# ESRC Grant Award R000223420

# The Mathematical Components of Engineering Expertise

# End of Award Report

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# SUMMARY OF RESEARCH RESULTS

### Mathematics in the structural engineering design process

The first phases of the project were concerned with developing an overview of the design work of engineers, progressively focusing our investigations on the particular specialism of structural engineers.

We identified three main classifications of work for structural engineers: *design*, *analysis* and *review*. The separation that the engineers made between analysis and design was ubiquitous and striking: “When I’m analysing I’m not designing, I’m doing the calculations that justify my design”. Furthermore, this separation is inherent in the career path of engineers: younger engineers are apprenticed over a period of years into the practice of engineering design, initially being given the relatively routine tasks of analysis, and gradually being given more responsibility for design. This suggests that over time, the mathematical profile of activities with which an individual engages tends to become less explicit (and performed), and more tacit (and performed by other people).

> *Finding 1: There is a division of mathematical labour which separates ‘analysis’ from ‘design’ in structural design work. As engineers become more experienced, their work roles shift from analysing (calculating) to designing.*

### Design and Analysis: Where is the mathematics?

We encountered a widespread view that the majority of experienced engineers, with their work focussed on design, did not do mathematics of any sophistication, and that, whilst it was important for engineers to develop an appreciation for advanced mathematics in university, it is something they would rarely be expected to use, especially after they have served their years of apprenticeship in analytical work.

However, we hypothesise that the belief of the engineers that few of the concepts they regularly use are mathematical in character is due to their having become so intimately bound up with the meanings of engineering practice. This transformation in the character of mathematics appears to be not simply a quantitative one, a replacement of mathematical activity by professional expertise and experience. To understand this more deeply, our research attempted to characterise what mathematical knowledge “remained” in design work, a crucial aspect being the “remainder” of mathematical knowledge which allows design engineers to understand, and make use of, mathematical work that is done by others.

> *Finding 2: What is consciously thought of as mathematics by engineers appears to be only the visible component of a larger body of mathematics in use. There is a qualitative, epistemological and cognitive restructuring of the mathematics used by engineers as it becomes ‘embedded’ in engineering expertise.*

## Division of mathematical labour and mathematical interfaces

The fact that the majority of design engineers can work without having to do advanced mathematics is due to the sophisticated distribution of expertise in engineering practice. We identified and studied three main areas of divided mathematical labour:

1. Ubiquitous computer programs as design tools
2. Codes of Practice which distil knowledge and expertise into a form usable for design
3. Analytical specialists acting as consultants to design practice

A design engineer at work on a project must interact with the first two of these areas, and in some cases, the third. As research progressed, we began to recognise the *mathematical interfaces* that exist between specialist domains of knowledge, and between the experts in those domains.

We have tried to characterise what kind of knowledge is necessary to make sense of a "hidden" calculation through an interface. There are situations where a visualisation of the inside workings of a mathematical calculation is *not* required to make an informed judgement about it: the judgement can come directly from engineering understanding. On the other hand, even in a multi-disciplinary design team with its own mathematical specialists, mathematical analysis cannot be a totally black box for any engineer who has to use a mathematical result and take responsibility for its use.

> *Finding 3: The team of design engineers is comprised of many specialisms, and only a minority of them are mathematics-based. Wherever there is a division of labour there is a need to communicate information across those divisions. Each division entails its own "interface": that is, the forms and language of communication between individuals across that division.*

## Characterising the embedding of mathematics in expert knowledge

An obvious question that arises is where does a design engineer's understanding of hidden calculations come from? Although the scope of this project does not allow a definitive answer, there are clues to be found in the expert structural engineer's repertoire of qualitative and quantitative ways of thinking about structures.

Approaches to analysing structures vary along a quantitative-qualitative spectrum, running from exact, explicitly mathematical or computer-based methods, through "rough calculations", to qualitative approaches based on "structural feel". Understanding comes through connecting across the different approaches, in the simplest instances cross-checking of, say, an exact answer against a rough calculation.

