Investigating Variability in Classroom Performance amongst Children Exhibiting Difficulties with Early Arithmetic

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Abstract

Researchers in both psychology and mathematics education acknowledge that children's mathematical performance can vary inexplicably from day to day, though there has been little detailed investigation of the form of variability discussed in this paper. The paper builds on research suggesting this might be a particular issue for children considered to have learning difficulties in mathematics. The children concerned were seven- to nine-year-olds taught together for mathematics in a small group with high levels of adult help in assessment, planning and teaching. Observational research was conducted, with the researcher making weekly visits over the course of a year. Findings synthesise a range of evidence for each child gathered both during planned assessment tasks and as part of routine classroom activity. The data are used to chart the performance of individuals over this period. Findings suggest that arithmetical capacities were not fixed and easily assessed, but varied from day to day. This variability is considered in some detail with the aim of offering explanations for perceived differences. Elements such as task presentation and subtle mathematical differences between tasks provide partial explanations. Many differences remain unexplained, and it is argued that variability is in *fact a feature of learning. Finally, implications for practice in assessing children and planning for their mathematical development, are considered.*

Introduction

The article uses detailed observational data gathered in a natural classroom situation, to compare children's responses to similar tasks over a period of time. While some variability may be expected, observation shows major variability, with children apparently demonstrating specific capacities only to 'lose' them later. The research questions are whether there is evidence for variability of the performance of individuals and whether it is possible to explain any variability. Variability can take many forms, for example between children or for individual children between different aspects of mathematics, different aspects of number or tasks presented in different ways. Another form of variability is between similar or identical tasks carried out by the same child on different occasions. This final type of variability is the main focus of this research although other types are relevant.

The research arises from a wider ethnographic study carried out in various settings. This particular research question arose in this setting from the concern of the adults who worked there. They expressed concern at the difficulty of assessing children during classroom activities due to perceived variability of their responses. The purpose of my investigation, therefore, was firstly to interrogate my existing data to see if there was evidence for this variability. A further question was whether variability really occurred over identical or similar tasks or whether it could in fact be explained by task differences.

The children concerned were all considered to have learning difficulties in mathematics and the article ends by considering whether variability is a particular issue for such children or is in fact a wider phenomenon. The paper's contribution is to show that classroom research confirms clinical psychological experiments on variability, suggesting that this may be integral to the very process of learning itself rather than an aberration requiring alternative explanations.

Background

Both psychologists and educational researchers note, frequently as an aside, that children's mathematical capacities are not fixed and easy to assess but vary markedly from day to day or even between similar tasks on the same day. The issue is not currently foregrounded in English schools, possibly because it conflicts with contemporary initiatives. For example, the National Numeracy Project (DfEE 1999) focuses on detailed learning objectives, often to be shared with children with the hope that they will be achieved within the lesson. It is stated explicitly (page 33) that assessment during every lesson should check that children have grasped the main teaching points and determine whether they can move on, or whether misunderstandings need to be addressed. The implication is that such decisions can reasonably be made in a lesson and that, in the medium term, records can be kept confirming which key objectives have been met. The current mathematics curriculum in England puts an emphasis on teaching number, though other aspects of mathematics are also included. Much of the detailed discussion about children with apparent difficulties in mathematics focuses on number (e.g. Dowker 2004, Wright et al. 2002). Other writers, (e.g. Gabb 2005) assert that pupils with special educational needs should have an appropriate diet of mathematics, not just restricted to basic number work. My own research in other settings suggests that some pupils who are apparently low attainers in mathematics can respond well to non-number tasks such as measuring or shape (Houssart 2004). However, most detailed studies of children considered to be low attainers concentrate on aspects of number.

A recent detailed discussion of individual differences in arithmetic (Dowker 2005) brings together findings confirming that many children show variability across different aspects of number. A central theme of Dowker's book is that it is not appropriate to talk about arithmetical ability, but rather arithmetical abilities which can be grouped in to several categories. It is suggested that there can be strong discrepancies in either direction between almost any two components. Although the focus is mainly on variability between aspects of arithmetic, some points are also made relevant to variability between occasions. For example, in relation to her studies of estimation, Dowker suggests that the 'know' or 'not know' dichotomy in relation to particular types of arithmetic is inadequate, and she refers to a 'zone of partial knowledge and understanding'. She provides many examples of individuals demonstrating uneven performance across different aspects of mathematics and uses the phrase 'cognitively uneven' for those who have verbal reasoning which is either much better or much worse than their spatial reasoning. Despite detailed discussion of the performance of individuals on particular aspects of arithmetic, the author stresses the difficulty in trying to break down arithmetic into components for the purposes of assessment and intervention.

