DISCUSSION PAPERS IN ECONOMICS

A model of delegated project choice with application to merger policy

by

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A Model of Delegated Project Choice With Application to Merger Policy[*]

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Abstract

We present a model in which a principal delegates the choice of project to an agent with different preferences. A project's characteristics are verifiable once presented to the principal, but the principal does not know how many projects are available to the agent. The principal chooses the set of projects which the agent can implement. Three frameworks are considered: (i) a static setting in which the set of available projects is exogenous to the agent but uncertain; (ii) a dynamic setting in which by expending effort the agent can affect the number of projects, and (iii) a dynamic setting in which the agent must wait for projects to materialize. The model is applied to the choice of welfare standard for merger policy.

1 Introduction

In this paper we present an analysis of optimal delegation of project choice to an agent with different preferences from those of the principal. An agent can propose a project to the principal, at which point the project's characteristics are observable to both parties. What is not observable to the principal are the number and characteristics of those projects which the agent could, but does not, propose. To give the agent an incentive to propose projects liked by the principal, the latter restricts the kinds of projects which the agent can implement. Three variants of this framework are analyzed: (i) a static setting in which the agent chooses a single project from an exogenous but uncertain number of projects available; (ii) the agent can determine the number of projects by exerting costly effort, and (iii) the agent must wait for desirable projects to materialize according to an exogenous arrival process.

An application of our model is to the choice of welfare standard for merger policy, and this is explained in detail in the next section. However, the model applies to

1

situations of delegated choice more generally and so our formal models in section 3 onwards are presented in a general principal-agent framework. For instance, it could apply to aspects of decision making within a firm. A CEO of a company may, within limits, delegate project choice to a more junior manager, where the CEO in interested in shareholder value (for example), while the manager enjoys private benefits from certain projects.

In the merger setting, policy is often presented as a binary choice between a total welfare or a consumer welfare standard. But it turns out to be technically tractable and more revealing to compute the *optimal* set of permitted projects rather than between two *ad hoc* policies, and then compare this optimal set to these benchmark alternatives. Indeed we will see that in some special cases the optimal delegation set coincides exactly with a benchmark policy: in our Poisson-Uniform example below, for instance, a total welfare-maximizing principal should adopt a consumer welfare standard if he anticipates that the agent has an average of four projects to consider over the relevant time horizon. More generally, we will see that as the number of available projects rises, the optimal delegation set will place progressively greater weight on consumer interests. Another reason why we focus on optimal delegation is that the optimum is often strikingly simple: in our second and third models where the agent searches or waits for projects, for instance, we show that optimal policy is always characterized by a *linear* rule.

Some other papers have examined aspects of optimal delegation when contingent transfers between principal and agent are ruled out (as in our model). Aghion and Tirole (1997) show how, depending on information structure and payoff alignment, it may be optimal for a principal to delegate decision-making power to a better-informed agent. The principal's loss of control over project choice can be outweighed by advantages in terms of encouraging the agent to develop and gather information about projects. In like vein Baker, Gibbons, and Murphy (1999), though they deny formal delegation of authority, examine informal delegation through repeated-game relational contracts. Even an informed principal able to observe project payoffs may refrain from vetoing ones that yield him poor payoffs in order to promote search incentives for the agent.

Other models of delegated choice include Armstrong (1995) and Alonso and Matouschek (2007), both building on the pioneering work by Holmstrom (1984). These models differ from ours in the form of asymmetric information and project specification. There, a project is characterized by a scalar parameter, and the principal restricts the range of possible projects to lie in an interval. The agent can potentially choose any project, but has private information about a payoff-relevant state of the world. For example, what if any discretion over price should a regulator unable to observe cost give to a profit-seeking monopolist? Alonso and Matouschek provide conditions for 'interval delegation' to be optimal – i.e. the optimal 'permission set' to allow the agent to choose from is a single interval (as with price cap regulation).

Like Alonso and Matouschek, our aim is to characterize the optimal permission set for the principal to allow the agent to choose from, but in the two-dimensional setting where the principal can observe both his own and the agent's payoff from the project proposed by the agent, but does not know what other projects may be available to the

agent. Like Aghion and Tirole, and Baker *et al.*, the provision of incentives for the agent to search (or wait) for projects is a factor that shapes optimal discretion in the variants of our model in sections 4 and 5, where the principal might optimally commit to allow some projects that are bad from his point of view. But, after discussing the merger application in section 2, we show in our first model with multiple projects in section 3 that it is generally optimal for the principal to commit to ban some projects that are good from his point of view so as to improve the chance of a better project being chosen by the agent.[1]

2   Welfare Standards in Merger Policy

An important debate in antitrust policy concerns the appropriate welfare standard to use when deciding whether to permit a merger (or some other form of conduct). The two leading contenders are a *total welfare* standard, where mergers are evaluated according to whether they increase the unweighted sum of producer and consumer surplus, and a *consumer welfare* standard, where only those mergers which improve consumer surplus are approved. Many economic commentators feel that antitrust policy should aim to maximize total welfare, whereas in most jurisdictions the focus is more on consumer welfare alone. See Farrell and Katz (2006) for an excellent overview of the issues.

One purpose of this paper is to examine a particular strategic reason, discussed by Lyons (2002) and Fridolfsson (2007), to depart from the regulator's true welfare standard, which is that a firm may have a *choice* of merger possibilities. A less profitable merger might be better for total welfare, but will not be chosen under a total welfare standard. To illustrate, consider Figure 1, which is similar to those presented in section IV.B in Farrell and Katz (2006).[2]

Here, u represents the gain in total profit resulting from a merger, while v measures the resulting gain (which may be negative) to consumers. Suppose that u and v are verifiable once a merger is proposed to the competition authority. If the regulator follows a total welfare standard, he will permit any merger which lies above negatively-sloped line in the figure. Suppose the firm has two mergers to choose from, depicted by ▲ and ★ on the figure. With a total welfare standard, the firm will choose the merger with the higher u payoff, i.e., the ▲ merger. However, the regulator would prefer the alternative ★ since that yields higher total welfare. If the regulator instead imposed a consumer welfare standard, so that only those mergers which lie above the horizontal line $v = 0$ are permitted, then the firm will be forced to choose the preferred merger. In this case, a regulator wishing to maximize total welfare is better off if he imposes a consumer welfare standard. As Farrell and Katz (2006, page 17) put it:

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[1] The problem we address is unrelated to the literature on "strategic" delegation, which examines how, under (arguably strong) assumptions about information and commitment in game theoretic settings, a principal might distort an agent's incentives away from the principal's true objective in order to induce favorable behaviour from other principal/agent pairs.

[2] The discussion in Farrell and Katz (2006) is a "reduced-form" version of the formal models in Lyons (2002) and Fridolfsson (2007).