The type of qualitative thinking that characterises the use of "feel" in design is exemplified by the concept of *load path*, the notion that the loads acting on a structure have to "flow down into the ground" like a kind of fluid. It is an extremely useful concept because it provides a way of thinking about a structure before any quantitative analysis is done.

Load path is closely-related to *structural geometry*: engineers use mathematics to carry around in a very compact form the shapes and magnitudes of the deformations of structural elements when loads are applied: understanding is "situated" in the sense that structural engineers think about geometrical and algebraic forms for what they mean in structural terms. From the outside observer's point of view, this can — we think mistakenly — be interpreted as an absence of mathematical knowledge.

*Finding 4: The mathematical components of engineering expertise operate in practice as a synergy of "pure" mathematical knowledge with knowledge of materials and structural geometry. Mathematics is seldom active and foregrounded in this relationship, but is embedded within design thinking. For example, there are powerful conceptions of the loads and forces in structures that can be characterised as situated abstractions — abstracted in the sense of providing general invariants of structures, yet situated within the tools and discourse of the practice.*

# Full Report of Activities and Results

# 1. Background

The Mathematical Components of Engineering Expertise project is the latest in a series of research studies conducted at the Institute of Education on the use of mathematics in vocational and professional workplaces. Throughout these investigations we have been concerned with both epistemological and psychological issues involved in relating mathematical and professional knowledge. The epistemological concern has led us to consider mathematical knowledge in its broadest sense, including the 'mathematisations' that are largely implicit in professional practice, not just those that consist of direct application of taught mathematical techniques. The current project has contributed to this effort by focusing on professional engineering design practice, a domain with a number of distinctive characteristics. This has afforded an opportunity to broaden our body of data, and has allowed us (modestly) to advance our theory-building efforts.

# 2. Objectives

The Aims and Objectives for the project were specified as follows in the original proposal:

**(1) To study the mathematical components of engineering practice in order to elaborate the relationships between mathematical abstraction and engineering expertise**

This objective has been achieved. For our methods of study, see the Methods section below, and for our findings, see the Results section and the attached Nominated Output.

**(2) To advance our theoretical work on abstraction, in particular on the notion of "situated abstraction", which seeks to clarify the boundaries between situated knowledge and mathematical abstraction**

This is in the course of being achieved as we write up our analyses for publication. During the course of the project we identified several key examples of situated knowledge and the existence of knowledge boundaries. These have been employed to develop preliminary analyses and published results, and are currently being elaborated further.

**(3) To assess the pedagogical implications of our findings, in the context of the current debate in the engineering profession concerning revisions to the teaching of mathematics to undergraduate engineering students**

Achieved as follows:

(1) In September 2001, we were invited by the Joint Board of Moderators (a group appointed by the professional Institutions of Civil, Structural and Building Services Engineers to accredit undergraduate degree courses) to contribute to their discussions about changes to degree course entry requirements for mathematics. We have remained in

contact with the chairman of the Board with a view to carrying out further research in this area (see Section 8).

(2) We have had extensive discussions with engineering educators at University College London and the Open University, with a view to developing project proposals that will build on the results of this project (see also Section 8).

# 3. Methods

As specified in the proposal, the methodology progressed through four phases. These are considered in order.

**Phase 1: Documentary analysis and introduction to the engineering firm (Months 1-2)**

At the outset, we approached a large, multidisciplinary civil and building engineering consultancy firm and established a working relationship with a director of the firm (*D*). During this phase we developed an audit of the 'visible mathematics' of engineering practice within the firm, the components of practice that formed the shared assumptions of engineers within it. This included documentary study of:

- standard engineering, and engineering mathematics, texts
- literature on engineering design
- common software employed, focusing on the specialism of structural engineering.

We conducted a series of interviews with academics (some face-to-face, some on the telephone, and some by email), to elicit expert commentary on our findings from the documentary study.