A major contributor to discussions of variability in arithmetic is the noted psychologist Siegler (1996) who focuses on children's strategies and how changes occur in their strategies and ways of thinking. He suggests that evidence for variability is present in the detail of much research but that, for several reasons, this remains peripheral and under-emphasised. A key reason he advances is that within Developmental Psychology variability *between age groups* is the main focus of attention; hence variability *within* groups is minimized. Although there is little work aimed directly at studying variability, clues to variability can be found by looking at the findings of apparently contradictory studies. In particular, work challenging Piaget (e.g. 1952, 1953) highlighted young children able to cope with concepts such as class inclusion and conservation at an earlier age than previously thought (e.g. McGarrigle and Donaldson 1974, McGarrigle, Grieve and Hughes 1978). Such findings are discussed by Siegler (1996) who suggests that recognizing variability of thinking is important in trying to reconcile evidence of young children's competence with evidence of their incompetence.

Within Education some writers touch on possible variability of performance, often in an assessment context. Black (1998) considers whether a pupil might perform differently on different days when discussing the reliability and validity of formal tests. He argues that this has received far less attention from researchers than other issues of reliability and validity. It could be argued that if variability was widely recognized this would undermine the system of formal assessment currently in use, and it is therefore perhaps not surprising that this is not a high profile issue. Discussions held with teachers (Watson 2000) indicate their strong belief that pupil performance varies from day to day. These teachers taught mathematics to children aged between ten and twelve years. Asked about how they reached judgments about their pupils' mathematics, roughly half of the thirty teachers said it was possible for pupils to be able to do some mathematics on one day but not on the next. Related work by the same author (Watson 2001, 2006) suggests that the whole issue of teacher assessment is problematic and that it is not possible to say for certain what a pupil knows.

Work aimed specifically at assessing pupils with learning difficulties in arithmetic also accepts that there is some variability (Wright, Martland and Stafford 2000). In this case the emphasis is again on variability of strategy, with the authors saying that children frequently use strategies that are less sophisticated than those of which they are capable. They give possible reasons for this, including the facts that a less sophisticated strategy may be easier or that some feature of the child's thinking prior to solving the current task may focus them on a less sophisticated strategy.

A related, currently more prominent issue in education is that pupils may perform differently according to how a task is presented. Clausen-May (2005) discusses pupils with different mathematics learning styles. She uses the outline VAK model, incorporating Visual, Auditory and Kinesthetic styles in a discussion of how pupils may exhibit learning differences in mathematics with preferences for tasks involving seeing, hearing or doing. She also suggests that classroom mathematics tasks may have a strong literacy base, disadvantaging some pupils. Different thinking styles in mathematics are also discussed by Chinn (2004) who uses the idea in the context of pupils considered to have learning difficulties in mathematics. Both writers' work is relevant to the current study because the data are drawn from a range of classroom tasks and it is possible that variability might be explained by pupils being asked to work on tasks presented in different ways.

There is therefore some support amongst both writers and teachers for the idea of variable performance. However, research on the issue is relatively sparse for methodological reasons; such studies are time-consuming and problematic. Most of the research evidence is provided by Siegler and his colleagues (Siegler and Jenkins 1989, Siegler 1996, Siegler and Stern 1998). These studies use micro genetic methods where individuals undertake the same task on several occasions, methods described in detail by Siegler and Crowley (1991) who argue that the concept of micro genetic methods and the rationale for using them go back for over eighty years.

This paper uses data gathered in a classroom whilst children undertake their normal activities. It is similar in some ways to micro genetic studies since it is longitudinal, with children frequently returning to similar tasks. It differs from them in that the researcher can not control the type or number of tasks carried out and because it is conducted in a classroom context rather than as a clinical experiment. This carries limitations in the number of times a child may work on each calculation but has the advantage of providing an opportunity to see whether Siegler and his colleagues' results in experimental situations can be replicated in a 'natural' classroom context.