"if we want to maximize gains in total surplus (northeasterly movements as shown in figure [1]) and firms always push eastwards, there is something to be said for someone adding a northerly force."
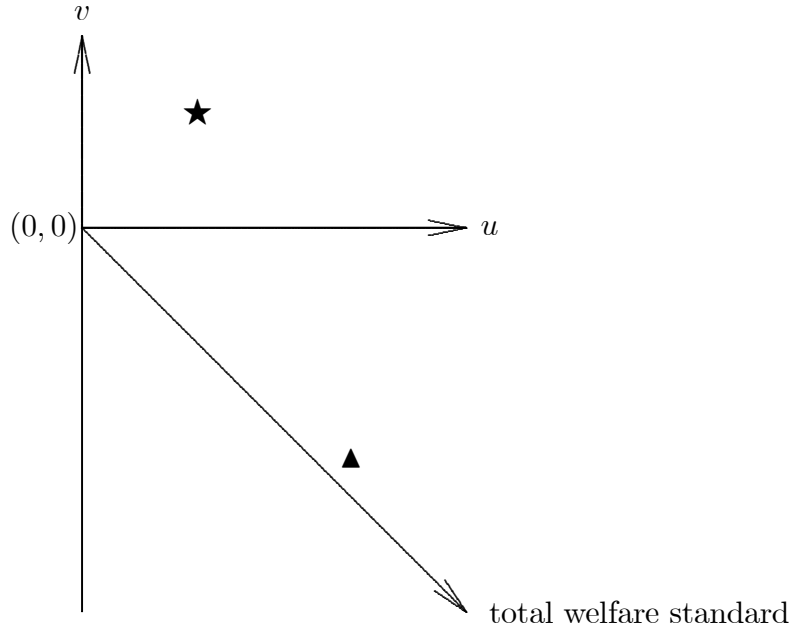

Figure 1: The Impact of Welfare Standard on Chosen Mergers

Nevertheless, there is a potential cost to adopting a consumer welfare standard: if the ▲ merger turns out to be the *only* possible merger then a consumer welfare standard will not permit this even though the merger will improve total welfare. Thus, the choice of welfare standard will depend on the number of possible mergers and the distribution of profit and consumer surplus gains for a possible merger. For instance, as Farrell and Katz observe, if efficiency gains from a merger take the form of reductions in fixed, not marginal, costs, any merger can only cause reductions in consumer surplus and so a consumer welfare standard would forbid all mergers (including those which increase total welfare). Our aim in this paper, as applied to the merger problem, is to examine in a systematic fashion how the number of available mergers and the distribution of profit and consumer surplus gains should determine the choice of welfare standard.

3 Choosing a Project

A *principal* delegates the choice of project to an *agent*. There may be several projects to choose from, although only one can be implemented over the relevant time horizon. We will consider three variants of the delegated choice problem: (i) a static setting in which the agent can choose one project from an exogenous but random number of available projects (as analyzed in this section); (ii) a search model in section 4 where the agent can choose the number of projects in a sequential manner by incurring a

cost for each new project; and (iii) a variant of the search model in section 5 where the agent (and principal) must wait for projects to materialize.

A *project* is fully described by two parameters, u and v. The agent's payoff if the type-(u, v) project is implemented is u, while the principal's payoff is $v + \alpha u$. Here, $\alpha \in [0, 1]$ represents the weight the principal places on the agent's interests, and v represents factors specific to the principal's interests. In the merger context, $\alpha = 1$ when the antitrust authority wishes to maximize total surplus. If (non-contingent) transfers are possible in the principal-agent context, then the principal will maximize $v + u$ in which case $\alpha = 1$.

Each project is an independent draw from the same distribution for (u, v). Since the agent will never implement a project with a negative payoff, we suppose that only non-negative u are realized. The marginal density of $u \geq 0$ is $f(u)$. The conditional density of v given u is denoted $g(v \mid u)$ and the associated conditional distribution function be $G(v \mid u)$. Here, v can be positive or negative.

The principal delegates the choice of project to the agent. (We assume that it is not possible, or credible, for the principal to give monetary incentives to the agent to choose a desirable project.) Once the agent selects a particular project from his set of possible projects, that project's characteristics are fully verifiable. The principal determines the set of permitted projects, $\mathcal{P}$ say, where

$$\mathcal{P} \subset [0, \infty] \times [-\infty, \infty] \ .$$

Thus, an agent can choose any available project with characteristics $(u, v) \in \mathcal{P}$. However, in all three of our models, for a given u the agent cares only about the *fraction* of projects which are permitted, i.e.,

$$\text{Prob}\{u \text{ is permitted}\} = \int_{(u,v) \in \mathcal{P}} g(v \mid u) dv \ ,$$

not the specific values of v which are permitted given u. Since the principal prefers higher v, all else equal, for any $\text{Prob}\{u \text{ is permitted}\}$ it is a dominant strategy for the principal to permit the highest v projects given u. Therefore, the optimal permitted set $\mathcal{P}$ is monotonic in v in the sense that the agent is permitted to choose any project (u, v) such that

$$v \geq r(u)$$

for some threshold function $r(u)$. Our aim is to determine the optimal rule $r(\cdot)$ in a variety of contexts.

In this first model, suppose the number of projects is random and the probability that the agents has exactly $n \geq 0$ possible projects is q_n. The realization of (u, v) for each project is described by f and g as above, and this is independent for each project among the n projects. In addition, (u, v) is distributed independently of n.

Suppose the principal chooses some threshold function $r(u)$. Define

$$x(u) = 1 - \int_u^\infty [1 - G(r(z) \mid z)] f(z) \ dz$$

to be the probability that a project either has agent payoff z less than u or is not permitted. (This is depicted as the shaded area in Figure 2 below.) Note that

$$x'(u) = f(u)[1 - G(r(u) \mid u)] . \tag{1}$$

If there are exactly $n \geq 1$ available projects, the probability that the agent's preferred permitted project has payoff no higher than u is $(x(u))^n$, and so the density of the agent's preferred permitted project is

$$\frac{d}{du}(x(u))^n = nf(u)[1 - G(r(u) \mid u)](x(u))^{n-1} .$$

(One of the n projects must lie in a vertical strip above $(u, r(u))$, which has probability $f(u)[1 - G(r(u) \mid u)]$, while the remaining $n-1$ projects must either have agent payoff lower than u or not be permitted—see Figure 2.)
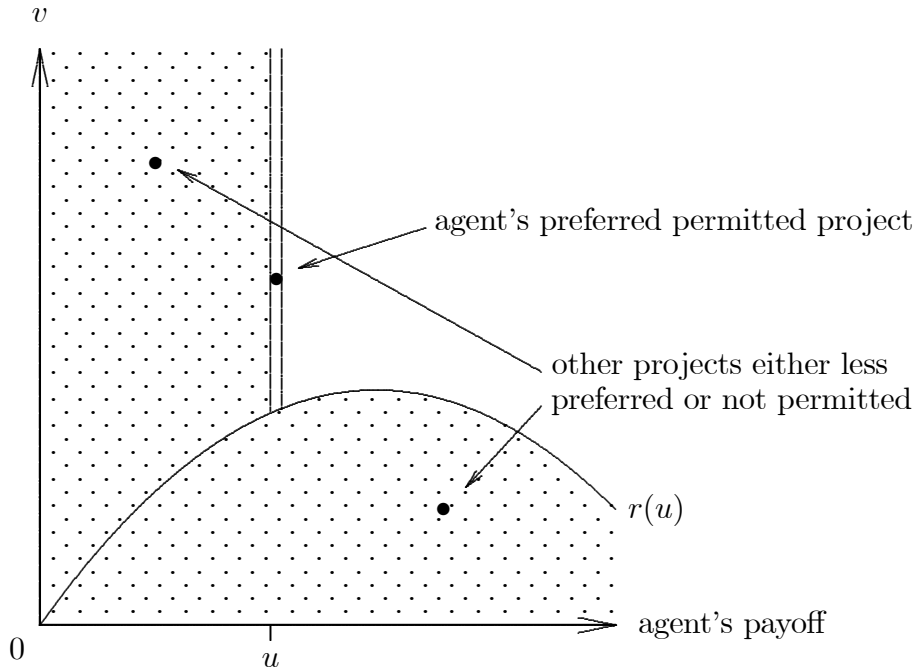

Figure 2: The Agent's Preferred Permitted Project

Summing over n implies that the density of the highest-u permitted project is

$$\frac{d}{du} \sum_{n=0}^{\infty} q_n (x(u))^n .$$

If we write $\phi(x) \equiv \sum_{n=0}^{\infty} q_n x^n$ for the *probability generating function* associated with the random variable n, it follows that the density of the highest-u permitted project is $\frac{d}{du}\phi(x(u))$.