We were fortunate to establish from the beginning a positive relationship with *D*, who provided access to different areas of the firm, and secured the cooperation of a wide range of engineers. Throughout the project we held regular discussions with *D* to evaluate our findings formatively, and to re-negotiate who and where to observe and interview next.

We decided in phase 1 to look beyond the general, visible mathematics of engineering design — which was originally intended for phase 2 — by carrying out a set of informal, exploratory interviews with structural engineers at the firm. Our rationale was that we wished to prepare the way for the observation ('tracking') phase, by gathering more detailed knowledge about the working practice within the firm. As it turned out, we learned a considerable amount about the general aspects of (structural) engineering practice through these interviews.

Towards the middle of month two, we wrote a series of short memos that summarised the interviews and identified emergent themes. As a result, we returned to a further interview with *D*, after which we decided to make a major change to our proposed methodology. In the proposal we stated that data from a single project over a period of months would form the focus of the data collection. However, it now emerged that this, although feasible, would not be the most productive route within the time frame of the project. Our concerns were twofold. First, to focus on only one or two particular design projects might not provide a representative sampling of engineering design practice within the firm, even within the specialism of structural engineering. Second, we were advised that although project meetings could be made available to us, it might not be profitable to spend so much time observing project meetings as we had proposed, since the kind of discussion that we could readily observe would be nearly all ‘project management’, rather than the technical discussion that we wanted to study.

As a result, we decided to adapt our methodology as follows. Instead of restricting ourselves to just one small part of the company, the company proposed to offer us much wider access. Its large size and work-range meant that by interviewing engineers and observing a subset of them on different projects at different points in project lifecycles, we could develop an overall picture of the evolution of projects, and the roles of mathematical work within them. We were convinced by *D* and the academic interviewees that design projects have a consistent structure and life cycle, and that this would allow us to piece together data from different projects into a coherent picture.

**Phase 2: Initial interviews (Months 2-3)**

We had already begun senior staff interviews in phase 1, and these were continued. Sampling of staff was undertaken to ensure as wide a range as possible of (a) type of projects in which they were involved, (b) age, experience and educational background and (c) work role within the practice — particularly to include specialist analysts and experts in computer software. These interviews were audio-taped and transcribed.

As the body of interview data grew, we began the task of identifying issues and themes, and progressively focused on a number of questions for further investigation. A cumulative diary was maintained throughout the project, consisting of interview summaries, short analytical memos, notes on background readings and web searches, etc. By the middle of month 3, we had developed a provisional epistemological analysis of the practice; versions of this were discussed with contacts in the firm, and a final map of the different components of the practice was drawn up, which illustrated how the different domains of work inter-related. This map was used as an organising instrument in phase 3. A “top level” version is shown at Figure 1: this illustrates the activities connected to *analysis* on which we began to focus.