Context and method

Data are drawn from a long-term research project carried out with four groups of children using ethnographic methods. The data considered were gathered from one group of children aged seven to nine years old. The twelve children in the group, who were drawn from different classes, were all considered to have learning difficulties as far as mathematics was concerned. The group was similar to other groups considered to be low attainers in mathematics (Denvir, Stolz and Brown 1982, Haylock 1991, Robbins 2000) in that they exhibited a range of apparent difficulties with a corresponding range of possible reasons.

The teacher was joined in mathematics lessons by two classroom assistants. Whilst researching, I visited the group for one mathematics lesson each week for a year, adopting a role similar to that of the classroom assistants and observed the children's responses to activities carried out with the whole group. While children worked alone or in smaller groups, I worked alongside them as requested by the teacher. I also occasionally conducted assessment activities with individual children as requested by the teacher. Detailed notes of children's responses to tasks were kept.

The data was analysed by examining field notes and extracting all those incidents which named individuals. These were reorganised to obtain a "personal record" for each child providing information drawn over a year for each individual, arranged chronologically. Incidents were coded on each personal record according to the aspect of mathematics concerned. The next step was to focus on examples where there were a large number of incidents for an individual featuring the same aspect of mathematics. These incidents were extracted and compared in order to examine variations across the year. This process is exemplified in the sections below with particular reference to one child, Claire, and with examples drawn from other children also shown for comparison.

Findings

Overview of Activities

Initial examination of the personal records for each child gave some indication of the curriculum covered by this group of children and in particular which aspects of mathematics were revisited several times. To illustrate this, the information from Claire's personal record has been tabulated to show which aspects of mathematics she was observed studying across each of the twenty-five weeks for which there were observations for her (see Table 1). Each mark on the table indicates aspects of mathematics covered in the notes, though these vary from brief mentions to records of whole lessons containing several activities on the same aspect. It is clear from the chart that the majority of time was spent on number rather than on other aspects of mathematics. There are only six entries covering data handling, shape and space and measures and most of these are very brief. This means it is impossible to draw any conclusions about whether Claire's performance on number tasks varied from her performance on other aspects. The shape and space observation for Week 11 related to Claire's use of a computer program concerned with tables facts. It had been noted that Claire negotiated the maze with apparent ease. This raises a slight possibility that Claire may be happier working with spatial tasks but there is insufficient data to examine this possibility. This was the same for all children, with the vast majority of observations being related to number. For this reason, the focus for the rest of this article will be on variability within number. It is worth noting, however, that any children in this group with strengths in other aspects of mathematics had few opportunities to demonstrate those strengths.

*{*Overleaf - *Table 1*: Summary of entries for Claire*}*

	Data Handling	Shape and Space	Measures	Money	Counting and ordering	Place Value	Counting in 2s, 5s and 10s.	Multiples of 2, 5 and 10	Odd and even numbers	Addition	Subtraction
Week 1		*			*						
Week 2						*				*	*
Week 3				*		*					*
Week 4										*	
Week 5							*			*	
Week 6					*					*	
Week 7			*			*	*	*			
Week 8				*		*	*	*		*	
Week 9	*							*			
Week 10	*										
Week 11		*	*				*	*			
Week 12					*	*		*			
Week 13					*		*		*		
Week 14							*				
Week 15						*		*		*	
Week 16				*	*					*	*
Week 17										*	
Week 18					*		*	*	*		
Week 19					*		*	*			
Week 20						*				*	
Week 21											*
Week 22				*							
Week 23				*			*				
Week 24										*	
Week 25				*						*	

Focusing on Number

Table 1 indicates a large amount of data concerning Claire's responses to aspects of mathematics which were revisited frequently throughout the year, such as addition, subtraction and counting in twos, fives and tens. The next step in analysis was to look separately at aspects of mathematics for which there was sufficient data. Observations for chosen aspects were tabulated with brief details included of the task involved. At this point tentative coding was used to indicate whether the task was completed correctly (√), incorrectly (X), or whether the response was mixed (-). Because this represents a fairly crude categorization, some comments were added to give details of the outcome. Where responses were almost entirely correct a tick was used, but any small errors were noted. Similarly, if responses were almost entirely incorrect, a cross was used and any correct response noted. A cross was also used when no response was given, but this was noted. The symbol for mixed response was used for a mixture of correct and incorrect answers and also for cases where work was completed with adult help. Table 2 shows Claire's response to addition tasks tabulated in this way and Table 3 shows her response to tasks involving counting in twos, fives and tens and multiples of two, five and ten.