As a final piece of notation, define

$$\mathcal{V}(r \mid u) \equiv E[v \mid u \text{ and } v \geq r]$$

6

to be the expected value of v given that the project has agent payoff u and that v is at least r. Clearly

$$[1 - G(r(u) \mid u)]\mathcal{V}(r \mid u) = \int_{r(u)}^{\infty} vg(v \mid u)dv.$$

The principal's payoff when the threshold rule $r(\cdot)$ is chosen is

$$\int_0^\infty [\mathcal{V}(r(u), u) + \alpha u]\frac{d}{du}\phi(x(u)) \ du$$

$$= \int_0^\infty [\mathcal{V}(r(u), u) + \alpha u][1 - G(r(u) \mid u)]f(u)\phi'(x(u)) \ du \ . \tag{2}$$

The principal's problem is to maximize (2) taking into account the relationship between and r and x in (1) and the endpoint constraint $x(\infty) = 1$.

To solve this classical calculus of variations problem, use the following variational argument. Fix some point u, and suppose that $r(\cdot)$ is increased by $\varepsilon > 0$ in the small neighborhood $[u - \frac{1}{2}\delta, u + \frac{1}{2}\delta]$ around u. This has two effects: (i) the "direct" effect of changing r at u in (2), and (ii) the "strategic" effect on x via (1). One can calculate that the direct effect (i) is

$$-\varepsilon\delta g(r(u) \mid u)f(u)\phi'(x(u))[r(u) + \alpha u] \ .$$

Note that this effect is negative whenever $r(u) + \alpha u > 0$. This is intuitive: for given u, the principal's payoff is maximized by permitting all desirable projects (i.e., projects with $v + \alpha u \geq 0$).

As for the strategic effect (ii), the local change in $r(\cdot)$ around u has no impact on $x(z)$ for $z > u$, but it increases $x(z)$ by $\varepsilon\delta g(r(u) \mid u)f(u)$ for all $z < u$. Therefore, effect (ii) is

$$\varepsilon\delta g(r(u) \mid u)f(u)\int_0^u [\mathcal{V}(r(z), z) + \alpha z][1 - G(r(z) \mid z)]f(z)\phi''(x(z)) \ dz \ .$$

Given that $\phi(\cdot)$ is necessarily convex and $\mathcal{V}(r(z), z) \geq r(z)$, a strongly sufficient condition for this effect to be positive is that $r(z) + \alpha z \geq 0$.

Putting these two effects together implies that at the optimum the threshold rule must satisfy

$$\phi'(x(u))[r(u) + \alpha u] = \int_0^u [\mathcal{V}(r(z), z) + \alpha z][1 - G(r(z) \mid z)]f(z)\phi''(x(z)) \ dz \tag{3}$$

for all u. In particular, we see that

$$r(0) = 0 \ .$$

This implies that the principal does not wish to limit the efficient projects available to the agent whose best project has only zero payoff, i.e., there is "no distortion at the bottom". The reason for this is that when $u = 0$ there is no strategic benefit

to restricting choice. (The strategic effect of raising $r(u)$ above $-\alpha u$ is to increase the probability that the agent will choose a smaller z, and this effect cannot operate when $u = 0$.) Therefore, only the direct effect is relevant, and this implies that choice should not be restricted.

Differentiating (3) implies that

$$r'(u) + \alpha = [\mathcal{V}(r(u) \mid u) - r(u)][1 - G(r(u) \mid u)]f(u)\frac{\phi''(x(u))}{\phi'(x(u))} . \qquad (4)$$

The right-hand side of (4) is necessarily non-negative. Therefore, since $r'(u) + \alpha \geq 0$ and $r(0) = 0$ it follows that

$$r(u) + \alpha u \geq 0$$

and so the principal never includes undesirable projects (i.e., projects with payoff $v + \alpha u < 0$) within the permitted set. Moreover, the gap between $r(u)$ and the efficient cut-off $-\alpha u$ widens with u. In particular, in the case $\alpha = 0$, the optimal $r(\cdot)$ is always monotonically increasing with u.

In the degenerate case where the agent never has more than one project (i.e., $q_0 + q_1 = 1$), $\phi'' = 0$ and so (4) implies that $r(u) + \alpha u \equiv 0$, and the principal permits all desirable projects. Outside this dull case, though, $\phi'' > 0$ and (4) implies that

$$r(u) + \alpha u > 0 \text{ if } u > 0 ,$$

and so it is optimal for the principal to exclude strictly desirable projects.

What is the intuition for why the principal wishes to exclude some desirable projects from the permitted set, whenever the agent sometimes has a choice of project? Suppose the principal initially allows all desirable projects, so that $r(u) \equiv -\alpha u$. If the principal increases $r(\cdot)$ slightly at some $u > 0$, the direct effect is approximately zero, since the principal is excluding projects about which he is almost indifferent (since $r(u) + \alpha u \approx 0$). But there is a strictly beneficial strategic effect: there is some chance that the agent's highest-u project is excluded by the modified permitted set, in which case there is a chance that he chooses another project which is permitted, say with $z < u$. This alternative project is unlikely to be marginal for the principal, and instead the principal will expect to get payoff $\mathcal{V}(r(z) \mid z) + \alpha z$, which is strictly positive when $r(z) = -\alpha z$. This argument indicates that the direction of optimal restriction is to restrict desirable projects, not to permit undesirable projects. Moreover, it is intuitive that the strategic effect is more important for higher u, since it applies over a wider range $z < u$, and this explains why the optimal gap $r(u) + \alpha u$ widens with u.

Expression (4) reveals that ϕ''/ϕ' is important for the form of the solution. A short list of examples for this includes:

- if the number of projects is known to be $n \geq 1$ for sure (so $q_n = 1$), then $\phi''(x)/\phi'(x) = (n-1)/x$;

- in n is geometric (so $q_n = (1-a)a^{n-1}$ for $n \geq 1$ and some parameter a) then $\phi''(x)/\phi'(x) = 2a/(1-ax)$;

8

- if n is Poisson with mean μ (so $q_n = e^{-\mu}\frac{\mu^n}{n!}$ for $n \geq 0$) then $\phi''(x)/\phi'(x) \equiv \mu$.

It is clear that the Poisson case is particularly simple, since (4) becomes a first-order differential equation in $r(u)$:

$$r'(u) + \alpha = \mu[\mathcal{V}(r(u) \mid u) - r(u)][1 - G(r(u) \mid u)]f(u) \ . \tag{5}$$

In all other cases, the variable $x(u)$ also plays a role, and we need to solve what is in effect a second-order differential equation in x, x' and x''.[3]

Before we solve this equation for some examples, consider the special case where the principal does not care about the realization of u, i.e., where $\alpha = 0$ and where v and u are independent. In this case, (5) simplifies to

$$\int_0^{r(u)} \frac{1}{[\mathcal{V}(r) - r][1 - G(r)]} dr = \mu F(u) \ ,$$

where $F(\cdot)$ is the cdf associated with the density $f(\cdot)$, we have written $\mathcal{V}$ and G as functions only of r (since there is no dependence on u), and we have used the fact that $r(0) = 0$. Since the left-hand side of the above is purely a function of $r(u)$, this expression shows that the threshold $r(u)$ depends on u only via the cdf $F(u)$. That is to say, for the principal it is only *ordinal* payoffs to the agent that matter, not their absolute value.[4] (For instance, if $r(u)$ is the optimal rule corresponding to the cdf $F(u)$ and if the environment changed so that u is doubled for each realization, then the optimal threshold rule for the new environment is just $r(\frac{1}{2}u)$.)