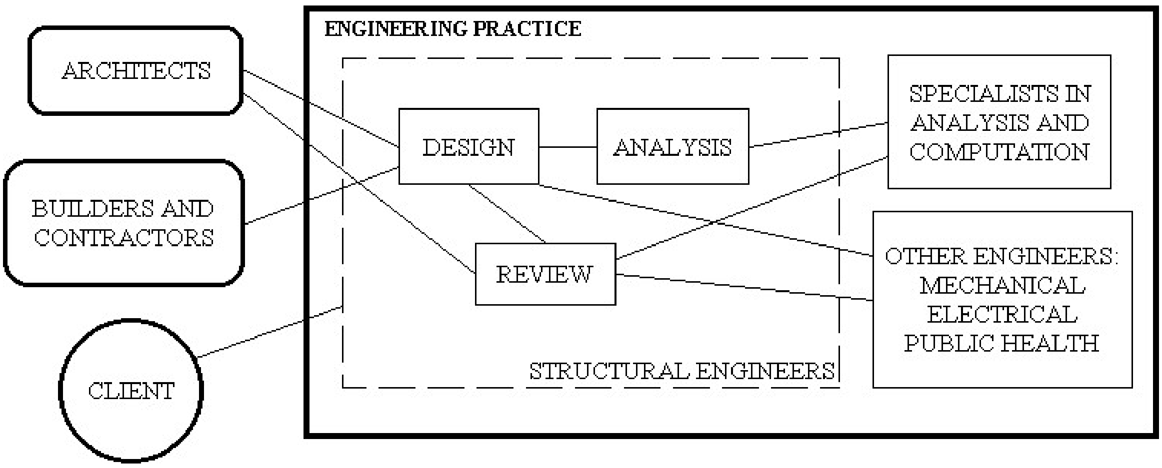


*Figure 1: Schematic diagram of the distribution of work in building design*

**Phase 3: Observation phase (Months 3-9)**

At the beginning of this phase, we made contact with a second engineering firm, and undertook several interviews with contacts there, in order to assess in a preliminary way the generalisability of what we had found in our primary firm.

At this point, we decided to focus almost entirely on structural engineers, with a few interviews in neighbouring disciplines. ‘Structurals’ were chosen because they are (1) core members of building design teams, (2) relatively sophisticated and frequent users of mathematics, compared with the related disciplines.

Data was collected as follows.

*Interviews:* We developed an interview schedule, which went through three iterations as our focus on issues became more refined. In the early part of the project we asked structural engineers in interviews to talk generally about the roles of calculation and ‘intuition’ in their design work, and what they considered to be the ‘mathematical elements’ of engineering design. On the basis of the common elements of their responses we were able to focus our questions (at the same time tuning into the language of the engineers) onto some key structural ideas (see Section 4 for a description of these), and asking how do mathematical ideas become ‘embedded’ within those ideas. Finally, in the last part of the project, when we broadened the range of interviewees from structural engineers to related engineering specialisms, we re-phrased our interview questions into more general terms, looking for analogous key ideas to those we had found from structural engineers.

In all the interviews, we requested the engineers to bring drawings and documents from one or two particular projects, and on some occasions we were able to receive demonstrations of the use of software packages. Part of each interview was focussed on these specific artefacts, and we asked the interviewees to describe ‘critical incidents’ relating to the documents and/or software.

Each interview was audio-taped and transcribed. Periodically, memos were written which identified questions and issues for further study: transcripts were reviewed independently by the researcher and project director and separate memos written that were subsequently synthesised.

*Project meetings*: We observed meetings over an initial 4-week period. These were audio-taped (given the dominantly verbal nature of the discourse, videotape was not used), and relevant portions transcribed. These meetings were, as we had been informed, mostly about project management, but they did provide a source of contacts and relevant issues that could be followed up subsequently in interviews.

*Project documents and computer files*: The interview and meeting data was enriched by working documents, computer outputs, and computer files from the projects under discussion.

*Email trails:* On a number of occasions we were able to follow interactions between specialist domains in the firm by tracing email conversations. We followed up these email trails by directly communicating with one or more participants by telephone or face-to-face interviews.

As the body of data and analytical memos grew, we were able to choose our data sources, including the choice of individuals to interview, according to the principles of theoretical sampling, in which we identified gaps in our data or particular issues for follow-up, and collected further data from sources chosen to inform our emerging theory and clarify hypotheses and categories. As part of this process, we developed a coding scheme for data analysis, which was taken as the starting point for the preliminary coding in phase 4.

### Phase 4: Analysis (Months 7-11)

All data were subject to a preliminary coding, in which we identified distinct episodes of observation, themes in interview data and email trails. A set of categories and hypotheses was drawn up, and all the data were subjected to a further coding on this basis. Once these categories were refined, we undertook a final coding, as well as synthesising diary and memo entries into provisional analytic descriptions of episodes and themes. Particular attention was paid to "breakdowns" and decision points; these were, as we had predicted in the proposal, informative, although given the limited time span of the project, they were not numerous.

