Route

The optimal routing strategy can be found by a means of an exhaustive method. That is to,

1. Find all possible routes.

2. Try to calculate $\boldsymbol{\varsigma}$ and hence $\boldsymbol{\mu}$ if $\boldsymbol{\varsigma}$ exist, for each route. $\boldsymbol{\varsigma}$ and $\boldsymbol{\mu}$ exist only for unsuppressed routes.

3. Calculate maximum mutual information for each unsuppressed route using $\boldsymbol{\mu}$ obtained above.

4. The best routing strategy is the one which achieve the maximum mutual information from 3 above.

The main disadvantage of this exhaustive method, however, is that it is practical only when few relays are available. The total number of possible routes, Q, is given by,

$$Q = \sum_{k=0}^{n} \frac{n!}{(n-k)!} \tag{6.33}$$

where n is the total number of available relays. Q grows rapidly as n increases. For $n = 7$ we have $Q = 13,700$. If n increases to 10 we have $Q = 9,864,101$. Next we look into some of the relationships that exist between routes. Our aim is to make discovery of an optimal routing strategy more practical. Some necessary definitions are made next. With the aid of lemmas to come, a systematic optimal route discovery procedure is to be proposed, finally.

When all possible routes between a pair of nodes are considered, several routes may have the same number of hops if more than one relay is available. Next we define a route based on the nodes taking part on transmission and their order rather than the number of hops, as we did so far.

Definition 6.4 (Transmission Route, $\mathfrak{R}$)**.** *A transmission route is an ordered set of nodes such that the first node plays the role of the source node, the last node is a destination node and other nodes relay source's message in order.*

For example, in the preceding sections we considered route $\mathfrak{R} = (0, 1 \ldots M)$ where node 0 is the source node, node M the destination node and nodes $1, 2 \ldots M-1$ are relay nodes.

We use $\mathbb{R}$ to denote the set of all possible routes between a pair of nodes. We continue to refer to nodes in the route by their order of transmission. When necessary, reference to nodes in the route is made using names given to nodes if they are not in a particular order yet, or when more than one route is formed by the same nodes. For example, we may refer to the two three-hop routes formed by a source, a destination and a pair of relays by $(\mathsf{S}, \mathsf{R}_1, \mathsf{R}_2, \mathsf{D})$ and $(\mathsf{S}, \mathsf{R}_2, \mathsf{R}_1, \mathsf{D})$.

Definition 6.5 (Sub-Route, $\mathfrak{R}(i,j)$)**.** *With regard to route $\mathfrak{R}$, sub-route $\mathfrak{R}(i,j)$ is the transmission route that has node i as a source node, node j as a destination node and nodes $i+1, i+2 \ldots j-1$ as relays, $i, j \in [0, M]$.*

For example, given route $\mathfrak{R} = (0, 1, 2, 3, 4)$, $\mathfrak{R}(1,3) = (1, 2, 3)$. Notice that for route $\mathfrak{R} = (0, 1 \ldots M)$, $\mathfrak{R} = \mathfrak{R}(0, M)$. That is, any route is a sub-route of itself.

Definition 6.6 (Leading Sub-Route, $\mathfrak{R}(j)$)**.** *With regard to route $\mathfrak{R}$, the leading sub-route $\mathfrak{R}(j)$ is the the sub-route $\mathfrak{R}(0, j)$.*

The smallest leading sub-route of route $\mathfrak{R}$ is $\mathfrak{R}(0)$, which contains only the source node.

Lemma 6.4 (Properties of Leading Sub-Routes)**.** *Leading sub-routes have the following properties with regard to their parent route:-*

1. *If it exists, ς_i, $i \leq j-1$; for a leading sub-route $\mathfrak{R}(j)$ is the same as that for the parent route.*
2. *Maximum mutual information achievable by a route is less than or equal to that achievable by any of its leading sub-routes.*

Proof. 1. From (6.8), ς_i is dependent on $\varsigma_1, \ldots \varsigma_{i-1}$ and $\gamma_{0,i} \ldots \gamma_i$, $i \leq j$. The presence of nodes $j+1, \ldots M$ in the parent route has no effect on ς_i.

2. Consider a M-hop route $\mathfrak{R}$, $M \geq 1$. Mutual information for $\mathfrak{R}$ is given by (6.1). Mutual information for any leading sub-route $\mathfrak{R}(j)$, $j \leq M$, is given by,

$$\mathcal{I}_j\left(\boldsymbol{\tau}\right) = \min_{k \in \{1,2,\ldots,j\}} \mathcal{I}_0^{(k)}\left(\boldsymbol{\tau}\right) \tag{6.34}$$

where $\mathcal{I}_0^{(k)}$ is given by (6.5). That can be substituted in (6.1) to get,

$$\begin{aligned}\mathcal{I}_M\left(\boldsymbol{\tau}\right) &= \min\left\{\mathcal{I}_j\left(\boldsymbol{\tau}\right), \min_{k=j+1,\ldots,M} \mathcal{I}_0^{(k)}\left(\boldsymbol{\tau}\right)\right\} \\ &\leq \mathcal{I}_j\left(\boldsymbol{\tau}\right). \end{aligned} \tag{6.35}$$

□

Definition 6.7 (Child Route, $\mathfrak{R}^{\mathrm{ch}}(\mathbf{H})$)**.** *With regard to route $\mathfrak{R}$, child route $\mathfrak{R}^{ch}(\mathbf{H})$ is another route which has the same source, the same destination and same relays as $\mathfrak{R}$; but with some of the relays removed. $\mathbf{H}$ is the set of removed relays.*

In general, maximum mutual information achievable by a route differs from that achievable by any of its children. An exception occurs if two or more relays have the same set of SNR to other nodes. Removing one of them from the route does not affect mutual information. We assert that if two routes achieve the maximum possible mutual information, then both are optimum routes. However, if one of the optimum routes is a child of the other, then we consider the child route to be optimal. The following lemma is useful in reducing candidate routes for optimal routing.

Lemma 6.5 (Feasible Routes)**.** *The optimal route can only be unsuppressed.*

Proof. We prove this lemma by showing that for each suppressed route there is a child route which is unsuppressed and achieves higher mutual information than the parent route.

Take suppressed route $\mathfrak{R}$. ς for this route does not exist and hence there is no conversion point. Without loss of generality assume that ς_{M-1} does not exist while all ς_j, $j = 0, 1 \ldots M-2$, exist. In light of Lemma 6.2, that means condition (6.9) is violated for $i = j = M-1$. Hence we have,

$$\sum_{k=0}^{M-2} \left[1 + \gamma_{k,M-1}\right]^{\varsigma_k} - (M-1) \leq \sum_{k=0}^{M-2} \left[1 + \gamma_{k,M}\right]^{\varsigma_k} - (M-1) \tag{6.36}$$

Rearranging and manipulating the above inequality, we get,

$$\mu_0 \log \left(\sum_{k=0}^{M-2} \left[1 + \gamma_{k,M-1}\right]^{\varsigma_k} - M \right) \leq \mu_0 \log \left(\sum_{k=0}^{M-2} \left[1 + \gamma_{k,M}\right]^{\varsigma_k} - M \right) \tag{6.37}$$

The left hand side of the above inequality is the maximum achievable mutual information by $\mathfrak{R}(M-1)$, which is a leading sub-route of $\mathfrak{R}$. The right hand side is the mutual information achievable by $\mathfrak{R}^{\text{ch}}(\{M-1\})$ using some arbitrary time allocation. $\mathfrak{R}^{\text{ch}}(\{M-1\})$ is a child of $\mathfrak{R}$ formed by removing node $M-1$. From Lemma 6.4 we know that maximum mutual information achievable by a leading sub-route is greater than that achievable by the parent route. Combined with (6.37), that means maximum mutual information achievable by $\mathfrak{R}$ is indeed less than or equal to the maximum mutual information achievable by $\mathfrak{R}(\{M-1\})$. Consequently, the suppressed route $\mathfrak{R}$ can not be the optimum route.

