

HAVING TO TEACH GEOMETRY: ON VISUALISATION AND TEACHERS' DEFENCES

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Abstract

The nature and presence of geometry in the school curriculum in England has undergone several changes over the past 40 years and today's curriculum includes some Euclidean-style geometric reasoning that was absent in the late 1980s and 1990s. Consequently, teachers of mathematics currently in secondary schools in England have a wide range of geometrical backgrounds, including many who did not have any school education in Euclidean geometry. This paper reports on an in-service geometry-for-teaching course and focuses on two themes – visualisation and psychoanalytic defences – that are briefly theorised and discussed in the context of teachers having to teach geometry.

Setting the scene

“Ah-ha, I see it now” a member of the class says softly, smiling, nodding slightly; another member of the class, face a resentful stare, cries out “I never see things, I’m rubbish at geometry”. What was, respectively, seen and not seen, was a two-dimensional Euclidean geometry theorem; those who saw, or did not see, were secondary mathematics teachers studying our ‘geometry for teaching’ MA module¹. The first teacher’s positive disposition contrasts with the second teacher’s disturbed and defensive state. In this paper, the source of data comes from the class of twelve school teachers who took this geometry module in the summer of 2008.

What’s the issue? Visualisation is central to geometrical reasoning, requires a different way of thinking and processing information to much school mathematics. Geometry is difficult to teach because of (a) the nature of visualisation for geometrical reasoning and the affective states that are conducive to visualisation, and (b) teachers’ lack of preparedness for this way of thinking, their consequent lack of confidence in modelling this way of thinking in front of their pupils in their classrooms and their resulting psychoanalytic defences that make it even more difficult for them to visualise geometrical theorems and relationships.

BACKGROUND

The mathematics curriculum in England has changed several times over the past 40 years and geometry has appeared in different guises. Post second world war, secondary education in England was generally divided into the academic grammar schools - where geometry was taught as the formal system of Euclid – and, for the majority of the age cohort, secondary-modern schools where learning for practical application predominated. By the late 1960s, influenced by the Modern Mathematics Movement (Howson, 1982), there were moves to “free geometry from the shackles of

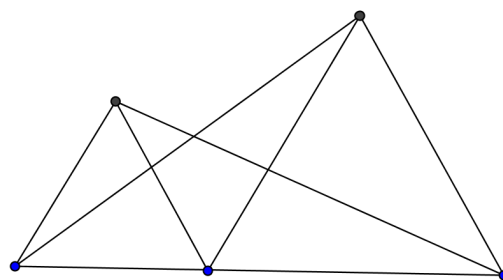
Euclid” so when the academic schools and the secondary schools merged (mostly by the early 1970s), geometry in schools in England was generally introduced through transformations (Howson, *op. cit.*) a pragmatic compromise between formality and applicability. Furthermore, this transformational geometry was often experienced descriptively (e.g., ‘what sort of rotation is that?’) rather than structurally (e.g. ‘what group represents the isometries of a square?’). More recently, concerns have been raised by mathematicians (Royal-Society, 2001) and mathematics educationalists (Hoyles & Küchemann, 2002) about young peoples’ competence in the key mathematical practice of providing proofs of conjectures in order to establish theorems. As elementary Euclidean geometry is an area of mathematics where proofs can be quite succinct, and visual/tangible/manipulable representations of theorems are frequently available, geometry has quite recently been returned to the curriculum (QCA, 2008). So in England we have teachers, usually in their mid-twenties to late-thirties, who learnt a descriptive geometry at school, who are now in the position of having to teach Euclidean geometry. These teachers are often in positions of seniority/responsibility, and, as one head of a school mathematics department remarked, it is “scary to find a big gap in my knowledge”. For many such teachers, mathematics has been experienced as being focused on numerical and algebraic concepts in which routinisation of procedures and understanding of constructs has been emphasised rather than a holistic, ‘whole picture’ view, characteristic of geometric thinking.

VISUALISATION IN GEOMETRIC REASONING

Turning now to geometrical thinking, in particular ‘(having a) visualisation’, by which I mean having a visual appraisal of a geometric configuration that leads directly to seeing (the truth of) a theorem or a key feature of a geometric relationship. The sense of the word visualisation I am using is in the spirit of René Thom’s “a theorem is above all the object of a vision” rather than in the sense of Van Hiele’s ‘level 1’ of geometric learning where ‘visualization’ refers to visual recognition of shapes but lack of awareness of their properties (Van Hiele, 1986). Van Hiele uses the term ‘insight’ for the experience that grasps a geometric result from an holistic appraisal. This term has a similar meaning to ‘visualisation’, the difference being that Van Hiele’s term requires a discursive potential which visualisation does not; as in Thom’s conception, the theorem is seen directly, (Rodd, 2000).