The preliminary coding progressed from attaching codes to individual utterances or events, into pattern coding of a smaller number of themes. When these themes were finalised, we returned to the corpus of data as a whole, and triangulated our initial findings with our primary contact *D* at the firm, and with some other of the engineers. We also asked the other engineering firm, and one of the engineering academics, to review our conclusions, and elaborated them on the basis of feedback received. Finally, we were fortunate to be invited in month 7 to present our provisional findings to a panel of expert users (see Section 2), providing an additional opportunity for feedback and revision.

# 4. Results

These results are arranged in sub-sections, which begin with observation and description and shift in emphasis through to analysis and theoretical findings.

### An overview of the structural engineering design process

An engineering design plan for a building — that is, a plan carried out in advance of its construction — is typically very complex, and created over months or even years. There are usually three main stages of design (termed 'concept', 'scheme' and 'detailed') which represent increasing levels of detail and commitment to the design, from a fuzzy initial idea to a clearer scheme design, where major decisions have been made but many details remain undecided, to the final design with all the details elaborated. The mathematical components of the design work vary widely across the stages and the particular roles of individuals and groups within the design team.

Within each stage of design, we have classified the structural engineers' work into three main areas, labelled in Figure 1 (above) as DESIGN, ANALYSIS and REVIEW. The interplay of DESIGN and REVIEW in structural engineering practice (in the form of meetings and discussions both formal and informal) was found to be largely routine, and not significantly different from practice within other industrial and commercial settings. For the current study, the distinguishing characteristic was in the area of ANALYSIS: that is, carrying out the calculations for a design (whether by computer or manually), *separate from* DESIGN itself. It was found that this separation was ubiquitous and explicit: as one engineer put it, "when I'm analysing I'm not designing, I'm doing the calculations that justify my design".

The separation leads to a significant *division of mathematical labour* in design projects: it is younger engineers who are performing the majority of the ANALYSIS (especially computer-based), whilst more experienced engineers tend to handle the broader tasks of DESIGN. This division arises because there is a standard career path by which the younger engineers are apprenticed over a period of years into the practice of engineering design, initially being given relatively routine tasks, and gradually being given more responsibility for DESIGN.

This finding, which emerged primarily from phase 1, throws light on the first project objective (see Section 2). It suggests that over time, the mathematical profile of activities with which an individual engages becomes less explicit (and performed), and more tacit (and performed by other people) as the focus of the engineer's work shifts from ANALYSIS to DESIGN.

> *Finding 1: There is a division of mathematical labour which separates 'analysis' from 'design' in structural design work. As engineers become more experienced, their work roles shift from analysing (calculating) to designing.*

**Design and Analysis: Where is the mathematics?**

DESIGN involves using the results of ANALYSIS, so it is not — in the way most engineers think about it — a quantitative, mathematical activity at all, beyond the most basic kind of numerical manipulation. Indeed, when this issue was probed in interview, some engineers stressed the 'art' of design, its qualitative, creative characteristics. The relationship between the qualitative and quantitative components of practice is a point of great current concern among professional engineers, not least since the sheer power of modern computer calculation means that nearly anything can be built, but calculation by itself does not lead to an understanding of what to build, in terms of quality or efficiency.

In phase 2, we encountered a ubiquitous view from senior engineers that the majority of structural engineers did not do mathematics of any sophistication in their professional careers. So, whilst it was important for graduate engineers to have an appreciation for advanced mathematics, it is something they would rarely be expected to use:

> Once you've left university you don't use the maths you learnt there, 'squared' or 'cubed' is the most complex thing you do. For the vast majority of the engineers in this firm, an awful lot of the mathematics they were taught, I won't say learnt, doesn't surface again.

However, this statement needs to be elaborated, as we encountered various slants and degrees of sophistication of mathematical expertise when we probed further into the engineers' use and thinking about mathematics. Engineers (in common with other non-specialist users of mathematics, such as the nurses studied in a previous project), might not consciously think that some of the concepts that they regularly use are mathematical in character, because they have become so intimately bound up with engineering practice. For example, for most structural engineers, it seems that geometry and trigonometry become embedded in practice, whereas much of calculus is 'not used' and is therefore consciously thought of as being 'mathematics'. However, this would not be true for those structural engineers, and engineers in other disciplines, who do use calculus regularly.