To generalize, given any M-hop route $\mathfrak{R}$, if ς_j, $1 < j \leq M-1$; does not exist, then it can be shown that the maximum mutual information achievable by sub-route $\mathfrak{R}(j+1)$ is equal to or less than that achievable by the same sub-route after removing node j. That also implies that the maximum mutual information achievable by $\mathfrak{R}$ is less than or equal to that of the child route $\mathfrak{R}(\{j\})$. Consequently, we may end up removing more than one relay from $\mathfrak{R}$ so that the child route formed is an unsuppressed route and the maximum mutual information achievable by $\mathfrak{R}$ is equal to or less than that achievable by the newly formed child route. We thus conclude that a suppressed route can not be the optimal route. $\square$

The implication of Lemma 6.5 is that we can restrict our search to unsuppressed routes without loss of optimality. That greatly reduces the search space. Our strategy now is twofold. We will devise a technique to construct an unsuppressed route from a given set of relays, if that is possible. On the other hand, we try to recognize as many suppressed routes as possible so that they are eliminated from the set of feasible routes.

6.4.2.1 Constructing an Unsuppressed Route

Lemma 6.6 (Recursively Constructing an Unsuppressed Route)**.** *Given a source node* S*, a destination node* D *and set of relays* $\{\mathsf{R}_1, \mathsf{R}_2, \dots \mathsf{R}_N\}$*, an unsuppressed route,* $(0, 1, \dots N+1)$*, may only be formed by a means of recursive procedure. Starting form* S*, the next relay in route is chosen from the set of unallocated relays,* $\check{\mathbb{R}}$*, based on the sub-route constructed so far. Given the unsuppressed sub-route* $\mathfrak{R}(q-1)$*,* $q = 1, 2, \dots N$*; node* q *in route is the one to solve the following maximization problem,*

$$\mathbb{P} \mapsto \left\{ \max_{\mathsf{R}_i \in \check{\mathbb{R}}} \sum_{k=0}^{q-1} [1 + \gamma_{k,\mathsf{R}_i}]^{\varsigma_k} \right. \tag{6.38}$$

Procedure terminates unsuccessfully if,

$$\sum_{k=0}^{q-1} [1 + \gamma_{k,q}]^{\varsigma_k} \leq \sum_{k=0}^{q-1} [1 + \gamma_{k,\mathsf{D}}]^{\varsigma_k} \tag{6.39}$$

Proof. An unsuppressed route must have a conversion point. That requires existence of $\boldsymbol{\varsigma}$. From Lemma 6.2, we can see that existence of ς_q (associated with node q) relies on $\varsigma_0, \varsigma_1, \dots \varsigma_{q-1}$. That surely implies that relays can only be allocated one at a time, starting from the primary relay.

Likewise existence of ς_{q+a}, $a = 1, \dots N-q$, relies on ς_q. To satisfy (6.9) we must have,

$$\sum_{k=0}^{q-1} \left[1 + \gamma_{k,\mathsf{R}_i}\right]^{\varsigma_k} - q > \sum_{k=0}^{q+a-1} \left[1 + \gamma_{k,\mathsf{R}_j}\right]^{\varsigma_k} - (q+a). \tag{6.40}$$

where $\mathsf{R}_i, \mathsf{R}_j \in \check{\mathbb{R}}$. That yields,

$$\begin{aligned} \sum_{k=0}^{q-1} \left[1 + \gamma_{k,\mathsf{R}_i}\right]^{\varsigma_k} &> \sum_{k=0}^{q+a-1} \left[1 + \gamma_{k,\mathsf{R}_j}\right]^{\varsigma_k} - a \\ &\overset{(1)}{\geq} \sum_{k=0}^{q-1} \left[1 + \gamma_{k,\mathsf{R}_j}\right]^{\varsigma_k} \end{aligned} \tag{6.41}$$

where (1) is correct since $\sum_{k=q}^{q+a-1} \left[1 + \gamma_{k,\mathsf{R}_j}\right]^{\varsigma_k} \geq a$. Hence, R_i must satisfy (6.38) if other relays in $\check{\mathbb{R}}$ are to follow in route.

Condition (6.9) must also be satisfied at the destination. That is,

$$\sum_{k=0}^{i-1} \left[1 + \gamma_{k,i}\right]^{\varsigma_k} - i > \sum_{k=0}^{N} [1 + \gamma_{k,\mathsf{D}}]^{\varsigma_k} - (N+1) \tag{6.42}$$

for $i = 1, 2 \ldots N$. This condition is broken if,

$$\begin{aligned} \sum_{k=0}^{i-1} \left[1 + \gamma_{k,i}\right]^{\varsigma_k} &\leq \sum_{k=0}^{N} [1 + \gamma_{k,\mathsf{D}}]^{\varsigma_k} - (N - i + 1) \\ &\overset{(2)}{\leq} \sum_{k=0}^{i-1} [1 + \gamma_{k,\mathsf{D}}]^{\varsigma_k} \end{aligned} \tag{6.43}$$

where (2) is correct since $\sum_{k=i}^{N} \left[1 + \gamma_{k,\mathsf{R}_j}\right]^{\varsigma_k} \geq N - i + 1$. Hence for the route to be unsuppressed (6.39) must not occur. □

Corollary 6.1 (Consequences of Lemma 6.6)**.** *As a result of Lemma 6.6,*

1. *To form an unsuppressed route, the primary relay must have the best channel from the source node compared to other relays and the destination.*
2. *Direct transmission is optimal if the destination node has the best channel from the source.*
3. *A given set of relays can form no more than one unsuppressed route.*
4. *A given set of relays is not always capable of forming an unsuppressed route.*

Proof. 1. To assign the primary relay we need to solve (6.38) with $q = 1$. That is,

$$\mathbb{P} \mapsto \left\{ \max_{\mathsf{R}_i \in \check{\mathbb{R}}} \gamma_{k,\mathsf{R}_i} \right. \tag{6.44}$$

This part of the corollary can also be proved independent of Lemma 6.6. As a result of Lemma 6.2, for ς_j, $j = 1, 2 \ldots M$, to exist inequality (6.9) must be satisfied. Take $i = 1$, we then have,

$$\begin{aligned} \gamma_{0,1} &> \sum_{i=0}^{j-1} [1 + \gamma_{i,j+1}]^{\varsigma_i} - j \\ &\overset{(1)}{=} \gamma_{0,j} + \sum_{i=1}^{j-1} [1 + \gamma_{i,j+1}]^{\varsigma_i} - j + 1 \\ &\overset{(2)}{\geq} \gamma_{0,j} \end{aligned} \tag{6.45}$$

where (1) and (2) are true because $\varsigma_0 = 1$ and $\sum_{i=1}^{j-1} [1 + \gamma_{i,j+1}]^{\varsigma_i} - j + 1 \geq 0$, respectively.