To make further progress we analyze a particular example in detail:

Uniform-Poisson example: Suppose that n follows a Poisson distribution with mean μ and that (u, v) is uniformly distributed on $[0, 1] \times [-1, 1]$. In this case, (5) becomes

$$r'(u) + \alpha = \frac{\mu}{4}(1 - r(u))^2 \ . \tag{6}$$

Since $r(0) = 0$, the solution to this equation satisfies

$$\int_0^{r(u)} \frac{1}{(1-r)^2 - \frac{4\alpha}{\mu}} dr = \frac{\mu}{4}u \ . \tag{7}$$

This expression is particularly easy to solve when $\alpha = 0$, in which case (7) becomes

$$\frac{\mu}{4}u = \frac{1}{1-r}\bigg]_0^{r(u)} = \frac{r(u)}{1 - r(u)}$$

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[3]The Poisson distribution is also somewhat plausible. For instance, in the merger context if a firm has many possible merger partners each of which has some independent small probability of wishing to merge, then the number of willing merger partners will approximately follow a Poisson distribution.

[4]This is quite general, and does not depend on the Poisson specification. Moreover, the argument applies to situations with correlation between v and u, provided that the conditional density g is defined in terms of the cdf $F(u)$ instead of u.

and so the threshold $r(\cdot)$ is given by[5]

$$r(u) = \frac{\mu u}{4 + \mu u} \quad . \tag{8}$$

Notice that $r(\cdot)$ is increasing in μ. As μ becomes large, so that the agent can choose from many options, $r(u)$ tends to 1 for $u > 0$, and so only projects with the highest payoff to the principal are permitted. As μ tends to zero, so that the agent is very unlikely to have any choice over the project, we see that $r(u)$ tends to zero and all desirable projects are permitted.

Next consider $\alpha > 0$, and write $A = \sqrt{4\alpha/\mu}$. Then (7) becomes

$$\frac{\mu}{4}u = \int_0^{r(u)} \frac{1}{(1-r)^2 - A^2} dr = \int_0^{r(u)} \frac{1}{(1-r+A)(1-r-A)} dr$$

$$= \frac{1}{2A} \int_0^{r(u)} \left\{ \frac{1}{(1-r-A)} - \frac{1}{(1-r+A)} \right\} dr = \frac{1}{2A} \log \frac{1-r+A}{1-r-A} \Bigg]_0^{r(u)}$$

$$= \frac{1}{2A} \log \frac{(1-r(u)+A)(1-A)}{(1-r(u)-A)(1+A)} \quad,$$

and so

$$\frac{(1-r(u)+A)(1-A)}{(1-r(u)-A)(1+A)} = e^{\mu A u/2} \quad .$$

Therefore, after some rearrangement,

$$r(u) = (1 - A^2) \frac{e^{\mu A u/2} - 1}{(1+A)e^{\mu A u/2} - (1-A)} \quad . \tag{9}$$

(As required, L'Hôpital's rule implies that (9) converges to (8) when $A \to 0$.) Notice in particular that when $A = 1$, so that $\mu = 4\alpha$, it is optimal to have the flat rule that only those projects which improve v are permitted. For instance, in the merger context, if the regulator wishes to maximize total welfare (so $\alpha = 1$), then if the expected number of feasible mergers is four in this example the regulator should enforce a pure consumer welfare standard.

We illustrate these solutions in Figure 3 for various μ with $\alpha = 1$. Thus, the agent is given less discretion over policy the more projects there are likely to be.

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[5]More generally, as discussed above, if u has cdf $F(u)$ the optimal threshold rule becomes

$$r(u) = \frac{\mu F(u)}{4 + \mu F(u)} \quad .$$

In particular, there is no reason to expect that $r(\cdot)$ is generally concave, as happens to be the case when u is uniformly distributed.
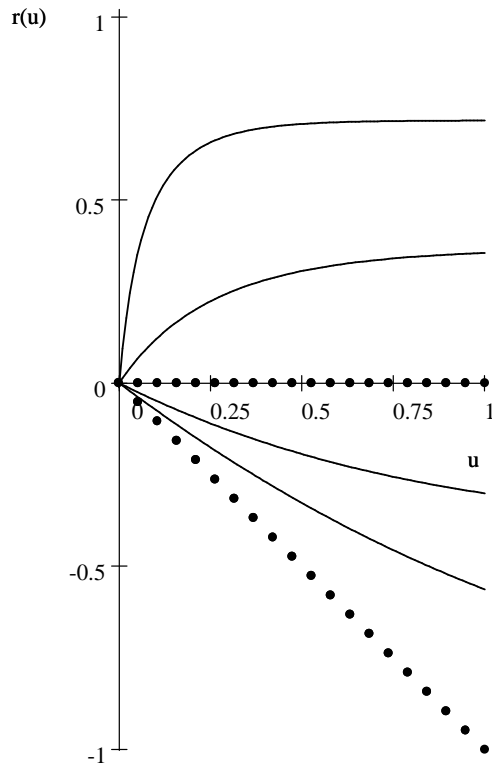
Figure 3: $r(u)$ for Uniform-Poisson example with $\alpha = 1$ and efficient cut-off (dotted), $\mu = 1, 2, 4$ (dotted), 10 and 50

In the next diagram we illustrate the solution with various α for $\mu = 4$. Thus we see that the more the principal cares about the utility of the agent, the more discretion the latter is given. (This is similar to Holmstrom (1984) and Armstrong (1995), where the more likely the agent's preferences were to be close to the principal's, the more discretion was given.)
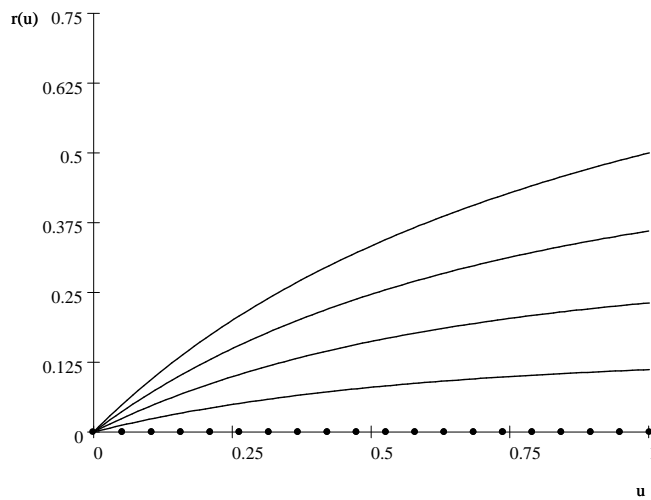

Figure 4: $r(u)$ for Uniform-Poisson example with $\mu = 4$ and $\alpha = 1$ (dotted), $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$ and 0

11

4 Searching for a Project

The previous model assumed that the number of projects was exogenous to the agent (but uncertain). Here and in the next section we suppose instead that the agent can determine the number of available projects in a dynamic search framework. There are two broad kinds of search model in the literature: (i) the searching agent is "active" and can instantaneously obtain a new draw by incurring a cost c, and (ii) the searching agent is "passive" and must wait for a new draw to materialize. In this section we investigate the first of these, while in section 5 we consider the alternative framework.