The transformation in the character of mathematics appears to be not simply a quantitative one, nor merely a replacement of mathematical activity by professional expertise and experience. To further explore this phenomenon of transformation, our study progressively focused on attempting to characterise what mathematical knowledge 'remained' in design work, and how it was embedded within it. A crucial aspect, which we turn to in the next section, is the 'remainder' of mathematical knowledge which allows design engineers to understand, and make use of, mathematical work that is done by others.

> *Finding 2: What is consciously thought of as mathematics by engineers appears to be only the visible component of a larger body of mathematics in*

*use. There is a qualitative, epistemological and cognitive restructuring of the mathematics used by engineers as it becomes 'embedded' in engineering expertise.*

**Division of mathematical labour and mathematical interfaces**

The fact that the majority of design engineers can work without having to do advanced mathematics is due to the sophisticated distribution of expertise in engineering practice. We have identified and studied three main areas of divided mathematical labour:

4. *Ubiquitous computer programs as design tools*

   The company uses a suite of standard programs for structural analysis, backed up by some more specialist in-house packages. There is an issue of automation: the suite could be made more integrated and automated, but the feeling of the user community was to resist this[1]. In terms of sheer volume of mathematical calculations, computers are the dominant labour force. The perceived relationship that the engineers had with the computers is ambiguous: most said that they did not entirely know what the computer was doing, but they understood (entirely adequately) what was required to provide it with input and to make sense of its output. The ability to judge the reasonableness of a result was considered a key component of this understanding.

5. *Codes of Practice which distil knowledge and expertise into a form usable for design*

   For structural engineers, the various UK and international Codes of Practice represent the baseline knowledge for practice[2]. They provide recommendations for the practical design of, for example, steel and concrete structures, based on a combination of accepted construction practice, experimental work on structures, and analytical knowledge. The somewhat marginal role of mathematics to design can be seen in these code documents: there is very little explicit mathematics or engineering theory present. Guidance comes mostly in the form of 'if-then' rules and (underived) algebraic formulae, which allow the engineer to substitute values of different parameters to calculate quantities required for the design.

6. *Analytical specialists acting as consultants to design practice*

   In the company observed, a small number of full-time specialists provide the analytical expertise necessary for the design work of the majority: just 2% of the professional workforce are regarded as analytical specialists. Specialists serve as a 'line of defence' for non-standard problems which do not fit the guidance provided by Codes. In such cases, designers cannot simply draw on the 'distilled' analysis in the Codes, and this distillation has to be undertaken by analysts, using some combination of 'first principles' analytical calculations and advanced computational techniques.

A design engineer at work on a project must interact with the first two of these areas, and in some cases, the third. As we collected more focused data, and progressed with the analyses, we began to recognise the *mathematical interfaces* that exist between specialist

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[1] Another example of that feeling, curious in the context of so much computer use, was to do with writing-up design calculations: this is still very often done by hand, because it is dangerous to trust a machine to do details, even the act of cut-and-paste in a word processor.

[2] Other engineering disciplines are less tightly bound by such official, nationally-regulated Codes, but 'codified knowledge' is crucial to all engineers whether it is prescribed and controlled (inter)nationally or locally to a particular specialist area, or particular company.

domains of knowledge, and between the experts in those domains[3]. (This idea is discussed more fully in the Nominated Output.)

We have tried to characterise what kind of knowledge is necessary to make sense of a ‘hidden’ calculation through an interface. We do not claim that a visualisation of the inside workings of a mathematical calculation is always required to make an informed judgement about it: the judgement can come directly from engineering understanding. One instance of this is in finite element calculations for structures, where the automatic element-generation algorithms can easily produce solutions that are spurious. This is an instance of the ‘reasonableness of output’ knowledge that we referred to above.