2. By setting $q = 1$ in (6.39) that construction of the unsuppressed channel fails if,

$$\gamma_{0,1} \leq \gamma_{0,\mathsf{D}}. \tag{6.46}$$

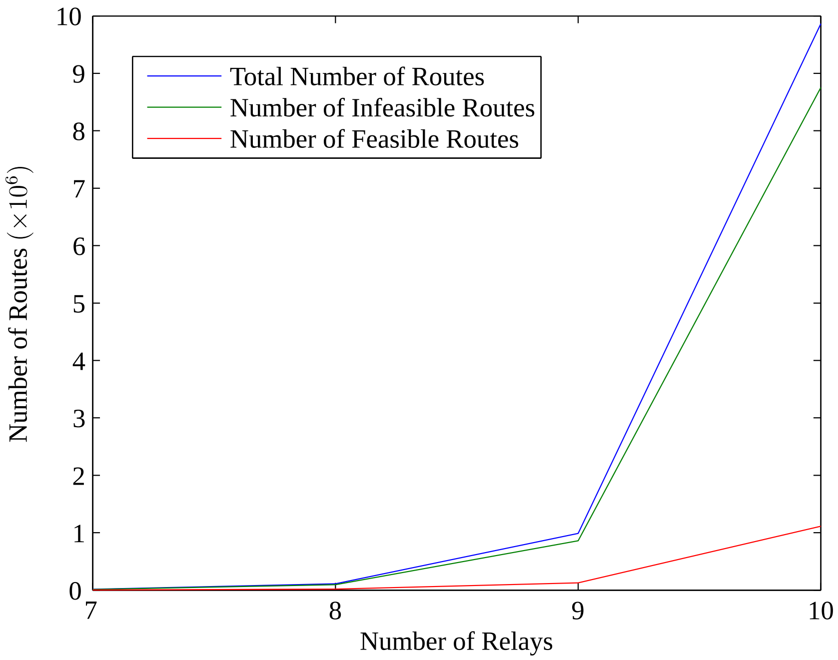


Figure 6.4: The number of candidate routes is reduced significantly by application of 1 in Corollary 6.1

In other words, there must be at least one relay with a channel from the source node better than that from the source to the destination, so that this relay can be assigned a primary relay role.

3. The recursive procedure proposed in Lemma 6.6 results in one and only one route.

4. Building an unsuppressed channel succeeds only if (6.39) is not encountered as each of the relays in $\mathbb{R}$ is being allocated.

□

Number 1 in the above corollary is particularly important. Its importance can be appreciated through an example. Assume $\mathsf{R}_1, \ldots \mathsf{R}_5$ are the available relays to help transmission from a source node S to a destination node D. $\gamma_{\mathsf{S},\mathsf{D}}$ is SNR from S to D. Further, assume that SNR from the source node to relays is such that $\gamma_{\mathsf{S},\mathsf{R}_1} > \gamma_{\mathsf{S},\mathsf{R}_2} > \gamma_{\mathsf{S},\mathsf{R}_3} > \gamma_{\mathsf{S},\mathsf{R}_4} > \gamma_{\mathsf{S},\mathsf{R}_5}$. If R_3, for example, is chosen to be the primary relay; then R_1 and R_2 can not be part of the route if we are trying to form an unsuppressed route. Any route which has R_3 as a primary relay is a suppressed route if it includes R_1 or R_2. As a result, the number of feasible routes considered is reduced from 16 according to (6.33) to 9. In general, if the primary relay is always appointed according to 1 in Corollary

6.1, the number of routes considered is given by,

$$\check{Q} = 1 + \sum_{j=0}^{n-1} \sum_{k=0}^{j} \frac{j!}{(j-k)!} \tag{6.47}$$

Figure 6.4 compares the total number of routes as given by (6.33) and the total number of routes after applying 1 in Corollary 6.1 in selecting the primary relay. We notice that application of the lemma significantly reduces the number of candidate routes.

6.4.2.2 Procedure to Find The Optimum Route

We are in a position to propose a procedure to find the optimum routing strategy.

We assume that there is a source node S which is willing to communicate with a destination node D. There is a set of relays $\{\mathsf{R}_1, \mathsf{R}_2, \dots \mathsf{R}_N\}$ available to help. The following procedure can be followed to find the optimum routing strategy.

1. We construct a list $\mathbf{R}_p$ which contains all eligible primary relays and the destination. According to 2 in Lemma 6.6, eligible primary relays are those which have signals from the source node better than that from the source node to the destination.

2. From the list formed in 1 above, each relay is selected once to be the primary relay.

3. Make a list of eligible secondary relays based on the chosen primary relay. According to 1 in Corollary 6.1, these are relays with SNR from the source which is not better than the primary relay.

4. Find all combinations of secondary relays.

5. For each combination of secondary relays try to construct an unsuppressed route following the procedure proposed by Lemma 6.6.

6. For each unsuppressed route successfully created in 5, find $\boldsymbol{\varsigma}$ and $\boldsymbol{\mu}$.

7. Calculate mutual information for each unsuppressed route constructed in 5 above.

8. Repeat steps 5-7 for each primary relay in the list.

9. Compare mutual information calculated each time in 7, to choose the best route.

6.4.3 Optimal Routing to Minimize Total Transmission Time

The two-hop relay channel studied in Chapter 3 and Chapter 4 is a special case of the M-hop channel studied in this chapter. Results presented in those chapters can be obtained here with $M = 2$. Consequently, we assume this extendibility applies to the

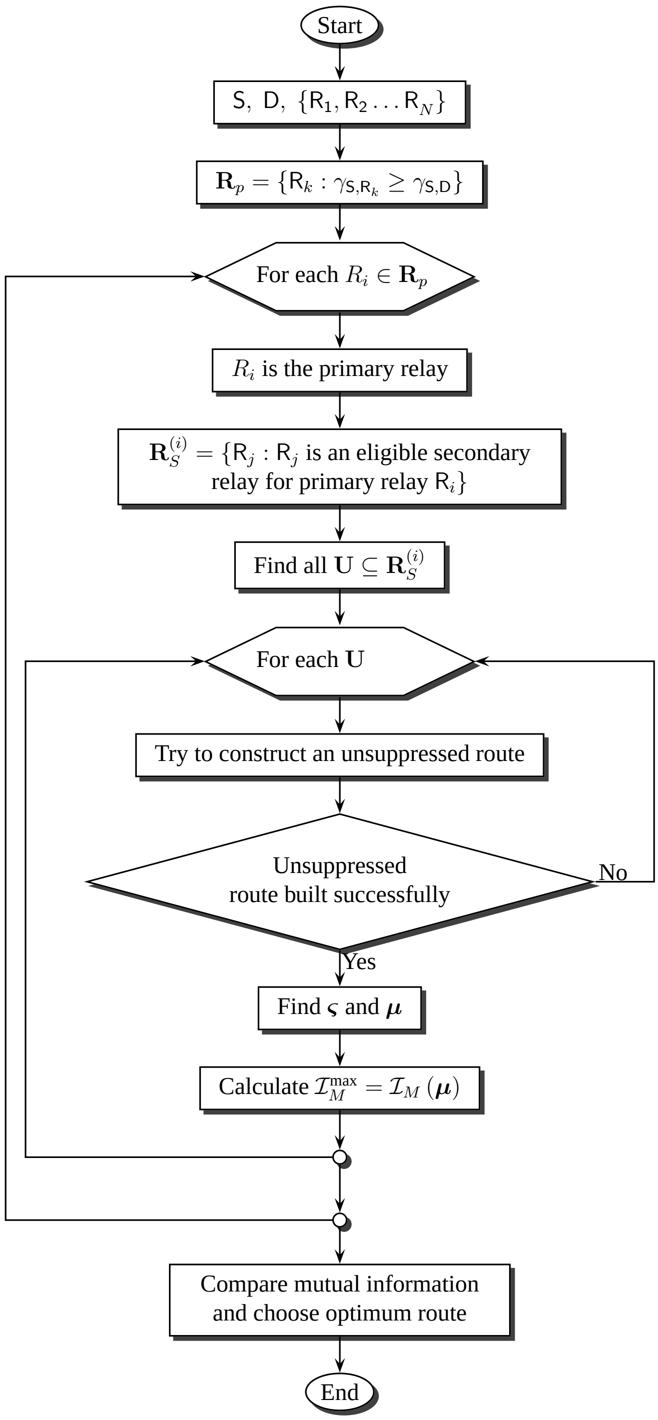


Figure 6.5: Flowchart explaining the optimum route selection procedure.

duality between optimization problems discussed earlier. As thus we take it as granted and seek no further proof.

