# Investigating the evolution of school mathematics through the lens of examinations: Developing an analytic framework

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## Background

Popular debate regarding whether standards in schools and in public examinations have changed over time occurs across the world. However the evidence presented in the media and mainstream policy documents generally lacks nuance and demonstrates lack of depth of understanding (Rushton *et al.*, 2011). The project “The Evolution of School Mathematics Discourse” seeks to contribute to our understanding of changes in school mathematics over the past 30 years in England by investigating the types of mathematical activities students engage in and the approaches to mathematical thinking that they are expected to take.

We take the public examinations taken at age 16 (minimum school leaving age) in all English schools as our window onto the nature of school mathematics because of their strong influence on curriculum and pedagogy. The symbiotic relationship between assessment and curriculum and pedagogy has been well documented (e.g. Broadfoot, 1996) and the impacts of assessment are considered to be especially strong in school systems with high levels of accountability based on national administered tests (such as England).

We adopt a discourse theoretic perspective that makes close links between the forms of language and other modes of communication used and the nature of mathematics and mathematical activity. In the first phase of the project we are developing an analytic framework that will allow us to compare the ways in which examination questions set in different years have been communicated. The development of the analytic framework draws on two approaches to discourse analysis, systemic functional linguistics and multimodal social semiotics (Halliday, 1978, 2003; Kress & van Leeuwen, 2001; Morgan, 1996, 2006) and the commognitive framework (Sfard, 2008). These two approaches to discourse agree that changing the textual form does not simply make the mathematics more or less difficult but also changes the ways in which a student may engage with the text and the possible ways of thinking about the mathematical content (cf Bezemer & Kress, 2008). To avoid the unhelpful dichotomisation of content and form, changes in *mathematical discourse* rather than *mathematics* or *mathematical knowledge* are considered*,* thus stressing that the issue in focus is that of the special form of communication that people engage in, with others or on their own, while solving mathematical problems or trying to learn a mathematical topic. The discursive definition transforms the question of change in mathematics education into one about change in observable characteristics of teaching-learning interactions, of students' and teachers' written work, of textbooks, examination papers and policy documents.

An important output from the project will be the development of validated analytical tools that can be used by examiners, curriculum developers and researchers to compare the types of mathematical demands made on students.

In the proposed paper for TSG28 we will detail the development of the framework and describe the analytical tool devised. This will be illustrated by analysis of selected examination questions. The second part of the paper will present some of the findings from the comparative analysis of the examination papers.

## Methodology

A sample of examination papers was selected from different years over the time period under consideration from two (of the three) examination bodies in England. Examinations were selected from eight years between 1980 and 2011. These data points were chosen, following a review of policy and curriculum developments during this period, for their potential to reflect changes in secondary school mathematics curriculum or in assessment policy. Other documents including syllabi and mark schemes were also collected.

Once the initial analysis using the framework had been completed, a focus group will be held with a select group of stakeholders (including examiners, teachers and mathematicians) to present, interrogate and validate the analysis. In a later phase of the project we intend to study the ways in which students respond to questions constructed to reflect differences that have been identified in the discourse analysis.

## Developing the analytical tool

The challenge in developing the analytic tool is to develop textual indicators that can be interpreted as indicative of what is expected of students and of their mathematical discourse. We start with two main categories of analysis: expectations about students' production - representing what is commonly called "the mathematics" - and the constructed relationships between student, the mathematics and the examiner - representing the pedagogy of the curriculum. These categories correspond roughly to Halliday’s ideational and interpersonal metafunctions of language (Halliday, 1978). The first category seeks to characterise the mathematical discourse the examinee is expected to be able to engage in when reading the text of the given task and then while writing a solution. It is sub-divided into four categories drawn from Sfard's characterisation of mathematical discourse (Sfard, 2008): vocabulary and syntax; visual mediators; routines; endorsed narratives. The second category seeks to identify how the student is positioned in relation to the mathematics and to the examiner.

In addition, we have considered complexity in the discourse of examination questions. Reducing complexity in examination questions has been the focus of much attention with the laudable aim of increasing accessibility for a wider range of students (see, for example, Fisher-Hoch, Hughes, & Bramley, 1997). However, our theoretical assumption that changes in the form of language are related to changes in the potential forms of mathematical activity leads us to investigate the wider effects of reduction of complexity. The first type of complexity we consider is the use of different meditational modes and the transitions made between them, for example from an equation (algebraic notation) to a table of values (numeric symbols) to a graph (Cartesian diagram). In addition to the question of the extent of multimodality, the issue in focus is the decisions to be made by the learners: Are the different modalities provided by the examiner or do they have to be produced by the examinee? Is the transition from one modality to another suggested by the examiner or is the initiative left to the problem-solver? We also consider the grammatical complexity of questions, asking how any differences in the approach to ‘readability’ may alter the nature of the mathematics and mathematical activity involved in reading and doing the task. These two focal aspects - the multimodality and the syntactical features of the examination tasks are indicative not only of the nature of the expected students' activity, but also of the properties of the mathematical discourse the students are likely to develop, if solving such tasks constitutes the core of their mathematical experience.

Within each category we have developed detailed indicators that allow us to make a quantifiable characterisation of differences between examination papers. Quantitative comparisons provide some insights on their own but also provide a frame on which to structure detailed qualitative comparisons and analysis of particular tasks and questions.

In the full paper for the TSG we will provide a fuller account of the analytic tools and their application.

## Significance of the study

The question of whether and in what way standards in qualifications have changed over time concerns policy makers, the media and the public in many countries, and comparative research in this area is fraught with challenges. The analytical work presented in this paper will characterise how school mathematics exams in England, and by inference school mathematics, have changed since 1980. An overview of some initial findings from the application of the framework to the English GCSE (General Certificate of Secondary Education) exam will be presented in the paper but the main focus in doing so will be to exemplify how the framework can be used rather than a detailed presentation of the findings.

While the analysis is rooted in the English context, the paper will be of interest to anyone involved in school- and degree-level mathematics education as the framework will present a rigorous and validated approach to considering the types of mathematics asked of students and the impact this may have on their ways of thinking as they respond to questions. This could be of significant use to those involved in producing examination questions and curriculum resources. The tools could be usefully applied within other English-speaking settings and have potential to be adapted for other languages.

## References

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