On the other hand, even in a multi-disciplinary design team with its own mathematical specialists, mathematical analysis cannot be a totally black box for any engineer who has to use a mathematical result and take responsibility for its use:

> Engineers have to some sort of intellectual visualisation of what is happening inside the black box, in order to decide which is the appropriate method [for a problem]. If they didn’t have that, we could only teach them rules, ‘use this method for that type of thing’. I would be very scared about that, the engineers have to understand what’s happening inside the black box, even though they’re not explicitly doing the calculations.

The issue was encapsulated in the words of one engineer who described how, after asking an analyst to work out some “quite complicated” maths, “once this guy had worked it out then it was within the range of us to understand what he had done at some level, to be able to use the results of it.”

> *Finding 3: The team of design engineers is comprised of many specialisms, and only a minority of them are mathematics-based. Wherever there is a division of labour there is a need to communicate information across those divisions. Each division entails its own ‘interface’: that is, the forms and language of communication between individuals across that division.*

**Characterising the embedding of mathematics in expert knowledge**

One question that arises is where this understanding of hidden calculation “at some level” comes from? The scope of this project does not allow a definitive answer. However, we have indicative data such as the following:

> When you’ve done a few of these kind of structures, you begin to understand them. So it’s not like there is a big equation in my head, but in the past to develop the knowledge and understanding of how these things work, then there were big equations in my head.

This changing nature of symbols and equations provides a pointer to the role that formal mathematical knowledge continues to play in shaping the ways engineers think about and analyse the objects of their practice, even when its ostensible role has diminished, and its character has transformed.

We observed that experienced structural engineers use an expert repertoire of qualitative and quantitative ways of thinking about structures.

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[3] We have had the opportunity to study interactions across mathematical interfaces in several ‘breakdowns’ in the process of design, that is disruptions to the smooth routine of practice which can expose normally hidden mathematical elements.

Approaches to analysing structures vary along a spectrum of detail, inversely with level of abstraction[4], that is, the most abstract is the least detailed, and vice versa — see Figure 2. The most detailed, and explicitly mathematical, approach is using the exact computer-based methods (or, in a few special cases, exact algebraic solutions); the most abstract approaches are ‘qualitative’, and there are in-between techniques that engineers call ‘rough calculations’ — for example, these might involve approximating the shape of a wall as nearly circular because then a simple formula can be applied to give an answer within 10% of the exact answer.


*Figure 2: The spectrum of abstraction and detail*

Understanding comes through connecting across the levels of detail, in the simplest instances cross-checking of, say, an exact answer against a rough calculation. Qualitative approaches are entwined with the notion of DESIGN, in distinction with the quantitative calculations of ANALYSIS (which are now largely in the realm of computer software). Another term for qualitative understanding often used by engineers is ‘structural feel’, which emphasises its (highly-prized) status as something intuitive.

The type of qualitative thinking that characterises the use of ‘feel’ in the design process is exemplified by the concept of *load path*, the notion that the loads acting on a structure have to “flow down into the ground” like a kind of fluid. It is a powerful, very physical concept[5], and extremely useful because it provides a way of thinking about a structure before any analysis is done, allowing judgements to be made about the validity of quantitative analysis of the structure:

> A load is applied and eventually it’s got to get back into the ground. It’s so fundamental to structural design that you have to be able to see what that route is in order to have a feeling, to be able to calculate, what sorts of loads and forces will be apparent in any particular member. Without a clear idea of the load path, you have nothing to judge what you’re getting from the computer.

Formal mathematical analysis is based on the assumption of static equilibrium, which assumes that *nothing* is moving in a stable structure, an assumption that appears to conflict with the load path concept. Nevertheless, load path allows predictions of behaviour that emerge from webbing together the actual properties of the material (e.g. steel beams) with the (mathematically-abstracted) forces with which they are associated[6].

Load path is closely-related to *structural geometry*: we found evidence for the idea that engineers use mathematics to carry around in a very compact form the shapes and

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[4] Note: This use of the term is drawn from computer science, and it is different to the concept of abstraction most usually discussed in mathematics education.

[5] Structural engineers usually treat load path as a qualitative metaphor. The concept *can* be formally described, but this is a specialist interest of structural theorists.