As before, the agent's payoff (gross of search costs) is u, the principal's payoff is a weighted sum of the agent's payoff (including search costs) and the expected value of a random variable v, where the relative weight on the agent's payoff by the principal is $\alpha \leq 1$. The principal determines a function $r(\cdot)$ such that any project with $v \geq r(u)$ is permitted. Suppose the agent is risk neutral. The agent will keep searching until he finds a permitted project which delivers his reservation utility, denoted U, where U is determined by the usual search equation:

$$c = \int_U^\infty (u - U)[1 - G(r(u) \mid u)]f(u) \ du \ . \tag{10}$$

(Integrating by parts gives $c = \int_U^\infty (1 - x(u)) \ du$, but the expression above keeps $r(u)$ explicit.) Here, the agent's expected payoff (before any project is observed) is just U. The payoff to the principal is $\alpha U + V$, where V is the expected value of v given that $u \geq U$ and (u, v) is permitted, i.e.,

$$V = \frac{\int_U^\infty \mathcal{V}(r(u) \mid u)x'(u) \ du}{1 - x(U)} = \frac{\int_U^\infty \left(\int_{r(u)}^\infty vg(v \mid u)dv \right) f(u) \ du}{1 - x(U)} \ . \tag{11}$$

The principal aims to maximize $\alpha U + V$ subject to (10) and the relationship between x and r in (1), i.e., he chooses $r(\cdot)$ and U to maximize

$$\mathcal{L} = \alpha U + V + \lambda \int_U^\infty (u - U)[1 - G(r(u) \mid u)]f(u) \ du \tag{12}$$

for some Lagrange multiplier λ. Using a similar variational argument to that in section 3, if r is increased by ε in some small neighborhood $[u - \frac{1}{2}\delta, u + \frac{1}{2}\delta]$ of $u > U$, $x(U)$ is increased by $\varepsilon\delta f(u)g(r(u) \mid u)$. Therefore, V in (11) is increased by

$$\frac{\varepsilon\delta f(u)g(r(u) \mid u)}{1 - x(U)} [V - r(u)]$$

while $\int_U^\infty (u - U)[1 - G(r(u) \mid u)]f(u) \ du$ is increased by $-\varepsilon\delta f(u)g(r(u) \mid u)[u - U]$. It follows that $\mathcal{L}$ in (12) is maximized, given U, when

$$r(u) = V - \gamma(u - U) \tag{13}$$

for a constant γ $(= (1 - x(U))\lambda)$.

One can see that $\gamma > 0$ by means of the following argument. Note that V is the expected value, for $u \geq U$, of $\mathcal{V}(r(u) \mid u)$, which is greater than $r(u)$ for each u. Thus V is greater than some average of $r(u)$. From (13), it follows that $\gamma > 0$. We deduce very generally that the optimal permitted set is defined by a downward-sloping linear threshold function $r(\cdot)$.

The solution to the principal's problem has two broad forms: either $U = 0$, so that the agent is left with no rent and implements the first permitted project found, or $U > 0$. To see the form of the solution, differentiate (12) to obtain

$$
\begin{aligned}
\frac{d\mathcal{L}}{dU} &= \alpha - \lambda(1 - x(U)) + \frac{dV}{dU} \\
&= (\alpha - \gamma) - \frac{[\mathcal{V}(r(U) \mid U) - V]x'(U)}{1 - x(U)} \\
&= (\alpha - \gamma) - \frac{f(U)}{1 - x(U)} \int_{r(U)}^{\infty} (v - V)\, g(v \mid U)\, dv \\
&= (\alpha - \gamma) - \frac{f(U)}{1 - x(U)} \int_{V}^{\infty} (1 - G(v \mid U))\, dv \ .
\end{aligned}
\tag{14}
$$

In particular, since we know already that $\gamma > 0$, if $\alpha = 0$ we see that the Lagrangean is always decreasing in U. Thus, for $\alpha = 0$ (or, more generally, when α is small) it is optimal for the principal to set $U = 0$.

There is a clear intuition for this result. When $\alpha = 0$, the principal aims to maximize the expected value of v subject to the agent being willing to search for a project. Suppose that $U > 0$ for some particular rule $r(u)$. Since $U > 0$ the principal can increase $r(\cdot)$ uniformly by some small amount and still give the agent an incentive to search for permitted projects. But the threshold U will fall as a result—see expression (10). The principal benefits in two ways from this policy change: (i) expected v rises for all u that would have been chosen beforehand (since r is higher), and (ii) the fact that U falls means that the principal has the chance to enjoy more projects, and these projects have higher than average v since $r(u)$ is a decreasing function (as is optimal). (Point (ii) is formally shown in expression (14) where we saw that dV/dU was negative.) We deduce it is never optimal to leave the agent with any rent when $\alpha = 0$.

For larger α it is optimal to choose $U > 0$. In these cases (14) is equal to zero, which in turn implies that

$$
\gamma < \alpha \ ,
$$

and the trade-off between u and v in the rule (13) is less steep than the principal's true trade-off α.

Given U, the two parameters V and γ in (13) are determined by the pair of simultaneous equations:

$$
V = \frac{\int_{U}^{\infty} \mathcal{V}(V - \gamma(u - U) \mid u)x'(u)\, du}{1 - x(U)} \ , \quad c = \int_{U}^{\infty} (u - U)[1 - G(r(u) \mid u)]f(u)\, du \ .
\tag{15}
$$

Once one solves this pair of equations, one can maximize $V + \alpha U$ over U. (In the case where it is optimal set $U > 0$, the optimal U is determined by the third equation (14) being equal to zero.) To investigate further, consider the following example.

Uniform example: Suppose again that (u, v) is uniform on the rectangle $[0, 1] \times [-1, 1]$. Here, we must have $c < \frac{1}{2}$ to have a chance of the agent being willing to search at all. When $r(u)$ is given by (13) the agent's reservation utility U satisfies

$$c = \frac{1}{2} \int_U^1 (u - U)(1 - r(u)) \, du = \frac{1}{4}(1 - V)(1 - U)^2 + \frac{1}{6}\gamma(1 - U)^3 . \quad (16)$$

Also,

$$V = \frac{\int_U^1 (1 - r^2) du}{2 \int_U^1 (1 - r) du} = \frac{(1 - V^2)(1 - U) + \gamma V(1 - U)^2 - \frac{1}{3}\gamma^2(1 - U)^3}{2(1 - V)(1 - U) + \gamma(1 - U)^2}$$

$$= \frac{(1 - V^2) + \gamma V(1 - U) - \frac{1}{3}\gamma^2(1 - U)^2}{2(1 - V) + \gamma(1 - U)} . \quad (17)$$

Rearranging (17) yields the simple relation

$$1 - V = \frac{\gamma}{\sqrt{3}}(1 - U) . \quad (18)$$

Substituting this value for γ into (16) yields this explicit formula for V:

$$V = 1 - \frac{2c}{k(1 - U)^2} , \quad (19)$$

where

$$k \equiv \frac{1}{2} + \frac{1}{\sqrt{3}} \approx 1.08 .$$

Since the principal's payoff is $V + \alpha U$, the principal will therefore choose U maximize

$$\alpha U - \frac{2c}{k(1 - U)^2} .$$

This is a decreasing function of U whenever

$$\alpha \leq \frac{4c}{k} \approx 3.7c , \quad (20)$$

in which case it is optimal to set $U = 0$ and so leave the agent with zero rent. The optimal permitted set is determined by $r(u) = V - \gamma u$, where V is given by (19) and γ is then given by (18), both with U set equal to zero. It follows that

$$r(u) = 1 - \sqrt{3}\frac{2c}{k}\left(u + \frac{1}{\sqrt{3}}\right) \quad (21)$$