In this section we seek the optimal routing strategy to minimize total transmission time while achieving minimum mutual information R. Taking advantage of the duality between optimization problems for the channel, we infer that the optimal route to maximize mutual information is also optimal for minimizing total transmission time. Further, optimal time allocation in the minimization problem is the conversion point on the operation line $\sum_i \tau_i = c^{\text{min}}$, i.e., $\boldsymbol{\mu}(c^{\text{min}})$.

Accordingly, the same techniques devised in previous sections can be applied equally to find the optimum route and $\boldsymbol{\mu}$. To find the optimal time allocation, recall that,

$$\boldsymbol{\mu}(c^{\text{min}}) = c^{\text{min}}\boldsymbol{\mu}. \tag{6.48}$$

which yields,

$$c^{\text{min}} = \frac{\mu_0(c^{\text{min}})}{\mu_0}. \tag{6.49}$$

$\tau_0 = \mu_0(c^{\text{min}})$ is the minimum time to allocate to the source node to achieve R at the primary relay. Thus

$$\mu_0(c^{\text{min}}) = \frac{R}{\log(1+\gamma_{0,1})}. \tag{6.50}$$

This readily implies a solution to the mutual information maximization over operation line $\sum_i \tau_i = c$.

6.4.4 Routing in The Physical Layer Versus Routing in The Network Layer

Conventionally, routing is seen as a network layer task. In the widely used cellular systems and alike, routing is taken care of by the wired part of the system. Wireless links connect base stations to end users, which are always a single-hop kind of links. The same routing protocols used for wired networks are continuing to work successfully in these networks [8]. With the emergence of other types of wireless networks which utilize multi-hop links, it was necessary to modify existing routing protocols [8], or devise new ones [75], taking into consideration aspects of transmission over the wireless medium and aspects of these new emerging networks. In particular, issues like fading, randomness in nodes' locations, non-existence or partial existence of infrastructure and rapid changes in topology due to nodes' mobility must be considered. Simulations showed that traditional routing protocols raise serious issues when applied to MANET [75].

In the thesis we considered routing in the physical layer. We are not novel in that as it was also considered by [35] and explicitly by [53]. With the emergence of cooperative techniques, it may be more appropriate to take cross-layer approaches when designing

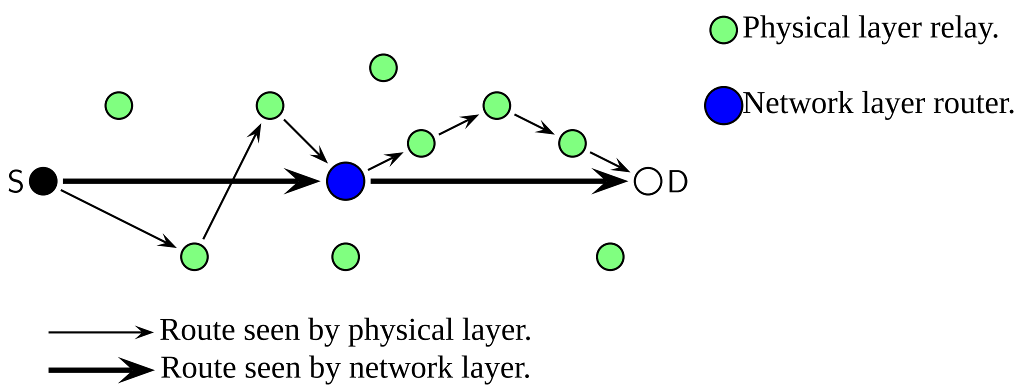


Figure 6.6: Example showing how relays and network routers can jointly establish a route from a source to a destination. From the network point of view there are only two links; one from the source to the router and another one from the router to the destination. The physical layer views it differently. Two multi-hop channels are made in the physical layer; one from the source node to the router and another from the router to the destination. Thus the router is sometimes considered as a destination and sometimes as a source node as viewed in the physical layer.

a system [47]. Cooperative routing considered in the thesis has the advantage of being maintainable. Cooperative transmission eliminates or reduces the number of single points of failure, a challenge to traditional network layer routing. Consequently, that leads to increased reliability and reduced overhead traffic needed to establish a new route every time nodes leave the network or change their locations.

One factor which makes routing in the physical layer possible is the the broadcast property of wireless transmission. Each node is virtually connected to all other nodes in the network without extra cost. Another factor is the ability of receivers to combine different signals to decode the transmitted message.

As routing in the network layer has its advantages too, choosing either method is subject to a considered scenario. In some cases it could be advantageous to combine both routing methods. Figure 6.6 shows an example where network routers and relays can possibly be joined together to maintain a connection between a source and a destination node. This way we reap advantages offered by each method.

6.5 Examples

The purpose of this section is to clarify some of the results in Section 6.2 and Section 6.3.

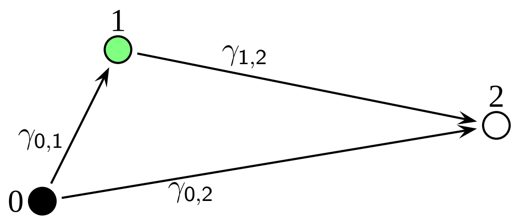


Figure 6.7: Example, two-hop relay channel.

6.5.1 Two-Hop Relay Channel

In this section we reproduce results stated in Chapter 3 and Chapter 4 using notation devised for this chapter. In the three-hop relay channel shown in Figure 6.7, node 0 is the source node, node 2 is the destination node and node 1 is the relay node. Given time allocation $\boldsymbol{\tau} = (\tau_0, \tau_1)$, mutual information between source and destination is,

$$\mathcal{I}_2(\boldsymbol{\tau}) = \min\left(\mathcal{I}_0^{(1)}(\boldsymbol{\tau}), \mathcal{I}_0^{(2)}(\boldsymbol{\tau})\right), \text{ bit/sec/Hz} \tag{6.51}$$

where,

$$\begin{cases} \mathcal{I}_0^{(1)}(\boldsymbol{\tau}) = \tau_0 \log(1+\gamma_{0,2}), \\ \mathcal{I}_0^{(2)}(\boldsymbol{\tau}) = \tau_0 \log\left(\gamma_{0,2} + [1+\gamma_{1,2}]^{\frac{\tau_1}{\tau_0}}\right). \end{cases} \tag{6.52}$$

According to Lemma 6.2, $\varsigma_0 = 1$. ς_1 exists if inequality (6.9) is correct for $j = 1$ and $i = 1$. I.e., if

$$\gamma_{0,1} > \gamma_{0,2}. \tag{6.53}$$

If $\boldsymbol{\varsigma}$ exists, then we may find the conversion point. First, (6.8) is used to find ς_1,

$$\varsigma_1 = \frac{\log(1+\gamma_{0,1}-\gamma_{0,2})}{\log(1+\gamma_{1,2})} \tag{6.54}$$

Then μ_0 is calculated using (6.12),

$$\mu_0 = \frac{1}{1+\varsigma_1}, \tag{6.55}$$

Finally (6.13) is used to calculate $\boldsymbol{\mu}$.