Week	Task details	Outcome	Outcome details
Week 2	Addition to 10	X	Does not answer
Week 4	Adding numerals from cards	X	Does not answer
Week 4	Adding spots on dominoes	X	Does not answer
Week 4	Totals to 10	X	Does not answer
Week 5	Number walls (written addition)	√	Had adult help with first few examples, then worked alone
Week 6	Pairs to 10 (oral activity)	-	Mixture of correct and incorrect answers
Week 6	Making 10 (written activity)	√	19 out of 20 calculations correct with no adult help
Week 8	Adding money (oral activity)	√	Correct answers, no help
Week 15	Addition to 20 (game format)	X	Not participating
Week 15	Number card addition to 20	-	Initially incorrect but corrected after adult help
Week 15	Addition dominoes	X	Not participating
Week 16	Make 10 worksheet (no help)	X	Mostly incorrect, heavy rubbing out
Week 16	Make 20 worksheet (no help)	X	Mostly incorrect, apparently tried to make use of a pattern
Week 16	Pairs to 20 (mental, then check with calculator)	X	Incorrect answer
Week 16	Addition to 20, dice game	X	Not participating
Week 17	Make 10 and make 20 worksheet	-	Completed with intensive adult help
Week 20	Addition to 20 using number cards	√	Mixture of instant correct responses and correct responses after counting on fingers
Week 24	Dice addition	-	Task interrupted
Week 25	Addition of money	√	All answers correct

	(oral activity)		

Table 2: Claire's response to addition tasks. Key to outcomes: __√= Mostly or entirely correct, – =Mixed response, X = Mostly or entirely incorrect or answers not given

Week	Task details	Outcome	Outcome details
Week 5	Counting in 5s (game)	X	Incorrect answers
Week 5	Counting in 5s (missing number game)	-	Correct answer with adult help
Week 7	Tape for 2x table	-	No response initially, joins in later, numbers are correct but out of step with the questions
Week 7	Counting in 2s (joint oral activity)	√	Claire counts correctly up to 30s then misses out 36
Week 8	Counting in 5s (joint activity)	-	Intermittent participation
Week 8	5x table tape	√	Joins in enthusiastically and correctly volunteers to recite it alone (not chosen)
Week 9	10x table tape	√	Joins in enthusiastically and correctly volunteers to say it alone
Week 11	2x, 5x and 10x tables as part of computer game	-	Needs extensive adult help to start with but later completes correctly by counting in 2s, 5s and 10s
Week 12	2x table tape	√	Recited correctly alone as assessment task
Week 13	Counting in 2s (joint oral activity)	-	Intermittent participation
Week 13	Counting in 2s worksheet	√	Mostly correct, a few small errors
Week 14	Counting in 5s (oral activity)	-	Intermittent participation
Week 14	Counting down in 10s	X	Incorrect answers
Week 14	Counting in 10s, missing number game	X	Incorrect answers

Week 15	10x table	√	Correctly recites alone as assessment activity
Week 18	Counting in 5s (joint oral activity)	-	Does not participate to start with, then joins in correctly
Week 18	Multiples of 5, card activity	-	Intermittent participation
Week 18	Multiples of 5 (writing on board)	√	Completed correctly
Week 18	Multiples of 2, 5 and 10 (worksheet)	-	Does not stay on task
Week 19	Counting in 2s, 5s and 10s	√	Completed correctly as individual assessment activity
Week 19	Counting in 2s oral activity (high-starting numbers)	-	Initially incorrect, completes with adult help
Week 23	Counting in 2s using coins	X	Incorrect answers, lots of adult encouragement

Table 3: Claire's response to tasks involving counting in 2s, 5s and 10s and multiples of 2s, 5s and 10s