14

where $0 \leq u \leq 1$.[6] Thus, for different values of c (21) traces out a family of linear, downward-sloping lines for $r(u)$ revolving about the point $(-1/\sqrt{3}, 1)$. See Figure 5 for the case $\alpha = 0$ (when condition (20) is always satisfied), where smaller c correspond to higher r. When $c \approx 0$, we have $r(u) \approx 1$ as expected. (This is like the "large μ" case in the previous model.)
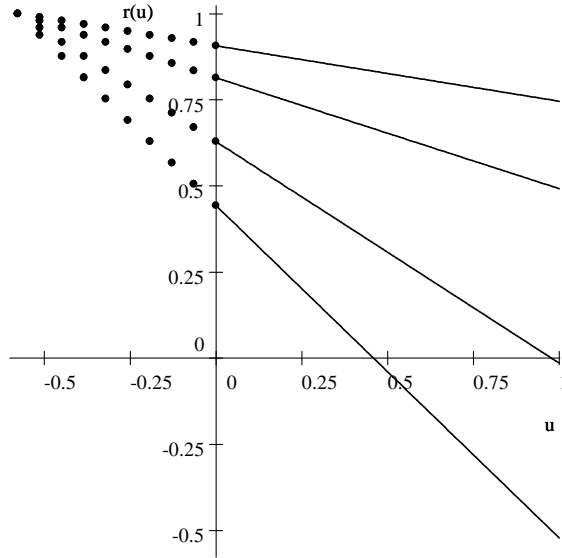

Figure 5: $r(u)$ for Uniform example with $\alpha = 0$ and $c = 0.05, 0.1, 0.2$ and 0.3

Some intuition for the linearity of $r(u)$ comes from noting that for each c the permitted set depicted in Figure 5 coincides with the acceptance threshold of a hypothetical searcher whose objective $\Omega = V + \gamma U$ is a weighted average of the payoffs of the principal and agent, with γ given by (21). The objective of the hypothetical searcher coincides with the Lagrangean of the principal in the problem above.

In the two models considered so far, when $\alpha = 0$ and with a uniform distribution for (u, v), we have derived two surprisingly simple families of threshold rules (see (8) and (21) above). In one respect optimal policy is similar in the two models: as projects are easier to come by for the agent (i.e., μ is larger in the first model or c is smaller in this second model), the permitted set of projects becomes progressively more restricted. In other respects, though, policy is dramatically different in the two settings. In the "choosing a project" model, the rules $r(u)$ start at $r(0) = 0$ and increase, and only desirable projects (i.e., $v \geq 0$) are permitted. In the search model $r(0) > 0$ and decreases, and it may be optimal to permit projects with negative payoff for the principal (as when $c = 0.3$ in Figure 5).

As we explained in section 3, the reason why the principal departs from the efficient rule ($r(u) \equiv 0$ in this case) in the first model is that when some marginally

---

[6]These solutions are valid only when $c \leq (1+1/\sqrt{3})/4 \approx 0.4$. This is to ensure that $r(u)$ does not hit the lower boundary $v = -1$. If $c > 0.4$, the optimum will involve $r(u)$ being a downward-sloping linear function which hits the lower boundary (and $r(u) \equiv -1$ beyond this point).

desirable high-u projects are excluded, this may induce the agent to choose a strictly desirable lower–u project instead. This benefit does not exist when $u = 0$, which explains why all desirable projects are permitted then. Furthermore, it is clear there can therefore be no incentive to include projects with a negative payoff to the principal. The reason to depart from the efficient cut-off rule is quite different in the search model. Here, when $\alpha = 0$ the principal wishes to maximize the expected value of v in the permitted set, subject to the agent being willing to engage in costly search for permitted projects. For a given expected value of v in the permitted set, the principal is indifferent about whether the threshold rule is upward or downward sloping; however, the agent's incentives to search are enhanced when higher-u projects are more likely to be permitted, i.e., when the rule is downward sloping. For the same reason, it can be optimal to permit the agent to choose projects with a negative payoff for the principal, if the search cost is large enough.[7]
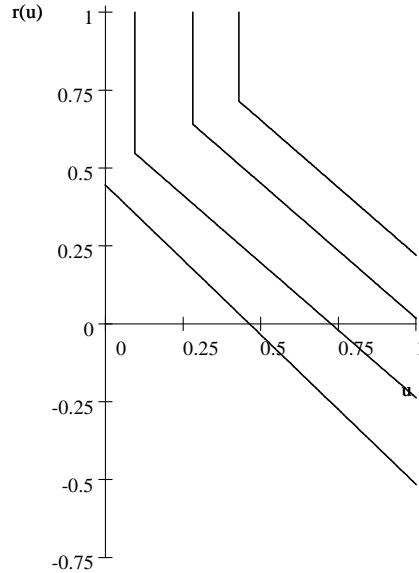

Figure 6: $r(u)$ for Uniform example with $\alpha = 1$ and $c = 0.05$, 0.1, 0.2, and 0.3

If (20) does not hold then $U > 0$ is optimal. Indeed optimal U and γ satisfy

$$ 1 - U = \left(\frac{4c}{k\alpha} \right)^{\frac{1}{3}} \ , \ \gamma = \frac{\sqrt{3}}{2}\alpha \ . $$

Note that γ here is independent of c and less than α (as we argued previously). Therefore, the permission rule is

$$ r(u) = 1 - (1 + \sqrt{3}) \left(\frac{c\alpha^2}{2k} \right)^{\frac{1}{3}} + \frac{\sqrt{3}}{2}\alpha(1 - u) \ . $$

---

[7]In this uniform example, although negative v may be permitted, the principal's *expected* payoff from a u project, $\mathcal{V}(r(u), u)$, is never negative. However, nothing rules out negative $\mathcal{V}(r(u), u)$ being optimal for some u with more general distributions.

In Figure 6 we show the permission sets when $\alpha = 1$, for the same search costs as in Figure 5. Since the agent will keep searching until a permitted project with $u > U$ is found, only that part of the rule with $u > U$ is relevant, and that part is depicted on the figure. (The principal can choose the linear rule without constraining $u > U$, so that the downward-sloping lines can be extended to the left until they reach the vertical axis, but the agent will never choose a permitted project to the left of the vertical lines shown.) In the merger context, if the principal wishes to maximize total welfare, Figure 6 suggests that a good approximation to optimal policy is to permit mergers which increase total welfare by some discrete threshold, where this threshold is higher when merger possibilities are less costly to discover.

5   Waiting for a Project

Suppose now that the agent is passive and must wait for projects to materialize. Specifically, suppose that a project emerges with probability $h \times dt$ in any small time interval dt, where h is exogenous. Suppose the principal chooses the rule $r(\cdot)$ which determines which projects are permitted, and suppose the agent chooses to wait until he obtains a permitted project with payoff u above some threshold U. Then the probability that a given project will be implemented is $1 - x(U)$, while the agent's expected payoff at the time the project is implemented is $B/(1 - x(U))$, where

$$B = \int_U^\infty u[1 - G(r(u) \mid u)]f(u) \ du \ .$$

Following this strategy, the agent will receive an acceptable project in a time interval dt with probability $h(1 - x(U)) \times dt$. This implies that the probability that the first acceptable project will arrive in the time interval $(t, t + dt)$ is

$$h(1 - x(U))e^{-h(1-x(U))t} \times dt \ .$$

If the agent discounts at the rate δ, his expected utility is

$$\int_0^\infty e^{-\delta t}h(1 - x(U))e^{-h(1-x(U))t}\frac{B}{(1 - x(U))}dt = \frac{hB}{h(1 - x(U)) + \delta} = \frac{B}{1 - x(U) + \Delta} \ ,$$

where to save notation we write $\Delta = \delta/h$. (The parameter Δ represents the net cost of foregoing an existing option, and ΔU plays a similar role to c in the previous search model.) The agent will choose the threshold U in order to maximize this utility, which has the first-order condition

$$U = \frac{B}{1 - x(U) + \Delta} \ .$$

In particular, U, the reservation utility, is also the agent's discounted payoff from following his optimal strategy (as is usual in search models). The above first-order condition can be written in a similar manner to (10) as

$$\int_U^\infty (u - U)[1 - G(r(u) \mid u)]f(u) \ du = \Delta U \ . \tag{22}$$

A significant difference between this setting and the previous search setting is that here the agent is inevitably left with some rent.