For example, in the diagram where on the horizontal, two equilateral triangles are constructed and two more line segments drawn in, it may be possible for you to see two congruent triangles; the fact of their identicalness can be (but might not be!) visualised, i.e. seen as true.

Recent work in psychology suggests that visualisation, in the sense outlined above,



employs affect in different ways from number/algebra. Linnenbrink and Pintrich, from an admittedly limited number of empirical studies, offer a general, though not universal principle: a positive mood tends to support a broad perspective that enables having an holistic appraisal of a situation, (and visualisation is a type of holistic appraisal of a geometric situation), while a negative mood tends to focus in on details (Linnenbrink & Pintrich, 2004). Evidence for this principle includes a study that found that young children learnt about shape more successfully when in a positive mood (*ibid.* p77), unlike middle school students' computational learning where "positive affect was related to lower levels of achievement" (*ibid.* p75). So Linnenbrink and Pintrich's cognitive-affective lens suggests that although the occurrence of instances of visualisation can't be predicted, the potential for having a holistic visual appraisal of a geometrical situation is linked to affective states like being in a positive mood which facilitates having a broad perspective.

This resonates with our work with the teachers on the course: in several sessions, we observed that a great deal of 'play-time' was needed for the participants to see geometrical configurations in more holistic, visual ways. I use 'play-time' to signal that the participants were free to explore the materials or the starting points in their own ways, that the atmosphere was relaxed and convivial and that, because over half the course was taught over whole days (Saturdays), we could give longer to a given task than had been anticipated in our planning (and cut other tasks). An example of a task that took time to be appraised visually was the following: we asked "in a unit cube, what is the distance from a vertex to the plane that is defined by the three adjacent vertices? (in class, a diagram was used). Though several participants were able to calculate an answer quickly with a familiar formula, some participants did not have this knowledge. Responding to this situation, we encouraged all participants to develop alternative approaches to employing a formula using different representations, like model-making. They worked in groups in a classroom where the atmosphere supported both discussion and contemplation and even those who 'knew the formula' remarked that this experience had helped them better understand the situation. In this vertex-to-the-plane task, the in-the-know participants were quick to calculate and they used the available time to develop a more visual approach to geometrical reasoning (rather than, for example, to hone their algebraic skills or to solve similar, more difficult problems). They were enjoying themselves, but, in the case reported, I cannot be sure whether or not their positive moods facilitated their own capacities for visualisation or whether the lack of time pressure was more important – clearly 'good mood' and 'lack of time constraints' are not independent.

This leads on to considering how teachers model doing mathematics, particularly problems like the one mentioned, in their classrooms. There are surely many reasons why teachers would quickly calculate, rather than explore different visual representations (other than the ever-present reason of curriculum pressures). In the 'white-board jungle' of the secondary classroom, teachers need to project a teacherly identity to their pupils and defend themselves from adolescent challenge. As a

mathematics specialist, this teacherly identity includes being a personifier of mathematical knowledge and so, for those whose geometrical confidence is weak, they might well rely on their fluent and to-the-point algebraic/numeric skills, rather than opening themselves to challenge by taking rather a long time to appraise a geometrical configuration.

These considerations lead to trying to understand more about the teacher's mind, in particular, defences that are invoked when having to teach geometry. This will hopefully start to explain why visualisation is difficult for teachers to do spontaneously in the classroom and difficult for teachers to educate their students to visualise.

IDEAS FROM PSYCHOANALYSIS TO FATHOM TEACHERS' RELATIONSHIPS WITH GEOMETRY

The psychoanalytic point of view has centrally the idea that one survives and defends oneself by reacting to phenomena by ingesting or by spitting out. These reactions start in infancy as experiences of the physical processes of taking in and expelling; then they develop into the psychological defence mechanisms of projective and introjective identification where, for example, the mother's mind is projected onto or is taken within. (There is also in psychoanalysis the family-historical dimension of becoming a mind that is tagged as the Oedipus Complex; this dimension is not pursued here, not that its importance is denied, but because integrating a teacher's family and background is beyond the scope of this paper.) In a rather ideal sense, using the psychoanalytic terms, a teacher projects something 'good' for pupils to ingest, then the teacher introjects the pupils' subsequent projections (that may be 'good' or 'bad') and the cycle continues.