Some observations can be made from these two tables. Firstly, it is clear that on several occasions, Claire did not provide answers to questions or did not participate in games or joint counting activities. It is not possible to say for certain whether Claire could have answered the questions or not and this presented a difficulty for the staff working with her. Towards the beginning of the year, for example, when she did not answer questions about the total number of spots on a domino in Week 4, the staff were concerned that she did not understand these very basic ideas and was unable to answer. Later in the year, they felt that Claire sometimes chose not to participate in tasks that she might be able to complete. There were also a few occasions when Claire arrived at lessons distressed about matters outside of mathematics and this may have been a factor in her lack of participation. The tables also show some general tendencies. For example, Table 2 suggests that Claire achieved more success in written addition tasks than in similar tasks presented in games or practical formats. Table 3 suggests that her counting and recitation of tables was better when she was picked to recite in front of the class or to an adult as an assessment activity. However, neither of these patterns applies entirely. Generally, both charts show a mixed performance across the weeks with Claire often failing to answer or answering incorrectly, sometimes on tasks similar to those she had completed previously. This will be considered in more detail later.

A similar process was carried out for other children and information is shown for Seth in Tables 4 and 5. Seth was one of the older children in the group, and one of the most successful at mathematics, although the adults remarked that he was not consistently successful. The charts for Seth differ from those for Claire in several ways. Firstly, it was very rare for Seth not to provide answers or participate in activities, so, when he did not answer correctly, incorrect answers were given which sometimes shed light on his difficulties. The chart for addition also indicates that Seth usually completed tasks correctly and sometimes did more than was expected, for example with his comments about addition of zero in Week 4 and his systematic recording of possibilities in Week 7. Although Seth's difficulties in Weeks 4, 14 and 17 have no obvious explanation, it is not surprising that he needed help with the second task in Week 20 as it was harder than those carried out previously. Seth also differs from Claire in that the two tables show a different picture. The table for place value suggests that he was less confident with this aspect and had some difficulty with tens and units. This observation is supported by other entries on Seth's personal record. For example, Seth had difficulties in identifying multiples of two, five and ten in Week 18. He ringed the number 205, suggesting it was a multiple of two, and when challenged about this said that it did end in two. When asked to ring multiples of ten, he ringed almost everything. Apparently, Seth's performance across aspects of number was somewhat uneven, with his confidence in addition not being matched with his understanding of place value. Care needs to be taken here as it is acknowledged that place value is often more problematic than addition (Cockburn 1999). However, Seth's performance on place value tasks was also compared with that of others in this group who completed many of them correctly yet had greater difficulties than Seth with addition tasks.

Week	Task Details	Outcome	Outcome details
Week 4	Adding spots on dominoes	X	Incorrect answers
Week 4	Domino addition	√	Discusses addition of zero
Week 5	Oral addition	√	Answers quickly and correctly in plenary
Week 7	Addition to twelve (written task)	√	All possibilities recorded systematically
Week 8	Finding coins for given totals	√	Correctly answered for totals of 10p, £1, 50p
Week 14	Addition to 100	-	Some errors initially, corrected with adult help
Week 15	Make twenty (number cards)	√	Completed correctly, comments on connection between 5+15 and 15+5
Week 16	Make ten worksheet (no help)	√	Completed without mistakes
Week 16	Make twenty worksheet (no help)	√	Completed without mistakes
Week 16	Make ten and make twenty worksheet	√	Appeared to complete easily
Week 16	Adding three numbers	√	Completed correctly
Week 17	Make ten and make twenty worksheet	-	Make ten part corrected easily, slows down and makes some mistakes on make twenty part
Week 20	Addition program on computer	√	Completed correctly and helped another child
Week 20	Two-digit addition using hundred square	-	Correct on 16+15, needed help with 32+33
Week 24	Adding four numbers	√	Completed correctly
Week 24	Dice addition	√	Completed correctly, number

			lines were available but Seth appeared to make use of known facts sometimes
Week 25	Oral addition	√	Seth appeared to find these questions (e.g. 3+2) easy

Table 4: Seth's response to addition tasks.

Week	Task Details	Outcome	Outcome Details
Week 2	Putting tiles on hundred square	-	Initially made mistakes but completed correctly after adult explanation
Week 2	Making numbers with arrow cards	X	Confused tens and units
Week 3	Putting tiles on hundred square	-	Made initial mistakes, corrected after adult help
Week 3	Questions about hundred square	-	Mixture of correct and incorrect answers
Week 7	Representing numbers with tens and units pieces	√	Completed correctly
Week 7	Hundred square jigsaw	√	Completed quickly and correctly
Week 8	Counting stick	X	Answered questions incorrectly
Week 16	Making numbers with cards	√	Could make and read three-digit numbers

Table 5: Seth's response to place value tasks.