In a similar manner, if V denotes the discounted expected value of v, then

$$V = \frac{\int_U^\infty \mathcal{V}(r(u) \mid u)x'(u) \, du}{1 - x(U) + \Delta} = \frac{\int_U^\infty \left(\int_{r(u)}^\infty vg(v \mid u)dv \right) f(u) \, du}{1 - x(U) + \Delta} \ .$$

The principal aims to maximize $\alpha U + V$ subject to (22), that is to say, he will choose U and $r(\cdot)$ to maximize

$$\mathcal{L} = \alpha U + V + \lambda \left[\int_U^\infty (u - U)[1 - G(r(u) \mid u)]f(u) \, du - \Delta U \right]$$

for some Lagrange multiplier λ. As in the previous search model, for given U, maximizing this with respect to $r(u)$ implies that

$$r(u) = V - \gamma(u - U) \tag{23}$$

for some constant γ $(= (1 - x(U) + \Delta)\lambda)$. The optimal permitted set is again very generally defined by a linear function $r(\cdot)$. Unlike the previous search model, however, here γ can be positive or negative.

Next, maximizing the Lagrangean with respect to U has the first-order condition

$$
\begin{aligned}
0 &= \alpha - \lambda\Delta - \lambda(1 - x(U)) + \frac{dV}{dU} \\
&= (\alpha - \gamma) - \frac{[\mathcal{V}(r(U) \mid U) - V]x'(U)}{1 - x(U) + \Delta} \\
&= (\alpha - \gamma) - \frac{f(U)}{1 - x(U) + \Delta} \int_{r(U)}^\infty (v - V) \, g(v \mid U) \, dv \\
&= (\alpha - \gamma) - \frac{f(U)}{1 - x(U) + \Delta} \int_V^\infty (1 - G(v \mid U)) \, dv \ . \tag{24}
\end{aligned}
$$

The final equality follows from (23) and integration by parts. Therefore, $\gamma < \alpha$ and the trade-off between u and v in the rule (23) is less steep than the principal's true trade-off α. (The same was true in the previous search model when $U > 0$.) For instance, if $\alpha = 0$ then an increasing linear rule is always optimal (in contrast to the previous search model where r was always decreasing).

In sum, the three parameters of interest, U, V and γ, are determined by the three equations, (24) together with

$$V = \frac{\int_U^\infty \mathcal{V}(V - \gamma(u - U) \mid u)x'(u) \, du}{1 - x(U) + \Delta} \tag{25}$$

and

$$\Delta U = \int_U^\infty (u - U)[1 - G(V - \gamma(u - U) \mid u)]f(u) \, du \ . \tag{26}$$

Uniform example: Again, specialize to the case where (u, v) is uniform on the rectangle $[0, 1] \times [-1, 1]$. Expression (26) becomes

$$4\Delta U = (1 - V)(1 - U)^2 + \frac{2}{3}\gamma(1 - U)^3 , \tag{27}$$

which is akin to (16) in the search model above. Expression (24) becomes

$$(\alpha - \gamma)\left[2(1 - V)(1 - U) + \gamma(1 - U)^2 + 4\Delta\right] = (1 - V)^2 . \tag{28}$$

Expression (25) becomes

$$V = \frac{\int_U^1 (1 - r^2)\, du}{2\int_U^1 (1 - r)\, du + \Delta}$$

$$= \frac{(1 - V^2)(1 - U) + \gamma V(1 - U)^2 - \frac{1}{3}\gamma^2(1 - U)^3}{2(1 - V)(1 - U) + \gamma(1 - U)^2 + 4\Delta}$$

which rearranges to give

$$4\Delta V = (1 - U)(1 - V)^2 - \frac{1}{3}\gamma^2(1 - U)^3 . \tag{29}$$

The three expressions (27), (28) and (29) can easily be solved numerically to find U, V and γ for any values for Δ and α. Figure 7 shows the threshold rule $r(u)$ when $\alpha = 1$ and Δ takes a number of values ranging from 1 down to $\frac{1}{20}$. The case with $\Delta = \frac{1}{10}$, for instance, corresponds approximately to a 10% discount rate and a single project expected to emerge per year.
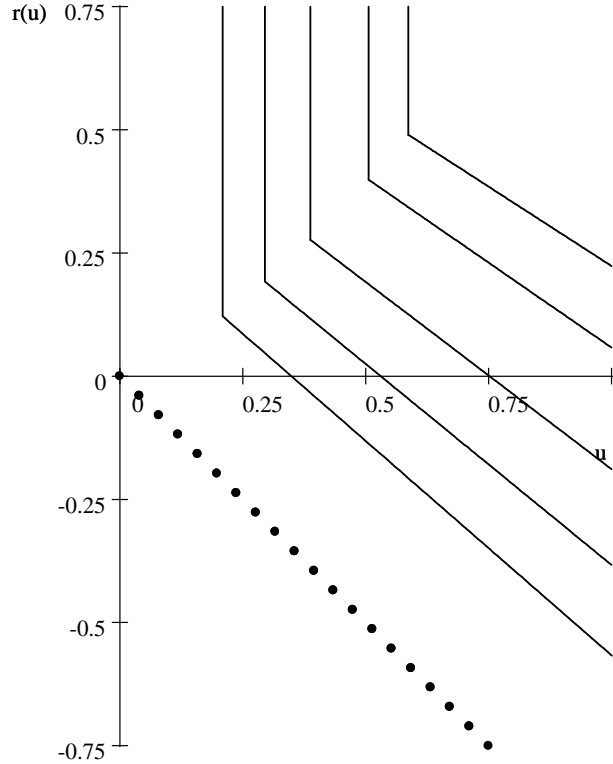

Figure 7: $r(u)$ for Uniform example with $\alpha = 1$ and efficient cut-off (dotted), $\Delta = 1$, $\frac{1}{2}, \frac{1}{4}, \frac{1}{10}$ and $\frac{1}{20}$

Smaller values of Δ, which correspond to more frequent projects or more patient actors, lead to higher $r(u)$. In the limit as $\Delta \to 0$, it is optimal to set $U = V = 1$ and $\gamma = 0$ (which solves the above three equations in this case). Thus, as is economically obvious, it is then optimal to wait for the perfect project with $u = 1$ and $v = 1$. As $\Delta \to \infty$, the optimal rule is simply the efficient cut-off rule $r(u) = -\alpha u$, and it is optimal to implement the first desirable project which appears. (Here, $U = V = 0$ and $\gamma = \alpha$ solve the three equations.)

As in the previous search model, in the merger context a good approximation to optimal policy is to permit mergers which increase total welfare by some discrete threshold, where this threshold is higher when merger possibilities are more frequent.