Projective and introjective identifications are linked with a person's 'states of mind' which are relevant to both affect and cognition. I am drawing on the Kleinian and post-Kleinian tradition, (Waddell, 1998), that names 'states of mind' in terms of life-stages: infancy, childhood, latency, adolescence, adulthood, old age, yet the "complexity [of states of mind] ... is that they are not naturally linked to the chronology of developmental stages" (*ibid.* p11); the states of mind are emotionally experienced at all ages (*ibid.* p8). Each of these states of mind is subject to fluctuations between that of self-interest and that of empathetic concern. Some self-interest (is perceived as) having a need to project (spit out) the bad and some self-interest (is perceived as) having a need to introject (drink in) the good. If self-interest, emotionally experienced as an infantile urge for survival, is not felt under threat, then the mind can project to others' interests and concerns and subsequently introject the feedback from those others, and develop empathy. Clearly, in teaching this concern for others is central, but in reality, sometimes teachers have to defend their own self as a priority. In our case here, the teachers who had to teach geometry to adolescents had to defend themselves doubly: against the typical (i.e., age-appropriate) challenge

from their students and their own lack of having been fed (sufficient) geometrical nutrients during their education.

The state of mind of an infant – that we all continue to feel throughout life – experiences objects (both people and events are called ‘object’) in extreme terms and ‘splits’, chaotically, between (what is experienced as) good and (what is experienced as) bad. This is referred to as ‘paranoid-schizoid’: anxiety (‘paranoid’) to do with self-preservation is felt as either all good or all bad (‘schizoid’). In contrast, the depressive (meaning ‘considered’ rather than ‘sad’) position occurs when the mind sees others as separate (unlike a newborn who does not distinguish itself from its mother), the mind is more balanced, more ambivalent and does not experience extreme feeling. The paranoid-schizoid position stimulates the person to defend themselves and the depressive position gives opportunity for developing relationships. What these defences or relationships are like depends on whether projective or introjective mechanisms are employed by the psyche and the nature of those mechanisms in the particular circumstance.

During the course, we noticed various ways that teachers defences manifest themselves. The examples presented in the next section illustrate (a) being blocked, (b) fantasising skill, and (c) identifying with the ‘Other’, here, pupils who are positioned as the not knowing.

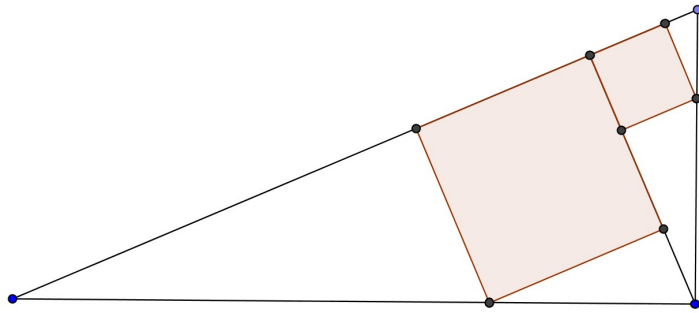
Defences related to geometry: examples from the course

- (a) Observation of the difficulty some of the teachers had with geometric reasoning started from the first session when the participants were given the Zome (Zome, 2008) materials to play with and get to know each other through companionable problem-posing/solving. We



observed that some teachers with strong mathematics identities and good qualifications started by counting as opposed to any process that could be said to be visualising. The photo shows one such student's first model; he was unable to tell how many sticks he'd used by relational means (for example, by seeing the shape made by a certain grouping of a few (some subitizable number) sticks and then recognising a pattern of groupings). He was not happy with his lack of geometrical seeing. Eventually he ‘gave in’ and reasoned numerically by pulling out the sticks one by one and counting. During this session, there was a very friendly atmosphere with people moving freely from talking with others (about shared tasks or on general chat) to pursuing an idea of their own, but it did not seem enough to facilitate visualisation. Yet, it was our first meeting of a masters course that self-consciously privileged mathematical knowledge and that may have set up expectations for performance that, consciously or subconsciously, produced a ‘blocking’ anxiety.