If channels are Rayleigh fading, we may find distribution and density function in order to evaluate average mutual information and outage probability. From (6.15) we have,

$$\mathcal{F}_{\mathcal{I}_2}(\boldsymbol{\tau}, r) = 1 - \left[1 - \mathcal{F}_{\mathcal{I}_0^{(1)}}(\boldsymbol{\tau}, r)\right]\left[1 - \mathcal{F}_{\mathcal{I}_0^{(2)}}(\boldsymbol{\tau}, r)\right]. \tag{6.56}$$

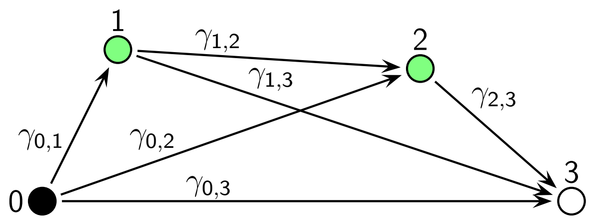


Figure 6.8: Example, three-hop relay channel.

To find $\mathcal{F}_{\mathcal{I}_2}$ it is thus necessary to find $\mathcal{F}_{\mathcal{I}_0^{(1)}}$ and $\mathcal{F}_{\mathcal{I}_0^{(2)}}$. From (6.23),

$$\begin{aligned}\mathcal{F}_{\mathcal{I}_0^{(1)}}(\boldsymbol{\tau}, r) &= \int_0^{2^{\frac{r}{\tau_0}}-1} \mathbf{f}_{\Xi_{0,1}}(x_0)\,\mathrm{d}x_0 \\ &= \mathcal{F}_{\Xi_{0,1}}\left(2^{\frac{r}{\tau_0}}-1\right) \\ &= 1 - e^{-\frac{\left[2^{\frac{r}{\tau_1}}-1\right]}{\Gamma_{0,1}}}. \end{aligned} \tag{6.57}$$

and

$$\begin{aligned}\mathcal{F}_{\mathcal{I}_0^{(2)}}(\boldsymbol{\tau}, r) &= \int_0^{2^{\frac{r}{\tau_0}}-1}\int_0^{2^{\frac{r}{\tau_0}}-1-x_0} \mathbf{f}_{\Xi_{0,2}}(x_0)\,\mathbf{f}_{\Xi_{1,2}}(x_1)\,\mathrm{d}x_1\,\mathrm{d}x_0 \\ &= \int_0^{2^{\frac{r}{\tau_0}}-1} \mathbf{f}_{\Xi_{0,2}}(x_0)\,\mathcal{F}_{\Xi_{1,2}}(2^{\frac{r}{\tau_0}}-1-x_0)\mathrm{d}x_0 \\ &= 1 - e^{-\frac{\left[2^{\frac{r}{\tau_0}}-1\right]}{\Gamma_{0,2}}} \\ &\qquad - \frac{1}{\Gamma_{0,2}}\int_0^{2^{\frac{r}{\tau_0}}-1} \exp\left(-\frac{\left(2^{\frac{r}{\tau_0}}-x_0\right)^{\frac{\tau_0}{\tau_1}}-1}{\Gamma_{1,2}} - \frac{x_0}{\Gamma_{0,2}}\right)\mathrm{d}x_0. \end{aligned} \tag{6.58}$$

$\mathcal{F}_{\mathcal{I}_2}$ is then obtained by substituting $\mathcal{F}_{\mathcal{I}_0^{(1)}}$ and $\mathcal{F}_{\mathcal{I}_0^{(2)}}$ in (6.56) which gives,

$$\begin{aligned}\mathcal{F}_{\mathcal{I}_2}(\boldsymbol{\tau}, r) = 1 - \exp&\left(-\left(\tfrac{1}{\Gamma_{0,2}}+\tfrac{1}{\Gamma_{0,1}}\right)\left(2^{\frac{r}{\tau_0}}-1\right)\right) \\ &- \frac{1}{\Gamma_{0,2}}e^{-\frac{2^{\frac{r}{\tau_0}}-1}{\Gamma_{0,1}}}\int_0^{2^{\frac{r}{\tau_0}}-1}\exp\left(-\frac{\left(2^{\frac{r}{\tau_0}}-x_0\right)^{\frac{\tau_0}{\tau_1}}-1}{\Gamma_{1,2}} - \frac{x_0}{\Gamma_{0,2}}\right)\mathrm{d}x_0, \end{aligned} \tag{6.59}$$

6.5.2 Three-Hop Relay Channel

In the three-hop relay channel shown in Figure 6.8, node 0 is the source node, node 3 is the destination node and nodes 1 and 2 are relay nodes. Given $\boldsymbol{\tau} = (\tau_0, \tau_1, \tau_2)$, mutual information between source and destination is given by,

$$\mathcal{I}_3(\boldsymbol{\tau}) = \min\left(\mathcal{I}_0^{(1)}(\boldsymbol{\tau}), \mathcal{I}_0^{(2)}(\boldsymbol{\tau}), \mathcal{I}_0^{(3)}(\boldsymbol{\tau})\right), \text{ bit/sec/Hz} \tag{6.60}$$

where,

$$\begin{cases} \mathcal{I}_0^{(1)}(\boldsymbol{\tau}) = \tau_0 \log(1+\gamma_{0,2}), \\ \mathcal{I}_0^{(2)}(\boldsymbol{\tau}) = \tau_0 \log\left(\gamma_{0,2} + [1+\gamma_{1,2}]^{\frac{\tau_1}{\tau_0}}\right), \\ \mathcal{I}_0^{(3)}(\boldsymbol{\tau}) = \tau_0 \log\left(\gamma_{0,3} + [1+\gamma_{1,3}]^{\frac{\tau_1}{\tau_0}} + [1+\gamma_{2,3}]^{\frac{\tau_2}{\tau_0}} - 1\right). \end{cases} \tag{6.61}$$

We have $\varsigma_0 = 1$. ς_1 exists if inequality (6.9) is correct for $j = 1$ and $i = 1$. I.e., if

$$\gamma_{0,1} > \gamma_{0,2}. \tag{6.62}$$

ς_2 exists if ς_1 exists and (6.9) is satisfied for $j = 1$ and $i = 1, 2$. That is,

$$\gamma_{0,1} > \gamma_{0,3} + [1+\gamma_{1,3}]^{\varsigma_1} - 1, \tag{6.63}$$

$$\gamma_{0,2} + [1+\gamma_{1,2}]^{\varsigma_1} - 1 > \gamma_{0,3} + [1+\gamma_{1,3}]^{\varsigma_1} - 1. \tag{6.64}$$

Existence of $\boldsymbol{\varsigma}$ means there is a conversion point. We first find ς_1 and ς_2 using (6.8). We have,

$$\varsigma_1 = \frac{\log(1+\gamma_{0,1}-\gamma_{0,2})}{\log(1+\gamma_{1,2})} \tag{6.65}$$

and

$$\varsigma_2 = \frac{\log\left(2+\gamma_{0,1}-\gamma_{0,3}-[1+\gamma_{1,3}]^{\varsigma_1}\right)}{\log(1+\gamma_{2,3})} \tag{6.66}$$

Then μ_0 is calculated using (6.12),

$$\mu_0 = \frac{1}{1+\varsigma_1+\varsigma_2} \tag{6.67}$$

Finally (6.13) is used to calculate $\boldsymbol{\mu}$.