We depict the impact of changing α in Figure 8, where smaller α corresponds to higher $r(\cdot)$. By manipulating expressions (27)–(29) one can show that γ is always positive, i.e., r is downward sloping, if $\alpha \geq \frac{1}{2}$ (as in Figure 7). For smaller α, γ has the same sign as

$$\Delta - \frac{(1 - 2\alpha)^3}{4\alpha(1 - \alpha)^2} \ .$$

In particular, if the above expression is zero (such as when $\alpha = \frac{1}{3}$ and $\Delta = \frac{1}{16}$ in the figure), the threshold rule $r(u)$ is exactly flat. (The corresponding condition in our first model for a flat rule was the simpler condition $\mu = 4\alpha$.)
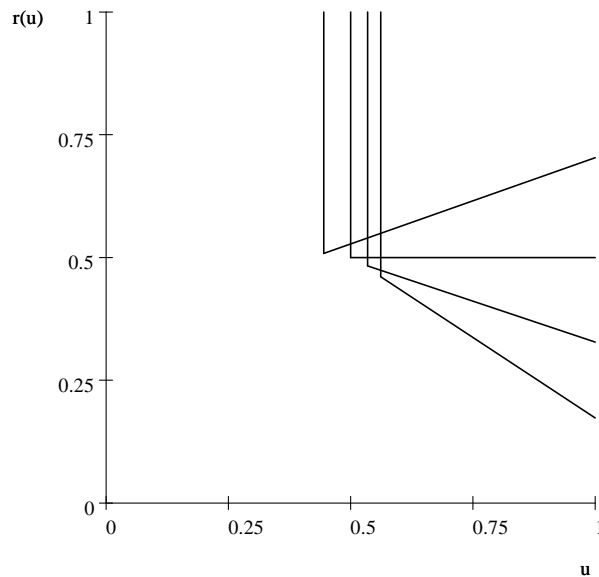

Figure 8: $r(u)$ for Uniform example with $\Delta = \frac{1}{16}$ and $\alpha = 0, \frac{1}{3}, \frac{2}{3}$ and 1

Figure 7 compared the optimal delegation policy with the efficient cut-off rule (represented as the dotted line). An alternative, and perhaps more appropriate, efficiency benchmark is the acceptance threshold for projects that the principal would adopt if acting (at no cost) in place of the agent. The principal in that situation would set an acceptance threshold W such that a project would be accepted if and only if

$w \equiv \alpha u + v \geq W$, where W satisfies the search condition

$$W = \frac{E[w \mid w \geq W] \cdot \Pr[w \mid w \geq W]}{\Delta + \Pr[w \mid w \geq W]} \ .$$

So the efficient acceptance condition is $v \geq r^*(u) \equiv W - \alpha u$. This differs in two respects from the $r(u) = V - \gamma(u - U)$ threshold that solved the delegation problem. First, since $\alpha > \gamma$, the slope is more negative with efficient acceptance. Second, efficiency involves acceptance of some projects with $u < U$, which the agent could not be induced to accept with delegated choice.[8] (It may also be noted that the intercept of $r(u)$ with the $u = 0$ axis, namely $V + \gamma U$, is less than W because $W \geq V + \alpha U > V + \gamma U$.)
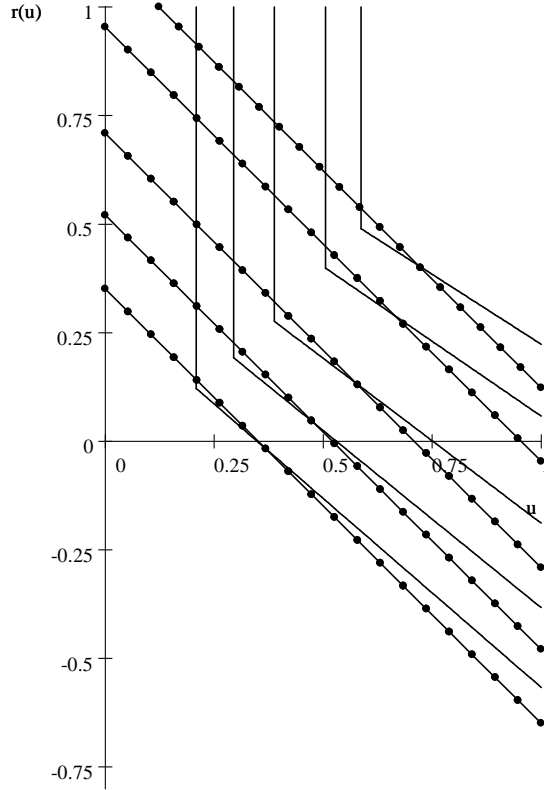

Figure 9: Comparing the efficient choice rule without delegation with the optimal delegation rule ($\alpha = 1$ and $\Delta = 1$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$ and $\frac{1}{20}$)

Figure 9 illustrates for the case $\alpha = 1$. This figure reproduces the optimal delegation sets from Figure 7, and overlays on these the efficient choice sets (drawn

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[8]Indeed if negative realisations of u are possible, unlike in our Uniform example, then if average delay is large enough, the first best for the principal will involve negative-u projects sometimes being chosen.

with dotted lines) which would be implemented if it were the principal choosing the projects. Clearly the efficient and optimally delegated sets are not quite "nested", although the chance that an agent would choose a project which the principal would not appears to be tiny in this set of examples. Indeed, in these examples with $\alpha = 1$ at least, if the principal has to delegate project choice, it appears that the principal does very well if he offers his own "non-strategic" choice set (the dotted lines in the figure) to the agent. Of course, the agent will not necessarily choose the first project that lies in this permitted set, but what will be chosen is very close to the optimal delegation set (the solid lines in the figure). Since a project lying in the principal's efficient acceptance set is more likely to be realized than a project in the agent's delegation set, delegation therefore involves more delay on average.

6 Conclusions

Proceeding from the motivating example of welfare standards in merger policy, we have explored the nature of optimal discretion for a principal to give to an agent in three related settings of delegated project choice. The principal's problem is to design the optimal set of permitted projects without knowing which projects are available to the agent—though being able to verify the characteristics of the project proposed by the agent—and with (contingent) transfers ruled out.

In the first setting the agent has a number (unknown to the principal) of projects to choose from. The optimal permission set excludes some projects that are good for the principal because the loss from excluding marginally good projects is outweighed by the expected gain from thereby inducing the choice of better projects. Solutions for the optimal set were derived for examples, most simply with a Poisson distribution over the number of projects.

In the second and third settings the agent searches and waits, respectively, for a project that is both permitted by the principal and meets the agent's own acceptance threshold. Here the optimal permission set is generally characterised by a simple linear relationship between the payoffs of principal and agent. In the searching model this relationship is negative. In order to encourage search, or to cover search costs efficiently, projects with higher agent payoffs are permitted for a wider range of principal payoffs, even to the extent that projects that are bad for the principal may be allowed. If the principal's preferences accord enough weight to agent utility, the optimal permission set includes projects that the agent will reject. The optimal permission set is in some ways akin to the acceptance threshold of a hypothetical searcher whose objective is a weighted average of principal and agent payoffs, but (at least if there are permitted projects that the agent rejects) with less weight on the agent's payoff than the principal accords.

Closely related findings emerge from the waiting model. (Indeed the searching and waiting models can be seen as instances of a more general framework with costly search and delay.) However, in the model of waiting there are always permitted projects that the agent rejects, and the relationship between principal and agent payoffs defining the optimal permission set may be positive or negative. When—

as in the waiting model and in the searching model if the principal values agent utility enough—there are permitted projects that the agent rejects, the principal's willingness to permit projects with high agent payoffs is curbed by the agent's resulting rejection of projects with low agent payoffs, which tend to have high principal payoffs.

In sum, our analysis has highlighted three aspects of optimal delegation of project choice. The first, from the model of project choice, is the exclusion of good projects to improve the chances of better projects being chosen. Second, from the model of project search, is the relatively greater tolerance of projects with high agent payoffs to encourage search. However, that model and the related model of waiting for projects illustrated thirdly that tolerance of such projects is muted by effects on the agent's own acceptance threshold—a widening of the set of permitted choices by the principal may cause some diminution of the set of projects from which the agent is willing to choose.

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