Analysis of Similar Tasks

Further analysis was carried out by extracting activities from each aspect of mathematics which were as similar as possible. This led to a series of shorter tables which tended to contain between five and ten activities. Carrying out this process for Claire revealed a series of tables which showed mixed progress. Two of these tables are shown as Table 6 and 7. Table 6 shows those addition tasks which required Claire to add up numbers rather than the harder tasks which involved finding numbers that added to a given total. This chart suggests that Claire's performance improved over the year, though with some variation across the weeks. It also shows, as mentioned before, that she was more likely to leave questions unanswered than to answer incorrectly. She did also sometimes need adult help to start with. For example, in the number walls activity in Week 5 she used a number line to add, and although she was secure about starting at the first number rather than zero, she needed reminding to count one on the first jump rather than on the starting number. After this initial help, Claire completed this written activity correctly. The tables for Claire suggest that she tended to participate less enthusiastically in practical and game activities. The detailed observations for Claire contained comments that support this. For example, in Week 4, Claire and some other children struggled with the introduction to addition via spots on dominoes. For this reason, the adults decided to abandon the planned worksheet and carry on with the practical introduction. At this point, Claire said 'Are we doing games all day?' Much later in the year, when work was being discussed at the beginning of a lesson, Claire made the comment 'I love sums, I love writing in my book.' It appears that Claire considered calculations written in books or on the whiteboard as 'real maths.' Although Claire's attitude was extreme, it is echoed in a more moderate form in the findings of Gregory, Snell, Dowker (1999) who carried out an international study about attitudes to mathematics and suggested that children may see written sums as a core aspect of the subject.

Table 7 deals with activities in which Claire was asked to count in twos or identify multiples of two. It is not surprising that Claire found the activity in Week 11 harder, as this computer game required her to multiply given numbers by two, whereas the normal activity was to chant multiplication facts in order only. The second task carried out in Week 19 was also harder than some of the others, as Claire was asked to count in twos starting from 74. It is less easy to explain Claire's difficulties in Week 23, when asked to count in twos as the teacher dropped 2p coins in a tin. 12p was dropped in the tin and Claire said the amount was 50p. The activity was repeated with intensive adult help. Looking at these activities for Claire suggests variability between occasions which can sometimes but not always be explained by looking at differences in the way tasks were presented or in the mathematics involved. In many cases, Claire's incorrect answers could not be explained by considering common errors or misconceptions and it is possible that she sometimes gave answers based on the idea of arithmetic as an arbitrary game as outlined by Ginsburg (1977). For some children, very similar activities were identified, often over consecutive weeks, to see if there was still variability.

There were several such cases where children were recorded successfully completing a mathematical task one week and then experiencing difficulty with a very similar task the following week. It was also sometimes recorded that children who did not appear to understand a piece of mathematics however it was presented in one lesson were able to cope with it in the next lesson. The following example concerns the performance of one child, Douglas, across similar tasks on three consecutive weeks.

Douglas was amongst those in the group having greatest difficulty with mathematics and his record reflected this. In an activity concerning addition pairs which made 20, Douglas did not answer when asked what should be added to 15. In a later activity in the same lesson, each child was given a number tile and asked to find another number tile to go with their tile to make 20. Douglas was given 19 but didn't answer. The teacher asked 'How many do you need to make 20?' There was no answer, so the teacher said 'Count on.' Douglas said, '20.' The teacher asked, 'How many?' and he said '20.' A number line was found and used to demonstrate. Eventually, Douglas gave the required answer: 1. This and similar incidents led me to conclude that Douglas could not provide the missing number in addition pairs. However, the following week I was proved wrong. The children were given a number and asked 'What has to be added to it to make 20?' They had calculators which they were allowed to check the addition with after the numbers had been suggested. 15 was given as the first number in the activity and Douglas correctly suggested 5 for the second number. The calculator was not used to give Douglas the answer, merely to check, though this incident was in keeping with others which suggested he did better in an activity involving technology even when the technology did not actually do the mathematics for him. This activity suggested to me that contrary to the evidence of the previous week, Douglas was able to understand the idea of pairs of numbers with a given total. The following week the idea of pairs to 20 was introduced in a different way. The children had a worksheet on which they had to ring and join pairs of numbers to total 20. Douglas made little progress with the sheet so I explained what he had to do and picked some individual numbers asking him for the pairs. The sheet was eventually completed with a very high level of help.