If channels are Rayleigh fading, we may find distribution and density function in order to evaluate average mutual information and outage probability. From (6.15) we have,

$$\mathcal{F}_{\mathcal{I}_3}(\boldsymbol{\tau}, r) = 1 - \left[1 - \mathcal{F}_{\mathcal{I}_0^{(1)}}(\boldsymbol{\tau}, r)\right]\left[1 - \mathcal{F}_{\mathcal{I}_0^{(2)}}(\boldsymbol{\tau}, r)\right]\left[1 - \mathcal{F}_{\mathcal{I}_0^{(3)}}(\boldsymbol{\tau}, r)\right]. \tag{6.68}$$

To find $\mathcal{F}_{\mathcal{I}_2}$ it is thus necessary to find $\mathcal{F}_{\mathcal{I}_0^{(1)}}$, $\mathcal{F}_{\mathcal{I}_0^{(2)}}$ and $\mathcal{F}_{\mathcal{I}_0^{(3)}}$. $\mathcal{F}_{\mathcal{I}_0^{(1)}}$ and $\mathcal{F}_{\mathcal{I}_0^{(2)}}$ are same as in (6.57) and (6.58). From (6.23),

$$\begin{aligned}\mathcal{F}_{\mathcal{I}_0^{(3)}}(\boldsymbol{\tau}, r) &= \int_0^{2^{\frac{r}{\tau_0}}-1} \int_0^{2^{\frac{r}{\tau_0}}-1-x_0} \int_0^{2^{\frac{r}{\tau_0}}-1-x_0-x_1} \mathbf{f}_{\Xi_{0,3}}(x_0)\, \mathbf{f}_{\Xi_{1,3}}(x_1)\, \mathbf{f}_{\Xi_{2,3}}(x_2)\, \mathrm{d}x_2\, \mathrm{d}x_1\, \mathrm{d}x_0 \\ &= \int_0^{2^{\frac{r}{\tau_0}}-1} \int_0^{2^{\frac{r}{\tau_0}}-1-x_0} \mathbf{f}_{\Xi_{0,3}}(x_0)\, \mathbf{f}_{\Xi_{1,3}}(x_1)\, \mathcal{F}_{\Xi_{2,3}}(2^{\frac{r}{\tau_0}} - 1 - x_0 - x_1) \mathrm{d}x_1\, \mathrm{d}x_0 \\ &= 1 - e^{-\frac{\left[2^{\frac{r}{\tau_0}}-1\right]}{\Gamma_{0,3}}} - \frac{1}{\Gamma_{0,3}} \int_0^{2^{\frac{r}{\tau_0}}-1} \exp\left(-\frac{\left(2^{\frac{r}{\tau_0}} - x_0\right)^{\frac{\tau_0}{\tau_1}} - 1}{\Gamma_{1,3}} - \frac{x_0}{\Gamma_{0,3}}\right) \mathrm{d}x_0 \\ &\quad - \frac{1}{\Gamma_{0,3}\Gamma_{1,3}} \frac{\tau_0}{\tau_1} \int_0^{2^{\frac{r}{\tau_0}}-1} \int_0^{2^{\frac{r}{\tau_0}}-1-x_0} (x_1+1)^{\frac{\tau_0-\tau_1}{\tau_1}} \\ &\qquad \exp\left(-\frac{\left(2^{\frac{r}{\tau_0}} - x_0 - x_1\right)^{\frac{\tau_0}{\tau_2}} - 1}{\Gamma_{2,3}} - \frac{(x_1+1)^{\frac{\tau_0}{\tau_1}} - 1}{\Gamma_{1,3}} - \frac{x_0}{\Gamma_{0,3}}\right) \mathrm{d}x_1 \mathrm{d}x_0. \end{aligned} \tag{6.69}$$

$\mathcal{F}_{\mathcal{I}_3}$ is then obtained by substituting $\mathcal{F}_{\mathcal{I}_0^{(1)}}$, $\mathcal{F}_{\mathcal{I}_0^{(2)}}$ and $\mathcal{F}_{\mathcal{I}_0^{(3)}}$ in (6.68).

6.6 Chapter Summary

In this chapter, results obtained in Chapter 3 and Chapter 4 are generalized for relay channels with any number of hops. Mutual information was first to consider. Channel classification into suppressed and unsuppressed has continued into this chapter, in addition to the concept of conversion point. In fading channels, average mutual information and outage probability are worked out after evaluating the cdf. Optimum routing strategy was also considered. Finally some examples are given for illustration purposes.

Chapter 7

Conclusions

In this chapter, the thesis is summarized, contributions are highlighted and potential future research is discussed.

7.1 Summary of The Thesis and Contributions

The thesis studied the D&F relay channel with half-duplex constraint on the relay(s). Relay channels are seen as a means to improve link reliability and increase throughput through spatial diversity. Due to hardware restrictions on wireless devices, relay nodes can only be operated on the half-duplex mode. D&F relaying over expandability has advantages when compared with its rival, A&F. Nevertheless, both suffer from inefficient utilization of spectrum resources due to the repetition nature.

The thesis aimed at producing analytical results to quantify channel performance, taking into consideration time allocation. In the thesis, mutual information is regarded as the primary performance measure for the AWGN channel. In the case of Rayleigh fading, average mutual information and outage probability are considered. Furthermore, optimal time allocation is sought for optimal operation. Results are always generated for the single relay two-hop channel and then generalized for any number of hops and a single relay per hop. A cooperating scheme was also proposed based on the three-node relay channel.

Here follows a list of contributions,

- A fundamental contribution of the thesis is to evaluate mutual information for the D&F relay channel for any arbitrary time allocation and any number of hops.

- Introduction of the conversion point concept and consequent classification of the channel into suppressed and unsuppressed channels offers a novel way to view the channel and understand the way it behaves.

- The working out of the distribution functions of mutual information in Rayleigh channels is an important tool to analyze the channel.

- As a result of obtaining distribution functions for the mutual information, average mutual information and outage probability are also evaluated.
- A solution is given to two optimization problems for the channel. Optimum time allocation is found to maximize mutual information as well as to minimize total transmission time.
- An important duality is established between optimization problems. It was shown that solution to one problem implies the solution for the other.
- Taking advantage of solutions to optimization problems for the three-node channel and duality between optimization problems, it was possible to generalize for the problem of optimal routing. A useful procedure is devised to significantly reduce the computational effort needed to find the optimal route when a large number of relays is available.
- A two-user cooperative model was proposed based on the three-node channel.
- Application of the two-user cooperative model to a testbed multi-user network demonstrated the concept of user-cooperative communication. More importantly, a framework is established addressing issues associated with user cooperation schemes such as partnership selection, fairness and performance analysis.

7.2 Future Work

Work presented in the thesis can be extended in several directions:-

- The thesis focused on producing analytical results. A more thorough analysis is needed to evaluate the feasibility of relaying in wireless networks, given the half-duplex constraint. In particular, more simulation results are needed to compare the optimum scheme with direct transmission and equal time allocation policies to establish the conditions under which one of them can be selected. Results presented in the thesis are the tools needed to carry out such work.
- Throughout the thesis we assumed a naive power policy and focused on time allocation. Results produced are very useful for high SNR regimes where spectrum is scarce. On the other hand, systems with optimized transmission power, studied in many other works, are useful for low SNR regimes where it is assumed that unlimited spectrum is available. It is useful however to consider globally optimized systems, that is, systems with optimized power and time allocation.
- Results for Rayleigh fading channel are not in a closed-form. If these results can be obtained in a closed-form, it can make them more practical. Otherwise, approximation or upper/lower bound can be useful as well.