- (b) About a third of the way through the course, we organised a computer-room Cabri session. The teachers had been asked whether they had had experience with Cabri and no one said they had not. Yet, when asked to produce a dynamic version of the figure illustrated² (in which a perpendicular is dropped from the vertex opposite the hypotenuse in a right angled triangle and two squares constructed, as shown), the level of expertise was much lower than anticipated. For example, several participants found it difficult to devise a method to construct the squares so they retained the required properties when dragged in Cabri. Also they resisted testing their constructions with the dragging capability. In this situation defences are employed to protect the professional self: the teachers were doubtless aware that mathematics teachers ‘ought’ to be skilled in pedagogical technologies, like Dynamic Geometry Software. However, they (several of the participants) had not ingested the details of how DGS generally, or Cabri in particular, is used in practice. Instead, their knowledge *that* there is such a technology was fantasised as knowledge *of* it as a tool for geometric thinking.



- (c) There is a tremendous exclusion potential in geometric thinking as it is hard for one person to get another to ‘see’ the way a configuration of a diagram or model yields an insight and this sets up various types of defences. For example, one of the participants got frustrated on several occasions when she was unable to visualise a theorem or geometrical relationship. On these occasions, she defended herself psychologically by saying that it was useful, in a way, to feel de-motivated and excluded (as a consequence of not visualising something), as it furthered empathy with her students: “its how the kids feel”. By using her lack of visualising as a positive feature vis. à vis. having good relationships with her students, this teacher seems to protect herself from not visualising.

The above three scenarios are examples of instances of defences against geometry that we captured in some way. There would have been other instances being employed that passed us by that might have been noticed by others or might now be noticed by ourselves in a subsequent course. Examples of behaviours that we might interpret as constituting a defence cannot be expected to occur predictably. Serendipity and the preparedness of the observers to notice such occurrences is what affords opportunities to mark such behaviours as data. This raises methodological considerations for further work in this area (which are outside the scope of this present paper).

FOR FURTHER DISCUSSION

On a tension between psychoanalytic and cognitive viewpoints

For those of us who teach or work at geometry, we know that the occurrence of a novel visualisation cannot be predicted, it is experienced as a creative leap – “ah-ha, I see it now!”. Dick Tahta wrote extensively about geometry and how geometry comes into being for people and in his chapter ‘Sensible Objects’ (Tahta, 2007) he draws on the work of the psychoanalyst Bion (*ibid.*, p210) to conceptualise how a visualisation might happen. Tahta conjectures that the position that is conducive to processing visual stimulus and projecting a visualisation is more akin to the paranoid-schizoid than the depressive one. The explanation is that taking in a ‘visual whole’ is like the all-or-nothing of the infant’s experience and that creativity requires the intensity of the infant’s state of mind: this sort of creativity is associated with holistic thinking. Thus, of the two positions, the paranoid-schizoid and the depressive, which fluctuate within us all, it is the former that is more conducive to visualisation. As the paranoid-schizoid position is orientated to survival, being demanding and defensive, so the creativity essential for visualisation, in a further, deeper way, stimulates the teacher’s defences.

This psychoanalytic take on conditions for visualisation contrasts with the cognitive psychologists’ ‘good mood’: the ‘paranoid-schizoid’ position is not associated with a relaxed pleasantness that ‘good mood’ signals! This should not be treated as a contradiction to fix but as something to consider and come to understand better. After all, can these psychoanalytic ‘positions’ be compared with moods?

Our situation is complicated because, in the classroom, a teacher of geometry has to operate as the parental figure with respect to teaching the pupils, which is associated with a considered (‘depressive’) position, as in the ‘reflective practitioner’ (Schön, 1983). And a teacher of geometry is also to be an autonomous geometrician and able to visualise aspects of geometric configurations, which at least according to Tahta is associated with an intense-holistic (‘paranoid-schizoid’) position. Furthermore, visualisation is not all there is to geometry. After all geometry - the study of space and shape and measurement - is a part of mathematics and mathematical language, notation and reasoning in general is pertinent to the geometrical domain.

All teachers ready themselves for their teaching, not only by planning lessons but by having been prepared and preparing themselves. Teachers’ preparation includes, of course, their own education. Returning to the particular situation of our teachers: what were the “loved and trusted resources” (Waddell, *op. cit.*, p 150) that were ingested as they learnt mathematics? In many of these teachers’ mathematical mind development, the loved and trusted resources, the reliable entities that they wanted to teach and share, were numbers and algebraic objects and routines, rather than geometric theorems. School mathematical work on number and algebra is usually routinisable and focussed on detail and thus it is more consonant with considered (depressive) analytic processes than it is with the broad sweep of geometrical

visualisation which involves intense-holistic (paranoid-schizoid) appraisal. In order to teach geometry, (i.e., in order to teach geometry in a way that includes working on opportunities for pupils to develop their visual appraisal of geometric situations), 'objects' of geometry need to be available and trusted too. Our intention in the course was to facilitate trust in the objects of geometry by play, camaraderie and pursuing a chosen geometric topic to depth (for an assignment). Play and camaraderie could be considered as good-mood enhancing practices and the assignment allowed the possibility for more intense experience.