Although it is not possible to be certain why this variability occurred, it seems likely that part of the explanation lies in the way the task was presented. Perhaps Douglas was motivated by calculator use and presenting the task in that context maximized his potential. Perhaps Douglas understood the underlying idea but declined to cooperate when the task was presented in other ways, or maybe he was genuinely unable to understand the format of the worksheet. Douglas was similar to Claire in responding differently when tasks were presented in different ways. This was true for others in the group. Some, in contrast to Claire, succeeded in mental calculation but had more difficulty when the same calculations were presented in written form. However, task presentation did not explain all or even most of the cases of variability. There were many examples where children were successful on a task one week but failed to carry out a similar task presented in a similar way the following week. Variability also occurred within extended tasks carried out on one occasion as shown in the examples which follow.

Week 4	Adding numerals from cards	X	Does not answer
Week 4	Adding spots on dominoes	X	Does not answer
Week 5	Number walls (written addition)	√	Had adult help with first few examples, then worked alone
Week 8	Adding money	√	Correct answers, no

	(oral activity)		help
Week 15	Addition dominoes	X	Not participating
Week 24	Dice addition	-	Task interrupted
Week 25	Addition of money (oral activity)	√	All answers correct

Table 6: Claire adds single-digit numbers.

Week 7	Tape for 2x table	-	No response initially, joins in later, numbers are correct but out of step with the questions
Week 7	Counting in 2s (joint oral activity)	√	Claire counts correctly up to 30s then misses out 36
Week 11	2x, 5x and 10x tables as part of computer game	-	Needs extensive adult help to start with but later completes correctly by counting in 2s, 5s and 10s
Week 12	2x table tape	√	Recited correctly alone as assessment task
Week 13	Counting in 2s (joint oral activity)	-	Intermittent participation
Week 13	Counting in 2s worksheet	√	Mostly correct, a few small errors
Week 19	Counting in 2s, 5s and 10s	√	Completed correctly as individual assessment activity
Week 19	Counting in 2s oral activity (high-starting numbers)	-	Initially incorrect, completes with adult help
Week 23	Counting in 2s using coins	X	Incorrect answers, lots of adult encouragement

Table 7: Claire counting in twos and multiples of two.

Analysis of Single Tasks

Activities in this classroom commonly involved counting in twos, fives or tens. Often this counting was carried out as a group but occasionally children were asked to count alone with the adults and other children listening. On one occasion Neil was asked to count alone in fives to 100s. He started slowly and deliberately. He started 5, 10, 15, 20, 13. Asked to try again, he said 20, 25, 40 and was then correct to 70. He was unsure whether 74 or 75 was next and was helped by the adults. He ended with 75, 80, 85, 100.

It was common for the teacher to complete a checklist concerning counting when children were asked to count alone in this way. However, it was not clear whether Neil could be recorded as able to count in fives to 100. There are several possible reasons for his slight difficulties. His slow start suggested that he may have been mentally adding on 5 every time. The 13 is harder to explain, but could have been a combination of skipping the 25 and confusing 13 with 30. Since Neil managed to count from 20 to 70 correctly, he may have been aware of the number pattern involved. However, he considered 74 as a possibility suggesting that he had not seen the pattern. Omitting 90 and 95 may have been a mathematical error, an accidental slip or a desire to get to 100 as requested.

On another occasion, the group was working on multiples of 5 and children had been invited to write multiples of 5 on the board. The numbers 55 and 80 were written and the teacher said one of those was also a multiple of 10. Michael quickly put his hand up and answered 80. The teacher praised him and asked him to write another multiple of 10 on the board. He wrote 56.

Michael's enthusiasm for answering the teacher's initial question and his correct answer of 80 had suggested to the adults that he could recognise multiples of 10. The teacher's intervention was important and was presumably designed to confirm Michael' understanding. However, it had the opposite effect, leaving us wondering if he understood multiples at all. His correct answer could have been a guess, especially since he had two numbers to choose from, though he seemed confident and was not obliged to answer. The 56 is harder to explain, though children occasionally did activities related to hundred squares where they were asked to identify the next number and writing 56 on the board near to 55 would have been correct if that had been the activity.