- Optimization carried out in the thesis produced solutions for the AWGN channel. These results are also useful for Rayleigh fading channels in the case of CSIT. It is also important to work out optimal or sub-optimal solutions for Rayleigh channels with only SCI at transmitters.

- Chapter 5 is important in that it offers a framework to work with user-cooperative systems. More useful cooperative schemes can be proposed. For example, there can be cooperating scenarios where some of the restrictions on partnership selection are relaxed. Optimum power and time allocation can also lead to more efficient cooperation.

- In complex communication systems such as the cooperative systems studied in the thesis, a cross-layer approach is necessary when analyzing the system. Procedures and techniques followed in the physical layer affect performance as measured from the data link layer or network layer perspective. For example, gain from the cooperative scheme proposed in Chapter 5 is somehow unrealistic unless overhead traffic introduced by partnership selection procedure is taken into account. Moreover, the rate at which network topology changes due to the node's mobility must be taken into account. Rapidly changing networks produce more overhead traffic. Mobility is considered to be a network layer property where, several mobility models are already proposed. Another example which explains the importance of cross-layer analysis is routing, as explained in Chapter 6.

- Although routing procedure proposed in Chapter 6 has significantly reduced efforts to find the optimum route, it might not be the best. More investigation could reveal an even cheaper one.

- All work on the relay channel contributes directly or indirectly to the efforts for finding capacity of the relay channel. Moreover, achievements on the relay case contribute to the theory of communication networks.

Appendix A

Second Derivative of $\mathcal{I}_0(\tau_\mathsf{R})$

In this appendix we seek $\mathrm{d}^2/\mathrm{d}\tau_\mathsf{R}^2\ \mathcal{I}_0(\tau_\mathsf{R})$ necessary to prove convexity of $\mathcal{I}_0(\tau_\mathsf{R})$ as stated in Lemma 4.1 in Section 4.2. The problem is to find,

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau_\mathsf{R}^2}\mathcal{I}_0\left(\tau_\mathsf{R}\right) = \frac{\mathrm{d}^2}{\mathrm{d}\tau_\mathsf{R}^2}(1-\tau_\mathsf{R})\log\left(\gamma_{\mathsf{S,D}} + \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\tau_\mathsf{R}}{1-\tau_\mathsf{R}}}\right). \tag{A.1}$$

The following differentiation rules are used,

1. $\frac{\mathrm{d}}{\mathrm{d}x}\log_a x = \frac{1}{x\ln a}$.

2. $\frac{\mathrm{d}}{\mathrm{d}x}a^x = a^x \ln a$.

- First derivative,

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}\tau_\mathsf{R}}(1-\tau_\mathsf{R})\log\left(\gamma_{\mathsf{S,D}} + \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\tau_\mathsf{R}}{1-\tau_\mathsf{R}}}\right) \\
&\quad = (1-\tau_\mathsf{R}) \times \frac{\left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\tau_\mathsf{R}}{1-\tau_\mathsf{R}}}\log\left(1+\gamma_{\mathsf{R,D}}\right)}{\left(\gamma_{\mathsf{S,D}} + \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\tau_\mathsf{R}}{1-\tau_\mathsf{R}}}\right)(1-\tau_\mathsf{R})^2} - \log\left(\gamma_{\mathsf{S,D}} + \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\tau_\mathsf{R}}{1-\tau_\mathsf{R}}}\right) \\
&\quad = \log\left(1+\gamma_{\mathsf{R,D}}\right)\frac{\left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\tau_\mathsf{R}}{1-\tau_\mathsf{R}}}}{\left(\gamma_{\mathsf{S,D}} + \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\tau_\mathsf{R}}{1-\tau_\mathsf{R}}}\right)(1-\tau_\mathsf{R})} - \log\left(\gamma_{\mathsf{S,D}} + \left[1+\gamma_{\mathsf{R,D}}\right]^{\frac{\tau_\mathsf{R}}{1-\tau_\mathsf{R}}}\right).
\end{aligned} \tag{A.2}$$

- Second derivative,

$$
\begin{aligned}
&\frac{\mathrm{d}^2}{\mathrm{d}\tau_{\mathsf{R}}^2}(1-\tau_{\mathsf{R}})\log\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)\\
&\quad=\frac{\mathrm{d}}{\mathrm{d}\tau_{\mathsf{R}}}\frac{[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\log\left(1+\gamma_{\mathsf{R,D}}\right)}{\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)(1-\tau_{\mathsf{R}})}-\log\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)\\
&\quad=[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\log(1+\gamma_{\mathsf{R,D}})\frac{\gamma_{\mathsf{S,D}}\log(1+\gamma_{\mathsf{R,D}})\ln 2+\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)(1-\tau_{\mathsf{R}})}{\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)^2(1-\tau_{\mathsf{R}})^3}\\
&\qquad-\frac{[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\log\left(1+\gamma_{\mathsf{R,D}}\right)}{\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)(1-\tau_{\mathsf{R}})^2}\\
&\quad=\frac{[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\log(1+\gamma_{\mathsf{R,D}})}{\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)^2(1-\tau_{\mathsf{R}})^3}\Bigg[\gamma_{\mathsf{S,D}}\log(1+\gamma_{\mathsf{R,D}})\ln 2\\
&\qquad\qquad+\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)(1-\tau_{\mathsf{R}})\\
&\qquad\qquad-\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)(1-\tau_{\mathsf{R}})\Bigg]\\
&\quad=\frac{[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\gamma_{\mathsf{S,D}}\left(\log(1+\gamma_{\mathsf{R,D}})\right)^2\ln 2}{\left(\gamma_{\mathsf{S,D}}+[1+\gamma_{\mathsf{R,D}}]^{\frac{\tau_{\mathsf{R}}}{1-\tau_{\mathsf{R}}}}\right)^2(1-\tau_{\mathsf{R}})^3}.
\end{aligned}
\tag{A.3}
$$

Appendix B

Derivation of $E[d^\alpha]$

The network simulated in Section 5.3.2 is parameterized by SNR. SNR is a function of $E[d^\alpha]$ where α is a constant representing the path loss exponent and d is a random variable representing the distance between any two nodes. In this appendix we demonstrate how to find $E[d^\alpha]$ for a particular network. In general

$$E[Z^m] = \int_{-\infty}^{\infty} z^m f_Z(z) \mathrm{d}z, \tag{B.1}$$

where f_Z is the pdf for the random variable Z.

On the other hand, the distance between any two nodes i and j is

$$d = \sqrt{(X_\mathrm{i} - X_\mathrm{j})^2 + (Y_\mathrm{i} - Y_\mathrm{j})^2},$$

where ordered pairs $(X_\mathrm{i}, Y_\mathrm{i})$ and $(X_\mathrm{j}, Y_\mathrm{j})$ mark the location of the nodes. If nodes are randomly positioned, as they are here, then d is a random variable with a distribution determined by the distribution of X_i, Y_i, X_j and Y_j. It is assumed in the simulated network that nodes i and j are positioned according to a Gaussian distribution, i.e., $X_\mathrm{i}, X_\mathrm{j}, Y_\mathrm{i}, Y_\mathrm{j} \sim \mathrm{N}(0, \sigma^2)$. Consequently, $(X_\mathrm{i} - X_\mathrm{j})$ and $(Y_\mathrm{i} - Y_\mathrm{j})$ also have zero-mean Gaussian distribution with variance $2\sigma^2$. As a result d is Rayleigh distributed: $d \sim \mathrm{Rayleigh}\left(\sqrt{2}\sigma\right)$. Applying the moment's formula for Rayleigh distributed random variables found in [74],

$$E[d^\alpha] = (4\sigma^2)^{\frac{\alpha}{2}} \Gamma\left(1 + \tfrac{\alpha}{2}\right) \tag{B.2}$$

where, $\Gamma(.)$ is the Gamma function. See also Problem 8 in [16] for a different case where source nodes have a different distribution from that of the destination nodes.