On professional identity

There have been some studies on primary teachers from a psychoanalytic perspective, (Hodgen, 2004). Primary teachers are generalists who are required to teach mathematics and very few who choose to teach younger children chose mathematics specialisms at university or even in high school. Stories of primary school teachers' psychic struggles to establish a mathematics teacher identity are, for all the interest in the detail, not that surprising; it makes sense to help primary teachers overcome their alienation from mathematics so that they can teach with honest enthusiasm and empathise, not just recognise, children's delight in their mathematical enquiries.

However, the teachers relevant to this discussion are secondary specialists who, by and large, have identities that include being a competent mathematician and 'in control' of their subject matter. They teach adolescents; the adolescent state of mind challenges order, unlike the state of mind typical of the younger 'latency' child, (the primary pupil), which craves order (Waddell, *op. cit.*, p11). So the site in which the secondary teachers have to preserve 'the self' is different from that of the primary teachers: the young people secondary teachers work with are likely to see lack of knowledge as a weakness (of course, any particular classroom situation could be different, but at secondary level some sophisticated craft knowledge is needed if a teacher positions themselves as not knowing). So what happens? Geometry is algebraised and visualisation marginalised and teachers tend to adopt the dominant culture and defend themselves against having to perform that which they do not practice.

In conclusion, on in-service provision

How do we resolve such conundrums as: being a mathematics subject leader but defensively unconfident in visual approaches? How do we work with teachers who avoid geometric thinking – either by being numerico-algebraicists or by being defensive-protective – yet want to engage with geometry or need to teach it?

In our course we took it as read that establishing good relationships between participants was an important foundation, not only for mutual support and informal peer teaching, but crucially for the motivating energy that pleasurable camaraderie brings. We found that tasks that seemed very simple, (e.g., compare the areas of the shaded regions in the rectangle and the triangle respectively, as illustrated), elicited conversations that, for example: explored the relationship between the visual and the

analytic, asked ‘how many different ways could the result be shown and which of these are proofs?’, and reflected on ‘how do we and our pupils develop our geometrical reasoning?’

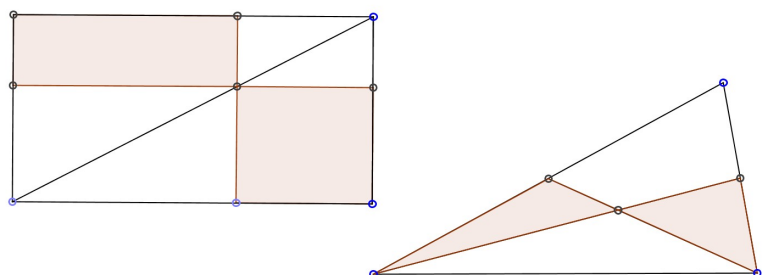
This paper has been concerned with teacher education in geometry in England, where teachers established in a successful career find themselves having to teach Euclidean geometry which they had not themselves learned at school. They have been charged to teach geometry to adolescents within a results-oriented culture, where there is ‘no time’ for waiting for the ‘insight’ either from teacher or student. These teachers are capable and confident of their capability in solving equations or performing mental arithmetic in front of their classes, but geometry, because of its visual aspect, seems particularly to need a holistic approach, involving non-routinisable ways of thinking that they have not experienced much themselves. Helping these teachers overcome their defensiveness and gain intellectual resources related to visualisation that they can trust to use in the classroom is a challenge in geometry teacher education.

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REFERENCES



¹ In this paper, the first person plural is used to describe aspects of the course that Dietmar Küchemann and I taught in the summer term 2008 for our MA in mathematics education at the Institute of Education, University of London; first person singular voice is used to develop the argument: teachers who have to teach geometry without much geometrical education themselves defend themselves against their lack of knowledge and this process of defending militates against their developing abilities to visualise geometrical theorems.

² A problem from John Rigby.