Summary of Findings

The above incidents are selected to show examples of different types of variability. Variability across lessons, as illustrated by Douglas, was common, and could sometimes be explained by factors such as task presentation. In many cases, more than one explanation was possible. Variability within individual incidents, as shown by Neil and Michael is harder to explain, suggesting that real difficulty exists in trying to determine from such an incident whether or not a child 'can do' a piece of mathematics or even whether they did it successfully on that occasion. In other words, in general there was no explanation available in terms of task presentation or indeed any other obvious factor for most cases of variability.

The adults who worked in the classroom were well aware of the issue of variability, especially for children such as Douglas and Claire, for whom this was a marked issue. My analysis of personal records indicates that variability was actually present for all children, though in some cases it was much less frequent and was not necessarily evident as part of normal classroom assessment. Discussions amongst the adults often attempted to explain variability in terms of factors such as task preference, lack of concentration and other personal or social factors. Such discussions often led to the crucial issue of what should be done next as far as planning with this group of children was concerned; a tension existed between reinforcement and repetition or moving on, with perceived pressure to move on to harder aspects of mathematics.

Issues of mastery and progression are related to this dilemma. Some believe it desirable to 'master' a piece of mathematics, i.e. by performing consistently. For some, 'mastery' is a pre-requisite for moving on. The idea of progression in mathematics is based for many on the belief that any new piece of mathematics can only successfully be learned when those preceding it have been understood thoroughly. However, my findings suggest that this may be an unrealistic aspiration.

Implications

These findings have clear implications for practitioners by casting doubt on the usefulness of assessments conducted on single occasions, especially if they are based on only one item only of each type. There are dangers in extrapolating such assessments to make statements about what a child 'can' or 'cannot' do. It could rather be argued that contradictory assessment can be more useful in diagnosing difficulties and in planning by indicating that children respond better to certain types of activity, or have difficulties with specific aspects of arithmetic or with particular methods. This information can be used to structure appropriate activities, and to inform adults about those activities they are likely to need particular help with. However, my findings suggest that it is unrealistic to expect busy classroom practitioners to compile detailed pictures of all children and reach appropriate conclusions. This is an extremely time-consuming task, and interpretation is problematic. Thus, although detailed assessment can inform teaching it can not be relied upon.

A further issue is whether the variability observed applies particularly to groups of children considered to have learning difficulties or whether it applies more widely. It is not possible to answer this question from this data, but it is useful to speculate about why variability in performance was apparently so marked in this classroom. One possible reason is that variability is more common amongst children who experience difficulties in learning arithmetic. Another is that the adults concerned were in the unusual position of being able to observe children closely as they worked on similar tasks over a long period of time and therefore more able to observe variability, which could be present but less noticeable in other situations. It is interesting to note that my analysis of personal records detected variability amongst all children, even those for whom it was not evident from normal classroom observation. It appears possible that variability is a natural part of learning and is present in classrooms, for all children.

Perhaps the key issue for practitioners is how to proceed in situations similar to the one described. Teachers need to make decisions about when to move on to harder aspects of mathematics and when to repeat or reinforce ideas. Some views of mathematics and learning point to the conclusion that individual aspects should be mastered to the point where performance is accurate and automatic before moving on. Perhaps the fact that the children concerned are considered to be low attainers makes this option more tempting. However, my findings suggest that aiming for mastery before moving on is unrealistic and likely to be demoralising for all concerned. In moving forward, however, teachers need to be aware that reminders are often required. This could be seen as a positive step with reinforcement taking place as required in order to enable progress to be made rather than being seen as an end in itself. Frequent repetition of tasks can also produce a reaction in some children. Using micro genetic methods with a group of ten children, Siegler and Jenkins (1989) had to stop working with two of the ten children because of the way they reacted to the repetition of tasks. One child became over-anxious about succeeding on the tasks, whereas the other apparently became bored and gave evidence of not trying. It is possible that the situation in our classrooms in which work is frequently repeated brings about a similar reaction in children. Ironically, this repetition is often carried out in a search for mastery and automaticity.

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