Appendix C

Derivation of $\mathcal{F}_\Omega$ and $\mathbf{f}_\Omega$

This appendix explains how to apply Lemma 3.7 in order to get $\mathcal{F}_\Omega$ and $\mathbf{f}_\Omega$. We have ,

$$\Omega \triangleq \gamma_{\mathsf{S,D}} + \Xi. \tag{C.1}$$

where, $\gamma_{\mathsf{S,D}}$ and Ξ are independent random variables. cdf and pdf for $\gamma_{\mathsf{S,D}}$ are given by,

$$\mathcal{F}_{\gamma_{\mathsf{S,D}}}(x) = \begin{cases} 1 - e^{-\frac{x}{\Gamma_{\mathsf{S,D}}}}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0, \end{cases} \tag{C.2}$$

and

$$\mathbf{f}_{\gamma_{\mathsf{S,D}}}(x) = \begin{cases} \frac{1}{\Gamma_{\mathsf{S,D}}} e^{-\frac{x}{\Gamma_{\mathsf{S,D}}}}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0, \end{cases} \tag{C.3}$$

respectively. Whereas cdf and pdf for Ξ are given by (3.60) and (3.61), respectively.

C.1 Finding $\mathcal{F}_\Omega$

From (C.1) and Lemma 3.7 we have,

$$\begin{aligned}
\mathcal{F}_\Omega(z) &= \int_{-\infty}^{\infty} \int_{-\infty}^{z-y} \mathbf{f}_{\Xi,\gamma_{\mathsf{S},D}}(x, y) \mathrm{d}x \mathrm{d}y \\
&= \int_{0}^{z-1} \int_{1}^{z-y} \mathbf{f}_{\Xi}(x) \mathbf{f}_{\gamma_{\mathsf{S},D}}(y) \mathrm{d}x \mathrm{d}y \\
&= \int_{0}^{z-1} \mathcal{F}_{\Xi}(z-y) \mathbf{f}_{\gamma_{\mathsf{S},D}}(y) \mathrm{d}y \\
&= \frac{1}{\Gamma_{\mathsf{S,D}}} \int_{0}^{z-1} \exp\left(-\frac{y}{\Gamma_{\mathsf{S,D}}}\right) - \exp\left(-\frac{(z-y)^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}} - 1}{\Gamma_{\mathsf{R,D}}} - \frac{y}{\Gamma_{\mathsf{S,D}}}\right) \mathrm{d}y \\
&= 1 - \exp\left(-\frac{z-1}{\Gamma_{\mathsf{S,D}}}\right) - \frac{1}{\Gamma_{\mathsf{S,D}}} \int_{0}^{z-1} \exp\left(-\frac{(z-y)^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}} - 1}{\Gamma_{\mathsf{R,D}}} - \frac{y}{\Gamma_{\mathsf{S,D}}}\right) \mathrm{d}y
\end{aligned} \tag{C.4}$$

supported over $[1, \infty)$.

C.2 Finding $\mathbf{f}_{\Omega}$

Differentiating $\mathcal{F}_{\Omega}(z)$ with respect to z gives $\mathbf{f}_{\Omega}(z)$. We need Leibniz integral rule

$$\frac{\mathrm{d}}{\mathrm{d}y}\int_{g_1(t)}^{g_2(t)}\mathbf{f}(x,y)\mathrm{d}x = \int_{g_1(t)}^{g_2(t)}\frac{\partial}{\partial y}\mathbf{f}(x,y)\mathrm{d}x + \frac{\mathrm{d}g_2(y)}{\mathrm{d}y}\mathbf{f}(g_2(y),y) - \frac{\mathrm{d}g_1(y)}{\mathrm{d}y}\mathbf{f}(g_1(y),y). \tag{C.5}$$

When we have a double integral, Leibniz rule is applied twice. We thus have,

$$\begin{aligned}
\mathbf{f}_{\Omega}(z) &= \frac{\mathrm{d}}{\mathrm{d}z}\mathcal{F}_{\Omega}(z) \\
&= \frac{\mathrm{d}}{\mathrm{d}z}\int_0^{z-1}\int_1^{z-y}\mathbf{f}_{\Xi}(x)\mathbf{f}_{\gamma_{\mathsf{S},D}}(y)\mathrm{d}x\mathrm{d}y \\
&= \int_0^{z-1}\frac{\partial}{\partial z}\left[\int_1^{z-y}\mathbf{f}_{\Xi}(x)\mathbf{f}_{\gamma_{\mathsf{S},D}}(y)\mathrm{d}x\right]\mathrm{d}y + 1\times\int_1^{z-(z-1)}\mathbf{f}_{\Xi}(x)\mathbf{f}_{\gamma_{\mathsf{S},D}}(z-1)\mathrm{d}x - 0 \\
&= \int_0^{z-1}\int_1^{z-y}\frac{\partial}{\partial z}\mathbf{f}_{\Xi}(x)\mathbf{f}_{\gamma_{\mathsf{S},D}}(y)\mathrm{d}x + 1\times\mathbf{f}_{\Xi}(z-y)\mathbf{f}_{\gamma_{\mathsf{S},D}}(y) - \mathrm{d}y \\
&= \int_0^{z-1}\mathbf{f}_{\Xi}(z-y)\mathbf{f}_{\gamma_{\mathsf{S},D}}(y)\mathrm{d}y \qquad \text{(C.6)} \\
&= \frac{1}{\Gamma_{\mathsf{S,D}}\Gamma_{\mathsf{R,D}}}\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}\int_0^{z-1}(z-y)^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}-1}e^{-\frac{(z-y)^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}}-1}{\Gamma_{\mathsf{R,D}}}}e^{-\frac{y}{\Gamma_{\mathsf{S,D}}}}\mathrm{d}y \qquad \text{(C.7)} \\
&= \frac{1}{\Gamma_{\mathsf{S,D}}\Gamma_{\mathsf{R,D}}}\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}\int_0^{z-1}(z-y)^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}-1}\exp\left(-\frac{(z-y)^{\frac{\tau_{\mathsf{S}}}{\tau_{\mathsf{R}}}}-1}{\Gamma_{\mathsf{R,D}}}-\frac{y}{\Gamma_{\mathsf{S,D}}}\right)\mathrm{d}y \qquad \text{(C.8)}
\end{aligned}$$

for $z \geq 1$.

It is also possible to find $\mathbf{f}_{\Omega}$ using convolution,

$$\begin{aligned}
\mathbf{f}_{\Omega}(z) &= \mathbf{f}_{\Xi} * \mathbf{f}_{\gamma_{\mathsf{S},D}} \\
&= \int_{-\infty}^{\infty}\mathbf{f}_{\Xi}(z-y)\mathbf{f}_{\gamma_{\mathsf{S},D}}(y)\mathrm{d}y \qquad \text{(C.9)}
\end{aligned}$$

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