[6] The attribution of ‘mythical’ chains of causality to formally non-causal situations has been studied by researchers in various areas of cognitive science, although not, as yet, within the context of mathematics education. This is a line of investigation that we have begun to pursue.

magnitudes of the deformations of structural elements when loads are applied: understanding is ‘situated’ in the sense that structural engineers think about a particular set of plane curves for what they mean in structural terms (and use idiosyncratic language to describe them). Although they may simultaneously know (and have certainly been exposed to) a large amount of mathematics, the active meanings are structural. There is no need, most of the time, to isolate out a ‘pure’ mathematical meaning: on the contrary, such meanings are invariably anchored with intimate knowledge of the properties of the material and structural form in use. From the outside observer’s point of view, this can easily be interpreted as an absence of mathematical knowledge; in reality though, there is a subtle and more complex synergy of mathematical and professional knowledge.

> *Finding 4: The mathematical components of engineering expertise operate in practice as a synergy of ‘pure’ mathematical knowledge with knowledge of materials and structural geometry. Mathematics is seldom active and foregrounded in this relationship, but is embedded within design thinking (and thus transformed in the process). For example, there are powerful conceptions of the loads and forces in structures that can be characterised as situated abstractions—abstracted in the sense of providing general invariants of structures, yet situated within the tools and discourse of the practice.*

# 5. Activities

The work of the project was presented at the following conferences:

- 5th British Congress of Mathematics Education, Keele, UK, July 2001 (Kent & Noss)
- 25th Psychology of Mathematics Education Conference, Utrecht, The Netherlands, July 2001 (Kent & Noss)
- 53rd Annual Meeting of the CIEAEM (Commission internationale pour l’étude et l’amélioration de l’enseignement des mathématiques), Verbania, Italy, July 2001 (Noss, plenary lecture)
- Meeting of the British Society for Research into Learning Mathematics, Southampton, UK, November 2001 (Kent & Noss)
- Second Annual Symposium on Engineering Education, organised by the Institution of Electrical Engineers, London, January 2002 (Kent)

The following conferences are forthcoming:

- AERA Conference, New Orleans, April 2002 (Noss)
- Fifth Congress of the International Society for Cultural Research and Activity Theory, Amsterdam, Netherlands, June 2002 (Noss)
- 26th Psychology of Mathematics Education Conference, Norwich, UK, July 2002 (Noss, plenary lecture)

# 6. Outputs

The interview transcripts from the project will be made available to other researchers via the ESRC Qualidata Archive.

Professor Noss was invited to give a plenary lecture at the CIEAEM conference in July 2001 (this is to be published in *Educational Studies in Mathematics*).

In February 2002, Professor Noss gave seminars on the project results in the USA, at Stanford University (School of Education) and University of California Berkeley (Graduate School of Education).

# 7. Impacts

We have made impacts with both academic engineers and mathematicians, and with professional engineering institutions.

*Academic*

- We have formed links with engineering educators at the Open University (Department of Telematics), on the theme of mathematics and computer software in engineering.
- We have shared our findings with the UK universities Learning and Teaching Support Network, Centres for Engineering (at Loughborough), and Mathematics, Statistics & Operations Research (Birmingham).

*Professional*

- We were invited to share our findings with the Joint Board of Moderators (see Section 2), and a further briefing paper was requested by them.

# 8. Future research priorities

We see a need for research in three areas:

**(1) A reassessment of the roles of mathematics and information technology in undergraduate engineering education**

At time of writing, we have a proposal for a pilot research study in this area under consideration by a charitable foundation. This work has received the support of the Joint Board of Moderators.

**(2) A longitudinal study on transformations in engineering expertise and mathematical knowledge.**

We are collaborating with engineering academics at the Open University and University College London on a project proposal in this area.

**(3) Understanding the roles of mathematics and information technology in the modern workplace**

This is the basis for a project proposal currently in preparation for the ESRC Teaching and Learning Research Programme, Phase 3.